Analysis of $B \to VV$ decays under $SU(3)_F \otimes SU(2)_{spin}$

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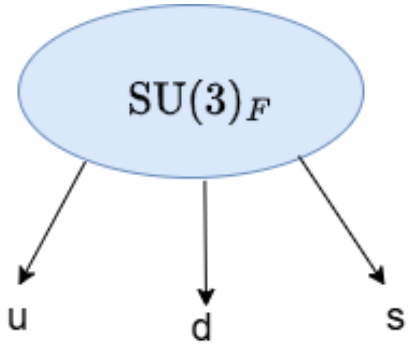
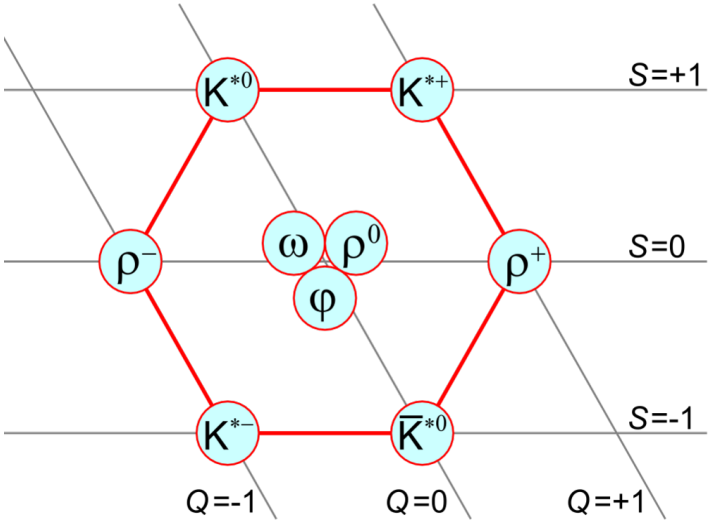
August 25, 2025

Introduction

- Tests of the Standard Model, how well are flavor symmetries such as isospin and $SU(3)_F$ realised in data
- Searches for New Physics





- $\frac{m_s - m_{u,d}}{m_b} \sim 0.03$
- $SU(3)_F$ broken in the Standard Model
- f_K/f_π - 1 $\sim$ 20 %
- πs (ρs) and Ks (K^*s) members of the $SU(3)_F$ octet and considered to be identical in the $SU(3)_F$ limit

Hadronic anomalies

- A long-standing puzzle in the four $B \to \pi K$ modes - $B^+ \to \pi^+ K^0$, $B^+ \to \pi^0 K^+$, $B^0 \to \pi^0 K^0$, $B^0 \to \pi^- K^+$
  $\to$ a certain inconsistency among the measurements of the branching ratios and CP asymmetries (direct and mixing-induced)

- Satisfy the isospin quadrilateral relation

$$\sqrt{2}A^{00} + A^{-+} = \sqrt{2}A^{0+} + A^{+0}$$

- Assumption motivated by QCD factorization - Discrepancy at the level of $\sim 3\ \sigma$

- Visit the $B \to \rho K^*$ modes to check for any puzzle

- $B \to \rho K^*$ modes comprise: $B^+ \to \rho^+ K^{*0}$ $(+0)$, $B^+ \to \rho^0 K^{*+}$ $(0+)$, $B^0 \to \rho^0 K^{*0}$ (00), $B^0 \to \rho^- K^{*+}$ $(-+)$

- Related by both isospin and $\mathrm{SU(3)_F}$

Hadronic anomalies

Anomalies in Hadronic B Decays

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In this paper, we perform fits to $B \to PP$ decays, where $B = \{B^0, B^+, B_s^0\}$ and the pseudoscalar $P = \{\pi, K\}$, under the assumption of flavor SU(3) symmetry [$\mathrm{SU}(3)_F$]. Although the fits to $\Delta S = 0$ or $\Delta S = 1$ decays individually are good, the combined fit is very poor: there is a 3.6σ disagreement with the $\mathrm{SU}(3)_F$ limit of the standard model ($\mathrm{SM}_{\mathrm{SU}(3)_F}$). One can remove this discrepancy by adding

Fit $\Delta S = 0$	$\lvert\widetilde{T}\rvert$	$\lvert\widetilde{C}\rvert$	$\lvert\widetilde{P}_{uc}\rvert$	$\lvert\widetilde{A}\rvert$
	4.0 ± 0.5	6.6 ± 0.7	3 ± 4	6 ± 5
	$\lvert\widetilde{PA}_{uc}\rvert$	$\lvert P_{tc}\rvert$	$\lvert PA_{tc}\rvert$	
	0.7 ± 0.8	0.8 ± 0.4	0.2 ± 0.4	
Fit $\Delta S = 1$	$\lvert\widetilde{T}'\rvert$	$\lvert\widetilde{C}'\rvert$	$\lvert\widetilde{P}'_{uc}\rvert$	$\lvert\widetilde{A}'\rvert$
	48 ± 14	41 ± 14	48 ± 15	81 ± 28
	$\lvert\widetilde{PA}'_{uc}\rvert$	$\lvert P'_{tc}\rvert$	$\lvert PA'_{tc}\rvert$	
	7 ± 4	0.78 ± 0.16	0.24 ± 0.04	
	$\lvert\widetilde{T}'/\widetilde{T}\rvert$	$\lvert\widetilde{C}'/\widetilde{C}\rvert$	$\lvert\widetilde{P}'_{uc}/\widetilde{P}_{uc}\rvert$	$\lvert\widetilde{A}'/\widetilde{A}\rvert$
	12 ± 4	6.6 ± 2.2	16 ± 22	14 ± 13
	$\lvert\widetilde{PA}'_{uc}/\widetilde{PA}_{uc}\rvert$	$\lvert P'_{tc}/P_{tc}\rvert$	$\lvert PA'_{tc}/PA_{tc}\rvert$	
	10 ± 13	0.97 ± 0.52	1.3 ± 2.7	

- $\Delta S = 0$ fit: p-value ~ 0.84
- $\Delta S = 1$ fit: p-value ~ 0.40
- Combined $\Delta S = 0$ and $\Delta S = 1$ fit: p-value $\sim 10^{-4}$
- Disagreement with $\mathrm{SM}_{\mathrm{SU}(3)_F}$ $\sim 3.6\ \sigma$

[Phys. Rev. Lett. 133, 211802 (2024) arXiv: 2505.11492]

Transversity amplitudes

3 helicity amplitudes - A_0, A_{11}, A_{-1-1}

$$A_{||} = \frac{A_{11} + A_{-1-1}}{\sqrt{2}} \quad A_{\perp} = \frac{A_{11} - A_{-1-1}}{\sqrt{2}}$$

$$A_0 = -\frac{S}{\sqrt{3}} + \sqrt{\frac{2}{3}} D$$

$$A_{||} = \sqrt{\frac{2}{3}} S + \sqrt{\frac{1}{3}} D \quad A_{\perp} = -P$$

A_0, $A_{||}$ $\rightarrow$ Symmetric, $A_{\perp}$ $\rightarrow$ Anti-Symmetric

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Space	×	Spin	×	$SU(3)_F$
Symmetric (l = 0,2)		Symmetric (s = 0,2)		Symmetric
Anti-symmetric (l = 1)		Anti-symmetric (s = 1)		Symmetric

Transversity amplitudes

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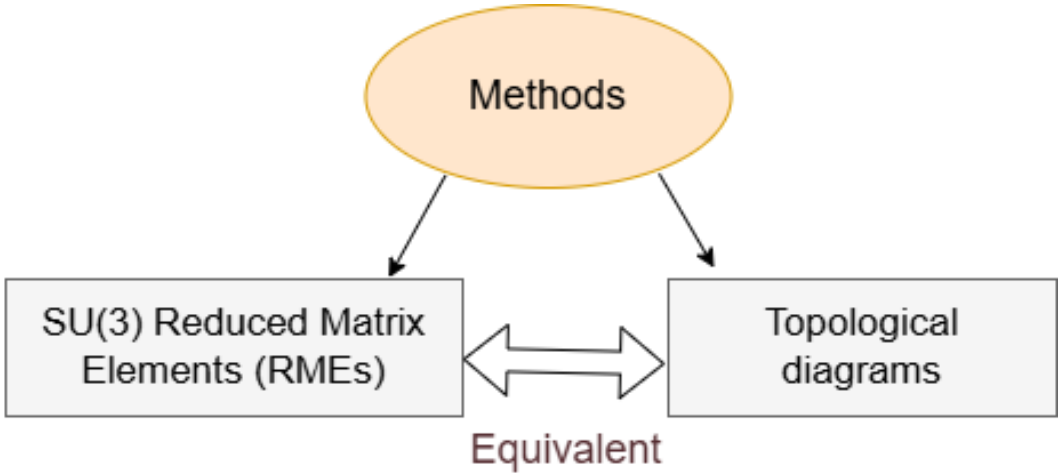
Symmetric (l = 0,2)		Symmetric (s = 0,2)		Symmetric
Anti-symmetric (l = 1)		Anti-symmetric (s = 1)		Symmetric



RMEs

- Application of the Wigner-Eckart theorem to the $B \to VV$ amplitudes under $\mathrm{SU(3)_F} \times \mathrm{SU(2)_{spin}}$ symmetry
- How each piece of $\langle VV|\mathcal{H}_W|B\rangle$ transforms
- Symmetric product of two octets $\to$ 5 Symmetric RMEs

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} \left(\lambda_u^{(q)} \sum_{i=1}^{2} c_i Q_i^{(q)} - \lambda_t^{(q)} \sum_{i=3}^{10} c_i Q_i^{(q)} \right)$$

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$$(3 \otimes 3^*) \otimes 3^* = 3^* \oplus 3^* \oplus 6 \oplus 15^*$$

$$3^* \otimes 3 = 1 \oplus 8_1$$

$$6 \otimes 3 = 8_2 \oplus 10$$

$$15^* \otimes 3 = 8_3 \oplus 10^* \oplus 27$$

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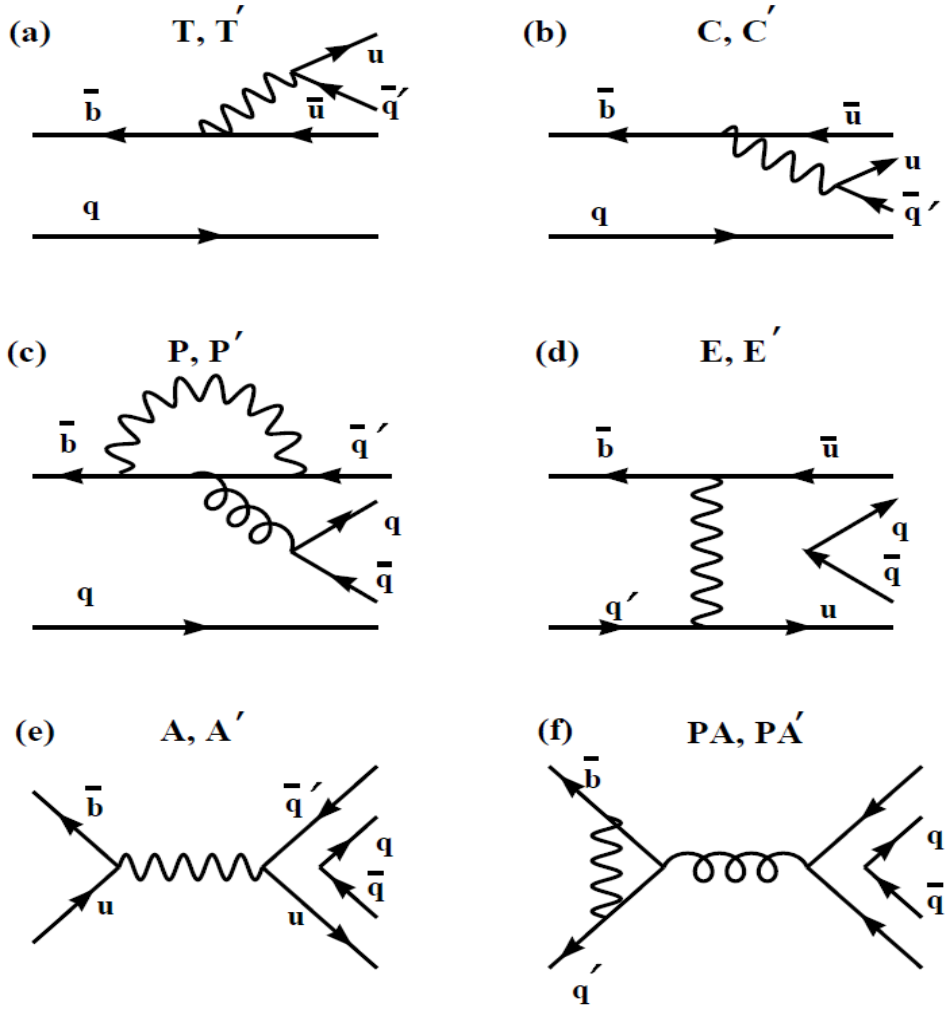
$$3^* \otimes 3 = 1 \oplus 8_1$$
$$6 \otimes 3 = 8_2 \oplus 10$$
$$15^* \otimes 3 = 8_3 \oplus 10^* \oplus 27$$

$$(8 \otimes 8)_S = 1 \oplus 8_S \oplus 27$$

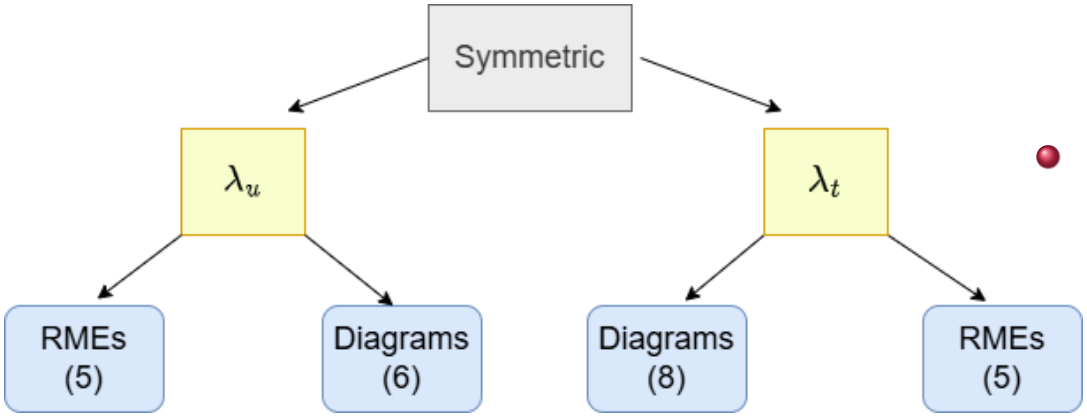
$$\langle 1||3^*||3\rangle_{(8\otimes 8)_S}, \langle 8||3^*||3\rangle_{(8\otimes 8)_S},$$
$$\langle 8||6||3\rangle_{(8\otimes 8)_S}, \langle 8||15^*||3\rangle_{(8\otimes 8)_S},$$
$$\langle 27||15^*||3\rangle_{(8\otimes 8)_S}$$

Diagrams

- $\lambda_u^{(q)}$: T (tree), C (color-suppressed tree), P_{uc} (penguin), A (annihilation), E (exchange) and PA_{uc} (penguin annihilation)
- $\lambda_t^{(q)}$: P_{tc}, PA_{tc}, P_{EW}^{T}, P_{EW}^{C}, $P_{EW}^{P_u}$, P_{EW}^{A}, P_{EW}^{E} and $P_{EW}^{PA_u}$



EWP-tree relations



- 3 Symmetric RMEs appear in both λ_u, λ_t parts
- Electroweak penguin diagrams in λ_t related to tree diagrams in λ_u sector after properly taking into account the Wilson coefficients- EWP-tree relations

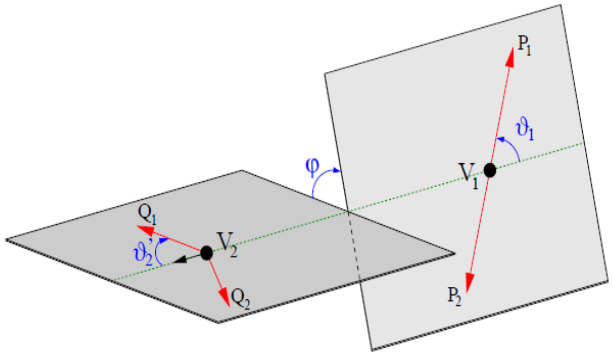
$$\widetilde{P^T_{EW}} = -\frac{3}{4}\left[\frac{c_9+c_{10}}{c_1+c_2}(\tilde{T}+\tilde{C}+\tilde{A}) + \frac{c_9-c_{10}}{c_1-c_2}(\tilde{T}-\tilde{C}-\tilde{A})\right]$$

$$\widetilde{P^C_{EW}} = -\frac{3}{4}\left[\frac{c_9+c_{10}}{c_1+c_2}(\tilde{T}+\tilde{C}-\tilde{A}) - \frac{c_9-c_{10}}{c_1-c_2}(\tilde{T}-\tilde{C}-\tilde{A})\right]$$

$$\widetilde{P^A_{EW}} = -\frac{3}{2}\frac{c_9+c_{10}}{c_1+c_2}\tilde{A}$$

Decay distribution

$$\frac{d\Gamma}{d\cos v_1\, d\cos v_2\, d\phi} \propto \Bigg(|A_0|^2 \cos^2 v_1 \cos^2 v_2 + \frac{|A_{||}|^2}{2} \sin^2 v_1 \sin^2 v_2 \cos^2 \phi + \frac{|A_\perp|^2}{2} \sin^2 v_1 \sin^2 v_2 \sin^2 \phi + \frac{\mathrm{Re}(A_0 A_{||}^*)}{2\sqrt{2}} \sin 2v_1 \sin 2v_2 \cos\phi - \frac{\mathrm{Im}(A_\perp A_0^*)}{2\sqrt{2}} \sin 2v_1 \sin 2v_2 \sin\phi - \frac{\mathrm{Im}(A_\perp A_{||}^*)}{2} \sin^2 v_1 \sin^2 v_2 \sin 2\phi \Bigg)$$



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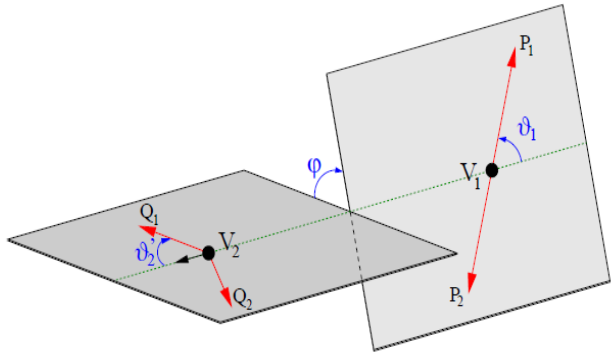


$$|A|^2 = |A_0|^2 + |A_{||}|^2 + |A_\perp|^2, \qquad |\bar{A}|^2 = |\bar{A}_0|^2 + |\bar{A}_{||}|^2 + |\bar{A}_\perp|^2$$

$$BR \propto (|A|^2 + |\bar{A}|^2) \qquad A_{CP} = \frac{|\bar{A}|^2 - |A|^2}{|\bar{A}|^2 + |A|^2} \qquad f_\lambda = \frac{|A_\lambda|^2}{|A_0|^2 + |A_{||}|^2 + |A_\perp|^2}$$

$$\delta_{||-0,\perp-0} = \mathrm{Arg}\left(\frac{A_{||,\perp}}{A_0}\right) \quad A_T^{(1)} \propto \frac{\mathrm{Im}(A_\perp A_0^*)}{|A_0|^2 + |A_{||}|^2 + |A_\perp|^2} \quad A_T^{(2)} \propto \frac{\mathrm{Im}(A_\perp A_{||}^*)}{|A_0|^2 + |A_{||}|^2 + |A_\perp|^2}$$

Results

- Full angular distribution in $B^0 \to \rho^0 K^{*0}$ mode
- Optimize $\chi^2 = \sum_i \frac{(\mathcal{O}_i^{exp} - \mathcal{O}_i^{th})^2}{(\Delta\mathcal{O}_i^{exp})^2}$. $\chi^2_{\rm min}/{\rm d.o.f} = 1.0/4$

Results for $\lambda = 0$ (in KeV)	$\lvert T^0\rvert$	$\lvert C^0\rvert$	$\lvert P^0_{uc}\rvert$	$\lvert \widetilde{P^0_{tc}}\rvert$
	12.0 ± 4.0	11.7 ± 2.3	24.9 ± 3.5	0.03 ± 0.06
Results for $\lambda = \lvert\rvert$ (in KeV)	$\lvert T^{\lvert\rvert}\rvert$	$\lvert C^{\lvert\rvert}\rvert$	$\lvert P^{\lvert\rvert}_{uc}\rvert$	$\lvert \widetilde{P^{\lvert\rvert}_{tc}}\rvert$
	31.0 ± 21.0	12.0 ± 10.0	25.9 ± 3.1	0.21 ± 0.18
Results for $\lambda = \perp$ (in KeV)	$\lvert T^\perp\rvert$	$\lvert C^\perp\rvert$	$\lvert P^\perp_{uc}\rvert$	$\lvert \widetilde{P^\perp_{tc}}\rvert$
	23.0 ± 11.0	17.0 ± 5.0	7.4 ± 3.5	0.16 ± 0.06

$$\frac{|C^0|}{|T^0|} = 0.98 \pm 0.38, \qquad \frac{|C^{||}|}{|T^{||}|} = 0.39 \pm 0.42, \qquad \frac{|C^\perp|}{|T^\perp|} = 0.74 \pm 0.41$$

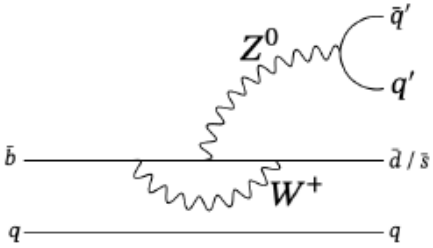
- From QCD factorization, $|C|/|T| \simeq 0.2 \to$ Imposing, we get $\chi^2_{\rm min}/{\rm d.o.f} = 4.9/7$

Conclusions

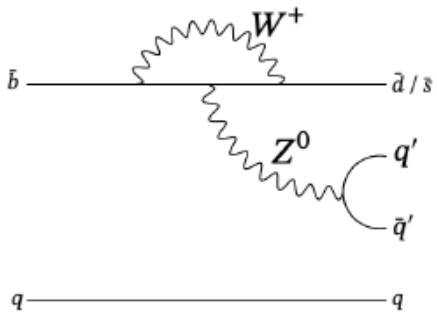
- Preliminary results seem to suggest that there is no puzzle in the $B \to \rho K^*$ modes - $\mathrm{SU(3)_F}$ symmetry is satisfied
- Far from being conclusive - Not all observables available experimentally
- Full angular distribution measured only in $B^0 \to \rho^0 K^{*0}$ mode
- This formalism sets up the stage to perform global fits with all $\Delta S = 0$ and 1 modes and test the $\mathrm{SU(3)_F}$ symmetry
- Full angular analysis in all four modes might shed light on tests of the various symmetries
- Data on suppressed observables such as CP asymmetries, TP asymmetries, etc in these channels might lead the way for NP searches


Thank
you!

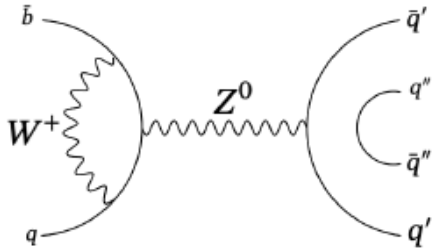
Back-up slides (EWP diagrams)



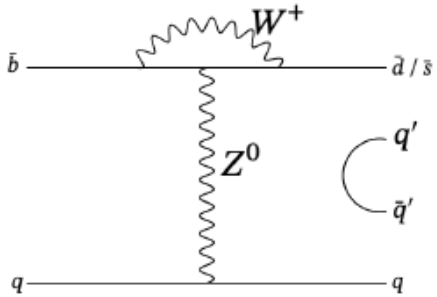
(a) P_{EW}^{T}



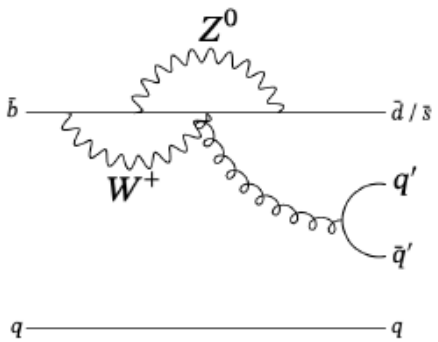
(b) P_{EW}^{C}



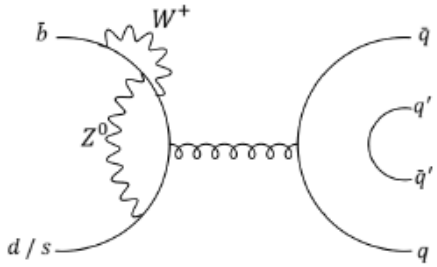
(c) P_{EW}^{A}



(d) P_{EW}^{E}



(e) $P_{EW}^{P_u}$



(f) $P_{EW}^{PA_u}$

Back-up slides

- Relationships between RMEs and diagrams for λ_u part:

$$A_1 = \frac{1}{2\sqrt{3}}(-3T_S + C_S - 8P_S - 8E_S - 24PA_S)$$
$$A_8 = \frac{1}{8}\sqrt{\frac{5}{3}}(-3T_S + C_S - 8P_S + E_S - 3A_S)$$
$$R_8 = \frac{\sqrt{5}}{4}(T_S - C_S + E_S - A_S)$$
$$P_8 = \frac{1}{8\sqrt{3}}(T_S + C_S + 5E_S + 5A_S)$$
$$P_{27} = -\frac{1}{2\sqrt{3}}(T_S + C_S)$$

- Relationships between RMEs and diagrams for λ_t part:

$$B_1 = -\frac{1}{6\sqrt{3}}\Big(24(P_{tc})_S + 72(PA_{tc})_S - 3(P^T_{EW})_S + (P^C_{EW})_S - 8(P^E_{EW})_S - 8(P^{P_u}_{EW})_S - 24(P^{PA_u}_{EW})_S\Big)$$
$$B_8 = \frac{1}{24}\sqrt{\frac{5}{3}}\Big(-24(P_{tc})_S + 3(P^T_{EW})_S - (P^C_{EW})_S + 3(P^A_{EW})_S - (P^E_{EW})_S + 8(P^{P_u}_{EW})_S\Big)$$
$$R_8 = \frac{\sqrt{5}}{12}\Big((P^T_{EW})_S - (P^C_{EW})_S - (P^A_{EW})_S + (P^E_{EW})_S\Big)$$
$$P_8 = \frac{1}{24\sqrt{3}}\Big((P^T_{EW})_S + (P^C_{EW})_S + 5(P^A_{EW})_S + 5(P^E_{EW})_S\Big)$$
$$P_{27} = -\frac{1}{6\sqrt{3}}\Big((P^T_{EW})_S + (P^C_{EW})_S\Big)$$

Back-up slides

Decay	BR	A_{CP}	S_{CP}	f_L	$\tilde{f}_\lambda$	$\tilde{A}_\lambda$	TPAs	pol obs
$B^+ \to \rho^0 K^{*+}$	$\surd$	$\surd$		$\surd$				
$B^+ \to \rho^+ K^{*0}$	$\surd$	$\surd$		$\surd$				
$B^0 \to \rho^- K^{*+}$	$\surd$	$\surd$		$\surd$				
$B^0 \to \rho^0 K^{*0}$	$\surd$	$\surd$			$\surd$ (3)	$\surd$ (3)	$\surd$ (4)	$\surd$ (6)