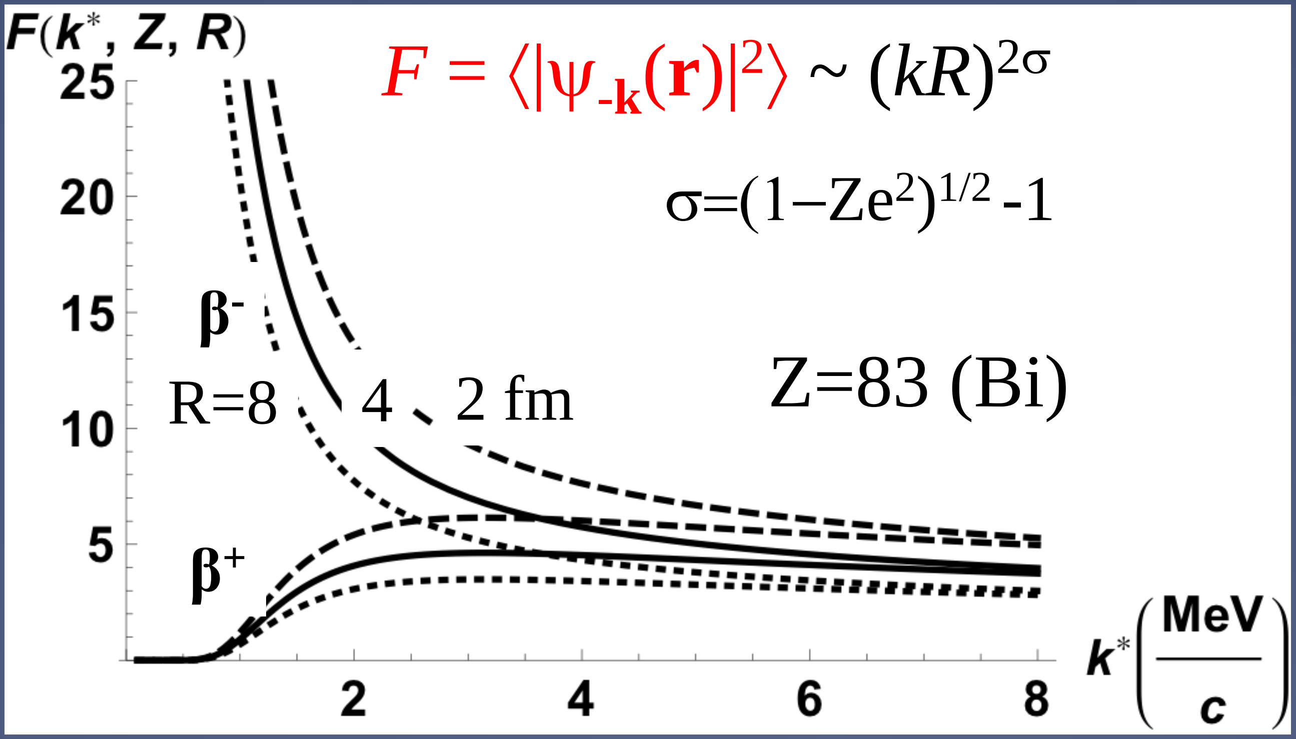
Femtoscopic Correlations and Final State Narrow Resonance Formation

R. Lednický @ JINR Dubna & IP ASCR Prague
P. Chaloupka and M. Šumbera @ NPI ASCR Řež

* Introduction
* Review of experimental results on $\pi^+\Xi^-$ & K^+K^- CF
* Narrow resonance FSI contributions to $\pi^+\Xi^-$ & K^+K^- CF's
* Conclusions

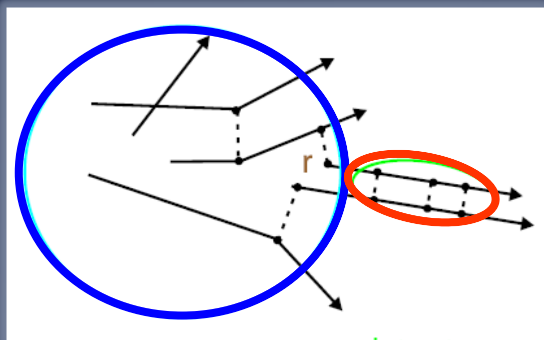
Fermi function F(k,Z,R) in β-decay

- Coulomb field of daughter nucleus decelerates **e**⁻ and accelerates **e**⁺ altering the shape of the ®–spectrum.
- The effect of the Coulomb field on **e**± distribution is included via Fermi function or Coulomb correction factor, F(k,Z,*R*) which is sensitive to nucleus radius.



R. Lednicky, Braz.J.Phys. 37 (2007) 939, e-Print: nucl-th/0702063

Correlation femtoscopy in a nutshell



Emitting source

Correlation function

Source function
(Pair separations distribution in the pair rest frame)

Two-particle w. f.(freeze-out independent, encodes FSI)

No FSI, QS only:

FSI is sensitive to source size r and scattering amplitude f
It complicates CF analysis but makes possible:

- ✔ Femtoscopy with nonidentical particles πK, πp, .. & Coalescence deuterons, ..
- ✔ Study "exotic" scattering $\pi\pi$, πK, KK, $\pi\Lambda$, $p\Lambda$, $\Lambda\Lambda$, ..
- ✔ Study relative space-time asymmetries delays, flow

Correlation asymmetries

Lednicky, Lyuboshitz, Erazmus, Nouais PLB 373(1996)30

- ✓ CF of identical particles is sensitive to terms even in **kr** (e.g. through <cos 2 **kr**>) i.e. measures only dispersion of the components of relative separation $\mathbf{r} = \mathbf{r}_1^* - \mathbf{r}_2^*$ in pair cms.

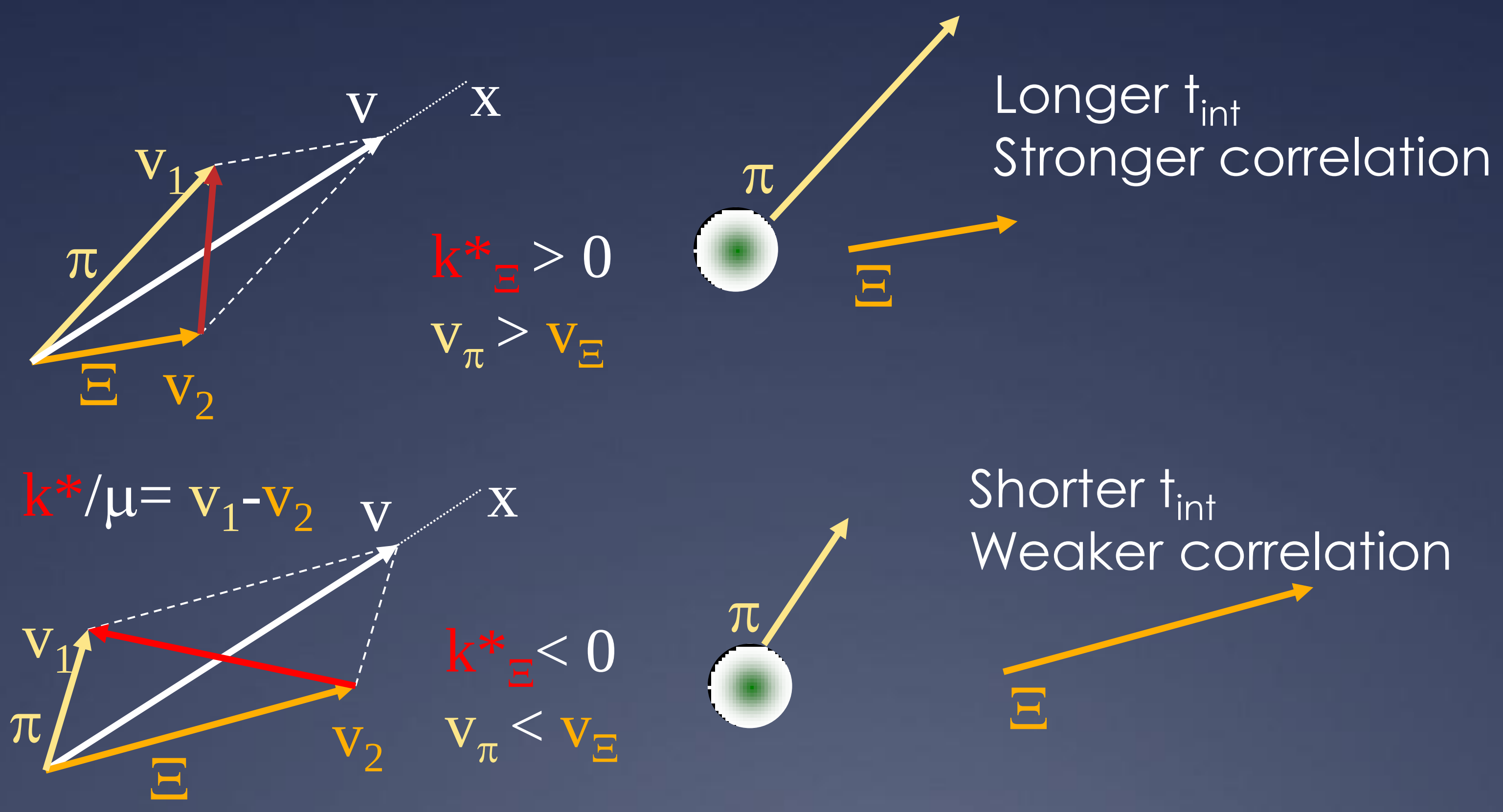
- ✓ CF of nonidentical particles is sensitive also to terms odd in **kr** □ measures also relative space-time asymmetries i.e. shifts □**r**□.

- ✓ Projection of the relative separation **r** in pair cms on the direction x: $\mathbf{r}_\mathbf{x}$□ □$x^* = x_1^* - x_2^*$. In LCMS (v_z=0) or x II v: □x^* = □$_T$(□x - v_T□t) □ CF asymmetry is determined by space and time asymmetries.

Simplified idea of CF asymmetry

(valid for Coulomb FSI)

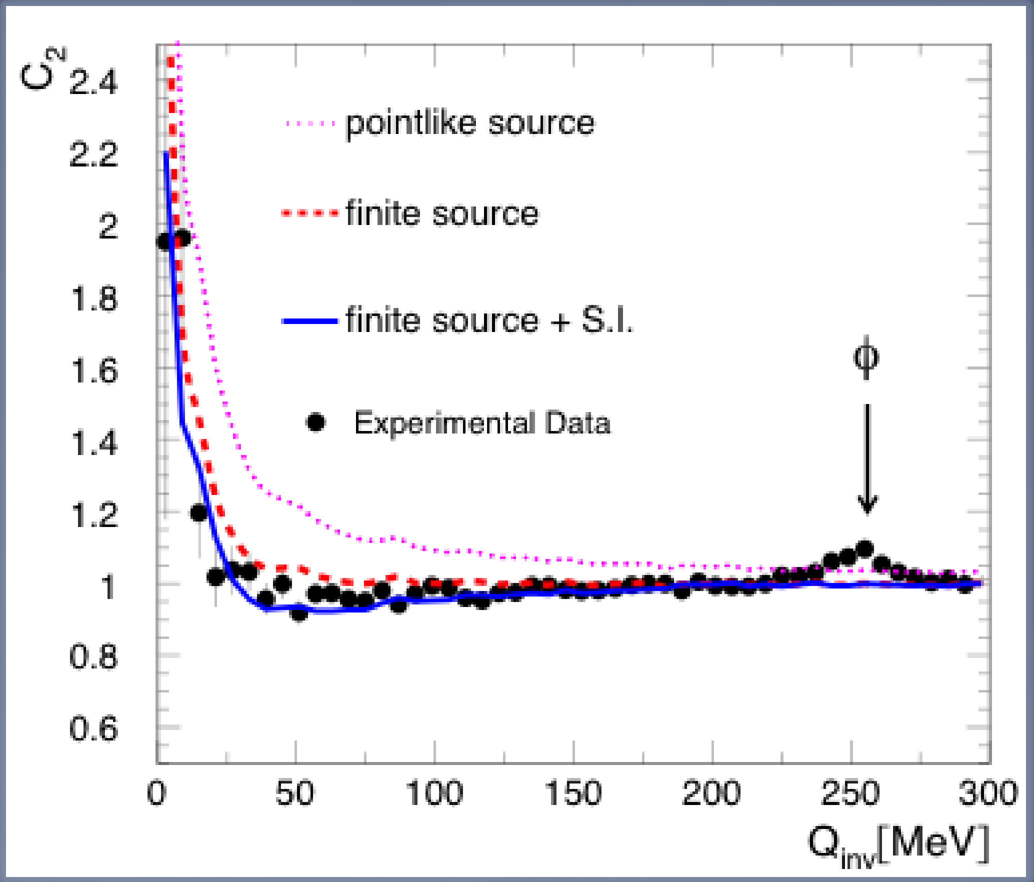
Assume □ emitted later than Ξ or closer to the center



K^+K^- correlations in Pb+Pb at SPS(NA49)

PLB 557 (2003) 157

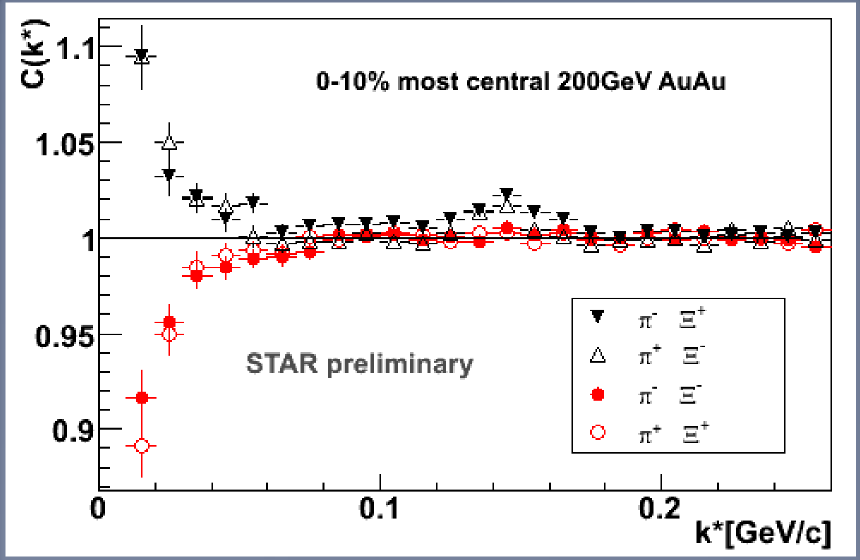
- Coulomb and strong FSI present: $\phi(1020)$, k^*=126 MeV/c, Γ=4.3 MeV
- Centrality dependence observed, particularly strong in the ϕ region; 0-5% CF peak value CF-1 ≈ 0.10 → 0.14 after purity correction
- 3D-Gaussian fit of 5% most central collisions CF's: out-side-long radii of 4-5 fm

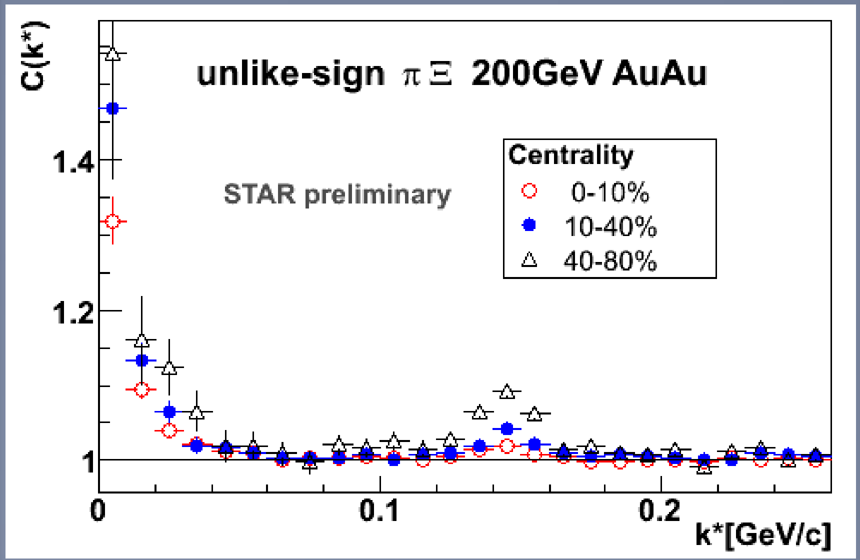


$\pi^-\Xi$ correlations in Au+Au at RHIC(STAR)

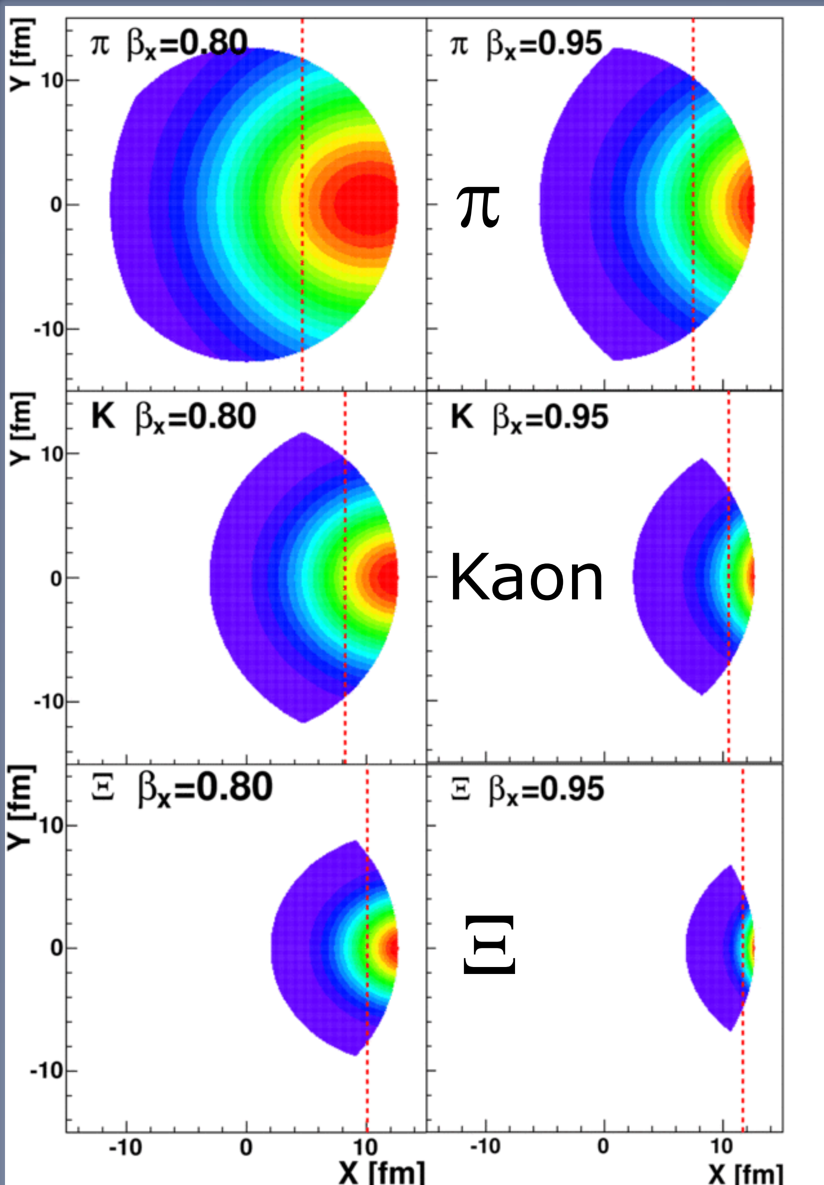
JPG 32(2006)S537
Braz.J.Phys. 37(2007)925
arXiv:1012.4591 [nucl-ex]

- Coulomb and strong FSI present $\Xi^*(1530)$, $k^*=146$MeV/c, $\Gamma=9.1$MeV
- No energy dependence seen
- Centrality dependence observed, quite strong in the Ξ^* region; 0-10% CF peak value CF-1 ≈ 0.025
- $r_0=[\frac{1}{2}(r_\pi^2+r_\Xi^2)]^{1/2} \approx 5$fm is in good agreement with the dominant contribution from r_π (≈7fm)





Effects of transverse flow



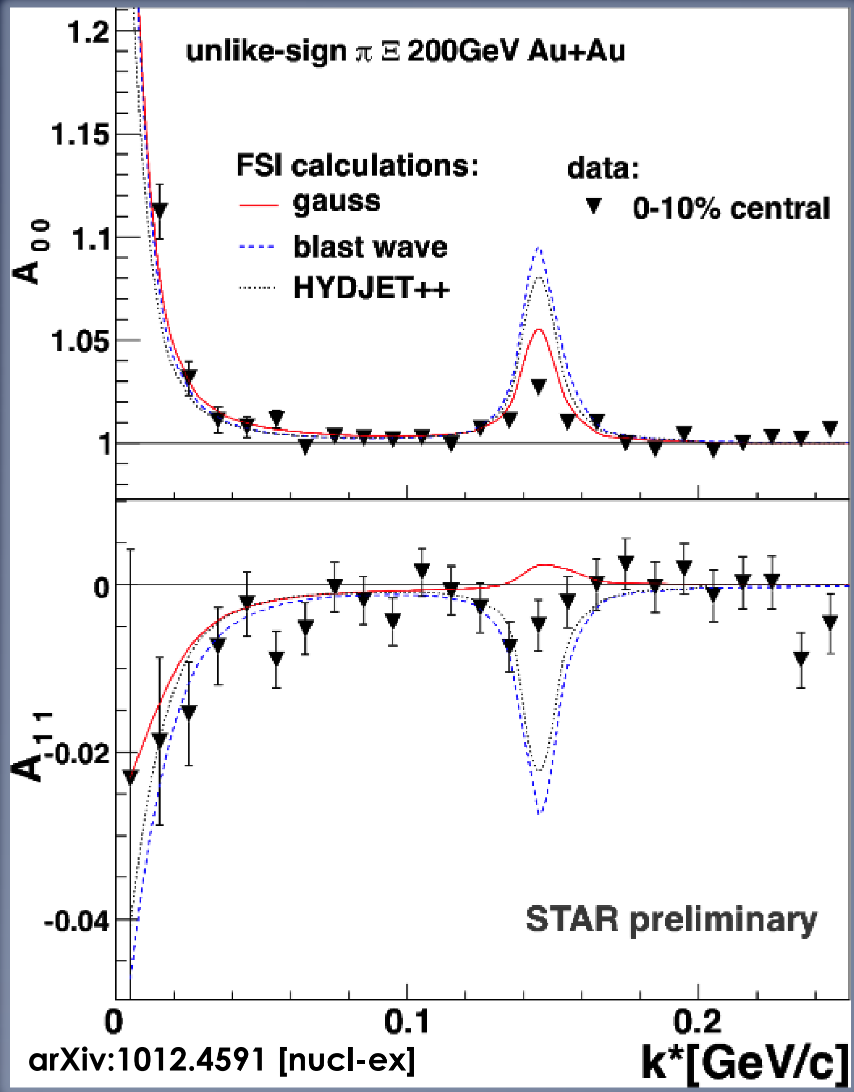
- Correlation between momentum and emission point causes effective reduction of source size and shift in average emission point
- Effect increases with m

Non-identical correlations test flow by measuring sizes and shifts of the sources

FSI model comparison

- Gaussian source predicts opposite sign of $A_{1,1}$
- Hydro based model with flow induced correlation between momenta and emission coordinates:
  - Blast wave or HYDJET++:

    particle production and resonance decays
- FSI model(S. Pratt, S. Petriconi, PRC 68 (2003) 054901)
- ☑ Qualitative reproduction of the Ξ^*region
  - overestimates both $A_{0\,0}$, $A_{1\,1}$
  - observed shift in $A_{1,1}$ agrees with flow scenario



Detailed comparison with hydro-kinetic model HYDJET++ (HKM)

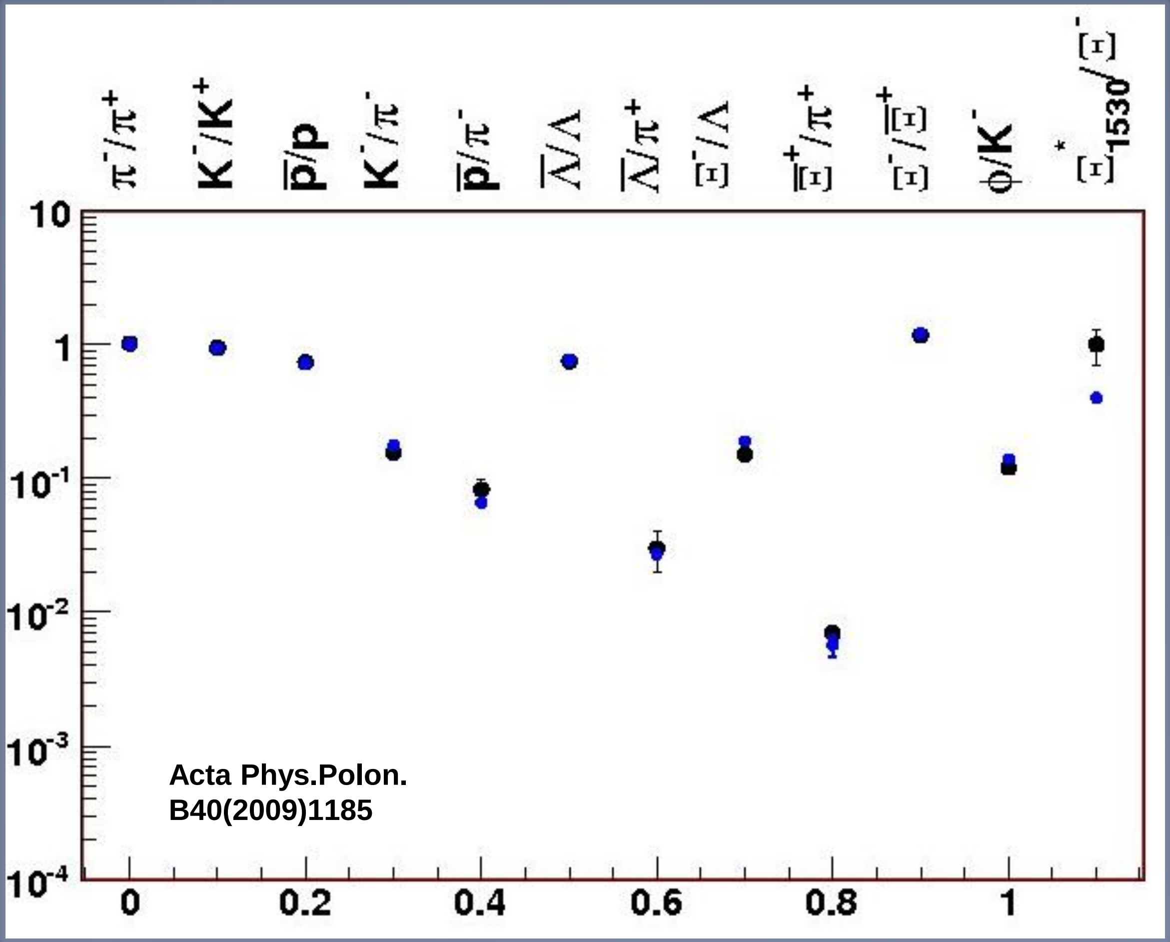
Acta Phys.Polon. B40(2009)1185

- Hydro expansion: Bjorken + Blast wave
- Effective volume of the fireball ~ $<N_{part}>$
- Particle production – statistical model
- Three different parameter sets tuned to reproduce spectra
  - $T_{ch}=T_{th}=165$ MeV
  - $T_{ch}=100$MeV, $T_{th}=165$MeV
  - Combined scenario of Ξ^* early freeze-out.
- Decays of hadronic resonances
- Fast generation of events

parameter	$T_{ch} = T_{th}$	$T_{ch} \neq T_{th}$
T_{ch},GeV	0.165	0.165
T_{th}, GeV	0.165	0.100
μ_B, GeV	0.028	0.028
μ_S, GeV	0.007	0.007
μ_Q, GeV	-0.001	-0.001
γ_S	1	1
τ, fm/c	7.0	8.0
$\Delta\tau$, fm/c	2.0	2.0
R, fm	9.0	10.0
η_{max}	2	2
ρ_u^{max}	0.65	1.1

N. Amelin *et al.*
Phys.Rev.C73:044909,2006
Phys.Rev.C74:064901,2006
Phys.Rev.C77:014903,2008
CPC 180 (2009) 779
http://uhkm.jinr.ru

Particle ratios from HKM



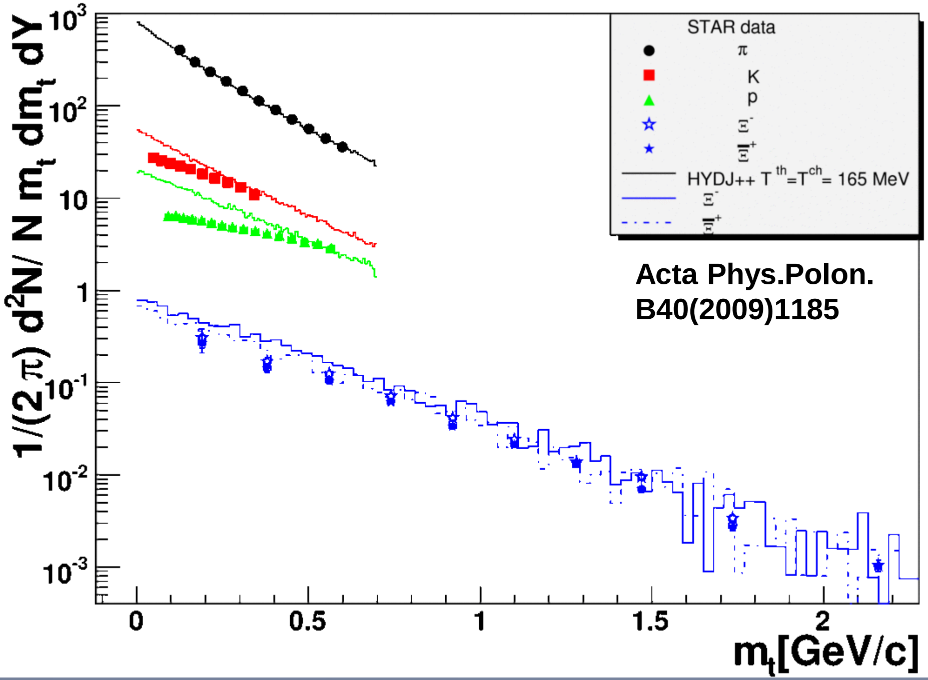
- ϕ/K^- described well
- Ξ^*/Ξ is enhanced
- Can this influence the correlation function (asymmetry)?

The particle number ratios from HYDJET++ (blue points), compared with the experimental data (black points).

Parameter	T_{ch} [MeV]	μ_B [MeV]	μ_S [MeV]	μ_Q [MeV]	γ_S
Value	0.165	0.028	0.016±0.009	-0.001	1.0

The m_t spectra from HKM

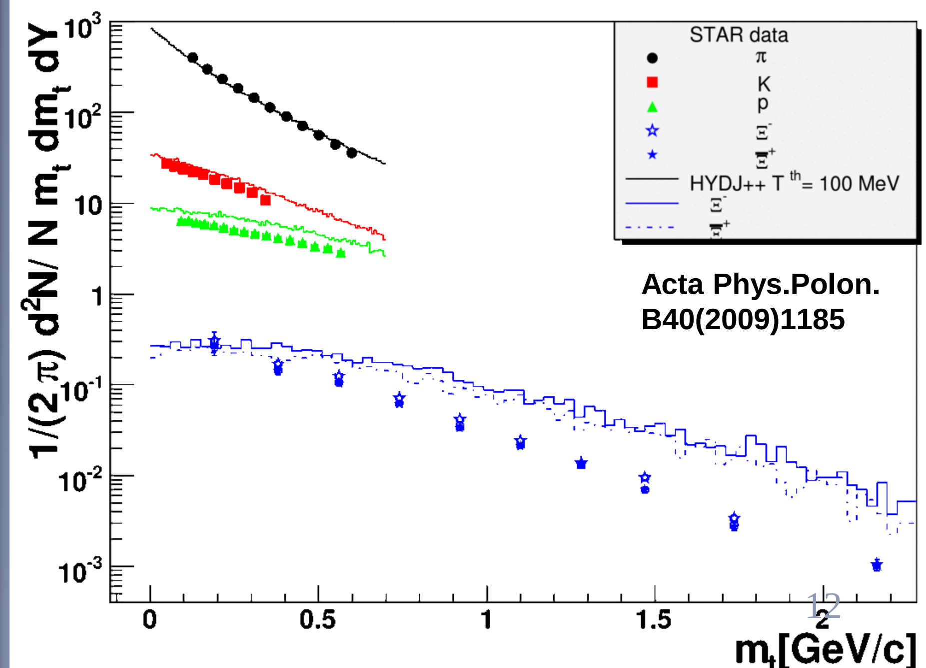
single freeze-out scenario



✓Single freeze-out describes Ξ

✓Separate freeze-out describes common particles

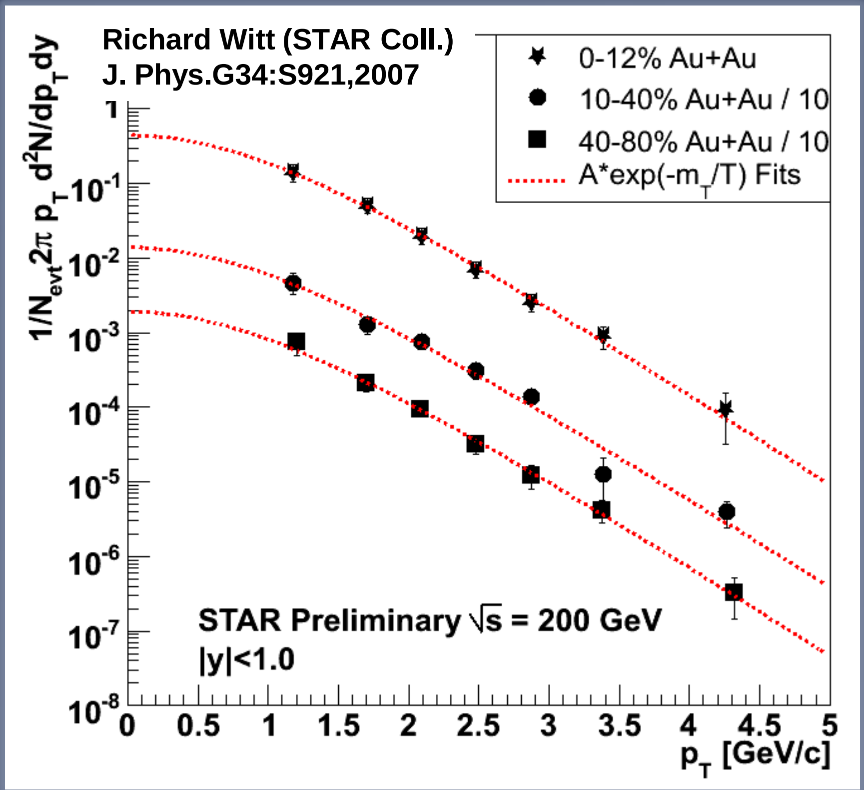
separate freeze-out scenario with T_{ch}=0.165 GeV & T_{th}=0.1 GeV

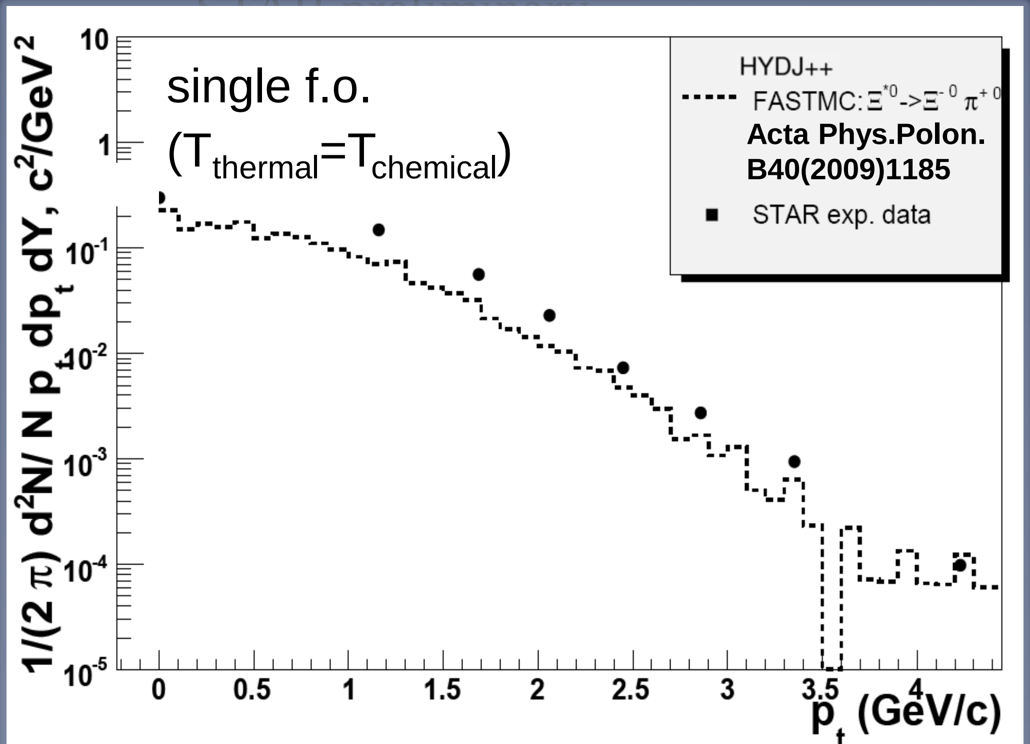


The model cannot simulate different conditions for different species....

..... but we can mix particles from both types of events

The p_t spectra of Ξ*(1530)





Centrality	Ξ*/Ξ Ratio
0-12%	**0.92 ± 0.28**
10-40%	**0.60 ± 0.12**
40-80%	**0.51 ± 0.12**

HKM

- reproduces shape of spectra with $T_{th}=T_{ch}$
- underestimates the yield

π-Ξ emission asymmetry from the model

"combined" events:

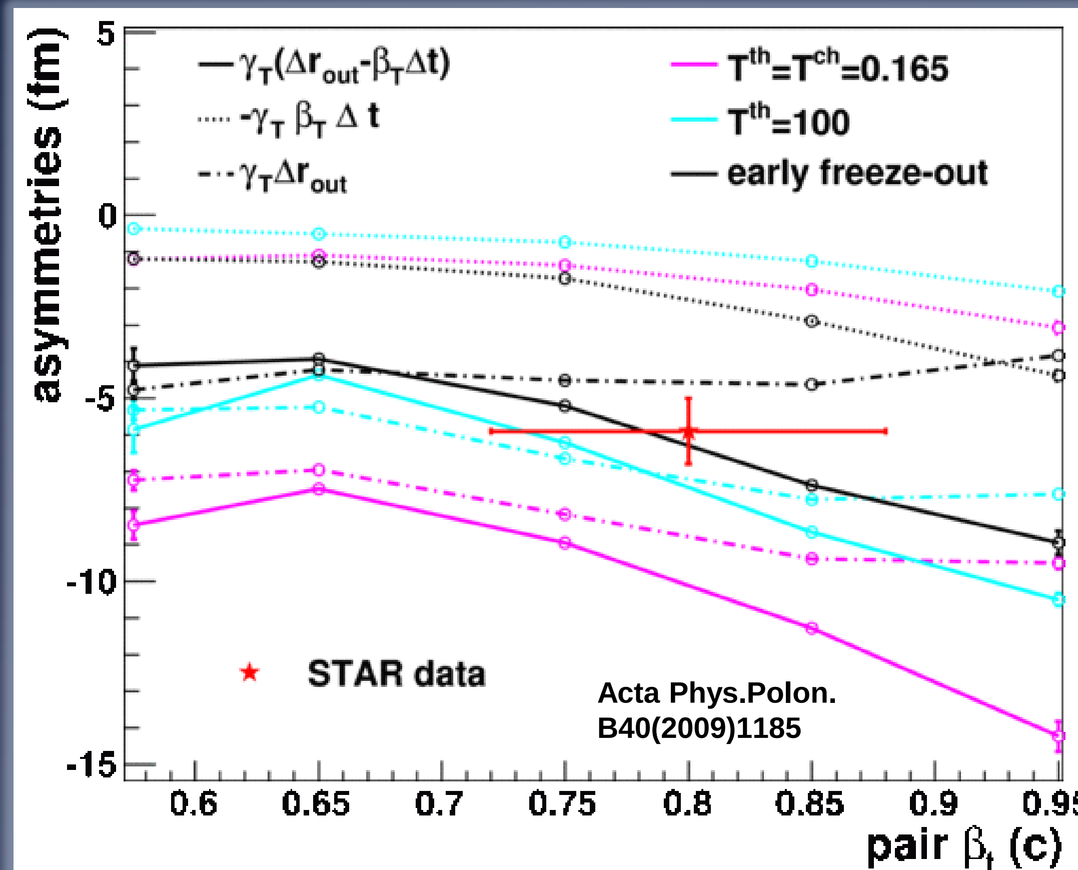
π from $T_{th} \neq T_{ch}$ events

and

Ξ from $T_{th} = T_{ch}$ events

- Difference between freeze-out scenarios.

- Decrease of the emission shift in case of Ξ freezing out at higher T_{th}.



Does the shift depend on Ξ*/Ξ ratio?

Femtoscopy approach to FSI resonance formation

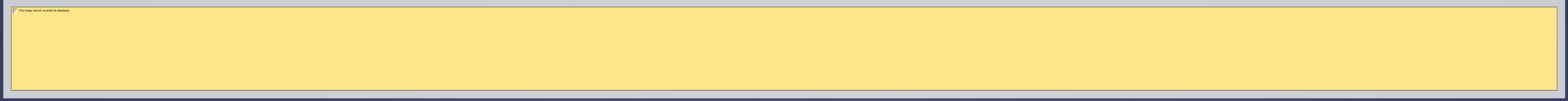
1. R. Lednicky, V.L. Lyuboshitz, SJNP 35 (1982) 770
2. R. Lednicky, V.L. Lyuboshitz, V.V. Lyuboshitz, Phys.At.Nucl. 61 (1998) 2050
3. S. Pratt, S. Petriconi, PRC 68 (2003) 054901
4. S. Petriconi, PhD Thesis, MSU, 2003
5. S. Bekele, R. Lednicky, Braz.J.Phys. 37 (2007) 994
6. B. Kerbikov, R. Lednicky, L.V. Malinina, P. Chaloupka, M. Sumbera, arXiv:0907.061v2
7. B. Kerbikov, L.V. Malinina, PRC 81 (2010) 034901
8. R. Lednicky, P. Chaloupka, M. Šumbera, in preparation

CF with narrow resonance close to the threshold

Smoothness approximation: smooth momentum dependence of the emission function ($W_P(\mathbf{r},\mathbf{k}) \approx W_P(\mathbf{r})$) compared to the FSI correlation effect at small $\mathbf{k}$:

Generalized smoothness approximation

- Caution: smoothness approx. is justified only for $k \ll 1/r_0$!!!
  □ For narrow resonance close to the threshold it must be modified:



- Smoothness approximation:

Incorporating **r-k** correlation in emission function

- BW simulations show that $b_{ij} = b_{ii}\delta_{ij}$ with $b_{ii} \approx b$ □

- Collective flow (b > 0) generates exponential suppression in the resonance region (k ≈ 150 MeV/c).

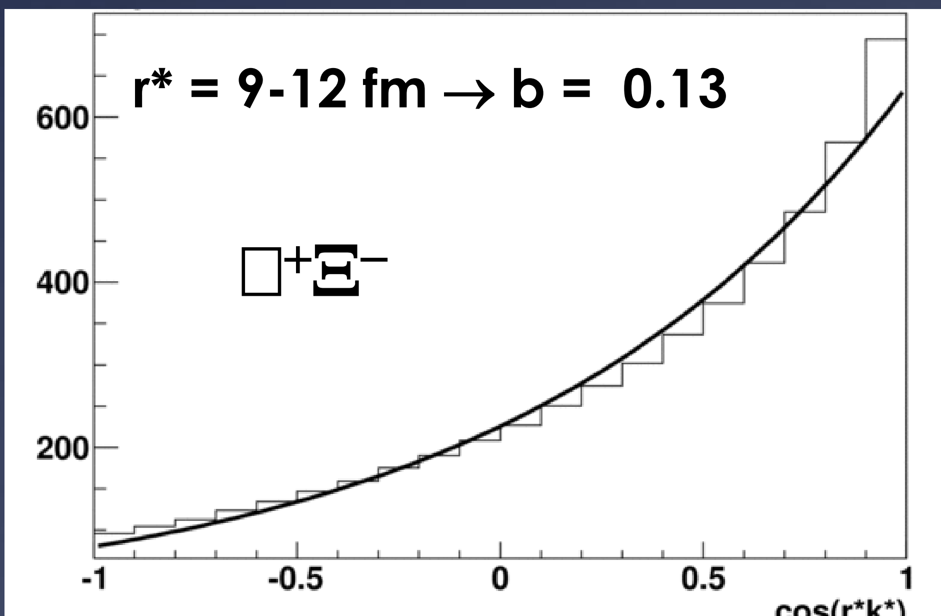
r-k correlation in the Ξ^*- and ϕ -resonance regions from HKM

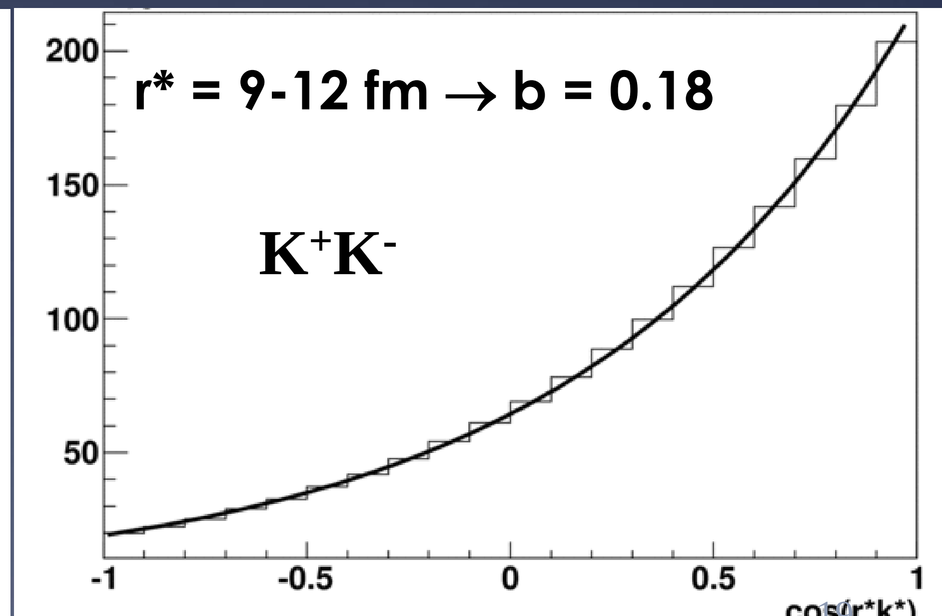
0-10% Au+Au @ $\sqrt{s_{NN}}$=200GeV

fitted by $W_P(\mathbf{r},\mathbf{k}) \sim \exp[-r^2/4r_0^2 + b k r \cos\theta]$

r*=0–27 fm: b($\pi^+\Xi^-$)=0.18÷0.08

r*=0–27 fm: b(K^+K^-)=0.32÷0.09





Approximate resonance FSI contribution

In good agreement with generalized smoothness approximation (see a figure later)

$$\Delta\mathcal{R} \approx W_P(0,\mathbf{k})\frac{2L+1}{\mathcal{N}}\frac{2\pi}{k}\left[\Re\delta_k f_L^{\alpha j;\alpha j} + \sum_{\alpha' m'} 2\Im\left(k_{\alpha'} f_L^{\alpha' m';\alpha j *}\delta_k f_L^{\alpha' m';\alpha j}\right)\right]$$

$$\exp[-\mathbf{b}^2 r_0^2 k^2 - (\Delta_{out}/2r_0)^2 - \mathbf{b}\, k_{out}\Delta_{out}]$$

Exponential suppression by the **r-k** correlation & out shift + correlation asymmetry: $\approx 1 - \mathbf{b}\, k_{out}\Delta_{out}$

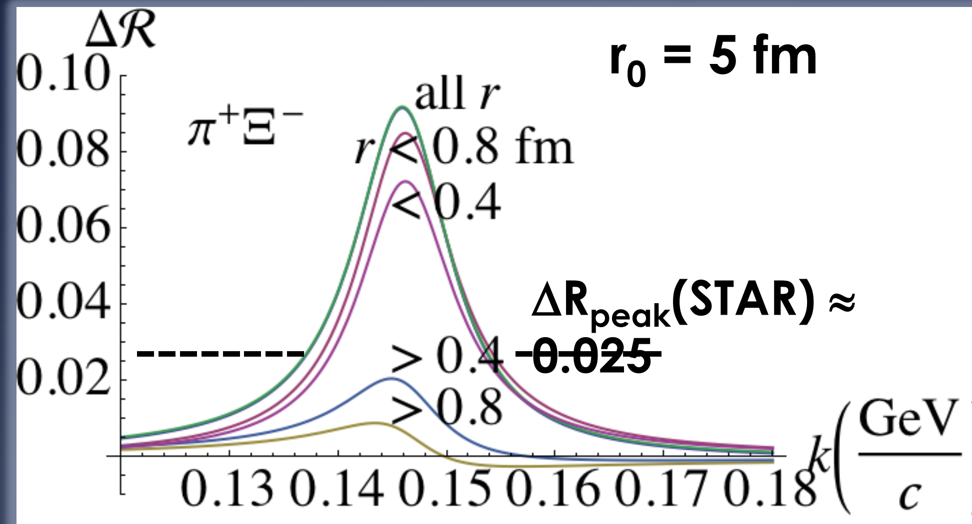
Compare with the correlation asymmetry in the Coulomb region ($k \to 0$) $\approx 1 + 2k_{out}\Delta_{out}/(k\,a)$

✓ same sign for oppositely charged particles ($a < 0$) and $b > 0$ (resulting from collective flow)

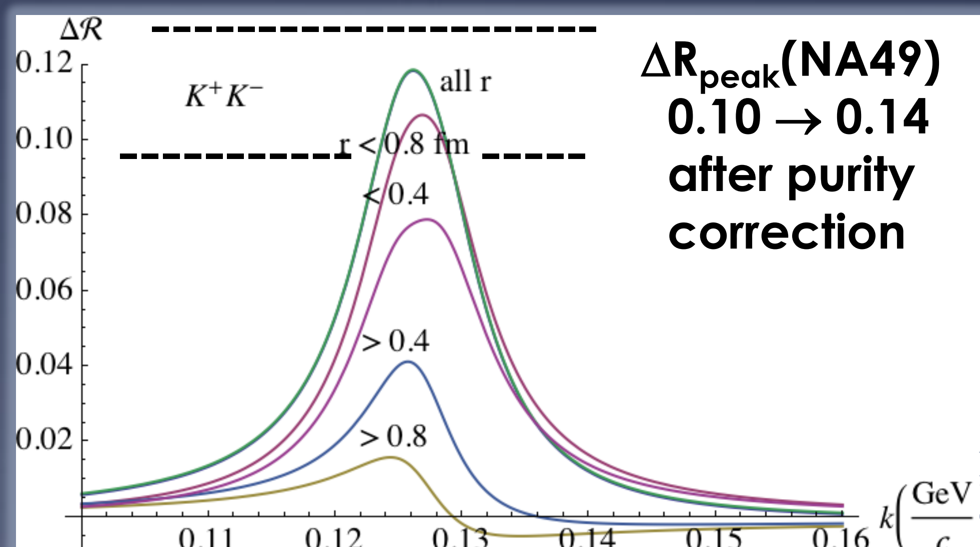
✓ as indicated by STAR $\pi^+\Xi^-$ CF

Resonance FSI contributions to $\pi^+\Xi^-$ & K^+K^- CF's

* Complete and corresponding inner and outer contributions of p-wave resonance (Ξ^*) FSI to $\pi^+\Xi^-$ CF for two cut parameters 0.4 and 0.8 fm and Gaussian radius of 5 fm
  FSI overestimates Ξ^* by a factor 4 (3) for r_0 = 5 (5.5) fm → factor 3 (2) if account for $\Delta_{out} \approx$ -6 fm



* The same for the p-wave resonance (ϕ) FSI contributions to K^+K^- CF
  FSI underestimates (overestimates) measured ϕ by 12 (20) % for r_0 = 5 (4.5) fm



* Little or no room for direct production of Ξ^*!!!

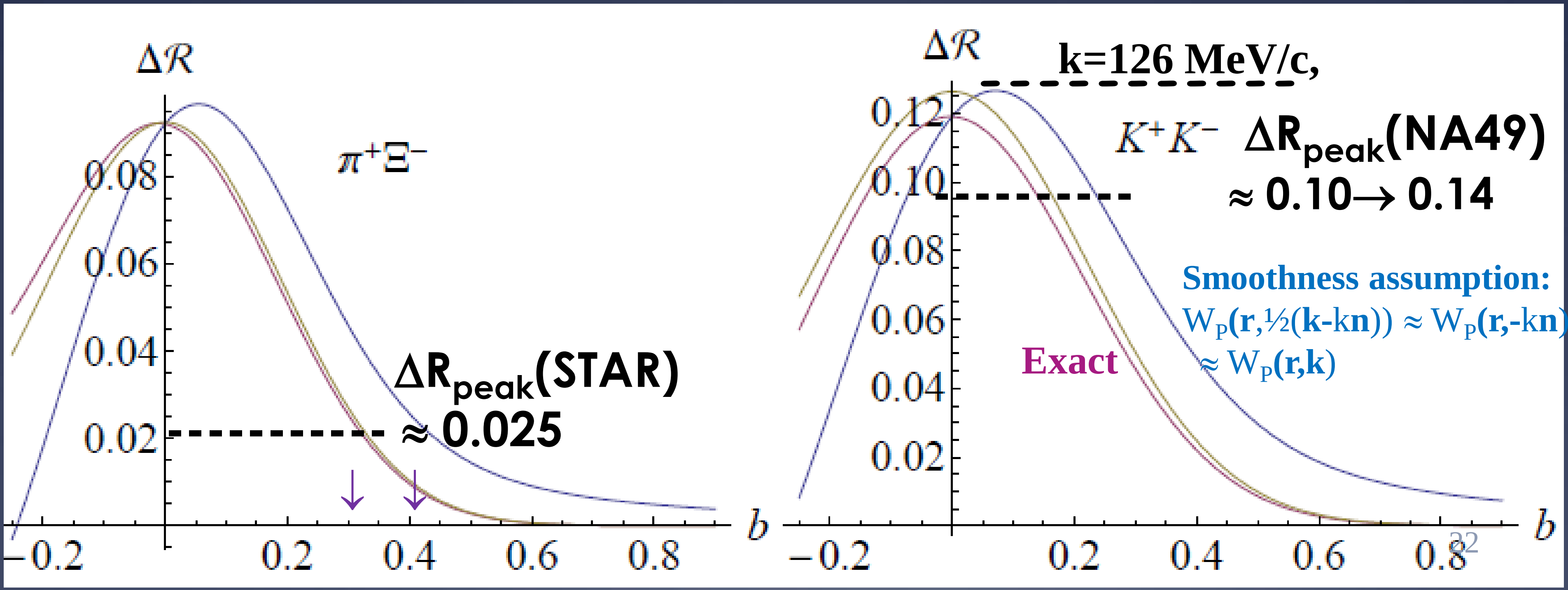
Resonance contribution vs r-k correlation parameter b

$W_P(\mathbf{r},\mathbf{k}) \sim \exp[-r^2/4r_0^2 + \mathbf{b}kr\cos\theta]$; θ = angle between **r** and **k**

CF suppressed by a factor $W_P(\mathbf{0},\mathbf{k}) \sim \exp[-\mathbf{b^2} r_0^2 k^2]$

To leave a room for a direct production

$b > 0.2$ is required for $\pi^+\Xi^-$ system



Summary

- Assumptions behind application of femtoscopic correlations formalism in HIC seem OK - up to a problem of the **r-k correlation** in the resonance region standard smoothness approximation needs to be generalized.
- The effect of narrow resonance FSI scales with inverse emission volume r_0^{-3}, compared to r_0^{-1} or r_0^{-2} scaling of the short-range s-wave FSI, thus being more sensitive to the space-time extent of the source. The higher sensitivity may be however disfavored by the theoretical uncertainty in case of a strong **r-k correlation**.
- The NA49 (K^+K^-) & STAR ($\pi^\pm\Xi^\pm$) correlation data from the most central collisions of heavy nuclei point to a strong **r-k correlation** which is needed to leave a room for a direct (thermal) production of near threshold narrow resonances to occur with non-zero probability.

Backup slides

Assumptions to derive "Fermi" formula for CF

$$\boxed{CF = \langle\, |\psi_{-\mathbf{k^*}}(\mathbf{r^*})|^2\, \rangle}$$

- two-particle approximation (small freezeout PS density f)

  ~ OK, <f> □□ 1 ? low p_t

- smoothness approx.: □□p□ □□ □q□$_{correl}$ □□$R_{emitter}$ □□ R_{source}

  ~ OK in HIC, R_{source}^2 □□ 0.1 fm^2□ p_t^2-slope of direct particles

- equal time approximation in PRF usually OK

  RL, Lyuboshitz'82 □ eq. time condition |t*| □□ □r*^{2} to several %

  - $t_{FSI} = d\delta/dE \;>>\; t_{prod}$

  t_{FSI} (s-wave) $= \mu f_0/k^* \;\Rightarrow\; k^* = ½q^* <<$ hundreds MeV/c

  ≈ typical momentum transfer in the production process

  RL, Lyuboshitz ..'98

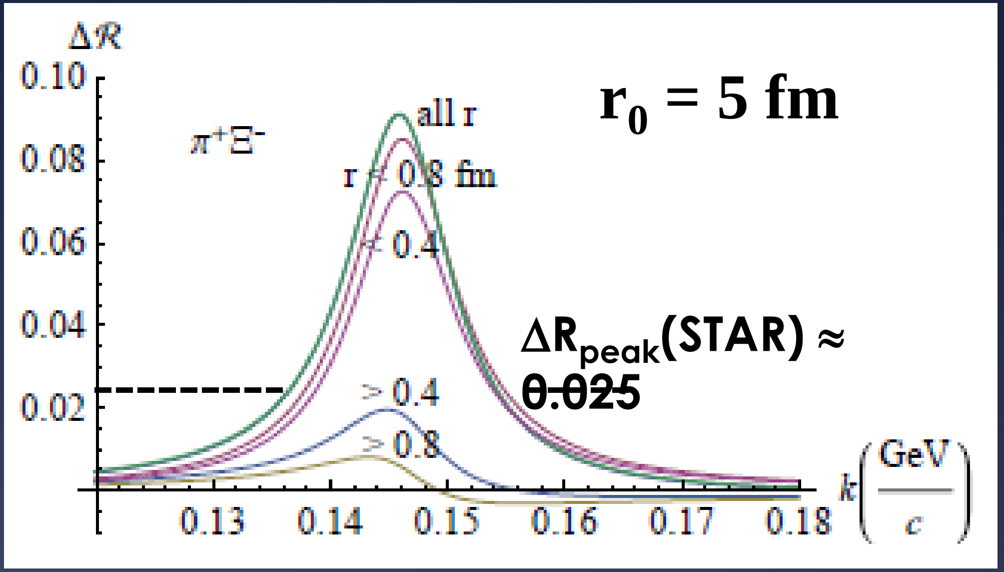
  & account for coupled channels within the same isomultiplet only:

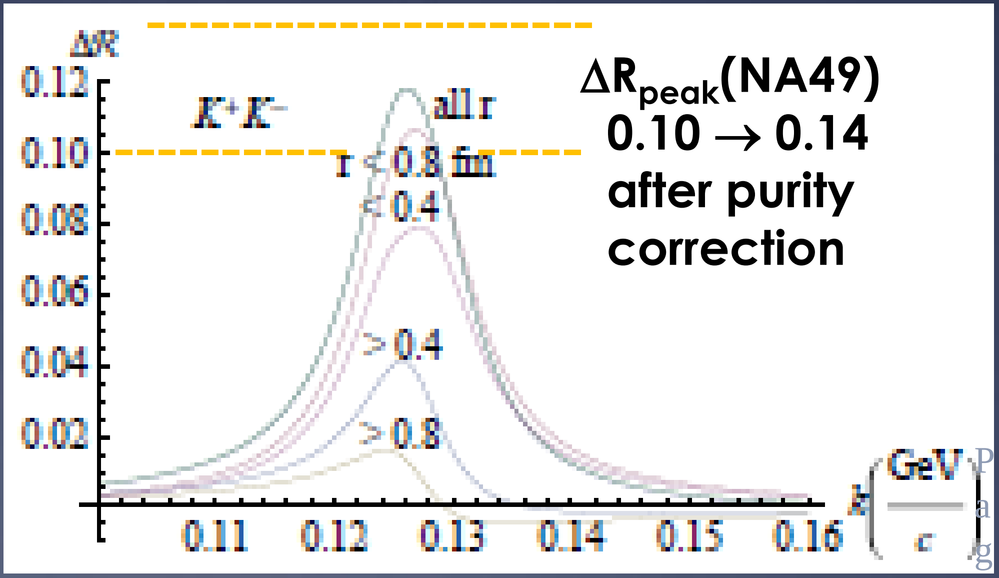
  $\pi^+\pi^- \leftrightarrow \pi^0\pi^0$, $\pi^-p \leftrightarrow \pi^0 n$, $K^+K^- \leftrightarrow K^0K^0$, ...

  t_{FSI} (resonance in any L-wave) $= 2/\Gamma \;\Rightarrow\; \Gamma$ □□ hundreds MeV/c

Resonance FSI contributions to $\pi^+\Xi^-$ & K^+K^- CF's

- Complete and corresponding inner and outer contributions of p-wave resonance (Ξ^*) FSI to $\pi^+\Xi^-$ CF for two cut parameters 0.4 and 0.8 fm and Gaussian radius of 5 fm → FSI contribution overestimates measured Ξ^* by a factor 4 (3) for r_0 = 5 (5.5) fm → factor 3 (2) if account for $\Delta_{out} \approx$ -6 fm
- The same for p-wave resonance (ϕ) FSI contributions to K^+K^- CF → FSI contribution underestimates (overestimates) measured ϕ by 12 (20) % for r_0 = 5 (4.5) fm
- Little or no room for direct production of Ξ^*!!!





Final State Interaction

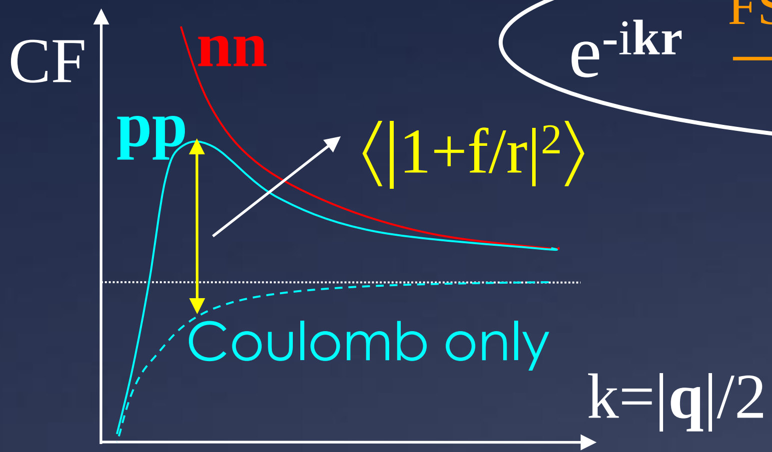
Similar to Coulomb distortion of β-decay Fermi'34: $\langle |\psi_{-\mathbf{k}}(\mathbf{r})|^2 \rangle$

Migdal, Watson, Sakharov, … Koonin, GKW, ...

$$e^{-i\mathbf{kr}} \xrightarrow{\text{FSI}} \psi_{-\mathbf{k}}(\mathbf{r}) \xrightarrow[\text{Coulomb}]{\text{s-wave strong FSI}} [\, e^{-i\mathbf{kr}} + \underbrace{f(k)}_{f_c\sqrt{A_c}\cdot(G_0+iF_0)} e^{ikr}/r \,]$$

$\underbrace{e^{i\delta_c}\sqrt{A_c}}$ Point-like Coulomb factor

$F = 1 + \dfrac{\mathbf{kr}+kr}{\underbrace{ka}} + \ldots$ Bohr radius



□ FSI is sensitive to source size r and scattering amplitude f
It complicates CF analysis but makes possible

✔ Femtoscopy with nonidentical particles πK, πp, .. &
Coalescence deuterons, ..

✔ Study "exotic" scattering ππ, πK, KK, πΛ, pΛ, ΛΛ, ..

✔ Study relative space-time asymmetries delays, flow

Caution: Smoothness approximation is justified only for small $k \ll 1/r_0$!!!

$$\mathrm{CF}(p_1,p_2) \approx \int d^3\mathbf{r}\, W_P(\mathbf{r},\mathbf{k})\, |\psi_{-\mathbf{k}}(\mathbf{r})|^2$$

should be generalized in resonance region k~150 MeV/c

$$\rightarrow \int d^3\mathbf{r}\, \{ W_P(\mathbf{r},\mathbf{k}) + W_P(\mathbf{r},½(\mathbf{k}-k\mathbf{n}))\, 2\mathrm{Re}[\exp(i\mathbf{kr})\Delta\psi_{-\mathbf{k}}(\mathbf{r})]$$

$$+W_P(\mathbf{r},-k\mathbf{n})\, |\Delta\psi_{-\mathbf{k}}(\mathbf{r})|^2 \}$$

where $\psi_{-\mathbf{k}}(\mathbf{r}) = \exp(-i\mathbf{kr}) + \Delta\psi_{-\mathbf{k}}(\mathbf{r})$ and $\mathbf{n} = \mathbf{r}/r$

The smoothness approximation

$$W_P(\mathbf{r},½(\mathbf{k}-k\mathbf{n})) \approx W_P(\mathbf{r},-k\mathbf{n}) \approx W_P(\mathbf{r},\mathbf{k})$$

is valid if one can neglect the k-dependence of $W_P(\mathbf{r},\mathbf{k})$, e.g. for $k \ll 1/r_0$

Modern correlation femtoscopy formulated by Kopylov & Podgoretsky

KP'71-75: settled basics of correlation femtoscopy in > 20 papers (for non-interacting identical particles)

- proposed CF= N^{corr}/N^{uncorr} & mixing techniques to construct N^{uncorr}
- argued that sufficiently smooth momentum spectrum allows one to neglect space-time coherence at small q* (smoothness approximation):

$$\left|\int d^4x_1 d^4x_2 \psi_{p1p2}(x_1,x_2)\ldots\right|^2 \rightarrow \int d^4x_1 d^4x_2 |\psi_{p1p2}(x_1,x_2)|^2\ldots$$

- clarified role of space-time production characteristics: shape & time source picture from various q-projections

Incorporating **r-k** correlation in emission function

Substituting the simple Gaussian emission function:

$$W_P(\mathbf{r},\mathbf{k}) = (8\pi^{3/2}r_0^{\,3})^{-1} \exp(-r^2/4r_0^{\,2})$$

by (θ = angle between **r** and **k**) :

$$W_P(\mathbf{r},\mathbf{k}) = (8\pi^{3/2}r_0^{\,3})^{-1} \exp(-\mathbf{b}^2 r_0^{\,2}k^2) \exp(-r^2/4r_0^{\,2} + \mathbf{b}kr\cos\theta)$$

Exponential suppression generated
in the resonance region
($k \approx 150$ MeV/c)
by a collective flow: $b > 0$

Incorporating **r-k** correlation in emission function

In the case of correlation asymmetry in the out-direction:

$$W_P(\mathbf{r},\mathbf{k}) = (8\pi^{3/2} r_0^{\,3})^{-1} \exp(-\mathbf{b}^2 r_0^{\,2} k^2 - \mathbf{b}\, k_{out}\, \Delta_{out})$$
$$\times \exp\{-[(r_{out}-\Delta_{out})^2 + r_{side}^{\,2} + r_{long}^{\,2}]/4r_0^{\,2} + \mathbf{b} k r \cos\theta\}$$

Note the additional suppression of $W_P(\mathbf{0},\mathbf{k})$ if $\Delta_{out} \neq 0$:

$W_P(\mathbf{0},\mathbf{k}) \sim \exp[-(\Delta_{out}/2r_0)^2]$ (~20% suppression if $\Delta_{out} \approx r_0$)

& correlation asymmetry even at $r \to 0$:

$$W_P(\mathbf{0},\mathbf{k}) \sim \exp(-\mathbf{b}\, k_{out}\, \Delta_{out}) \approx 1 - \mathbf{b} k_{out} \Delta_{out}$$