Thermal corrections to Dark Matter Annihilation Cross Sections at NLO

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based on

- *NLO thermal corrections to dark matter annihilation cross sections: a novel approach*; Phys.Rev.D 110 (2024) 3, 036006 (arXiv: 2404.15987 [hep-ph]);
- *Thermal corrections to dark matter annihilation with real photon emission/absorption*; arXiv: 2506.07663 [hep-ph] (2025);

Prabhat Butola, D. Indumathi, Pritam Sen

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Overview

Introduction/Motivation

- The existence of dark matter (DM) is widely accepted today.
- The relic density is extremely precisely measured: $\Omega_{cdm} = 0.265(7)$.
  It is governed by the DM annihilation cross section via

$$\chi + \overline{\chi} \leftrightarrow f_{SM} + \overline{f}_{SM} \quad (\chi + f_{SM} \leftrightarrow \chi + f_{SM}) \; .$$

- The DM density evolves according to the Boltzmann equation:

$$\frac{dn_\chi}{dt} + 3\mathcal{H} n_\chi = \langle \sigma_{\chi\bar{\chi}\to f\bar{f}} . v_{rel} \rangle_T (n_\chi^{eq} . n_{\bar{\chi}}^{eq} - n_\chi . n_{\bar{\chi}}) \; .$$

- $\langle \sigma v \rangle_T$ is thermally averaged, but σ itself is usually calculated at zero temperature in quantum field theory; no longer acceptable.
- Both higher order corrections and thermal effects need to be included. First calculated by *Beneke et al., JHEP 1410 (2014) 45; JHEP 09 (2016) 031*; they also showed the processes are IR finite to NLO.
- Here we provide an improved and independent calculation using the Grammer Yennie technique for isolating IR divergences.

Typical class of DM models

- The relevant Lagrangian is

  (A. DiFranzo et al. JHEP 11 (2013) 014, arXiv: 1308.2679 [hep-ph].)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{f}\left(i\not{D} - m_f\right)f + \frac{1}{2}\overline{\chi}\left(i\not{\partial} - m_\chi\right)\chi \\ + \left(D^\mu\phi\right)^\dagger\left(D_\mu\phi\right) - m_\phi^2\phi^\dagger\phi + \left(\lambda\,\overline{\chi}P_L f^-\phi^+ + \mathrm{h.c.}\right)\ .$$

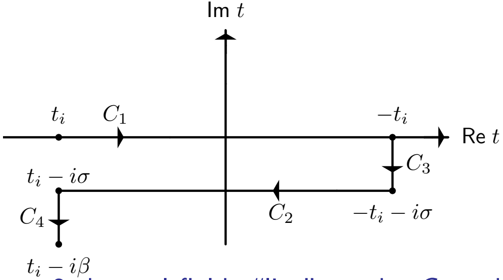
- Here $f = (f^0, f^-)^T$ is left-handed fermion doublet; $\phi = (\phi^+, \phi^0)^T$ is an additional scalar doublet; χ is $[SU(2) \times U(1)]$ singlet Majorana fermion.
- Assume bino to be TeV-scale DM, so freeze-out occurs *after* electro-weak phase transition; so only EM interactions relevant at IR.
- So we have a theory of fermions and scalars interacting with photons in a heat bath at finite temperature.

Real time formulation of thermal field theory

Path C must go through all the arguments of the Green function. Then

$$
\begin{aligned}
\text{Propagator } D_C(x,x') &= \theta_C(t-t')D_C^{>}(x,x') + \theta_C(t'-t)D_C^{<}(x,x') \, , \\
\text{with } D_C^{>}(x,x') &= \langle \hat{\phi}(x)\hat{\phi}(x')\rangle_\beta \, , \\
\text{and } D_C^{<}(x,x') &= D_C^{>}(x',x) = \langle \hat{\phi}(x')\hat{\phi}(x)\rangle_\beta \, .
\end{aligned}
$$

C defined so that *Im t* codes for the inverse temperature, $\beta = 1/T$.



Type-1 and type-2 thermal fields "live" on the C_1 and C_2 paths.

Periodic B.C. (KMS) $\varphi(t_0) = \pm\varphi(t_0 - i\beta)$ for boson/fermion fields .

Thermal Feynman rules

- The Scalar propagator

$$i\mathcal{S}^{t_a,t_b}_{\rm scalar}(p,m) = \begin{pmatrix} \Delta(p) & 0 \\ 0 & \Delta^*(p) \end{pmatrix} + 2\pi\delta(p^2-m^2)N_{\rm B}(|p^0|)\begin{pmatrix} 1 & e^{|p^0|/(2T)} \\ e^{|p^0|/(2T)} & 1 \end{pmatrix}$$

$$\Delta(p) = i/(p^2-m^2+i\epsilon);\ t_a, t_b = 1, 2.$$

- The Photon Propagator

$$i\mathcal{D}^{t_a,t_b}_{\mu\nu}(k) = -ig_{\mu\nu} D^{t_a,t_b}(k) = -ig_{\mu\nu}\, S^{t_a,t_b}_{\rm scalar}(k,0)\ .$$

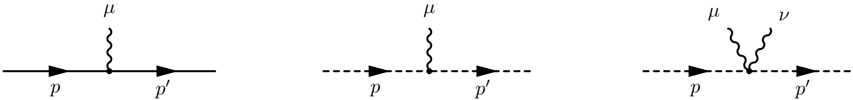
- The Fermion propagator (zero chemical potential)

$$i\mathcal{S}^{t_a,t_b}_{\rm ferm}(p,m) = \begin{pmatrix} S & 0 \\ 0 & S^* \end{pmatrix} - 2\pi S'\delta(p^2-m^2)N_{\rm F}(|p^0|)\begin{pmatrix} 1 & \epsilon(p_0)e^{|p^0|/(2T)} \\ -\epsilon(p_0)e^{|p^0|/(2T)} & 1 \end{pmatrix}$$

$$\equiv (\not p + m)\begin{pmatrix} F_p^{-1} & G_p^{-1} \\ -G_p^{-1} & F_p^{*-1} \end{pmatrix},$$

where $S = i/(\not p - m + i\epsilon)$, and $S' = (\not p + m)$.

Thermal Feynman rules: 2



a. Fermion-Photon Vertex   b. Scalar-Photon Vertex   c. Seagull Vertex

$(-ie\gamma_\mu)(-1)^{t_\mu+1}$ $[-ie(p_\mu + p'_\mu)](-1)^{t_\mu+1}$ $[+2ie^2 g_{\mu\nu}](-1)^{t_\mu+1}$.

- The fermionic number operator
$$N_{\mathrm{F}}(|p^0|) \equiv \frac{1}{\exp\{|p^0|/T\}+1} \xrightarrow{p^0\to 0} \frac{1}{2},$$
- The bosonic number operator
$$N_{\mathrm{B}}(|k^0|) \equiv \frac{1}{\exp\{|k^0|/T\}-1} \xrightarrow{k^0\to 0} \frac{T}{|k^0|}.$$

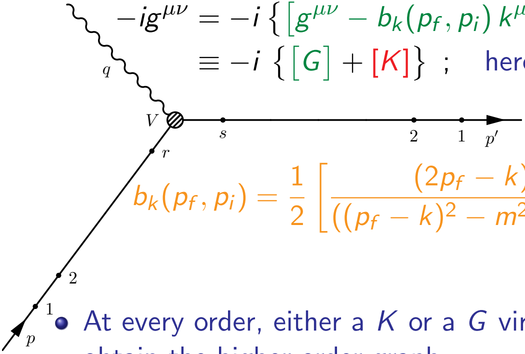
$$i\mathcal{D}^{ab}(k) = \left[\frac{i}{k^2+i\epsilon} \pm 2\pi\delta(k^2)N_B(|k^0|)D_T^{ab}\right].$$

- So leading IR divergence in the finite temperature part is linear rather than logarithmic (as at $T=0$). Consequently, there is a residual logarithmic sub-divergence at finite temperature.

IR behaviour of QED: Grammer, Yennie Phys. Rev. D8, 4332 (1973)

- Consider a virtual photon insertion (vertices on final (f) or initial (i) lines) on a generic $n = (r+s)^{th}$ order graph.
- Rearrange photon propagator into IR-divergent (K) plus IR-finite (G):

$$-ig^{\mu\nu} = -i\left\{\left[g^{\mu\nu} - b_k(p_f,p_i)\,k^\mu k^\nu\right] + \left[b_k(p_f,p_i)k^\mu k^\nu\right]\right\} \;,$$
$$\equiv -i\;\left\{\left[G\right] + \left[K\right]\right\}\;; \quad \text{here } b_k \text{ is a function of } (k, p_f, p_i).$$



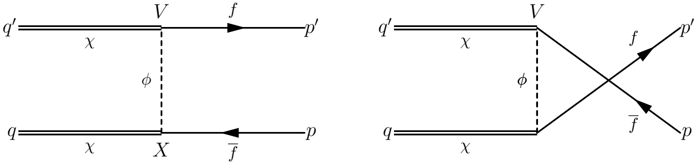
$$b_k(p_f,p_i) = \frac{1}{2}\left[\frac{(2p_f-k)\cdot(2p_i-k)}{((p_f-k)^2-m^2)((p_i-k)^2-m^2)} + (k \leftrightarrow -k)\right] \;.$$

- At every order, either a K or a G virtual photon can be inserted to obtain the higher order graph.

The $\chi\chi \to f\bar{f}$ cross section

- Notation: CM frame:

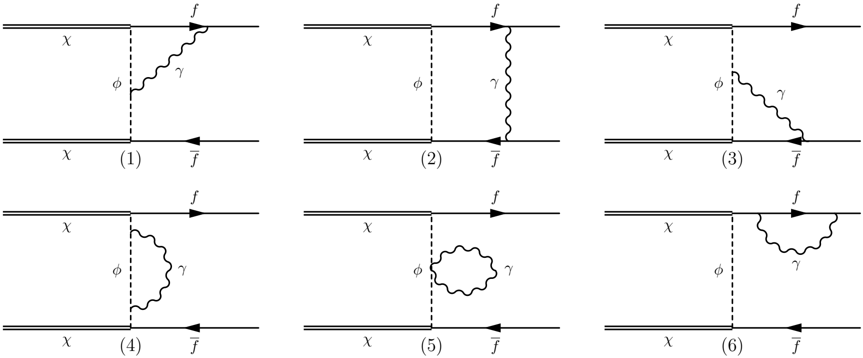
$$q'^{\mu} = (H, 0, 0, Q) \; ; p'^{\mu} = (H, P\sin\theta, 0, P\cos\theta)$$
$$q^{\mu} = (H, 0, 0, -Q) \; ; p^{\mu} = (H, -P\sin\theta, 0, -P\cos\theta) \; .$$



$$\sigma_{LO}^{\text{Majorana,Heavy Scalar}} = \frac{1}{12\pi s}\frac{P}{Q}\frac{\lambda^4}{m_\phi^4}\left[8H^2(H^2 - m_\chi^2) + m_f^2(5m_\chi^2 - 2H^2)\right] \; ,$$

- Heavy scalar assumption: $m_\phi \gg m_\chi \geq m_f$, $H = \sqrt{s}/2 \geq m_\chi$.
- $H^2 - m_\chi^2 = Q^2$, so Majorana contribution helicity suppressed in the s-wave limit.

Thermal Virtual corrections at NLO



- The contribution at order $\mathcal{O}(\alpha)$ arises from product of these and LO matrix elements.
- These are t-channel diagrams; will have u and cross tu channels also.
- Diagrams 2, 6, and 7 (antifermion self energy analogue of 6) are leading order in heavy scalar mass.
- Thermal contributions arise from thermal parts of photon and fermion propagator, taken one at a time.

Key symmetry argument

• The integration over the photon momentum k can be simplified:

$$
\begin{aligned}
A &\equiv 2\pi \int \mathrm{d}^4 k \delta(k^2)\ F(k^0, K, \Omega_k)\ , \\
&= 2\pi \int_{-\infty}^{\infty} \mathrm{d}k^0 \int_0^{\infty} K^2 \mathrm{d}K \int \mathrm{d}\Omega_k \left[\delta((k^0)^2 - K^2)\right] F(k^0, K, \Omega_k)\ , \\
&= 2\pi \int \mathrm{d}k^0 \int \mathrm{d}\Omega_k \int K^2 \mathrm{d}K \, \frac{\left[\delta(k^0 - K) + \delta(k^0 + K)\right]}{|2k^0|} F(k^0, K, \Omega_k)\ , \\
&= \pi \int \mathrm{d}\Omega_k \left[\int_0^{\infty} |k^0| \mathrm{d}k^0 F(k^0, k^0, \Omega_k) + \int_{-\infty}^{0} |k^0| \mathrm{d}k^0 F(k^0, -k^0, \Omega_k)\right]\ , \\
&\equiv \pi \int \mathrm{d}\Omega_k \int_0^{\infty} \omega \mathrm{d}\omega \left[F_+(\omega, \omega, \Omega_k) + F_-(-\omega, \omega, \Omega_k)\right]\ ,
\end{aligned}
$$

and the leading thermal contribution arises from

$$
\int_0^{\infty} \omega \mathrm{d}\omega \, n_B(\omega) = \frac{\pi^2 T^2}{6}\ .
$$

Total NLO Thermal Virtual Contribution

- The thermal contribution arises from thermal corrections to the photon and fermion propagators; called thermal photon and thermal fermion corrections.

$$\sigma_{NLO}(s) = \frac{1}{32s(2\pi)^4}\,\frac{P}{Q}\left[\int \omega \mathrm{d}\omega\; n_B\, Int^{tt+uu-tu}_{Total,\gamma T} + \int K_t \mathrm{d}\omega_t\; n_F\, Int^{tt+uu-tu}_{Total,(f+\bar{f})T}\right]$$

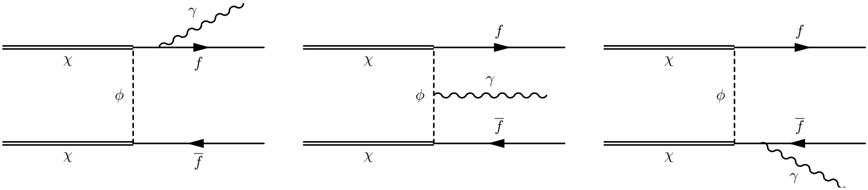
Writing $\quad \sigma v = a + b v^2 + \cdots\;,$

which are the s-wave and higher contributions,

$$\sigma^{Virtual}_{NLO}(s\text{-wave}) = \frac{\alpha\lambda^4}{256v}\,\frac{m_f^2\,T^2}{m_\chi^2}\left[-\frac{8}{m_\phi^4} + \frac{32m_\chi^2}{3m_\phi^6} + \frac{128m_\chi^4}{3m_\phi^8}\right] + \mathcal{O}(v^2)\;.$$

- Helicity suppression noticed for Majorana DM at NLO as well.
- These calculations was first done by Beneke et al. (*JHEP 10 (2014) 045*).

Thermal Real Corrections at NLO



- The thermal correction from these diagrams arise from the phase space: both emission and absorption diagrams exist, in contrast to zero temperature case.

$$dP_p = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \left(2\pi\delta(p^2 - m_p^2)\right) \left[\theta(p^0) \pm n(|p^0|)\right] \; .$$

- $n \equiv n_B \, (n_F)$, the Bose (Fermi) distribution function, for bosons and fermions respectively, with the $+\,(-)$ sign corresponding to bosons (fermions)
- We take only one particle (photon/fermion/anti-fermion) to be thermal at a time.

The real DM annihilation cross section

- The real photon cross section is given by 21 contributions:

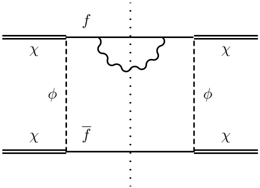
$$\sigma_{NLO}^{Real} = \left[\sum_{i=1}^{6} \sigma_{i,i}^{Real} + (-1)^{S_p} \sum_{i<j}^{6} \sigma_{i,j}^{Real} \right] ,$$

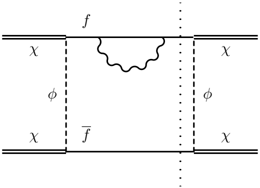
  each being a sum of contributions due to thermal photon, fermion, and anti-fermion:

$$\sigma_{i,j}^{Real} = \sigma_{i,j}^{\gamma,Real} + \sigma_{i,j}^{f,Real} + \sigma_{i,j}^{\bar{f},Real} .$$

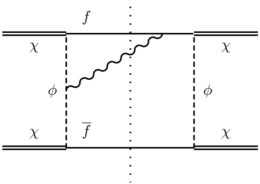
- Example: Square of the first diagram gives $|\mathcal{M}_1|^2 \sim \sigma_{1,1}$ while the product of the first and second gives $\mathcal{M}_1\mathcal{M}_2^\dagger + \mathcal{M}_2\mathcal{M}_1^\dagger \sim \sigma_{1,2}$.
- Before we calculate the finite part of the cross section, we discuss the cancellation of soft and collinear divergences.

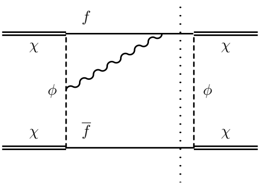
The correspondence between real and virtual corrections





Example 1 : $\sigma^{Real}_{1,1}$ *(Left)* $\sigma^{Virtual}_{1,1}$ *(Right)*





Example 2 : $\sigma^{Real}_{1,2}$ *(Left)* $\sigma^{Virtual}_{1,2}$ *(Right)*

- IR divergences occur only when the photon is taken to be thermal
- There is no IR divergence when fermion/anti-fermion are thermal

Cancellation of IR Divergences

- Recall IR divergences cancels between K and $\widetilde{K}$

i,j	**Real** $Int_{i,j}^{\widetilde{K}}$ from $Int_{i,j}^{\gamma}$	**Virtual** $Int_{i,j}^{K_{div}}$	$Int_{i,j}^{\widetilde{K}_{fin}} = Int_{i,j}^{\widetilde{K}} + Int_{i,j}^{K_{div}}$
$1,1$	$m_\chi^2\left(\frac{32m_\chi^2-8m_f^2}{\omega m_\phi^4}\right)-\omega\left(\frac{32m_\chi^2-8m_f^2}{m_\phi^4}\right)$	$m_\chi^2\left(\frac{32m_\chi^2-8m_f^2}{\omega m_\phi^4}\right)$	$-\omega\left(\frac{32m_\chi^2-8m_f^2}{m_\phi^4}\right)$
$2,2$	$-\omega m_\chi^4\left(\frac{128m_\chi^2+96m_f^2}{m_\phi^8}\right)$	–	$-\omega m_\chi^4\left(\frac{128m_\chi^2+96m_f^2}{m_\phi^8}\right)$
$3,3$	$-m_\chi^2\left(\frac{32m_\chi^2-8m_f^2}{\omega m_\phi^4}\right)-\omega\left(\frac{32m_\chi^2-8m_f^2}{m_\phi^4}\right)$	$m_\chi^2\left(\frac{32m_\chi^2-8m_f^2}{\omega m_\phi^4}\right)$	$-\omega\left(\frac{32m_\chi^2-8m_f^2}{m_\phi^4}\right)$
$1,2$	0	0	0
$1,3$	$\omega\left(\frac{32m_\chi^2-32m_f^2}{m_\phi^4}\right)$	0	$\omega\left(\frac{32m_\chi^2-32m_f^2}{m_\phi^4}\right)$
$2,3$	$\omega m_\chi^2\left(\frac{256m_\chi^2-32m_f^2}{m_\phi^6}\right)$	0	$\omega m_\chi^2\left(\frac{256m_\chi^2-32m_f^2}{m_\phi^6}\right)$

Table: Cancellation of IR divergences between K and $\tilde{K}$ for t-channel; similar cancellations occur for u and tu channels also.

Cancellation of Collinear divergences

Thermal Photon Collinear Contribution		
i,j	Real	Virtual
1, 1	$-\omega\frac{48m_\chi^2}{3m_\phi^4}L_R$	$\omega\frac{48m_\chi^2}{3m_\phi^4}L_V$
2, 2	0	–
3, 3	$-\omega\frac{48m_\chi^2}{3m_\phi^4}L_R$	$\omega\frac{48m_\chi^2}{3m_\phi^4}L_V$
1, 2	$-\omega\frac{32}{3m_\phi^6}(6m_\chi^4 - 3m_\chi^2 m_f^2)L_R$	$\omega\frac{32}{3m_\phi^6}(6m_\chi^4 - 3m_\chi^2 m_f^2)L_V$
1, 3	$-\frac{16(4m_\chi^4 - 2m_\chi^2 m_f^2)}{\omega m_\phi^4}L_R - \omega\frac{16m_f^2}{m_\phi^4}L_R$	$\frac{16(4m_\chi^4 - 2m_\chi^2 m_f^2)}{\omega m_\phi^4}L_V + \omega\frac{16m_f^2}{m_\phi^4}L_V$
2, 3	$-\omega\frac{32m_\chi^2(2m_\chi^2 - m_f^2)}{m_\phi^6}L_R$	$\omega\frac{32m_\chi^2(2m_\chi^2 - m_f^2)}{m_\phi^6}L_V$

Table: *Collinear divergences in various terms of the real and virtual* thermal photon *contributions for t-channel when* $L_V = L_R$; *similar cancellations occur for u and tu channels also.*

Cancellation of Collinear divergences from thermal photons

• Both virtual (L_V) and real photon corrections (L_R, L'_R) exhibit collinear divergences

$$
\begin{aligned}
L_V &= \log\left[\frac{H-P}{H+P}\right] \approx \log\left[\frac{m_f^2}{4H^2-m_f^2}\right] \ , \\
L_R &= \frac{1}{2}\left(\log\left[\frac{m_f^2}{4H^2-4H\omega-m_f^2}\right] + \log\left[\frac{m_f^2}{4H^2+4H\omega-m_f^2}\right]\right) \ , \\
L'_R &= \frac{1}{2}\left(\log\left[\frac{m_f^2}{4H^2-4H\omega-m_f^2}\right] - \log\left[\frac{m_f^2}{4H^2+4H\omega-m_f^2}\right]\right) \ .
\end{aligned}
$$

Expanding,

$$
L_R \approx \frac{1}{2}\left[2\log\left[\frac{m_f^2}{4H^2-m_f^2}\right] + \mathcal{O}(\omega)^2\right] \equiv L_V + \mathcal{O}(\omega)^2 \ ,
$$

while all L'_R terms cancel amongst themselves.

Cancellation of Collinear divergences from thermal fermions

- Logs appearing in real photon contribution:

$$L_{R_f} = \frac{1}{2}\left(\log\left[\frac{2H(\omega_t - K_t) - m_f^2}{2H(\omega_t + K_t) - m_f^2}\right] + \log\left[\frac{2H(\omega_t - K_t) + m_f^2}{2H(\omega_t + K_t) + m_f^2}\right]\right) \ ,$$

$$L'_{R_f} = \frac{1}{2}\left(\log\left[\frac{2H(\omega_t - K_t) - m_f^2}{2H(\omega_t + K_t) - m_f^2}\right] - \log\left[\frac{2H(\omega_t - K_t) + m_f^2}{2H(\omega_t + K_t) + m_f^2}\right]\right) \ ,$$

- Logs appearing in virtual photon contribution:

$$L_{V_f} = \frac{1}{2}\left(\log\left[\frac{H\omega_t - K_t P' - m_f^2}{H\omega_t + K_t P' - m_f^2}\right] + \log\left[\frac{H\omega_t - K_t P' + m_f^2}{H\omega_t + K_t P' + m_f^2}\right]\right) \ ,$$

$$L'_{V_f} = \frac{1}{2}\left(\log\left[\frac{H\omega_t - K_t P' - m_f^2}{H\omega_t + K_t P' - m_f^2}\right] - \log\left[\frac{H\omega_t - K_t P' + m_f^2}{H\omega_t + K_t P' + m_f^2}\right]\right) \ .$$

Again, expanding,

$$L_{R_f} \approx \frac{1}{2}\left(2\log\left[\frac{\omega_t - K_t}{\omega_t + K_t}\right]\right) \approx \log\left[\frac{m_f^2}{4\omega_t^2 - m_f^2}\right] \approx {\color{red} L_{V_f}} \ .$$

Cancellation of Collinear divergences: thermal fermions

Thermal Fermion Collinear Contribution		
i, j	Real	Virtual
1, 1	$-\omega_t \frac{16 m_\chi^2}{m_\phi^4} L_{R_f}$	$\omega_t \frac{16 m_\chi^2}{m_\phi^4} L_{V_f}$
2, 2	0	–
3, 3	0	0
1, 2	$-\omega_t \frac{32 m_\chi^2 (2m_\chi^2 - m_f^2)}{m_\phi^6} L_{R_f}$	$\omega_t \frac{32 m_\chi^2 (2m_\chi^2 - m_f^2)}{m_\phi^6} L_{V_f}$
1, 3	$\omega_t \frac{8(4m_\chi^2 - 3m_f^2)}{m_\phi^4} L_{R_f}$	$-\omega_t \frac{8(4m_\chi^2 - 3m_f^2)}{m_\phi^4} L_{V_f}$
2, 3	0	0

Table: *collinear divergences in various terms of real and virtual contributions from* thermal fermions *for t-channel*

Cancellation of Collinear divergences: thermal anti-fermions

Thermal Anti-Fermion Collinear Contribution		
i, j	Real	Virtual
$1, 1$	0	0
$2, 2$	0	$-$
$3, 3$	$-\omega_t \frac{16 m_\chi^2}{m_\phi^4} L_{R_f}$	$\omega_t \frac{16 m_\chi^2}{m_\phi^4} L_{V_f}$
$1, 2$	0	0
$1, 3$	$\omega_t \frac{8(4m_\chi^2 - 3m_f^2)}{m_\phi^4} L_{R_f}$	$-\omega_t \frac{8(4m_\chi^2 - 3m_f^2)}{m_\phi^4} L_{V_f}$
$2, 3$	$-\omega_t \frac{32(2m_\chi^2 - m_f^2)}{m_\phi^6} L_{R_f}$	$\omega_t \frac{32(2m_\chi^2 - m_f^2)}{m_\phi^6} L_{V_f}$

Table: *collinear divergences in various terms of real and virtual contributions from* thermal anti-fermions *for t-channel.*

Finite remainder from $\widetilde{G}$ and thermal fermions

i,j	**Real** $Int^{\widetilde{G}}_{i,j}$ from $Int^{\gamma}_{i,j}$	**Real** $Int^{f}_{i,j}$	**Real** $Int^{\bar{f}}_{i,j}$
1, 1	$\omega(\frac{32m_\chi^2}{m_\phi^4})$	$-K_t(\frac{32m_\chi^2-12m_f^2}{m_\phi^4})$	0
2, 2	$\omega m_\chi^4(\frac{256m_\chi^2-96m_f^2}{m_\phi^8})$	0	0
3, 3	$\omega(\frac{32m_\chi^2}{m_\phi^4})$	0	$-K_t(\frac{32m_\chi^2-12m_f^2}{m_\phi^4})$
1, 2	$\omega m_\chi^2(\frac{64m_\chi^2-32m_f^2}{m_\phi^6})$	$K_t m_\chi^2(\frac{64m_\chi^2}{m_\phi^6})$	0
1, 3	$-\omega(\frac{64m_\chi^2-32m_f^2}{m_\phi^4})$	$-K_t(\frac{32m_\chi^2}{m_\phi^4})$	$-K_t(\frac{32m_\chi^2}{m_\phi^4})$
2, 3	$-\omega m_\chi^2(\frac{192m_\chi^2}{m_\phi^6})$	0	$K_t m_\chi^2(\frac{64m_\chi^2}{m_\phi^6})$

Table: *The finite real photon contributions from various diagrams for t-channel.*

The total thermal real photon cross section

From thermal finite part of photons and fermions, upto $\mathcal{O}(T^2, m_f^2)$:

$$\sigma_{NLO}^{Real,{\color{red}Majorana}}(\text{s-wave}) = \frac{\alpha|\lambda|^4}{128 v_{rel}} \frac{T^2}{m_\chi^2} \, {\color{red}m_f^2} \left[\frac{16}{3m_\phi^4} + \frac{160 m_\chi^4}{9 m_\phi^8}\right] ,$$

so the total thermal contribution is

$$\sigma_{NLO}^{Real+Virtual,{\color{red}Majorana}} \, v_{rel} = \frac{\alpha|\lambda|^4}{128} \frac{T^2}{m_\chi^2} \, {\color{red}m_f^2} \left[-\frac{8}{3m_\phi^4} + \frac{32 m_\chi^2}{3 m_\phi^6} + \frac{544 m_\chi^4}{9 m_\phi^8}\right] .$$

- The total contribution at NLO remains helicity suppressed.
- The full results available online as Mathematica notebooks.
- When $m_f \to 0$ limit, $a(T^2) = 0$; however, the coefficient $b \neq 0$; first obtained by Beneke et al. (*JHEP 1410 (2014) 45; JHEP 09 (2016) 031*).
- In particular, when $m_f = 0$, the thermal photon as well as thermal fermion contribution vanishes. This also agrees with Beneke's result.
- At T^4, Beneke obtained $a(T^4) \neq 0$; also not proportional to mf^2, we are calculating this.

Some comments

- In the s-wave limit, the ratio of thermal NLO contribution to the LO cross section is:

$$\frac{\sigma_{NLO}}{\sigma_{LO}} = -\frac{2\alpha\pi}{3}\,\frac{T^2}{m_\chi^2}\;.$$

- DM decouples in freeze-out scenario for $1/20 < T/m_\chi < 1/10$; hence the thermal correction at freeze-out is miniscule!
- In freeze-in scenario DM decouples for $1/10 < T/m_\chi < 1$ where the finite temperature correction can be more significant.
- For $\sigma v = a + bv^2 + cv^4 + \cdots$, we have $a = a_0 + a_T(T^2/m_\chi^2)$, so that

$$\langle \sigma v \rangle = \frac{\int \sigma v e^{-E_1/T} e^{-E_2/T} d^3p_1 d^3p_2}{\int e^{-E_1/T} e^{-E_2/T} d^3p_1 d^3p_2}\;,$$

so that

$$\langle \sigma(T \neq 0) v \rangle_T = a_0 + \frac{3b}{2}\frac{T}{m_\chi} + \left(\frac{15c}{8} + a_T\right)\frac{T^2}{m_\chi^2} + \cdots\;,$$

Note that the coupling λ is unknown!

Outlook

- Very few papers discuss thermal corrections to cross sections of astrophysical interest. The field is growing rapidly, however.
- Many questions about the consistency of thermal field theories. We have explicitly shown the cancellation of soft and collinear divergences at NLO; earlier showed cancellation of soft IR divergences to all orders in perturbation theory.
- The structure of the collinear divergences (epecially from thermal fermions) is unusual and has been observed for the first time.
- The actual numerical contribution to the relic density calculation from the thermal terms is small but their inclusion is a matter of consistency.
- Their impact (or otherwise!) will only be known after direct detection of the DM (with a measurement of λ) or if there are significant temperature dependent effects in the dark matter density distribution in the Universe. At present we do not know this.
- Even so, it is likely that such thermal corrections are relevant more for freeze-in than freeze-out scenarios.

Thank You