Intodoctory review

Bose-Einstein correlations in small systems

Csörgő, Tamás

MTA KFKI RMKI, Budapest, Hungary

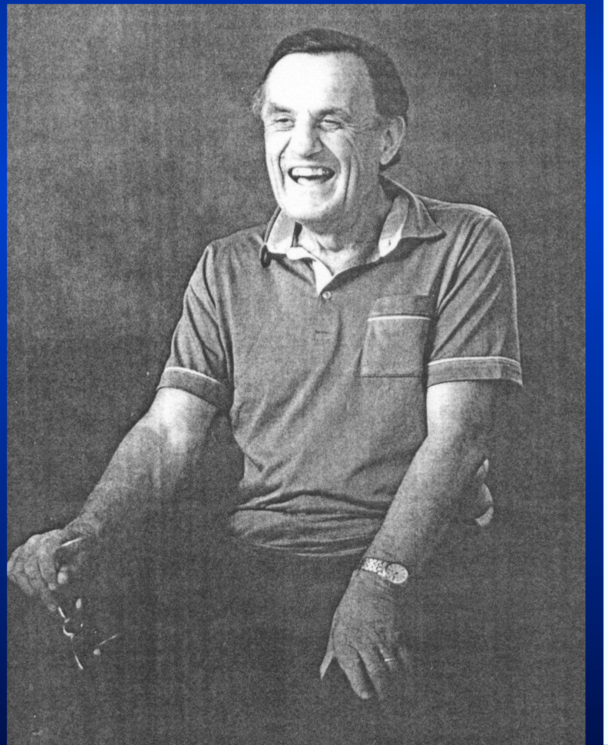
HBT story, intensity interferometry
GGLP, Bose-Einstein Correlations
Famous quotes vs data

Functional dependence (proper variables = ?)
Shape analysis (proper shape = ?)
„Invisible gorillas” of Bose-Einstein correlations

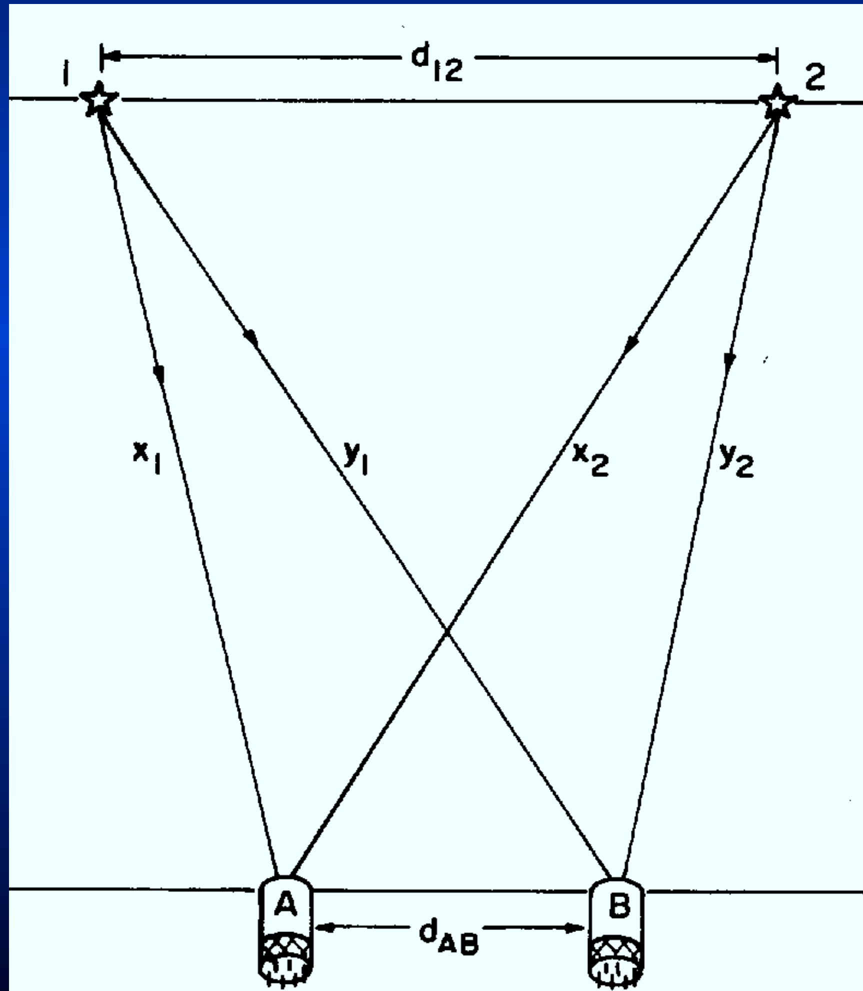
From BIG to small: first look at stars

- Intensity interferometry in radio astronomy
- Angular diameter of a main sequence stars (1954)
- Two persons: Robert Hanbury Brown and Richard Q. Twiss (radio engineers)

Figure 10.1 The first stellar intensity interferometer; the pilot model of the stellar intensity interferometer at Jodrell Bank in 1955. Two Army searchlights were used to make the first measurement of the angular diameter of a main sequence star (Sirius).



R. Hanbury Brown



Famous quotes

„Interference between two different photons can never occur."
P. A. M. Dirac, The Principles of Quantum Mechanics, Oxford, 1930

„In fact to a surprising number of people the idea that the arrival of photons at two separated detectors can ever be correlated was not only heretical but patently absurd, and they told us so in no uncertain terms..."

„I was a long way from being able to calculate, whether it would be sensitive enough to measure a star. ... my education in physics had stopped far short of the quantum theory. Perhaps just as well, ignorance is sometimes a bliss in science"
Both from R. H. Brown, Boffin: a personal story of radar, radio astronomy and quantum optics (Taylor & Francis, 1991)

„R- H. Brown preferred his middle name to his first name. Therefore, instead of using Robert H. Brown ... he used R. Hanbury Brown instead. This practice inevitably led to the citation of Brown as Hanbury Brown and even Hanbury-Brown ... he is correctly listed under his last name Brown in ... encyclopedias" J. Rayford Nix, nucl-th/9801029

„The second term is positive definite, i. e. the correlation function can not ... oscillate around unity. ... If you see such a result in the literature, it is wrong."
„The good exponential fits from pp and e^+e^- data are ... purely accidental and without physical meaning. The variable Q_{inv} should not be used for fitting HBT data."
Both from U. Heinz, Proc. NATO ASI, Dronten, 1996, nucl-th/9609029

GFGHKP → GGLP

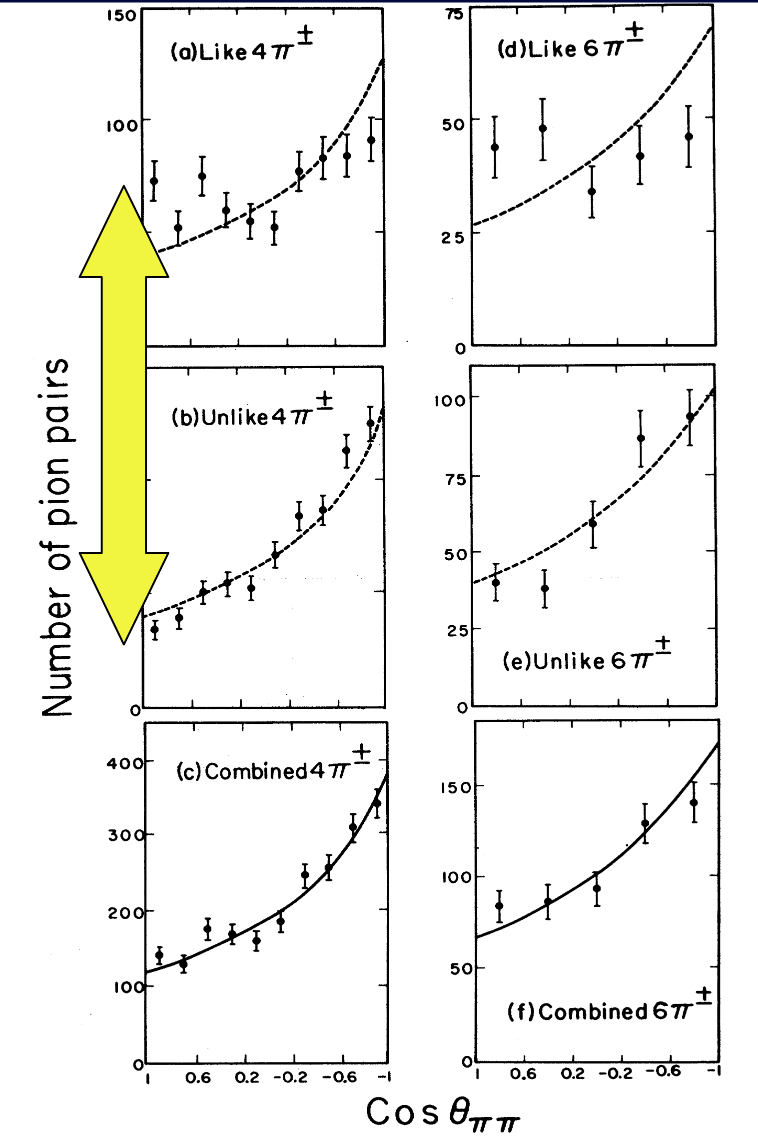


FIG. 1. Distribution of angles between pion pairs as a function of $\cos\theta_{12}$. The curves correspond to calculations on the Lorentz-invariant phase-space (LIPS) model. The deviations of the experimental distribution from the LIPS model are discussed in the text.

G. Goldhaber, W. B. Fowler, S. Goldhaber, T.F. Hoang, T. E. Kalogeropoulos, W. M. Powell,
Phys. Rev. Lett. 3 (1959) 181

Searching for ρ meson in p+p annihilation at $\sqrt{s_{NN}}$ = 2.1 GeV in a bubble-chamber experiment

observe enhancement of like-sign pion pairs in the same hemisphere. Attribute it to pion correlations. „Work is in progress ... involve the radius R of the interaction volume and may enable one to determine its value."

G. Goldhaber, S. Goldhaber, W-Y. Lee, A. Pais
Phys. Rev. 120, 300 (1960)

Explanation: Bose-Einstein statistics of pions

Introduce the concept of

Bose-Einstein correlations

GFGHKP → GGLP

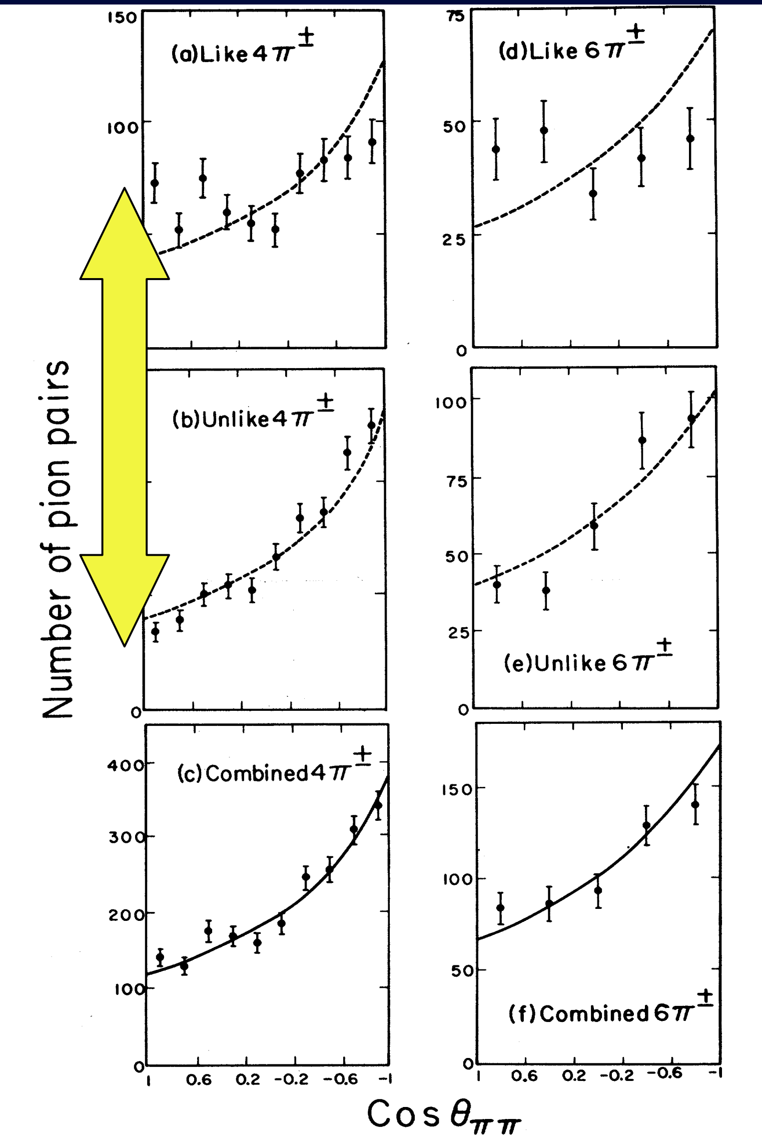


FIG. 1. Distribution of angles between pion pairs as a function of $\cos\theta_{12}$. The curves correspond to calculations on the Lorentz-invariant phase-space (LIPS) model. The deviations of the experimental distribution from the LIPS model are discussed in the text.

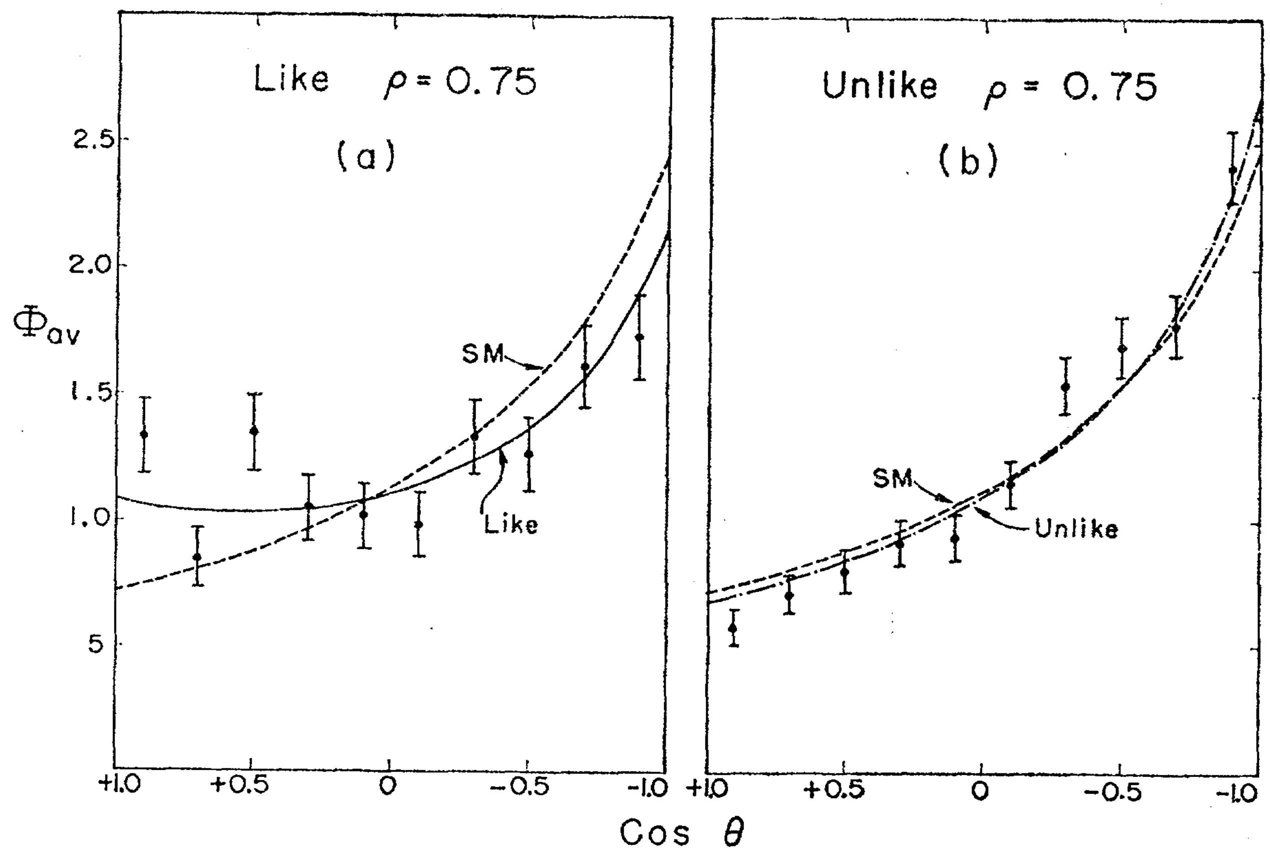


FIG. 6. The functions $\Phi_{av}(\cos\theta)$ computed at $\rho=0.75$ are compared with the experimental distribution of angles between pion pairs. Figures 6(a) and 6(b) give the distributions for like and unlike pions respectively. Also shown in each is the curve for $\Phi_{av}{}^{SM}(\cos\theta)$, the statistical distribution, without the effect of correlation functions. Here Φ_{av} represents an average of Φ_4, Φ_5, and Φ_6, weighted according to the individual charge channels. The experimental data comes from reference 1 (see also Table I, footnote a).

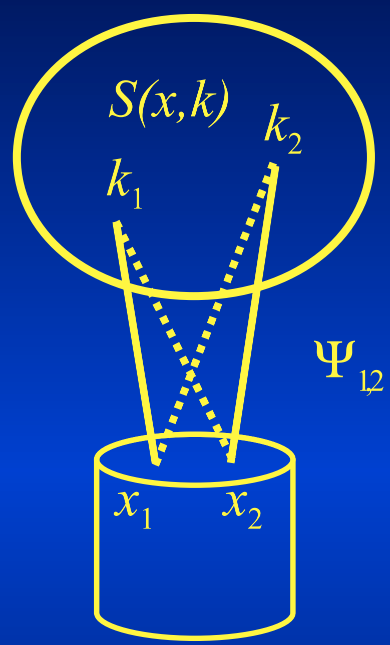
Bose-Einstein or HBT correlations

Two plane-waves:

Bosons:

BE symmetrization

$$\Psi_1 = \mathrm{e}^{-ik_1x_1}$$
$$\Psi_2 = \mathrm{e}^{-ik_2x_2}$$



$$\Psi_{1,2} = \frac{1}{\sqrt{2}}(\mathrm{e}^{-ik_1x_1}\mathrm{e}^{-ik_2x_2} + \mathrm{e}^{-ik_1x_2}\mathrm{e}^{-ik_2x_1})$$

$$N_1(k_1) = \int S(x_1,k_1)|\Psi_1|^2 \mathrm{d}x_1$$

S(x,k): source distribution.

Picture: for HBT, formulas for femtoscopy x ↔ k

Two-particle spectrum (momentum-distribution):

$$N_2(k_1,k_2) = \int S(x_1,k_1)S(x_2,k_2)|\Psi_{1,2}|^2 \mathrm{d}x_1\mathrm{d}x_2$$

Approximations: Plane-wave, no multiparticle symmetrization, thermalization …

Kopylov, Podgoretskii, Lednicky

G. Goldhaber, S. Goldhaber, W-Y. Lee and A. Pais (GGLP) :

explain a HBT like effect in p+p reactions at

Kopylov, Podgoretskii, Dubna school

Start to use correlations as a tool to measure sizes

G.I. Kopylov: Like particle correlations as a tool...

Phys. Lett. B 50, 474 (1974)

Interference of particles emitted by moving sources

G.I. Kopylov, M. I. Podgoretsky

Yad.Fiz.18:656-666 (1973)

Yano, Koonin, Podgoretsky (YKP) parametrization

Non-identical particle interferometry: effects of fsi

Sequence of particle emission in principle can be obtained

R. Lednicky, Ljuboshitz

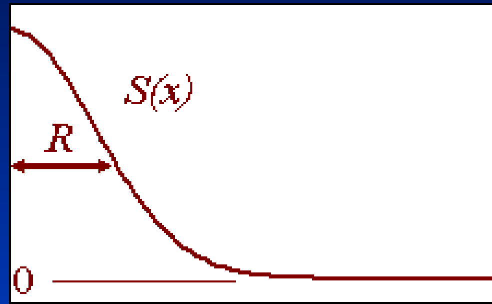
Yad.Fiz.35:1316-1330,1981

Lednicky: Coined the name of Femtoscopy (nucl-th/0112011)

Again, what brings us all this?

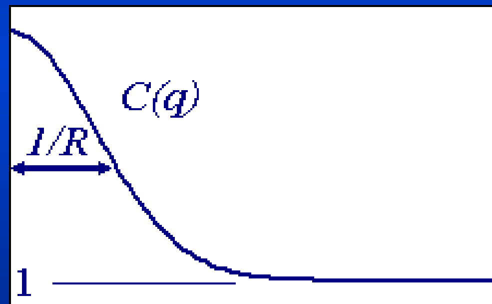
If the source is approximated with Gaussian:

$$S(x) \sim \exp\left(-\frac{r_x^2}{2R_x^2} - \frac{r_y^2}{2R_y^2} - \frac{r_z^2}{2R_z^2}\right)$$



Then the correlation function is also Gaussian:

$$C(q) - 1 \sim \exp\left(-q_x^2 R_x^2 - q_y^2 R_y^2 - q_z^2 R_z^2\right)$$



- **These are the so-called HBT radii**

If transformed to the out-side-long system (not invariant)

Out: direction of the mean transverse momentum of the pair

Side: orthogonal to out

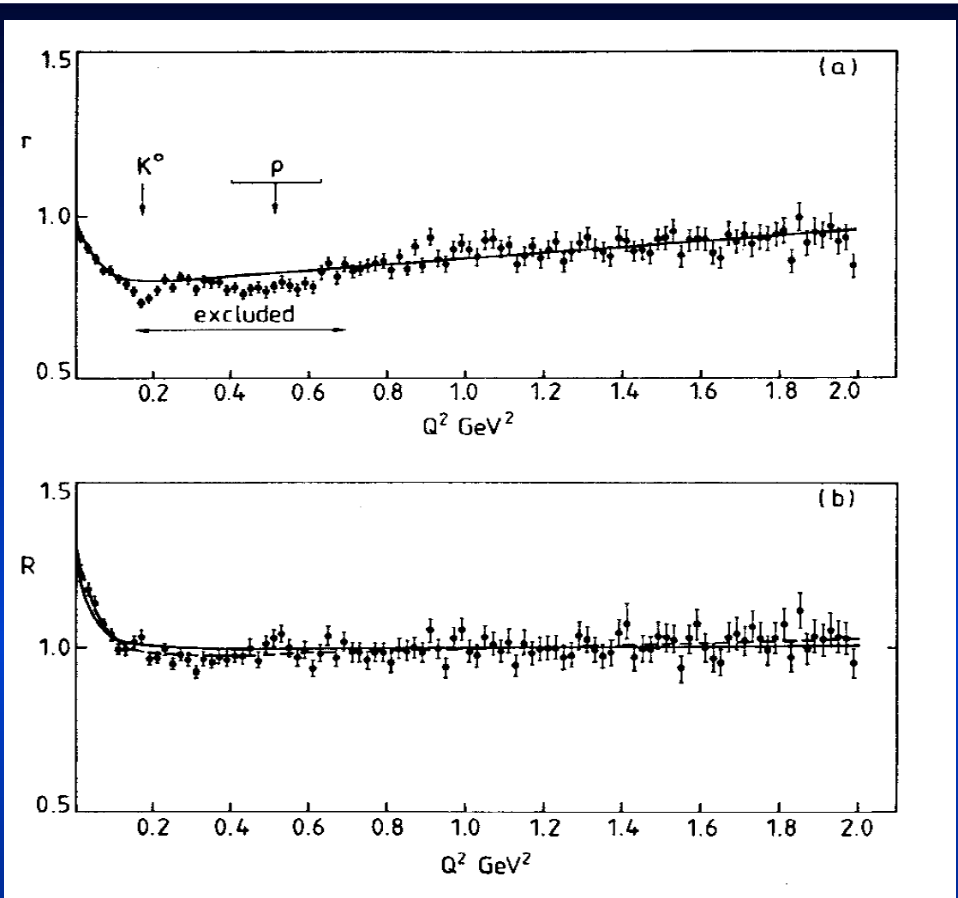
Long: beam direction

$$C(q) = 1 + \lambda \exp\left(-q_o^2 R_o^2 - q_s^2 R_s^2 - q_l^2 R_l^2\right)$$

Not necessarily reflecting the geometrical size

- Yu. Sinyukov: Lenghts of homogeneity ...

Bose-Einstein in e^+e^-: TASSO



8. Conclusions

We summarise our conclusions as follows:

(i) The TASSO data on Bose-Einstein correlation of like sign pairs of charged particles are well represented in two dimensional arrays by the simple function

$$C_2 \approx 1 + \alpha e^{-\beta Q^2} \qquad (8.1)$$

with $\beta \approx 13$ GeV^{-2}.

(ii) To the extent that the observed Bose-Einstein correlation is a function only of Q^2, the source must be spherically symmetric when viewed from the rest frame of any pair the members of which are close in momentum. This does not imply a spherically symmetric source in the event frame.

(iii) The principal features of Bose-Einstein correlation exhibited by triplets of like sign particles are readily explained in terms of the results obtained for pairs.

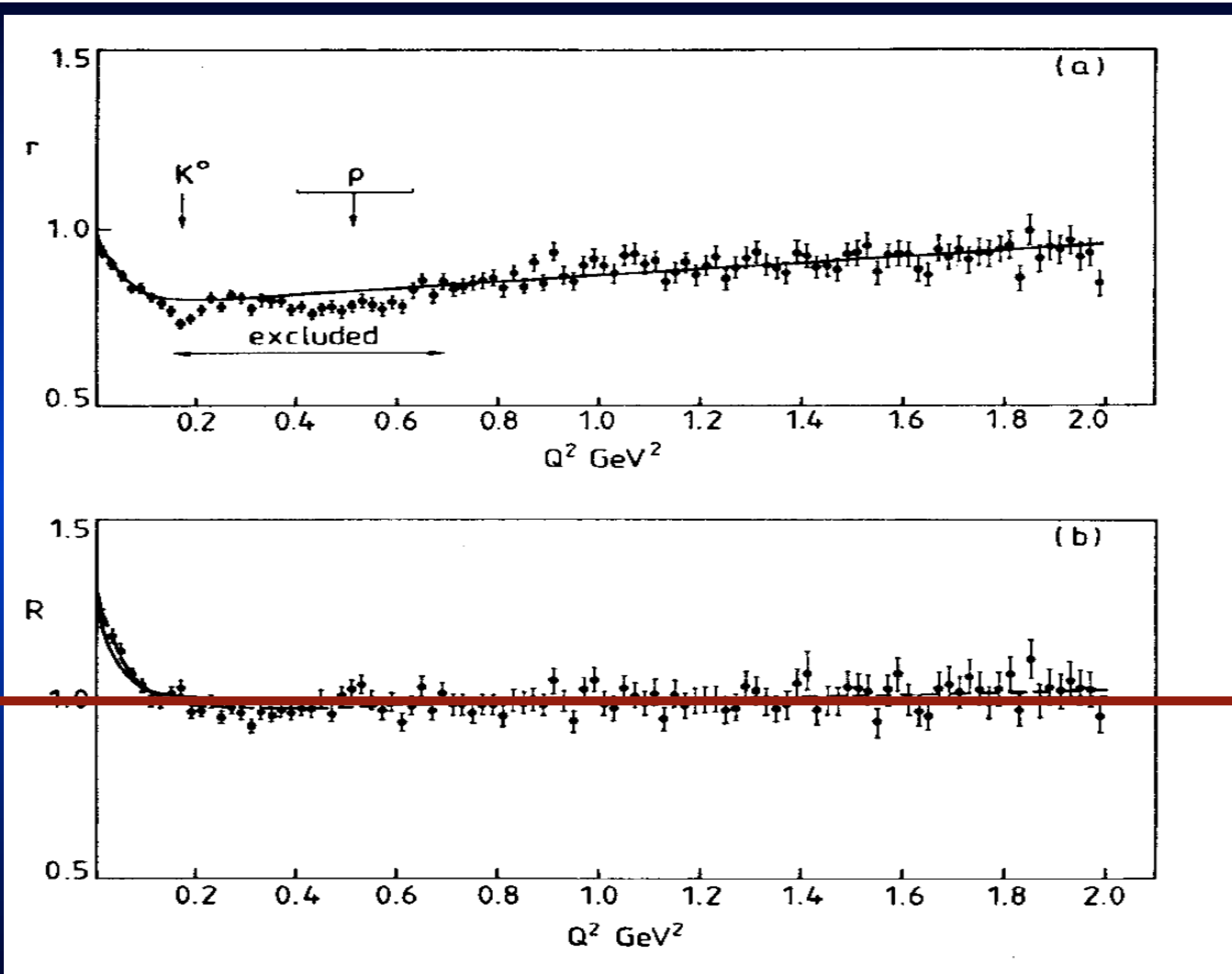
Fig 1. from M.Althoff et al, TASSO Collaboration, Z. Phys. C30:355 (1986)
(a): like-sign correlations (b): corrected by (+-) and Monte Carlo

point out an observation of a Q_{inv} dependence

approximately Gaussian phase

direct relation to L3 results (W. Metzger, next talk) – a dip too!

Bose-Einstein in e^+e^-: TASSO



Oops. A dip, too!

Unexpected, the dip was excluded from the fit... Is it there or not? See L3/CMS talks

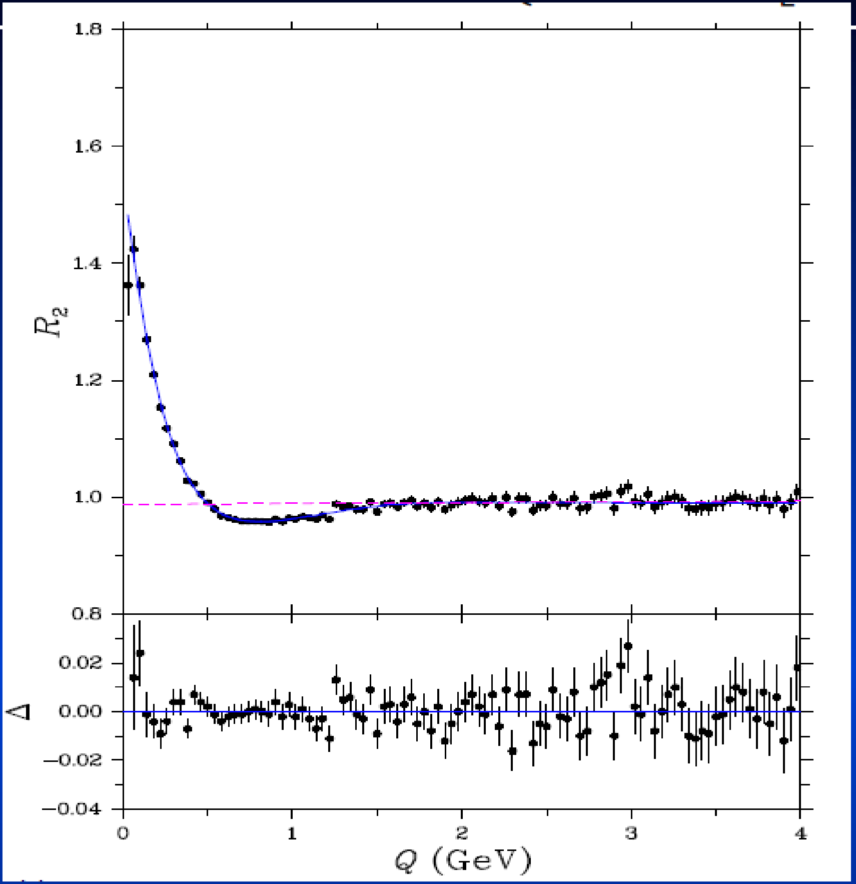
„Positive definitness” of Bose-Einstein

Usual derivation is based on
several assumtions.

Check all of them experimentally.

One of them fails for jet physics:
smoothness approximation

if not 1 + positive definit form:
→ look out for jets!
e^+e^- at L3/LEP and pp at CMS/LHC data
vs theory of nucl-th/9609026



Assuming the generalized Wick theorem (9) the correlation function (8) can be written as

$$C(p_a, p_b) = 1 \pm \frac{|\langle \hat{a}^+_{\boldsymbol{p}_a} \hat{a}_{\boldsymbol{p}_b} \rangle|^2}{\langle \hat{a}^+_{\boldsymbol{p}_a} \hat{a}_{\boldsymbol{p}_a} \rangle \langle \hat{a}^+_{\boldsymbol{p}_b} \hat{a}_{\boldsymbol{p}_b} \rangle} \,. \tag{13}$$

Note that the second term is positive definite, i.e. the correlation function cannot, for example, oscillate around unity. [If you see such a behaviour in the literature [8] (and the authors did not include final state interactions) it is wrong.]

Experiments: UA1, NA22, L3, OPAL…

GFGHKP: 30 in. Bubble chamber experiment, p+p, $\sqrt{s_{NN}}$ = 2.1 GeV, LRL,
2500 events, 2106+532 = 2638 total number of pairs

EHS/NA22: Bubble chamber experiment at CERN SPS $\sqrt{s_{NN}}$ = 22 GeV, SPS
25 k π^+p and 29k K^+p events

UA1: p+p experiment at CERN SppS sqrt(s) = 630 GeV
p_T > 0.15 GeV/c, $|\eta| < 3$, 45° < $|\phi|$< 135°
1.2 x 10^6 NSD events, $|\Delta k|$ ~ 8 MeV

L3, OPAL, ALEPH, DELPHI:
$e^+ e^-$ annihilations at LEP. 2 jets and 3 + jets, $\sqrt{s_{NN}}$ = 91.2 GeV
~ 10^6 events (hadronic Z^0 decays) + …

UA1: Non-Gaussian distributions

Correlations do NOT have to be Gaussian

Non-Gaussian tails in 630 GeV p+p

Log scale in q, many low q bins

Partial coherence: 1 + 2 terms

Best Gaussians/exponentials FAIL

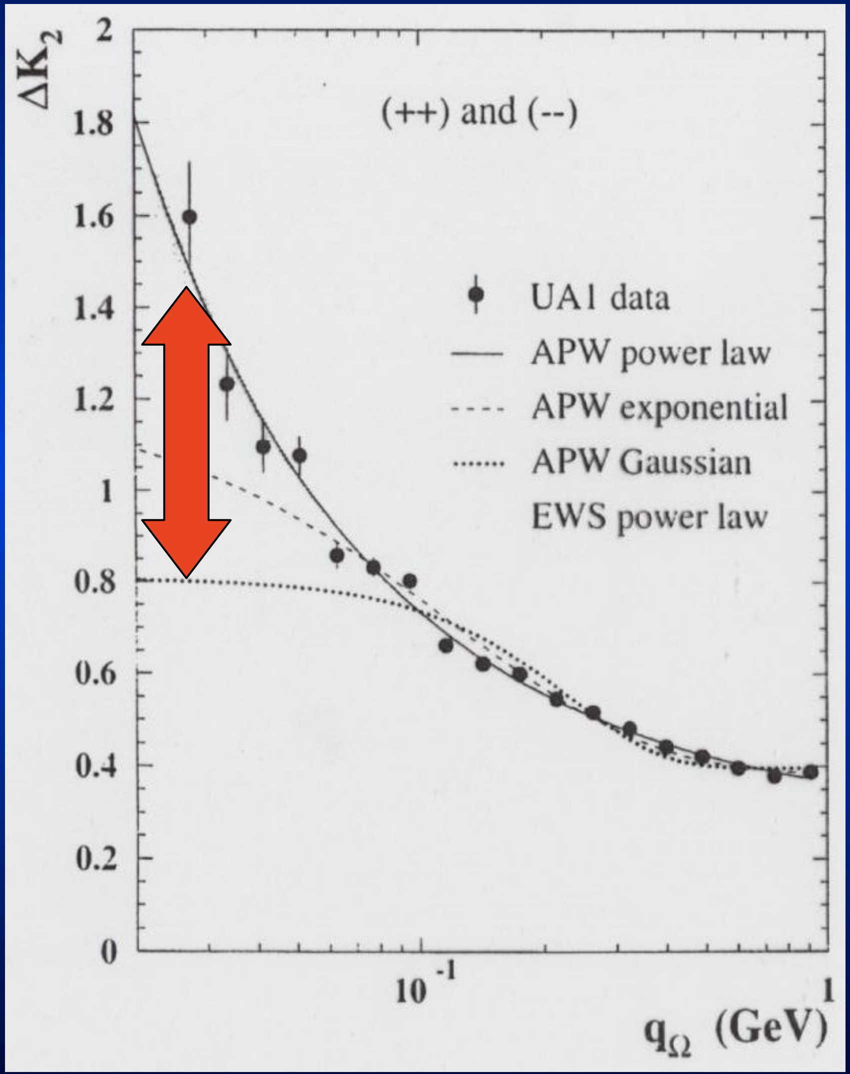
$$\begin{aligned} \text{Gaussian:} \quad & d_{ij} = \exp(-r^2 q_{ij}^2)\,, \\ \text{exponential:} \quad & d_{ij} = \exp(-r q_{ij})\,, \\ \text{power law:} \quad & d_{ij} = q_{ij}^{-\alpha}\,. \end{aligned}$$

$$k_2^{\rm th} \equiv \frac{C_2}{\rho_1 \otimes \rho_1} = 2\lambda(1-\lambda) d_{12} + \lambda^2 d_{12}^2\,,$$

Gaussian assumption →

meaningless results (CL<0.1 %)

How to check, if the correlation function is really Gaussian ?



Eggers, Lipa, Buschbeck, hep-ph/9702235

APW: Andreev, Plümer, Weiner, Int. J. Mod. Phys. A8 4577 (1993).

UA1: Search for partial coherence fails

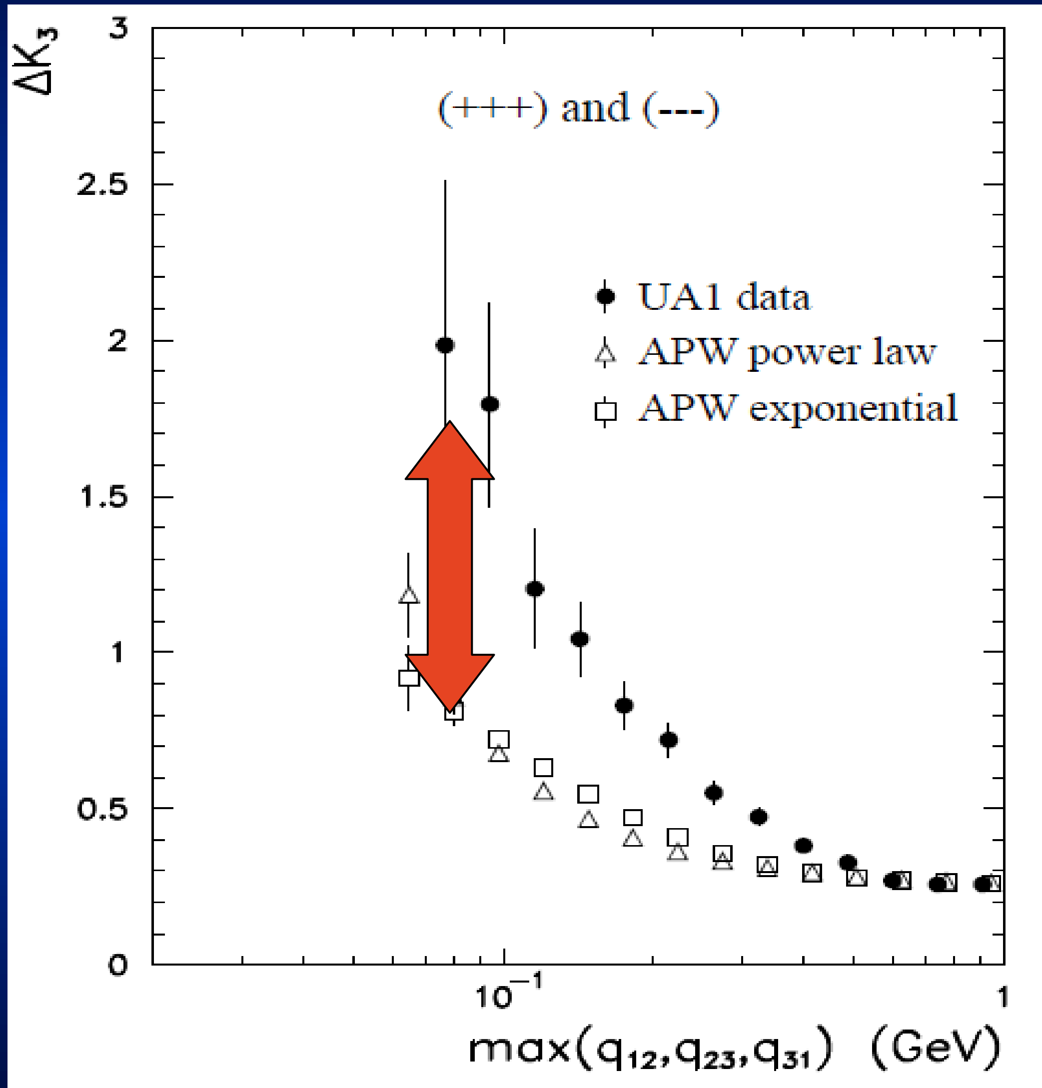
Correlations are NOT due to partial coherence alone

2rd and 3rd order correlations
in 630 GeV p+p NSD events

$$\begin{aligned} \text{Gaussian:} \quad & d_{ij} = \exp(-r^2 q_{ij}^2)\,, \\ \text{exponential:} \quad & d_{ij} = \exp(-r q_{ij})\,, \\ \text{power law:} \quad & d_{ij} = q_{ij}^{-\alpha}\,. \end{aligned}$$

$$k_2^{\text{th}} \equiv \frac{C_2}{\rho_1 \otimes \rho_1} = 2\lambda(1-\lambda) d_{12} + \lambda^2 d_{12}^2\,,$$

$$k_3^{\text{th}} \equiv \frac{C_3}{\rho_1 \otimes \rho_1 \otimes \rho_1} = 2\lambda^2(1-\lambda)[d_{12}d_{23} + d_{23}d_{31} + d_{31}d_{12}] + 2\lambda^3 d_{12}d_{23}d_{31}\,,$$



3rd order correlation: stronger,
than from 2rd order + partial coh.

→ **How to check, if the source has some partial coherence or not?**

Eggers, Lipa, Buschbeck, hep-ph/9702235
APW: Andreev, Plümer, Weiner, Int. J. Mod. Phys. A8 4577 (1993).

Model independent shape analysis

Advantage and/or disadvantage:

Analyse, quantify correlations model independently

Only two assumptions:

The correlations are centered around some point (Q = 0)

They are short-range type

- **Long range correlations can be removed or measured independently**

Expansion methods to test:

- **Is it Gaussian ? → Edgeworth expansions**
- **Is it Exponential ? → Laguerre expansions**
  - **Based on complete orthog. set of functions**
  - **T. Cs. and S. Hegyi, hep-ph/9912220**
    - » **Not 1+ pos definite**
    - » **Not connected to a source model**
    - » **Breaks down on L3 data (middle range dip)**
    - » **see W. Metzger's and M. de Kock's talks**

Is it Gaussian? → Edgeworth Expansion

Model independent, in e+e-, h+p, and in heavy ion reactions:

$$t = \sqrt{2} Q R_E,$$
$$w(t) = \exp(-t^2/2),$$
$$\int_{-\infty}^{\infty} dt \, \exp(-t^2/2) H_n(t) H_m(t) \propto \delta_{n,m},$$

$$H_1(t) = t,$$
$$H_2(t) = t^2 - 1,$$
$$H_3(t) = t^3 - 3t,$$
$$H_4(t) = t^4 - 6t^2 + 3, \; \ldots$$

$$C_2(Q) = \mathcal{N} \left\{ 1 + \lambda_E \exp(-Q^2 R_E^2) \times \left[1 + \frac{\kappa_3}{3!} H_3(\sqrt{2} Q R_E) + \frac{\kappa_4}{4!} H_4(\sqrt{2} Q R_E) + \ldots \right] \right\}.$$

$$H_n(t) = \exp(t^2/2) \left(-\frac{d}{dt} \right)^n \exp(-t^2/2).$$

T. Cs., S. Hegyi, hep-ph/9912220

Exponential? → Laguerre expansion

Model independent, in e+e-, h+p, and in heavy ion reactions:

$$t = QR_L,$$
$$w(t) = \exp(-t),$$
$$\int_0^\infty dt\, \exp(-t) L_n(t) L_m(t) \propto \delta_{n,m},$$

$$L_0(t) = 1,$$
$$L_1(t) = t - 1,$$
$$L_2(t) = t^2 - 4t + 2, \ \ldots .$$

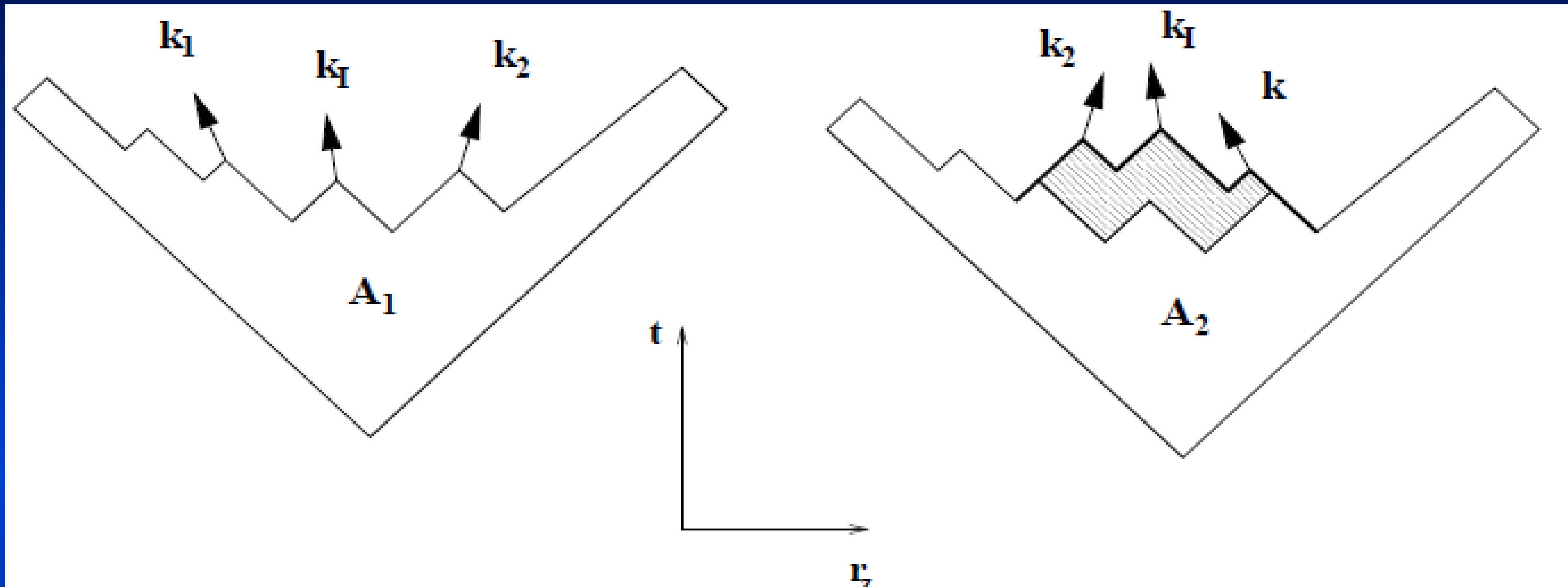
$$C_2(Q) = \mathcal{N}\left\{1 + \lambda_L \exp(-QR_L)\left[1 + c_1 L_1(QR_L) + \frac{c_2}{2!} L_2(QR_L) + \ldots\right]\right\}$$

$$L_n(t) = \exp(t) \frac{d^n}{dt^n} (-t)^n \exp(-t).$$

Is it Levy? See the talk of M. de Kock!

Andersson-Hoffmann model

Applied in e+e- reactions:

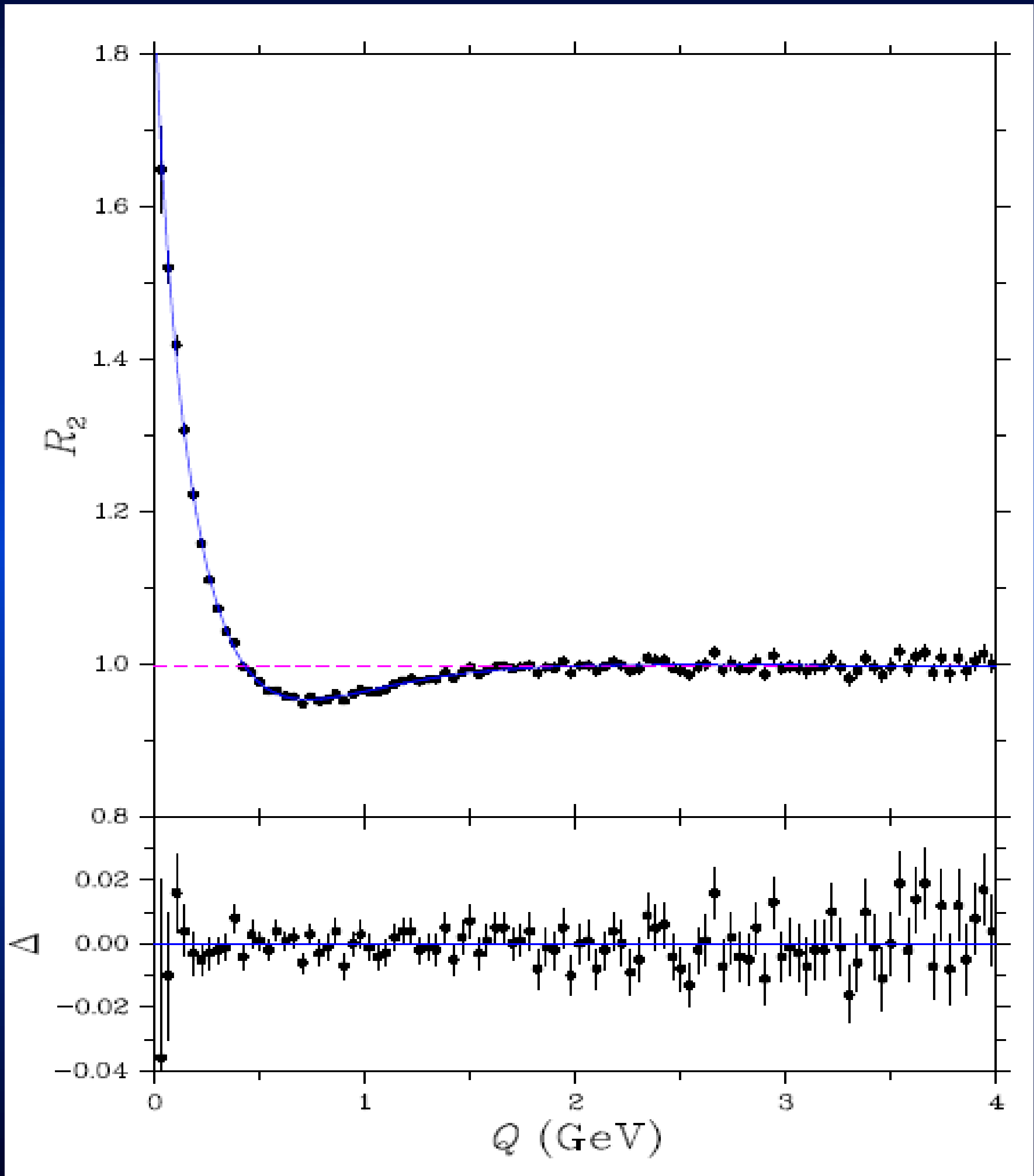


$$M \sim \exp[i(\kappa + ib/2)A_1] + \exp[i(\kappa + ib/2)A_2]\,,$$

$$\mid M \mid^2 \sim [\exp(-bA_1) + \exp(-bA_2)] \cdot \left[1 + \frac{\cos(\kappa \Delta A)}{\cosh(b\Delta A/2)}\right]$$

Suggests: Oscillation (dip), elongation of the source, approx Qinv

Published L3 results:



Published L3 result:
dip is significant

Confirms
Earlier TASSO result

Is it only in e^+e^-?
Not! CMS found it too.

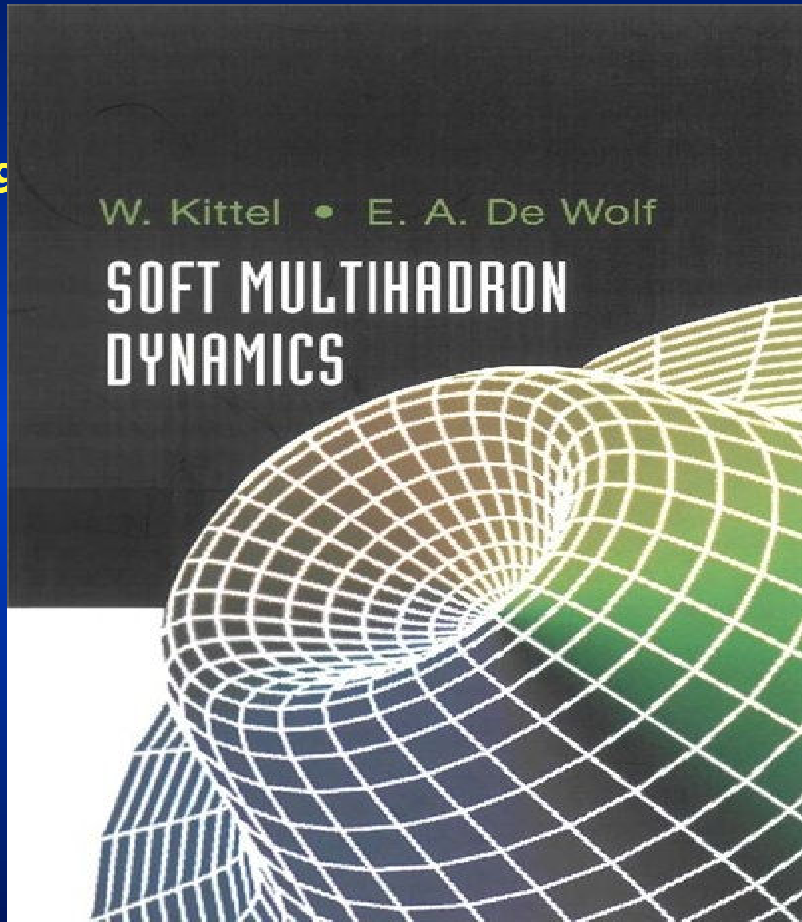
For more details:

See W. Metzger's and
S. Padula's talks

Silently kills 99% of models

Metareview

- **W. Kittel and E. de Wolf,**
  - **Soft Multihadron Dynamics, World Scientific (2005) 652 p**
- **B. Lörstad,**
  - **Int.J.Mod.Phys.A4:2861, 1989**
- **W. A. Zajc,**
  - **NATO Adv.Study Inst.Ser.B Phys.303:435-459,199**
- **M. Lisa, S. Pratt, , R. Soltz, U. A. Wiedemann**
  - **Ann.Rev.Nucl.Part.Sci.55:357-402,2005**
- **T. Cs.**
  - **hep-ph/0001233**
  - **J.Phys.Conf.Ser.50:259, 2006**
- **R. M. Weiner**
  - **Phys.Rept.327:249-346,2000**
- **U. Heinz and U. A. Wiedemann**
  - **Phys.Rept.319:145-230,1999**
- **W. Kittel,**
  - **AIP Conf.Proc.828:519-524,2006 – beyond Gaussian**
- **W. Kittel,**
  - **Hep-ph/9905394 – a critical (p)review**
- **M. G. Bowler: Are the observed Bose-Einstein correlations possible?**
  - **Marburg LESIP IV 1990:2-15 (QCD161:I972:1990)**



どうも有難う御座いました