# Quarkonium at *T*>0

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# Quarkonium in Heavy Ion Collisions

**Production from q,g** $\sigma_{\bar{Q}Q},\ \tau_{\rm form}$

**Interaction w/ hot medium** $\rho(\omega, \boldsymbol{k})$

**Tools**

| | |
|---|---|
| Lattice QCD | Static properties |
| Resummed perturbation / Effective Field Theories | Weak coupling / Scale hierarchy |
| | Qualitative guide for V(r,T) |
| Potential Model | Schrödinger Eq., need V(r,T) |
| Sum Rules | OPE, need QCD condensates |
| AdS/CFT | A guide from Strong coupling |

**Hadronization, interacting w/ hadrons** $\sigma_{\bar{Q}Q+X}$

Mechanism? $T(x^\mu),\ f(x,p)$

**dilepton spectrum → $p_t$ distribution, $R_{AA}$, $v_2$, etc**

# $\rho(\omega)$ from lattice QCD

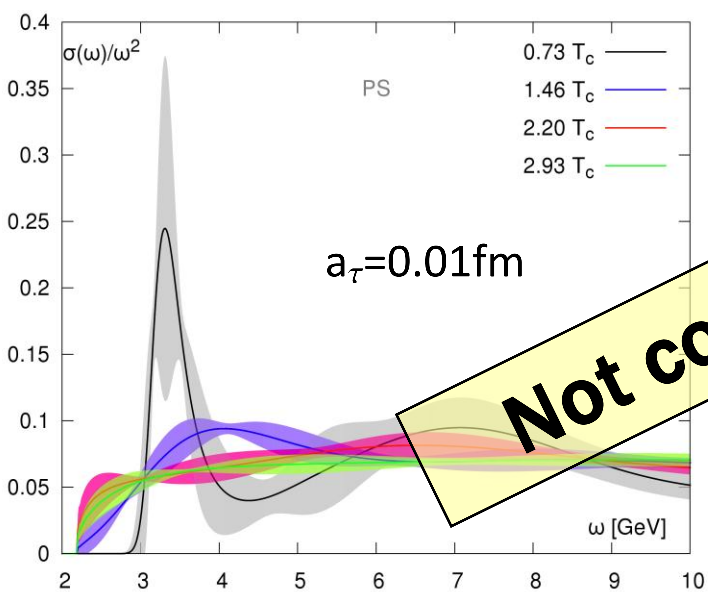
- **Imaginary time correlator + Maximum Entropy Method**

$$G(\tau,T)=\int d^3\boldsymbol{x}\langle J(\tau,\boldsymbol{x})J(0,\boldsymbol{0})\rangle=\int_0^\infty d\omega\frac{\cosh[\omega(\tau-\beta/2)]}{\sinh(\omega\beta/2)}\rho(\omega,T)$$

**Solving inverse problem via Maximum Entropy Method (Asakawa-Hatsuda)**

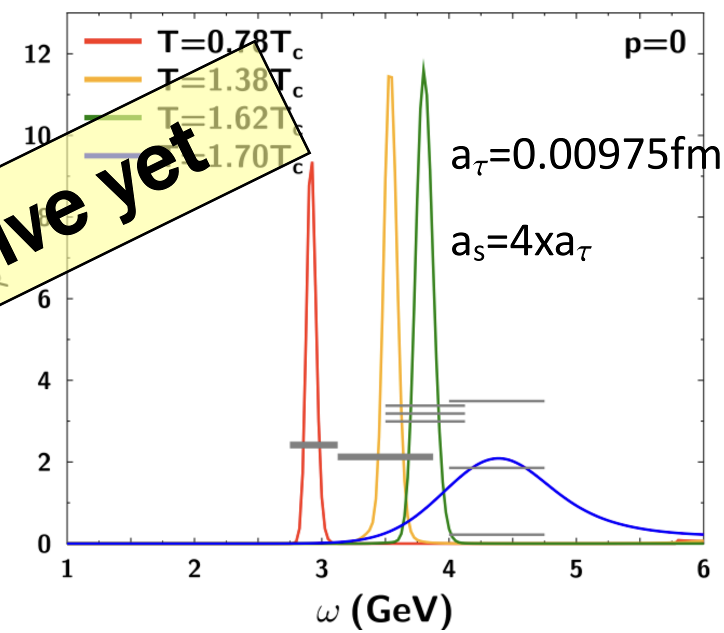
Ding et al., (BNL-Bielefeld) $128^3$x96

Nonaka et al., (Osaka-Nagoya) $64^3$x96




$a_\tau$=0.01fm

**Not conclusive yet**

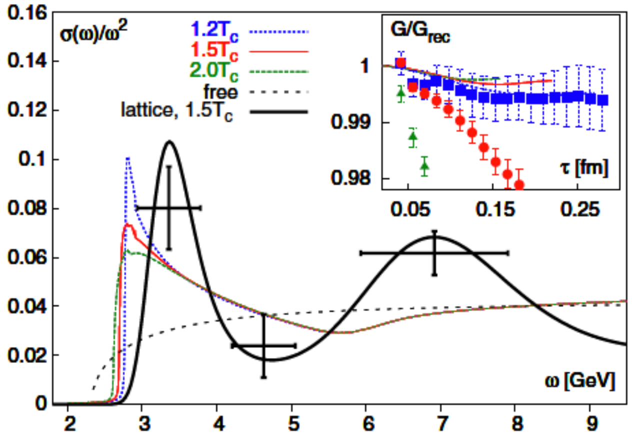



$a_\tau$=0.00975fm

$a_s$=4x$a_\tau$

# What does the peak mean?

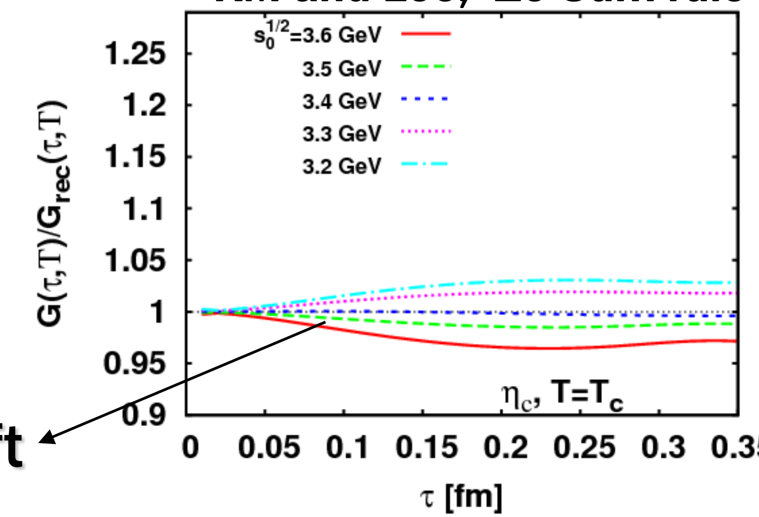
## Potential model analysis (Mocsy-Petreczky ’08)




Threshold enhancement above $T_c$

Note : width in MEM does not have definite physical meaning

KM and Lee, ’10 Sum rule




## Lattice observable : G($\tau$)

No change in G($\tau$)

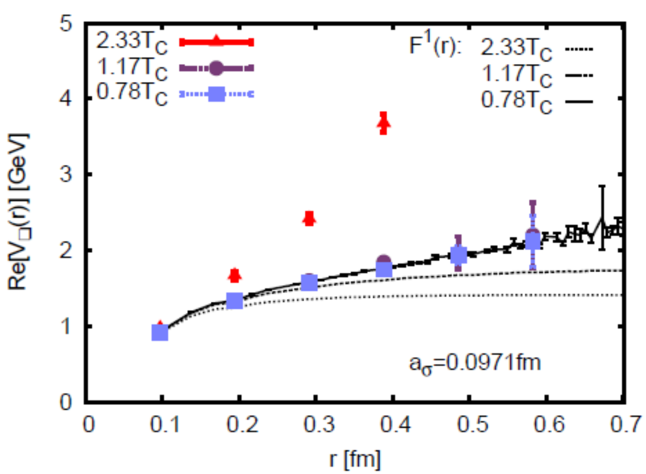
$\rightarrow \rho(\omega,T{=}0) \cancel{=} \rho(\omega,T)$?

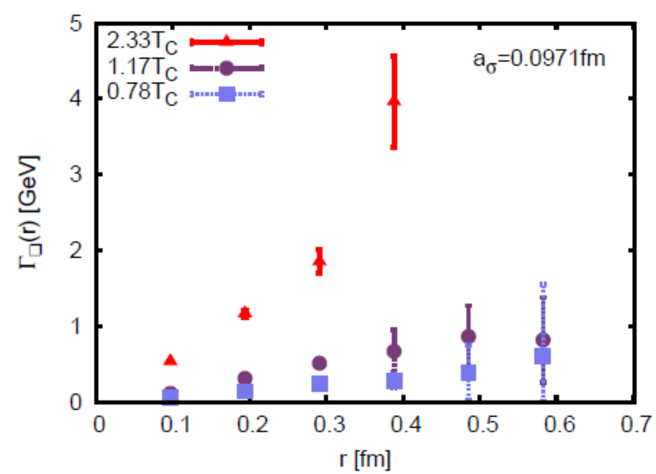
Corresponds to -50 MeV mass shift

~100 MeV modification cannot be resolved by lattice

# Deriving V(*r*,*T*)

- **Weak coupling calculations**
  - **Emergence of the imaginary part** (Landau damping, Singlet→Octet breakup)
  - **Even analytic result from EFT(pNRQCD) given** (Brambilla et al., ’10) $m_Q \gg m_Q\alpha_s \gg T \gg m_Q\alpha_s^2 \gg m_D (\simeq gT)$
- **Lattice QCD** (Rothkopf-Hatsuda-Sasaki ’11)
  - **Applying extraction procedure from Wilson loop**
    - Resummed perturbative approach by M.Laine ‘07

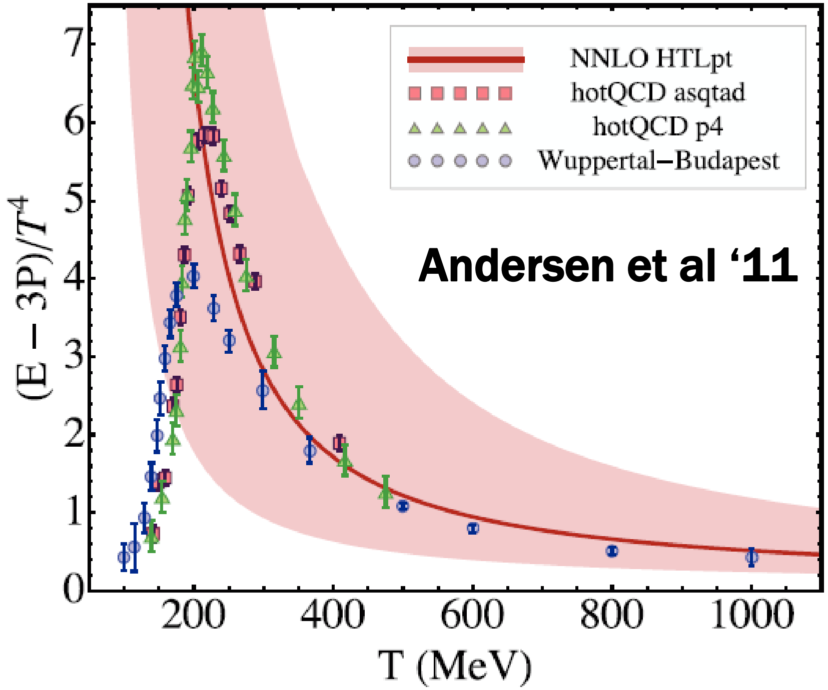


# Property of medium

- Eq. of state
  - 3-loop HTL : $T>2T_c$
- Lifetime of QGP
  - 4-5 fm/$c$ at RHIC
  - Longer at LHC

(←HBT measurement)




$\Gamma$ ~ 50 MeV is large enough to "melt" charmonia

**Strongly coupled nature & Estimation of width are indispensable for quarkonium physics near Tc**

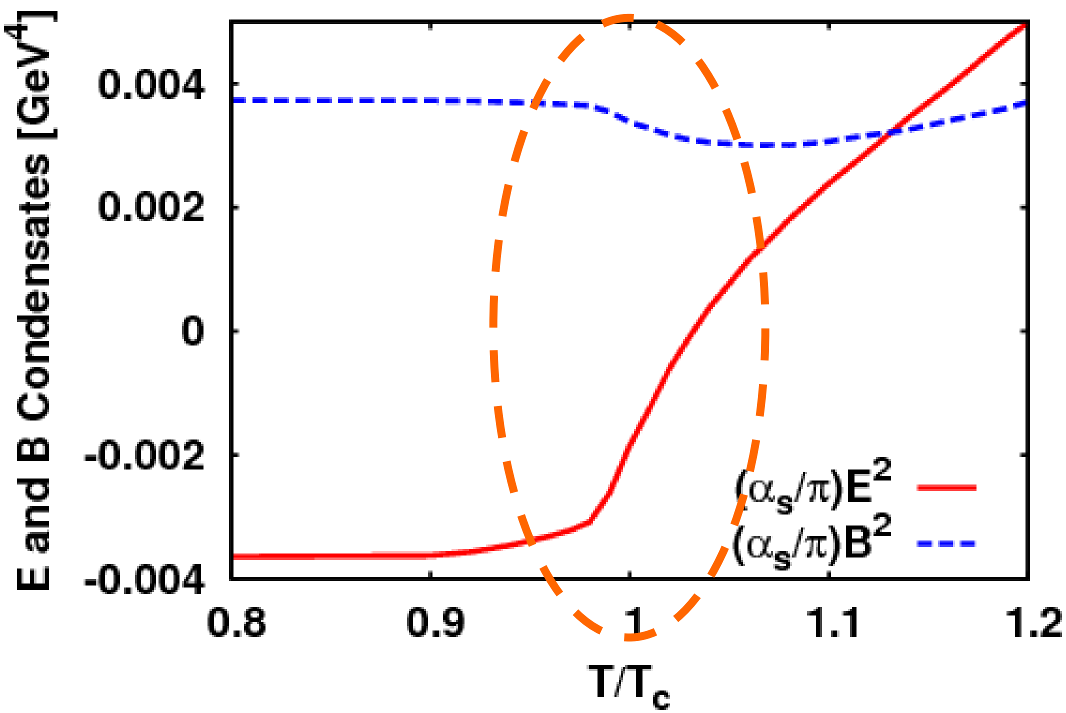
# Characterizing medium w/ local operators

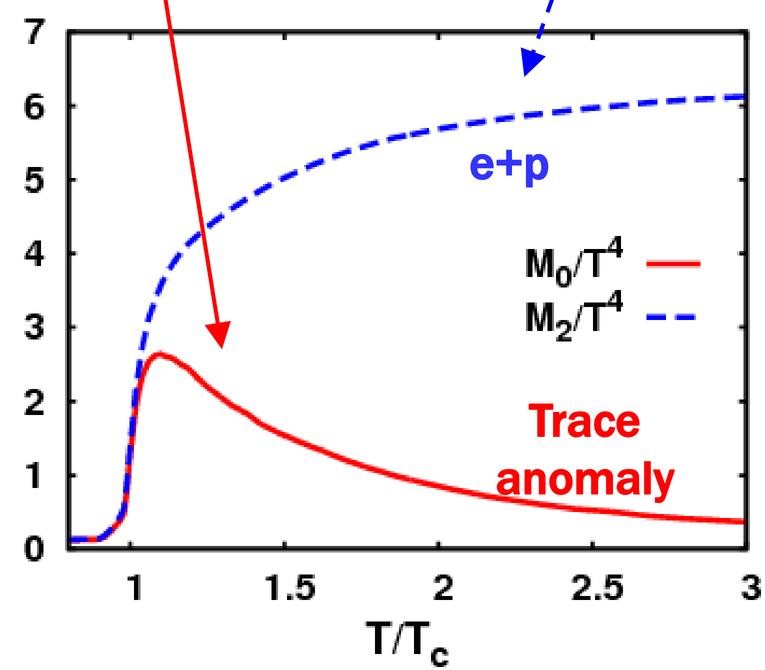
$$\left\langle \frac{\alpha_s}{\pi} \boldsymbol{E}^2 \right\rangle_T \quad \cdots \quad M_0(T) + \frac{3}{4} \frac{\alpha_s^{\mathrm{eff}}(T)}{\pi} M_2(T)$$

**Sudden Change across $T_c$**

$$\left\langle \frac{\alpha_s}{\pi} \boldsymbol{B}^2 \right\rangle_T \quad \cdots \quad M_0(T) + \frac{3}{4} \frac{\alpha_s^{\mathrm{eff}}(T)}{\pi} M_2(T)$$

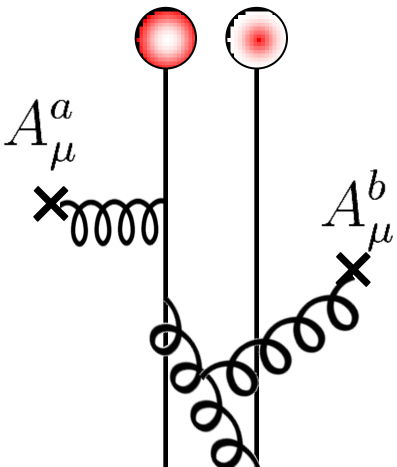
**Almost constant across $T_c$**



# From local operators to quarkonium : OPE

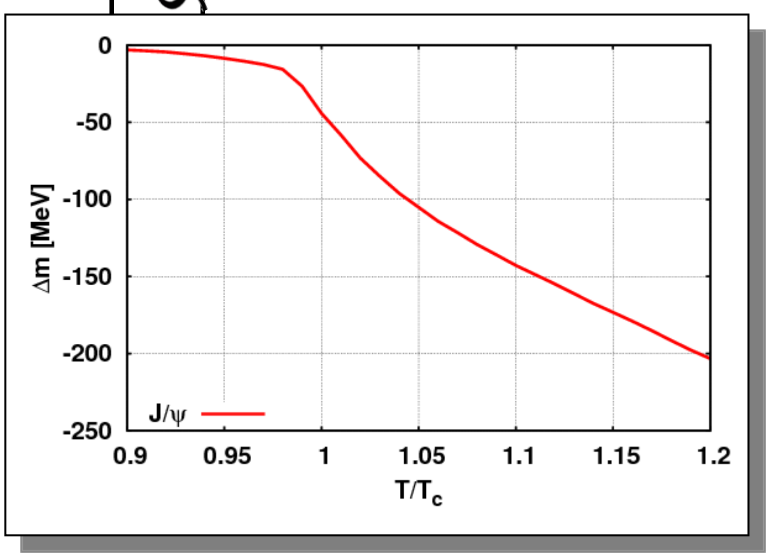
## Treating as a short-distance process (Peskin ’79)

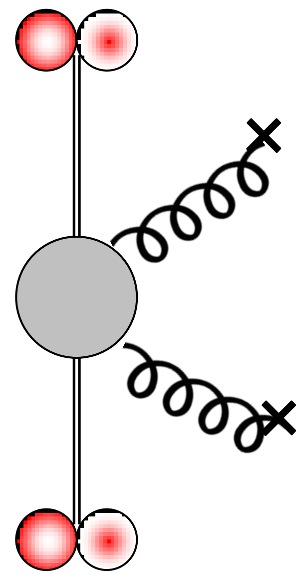



One-gluon exchange

$k \sim \epsilon$

Ext. soft gluon



Info. on scale < $\epsilon$

$$\sim \sum C_i \langle \mathcal{O}_i \rangle$$

info. on scale > $\epsilon$

Leading order gives mass shift (2nd order Stark)

$$\Delta m_{\bar{Q}Q} = -\frac{7\pi^2}{18}\frac{a_0^2}{\epsilon}\left\langle \frac{\alpha_s}{\pi}\Delta \boldsymbol{E}^2 \right\rangle_T$$

# Imaginary part : QCD sum rules

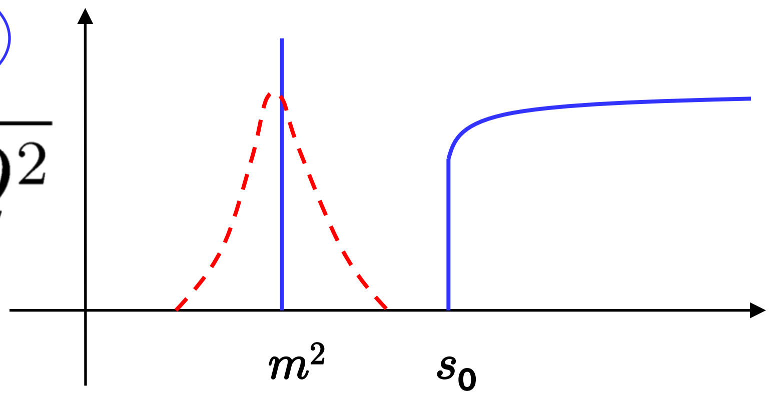
Shifman-Vainshtein-Zakharov '79

$$i \int d^4x e^{iqx} \langle T[j^J(x) j^J(0)] \rangle$$

$q^2$=-$Q^2$

$$\Pi(Q^2) = \int_0^\infty ds \frac{\rho(s)}{s+Q^2}$$

"Hadronic" spectral function




**OPE at large $Q^2$**

**Matching to obtain $m^2$ , $s_0$ and $\Gamma$**

$$\simeq C_{\text{pert}}(Q^2) + C_G \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle + \cdots$$

**at dim.d :**

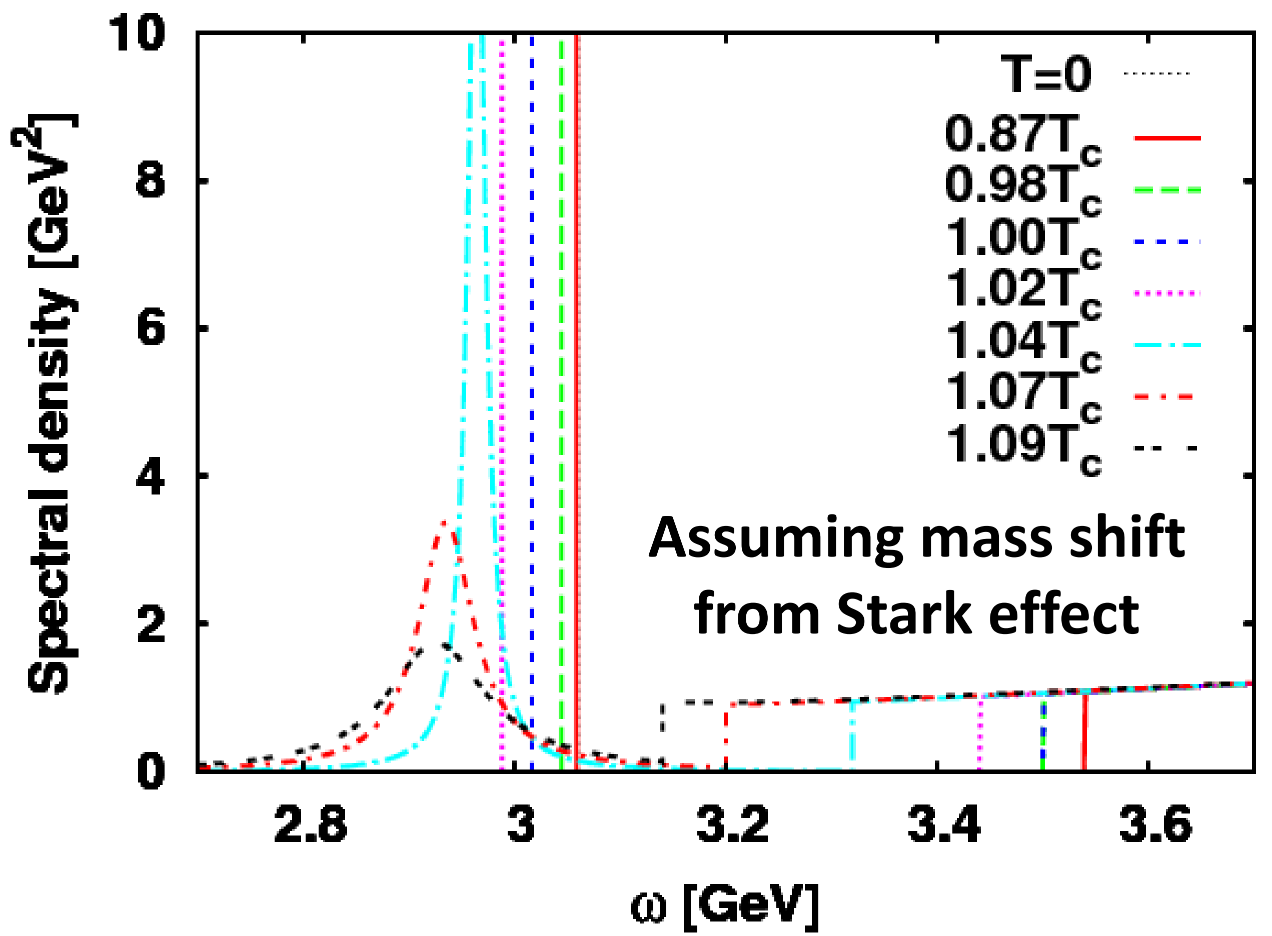
$$\frac{\langle G^d \rangle}{(4m_Q^2 + Q^2)^2} \longrightarrow$$

**Better convergence expected for heavier quark mass and $Q^2$ >0**

# Results from “Pole+Continuum” ansatz

(Breit-Wigner + pQCD)

KM and Lee, PRD‘10



# Beyond the Ansatz : QCDSR meets MEM

$$\boxed{\mathcal{M}^J(\nu) = \int dx^2 e^{-\nu x^2} \rho^J(2m_Q x, T)}$$

**Input from T-dep OPE**

**Output of MEM**

- **Compared with the imaginary time correlator (lattice)**

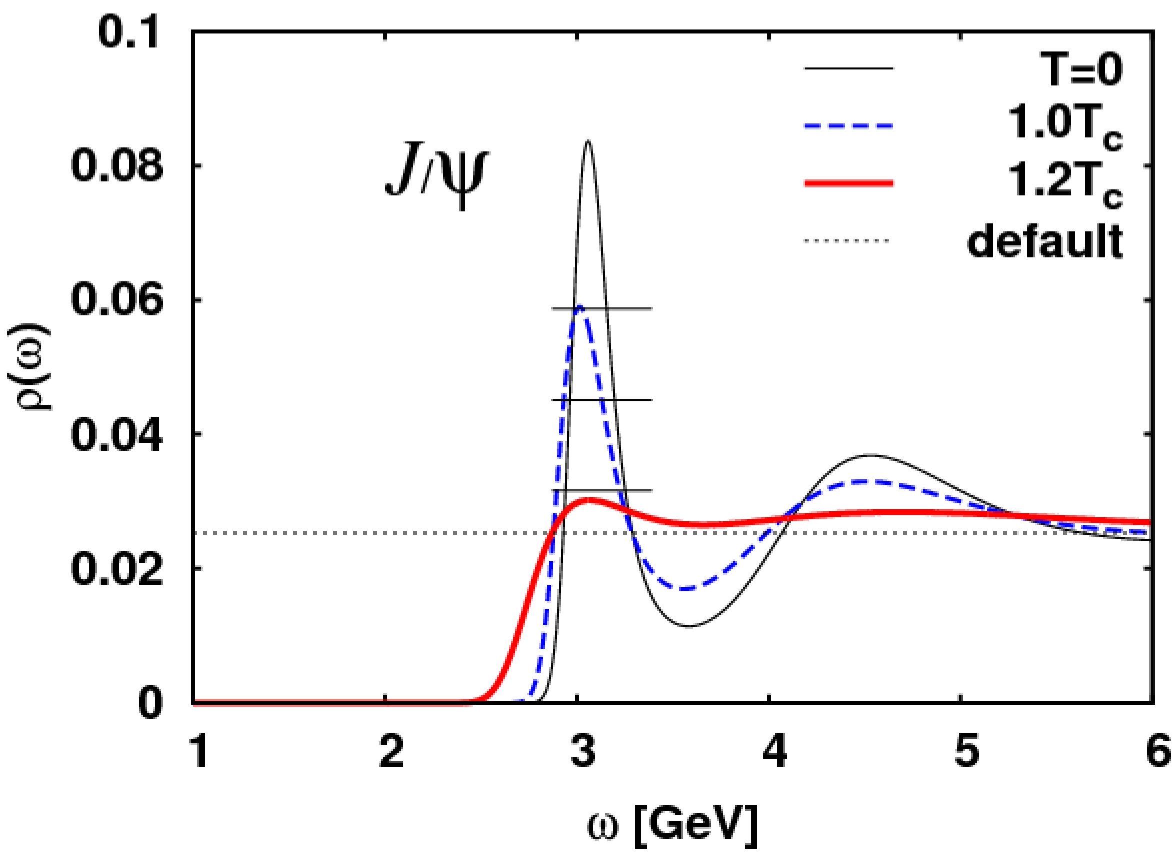
$$G(\tau, T) = \int_0^\infty d\omega \frac{\cosh[\omega(\tau - 1/(2T))]}{\sinh(\omega/(2T))} \rho(\omega, T)$$
$$\sim e^{-\omega\tau} \quad (T = 0)$$

  - Discretized vs Continuum : take as many points as we want!
  - Temperature dependent $\tau$ range vs independent $\nu$ range
  - Temperature dependent kernel vs independent kernel
  - Exact for all $\tau$ vs restricted $\nu$ range by convergence of OPE
  - NRQCD dispersion relation is similar to Borel sum rule

# Spectral function from QCDSR+MEM

P.Gubler, KM, M.Oka PRL'11

P.Gubler, K.Suzuki, KM, M.Oka, in preparation




$$0.78 \leq \nu \leq 8.03$$

**Typical Borel mass** **from OPE convergence**

Resolution of the width not sufficient

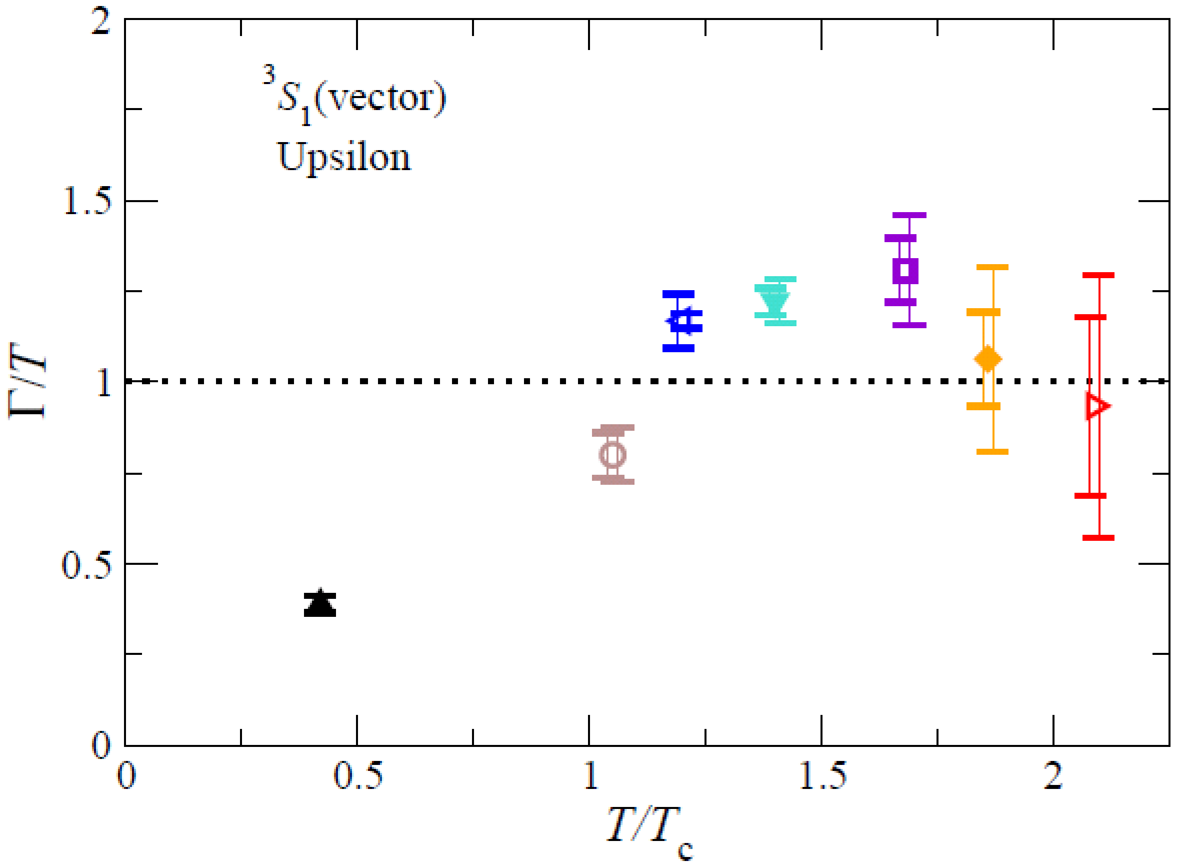
Peak reduction statistically significant

For technical details of QCDSR+MEM, Gubler and Oka, PTP124,995 ('10)

# Υ is more important at LHC

## Most of tools are expected to work!

- Lattice NRQCD (Aarts et al., 1109.4496)

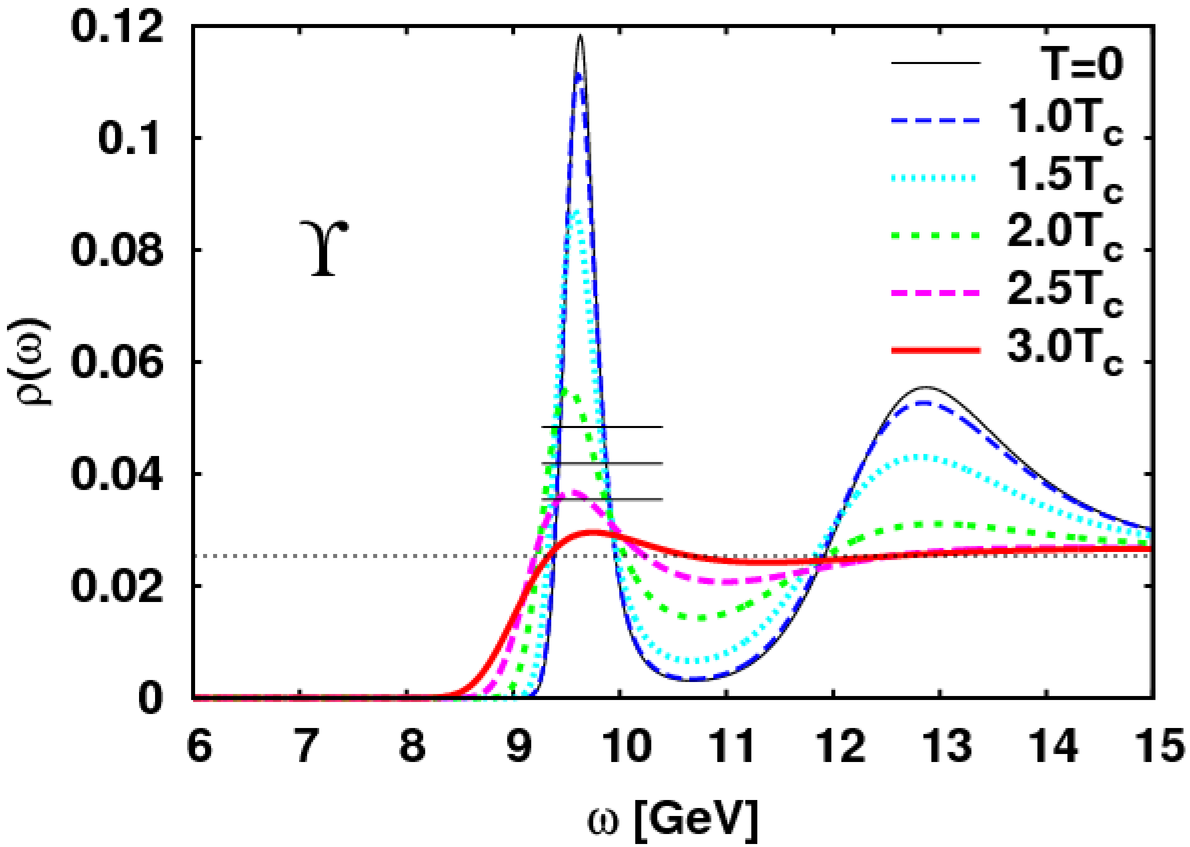



weak coupling pNRQCD

Vacuum Υ mass

# Υ is more important at LHC

- **Most of tools are expected to work!**
  - QCDSR+MEM




Caveat :

Peak = 1S+2S+3S

(Unlike charmonium)

# Summary and Outlook

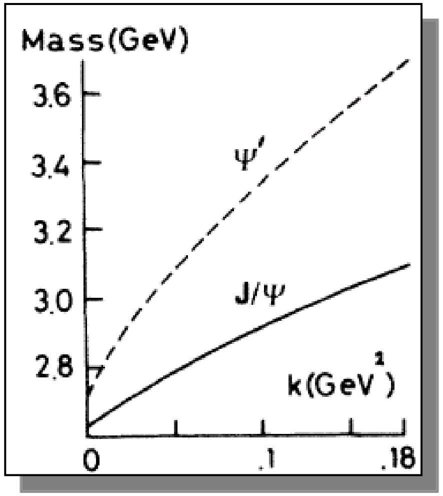
- Broadening sets in above Tc : dominated by gluonic dissociation
  - Even if spectral peak survives, $J/\psi$ cannot live long
- Charmonium : sensitive to change near Tc
  - Characterized by local operators (gluon condensate)
  - Strong coupling approach necessary
- Bottomonium : modification at T>2Tc
  - Weak coupling approach expected to work
  - LHC : nice testing ground for the theories

## Thank you!

# Backup

# 25 Years Ago…

- **Change of confinement potential : Mass shift of $J/\psi$ etc** (Hashimoto et al., '86)




$$V(r, T=0) = -\frac{\alpha}{r} + \sigma r$$

$$\sigma(T) = \sigma_0 \left(1 - \frac{T}{T_c}\right)^b$$

- **Debye screening in QGP : No bound state can exist in QGP – $J/\psi$ suppression** (Matsui-Satz '86)

$$V(r, T > T_c) = -\frac{\alpha}{r} e^{-m_D r}$$

**Both based on the change of force btw Q and Qbar**

# Relation to Confinement

- **Area/Perimeter law of Wilson loop in Lattice**

  **(Manousakis-Polonyi '87)**

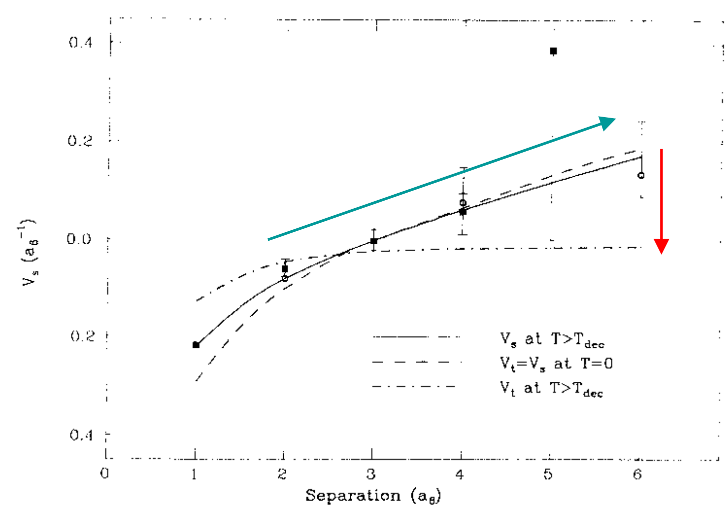



FIG. 1. The correlation function of spacelike Polyakov lines

  - S-T loop : Area→Perimeter
  - S-S loop : Area→Area

- **OPE for Wilson loop**

  **(Shifman '80)**

$$W(\text{S-T}) \simeq 1 - \left\langle \frac{\alpha_s}{\pi} \boldsymbol{E}^2 \right\rangle (ST)^2 + \cdots$$

$$W(\text{S-S}) \simeq 1 - \left\langle \frac{\alpha_s}{\pi} \boldsymbol{B}^2 \right\rangle (SS)^2 + \cdots$$

**Sudden change of <$E^2$> : coming from Confinement-deconfinement transition**

# QCD Sum Rules for Heavy Quarkonium

- **Correlator in momentum space**

$$\Pi^J(q^2) = i\int d^4x e^{iqx}\langle T[j^J(x)j^J(0)]\rangle$$

$$j^P = i\bar{c}\gamma_5 c, \quad j^V_\mu = \bar{c}\gamma_\mu c$$
$$j^S = \bar{c}c, \quad j^A_\mu = (q_\mu q_\nu/q^2 - g_{\mu\nu})\bar{c}\gamma^\nu\gamma_5 c$$

$$\Pi^{P,S}(q^2) = q^2\tilde{\Pi}^J(q^2)$$
$$\Pi^{V,A}_{\mu\nu}(q^2) = (q_\mu q_\nu - q^2 g_{\mu\nu})\tilde{\Pi}^J(q^2)$$

- **Take spacelike momentum : $q^2 = -Q^2 < 0$** $\quad \tilde{\Pi} = \tilde{\Pi}^R$
  - OPE and truncation valid for : $4m_Q^2 + Q^2 > (\Lambda_{\rm QCD} + aT)^2$
  - Up to dim.4, rough estimation of dim.6
  - Temperature effect only through condensates

Hatsuda-Koike-Lee '93

- **Meson at rest with respect to medium: $q = (\omega, 0)$**

# OPE side

## Borel transformed correlator

$$\nu = 4m_Q^2/M^2$$

$$\mathcal{M}(\nu) = \lim_{\substack{Q^2/n \to M^2, \\ n, Q^2 \to \infty}} \frac{(Q^2)^{n+1}\pi}{n!} \left( -\frac{d}{dQ^2} \right)^n \Pi(Q^2)$$

$$= e^{-\nu} A(\nu)[1 + \alpha_s(\nu) a^J(\nu) + b^J(\nu)\phi_b(T) + c^J(\nu)\phi_c(T) + d^J(\nu)\phi_d(T)]$$

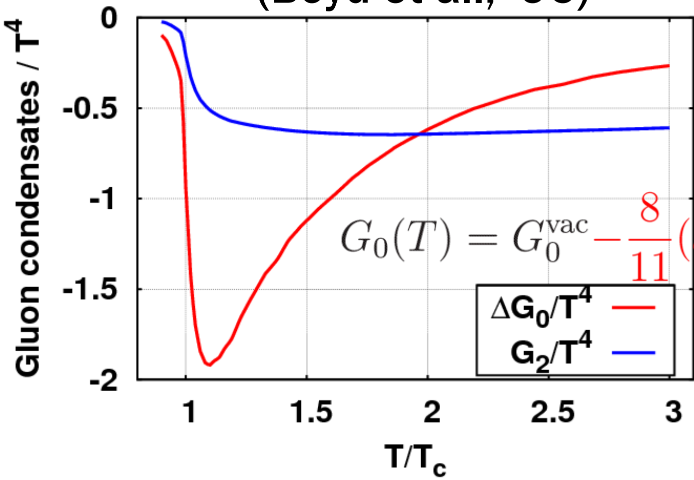
$$\phi_\mathrm{b} = \frac{4\pi^2}{9(4m_Q^2)^2} G_0, \quad \phi_\mathrm{c} = \frac{4\pi^2}{3(4m_Q^2)^2} G_2$$

**Requirement : Convergence**

**Dim.6 < 20% of OPE**

**Crude estimation by instanton liquid model**

**Gluon condensates in pure SU(3) LQCD**
**(Boyd et al., '96)**



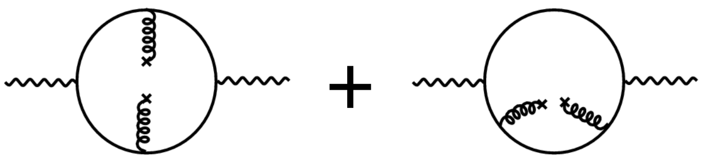

$$G_0(T) = G_0^{\mathrm{vac}} - \frac{8}{11}(\varepsilon - 3p)$$

$$G_2(T) = -\frac{\alpha_s(T)}{\pi}(\varepsilon + p)$$

$$\phi_d = \frac{\langle g^3 f G^3 \rangle}{(4m_Q^2)^3}$$

# Gluon condensates in medium

## Appearance of the twist-2 gluon operator



$$= C_{G_0}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + C_{G_2} \frac{q^\mu q^\nu}{q^2} \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\alpha} G^{a\,\alpha}_{\nu} \right\rangle$$

## Relation to thermodynamic quantities

- Trace anomaly + traceless/symmetric term

$$\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\alpha} G^{a\alpha}_{\nu} \right\rangle = \left( u_\mu u_\nu - \frac{1}{4} g_{\mu\nu} \right) G_2(T) + \frac{1}{4} g_{\mu\nu} G_0(T)$$

- Energy-momentum tensor (caveat : pure gauge!)

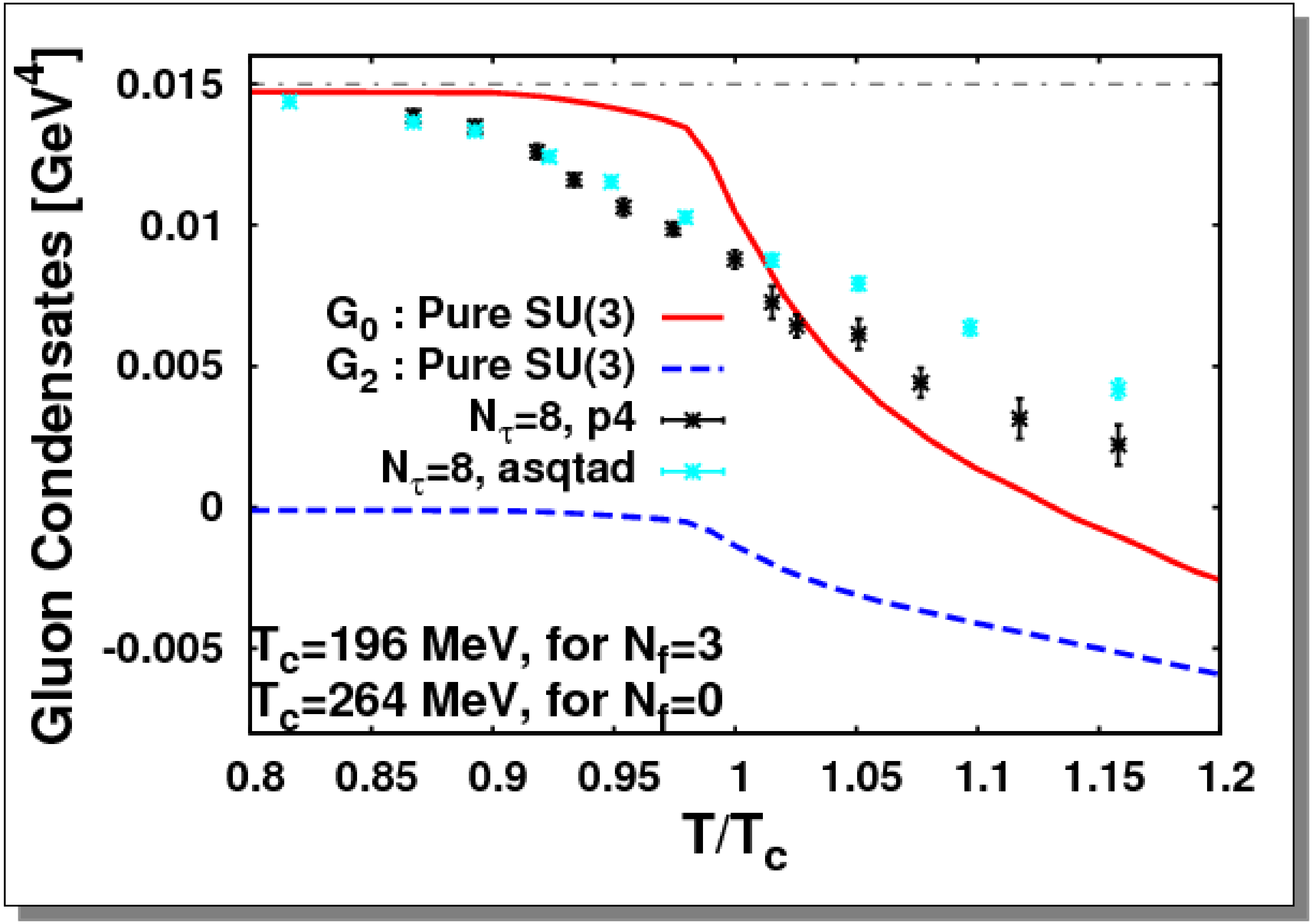
$$\langle T^\mu_\mu \rangle = \left\langle \frac{\beta(g)}{2g} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle = \varepsilon - 3p$$

Trace anomaly

$$T^{\mu\nu} = -G^{a\mu\alpha} G^{a\nu}_{\ \ \alpha} = (\varepsilon + p) u^\mu u^\nu$$

symmetric & traceless

# Gluon condensates




- **Smoother but same amount of change in Full QCD**

Full QCD : Bazavov et al., '09

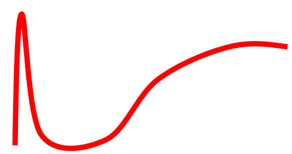
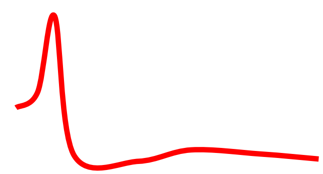
# Borel Transformation in QCDSR

- **Large $Q^2$ limit + Probing resonance (large n)**
  - **Suppression of high energy part of $\rho(s)$**

$$\mathcal{M}(M^2) = \lim_{\substack{Q^2/n \to M^2, \\ n, Q^2 \to \infty}} \frac{(Q^2)^{n+1}\pi}{n!} \left(-\frac{d}{dQ^2}\right)^n \Pi(Q^2)$$

$$= \int_{4m_Q^2}^{\infty} \underline{ds e^{-s/M^2} \mathrm{Im}\Pi(s)} \Leftarrow$$

**Dispersion relation**

$e^{-s/M^2} \times$  **=** 

$$\rho^{\mathrm{ph}}(s) = \frac{1}{\pi}\frac{f\Gamma\sqrt{s}}{(s-m^2)^2 + s\Gamma^2} + \frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{pert}}(s)\theta(s-s_0)$$

  - **Solve**

$$-\frac{\frac{1}{\partial(1/M^2)}[\mathcal{M}(M^2) - \mathcal{M}^{\mathrm{cont}}(M^2)]}{\mathcal{M}(M^2) - \mathcal{M}^{\mathrm{cont}}(M^2)} = \frac{\int_{4m_Q^2}^{\infty} ds\, s\, e^{-s/M^2} \rho^{\mathrm{pole}}(s)}{\int_{4m_Q^2}^{\infty} ds\, e^{-s/M^2} \rho^{\mathrm{pole}}(s)}$$

$$(= m^2 \quad \text{if} \quad \rho^{\mathrm{pole}}(s) = f\delta(s-m^2))$$