The Weak Gravity Conjecture in Anti-de Sitter Space

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Based on:
Work in progress,
arXiv:2503.05862 PL, A. Mininno, G. Shiu
arXiv:2409.02197 PL, G. Shiu

The Weak Gravity Conjecture

WGC: Gravitational interaction is weaker than U(1) gauge interaction

A consistent quantum gravity theory (with U(1) gauge fields) should admit a state (particle) with $\frac{gq}{\kappa m} > z_c$

Motivations

Black hole decay: **extremal** BHs ($T_H = 0$) should be unstable

WGC as an obstruction to global symmetry

WGC in AdS $\longleftrightarrow$ CFT

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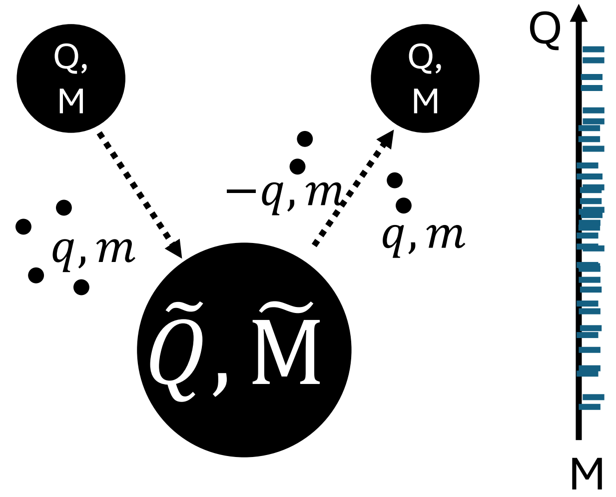
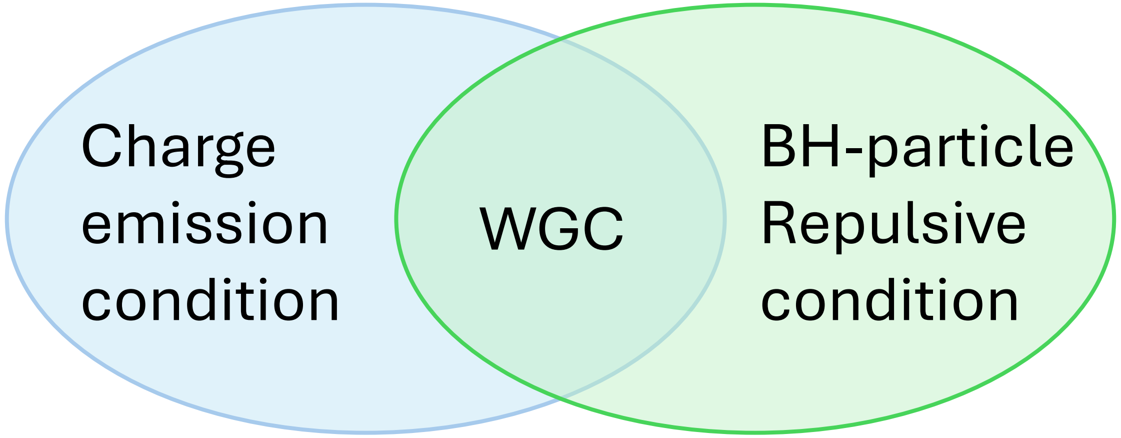


Results: how & what

1. Prescription for computation of the WGC



2. WGC in AdS space ($\Lambda < 0$)

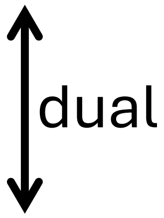
Conjecture (AdS Weak Gravity Conjecture). Given any U(1) gauge field coupled to Einstein–Maxwell gravity, there must exist a particle of charge $\mathfrak{q}$ and mass $\mathfrak{m}$ such that

$$\frac{g_{\mathrm{U(1),D}}\mathfrak{q}}{\mathfrak{m}} \geq \sqrt{\gamma}\frac{1}{M_{\mathrm{Pl,D}}^{\frac{D-2}{2}}}\sqrt{\frac{1+\frac{(D-1)(D-2)}{(D-3)^2}\eta^2}{1+\frac{D-1}{D-3}\eta^2}}\,, \quad \text{with } \eta = \frac{r_h}{\ell_{\mathrm{AdS,D}}}\,, \qquad \gamma = \frac{D-3}{D-2}$$

Results: extensions and implications

3. “Dual” relation between (charge-to-mass ratios of) BH and WGC particles

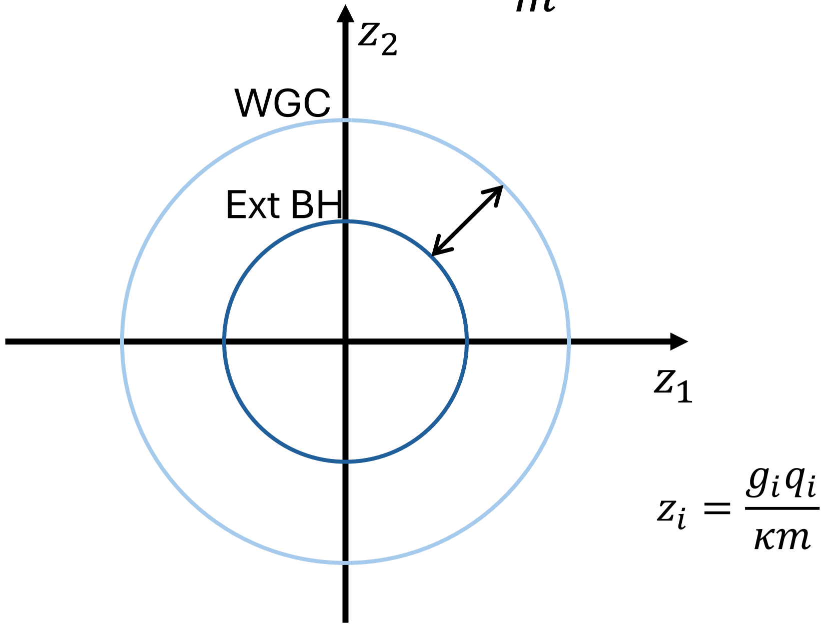
Extremality
(BH, gravity)



WGC
(particle, EFT)

BH: $\frac{Q}{M} < Z_c$

Charged state: $\frac{q}{m} > z_c$

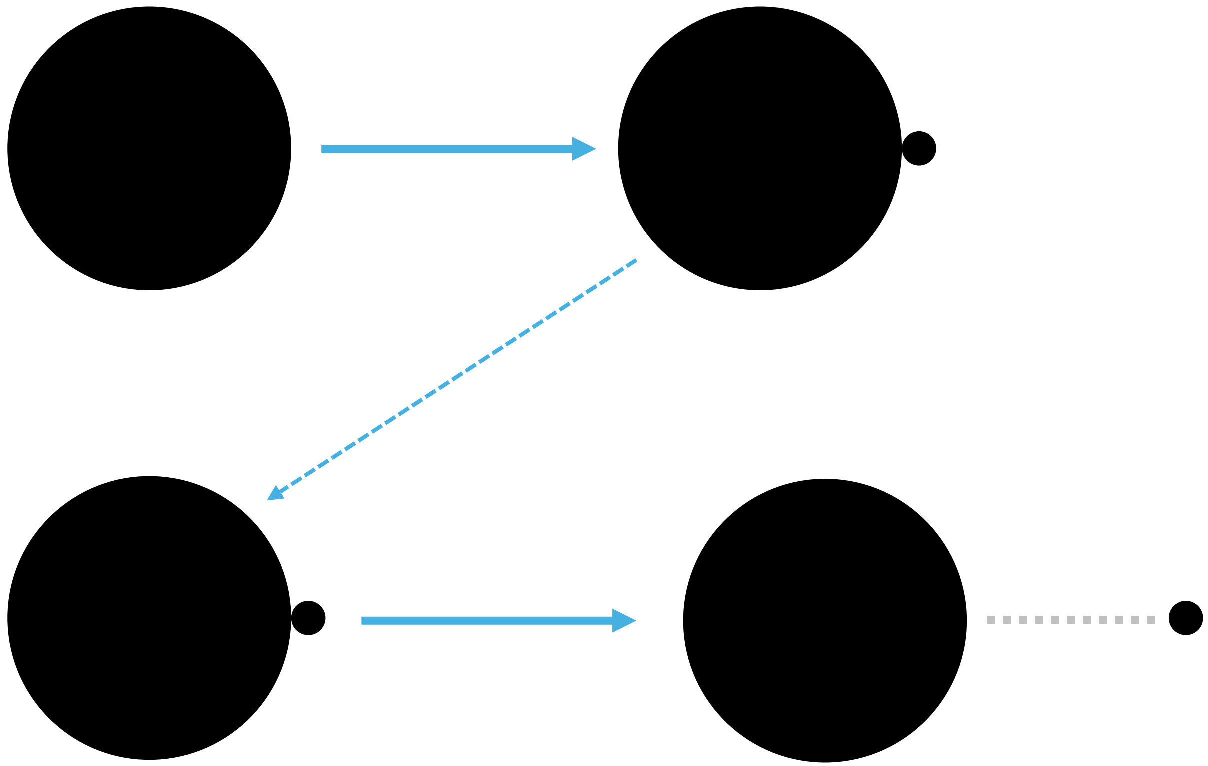


4. Stronger condition (tower WGC) in the presence of Kaluza-Klein U(1) and Λ

WGC = charge radiation ∩ Repulsiveness

Decay of an extremal black hole

Quantum:
(Production)



Schwinger effect
Charged emission

Classical:
(Separation)

Repulsive force
between BH-particle

Schwinger effect of extremal AdS BH

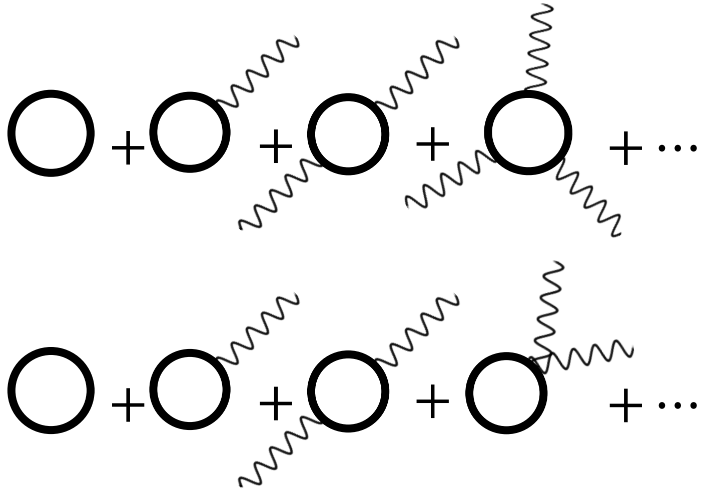
The Schwinger effect: Instability of vacuum with background gauge & gravitation fields

$$\langle 0, A | 0, A \rangle = e^{iS_{\mathrm{eff}}[g,A]} = \int \mathcal{D}\Phi \; \mathrm{e}^{iS[g,A,\Phi]}$$

Charged particle production rate

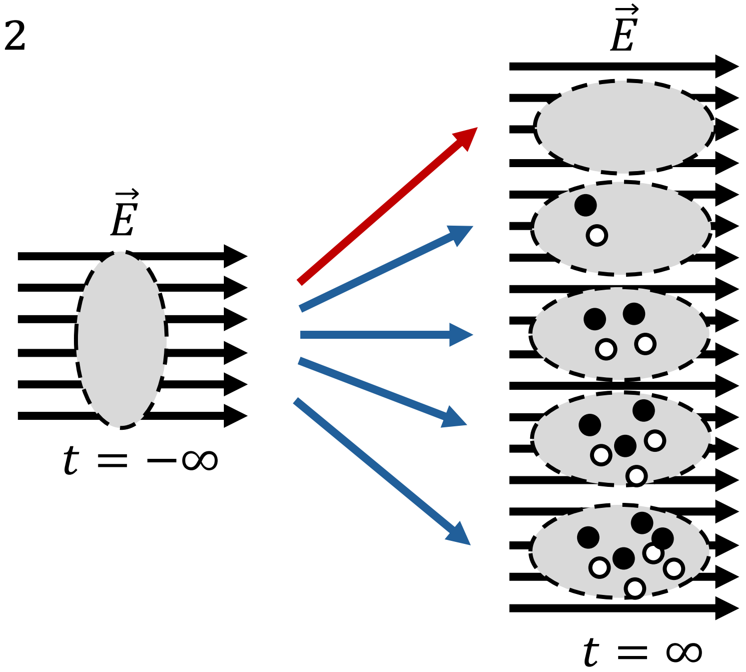
$$\Gamma \sim 2\mathrm{Im}S_{\mathrm{eff}} = 1 - |\langle 0, A | 0, A \rangle|^2$$

$e^{iS_{\mathrm{eff}}[A]} =$

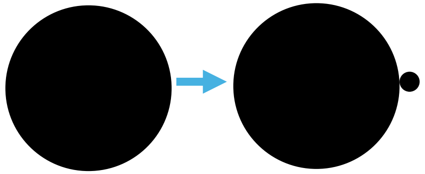


QED

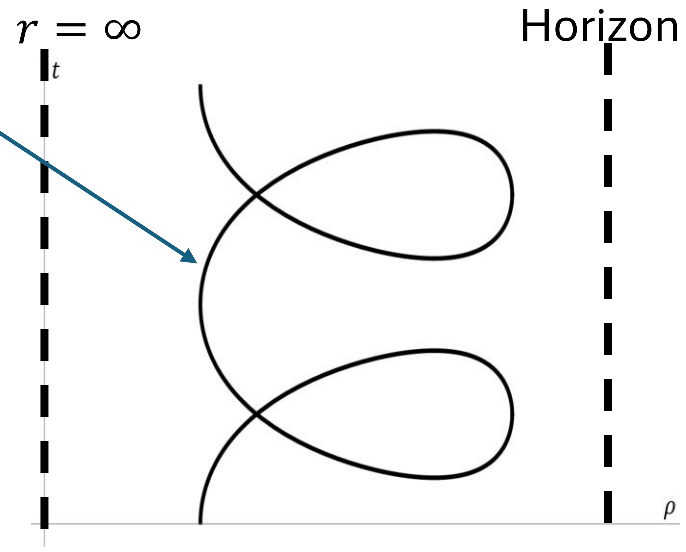
Scalar QED



Structure of the Schwinger production rate



$$\Gamma = 2\mathrm{Im}S_{\mathrm{eff}} = \mathrm{Im}(\mathcal{A}e^{-S_{\mathrm{Inst}}})$$



Where $\mathcal{A} = \frac{2i}{\bar{s}}|\det \mathfrak{H}|^{-\frac{1}{2}}$

$\mathcal{A}$ dictates if the Schwinger effect can take place

$\mathcal{A} \to 0$: shut-off of charged emission (as $z = \frac{q}{m} \to z_{\mathrm{critical}}$)

Final result:

$$\frac{gq}{\kappa_D m} \geq \sqrt{\gamma}\mathcal{R}(\eta), \qquad \eta = \frac{r_h}{\ell_{AdS}}, \gamma = \frac{D-3}{D-2}$$

$$\boxed{\mathcal{R}(\eta) = \sqrt{\frac{1 + \frac{(D-1)(D-2)}{(D-3)^2}\eta^2}{1 + \frac{D-1}{D-3}\eta^2}}}$$

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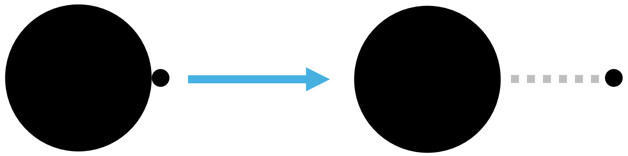
BH-particle repulsive force



Charged particle geodesic

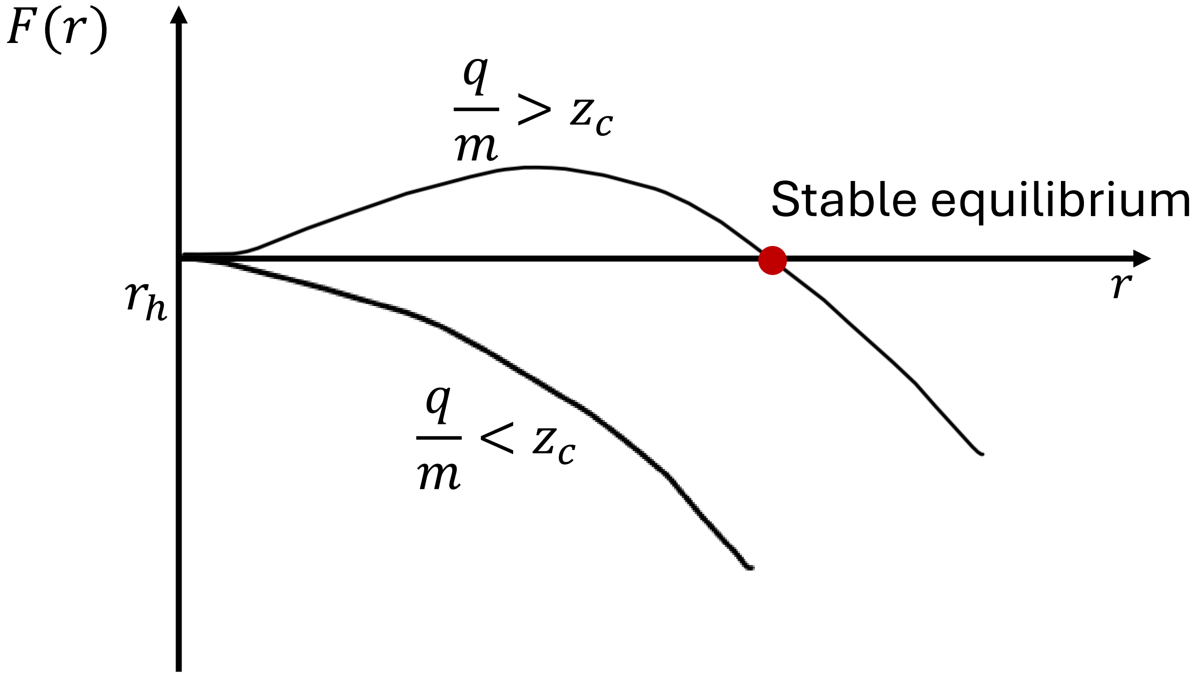
$$\ddot{\xi}^{\mu} + \Gamma^{\mu}_{\rho\sigma}\dot{\xi}^{\rho}\dot{\xi}^{\sigma} - \frac{q}{m} g^{\mu\rho} F_{\rho\nu}\dot{\xi}^{\nu} = 0$$

Particle worldline: $\dot{\xi}^{\mu}$

The force density

$$F(r) \equiv \ddot{r} = -\Gamma^{1}_{00} f^{-1} + \frac{q}{m} f^{\frac{1}{2}} A_t'$$

Require $\exists \delta, F(r) \geq 0$ for $r \in [r_h, r_h + \delta]$



$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{D-2}$$

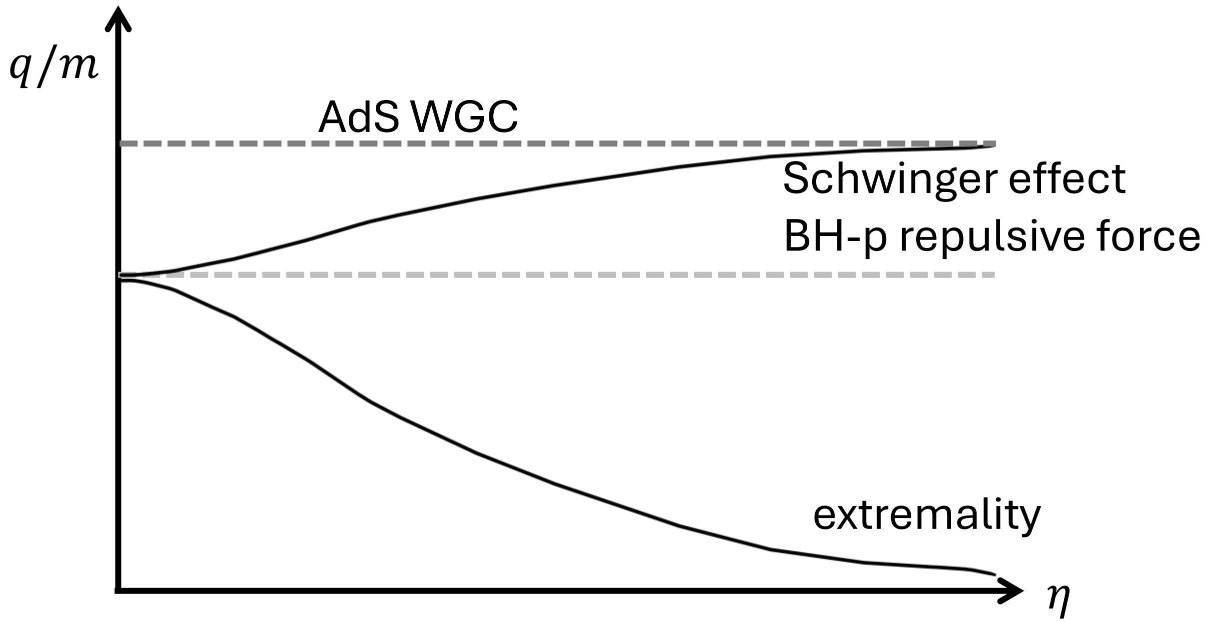
$$f(r) = 1 - \frac{2M}{r^{D-3}} + \frac{Q^2}{r^{2(D-3)}} + \frac{r^2}{\ell^2_{AdS}}$$

$$A = \left(\frac{g^2 Q}{(D-3)\omega_{D-2} r^{D-3}} + C\right) dt$$

$$Q = \frac{\sqrt{\gamma}}{(D-3)\omega_{D-2}} \frac{gQ}{M_P^{\frac{D-2}{2}}}$$

The WGC in AdS space

$$\underbrace{\frac{gq\kappa_D}{m} \geq \sqrt{\gamma}}_{\text{Minkowski}} \underbrace{\sqrt{\frac{1+\frac{(D-1)(D-2)}{(D-3)^2}\eta^2}{1+\frac{D-1}{D-3}\eta^2}}}_{\mathcal{R}(\eta),\ \text{Modification due to } \Lambda<0}, \qquad \eta = \frac{r_h}{\ell_{AdS}}$$



Hint towards a dual relation:
A stronger requirement on the particle charge-to-mass ratio compensates for the lowering of the black hole charge-to-mass ratio in AdS

Multiple U(1)s

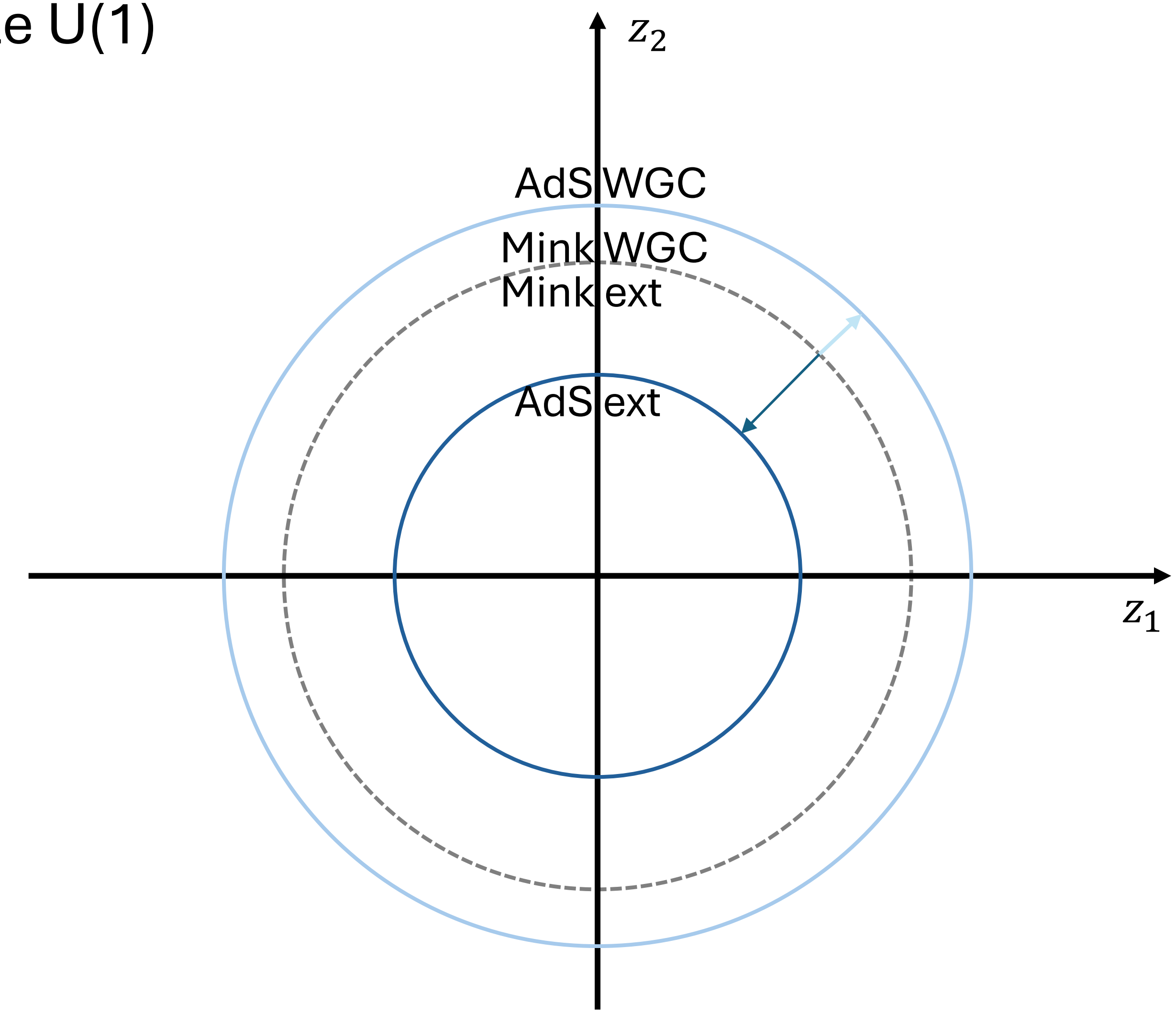
The EFT can contain more than a single U(1)

Einstein+multiple Maxwell+Λ:

$$Q \to \vec{Q} = (Q_1, \dots, Q_2)$$

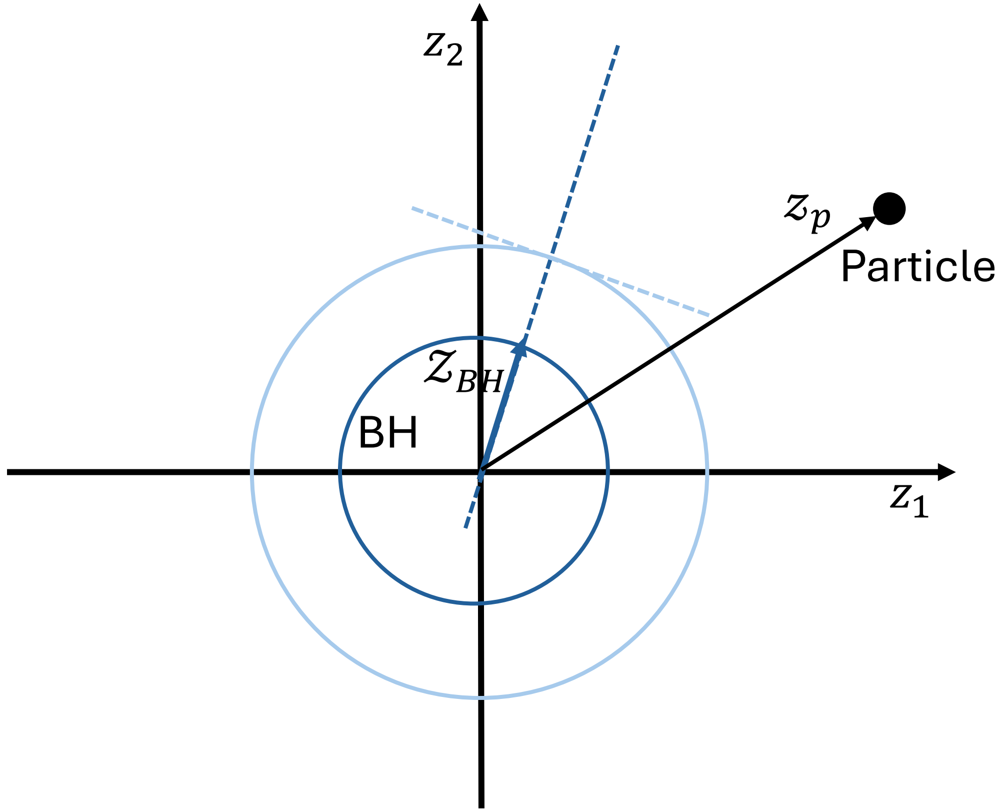
$$\frac{Q}{M} \to \frac{|\vec{Q}|}{M}$$

$$z > \mathcal{R} \to \vec{z} \cdot \hat{Z} > \mathcal{R}$$



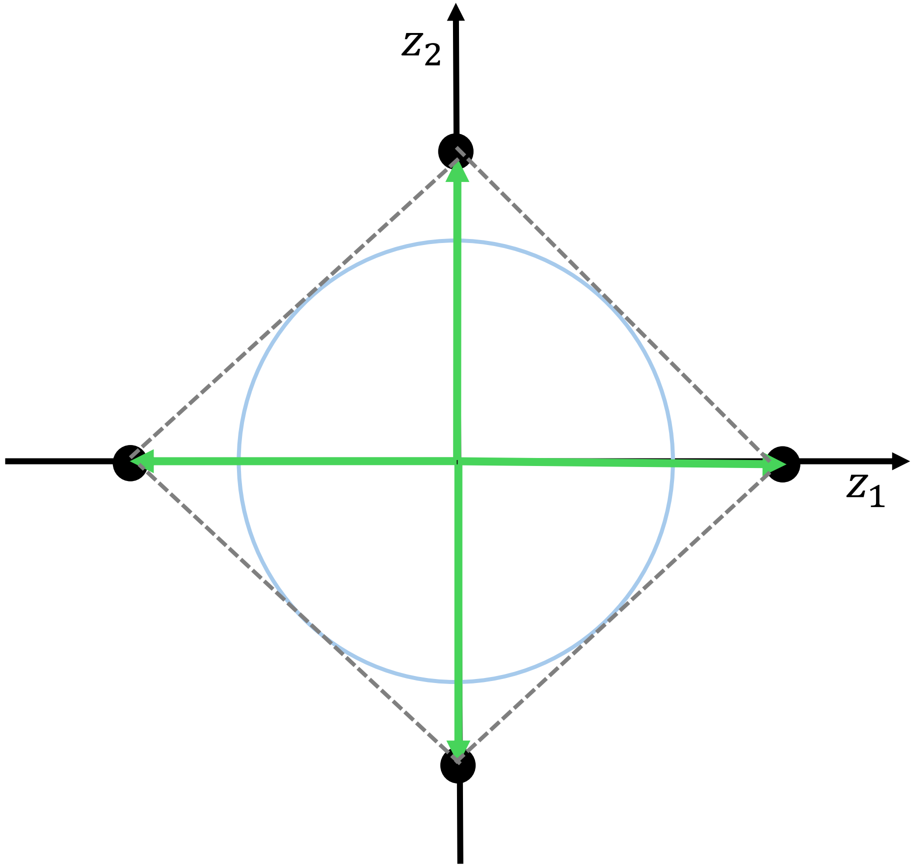
Convex hull condition

$\vec{z} \cdot \hat{Z} \geq \mathcal{R}(\eta)$: particle $\vec{z}$ can lead to decay of extremal BH $\vec{Z}$

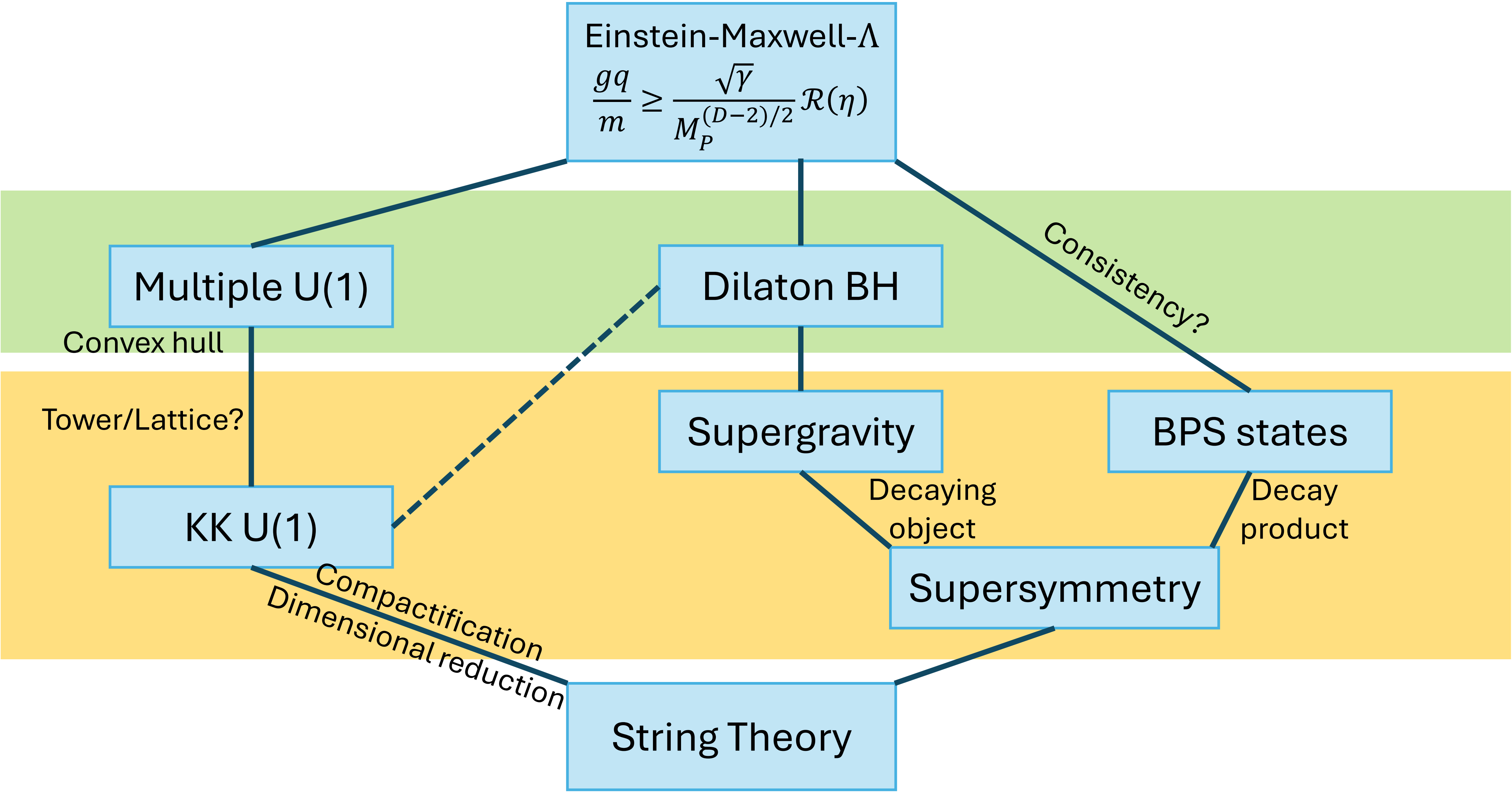


Ext BH $\{\vec{Z}\}$ hypersurface $\xrightarrow[\text{Dual relation}]{\text{Decay condition}}$ particle $\{\vec{z}\}$ hypersurface

Convex hull WGC: The convex hull formed by all charged states should cover the dual WGC hypersurface (all ext BH can decay)



Bridging the EFT and the UV



Kaluza-Klein reduction

U(1) gauge fields might have UV origins. Dimensional reduction generates additional U(1) gauge fields and scalars

$$G_{AB} = \begin{pmatrix} e^{\frac{\lambda}{d-2}} g_{\mu\nu} + e^{-\lambda} r_{S^1}^2 B_\mu B_\nu & e^{-\lambda} r_{S^1} B_\mu \\ e^{-\lambda} r_{S^1} B_\nu & \mathrm{e}^{-\lambda} \end{pmatrix}$$

U(1) gauge field: B_μ, field strength $H = dB$

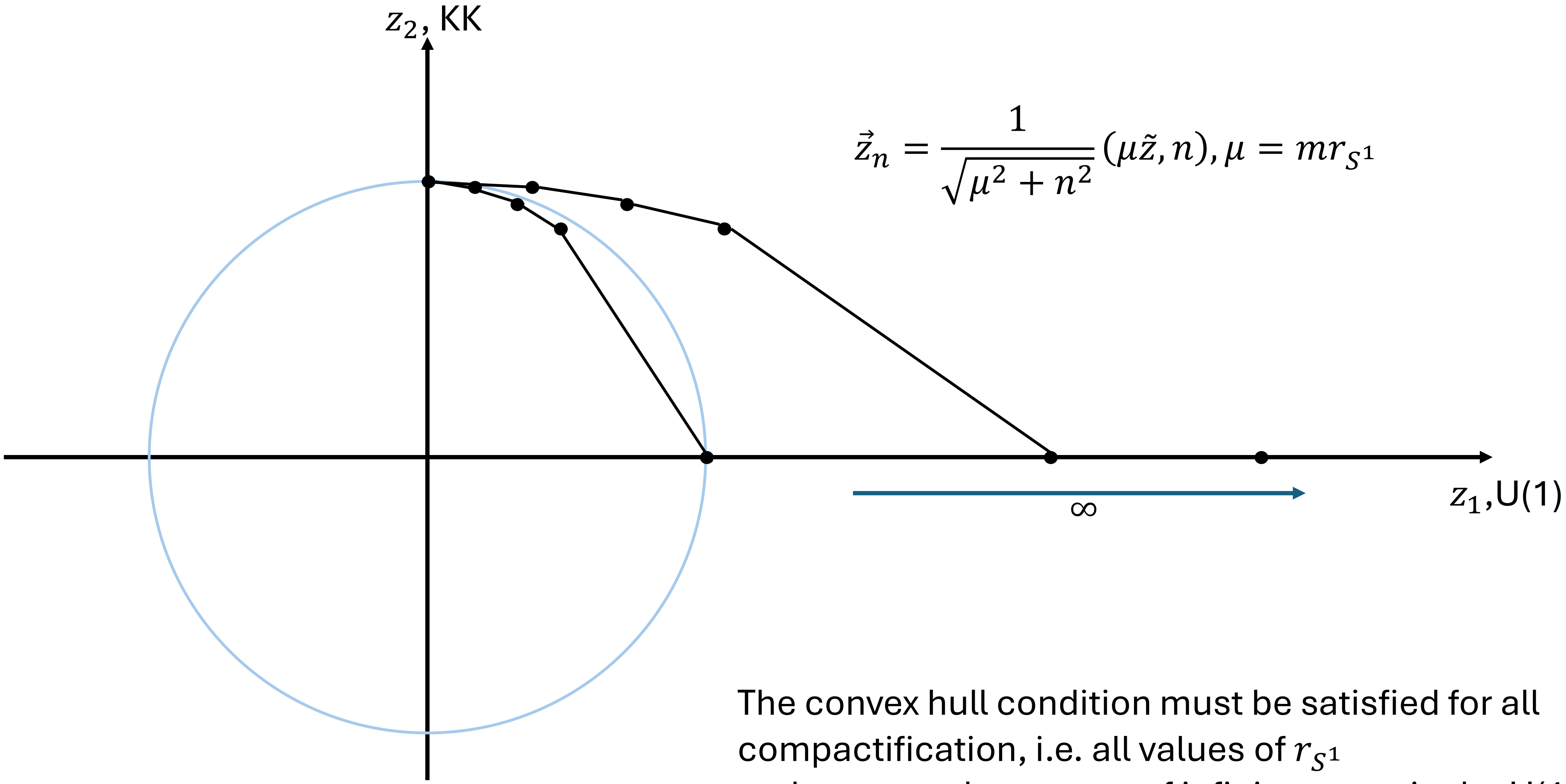
Radion field: λ

Radius of internal space: r_{S^1}

Einstein-Maxwell-dilaton-(Λ):

$$S = S_{EH} + S_\lambda - \frac{1}{4g_{KK}^2} \int d^d x \sqrt{-g}\, e^{-\frac{d-1}{d-2}\lambda} H^2 - \frac{1}{4g_{U(1)}^2} \int d^d x \sqrt{-g}\, e^{-\frac{\lambda}{d-2}} F^2$$

KK reduction and tower WGC

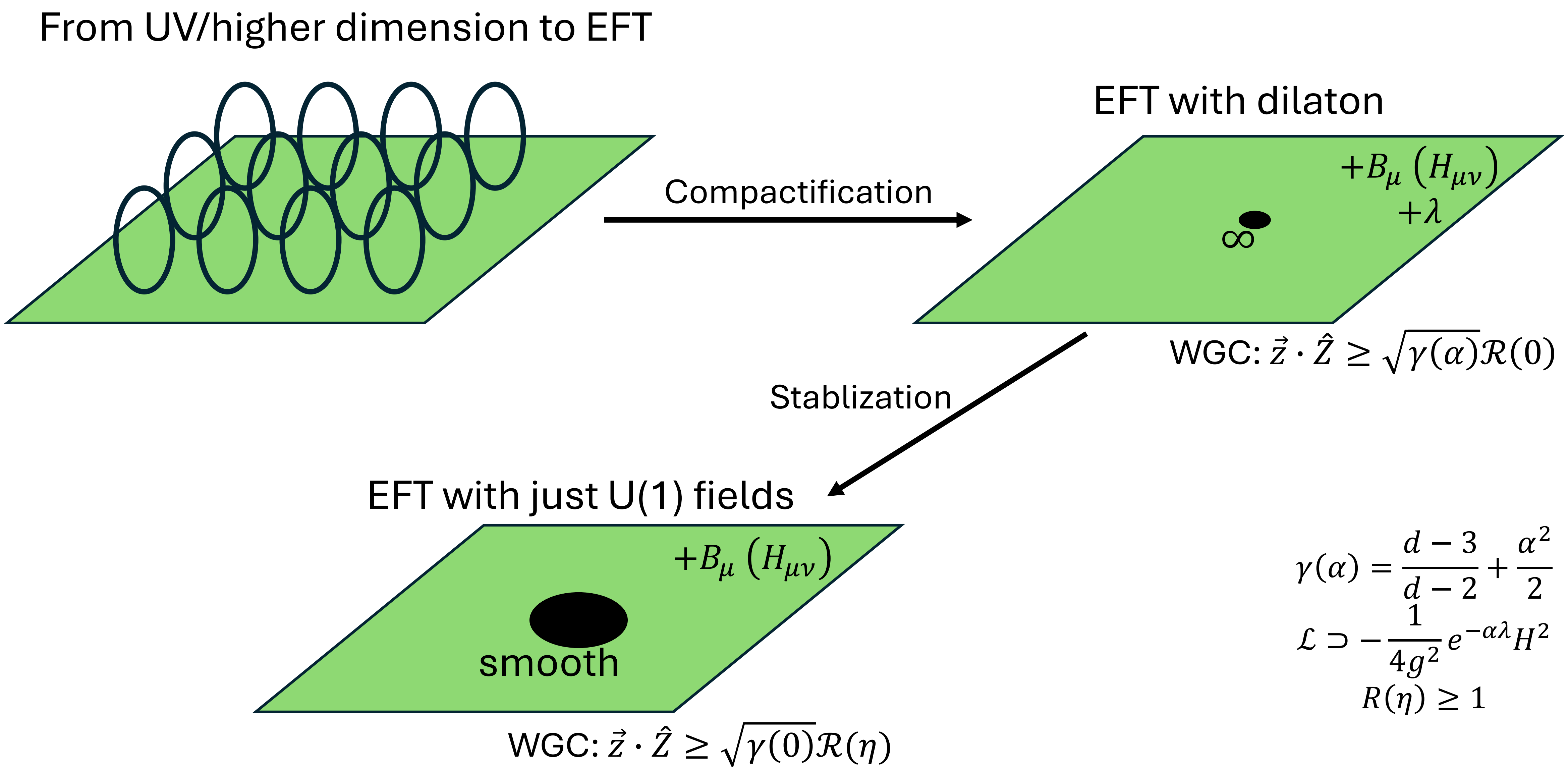


$$\vec{z}_n = \frac{1}{\sqrt{\mu^2 + n^2}}(\mu\tilde{z}, n), \mu = mr_{S^1}$$

The convex hull condition must be satisfied for all compactification, i.e. all values of r_{S^1}

$\Rightarrow$ there must be a tower of infinite states in the U(1) direction

Tower WGC in AdS?

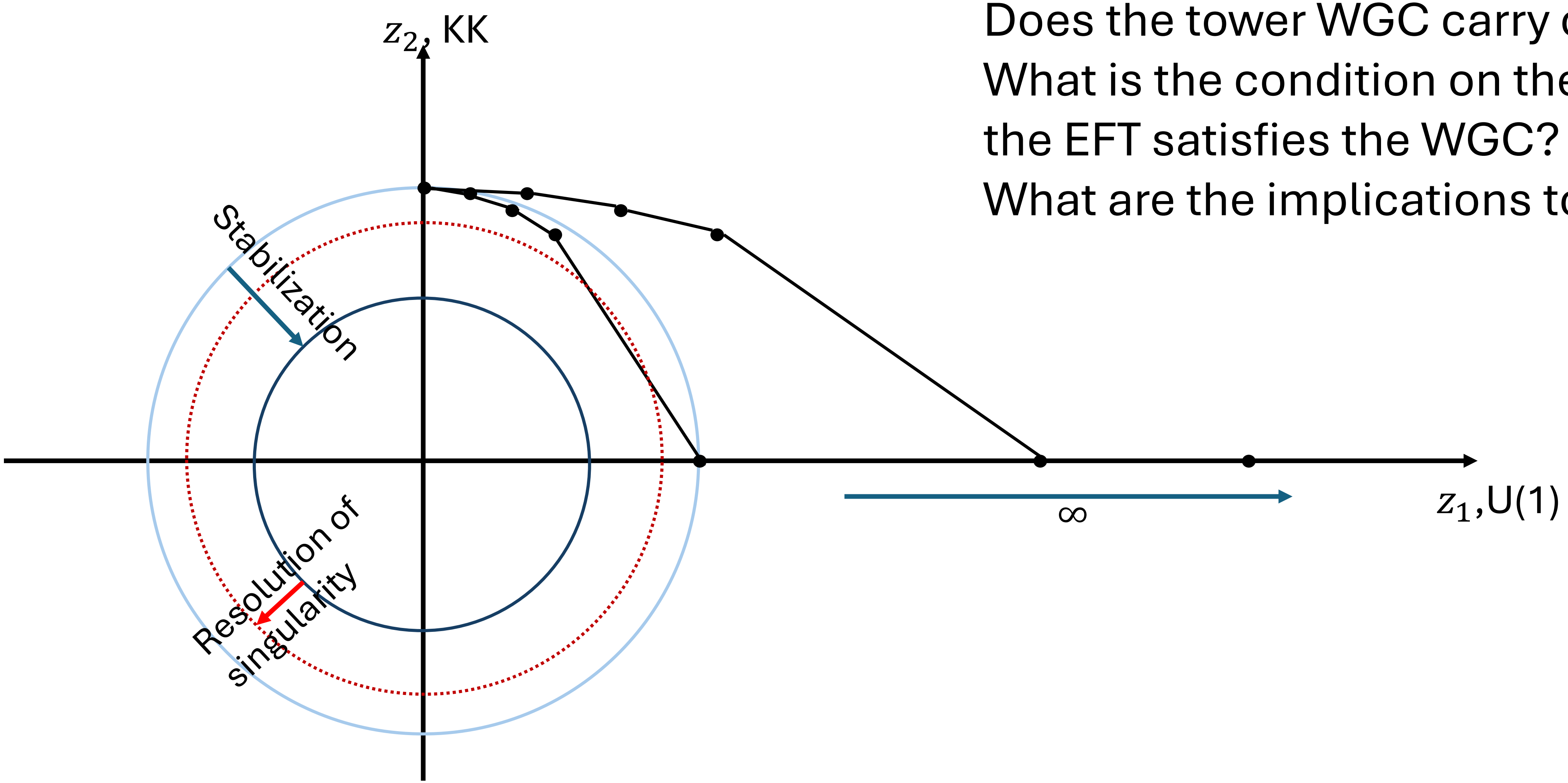


$$\gamma(\alpha) = \frac{d-3}{d-2} + \frac{\alpha^2}{2}$$

$$\mathcal{L} \supset -\frac{1}{4g^2}e^{-\alpha\lambda}H^2$$

$$R(\eta) \geq 1$$

Tower WGC in AdS?



Paper to appear:
Does the tower WGC carry on to AdS space?
What is the condition on the parent theory if the EFT satisfies the WGC?
What are the implications to the dual CFT?

Summary

Extremality
(BH, gravity)

$\Big\updownarrow$ dual

WGC
(particle, EFT)

Conjecture (AdS Weak Gravity Conjecture). Given any U(1) gauge field coupled to Einstein–Maxwell gravity, there must exist a particle of charge $\mathfrak{q}$ and mass $\mathfrak{m}$ such that

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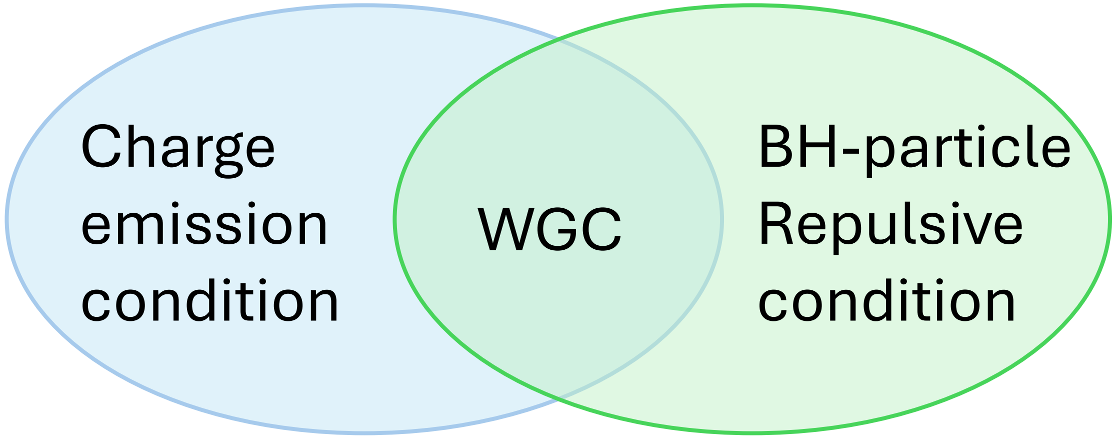


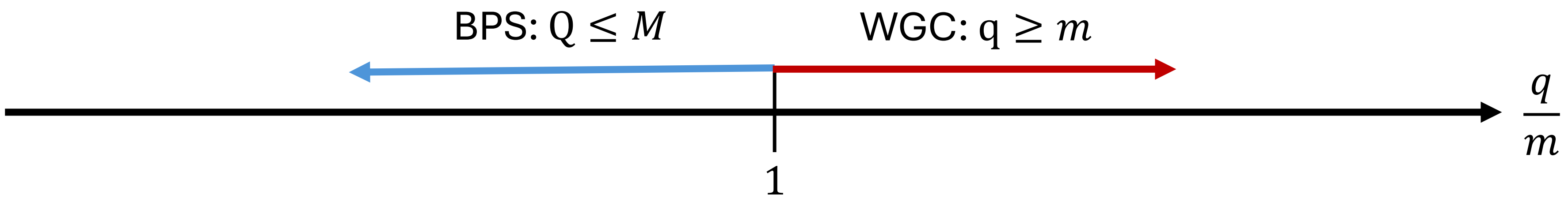
Towards the UV:
Convex hull, tower WGC with $\Lambda < 0$

Thank you!

Extensions: Supersymmetry

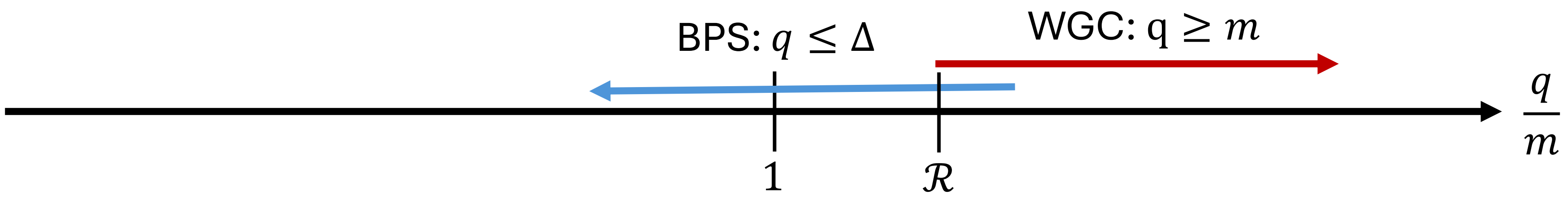
Is the WGC consistent with supersymmetric?

In Minkowski, the BPS bound overlaps the WGC at saturation



Witten 98

In AdS space, $m \to \Delta$, *BPS* in AdS $q \leq \Delta$



Overlap when $D - 1 < \Delta \leq (D-2)(D-1) + \frac{(D-3)^2}{\eta^2}$

$$\boxed{m^2 \ell_{AdS}^2 = \Delta^2 (1 - (D-1)\Delta^{-1})}$$