Review of HVP calculations via e^+e^- measurements

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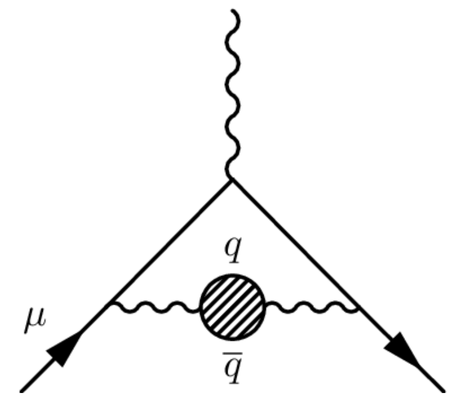
Outline of the talk

1. Introduction
2. How are the HVP calculations performed?
3. What are the issues?
4. New BABAR measurement
5. Summary and outlook

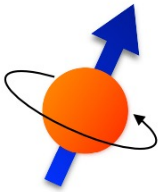


HVP
Hadronic Vacuum Polarization

Introduction

For a charged lepton with charge q_ℓ and mass m_ℓ, its magnetic moment is connected to its spin by a g_ℓ factor:

$$\vec{\mu}_\ell = g_\ell \left(\frac{q_\ell}{2m_\ell} \right) \vec{S}_\ell$$



Dirac predicted $g_\ell = 2$ for electrons in 1928 [Proc. Roy. Soc. Lond. A 118, 351 (1928)]

Quantum corrections of the SM from all three sectors (QED, strong and weak) modify the value of g_ℓ by ~ 0.002

The anomalous magnetic moment a_ℓ was introduced to quantify the deviation from 2:

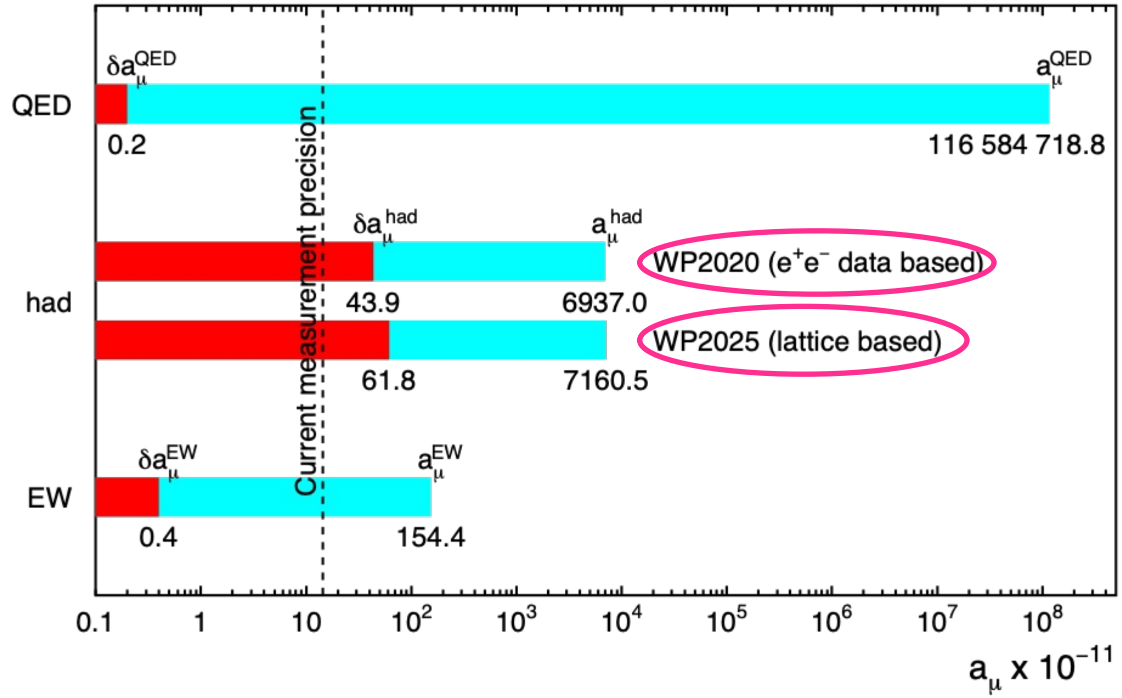
$$a_\ell = \frac{g_\ell - 2}{2} \sim 0.001$$

a_μ one of the most precisely predicted and measured (➔ talk of James Mott) observables in particle physics

a_e also precisely predicted and measured but less sensitive to new physics effects

Different Components of the SM Contributions

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm had} + a_\mu^{\rm EW} \quad \sim 0.001$$



$$a_\mu^{\rm had} = a_\mu^{\rm HVP\ LO} + a_\mu^{\rm HVP\ HO} + a_\mu^{\rm HLbL}$$

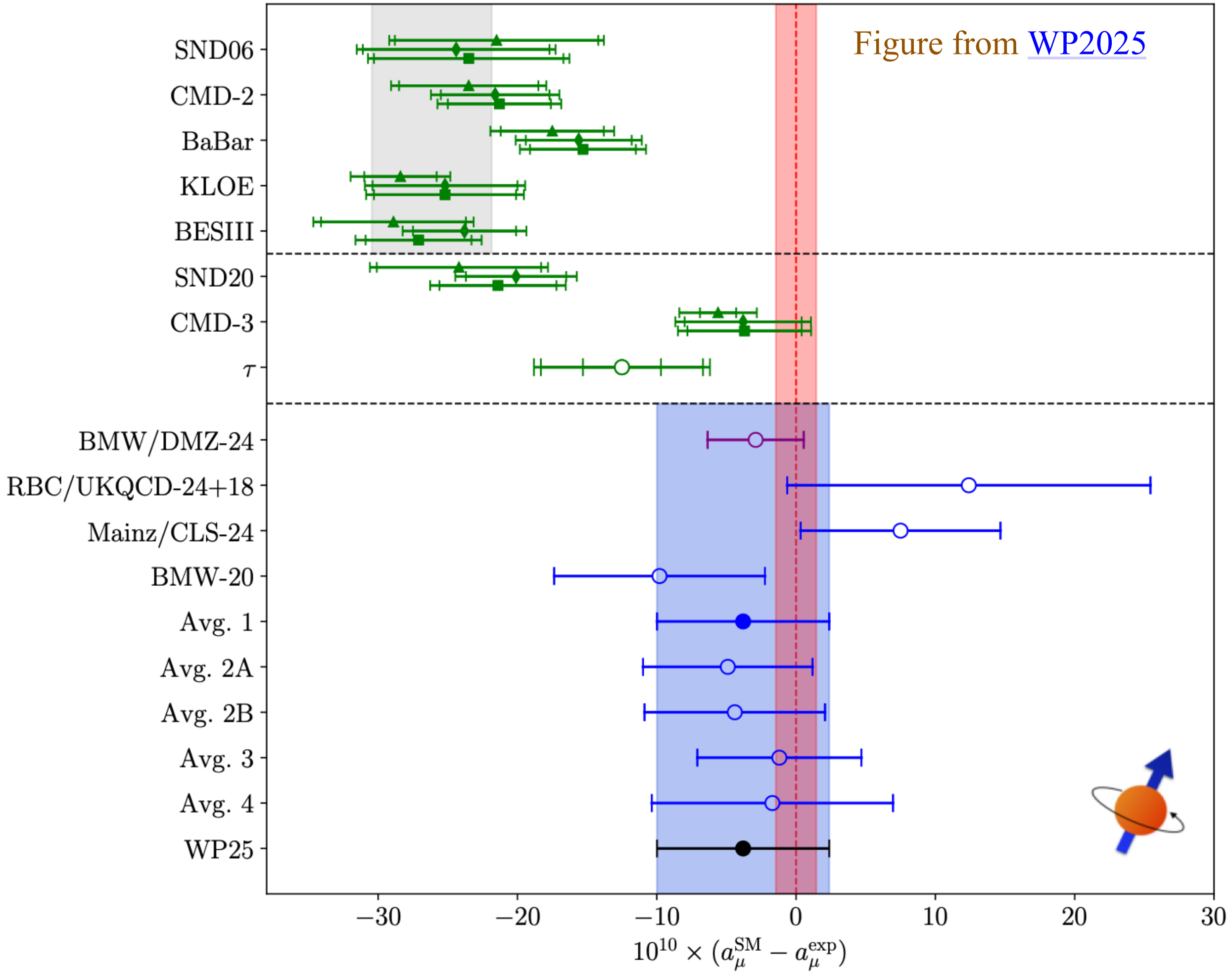
had	HVP LO	HVP HO	HLbL
		$a_\mu[10^{-11}]$ (unc)	
WP2020	6931 (40)	−85.9 (0.7)	92 (18)
WP2025	7132 (61)	−87.2 (1.3)	115.5 (9.9)

WP2020: Phys. Rept. 887 (2020) 1
WP2025: arXiv:2505.21476

WP stands for the white paper of the muon g-2 theory initiative

- ➢QED has by far the largest contribution
- ➢QED and EW both calculable with high precision within the SM
- ➢HVP LO, of non-perturbative nature at low energies, has the largest uncertainty
  - ➔ the focus of studies since decades

The Impact of HVP Calculations on Muon g-2



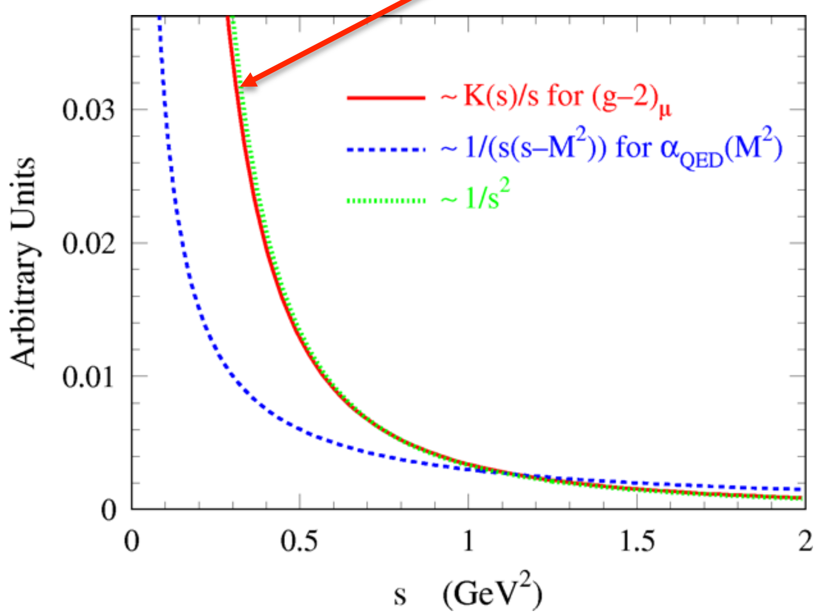
- ➢ The e^+e^- based prediction shows a discrepancy of $>5\sigma$ with the measurement
- ➢ The lattice based prediction, though less precise, agrees with the measurement
- ➢ 90% of the change due to HVP from e^+e^- to lattice, while 10% from HLbL
  - ➔ How is the e^+e^- data-based HVP calculation performed?
  - ➔ What are the issues with the e^+e^- data?

- Grey band: the e^+e^- based WP2020 SM precision
- Blue band: the lattice based WP2025 SM precision
- Red band: the measurement precision
- Green error bars: three slightly different evaluations
- For tau → backup slides 25 & 26

e^+e^- Data-Based HVP Calculation

Based on analyticity and unitarity, the LO HVP contribution can be calculated using the dispersion relation [1] over $e^+e^- \to$ hadrons cross sections

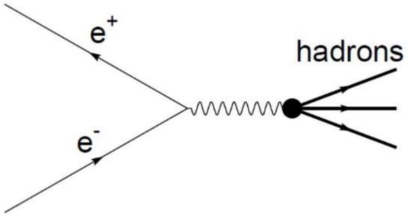
$$a_\mu^{\text{HVP LO}} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds \boxed{\frac{K(s)}{s}} R(s)$$



Bare: $\sigma^{(0)}(s) = \sigma(s)(\alpha/\alpha(s))^2$

$$12\pi\,\mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^0\,[e^+e^- \to \text{hadrons}\,(\gamma_{FSR})]}{\sigma_{pt}} \equiv R(s)$$

Im[ ] ∝ | hadrons |^2



The QED kernel K(s) [2] has such an *s* dependence that low energy data contribute most:

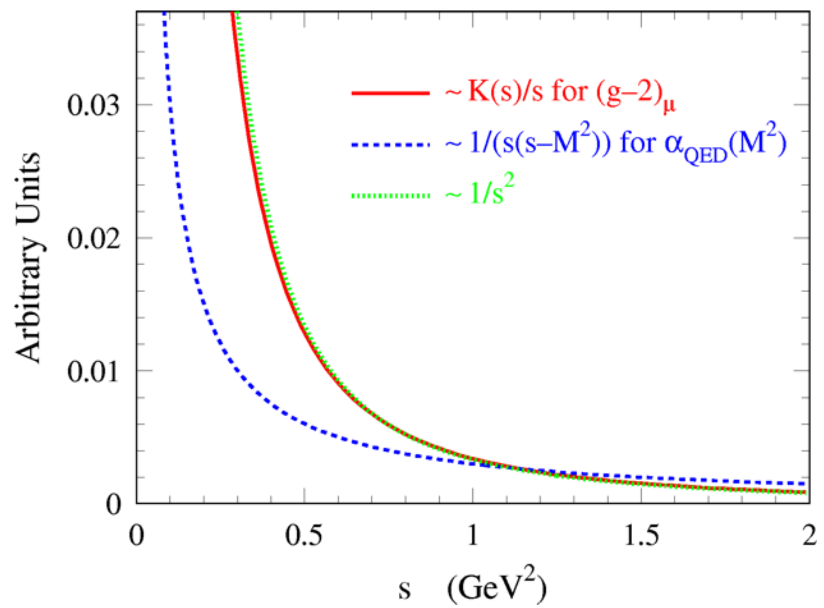
→ The precision is driven by that of $e^+e^- \to$ hadrons

[1] Bouchiat and Michel, 1961
[2] Brodsky, de Rafael, 1968

HVP Contribution to Running QED $\alpha(s)$

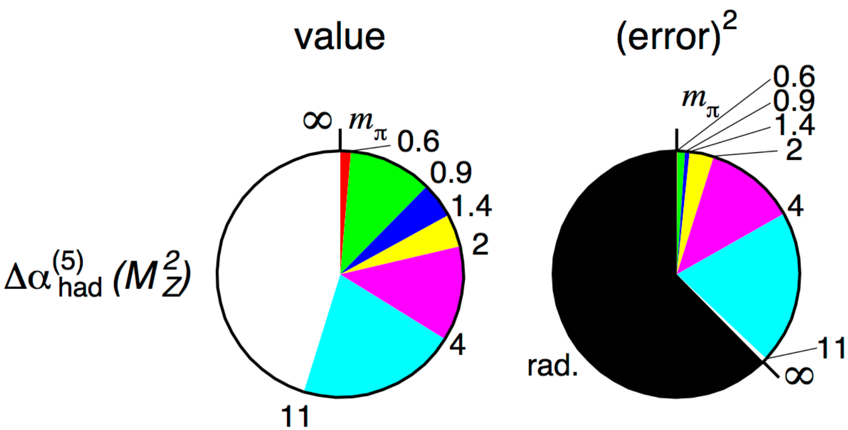
$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha_{\rm lep}(s) - \Delta\alpha_{\rm had}(s)}$$

$$\Delta\alpha_{\rm had}(s) = -\frac{\alpha(0)}{3\pi}\mathrm{Re}\int_{4m_\pi^2}^{\infty} ds' \frac{R(s')}{s'(s'-s-i\epsilon)}$$



R(s): the same input as for a_μ LO HVP except that the higher energy part receives relatively larger weights

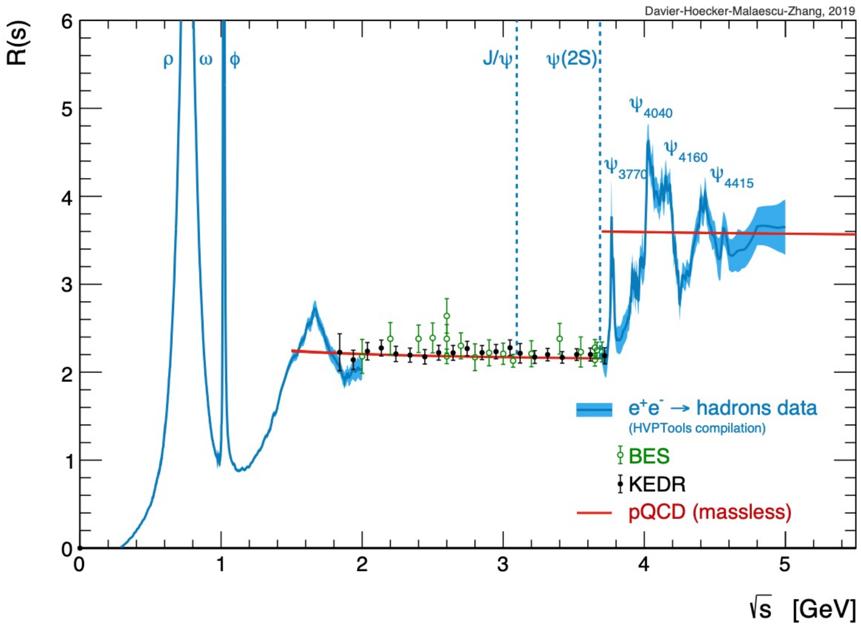
The current precision for $\Delta\alpha_{\rm had}(s)$ is a limiting factor for stringent SM consistency tests.

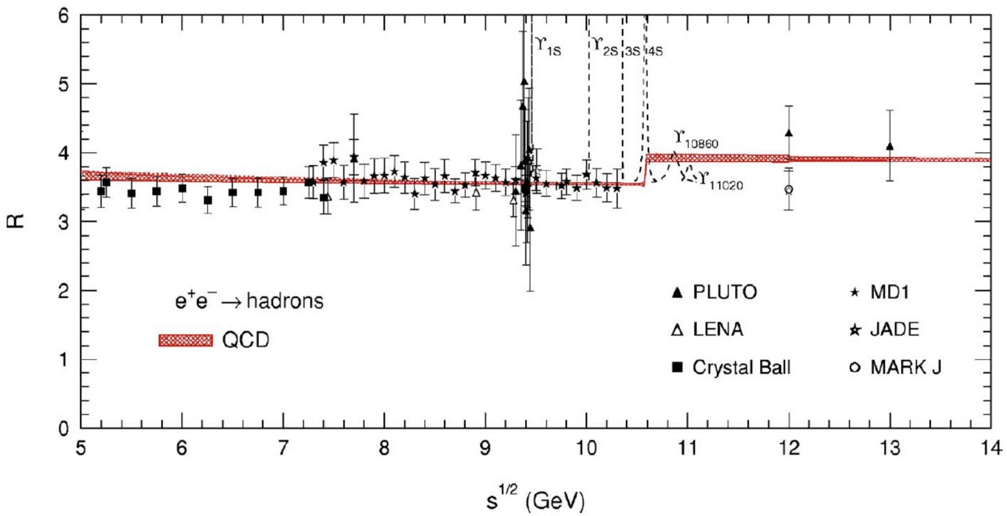


Figures from KNT18 Phys. Rev. D97 (2018) 114025

R(s) Data

Figures from DHMZ19, EPJC80 (2020) 241





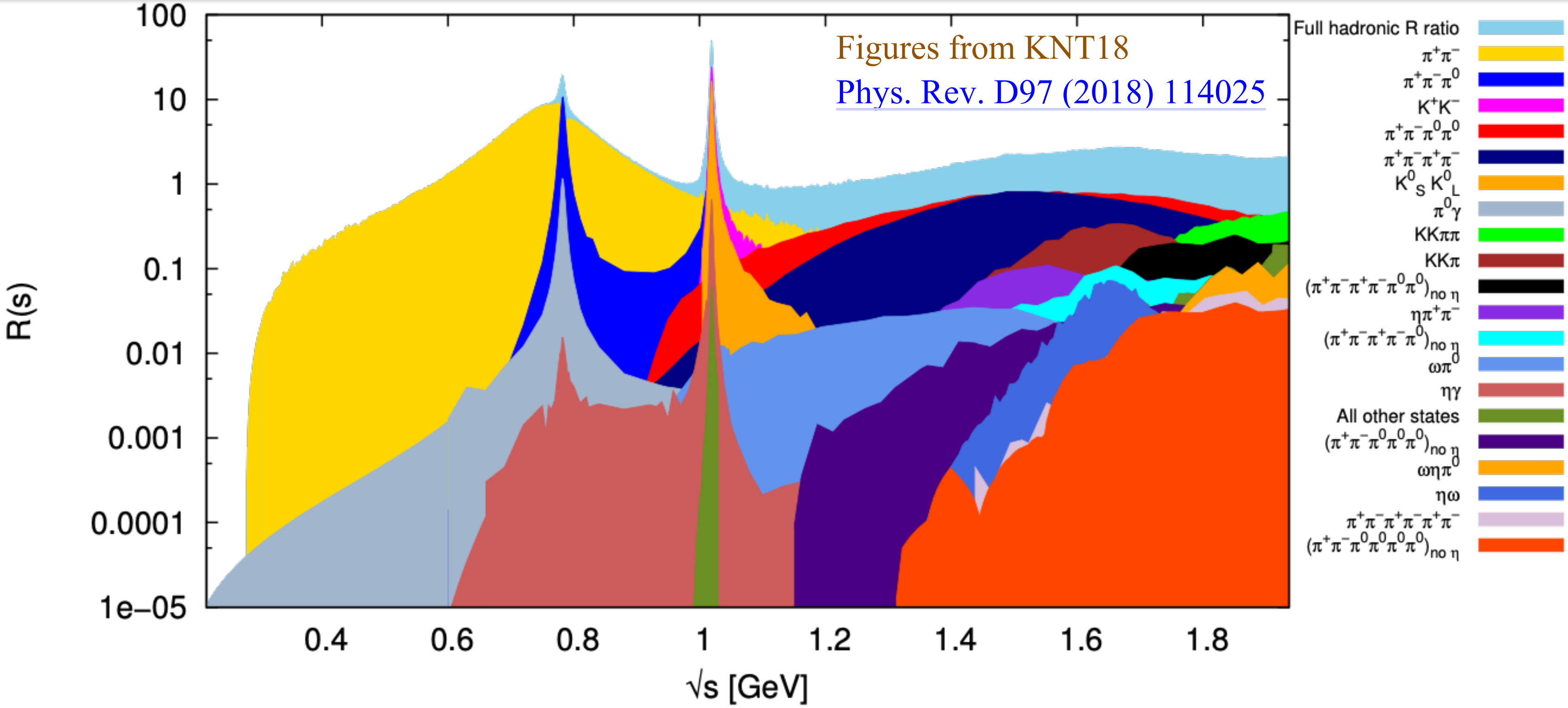
- $[\pi^0\gamma$-1.8 GeV]
  - Over 30 exclusive channels contributes (➔next slide)
  - Unmeasured channels estimated ~0.016% using isospin relations
  - data combination and correlation with one channel and between different channels handled with HVPTools

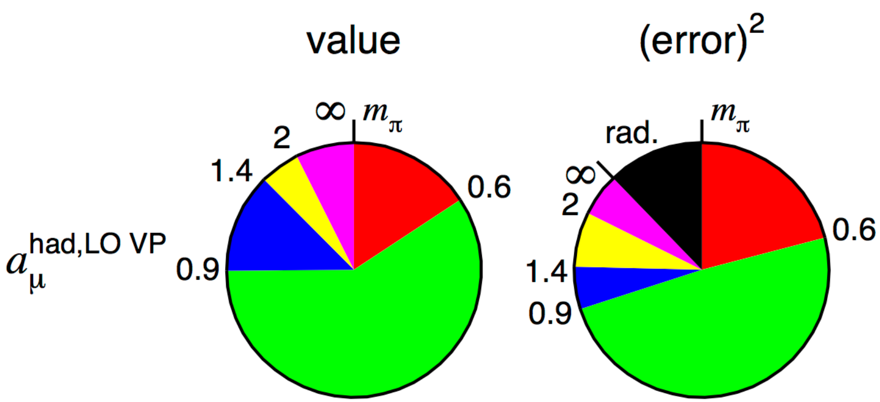
- [1.8-3.7] GeV
  - good agreement between data and pQCD calculation
    ➛ use 4-loop pQCD
  - J/ψ, $\psi(2s)$: Breit-Wigner integral

- [3.7-5] GeV
  Rich resonance structures, use data

- >5 GeV
  use 4-loop pQCD calculation

Exclusive Channels Below ~1.8 GeV





Many exclusive channels contribute

The weight of the kernel function and cross section shape are such that the low energy below ~1.4 GeV has the largest contribution to LO HVP and its error

$e^+e^- \to \pi^+\pi^-$ contributes ~73%

List of Measurements of the $\pi^+\pi^-$ Channel

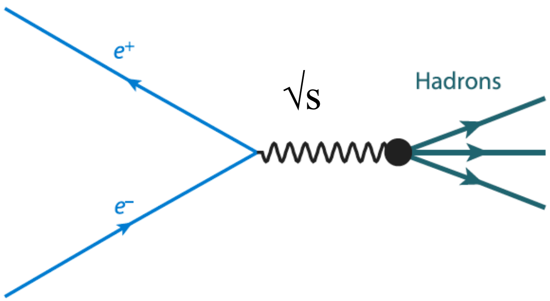
Experiment	N_{data}	Energy range [GeV]	δ(stat.) [%]	δ(syst.) [%]
Beam energy scan				
DM1 (1978)	16	0.483 – 1.096	6.6 – 40	2.2
TOF (1981)	4	0.400 – 0.460	14 – 20	5
OLYA (1979, 1985)	2 + 77	0.400 – 1.397	2.3 – 35	4
CMD (1985)	24	0.360 – 0.820	4.1 – 10.8	2
DM2 (1989)	17	1.350 – 2.215	17.6 – 100	12
CMD2 (2003)	43	0.611 – 0.962	1.8 – 14.1	0.6
SND (2006)	45	0.390 – 0.970	0.5 – 2.1	1.2 – 3.8
CMD2_{low} (2006)	10	0.370 – 0.520	4.5 – 7	0.7
CMD2_{rho} (2006)	29	0.600 – 0.970	0.5 – 4.1	0.8
$\text{CMD2}_{\text{high}}$ (2006)	36	0.980 – 1.380	4.5 – 18.4	1.2 – 4.2
SND (2020)	36	0.525 – 0.883	0.4 – 6.0	0.8 – 1.2
CMD3 (2023)	151	0.32 – 1.2	0.2 – 6.8	0.7 – 2.0
ISR (initial state radiation) photon / radiative return				
KLOE (2008)	60	0.592 – 0.975	0.3 – 1.2	0.9
BABAR (2009)	337	0.3 – 3.0	0.9 – 10.5 ($<$ 1.4 GeV)	0.5 – 1.4 ($<$ 1.4 GeV)
KLOE (2010)	75	0.316 – 0.922	0.6 – 17.6	1.3 – 10
KLOE (2012)	60	0.592 – 0.975	1.8 – 2.6	0.9
BESIII (2015)	60	0.6 – 0.9	2.0 – 4.3	0.9
CLEO (2017)	35	0.3 – 1.0	2 – 20	1.5
BABAR (2025)	311	0.28 – 1.4	down to 0.5	down to 0.3

A Few Comments

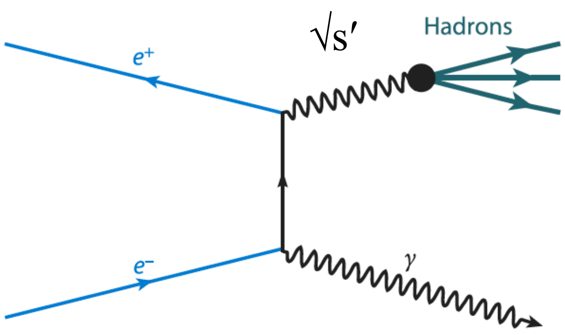
- Two types of measurements: traditional beam energy scan method vs. radiative return method (➔ slide 11)
- Old measurements before 2000 were no longer competitive in precision
- Corrections needed for those measurements not measuring the bare cross sections
- Only few measurements were performed in ratio $\pi\pi/\mu\mu$ (syst error cancellation & avoiding dependence on ee luminosity measurement etc)
- Even fewer measurements were performed in blinded ways
- New measurements have increasing precision, however large discrepancy already observed between BABAR (2009) and KLOE (2008-2012) for WP2020
- Since WP2020, three new measurements: SND (2020), CMD3 (2023) and BABAR (2025)
- New measurement CMD3 larger than all other measurements, including CMD2
- New BABAR (2025) measurement essentially independent of BABAR (2009)

Beam Energy Scan vs Radiative Return

1. Beam energy scan method (varying √s)
   - Advantages:
     - Well defined √s
     - Good energy resolution ~10^{-3}√s
   - Disadvantages:
     - Energy gap/density between two scans
     - Low luminosity at low energies
     - Limited √s range of a given experiment



2. Radiative return method (fixed √s but varying √s′)
   - Advantages:
     - Continuous cross section measurement over a broad energy range down to threshold
     - Large acceptance for hadrons if ISR detected at large angle
     - $\sigma(e^+e^- \rightarrow \text{hadrons})$ may be measured over $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ thus reducing some syst uncertainties
   - Disadvantages:
     - Require high luminosity to compensate higher order in α

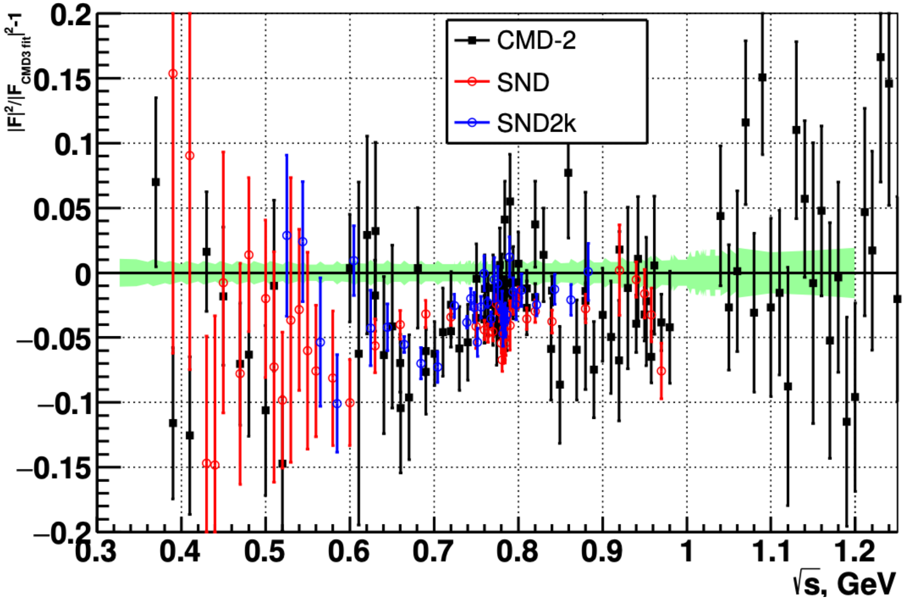


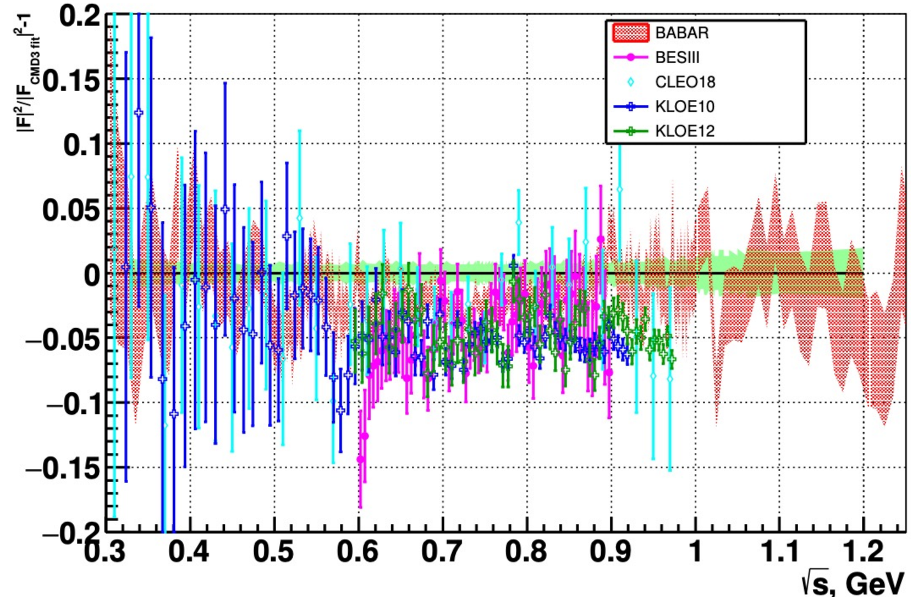
$s' = s\,(1 - x)$
$x = 2E^*_\gamma / \sqrt{s}$

Back

Comparison of CMD3 with Other Measurements

Figures from CMD3, Phys. Rev. D 109 (2024) 112002



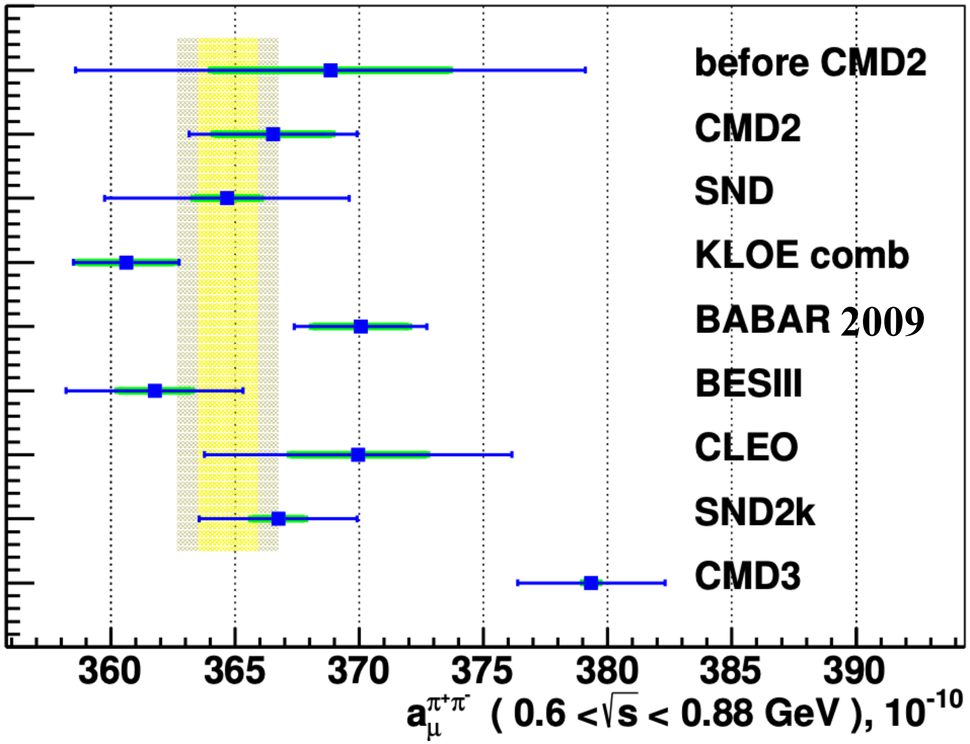


- CMD3 (green band shows its systematic uncertainty) in fair agreement with CMD2 for >1 GeV
- CMD3 tends to be higher than CMD2, SND, SND2k elsewhere
- Energy scan measurements have uneven density over √s

- CMD3 (green band shows its systematic uncertainty) in fair agreement with BABAR and KLOE at low energies
- CMD3 & BABAR also agree >0.9 GeV
- CMD3 higher than all others in the rho peak region
- CMD3-KLOE largest discrepancy
- KLOE limited <1 GeV since √s=1.02 GeV

Comparison in terms of $a_\mu[2\pi$, 0.6-0.88 GeV]

Figures from CMD3, Phys. Rev. D 109 (2024) 112002



Experiment	$a_\mu[2\pi, 10^{-10}]$ (unc)
Before CMD2	368.8 (10.3)
CMD2	366.5 (3.4)
SND	364.7 (4.9)
KLOE	360.6 (2.1)
BABAR 2009	370.1 (2.7)
BESIII	361.8 (3.6)
CLEO	370.0 (6.2)
SND2k	366.7 (3.2)
CMD3	379.3 (3.0)

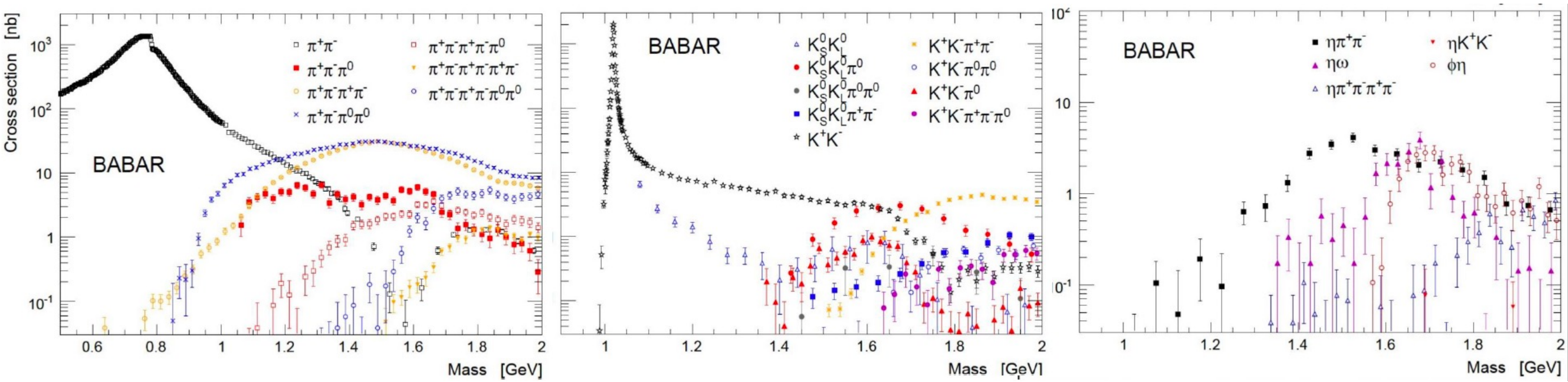
- Vertical bands similar to WP2020: the yellow one corresponds to average without CMD3; the grey one includes BABAR-KLOE discrepancy
- Green error bars show the stat error; blue one the total error

- KLOE-BABAR discrepancy clearly shown
- New CMD3 more discrepant with KLOE
- These discrepancies prevented a new data-driven HVP prediction for WP2025

- Shape differences may partially cancel in integrated a_μ

Other Channels, e.g. Those Measured by BABAR

There are many exclusive channels (~ 40 processes) contributing to HVP
They are measured by BABAR in a single experiment from threshold up to 4.5 GeV
Here are some examples shown below 2 GeV



New BABAR measurements since WP2020 not included in the plots are: $\pi^+\pi^-\pi^0$, $2(\pi^+\pi^-)\pi^0\pi^0+\pi^0/\eta$, $\pi^+\pi^-\pi^0\pi^0\pi^0+\pi^0/\eta$, $K^+K^-\pi^0\pi^0\pi^0$, $K_S^0K^\pm\pi^\mp\pi^0\pi^0$, $K_S^0K^\pm\pi^\mp\pi^+\pi^-$ (see references listed on slide 24)

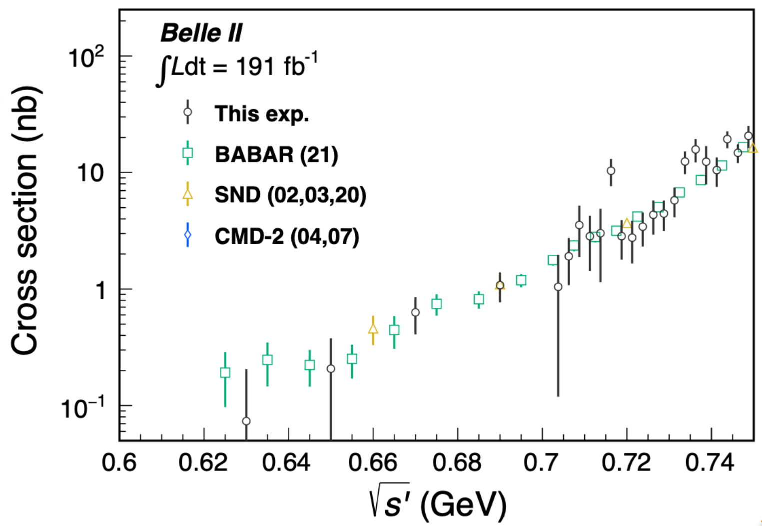
Relative contributions to a_μ from unmeasured channels (estimated based on isospin symmetry)

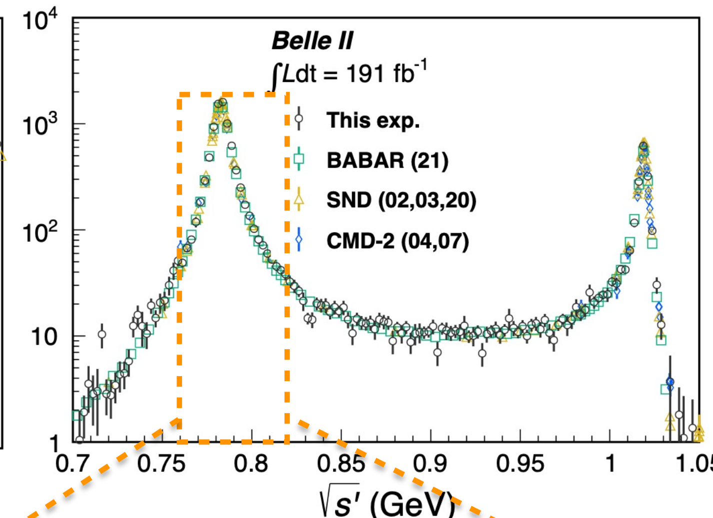
→ (0.87 ± 0.15) % (DEHZ 2003)
→ (0.69 ± 0.07) % (DHMZ 2010)
→ (0.09 ± 0.02) % (DHMZ 2017)
→ (0.016 ± 0.016) % (DHMZ 2019)

(*Nearly complete set of exclusive measurements from BABAR*)

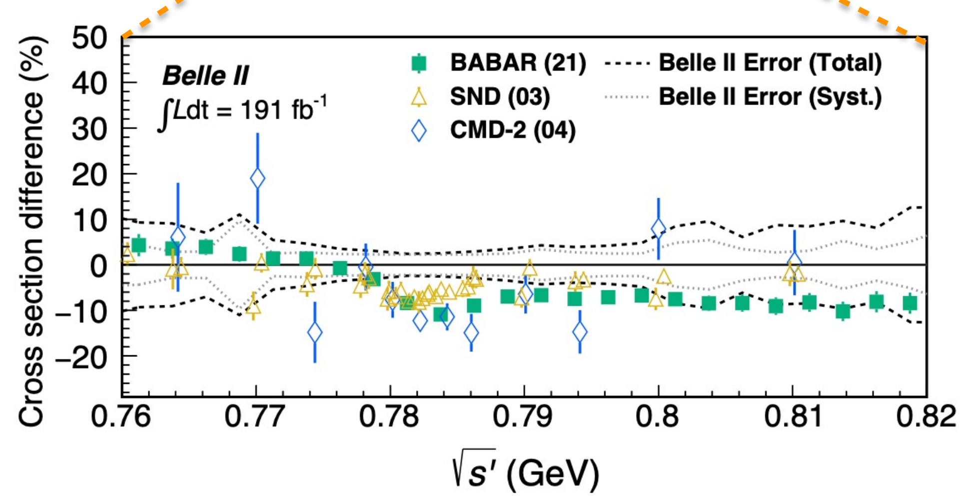
Tension in Other Channels ($\pi\pi\pi^0$)

Figures from Belle II,
Phys. Rev. D 110 (2024) 112005



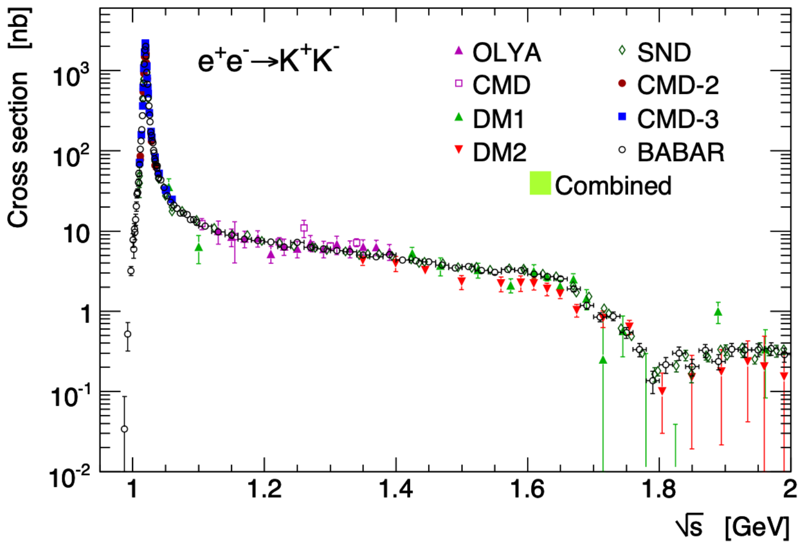


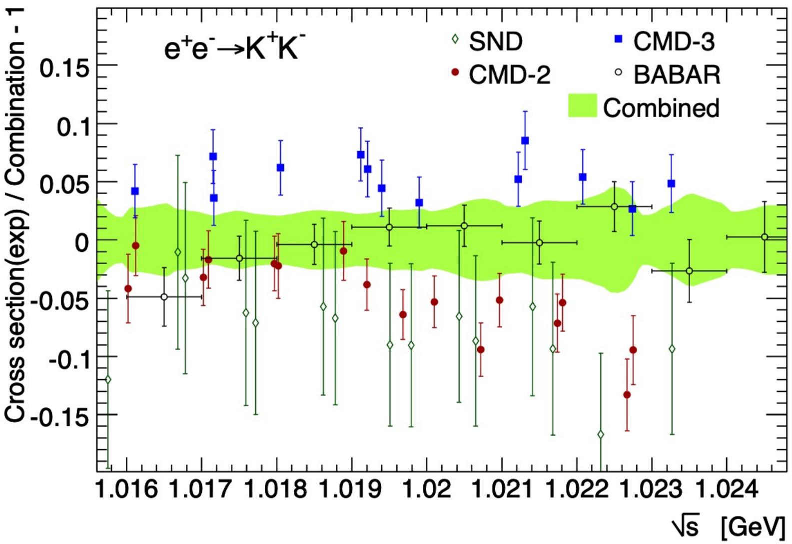
- This channel contributes ~10% to LO HVP, 2nd largest channel
- New measurement from Belle II (0.62-3.5 GeV, 191 fb^{-1}) compared with BABAR (0.62-3.5 GeV, 469 fb^{-1}), SND and CMD2
- Some tension beyond the quoted uncertainties



Tension in Other Channels (KK)

Figures from DHMZ, EPJC80 (2020) 241





Several measurements with different precisions, CMD2 and CMD3 do not agree within the quoted uncertainties!

This channel (3rd largest channel) contributes 3.3% to LO HVP and 1.2% to uncertainty-squared.

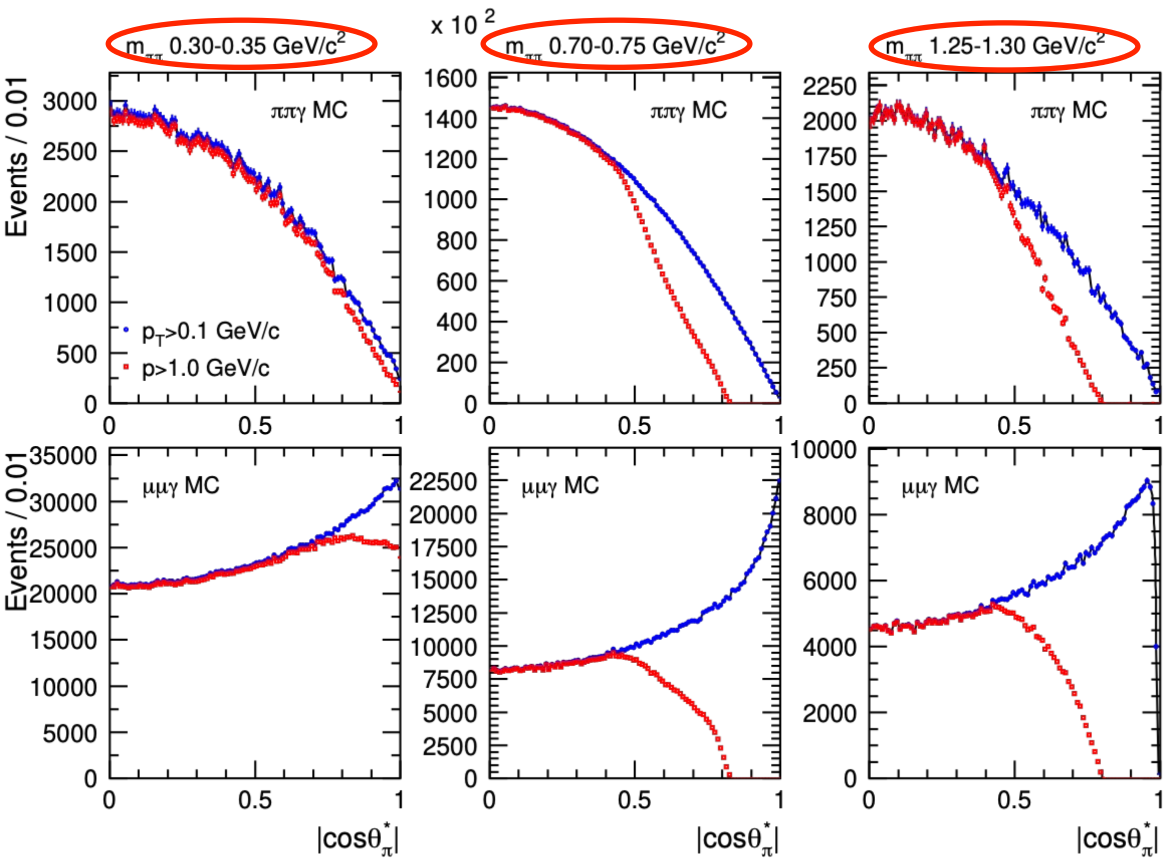
New BABAR 2025 $e^+e^- \to \pi^+\pi^-/\mu^+\mu^-$ Measurements

2025 analysis

- Full data sample (460 fb^{-1})
- Exploit different angular distributions to separate $\pi\pi/\mu\mu$ final states
- Track $p_T > 0.1$ GeV/c
- Signal MC: Phokhara

2009 analysis

- Partial data sample (232 fb^{-1})
- Use particle identification to separate $\pi\pi/\mu\mu$ final states ➔ dominant error source
- Track p > 1 GeV/c
- Signal MC: AfkQED



➔ 2025 analysis improved with ×2 larger data sample lower p(T) selection, and a different $\pi\pi/\mu\mu$ separation method

➔ Different systematic uncertainties

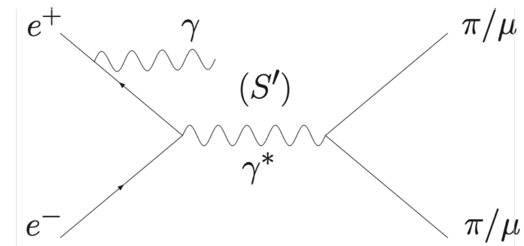
➔ See Léonard Polat's talk for more details

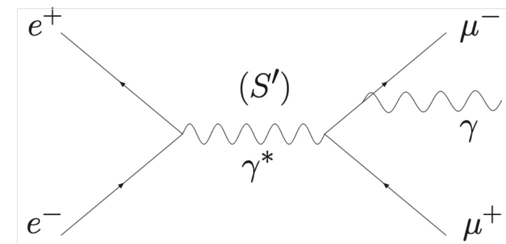
BABAR High Order Radiation Study 2023

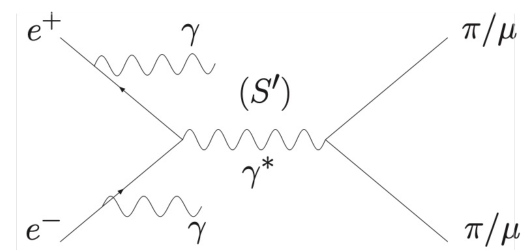
- Using full data sample (460 fb^{-1}), BABAR studied $e^+e^- \to \mu^+\mu^-$ and $\pi^+\pi^-$ processes with up to 3 hard radiation photons at small or large angles in the initial or final states in 2023 (Phys. Rev. D 108 (2023) L111103)

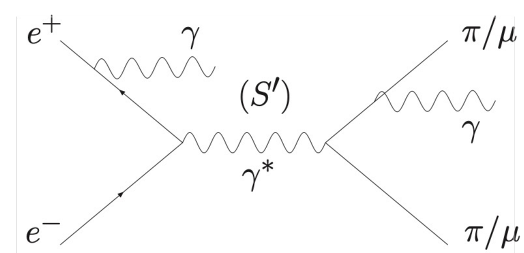
- Two key observations:
  - NNLO radiation (~3.5%) observed in data
  - NLO Phokhara predicts ~25% too high rate for hard NLO small angle ISR photons

- Consequences (DHLMZ, Eur. Phys. J. C 84 (2024) 721):
  - BABAR measurement unaffected due to loose event selection and data-driven efficiency corrections
  - Other analyses relying heavily on Phokhara may be affected as discussed in DHLMZ; they are under investigation by BESIII and KLOE







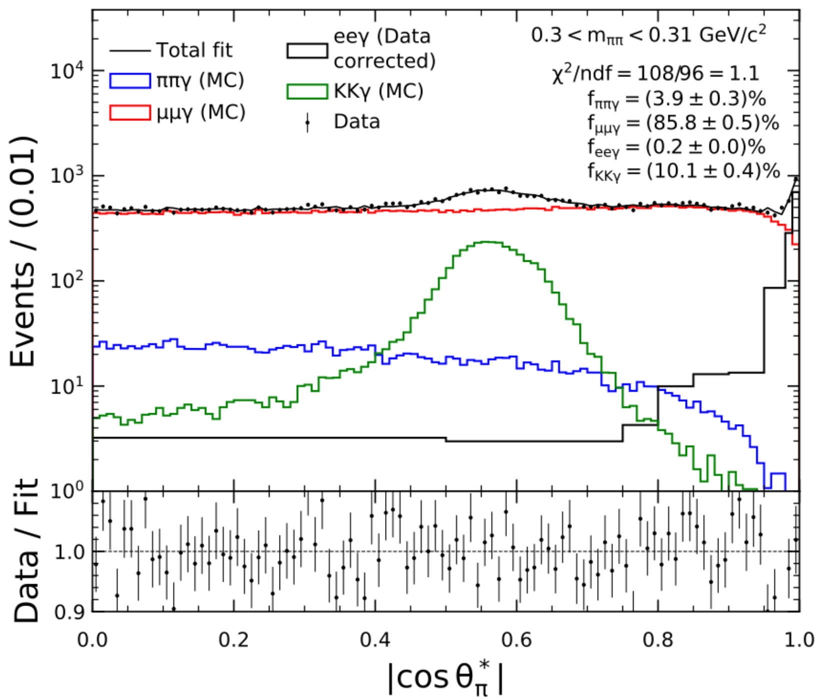


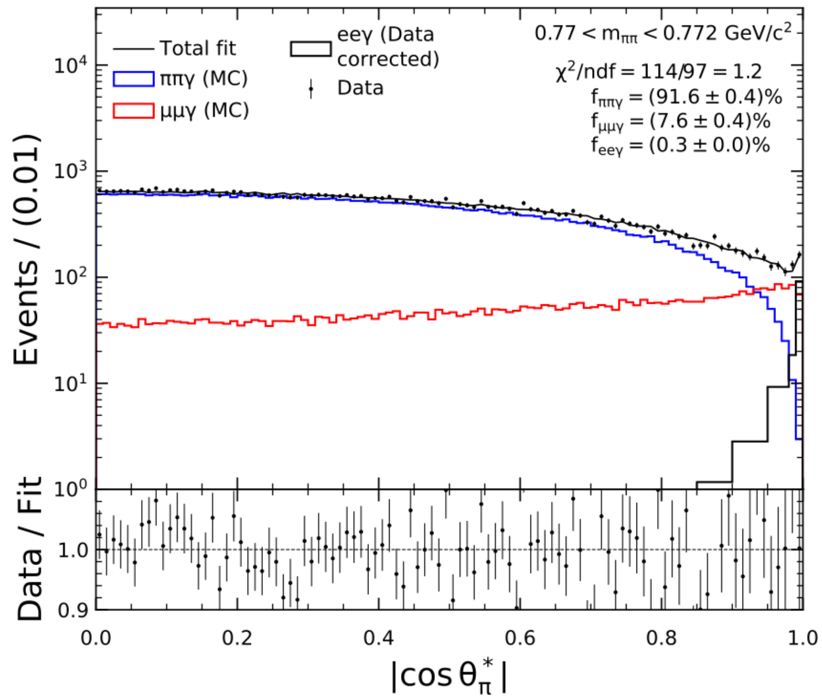
Key Analysis Steps in the 2025 Analysis

- Background reduction/suppression
- Evaluate data/MC differences (& uncertainties) affecting $|\cos\vartheta^*|$ template and mass distributions (e.g. trigger, tracking, event selections)
- Kinematic/template fits for separating $\pi\pi/\mu\mu$
- Blinded analysis with three blinding factors ($\varepsilon_{\text{trigger}}$, $\varepsilon_{\text{tracking}}$ and normalization of the fitted $\pi\pi/\mu\mu$ mass spectra) in two steps

Fit strategy for a given mass bin:

- Fit $|\cos\vartheta^*|$ in 0.9-1 to get norm. of $ee\gamma$ (its template derived from data)
- Fit $|\cos\vartheta^*|$ in 0-0.9 to separate $\pi\pi/\mu\mu$ (also KK for mass < 0.4 GeV/c^2)
- Extrapolate to 1 to get full $\pi\pi/\mu\mu$

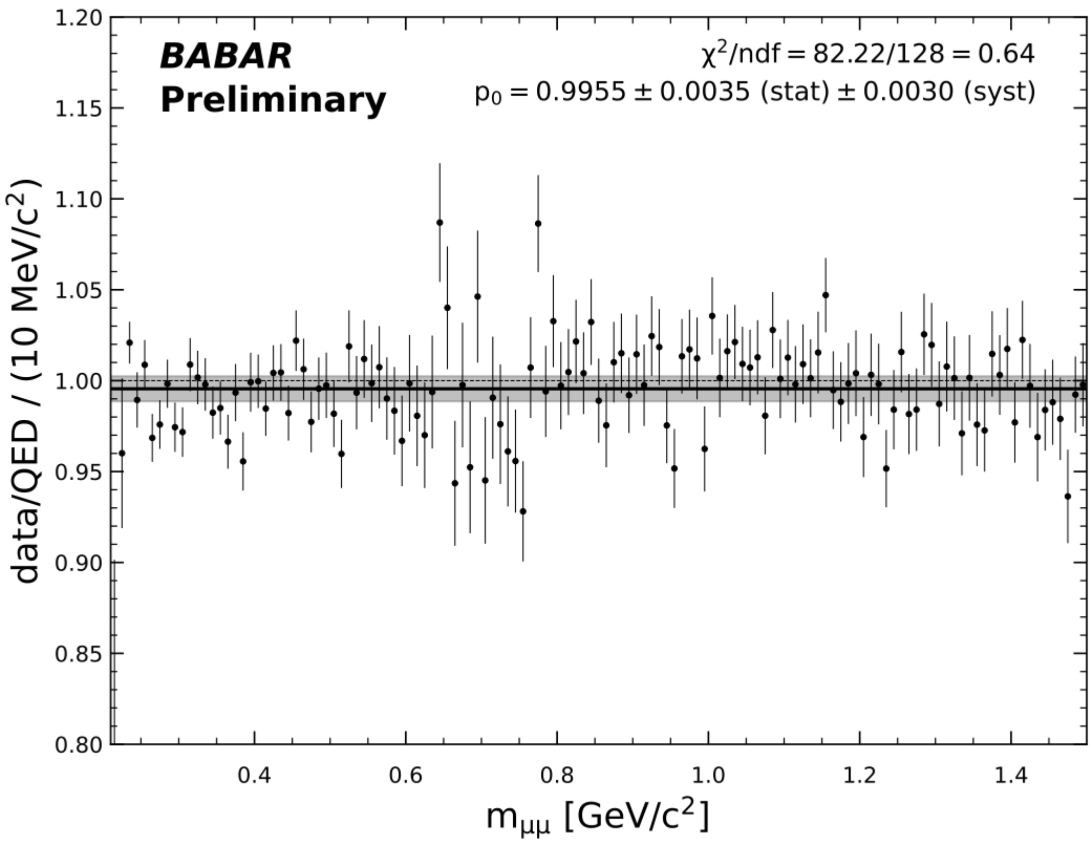




First Step Unblinding: QED Test

- When all data/MC corrections and their uncertainties have been evaluated
- Unblind the $\mu\mu$ mass spectrum over QED prediction for a QED test:

$$R_{\mu\mu} = 0.9955 \pm 0.0035_{\text{stat}} \pm 0.0030_{\text{fit}} \pm 0.0033_{\gamma\ \text{ISR}} \pm 0.0043_{\text{lumi ee}}\ [0.0071]_{\text{total}}$$



Non-trivial test as each data/MC correction affects the $\mu\mu/\pi\pi$ separation and $\mu\mu$ (& $\pi\pi$) mass spectrum

The relatively large fluctuation in the "rho" mass region has been investigated

Closure test shows that the $\pi\pi/\mu\mu$ separation procedure does not introduce any systematic bias due to uneven event yields of the two channels

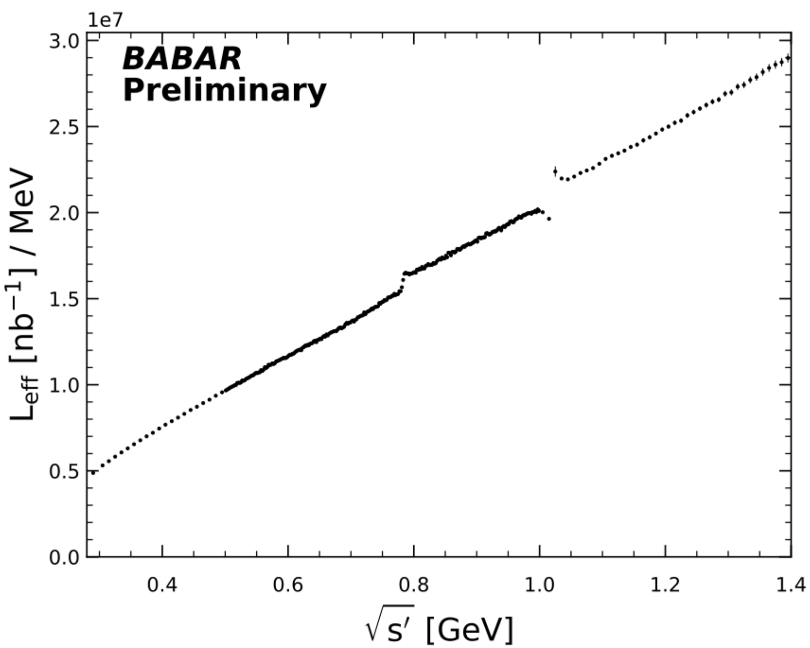
Second Step Unblinding

- With the successful QED test, effective ISR luminosity is determined from unfolded $\mu\mu$ mass spectrum

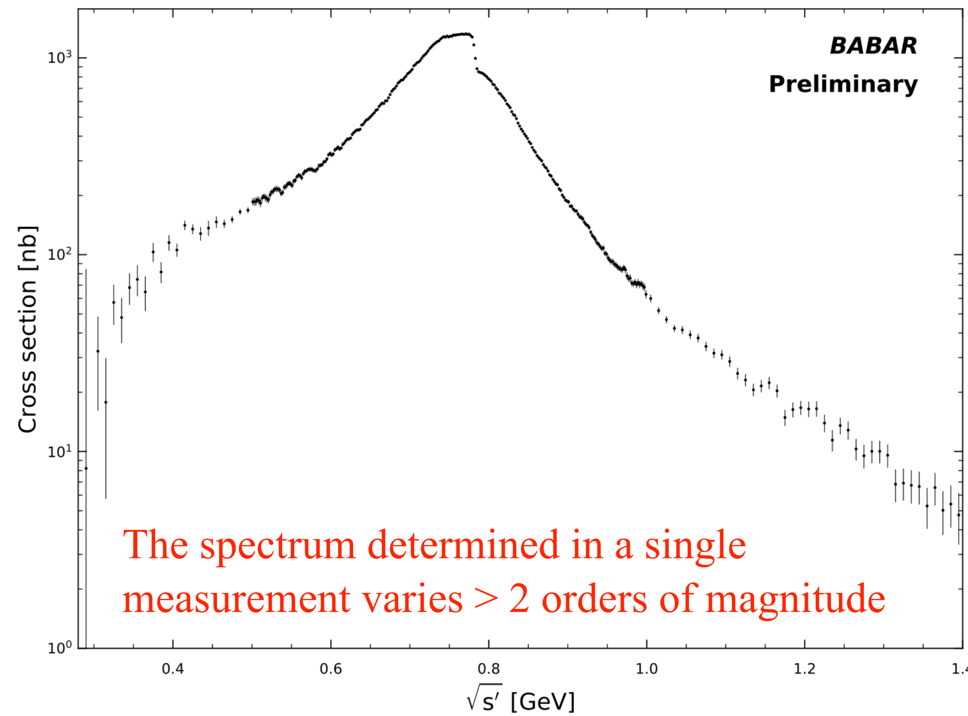
$$\frac{dL^{\text{eff}}_{\text{ISR}}}{d\sqrt{s'}} = \frac{\frac{dN_{\mu\mu}}{d\sqrt{s'}}}{\epsilon_{\mu\mu}(\sqrt{s'})\sigma^0_{\mu\mu}(\sqrt{s'})}$$

- Unblinded preliminary $\pi\pi$ mass spectrum is derived

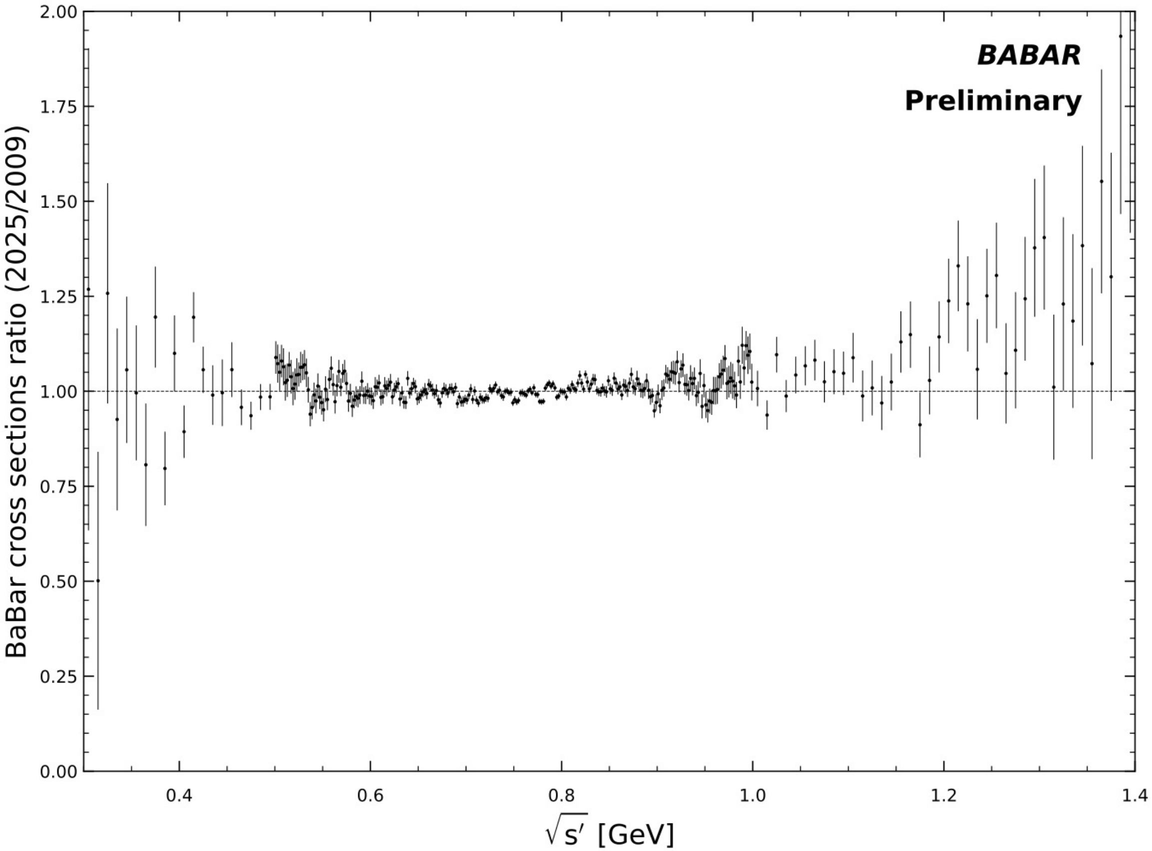
$$\sigma^0_{\pi\pi}(\sqrt{s'}) = \frac{\frac{dN_{\pi\pi}}{d\sqrt{s'}}}{\epsilon_{\pi\pi}(\sqrt{s'})\frac{dL^{\text{eff}}_{\text{ISR}}}{d\sqrt{s'}}}$$



The discontinuities reflect the VP contributions



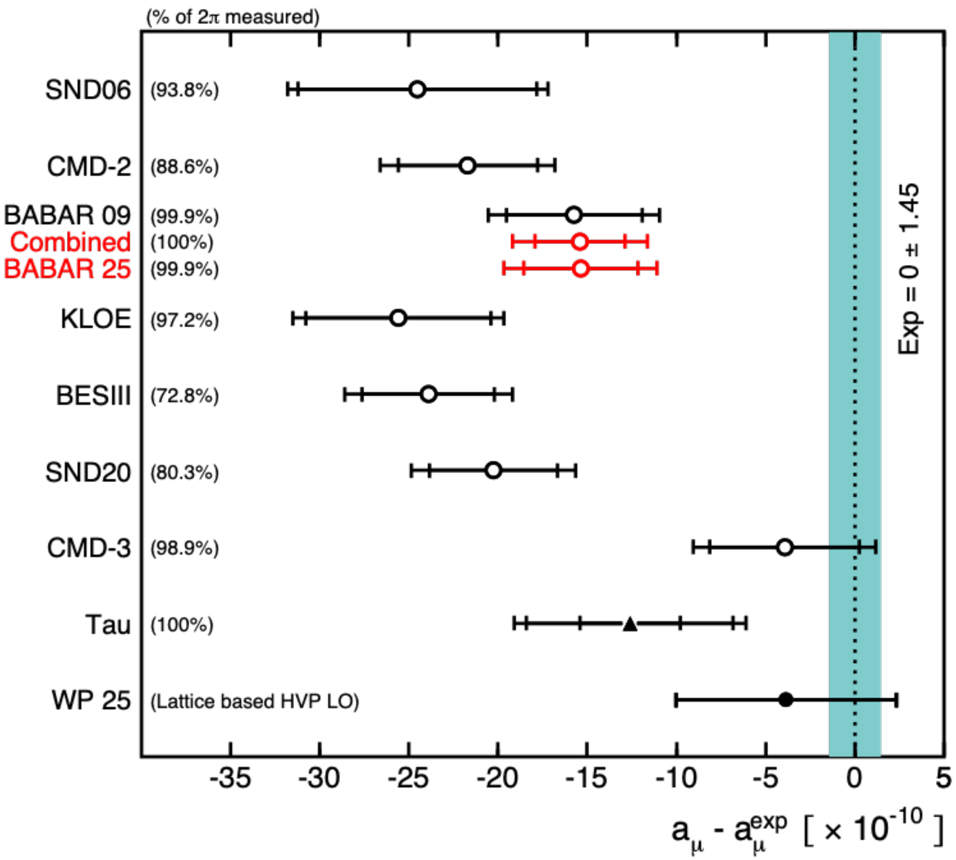
Comparison 2025 with 2009



Energy range [GeV]	2025 $a_\mu[2\pi, 10^{-10}]$ (stat)(syst)	2009 $a_\mu[2\pi, 10^{-10}]$ (stat)(syst)
Below 0.5	58.0 (9.4%)(3.0%)	57.6 (1.1%)(0.96%)
0.5–1.4	456.2 (0.49%)(0.37%)	455.6 (0.46%)(0.57%)

- The preliminary 2025 energy spectrum in excellent agreement with that of 2009
- The new results are more precise in the rho peak region while the 2009 results are more precise at low and high energies
- Non-trivial agreement given different $\pi\pi/\mu\mu$ separation methods, data samples, fully blinded strategies
- Preliminary 2025-2009 combination: a_μ [2π, <1.8 GeV] = 514.4×10^{-10} (0.49%) provides the most precise measurement in the two-pion channel

Summary and Outlook



- Inner errors: uncertainty of the 2π channel <1.8 GeV (non-100% part from combined 2π data)
- Outer one: uncertainty of other components
- Intermediated one for tau: uncertainty of inflated isospin breaking corrections

- The e^+e^- based LO HVP calculation mainly contributed by 2π channel
- There is however large discrepancy among different measurements which prevents a new WP2025 prediction
- WP2025 SM prediction based on lattice LO HVP calculations
- New BABAR measurement in fully blinded ways confirms the previous one
- The BABAR 2009-2025 combination: the most precise 2π measurement

➢ Need better MC event generators and more precise and independent measurements from other experiments to clarify the situation

➢ The difference between the e^+e^- data-based HVP calculation and the lattice-based one also needs to be understood before a firm conclusion whether there is or not any new physics in the muon g-2 observable

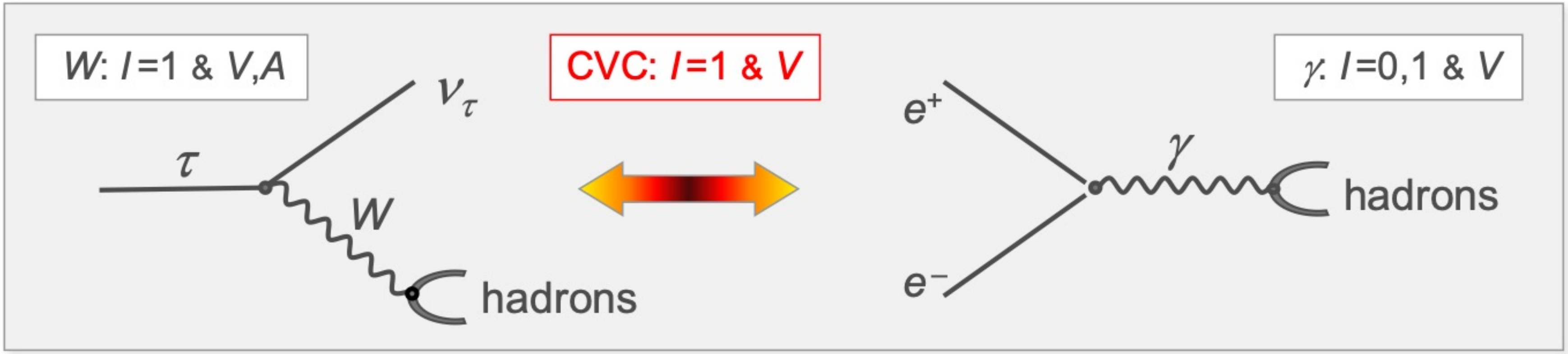
➢ Theoretical and experimental efforts are ongoing, a new plenary workshop of the muon g-2 Theory Initiative will take place on 8-12 Sept. at Orsay, France (link)

References for recent BABAR Measurements

- Measurement of additional radiation in the initial-state-radiation processes $e^+e^- \to \mu^+\mu^-\gamma$ and $e^+e^- \to \pi^+\pi^-\gamma$ at BABAR, Phys. Rev. D 108 (2023) 11, L111103
- Study of the process $e^+e^- \to \pi^+\pi^-\pi^0$ using initial state radiation with BABAR, Phys. Rev. D 104 (2021) 11, 112003
- Study of the reactions $e^+e^- \to 2(\pi^+\pi^-)\pi^0\pi^0\pi^0$ and $e^+e^- \to 2(\pi^+\pi^-)\pi^0\pi^0\eta$ at center-of-mass energies from threshold to 4.5 GeV using initial-state radiation, Phys. Rev. D 103, 092001 (2021)
- Study of the reactions $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0\pi^0$ and $\pi^+\pi^-\pi^0\pi^0\pi^0\eta$ at center-of-mass energies from threshold to 4.5 GeV using initial-state radiation, Phys. Rev. D 104 (2021) 11, 112004
- Study of the reactions $e^+e^- \to K^+K^-\pi^0\pi^0\pi^0$, $e^+e^- \to K^0_SK^\pm\pi^\mp\pi^0\pi^0$, and $e^+e^- \to K^0_SK^\pm\pi^\mp\pi^+\pi^-$ at center-of-mass energies from threshold to 4.5 GeV using ISR, Phys. Rev. D 107 (2023) 7, 072001

Alternative Calculation using τ Data

Proposed by Alemany-Davier-Hoecker (ADH), EPJC 2 (1998) 123



Hadronic physics factorises in Spectral Functions:
Isospin symmetry connects I=1 e^+e^- cross section to vector τ spectral functions

Fundamental ingredient relating long distance (resonances) to short distance description (QCD)

$$\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s}\,\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]$$

$$\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right] \propto \frac{BR\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}{BR\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]}\,\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}\,\frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+s/m_\tau^2\right)}$$

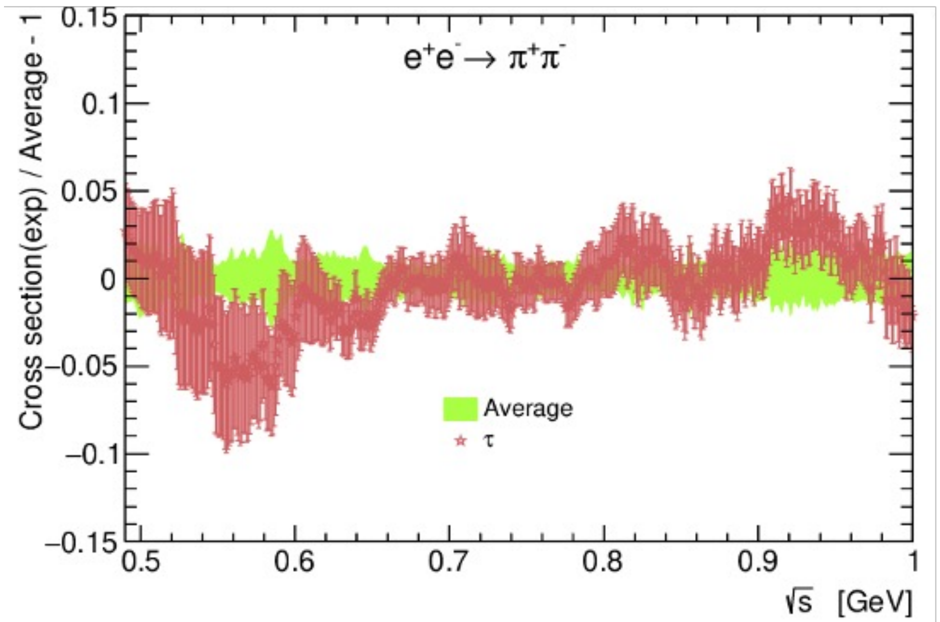
Branching fractions | Mass spectrum | Kinematic factors (PS)

Alternative Calculation using τ Data

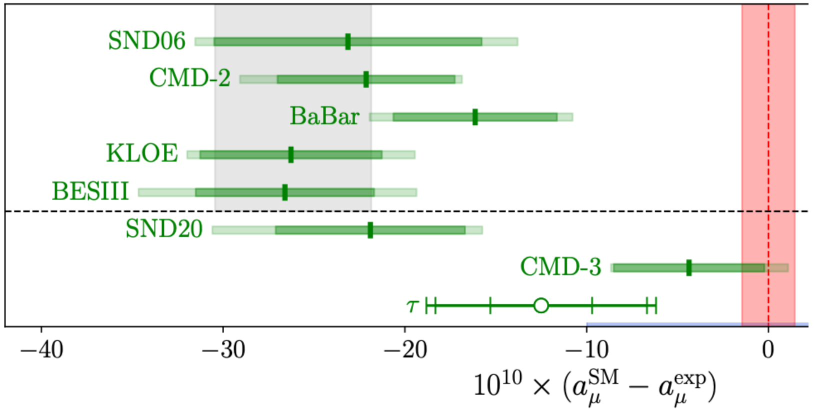
- Tau spectral functions (SFs) for the $\pi\pi^0$ channel measured by ALEPH (2005, 2013), Belle (2008), CLEO (2000), OPAL (1999) are in good agreement and combined
- Isospin Breaking (IB) corrections need to be applied to tau SFs before being used for HVP

$$v_{1,\pi^-\pi^0\nu_\tau} \propto \frac{B_{\pi\pi^0}}{B_e}\frac{dN_{\pi\pi^0}}{N_{\pi\pi^0}ds}\frac{m_\tau^2}{(1-s/m_\tau^2)^2(1+2s/m_\tau^2)}\frac{R_{\rm IB}(s)}{S_{\rm EW}}$$

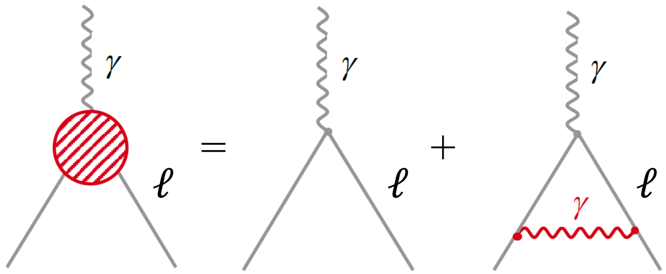
$$\frac{R_{\rm IB}(s)}{S_{\rm EW}} \quad \text{with} \quad R_{\rm IB}(s)=\frac{{\rm FSR}(s)}{G_{\rm EM}(s)}\frac{\beta_0^3(s)}{\beta_-^3(s)}\left|\frac{F_0(s)}{F_-(s)}\right|^2$$



- Current IB corrections are somewhat uncertain with their uncertainties inflated in WP2025
- New data-driven corrections (arXiv:2504.21476)
- Nevertheless the combined tau SF after corrections are in fair agreement with BABAR+CMD3+tau combination both in shape and for a_μ



QED Contribution



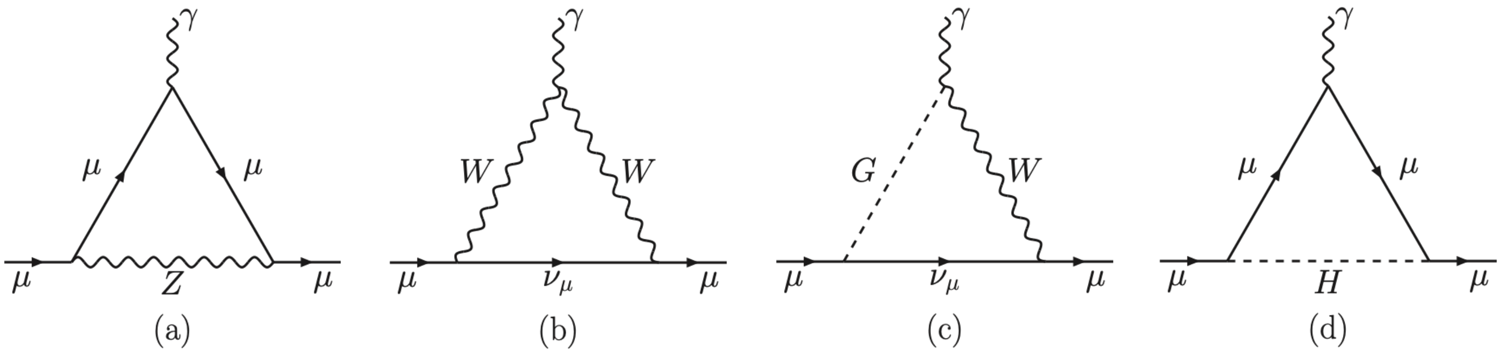
+ 2-loop + 3-loop + 4-loop + 5-loop + …

$$a_\ell^{\mathrm{QED}} = 0 + \frac{\alpha}{2\pi} + A_2\left(\frac{\alpha}{\pi}\right)^2 + A_3\left(\frac{\alpha}{\pi}\right)^3 + A_4\left(\frac{\alpha}{\pi}\right)^4 + A_5\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

No. of Feynman diagrams involved: 1 7 72 891 12 672

The QED field theory is tested by a_e measurements to an unprecedented tenth-order

EW Contribution



The one-loop EW involving Z, W, gauge and Higgs bosons are shown above

The EW contribution is heavily suppressed by the mass of the bosons
But both the one- and two-loop contributions are known with an overall precision of 0.26%