Entanglement and Beyond: Quantum Information Properties of On-shell Scattering

Based on 2405.08056 and 2410.22414 with Ian Low,
and 2502.17550 and 2503.03098 with Qiaofeng Liu and Ian Low

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Entanglement entropy in particle scattering

- Entanglement is a key phenomenon in quantum theories
- Questions about final states from particle scattering:
  - What kind of theory results in maximized or minimized entanglement?
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Maximal or minimal entanglement leads to unique theories

- Maximal entanglement in QED

  Cervera-Lierta, Latorre, Rojo, Rottoli, 1703.02989; Fedida, Serafini, 2209.01405; ...

- Minimal entanglement in classical BH scattering, strong interactions, flavor physics, and BSM theories

  Beane, Kaplan, Klco, Savage, 1812.03138; Aoude, Chung, Huang, Machado, Tam, 2007.09486; Low, Mehen, 2104.10835; Liu, Low, Mehen, 2210.12085; Carena, Low, Wagner, Xiao, 2307.08112; Thaler, Trifinopoulos, 2410.23343; Núñez, Cervera-Lierta, Latorre, 2504.15353...

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Hints that the EFT minimizes entanglement resulted from scattering

Aoude, Chung, Huang, Machado, Tam, 2007.09486

A universal relation

For AB $\to$ anything, select the elastic final state AB and consider the entanglement between A and B

$$\mathcal{E}^{\mathsf{f}}_{n,\mathsf{T/R}} \doteq \frac{n}{n-1} I_0 \, \sigma_{\mathsf{el}}$$

Low, **ZY**, 2405.08056, 2410.22414

- $\mathcal{E}^{\mathsf{f}}_{n,\mathsf{T/R}}$ is the nth Tsallis or Rényi entropy
- The result is of the leading order in the **plane wave limit**, i.e. for initial state **wave packets** approaching momentum eigenstates
  - Realizing concretely an expansion of $\delta_{\mathsf{p}}/|\vec{k}|$
- σ_{el} is the **total elastic cross section**; **non-perturbative** in coupling strength for any theory with at least 2 degrees of freedom
- I_0 is theory independent; size given by the inverse of the transverse **area** for the initial wave packets in the position space:

$$I_0 \sim 1/A$$

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An area law?

$$\mathcal{E}^{\mathsf{f}}_{n,\mathsf{T/R}} \doteq \frac{n}{n-1} I_0 \, \sigma_{\mathsf{el}}$$

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$\mathcal{E}^{\mathsf{f}}_{\mathsf{T/R}} \propto \sigma_{\mathsf{el}}$: What we know about inclusive cross sections:

- Bound on growth of total cross section: $\log^2 s$

  Froissart, 1961; Martin, 1963

- Actual growth approaches $\log^2 s$ for large s, with $\sigma_{\mathsf{el}}/\sigma_{\mathsf{tot}} \to 1/2$

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- Observed in experiments

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Is there an area law here? Connection to black hole thermodynamics (c.f. Bekenstein–Hawking)?

Beyond entanglement

Entanglement is not enough to guarantee advantage of quantum computing

- Gottesman-Knill theorem: Classical algorithms can efficiently simulate stabilizer states going through quantum circuit consisting of Clifford gates
- Non-stabilizerness, i.e. **magic**, of states act as quantum resource
  - What kind of theory results in maximized or minimized magic?
  - Can constraints related to quantum computing serve as underlying principles?

Magic in particle scattering

Measure: Stabilizer Rényi entropy (SRE)

Leone, Oliviero, Hamma, 2106.12587

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Studies begin on magic resulting from particle scattering, in contexts of nuclear interaction, collider physics, neutrino physics...

Robin, Savage, 2405.10268; White, White, 2406.07321; Chernyshev, Robin, Savage, 2411.04203; Gargalionis, Moynihan, Trifinopoulos, Wallace, White, 2508.14967

Maximal magic for 2-qubit

What is the maximal SRE for a 2-qubit system?

- Previous bound: $\log 5/2 \approx 0.916$. It is known that the 2-qubit system cannot saturate the bound

  Cuffaro, Fuchs, 2412.21083

- Our new bound: $\log 16/7 \approx 0.827$

  Liu, Low, **ZY**, 2502.17550

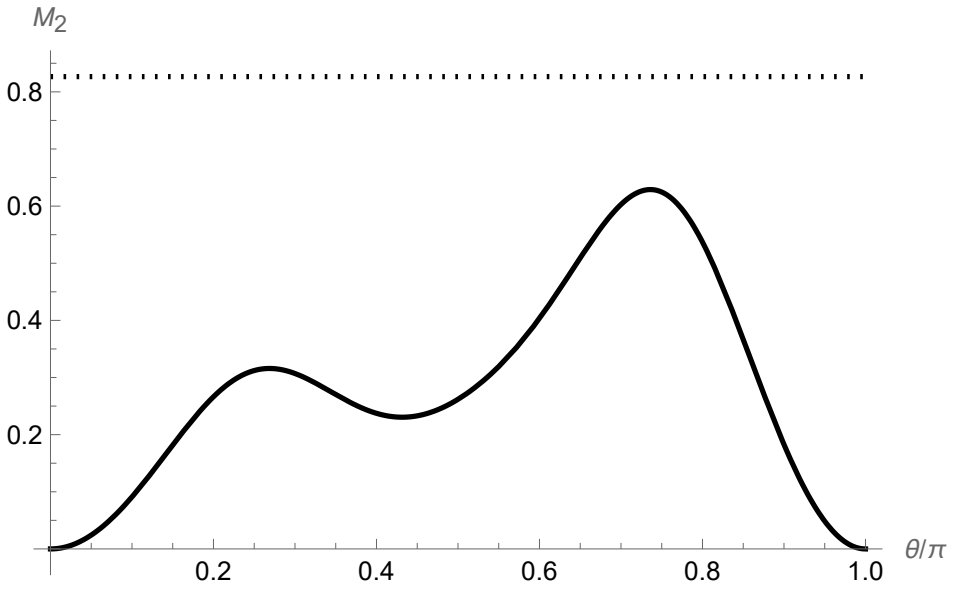
- States that saturate this bound: fiducial states for Weyl-Heisenberg mutually unbiased bases

  Feng, Luo, *Commun.Theor.Phys.* 77 (2025) 1, 015102

  - They contain a certain amount of entanglement

Magic in QED

How much magic can be generated in a $2 \to 2$ QED process, in terms of the spin as 2 qubits?
One example: $e^-\mu^- \to e^-\mu^-$, high energy limit

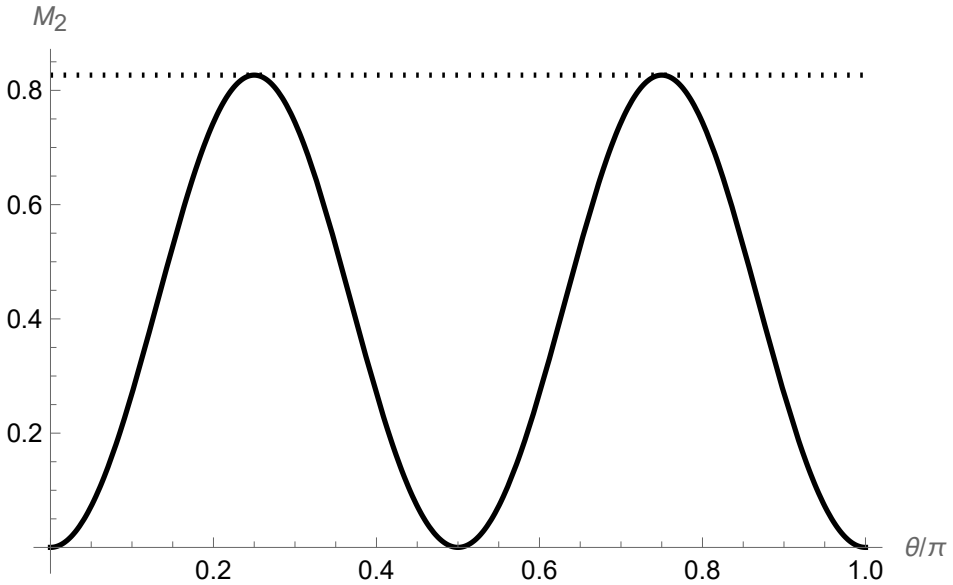


- Initial state: $(|\uparrow\uparrow\rangle + i|\uparrow\downarrow\rangle)/\sqrt{2}$
- $\max M_2 \approx 0.629$

Liu, Low, **ZY**, 2503.03098

Largest magic in QED

$\mu^-\mu^+ \to e^-e^+$, low energy limit



- Initial state: $(|\uparrow\uparrow\rangle + i|\uparrow\downarrow\rangle)/\sqrt{2}$
- The bound of $\log 16/7$ is saturated in the $m_e/m_\mu \to 0$ limit

Liu, Low, **ZY**, 2503.03098

Outlook

Universal increase of entanglement w.r.t. cross sections

- Beyond elastic scattering?
- Infrared divergence: Decoherence effects?

Aoude, Barr, Maltoni, Satrioni, 2504.07030

Magic: Something new to consider

- Is SM special in the magical sense?
- Basis dependence