

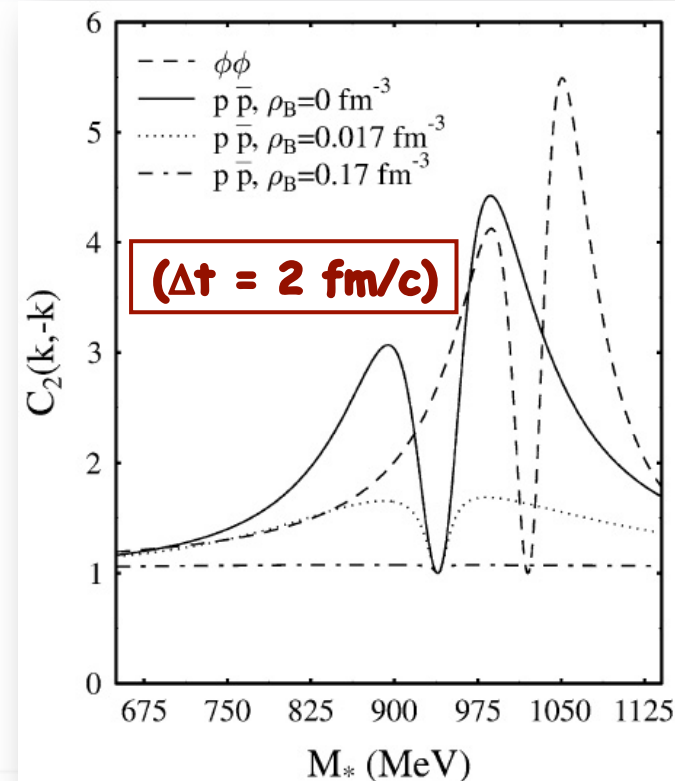
Phenomenology of the Squeezed Hadronic Correlations at RHIC Energies

Sandra S. Padula

Danuce M. Dudek & Otávio Socolowiski, Jr.

About a decade ago...

- Late 90' s: Back-to-Back Correlations (BBC) among boson-antiboson pairs \rightarrow shown to exist if the masses of the particles were modified in a hot and dense medium [Asakawa, Csörgő & Gyulassy, P.R.L. 83 (1999) 4013]
- Shortly after \rightarrow similar BBC shown to exist among fermion-antifermion pairs with medium modified masses [Panda, Csörgő, Hama, Krein & SSP, P. L. B512 (2001) 49]



\rightarrow Properties:

- Similar positive correlations with unlimited intensity of both fBBC and bBBC
- Similar formalism for both bosonic (bBBC) and fermionic (fBBC) Back-to-Back
- Expected to appear for $p_T \leq 1-2 \text{ GeV}/c$

Outline



- **Brief review on squeezed correlations**
- **The effects of time emission parametrization on squeezing: instantaneous, Lorentzian & Lévy distributions**
- **Predictions of the model**
- **Comparison with first preliminary data (PHENIX, WPCF 2009)**



Full Correlation Function ($\pi^0\pi^0, \phi\phi$)

$$\langle a_{k_1}^\dagger a_{k_2}^\dagger a_{k_1} a_{k_2} \rangle = \langle a_{k_1}^\dagger a_{k_1} \rangle \langle a_{k_2}^\dagger a_{k_2} \rangle \pm \langle a_{k_1}^\dagger a_{k_2} \rangle \langle a_{k_2}^\dagger a_{k_1} \rangle$$

NOTATION

$$\left\{ \begin{array}{l} N_1(\vec{k}_i) = \omega_{k_1} \frac{d^3 N}{d^3 k} \equiv G_c(\vec{k}_i, \vec{k}_i) \equiv G_c(i, i) = \omega_{k_1} \langle a_{k_i}^\dagger a_{k_i} \rangle \quad \text{(Spectra)} \\ G_c(\vec{k}_1, \vec{k}_2) \equiv G_c(1, 2) = \sqrt{\omega_{k_1} \omega_{k_2}} \langle a_{k_1}^\dagger a_{k_2} \rangle \quad \text{(Chaotic amplitude} \rightarrow \text{HBT)} \end{array} \right.$$



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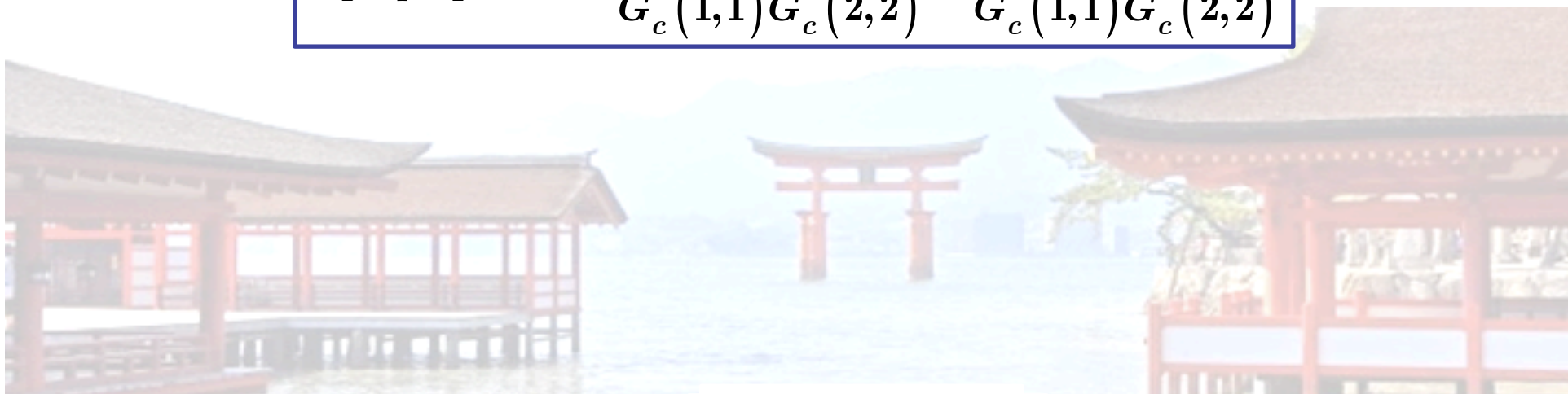
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$$\Omega_k^2 = m_*^2 + \vec{k}^2 = \omega_k^2 - \delta M^2$$

Bogolyubov transformation

$$\begin{cases} a_k^\dagger = c_k^* b_k^\dagger + s_{-k} b_{-k} \\ a_k = c_k b_k + s_{-k}^* b_{-k}^\dagger \end{cases}$$

$$f_k = \frac{1}{2} \ln(\omega_k / \Omega_k) \leftrightarrow \text{Squeezing parameter}$$

$$c_k = \cosh[f_k] \quad s_k = \sinh[f_k]$$

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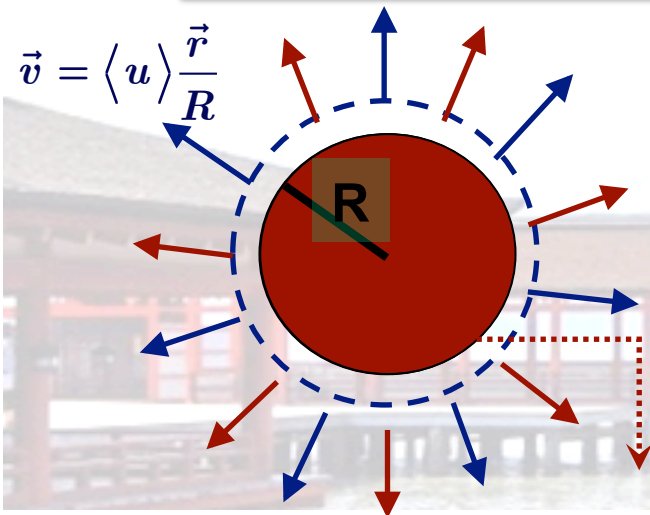
limit of no-squeezing: $\Omega_k \rightarrow \omega_k$
 $\Rightarrow f_k \rightarrow 0 \Rightarrow s_k \rightarrow 0 \wedge c_k \rightarrow 1$

$$c_k = \cosh[f_k] \quad s_k = \sinh[f_k]$$

Finite system expanding with non-relat. flow

- Non-relativistic flow
- Neglecting flow effects on squeezing parameter $f_{i,j}$
- Simplest finite squeezing vol. profile \rightarrow analytical calculations: 3-D Gaussian \rightarrow circular cross-sectional area of radius R

$$\approx \exp[-\vec{r}^2 / (2R^2)]$$

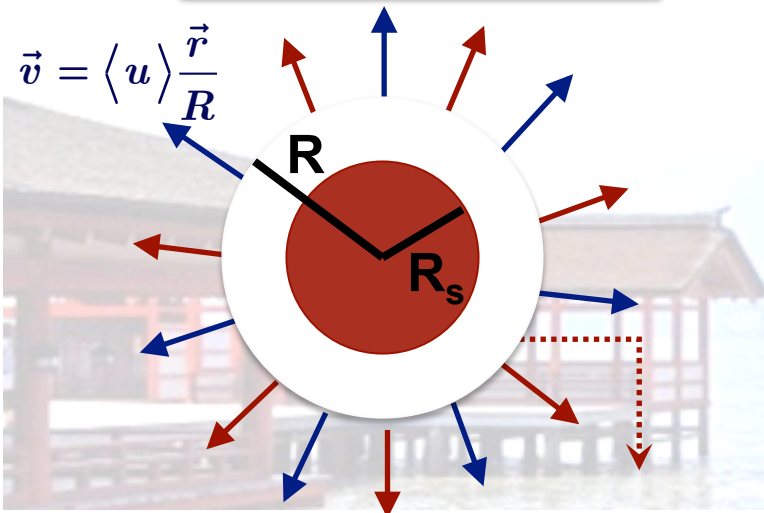


Region where mass-shift is non-vanishing

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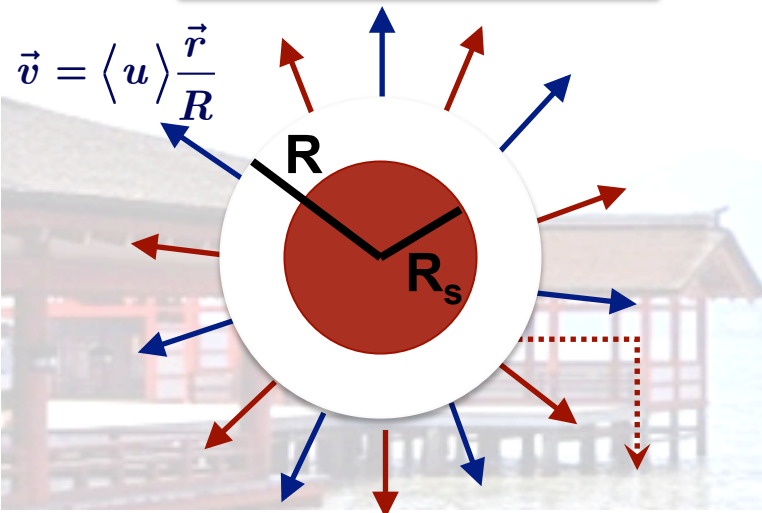


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Region where mass-shift is non-vanishing

Freeze-out

$$\tilde{F}(\Delta\tau) = \int dt E_{i,j} F(\tau_f) e^{-iE_{i,j}(\tau - \tau_0)} d\tau_f$$

- Sudden freeze-out [$\delta(\tau - \tau_0)$]

$$\tilde{F}(\Delta\tau) = E_{i,j} e^{-2iE_{i,j} \cdot \tau_0}$$

- Lorentzian distrib. [$\theta(\tau - \tau_0) e^{-(\tau - \tau_0)/\Delta\tau} / \Delta\tau$]
(finite emission)

$$\tilde{F}(\Delta\tau) = \frac{E_{i,j}}{[1 + (E_{i,j} \Delta\tau)^2]}$$

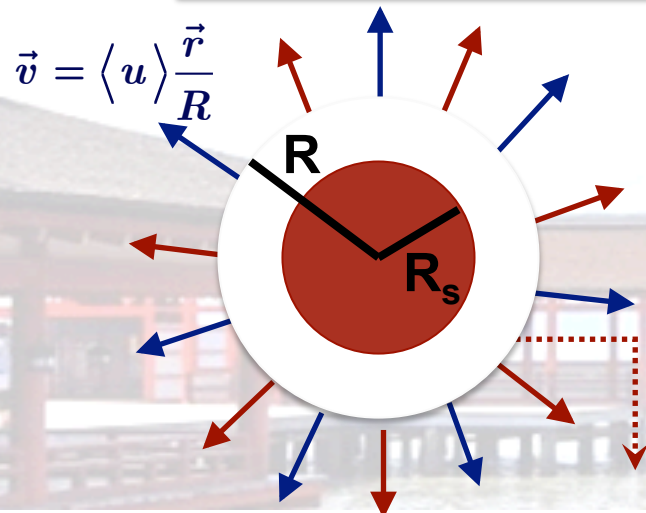
- Lévy-type distrib. (fits of PHENIX correlat.)

$$\tilde{F}(\Delta\tau) = E_{i,j} \exp\{-[\Delta\tau(\omega_1 + \omega_2)]^\alpha\}$$

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$$n_{i,j}(x) \sim \exp\left[-\left(K_{i,j}^\mu u_\mu - \mu(x)\right) / T(x)\right]$$

Hydro parameterization \rightarrow

$$\frac{\mu(x)}{T(x)} = \frac{\mu_0}{T(x)} - \frac{\vec{r}^2}{2R^2}$$

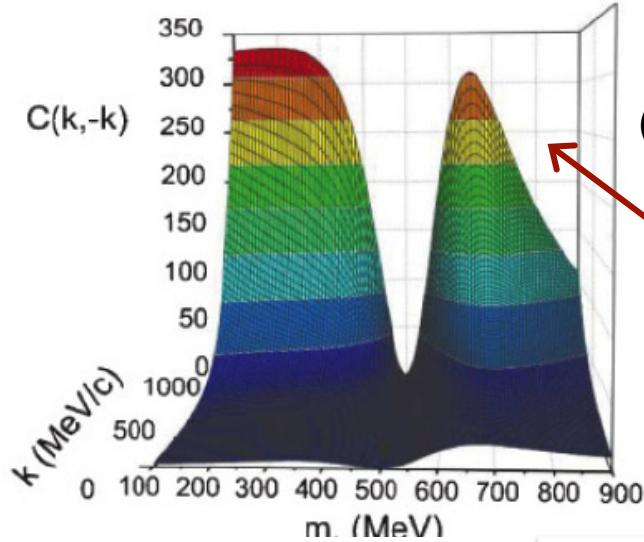
$$u^\mu = \gamma(1, \vec{v}) \quad ; \quad \vec{v} = \langle u \rangle \frac{\vec{r}}{R}$$

$$\gamma = (1 - \vec{v}^2)^{-1/2} \approx 1 + \frac{1}{2}\vec{v}^2 \quad [\mathcal{O}(v^2)]$$

K^+K^- BBC pairs \rightarrow m_* scan

Instant time emission

$R = 7$ fm $T = 177$ MeV $\Delta t = 0$ fm/c $\langle u \rangle = 0.5$

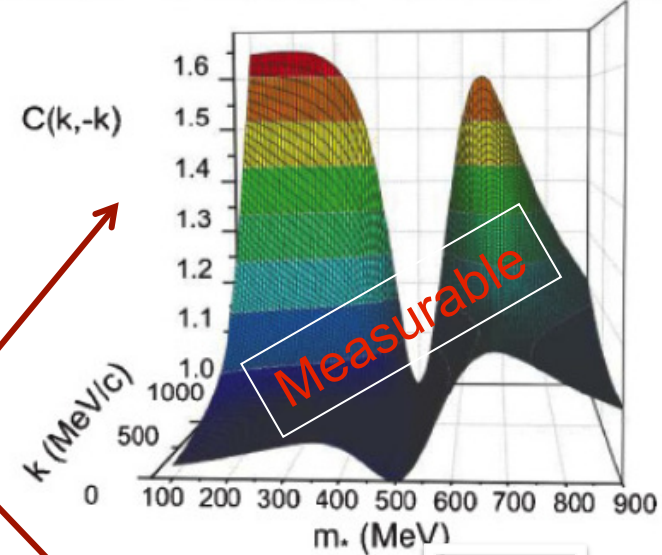


(a)

Maximal values for each $(m_*, |k|)$, i.e., for strict back-to-back pairs [\sim equiv. HBT intercept of correlation function]

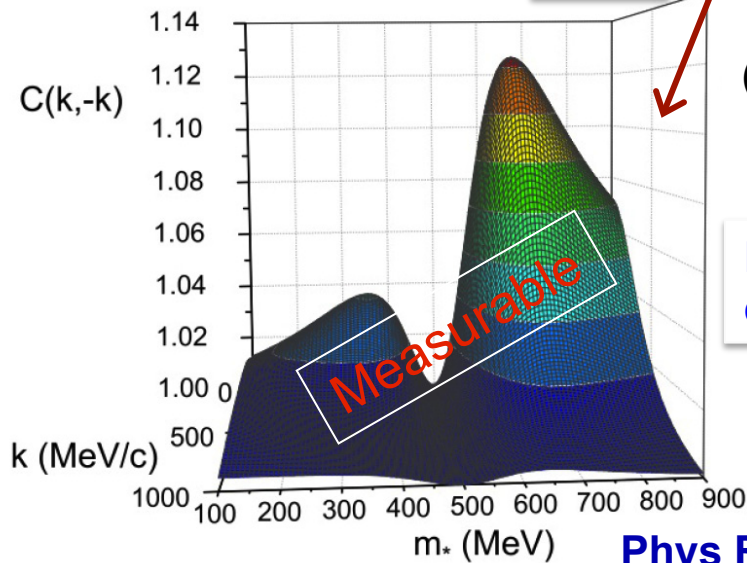
Lorentzian time distr.

$R = 7$ fm $T = 177$ MeV $\Delta t = 2$ fm/c $\langle u \rangle = 0.5$



(b)

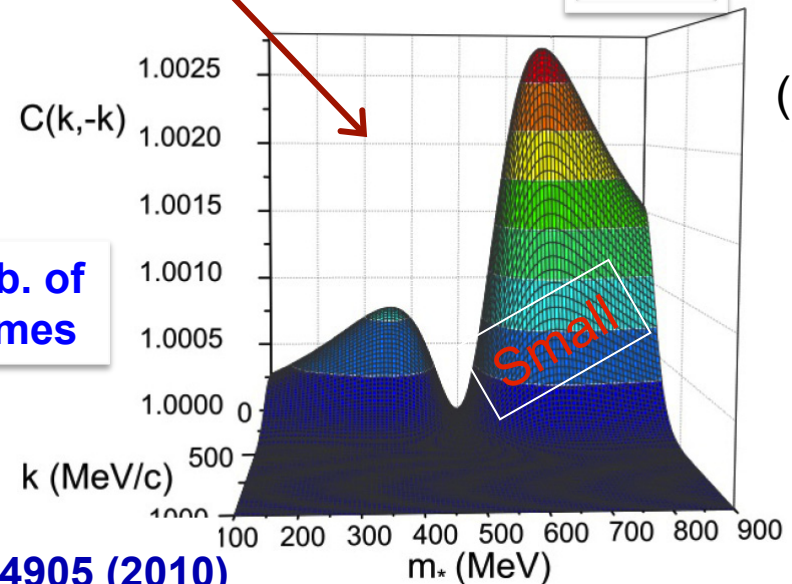
$R = 7$ fm $T = 177$ MeV $\Delta t = 1$ fm/c $\alpha = 1$ $\langle u \rangle = 0.5$



(c)

Lévy distrib. of emission times

$R = 7$ fm $T = 177$ MeV $\Delta t = 1$ fm/c $\alpha = 1.35$ $\langle u \rangle = 0.5$



(d)

For the experimental search

m_* scan \Rightarrow

- ◆ Necessary \rightarrow to determine maximum values of squeezed correlation function

$$C_s(m_*, \vec{k}_1 = -\vec{k}_2 = \vec{k})$$

- ◆ $m_* \rightarrow$ in-medium, not measurable
- ◆ Momenta \rightarrow measurable with finite precision

$$|\vec{k}_1| - |\vec{k}_2| \leq \Delta \vec{k} \neq 0$$

Two main steps:

- 1) Rewrite theoretical $C_s(k_1, k_2)$ in terms of K and q :

$$\begin{aligned} 2 * \vec{K}_{i,j} &= (\vec{k}_i + \vec{k}_j) \\ \vec{q}_{i,j} &= (\vec{k}_i - \vec{k}_j) \end{aligned}$$

The effect is maximal for

$$\vec{k}_1 = -\vec{k}_2 = \vec{k}$$

i.e., for $\vec{K}_{12} = 0 \rightarrow$ study for different values of q

- 2) Combine particle-antiparticle pair
 - \rightarrow Theory: generate (k_1, k_2) -simulation
 - \rightarrow Experiment: combine pairs
(Same event)/(Mixed event)

For the experimental search

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$$|\vec{k}_1| - |\vec{k}_2| \leq \Delta \vec{k} \neq 0$$

- ◆ Relativistic extension (M. Nagy)

$$Q_{back} = (\omega_1 - \omega_2, \vec{k}_1 + \vec{k}_2) = (q_{12}^0, 2\vec{K}_{12})$$

$$Q_{bbc}^2 = -Q_{back}^2 \approx (2\vec{K}_{12})^2$$

Non-relativistic limit

Two main steps:

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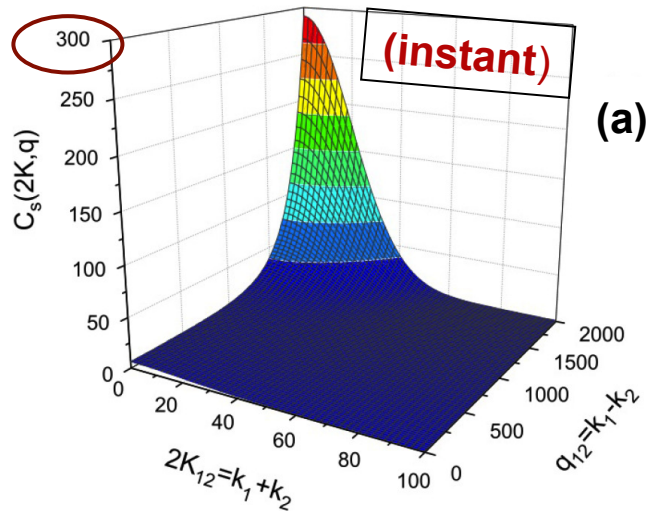
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 - \rightarrow Experiment: combine pairs (S event)/(D event)

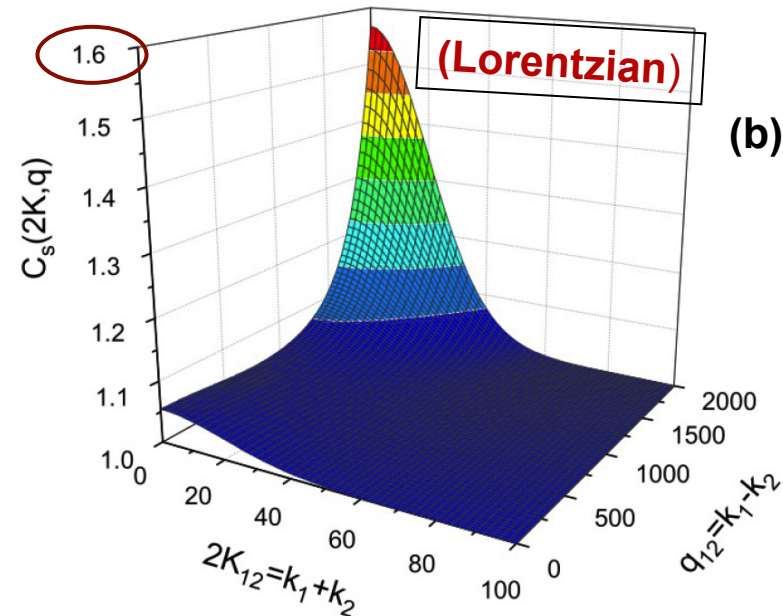
Squeezed correlations of K^+K^- pair: predictions

Choose first maximum of $C_s(k_1, k_2)$ in $(m_*, \vec{k}_1 = -\vec{k}_2 = \vec{k})$ plane $\leftrightarrow m_* = 350\text{MeV}$

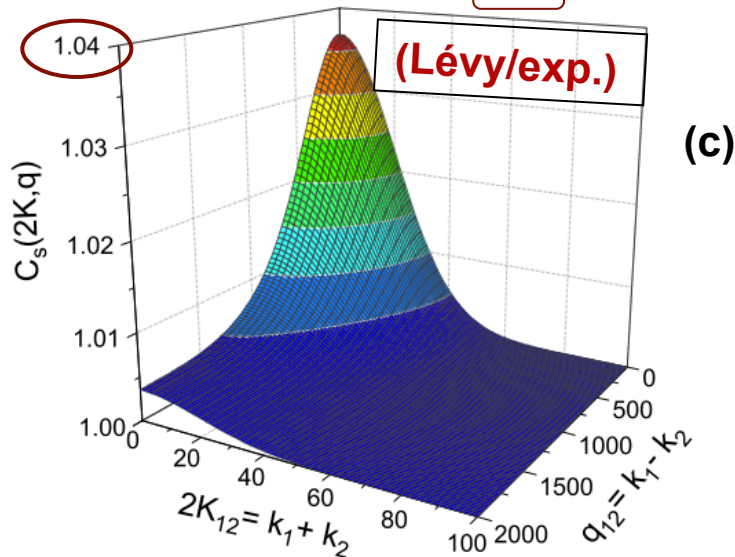
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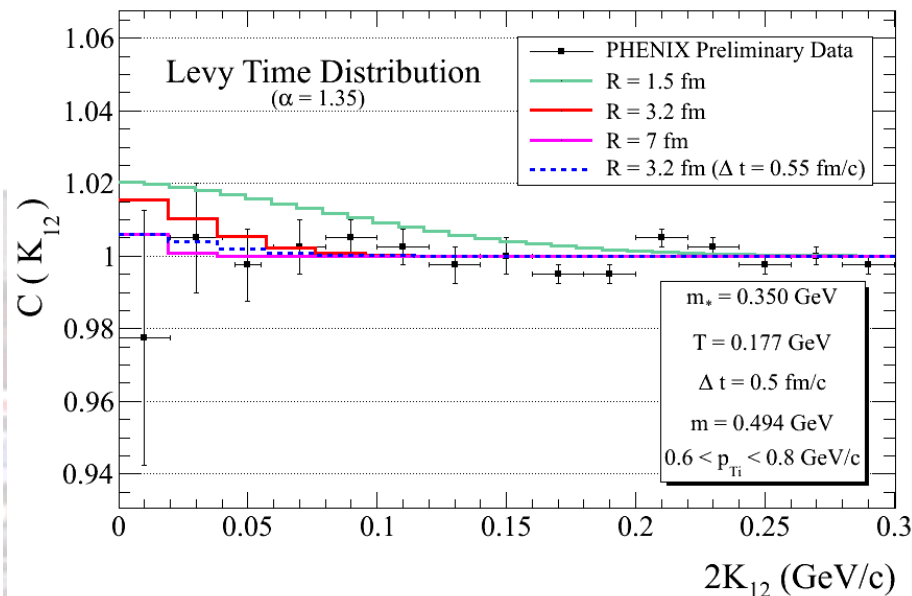
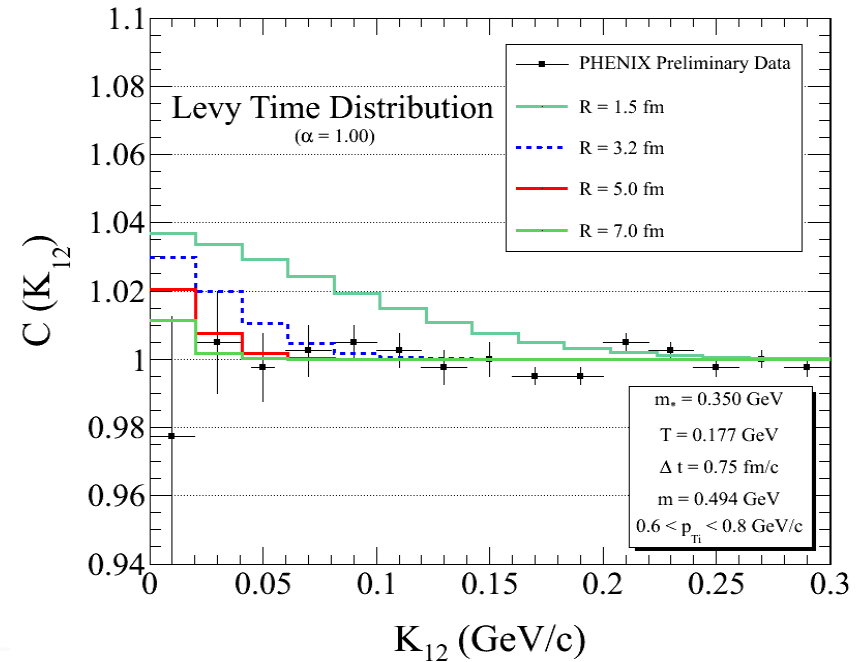
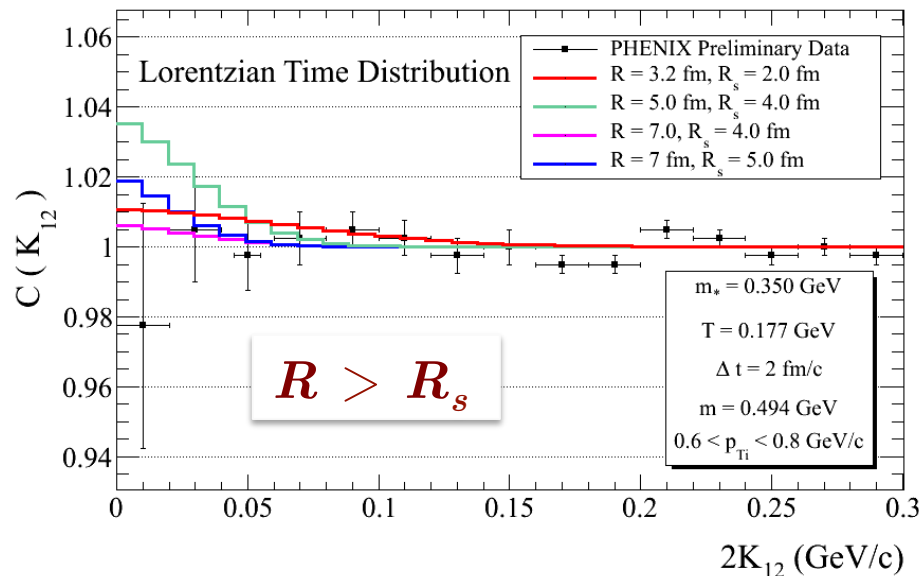
R = 7 fm T = 177 MeV $m_* = 350$ MeV $\alpha = 1$ $\Delta t = 1$ fm/c $\langle u \rangle = 0.5$



- Time emission distribution \rightarrow unknown
- Possibilities:
 - Instant emission (delta function)
 - Lorentzian emission \rightarrow reduces signal by ~ 20 times
 - Lévy ($\alpha = 1 \leftrightarrow$ exponential emission) \rightarrow reduces signal by \sim factor of 300 (but measurable)

All above predictions \leftrightarrow expanding systems

Comparison to preliminary data



Partial conclusions

- Evidence of mass modification \rightarrow may be inconclusive, but not negative!
- Time emission distribution:
 - If **Lorentzian** \rightarrow mass modification occurring in smaller part of the system
 - If **Lévy** distribution (either w/ $\alpha_{Levy} = 1$ (exponential) or $\alpha_{Levy} = 1.35 \rightarrow$ mass modified all over the system)

First experimental search for hadronic squeezed states



- At the WPCF 2009 → Martón Nagy showed
– PHENIX *Preliminary* data

$$C_s(k_1, k_2) \text{ vs. } (|k_1 + k_2|, |k_1 - k_2|)$$

- $K^+ K^-$, $p\bar{p}$ and $\pi^+ \pi^-$.

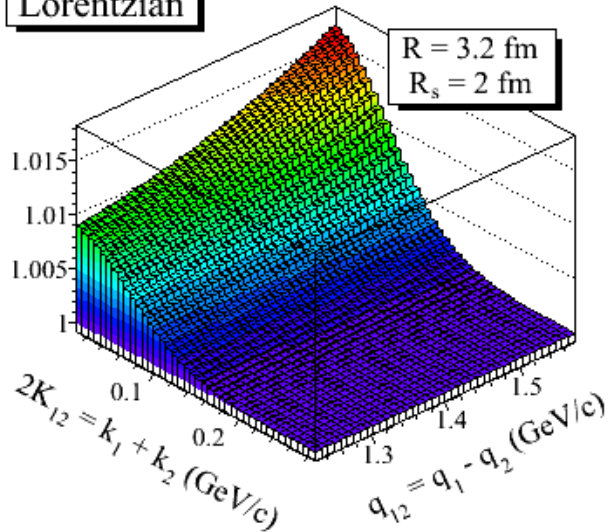
(compatible with our model for bosons)

- How do the results compare with our model?

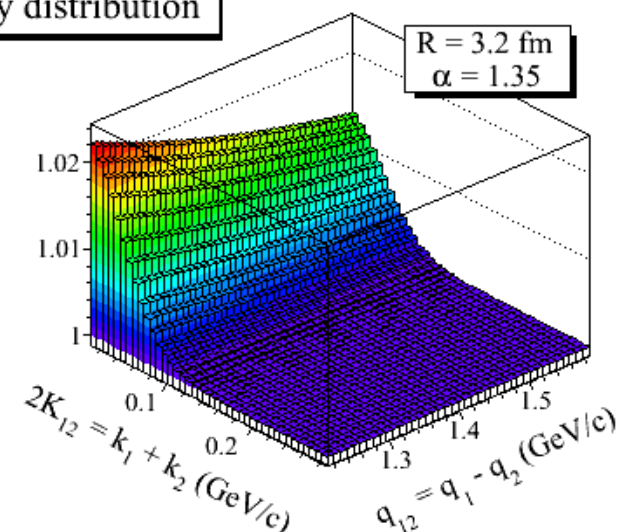
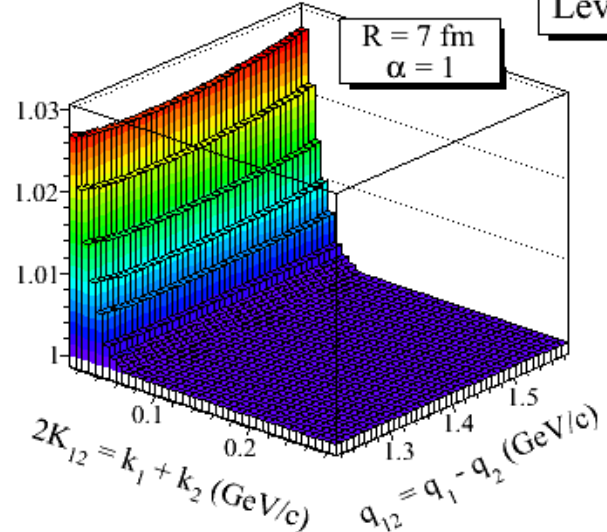
2-D $C_s(K_{12}, q_{12}) \rightarrow$ disentangle scenarios

$m_* = 0.350$ GeV, $T = 0.177$ GeV, $\langle u \rangle = 0.5$, $m = 0.494$ GeV

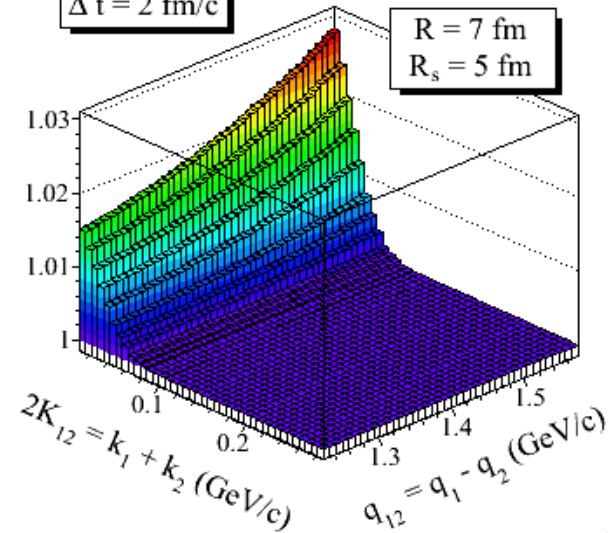
Lorentzian



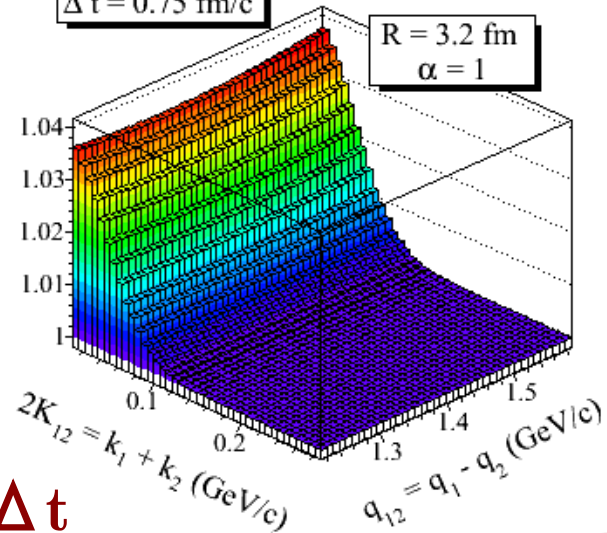
Levy distribution



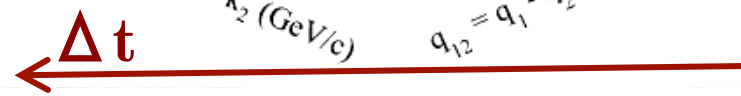
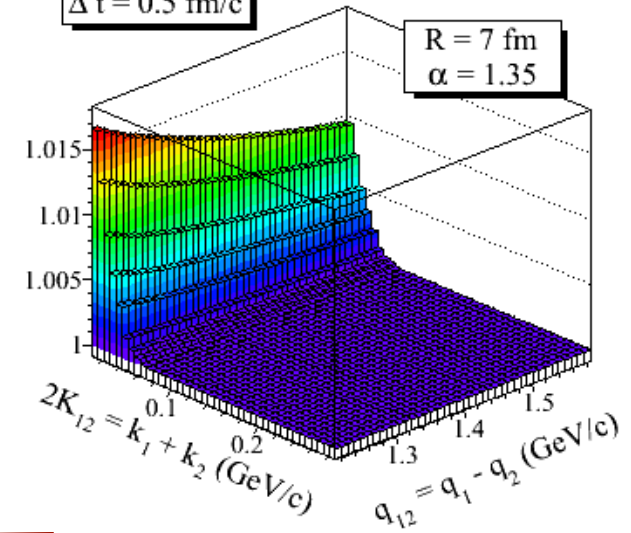
$\Delta t = 2$ fm/c



$\Delta t = 0.75$ fm/c



$\Delta t = 0.5$ fm/c



Comments & Conclusions



– 1 Considering $K^+ K^-$ squeezed correlations

- Focused on the role of the time emission interval
 - Lorentzian time factors \rightarrow strongly suppresses squeezing signal
 - Lévy time factor \rightarrow reduces the intensity even more drastically:
 - » $\alpha = 1$; $\Delta\tau = 1\text{fm}/c$ \rightarrow intensity still measurable
 - » $\alpha = 1.35$; $\Delta\tau = 1\text{fm}/c$ \rightarrow too small
- However, a priori, emission process is unknown

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– 2 Comparison with PHENIX preliminary data

- Results inconclusive but do not exclude the signal so far:
 - » Lorentzian in time \rightarrow compatible w/ squeezing all over $R_s < R$
 - » Lévy distrib. \rightarrow compatible w/ squeezing over R
- Higher statistics required \rightarrow perhaps we already have the signal:
 - » $C_s(k_1 + k_2, k_1 - k_2)$ vs. $(k_1 + k_2, k_1 - k_2)$ \rightarrow enhances differences

EXTRA SLIDES



Correlation for strict BBC pairs

$$C_s(k, -k) \sim 1 + \frac{|c_0|^2 |s_0|^2 R^6}{|s_0|^4 R^6} \sim 1 + \frac{|c_0|^2}{|s_0|^2} \xrightarrow[m_0 \rightarrow 1]{m \sim m_*} \text{div.}$$

$$2 * K_{i,j}^\mu = (k_i + k_j)$$

$$q_{i,j}^\mu = (k_i - k_j)$$

$$\langle u \rangle = 0 ; |k| \gg 1$$

$$k_2 = -k_1 = k$$

$$C_s(k, -k) = 1 + \left\{ |c_0| |s_0| \left[R^3 + 2 \left(\frac{R^2}{\left(1 + \frac{m^2 \langle u \rangle^2}{m_* T}\right)} \right)^{\frac{3}{2}} \exp\left(-\frac{m_*}{T} - \frac{k^2}{2m_* T}\right) \right]^2 \right\} \times$$

$$\left\{ |s_0|^2 R^3 + \left(|c_0|^2 + |s_0|^2 \right) \left(\frac{R^2}{\left(1 + \frac{m^2 \langle u \rangle^2}{m_* T}\right)} \right)^{\frac{3}{2}} \exp\left(-\frac{m_*}{T} - \frac{k^2}{2m_* T} + \frac{m^2 \langle u \rangle^2 k^2 / m_*^2}{\left(1 + \frac{m^2 \langle u \rangle^2}{m_* T}\right) 2T^2}\right) \right\}^{-2}$$

Squeezed Correlation vs. k_1 & k_2

$$2 * \vec{K} = \vec{k}_1 + \vec{k}_2$$

$$\vec{q} = \vec{k}_1 - \vec{k}_2$$

$$G_s(k_1, k_2) = \frac{E_{1,2}}{(2\pi)^{3/2}} c_{12} s_{12} \left\{ \begin{array}{l} R^3 \exp\left(-\frac{R^2(k_1 + k_2)^2}{2}\right) + 2 n_0^* R_*^3 \exp\left(-\frac{(k_1 - k_2)^2}{8m_*T}\right) \times \\ \exp\left[\left(-\frac{im\langle u \rangle R}{2m_*T_*} - \frac{1}{8m_*T_*} - \frac{R_*^2}{2}\right)(k_1 + k_2)^2\right] \end{array} \right\}$$

$$2 * \vec{K} = \vec{k}_1 + \vec{k}_2$$

Remember: $2 * \vec{K} = \vec{k}_1 + \vec{k}_2$, $\vec{q} = \vec{k}_1 - \vec{k}_2$

$$G_c(k_i) = \frac{E_{i,i}}{(2\pi)^{3/2}} \left\{ |s_{ii}|^2 R^3 + n_0^* R_*^3 \left(|c_{ii}|^2 + |s_{ii}|^2 \right) \exp\left(-\frac{k_i^2}{2m_*T_*}\right) \right\}$$

$$R_* = R \sqrt{\frac{T}{T_*}}$$

$$C_s(\vec{k}_1, \vec{k}_2) = 1 + \frac{|G_s(\vec{k}_1, \vec{k}_2)|^2}{G_c(\vec{k}_1, \vec{k}_1) G_c(\vec{k}_2, \vec{k}_2)}$$

$$T_* = \left(T + \frac{m^2 \langle u \rangle^2}{m_*} \right)$$

Formalism (fermions)

$$H = H_0 + H_I \quad ; \quad H_0 = \int d\vec{x} : \bar{\psi}(x) (-i\vec{\gamma} \cdot \vec{\nabla} + M) \psi(x) :$$

$$\psi(x) = \frac{1}{V} \sum_{\lambda, \lambda', \vec{k}} (u_{\lambda, \vec{k}} a_{\lambda, \vec{k}} + v_{\lambda', -\vec{k}} a_{\lambda', -\vec{k}}^\dagger) e^{i\vec{k} \cdot \vec{x}}$$

$$\langle a_{k_1}^\dagger a_{k_2}^\dagger a_{k_1} a_{k_2} \rangle = \langle a_{k_1}^\dagger a_{k_1} \rangle \langle a_{k_2}^\dagger a_{k_2} \rangle - \langle a_{k_1}^\dagger a_{k_2} \rangle \langle a_{k_2}^\dagger a_{k_1} \rangle + \langle a_{k_1}^\dagger a_{k_2}^\dagger \rangle \langle a_{k_1} a_{k_2} \rangle$$

- **System described** by quasi-particles \rightarrow medium effects taken into account through self-energy function
- For spin-1/2 particles under mean fields in a many body system:

$$\Sigma^s + \gamma^0 \Sigma^0 + \gamma^i \Sigma^i \quad \rightarrow \text{to be determined by detailed calculation}$$

- $\Sigma^s \rightarrow$ notation: $\Sigma^s(k) = \Delta M(k)$
- $\Sigma^1 \rightarrow$ very small \rightarrow neglected
- $\Sigma^0 \rightarrow$ weakly-dependent on momentum \rightarrow totally thermalized medium:
 $m_* = m - \Sigma^0 \rightarrow$ (results for net baryon number)
- Hamiltoniana $H_1 \rightarrow$ describes a system of quasi-particles with mass-dependent momentum $m_* = m - \Delta M(k)$

bBBC & fBBC - formalism summary

• Bosonic BBC

$$c_k = \cosh[f_k] ; s_k = \sinh[f_k]$$

$$\begin{cases} a_k^\dagger = c_k b_k^\dagger + s_{-k} b_{-k} \\ a_k = c_k b_k + s_{-k}^* b_{-k}^\dagger \end{cases}$$

$$\begin{cases} f_k \equiv r_k^{ACG} = \frac{1}{2} \log \left(\frac{\omega_k}{\Omega_k} \right) \\ \omega_k^2 = m^2 + \vec{k}^2 \\ \Omega_k^2 = \omega_k^2 - \delta M^2(|k|) \\ m_*^2 = m^2 - \delta M^2(|k|) \end{cases}$$

• Fermionic BBC

$$c_k = \cos[f_k] ; s_k = \sin[f_k]$$

$$\begin{pmatrix} a_{\lambda,k} \\ \tilde{a}_{\lambda',-k}^\dagger \end{pmatrix} = \begin{pmatrix} c_k & \frac{f_k}{|f_k|} s_k A \\ -\frac{f_k^*}{|f_k|} s_k^* A^\dagger & c_k^* \end{pmatrix} \begin{pmatrix} b_{\lambda,k} \\ \tilde{b}_{\lambda',-k}^\dagger \end{pmatrix}$$

$$A = [\chi_\lambda^\dagger(\sigma \cdot \hat{k}) \tilde{\chi}_{\lambda'}] ; A^\dagger = [\tilde{\chi}_{\lambda'}^\dagger(\sigma \cdot \hat{k})^\dagger \chi_\lambda]$$

$$\tilde{\chi}_{\lambda'} = -i\sigma^2 \chi_{\lambda'} ; \hat{k} = \vec{k}/|\vec{k}|$$

$$\tan(2f_k) = -\frac{|k| \Delta M(k)}{\omega_k^2 - \Delta M(k)M}$$

$$m_*(k) = m - \Delta M(k)$$

$$\omega_k^2 = m^2 + \vec{k}^2 ; \Omega_k^2 = m_*^2 + \vec{k}^2$$