

Quantum Entanglement and Bell Inequality Violation in Semi-Leptonic Top Decays

Tao Han, Matthew Low, Tong Arthur Wu

[2310.17696](https://arxiv.org/abs/2310.17696)



University of
Pittsburgh



Motivation

Questions:

1. How strong is the entanglement between $t\bar{t}$ in LHC?
2. Does $t\bar{t}$ violate Bell's inequality?

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Interesting property of
quantum state

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2. Does $t\bar{t}$ violate **Bell's inequality**?



Refutes local hidden variable theory

⇒ If try to understand quantum with classical theory
⇒ must be some **nonlocal** theory

Quantum state

Density matrix

$$\rho = \sum_i n_i |\phi_i\rangle \langle \phi_i|$$

$$\langle \mathcal{O} \rangle = \text{Tr}(\mathcal{O}\rho)$$

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Production density matrix for $t\bar{t}$

$$\rho_{ab,\bar{a}\bar{b}} \propto R_{ab,\bar{a}\bar{b}} = \overline{\sum_{\text{initial}} \mathcal{M}(XY \rightarrow t_a \bar{t}_{\bar{a}}) \mathcal{M}^*(XY \rightarrow t_b \bar{t}_{\bar{b}})}$$

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Decomposition of two-qubit density matrix

$$\rho = \frac{1}{4} \left(\mathbb{I}_4 + \underbrace{P_i^A}_{\text{Polarization vector}} (\sigma_i \otimes \mathbb{I}_2) + \underbrace{P_i^B}_{\text{Polarization vector}} (\mathbb{I}_2 \otimes \sigma_i) + \underbrace{C_{ij}}_{\text{Spin correlation matrix}} (\sigma_i \otimes \sigma_j) \right)$$

Entanglement

Concurrence

$$\mathcal{C}(\rho) = \begin{cases} \frac{1}{2} \max(|C_1 + C_2| - 1 - C_3, 0), & C_3 \leq 0 \\ \frac{1}{2} \max(|C_1 - C_2| - 1 + C_3, 0), & C_3 \geq 0 \end{cases}$$

C_i : eigenvalues of $\{C_{ij}\}$

$$0 \leq \mathcal{C}(\rho) \leq 1$$

↓
No entanglement

↓
Maximumly entangled

Bell's inequality

$$\langle A_1 B_1 \rangle - \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle + \langle A_2 B_2 \rangle \leq 2$$

E.g., choosing

$$A_1 = \sigma_1, \quad A_2 = \sigma_3, \quad B_1 = \pm \frac{1}{\sqrt{2}}(\sigma_1 + \sigma_3), \quad B_2 = \pm \frac{1}{\sqrt{2}}(-\sigma_1 + \sigma_3)$$

$$\Rightarrow |C_{11} \pm C_{33}| \leq \sqrt{2}$$

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$\rho_{ab,\bar{a}\bar{b}}$ can be extracted from the angular distribution of decay product

$$\sigma(XY \rightarrow t\bar{t} \rightarrow (A_1A_2A_3)(B_1B_2B_3)) = \int d\Omega^A d\Omega^B \left(\frac{d\Gamma_{a\bar{a}}}{d\Omega^A} \right) R_{ab,\bar{a}\bar{b}} \left(\frac{d\bar{\Gamma}_{b\bar{b}}}{d\Omega^B} \right)$$

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$$\frac{d\Gamma_{a\bar{a}}}{d\Omega} \propto \delta_{a\bar{a}} + \kappa \sigma_{a\bar{a}}^i \Omega^i$$

Spin analyzing power

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Spin analyzing power

$$\Rightarrow \frac{1}{\sigma} \frac{d\sigma}{d\Omega^A d\Omega^B} = \frac{1}{(4\pi)^2} \left(1 + \kappa^A P_i^A \Omega_i^A + \kappa^B P_i^B \Omega_i^B + \kappa^A \kappa^B \Omega_i^A C_{ij} \Omega_j^B \right)$$

Direction of A, B

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Spin analyzing power

$$\Rightarrow \frac{1}{\sigma} \frac{d\sigma}{d(\cos \theta_i^A \cos \theta_j^B)} = -\frac{1 + \kappa^A \kappa^B C_{ij} \cos \theta_i^A \cos \theta_j^B}{2} \log \left| \cos \theta_i^A \cos \theta_j^B \right|$$

Polar angle of A with respect to the i-th axis

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$$\Rightarrow C_{ij} = \frac{4}{\kappa^A \kappa^B} \frac{N(\cos \theta_i^A \cos \theta_j^B > 0) - N(\cos \theta_i^A \cos \theta_j^B < 0)}{N(\cos \theta_i^A \cos \theta_j^B > 0) + N(\cos \theta_i^A \cos \theta_j^B < 0)}$$

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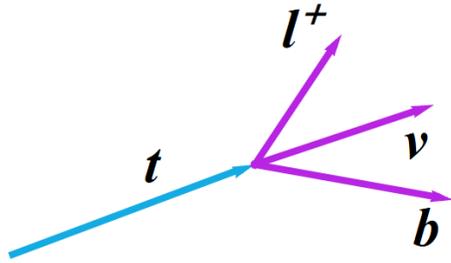
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$$\delta C_{ij} \propto \frac{1}{\kappa^A \kappa^B} \frac{1}{\sqrt{N}}$$

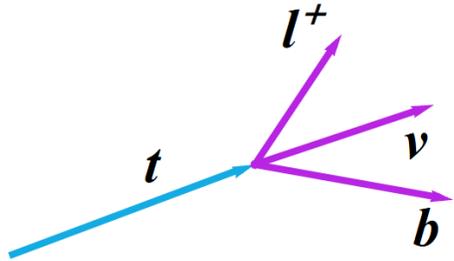
Which decay channel?



Leptonic decay:

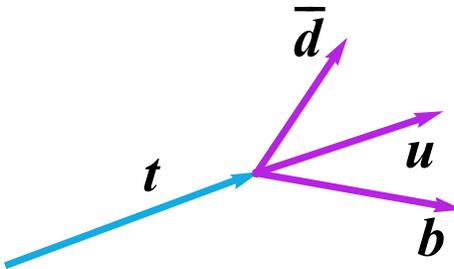
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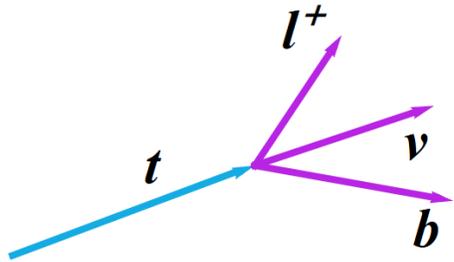
Hadronic decay:

$$\kappa_{\bar{d}} = 1.0$$

$$\kappa_u = 0.34$$

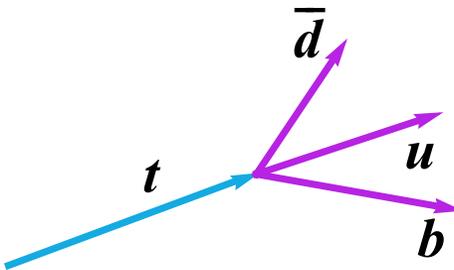
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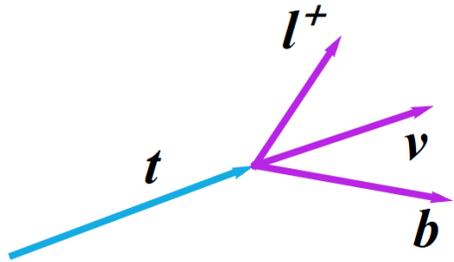
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optimized direction:

$$\vec{\Omega}_{\text{opt}}(\cos \theta_W) = P_{d \rightarrow p_{\text{soft}}}(\cos \theta_W) \hat{p}_{\text{soft}} + P_{d \rightarrow p_{\text{hard}}}(\cos \theta_W) \hat{p}_{\text{hard}}$$

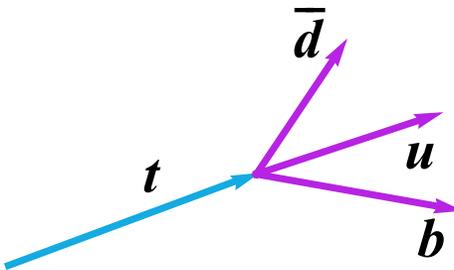
(arXiv:1401.3021)

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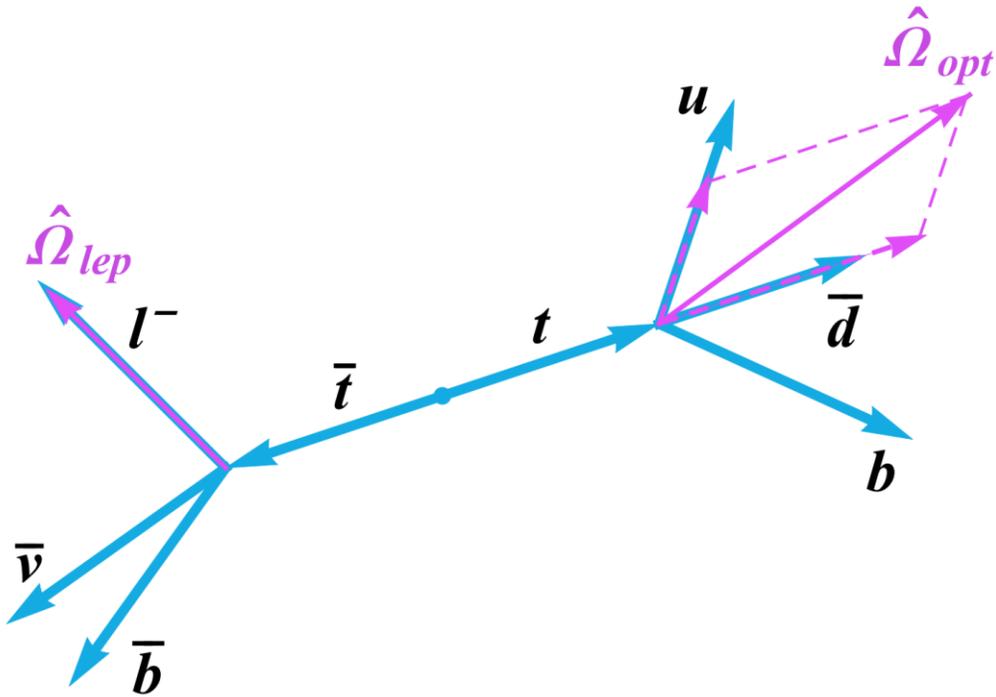
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$$\Rightarrow \kappa_{\text{opt}} = 0.64$$

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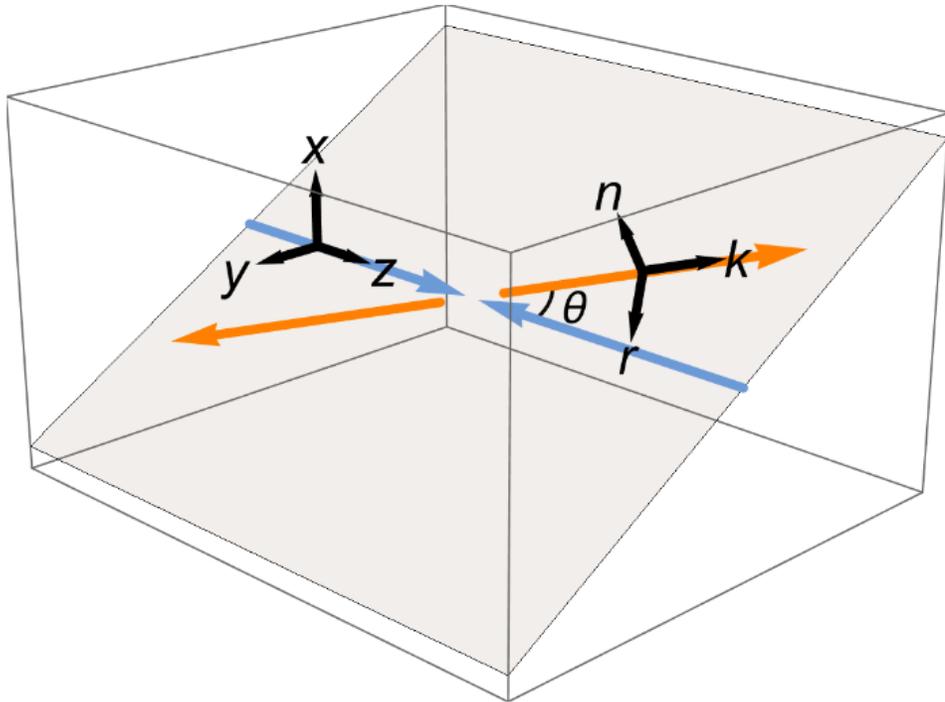
Semi-Leptonic Decay



Larger branching ratio, higher significance

$$\frac{\text{significance}(t\bar{t} \rightarrow lj)}{\text{significance}(t\bar{t} \rightarrow ll)} = \frac{\kappa_l \kappa_{opt}}{\kappa_l \kappa_l} \sqrt{\frac{\text{BR}(t\bar{t} \rightarrow lj)}{\text{BR}(t\bar{t} \rightarrow ll)}} \\ \approx 0.64 \times \sqrt{6} = 1.6$$

Which basis?



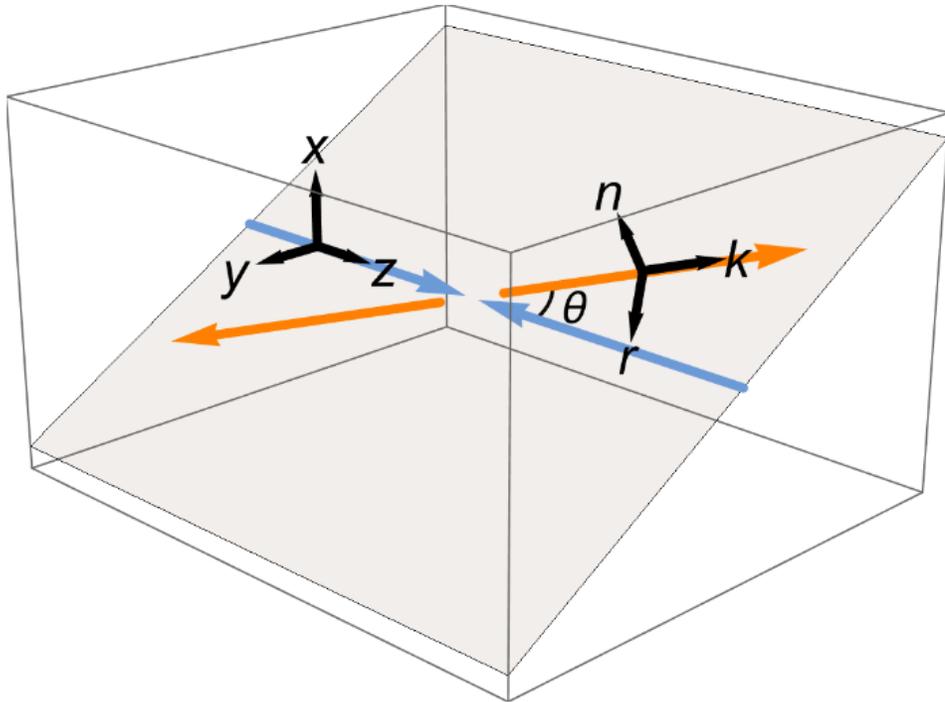
Beam basis $\{x, y, z\}$:

Cancellation leads to null result

Helicity basis $\{r, k, n\}$ is more sensitive!

Maintain the entanglement with no cancellations
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Which basis?



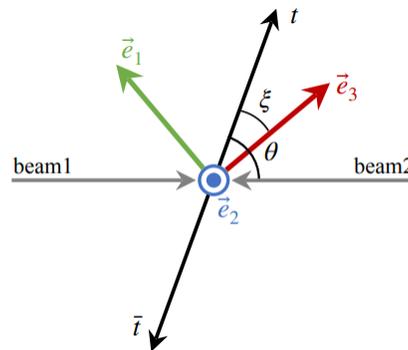
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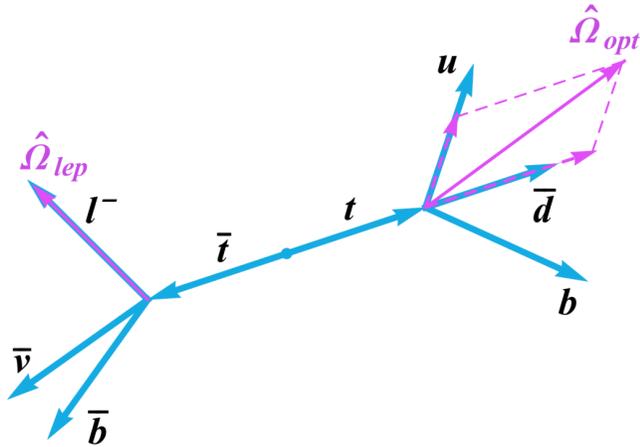
Optimal basis to optimize entanglement!



arxiv:2407.01672

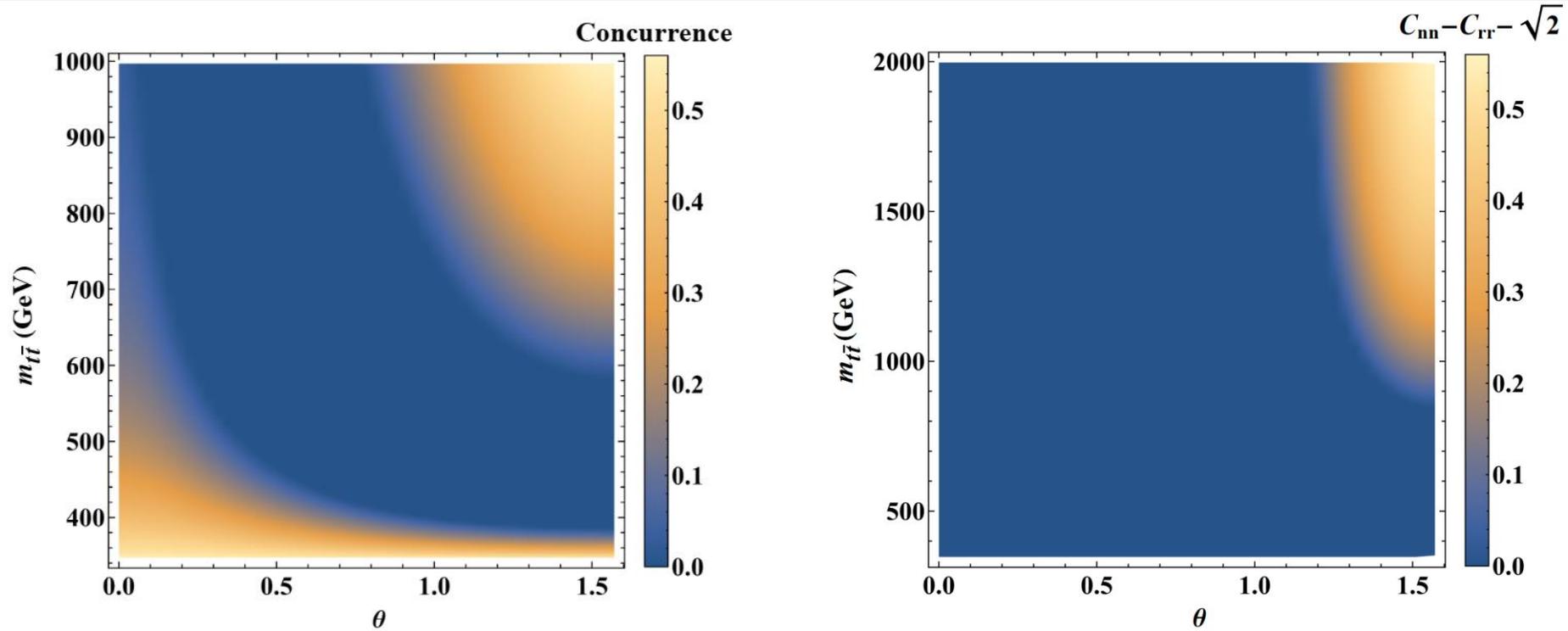
Kun Cheng, Tao Han, Matthew Low

Outline

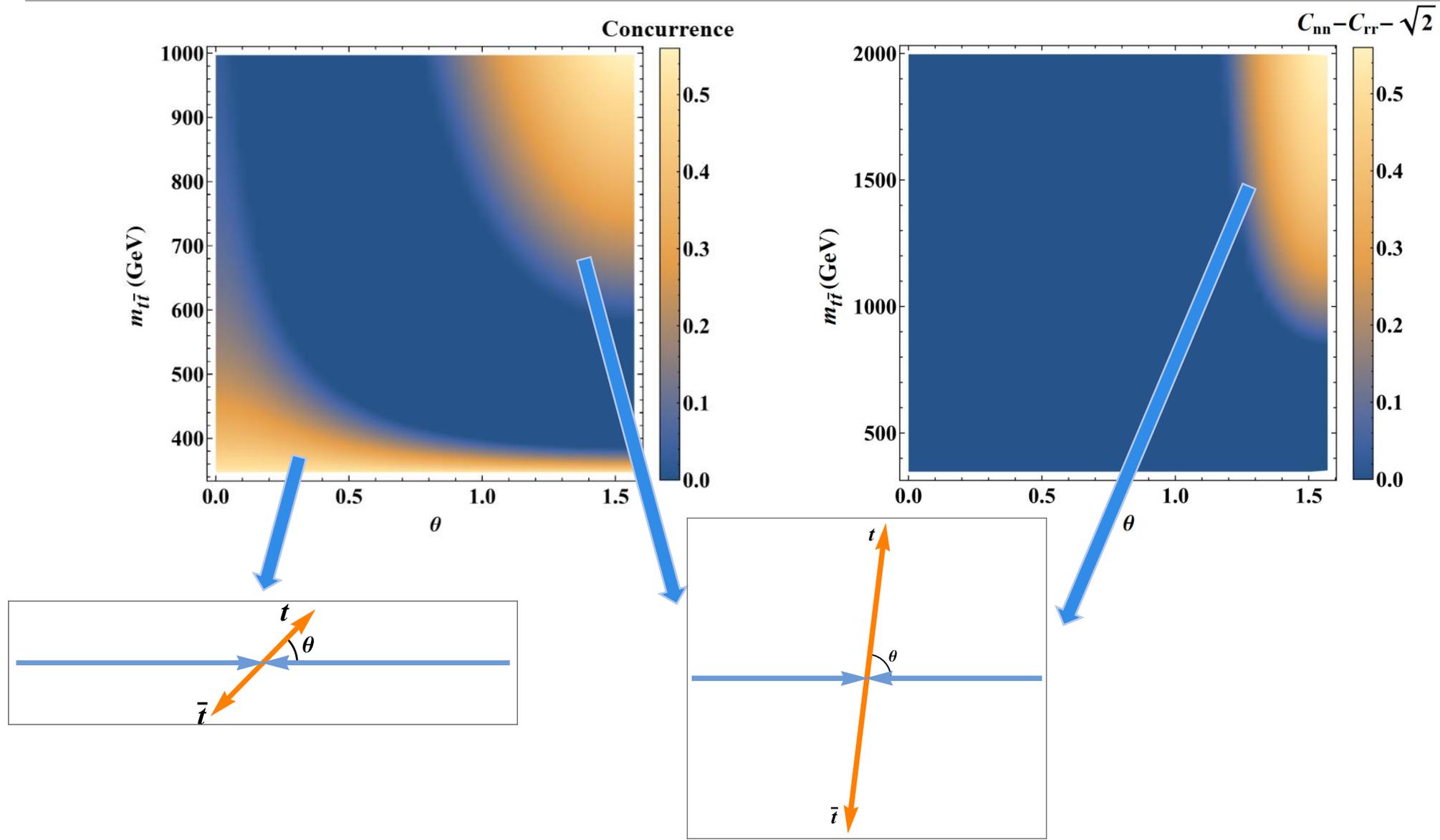


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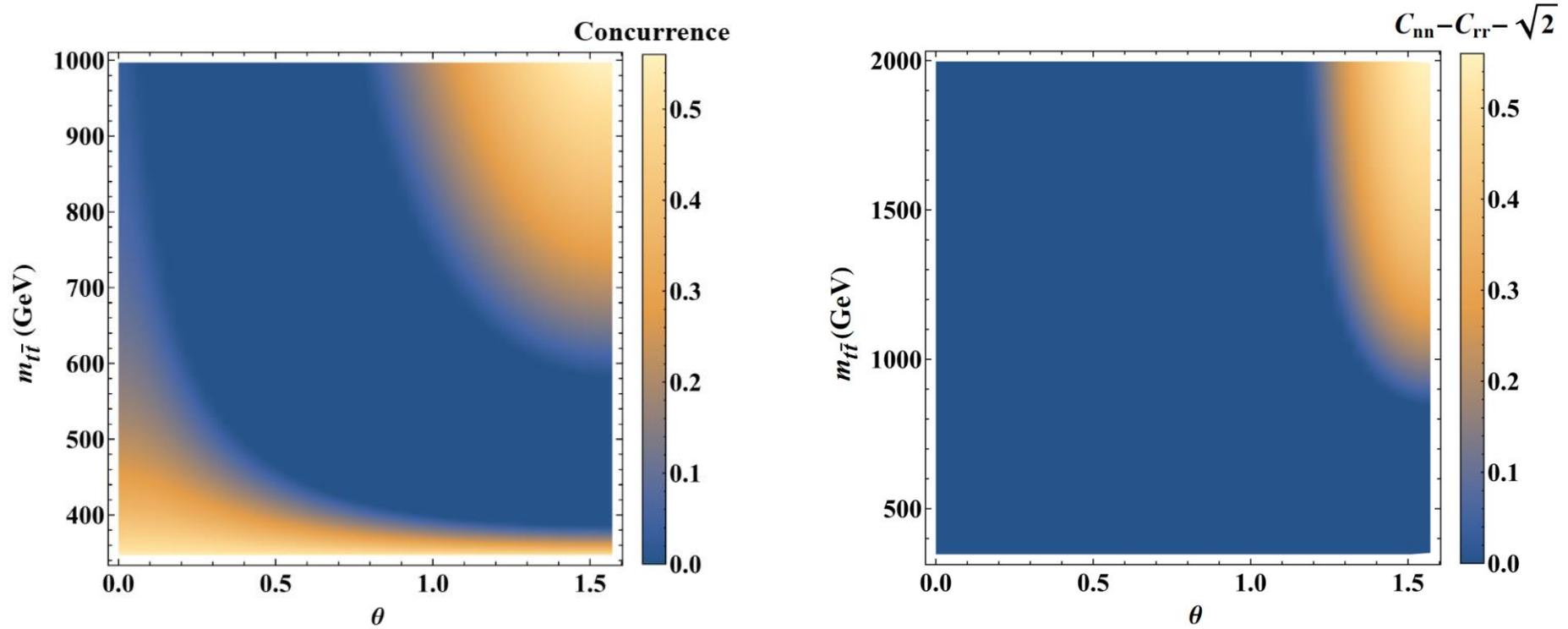
Theoretic & parton level result



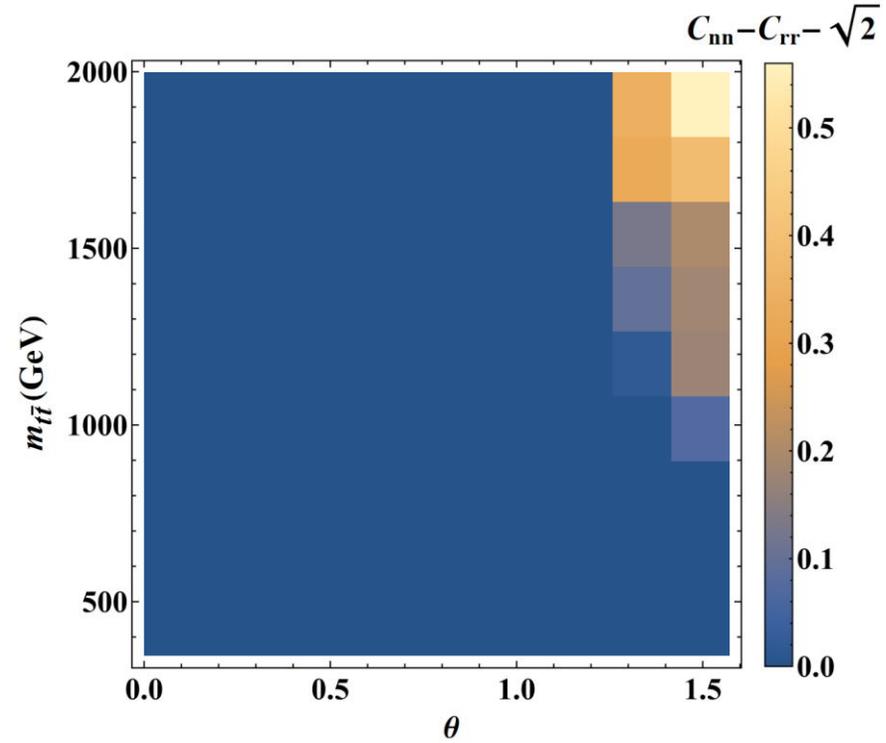
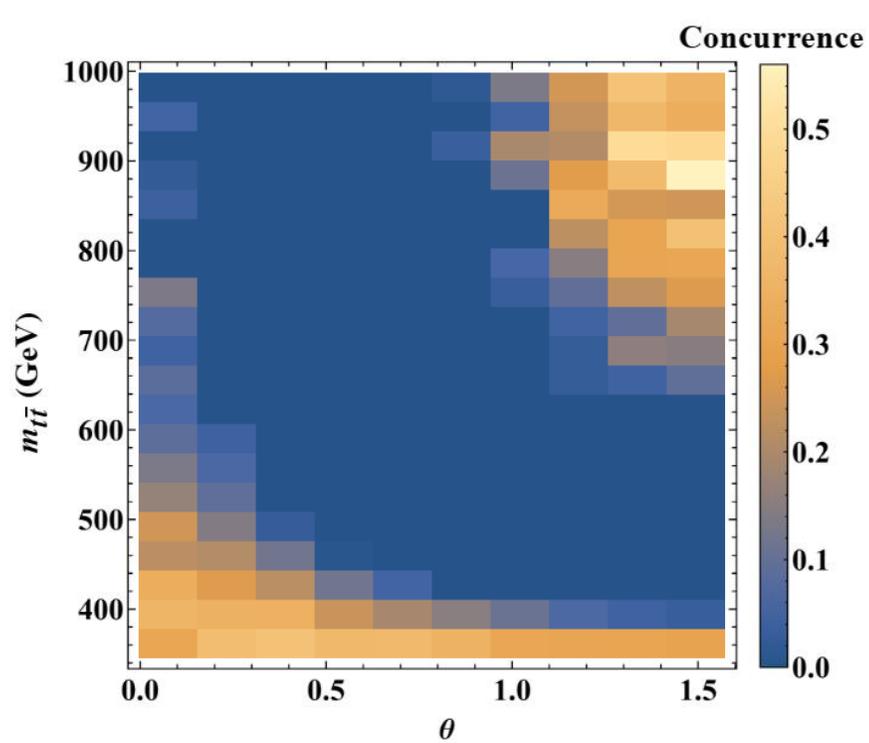
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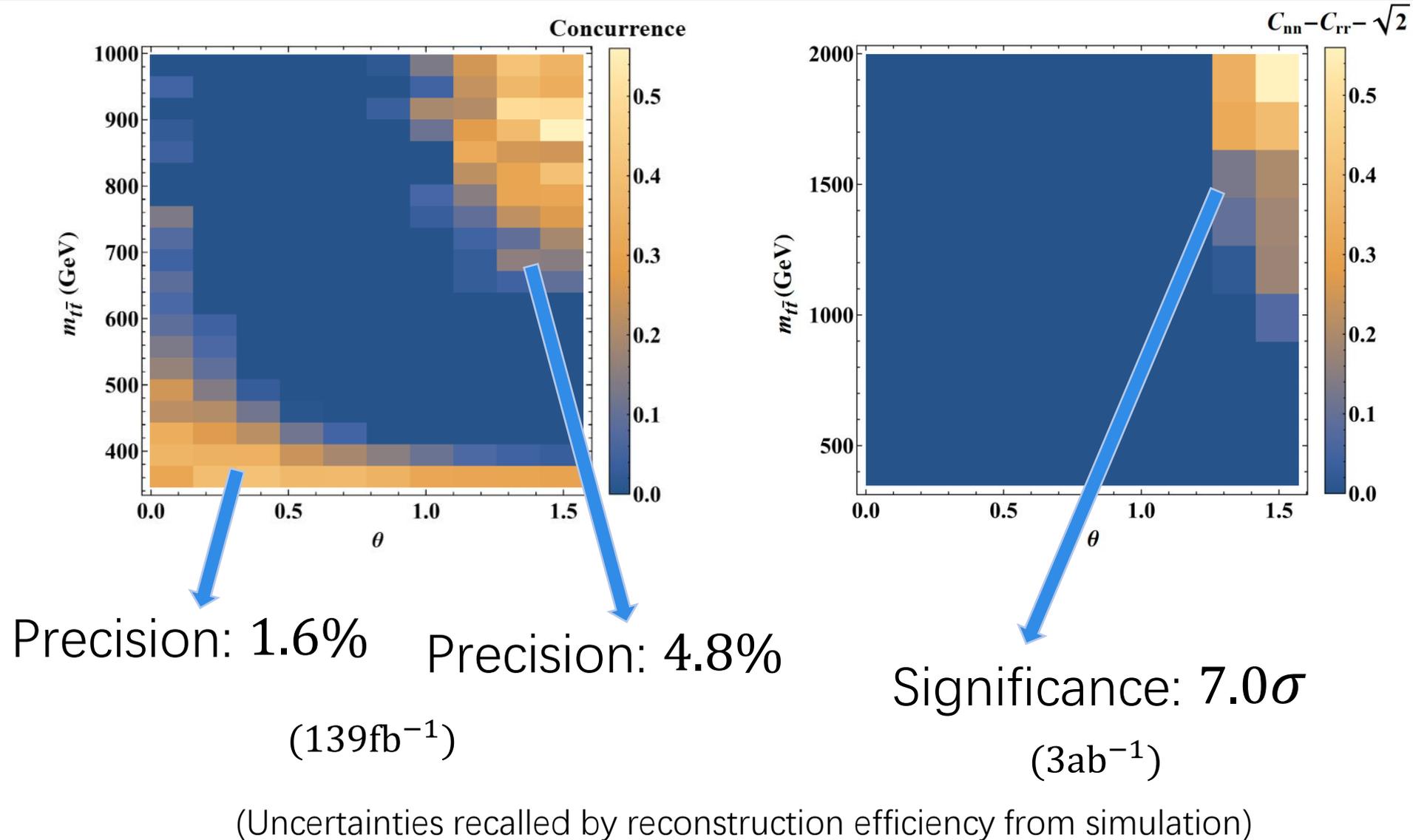
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Reconstruction

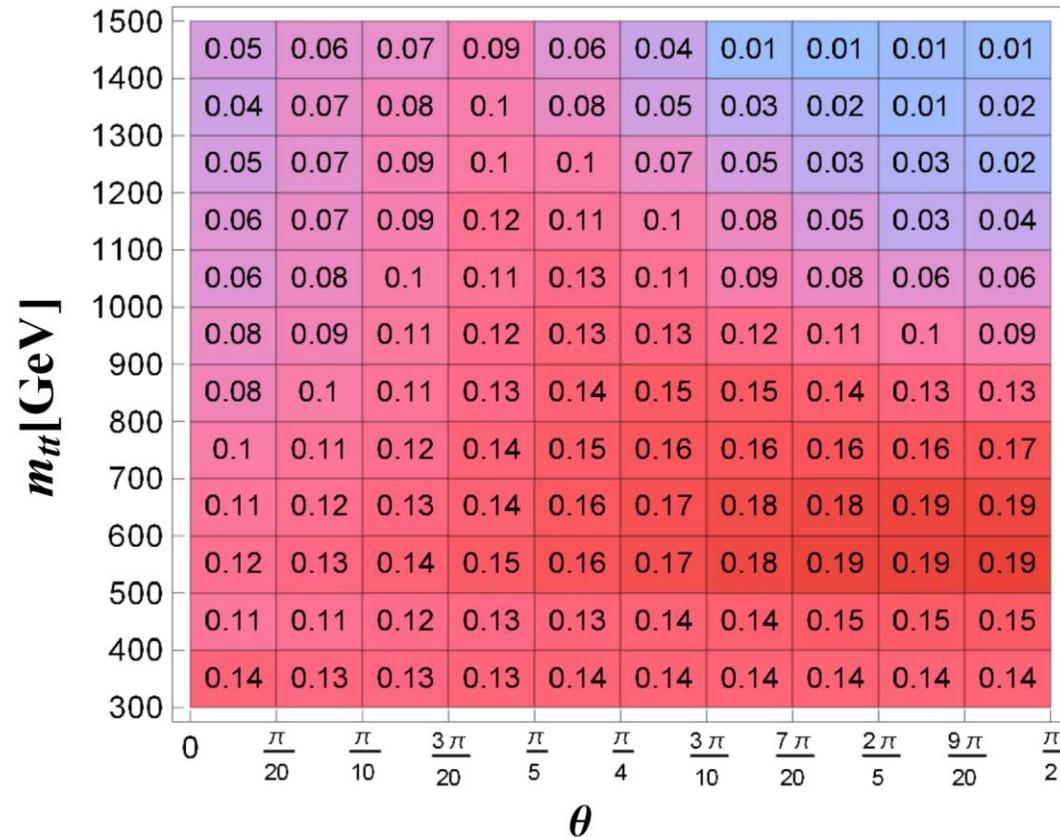
Full detector simulation with Madgraph 5 + Pythia 8 + Delphes 3

Reconstructed based on the pseudo-top algorithm.

Reconstruction

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Difficulties: ① Low efficiency



We need:
angular distribution in all directions

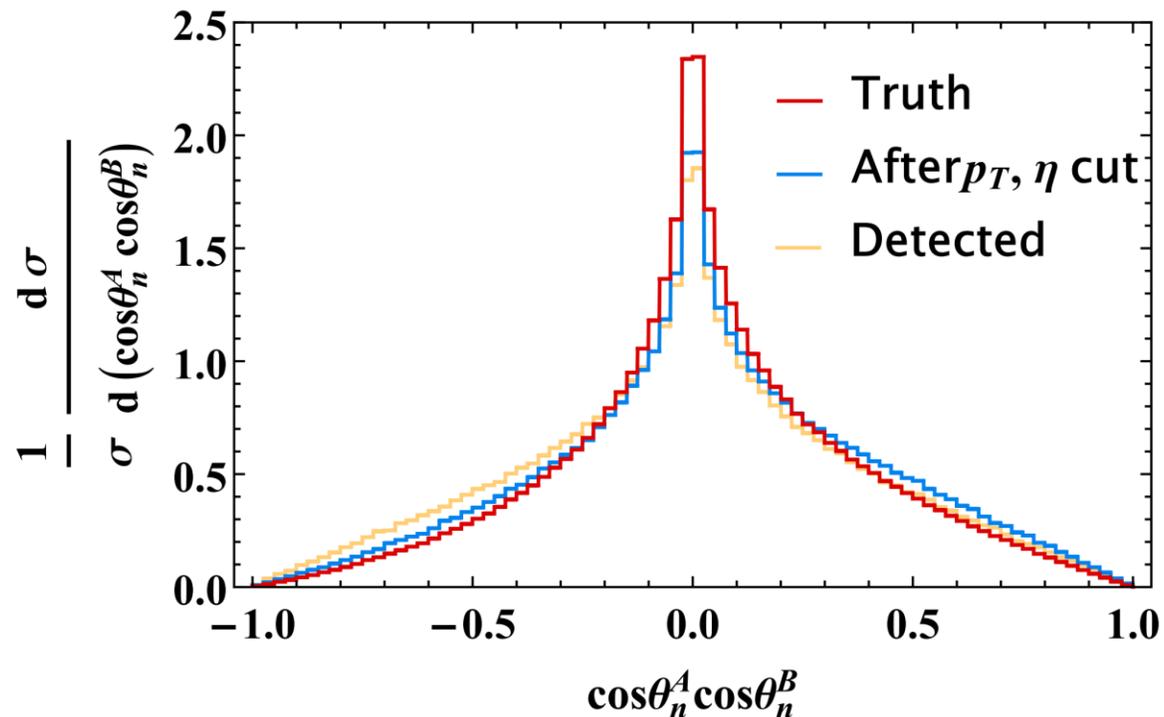
p_T, η cut:
throw away most of events :(

⇒ Efficiency: 0.01-0.2

Reconstruction

Full detector simulation with Madgraph 5 + Pythia 8 + Delphes 3
Reconstructed based on the pseudo-top algorithm.

Difficulties: ② Smearing angular distribution



An asymmetry of 0.5 was smeared to 0.05
:(

Reconstruction

Solution: Parametric Fit

Detector effects can be quantified by the Response Matrix R :

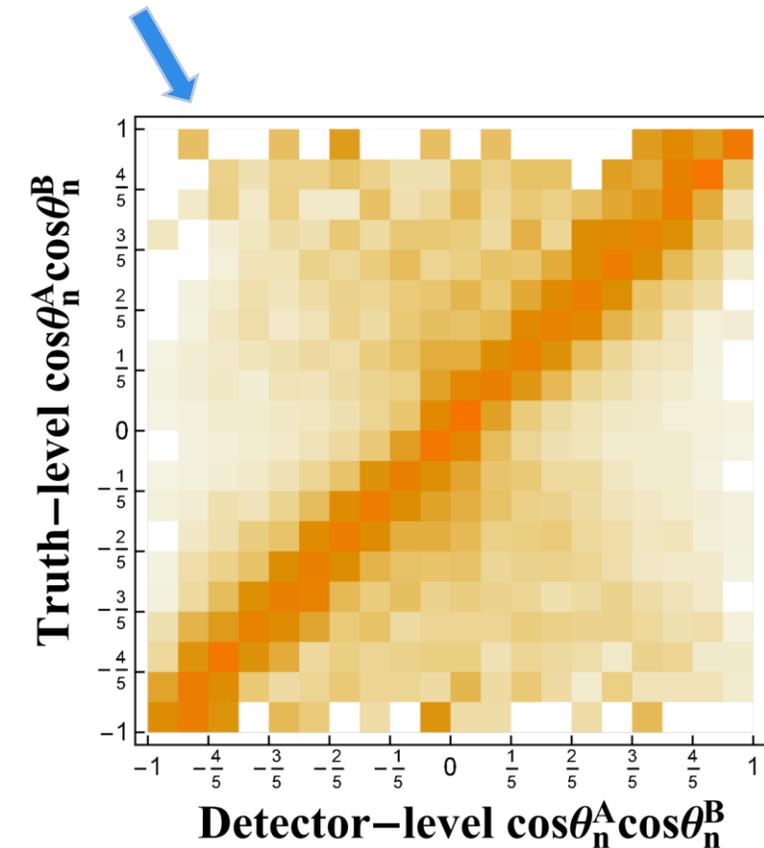
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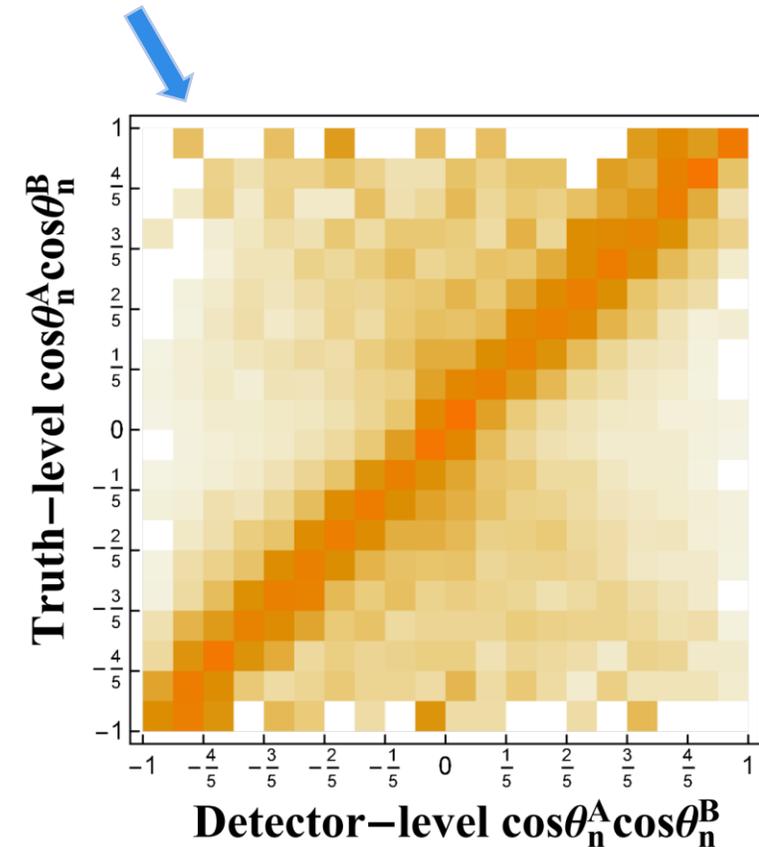
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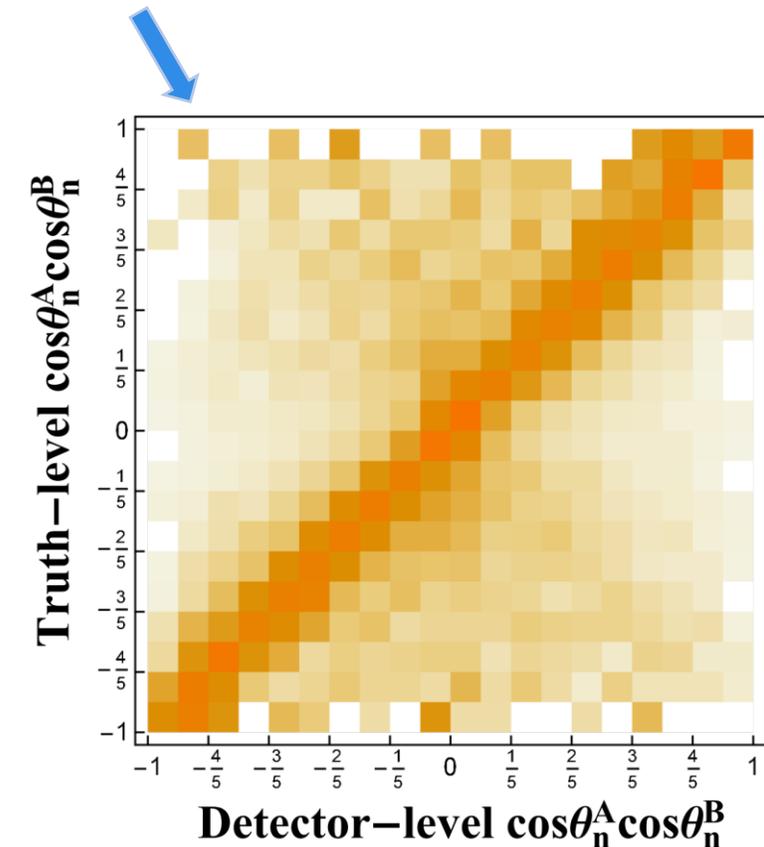
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Fitting P to D solves the problem :)



Result

Entanglement

	Result(139fb^{-1})	Precision
Boosted	0.276 ± 0.026	9.5%
Threshold	0.261 ± 0.008	3.0%

Bell's inequality violation

Result(3ab^{-1})	Significance
0.23 ± 0.06	4.1σ

Conclusion

1. Semi-leptonic decay is much more sensitive than full leptonic decay.
2. There is strong detector effect in reconstructing spin correlation matrix.

Thank you!