Bose-Einstein Correlations, the τ-model, and jets in e^+e^- annihilation

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Outline

1. Old stuff — Eur. Phys. J. C (2011) 71:1648

   25 pages — quickly summarize

2. New stuff

BEC Introduction

$$R_2 = \frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)} = \frac{\rho_2(Q)}{\rho_0(Q)}$$

Assuming particles produced incoherently
with spatial source density $S(x)$,

$$R_2(Q) = 1 + \lambda|\widetilde{S}(Q)|^2$$

where $\widetilde{S}(Q) = \int \mathrm{d}x\, e^{iQx} S(x)$ – Fourier transform of $S(x)$
$\lambda = 1$ — $\lambda < 1$ if production not completely incoherent

Assuming $S(x)$ is a Gaussian with radius $r \Longrightarrow$

$$R_2(Q) = 1 + \lambda\, \mathrm{e}^{-Q^2 r^2}$$

intro

The L3 Data

- $e^+e^- \longrightarrow$ hadrons at $\sqrt{s} \approx M_Z$
- about $36 \cdot 10^6$ like-sign pairs of well measured charged tracks from about $0.8 \cdot 10^6$ events
- about $0.5 \cdot 10^6$ 2-jet events — Durham $y_{\mathrm{cut}} = 0.006$
- about $0.3 \cdot 10^6 > 2$ jets, "3-jet events"
- use mixed events for reference sample, ρ_0

Results – ‘Classic’ Parametrizations

$R_2 = \gamma \cdot [1 + \lambda G] \cdot (1 + \epsilon Q)$

- Gaussian

  $G = \exp\left(-(rQ)^2\right)$

- Edgeworth expansion

  $G = \exp\left(-(rQ)^2\right) \cdot \left[1 + \frac{\kappa}{3!} H_3(rQ)\right]$

  Gaussian if $\kappa = 0$

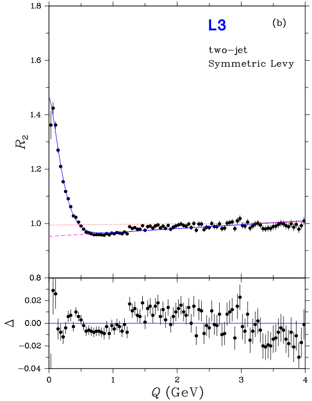
  $\kappa = 0.71 \pm 0.06$

- symmetric Lévy

  $G = \exp\left(-|rQ|^\alpha\right)$

  $0 < \alpha \leq 2$

  $\alpha = 1.34 \pm 0.04$



	Gauss	Edgew	Lévy
CL:	10^{-15}	10^{-5}	10^{-8}

Poor χ^2. Edgeworth and Lévy better than Gaussian, but poor.
Problem is the dip of R_2 in the region $0.6 < Q < 1.5\,\mathrm{GeV}$

The τ-model

T.Csörgő, W.Kittel, W.J.Metzger, T.Novák, Phys.Lett.**B663**(2008)214
T.Csörgő, J.Zimányi, Nucl.Phys.**A517**(1990)588

- Assume avg. production point is related to momentum:
$$\overline{x}^\mu(p^\mu) = a\tau p^\mu$$
where for 2-jet events, $a = 1/m_t$
$$\tau = \sqrt{\overline{t}^2 - \overline{r}_z^2} \text{ is the “longitudinal” proper time}$$
and $m_t = \sqrt{E^2 - p_z^2}$ is the “transverse” mass
- Let $\delta_\Delta(x^\mu - \overline{x}^\mu)$ be dist. of prod. points about their mean, and $H(\tau)$ the dist. of τ. Then the emission function is
$$S(x,p) = \int_0^\infty d\tau H(\tau)\delta_\Delta(x - a\tau p)\rho_1(p)$$
- In the plane-wave approx. F.B.Yano, S.E.Koonin, Phys.Lett.**B78**(1978)556.
$$\rho_2(p_1,p_2) = \int d^4x_1 d^4x_2 S(x_1,p_1)S(x_2,p_2)\left(1+\cos\left([p_1-p_2][x_1-x_2]\right)\right)$$
- Assume $\delta_\Delta(x - a\tau p)$ is very narrow — a δ-function. Then
$$R_2(p_1,p_2) = 1 + \lambda \,\mathrm{Re}\widetilde{H}\left(\frac{a_1 Q^2}{2}\right)\widetilde{H}\left(\frac{a_2 Q^2}{2}\right), \quad \widetilde{H}(\omega) = \int d\tau H(\tau)\exp(i\omega\tau)$$

BEC in the τ-model

- Assume a Lévy distribution for $H(\tau)$
  Since no particle production before the interaction, $H(\tau)$ is one-sided.
  Characteristic function is
  $$\widetilde{H}(\omega) = \exp\left[-\frac{1}{2}\left(\Delta\tau|\omega|\right)^{\alpha}\left(1 - i\,\mathrm{sign}(\omega)\tan\left(\frac{\alpha\pi}{2}\right)\right) + i\,\omega\tau_0\right] \;, \quad \alpha \neq 1$$
  where
    - α is the index of stability;
    - τ_0 is the proper time of the onset of particle production;
    - $\Delta\tau$ is a measure of the width of the distribution.
- Then, R_2 depends on Q, a_1, a_2

$$R_2(Q, a_1, a_2) = \gamma\left\{1 + \lambda\cos\left[\frac{\tau_0 Q^2(a_1+a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha}+a_2^{\alpha}}{2}\right] \cdot \exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha}+a_2^{\alpha}}{2}\right]\right\} \cdot (1 + \epsilon Q)$$

BEC in the τ-model

$$R_2(Q,a_1,a_2) = \gamma\left\{1+\lambda\cos\left[\frac{\tau_0 Q^2(a_1+a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha}+a_2^{\alpha}}{2}\right]\cdot\exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha}+a_2^{\alpha}}{2}\right]\right\}\cdot(1+\epsilon Q)$$

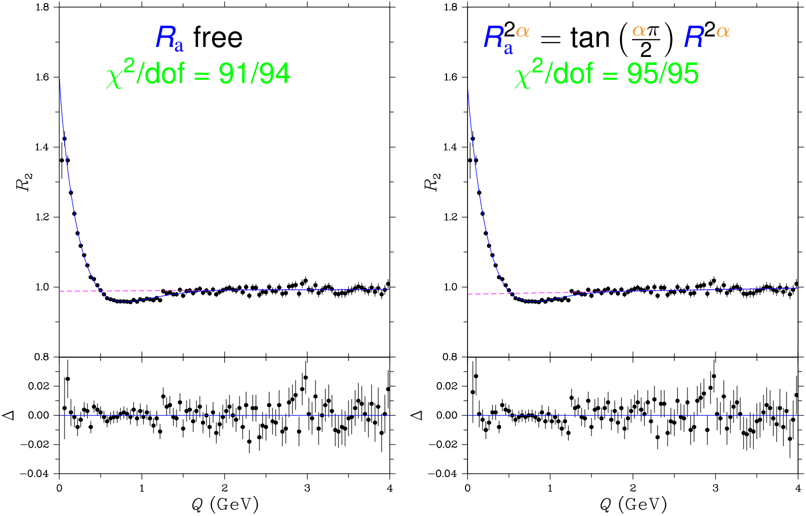
Simplification:

- effective radius, R, defined by $R^{2\alpha} = \left(\frac{\Delta\tau}{2}\right)^{\alpha}\frac{a_1^{\alpha}+a_2^{\alpha}}{2}$
- Particle production begins immediately, $\tau_0 = 0$
- Then
  $$R_2(Q) = \gamma\left[1+\lambda\cos\left((R_a Q)^{2\alpha}\right)\exp\left(-(RQ)^{2\alpha}\right)\right]\cdot(1+\epsilon Q)$$
  where $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right)R^{2\alpha}$

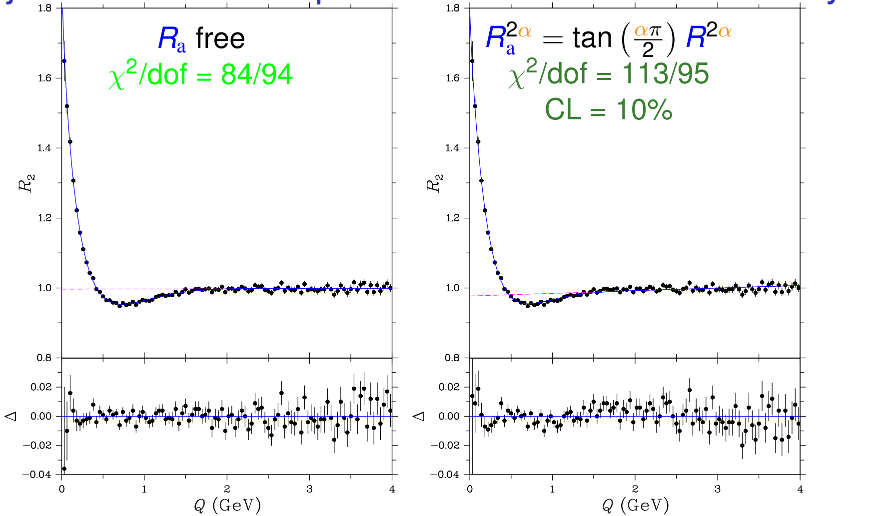
Compare to sym. Lévy parametrization:

$$R_2(Q) = \gamma\left[1+\lambda\qquad\qquad\exp\left[-|rQ|^{\alpha}\right]\right](1+\epsilon Q)$$

2-jet Results on Simplified τ-model from L3 Z decay



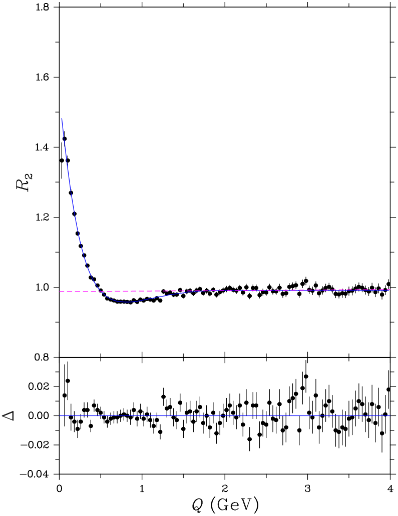
3-jet Results on Simplified τ-model from L3 Z decay



R-summary

Full τ-model for 2-jet events — $a = 1/m_t$

$$R_2(Q, m_{t1}, m_{t2}) = \gamma \left\{ 1 + \lambda \cos\left[\frac{\tau_0 Q^2 (m_{t1}+m_{t2})}{2(m_{t1} m_{t2})} + \tan\left(\frac{\alpha\pi}{2}\right) \left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{m_{t1}^{\alpha} + m_{t2}^{\alpha}}{2(m_{t1} m_{t2})^{\alpha}} \right] \cdot \exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{m_{t1}^{\alpha} + m_{t2}^{\alpha}}{2(m_{t1} m_{t2})^{\alpha}} \right] \right\} \cdot (1 + \epsilon Q)$$

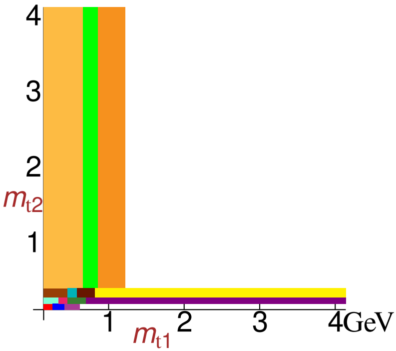


- Fit $R_2(Q)$ using avg m_{t1}, m_{t2} in each Q bin, $m_{t1} > m_{t2}$
- $\tau_0 = 0.00 \pm 0.02$ so fix to 0
- $\chi^2/\text{dof} = 90/95$

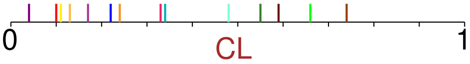
fit

Full τ-model for 2-jet events

- τ-model predicts dependence on m_t, $R_2(Q, m_{t1}, m_{t2})$
- Parameters α, $\Delta\tau$, τ_0 are independent of m_t
- λ (strength of BEC) can depend on m_t



- divide m_{t1}-m_{t2} plane in regions (equal statistics)
- in each region fit $R_2(Q)$ using avg m_{t1}, m_{t2} in each Q bin with α, $\Delta\tau$, fixed to values found for entire plane and $\tau_0 = 0$ fits



Elongation?

- Previous results using fits of Gaussian or Edgeworth found (in LCMS) $R_{\mathrm{side}}/R_{\mathrm{L}} \approx 0.64$
- But we find that Gaussian and Edgeworth fit $R_2(Q)$ poorly
- τ-model predicts no elongation and fits the data well
- Could the elongation results be an artifact of an incorrect fit function?

  or is the τ-model in need of modification?
- So, we modify *ad hoc* the τ-model description to allow elongation (more on this later)
- and find $R_{\mathrm{side}}/R_{\mathrm{L}} = 0.61 \pm 0.02$ – elongation is real
- Perhaps, a should be different for transverse/longitudinal

$$\overline{x}^{\mu}(p^{\mu}) = a\tau p^{\mu}, \qquad a = 1/m_{\mathrm{t}} \text{ for 2-jet}$$

restQdep

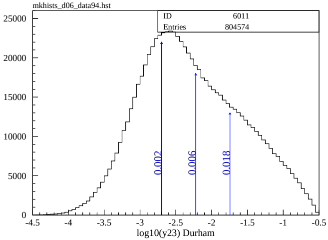
Outline

New stuff — very preliminary

Are BEC sensitive to jet structure?

Jets

- Jets — Durham algorithm
- y_{23} is value of y_{cut} where number of jets changes from 2 to 3
- force event to have 3 jets
- define regions of y_{23}:

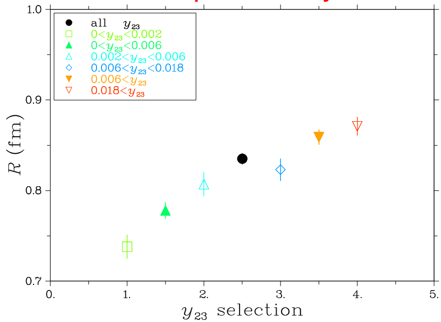


$y_{23} < 0.002$	narrow two-jet	or		
$0.002 < y_{23} < 0.006$	less narrow two-jet		$y_{23} < 0.006$	two-jet
$0.006 < y_{23} < 0.018$	narrow three-jet		$0.006 < y_{23}$	three-jet
$0.018 < y_{23}$	wide three-jet			

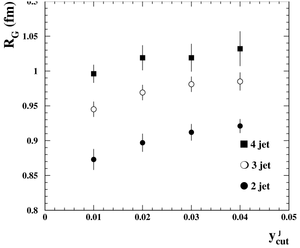
To stabilize fits against large correlation of α, R, fix $\alpha = 0.443$

Jets

L3 preliminary



OPAL: Z.Phys. C72 (1996) 389



- R increases as number of jets increases
- or as jets become more separated

Jets - Elongation

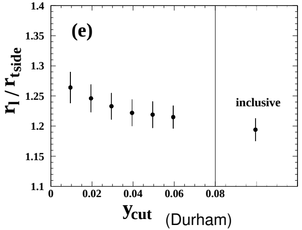
Results in LCMS frame: Longitudinal = Thrust axis ▸ LCMS

	R_L / R_{side}
L3	$1.25 \pm 0.03^{+0.36}_{-0.05}$
OPAL	$1.19 \pm 0.03^{+0.08}_{-0.01}$

(ZEUS finds similar results in ep)
~25% elongation along thrust axis

OPAL:

Elongation larger for narrower jets



(Durham)

LCMS and the Simplified τ-model

Consider 2 frames:

1. LCMS: $$Q^2 = Q_L^2 + Q_{side}^2 + Q_{out}^2 - (\Delta E)^2$$ $$= Q_L^2 + Q_{side}^2 + Q_{out}^2 \left(1-\beta^2\right), \quad \beta = \frac{p_{1out}+p_{2out}}{E_1+E_2}$$
2. LCMS-rest: $$Q^2 = Q_L^2 + Q_{side}^2 + q_{out}^2, \quad q_{out}^2 = Q_{out}^2\left(1-\beta^2\right)$$

q_{out} is Q_{out} boosted (β) along out direction to rest frame of pair

In simplified τ-model, replace R^2Q^2 by

1. $A^2 = R_L^2 Q_L^2 + R_{side}^2 Q_{side}^2 + \rho_{out}^2 Q_{out}^2$
2. $B^2 = R_L^2 Q_L^2 + R_{side}^2 Q_{side}^2 + r_{out}^2 q_{out}^2$

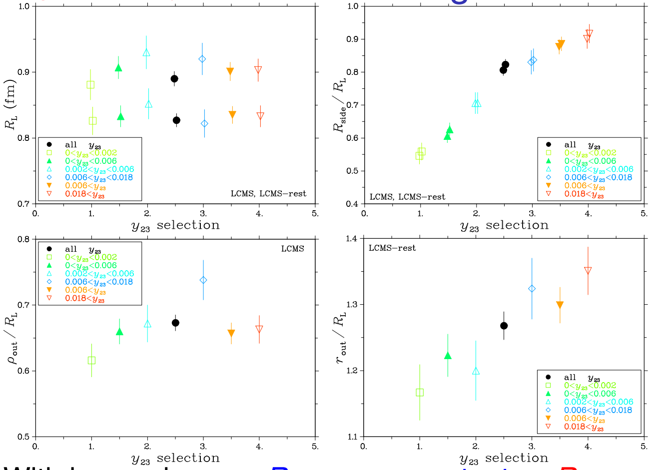
Then in τ-model, for case 1:

$$R_2(Q_L, Q_{side}, Q_{out}) = \gamma\left[1 + \lambda \cos\left(\tan\left(\frac{\alpha\pi}{2}\right) A^{2\alpha}\right)\exp\left(-A^{2\alpha}\right)\right]$$
$$\cdot\left(1 + \epsilon_L Q_L + \epsilon_{side} Q_{side} + \epsilon_{out} Q_{out}\right)$$

and comparable expression for case 2, $R_2(Q_L, Q_{side}, q_{out})$

L3 preliminary

Jets - Elongation



Note:

$R_{\rm side} < R_{\rm L}$

$r_{\rm out} > R_{\rm L}$

Not azimuthally symmetric not even for narrow 2-jet trigger bias??

With increasing y_{23}, $R_{\rm L}$, $\rho_{\rm out} \approx$ constant, $R_{\rm side}$, $r_{\rm out}$ increase

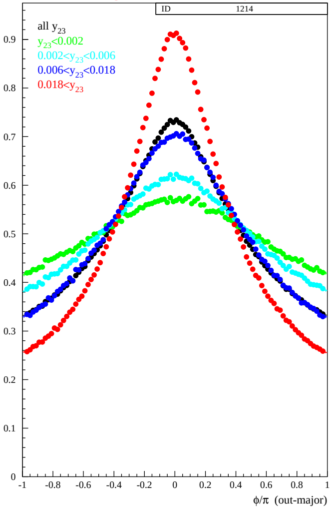
narrow 2-jet limit: $R_{\rm side} \approx R_{\rm L}/2$ $r_{\rm out} \approx 1.1 R_{\rm L}$

wide 3-jet limit: $R_{\rm side} \approx R_{\rm L}$ $r_{\rm out} \approx 1.4 R_{\rm L}$

ϕ major-out

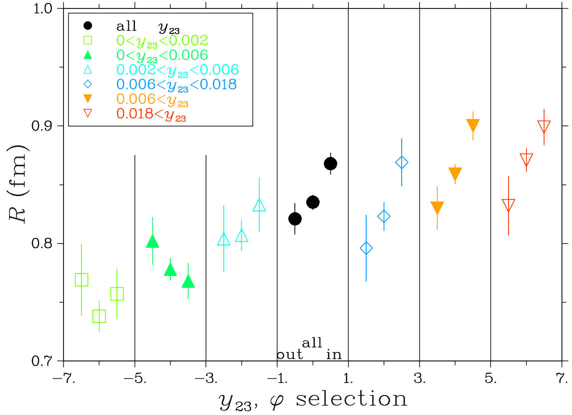
- event plane $\equiv$ (thrust,major)
- out direction tends to be in the event plane out $\approx$ in
- side direction tends to be out of the plane side $\approx$ out
- this tendency increases with y_{23}
- suggests that lack of azimuthal symmetry is due to difference in fragmentation in and out of the event plane

L3 preliminary



in/out of event plane

use only tracks with ϕ(trk-major) $< 45°$ in plane
ϕ(trk-major) $> 45°$ out of plane



L3 preliminary

R larger in the event plane

New Stuff — Summary

	narrow 2-jet limit	wide 3-jet limit
$R \approx$	0.7 fm	0.9 fm
$R_{\mathrm{L}} \approx$	0.9 fm – constant	
$\rho_{\mathrm{out}} \approx$	0.66 fm – constant	
	$R_{\mathrm{side}} \approx R_{\mathrm{L}}/2$	$R_{\mathrm{side}} \approx R_{\mathrm{L}}$
	$r_{\mathrm{out}} \approx 1.1 R_{\mathrm{L}}$	$r_{\mathrm{out}} \approx 1.4 R_{\mathrm{L}}$

out direction $\approx$ in event plane

side direction $\approx$ out of event plane

R in $\approx R$ out of plane	R in $> R$ out of plane

Acknowledgments

- Tamás Novák, Tamás Csörgő, Wolfram Kittel were instrumental for the 'Old Stuff'
- I take full responsibility for the 'New Stuff'

A Comment

- τ-model is closely related to a string picture
  - strong *x*-*p* correlation
  - fractal - Lévy distribution
- CMS finds BEC in pp at 0.9 and 7 TeV are described by simplified τ-model formula JHEP 05 (2011) 029
- suggests that BEC in pp is (mostly) from string fragmentation

Emission Function of 2-jet Events.

In the τ-model, the emission function in configuration space is

$$S(\vec{x},\tau) = \frac{1}{\overline{n}}\frac{\mathrm{d}^4 n}{\mathrm{d}\tau\mathrm{d}\vec{x}} = \frac{1}{\overline{n}}\left(\frac{m_\mathrm{t}}{\tau}\right)^3 H(\tau)\rho_1\left(\vec{p} = \frac{m_\mathrm{t}\vec{x}}{\tau}\right)$$

For simplicity, assume $\rho_1(\vec{p}) = \rho_\mathrm{y}(y)\rho_{p_\mathrm{t}}(p_\mathrm{t})/\overline{n}$

(ρ_1, ρ_y, ρ_{p_t} are inclusive single-particle distributions)

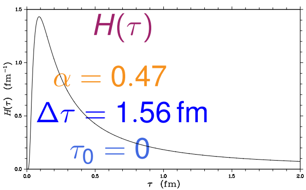
Then $S(\vec{x},\tau) = \frac{1}{\overline{n}^2}H(\tau)G(\eta)I(r)$

Strongly correlated $x, p \Longrightarrow$

$$\eta = y \qquad r = p_\mathrm{t}\tau/m_\mathrm{t}$$

$$G(\eta) = \rho_\mathrm{y}(\eta) \qquad I(r) = \left(\frac{m_\mathrm{t}}{\tau}\right)^3 \rho_{p_\mathrm{t}}(rm_\mathrm{t}/\tau)$$

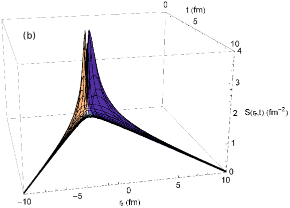
expt. – Factorization OK



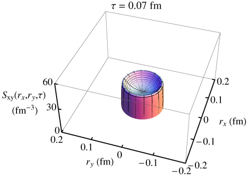
So, using experimental $\rho_\mathrm{y}(y)$, $\rho_{p_\mathrm{t}}(p_\mathrm{t})$ dists.
and $H(\tau)$ from BEC fits,
we can reconstruct S.

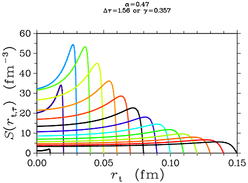
Emission Function of 2-jet Events.

Integrating over *r*,



Integrating over *z*,





"Boomerang" shape

Particle production is close to the light-cone

α_s

- LLA parton shower leads to a fractal in momentum space fractal dimension is related to α_s Gustafson *et* al.
- Lévy dist. arises naturally from a fractal, or random walk, or anomalous diffusion Metzler and Klafter, Phys.Rep.**339**(2000)1.
- strong momentum-space/configuration space correlation of τ-model $\Longrightarrow$ fractal in configuration space with same α
- generalized LPHD suggests particle dist. has same properties as gluon dist.
- Putting this all together leads to Csörgő *et* al.

$$\alpha_s = \frac{2\pi}{3}\alpha^2$$

- Using our value of $\alpha = 0.47 \pm 0.04$ yields $\alpha_s = 0.46 \pm 0.04$
- This value is reasonable for a scale of 1–2 GeV, where production of hadrons takes place *cf.*, from τ decays $\alpha_s(m_\tau \approx 1.8\,\mathrm{GeV}) = 0.34 \pm 0.03$ PDG

BEC Introduction

q-particle density $\rho_q(p_1,\ldots,p_q) = \frac{1}{\sigma_{\text{tot}}}\frac{\mathrm{d}^q\sigma_q(p_1,\ldots,p_q)}{\mathrm{d}p_1\ldots\mathrm{d}p_q}$

2-particle correlation: $\frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)}$

To study only BEC, not all correlations,
let $\rho_0(p_1,p_2)$ be the 2-particle density if no BEC
($= \rho_2$ of the 'reference sample') and define

$$R_2(p_1,p_2) = \frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)} \cdot \frac{\rho_1(p_1)\rho_1(p_2)}{\rho_0(p_1,p_2)} = \frac{\rho_2(p_1,p_2)}{\rho_0(p_1,p_2)}$$

Since 2-π BEC only at small Q

$$Q = \sqrt{-(p_1-p_2)^2} = \sqrt{M_{12}^2 - 4m_\pi^2}$$

integrate over other variables: $R_2(Q) = \frac{\rho(Q)}{\rho_0(Q)}$

intro

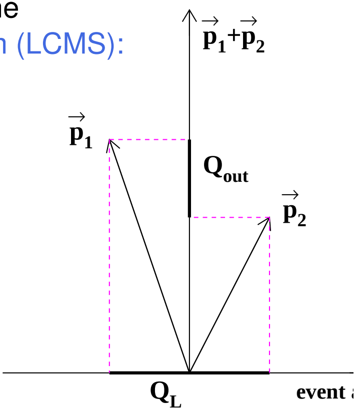
LCMS

The usual parametrization assumes a symmetric Gaussian source

But, there is no reason to expect this symmetry in $e^+e^- \to q\bar{q}$.

Therefore, do a 3-dim. analysis in the

Longitudinal Center of Mass System (LCMS):



Boost each π-pair along event (thrust) axis $p_{L1} = -p_{L2}$

$\vec{p}_1 + \vec{p}_2$ defines 'out' axis

$Q_{side} \perp (Q_L, Q_{out})$

advantage

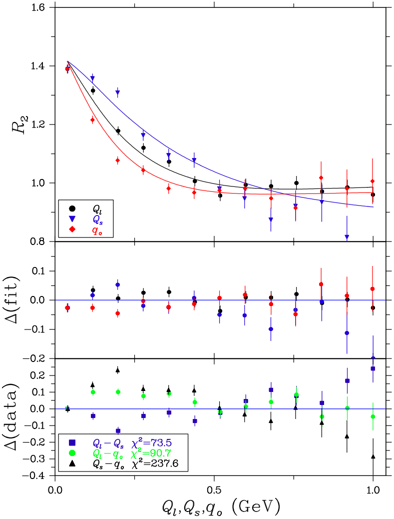
LCMS

Advantages of LCMS:

$$
\begin{aligned}
Q^2 &= Q_L^2 + Q_{\mathrm{side}}^2 + Q_{\mathrm{out}}^2 - (\Delta E)^2 \\
&= Q_L^2 + Q_{\mathrm{side}}^2 + Q_{\mathrm{out}}^2 (1 - \beta^2) \qquad \text{where } \beta \equiv \frac{p_{\mathrm{out}\,1} + p_{\mathrm{out}\,2}}{E_1 + E_2}
\end{aligned}
$$

Thus, the energy difference,
and therefore the difference in emission time of the pions couples only to the out-component, Q_{out}.
Thus,
Q_L and Q_{side} reflect only spatial dimensions of the source
Q_{out} reflects a mixture of spatial and temporal dimensions.

$\boldsymbol{Q}$ Dependence



$R_2(Q_L, Q_{side}, q_{out})$ vs.

Q_L for $Q_{side}, q_{out} < 0.08$ GeV

Q_{side} for $Q_L, q_{out} < 0.08$ GeV

q_{out} for $Q_L, Q_{side} < 0.08$ GeV

Dependence on components of Q is preferred.

$r_{out} > R_L > R_{side}$

Not azimuthally symmetric

elong

Summary of Simplified τ-model

	α	R (fm)	R_a (fm)	CL
2-jet	$0.41 \pm 0.02^{+0.04}_{-0.06}$	$0.79 \pm 0.04^{+0.09}_{-0.19}$	$0.69 \pm 0.04^{+0.21}_{-0.09}$	57%
3-jet	$0.35 \pm 0.01^{+0.03}_{-0.04}$	$1.06 \pm 0.05^{+0.59}_{-0.31}$	$0.85 \pm 0.04^{+0.15}_{-0.05}$	76%
3-jet	$0.41 \pm$ fixed	0.93 ± 0.03	0.76 ± 0.01	38%
2-jet	$0.44 \pm 0.01^{+0.05}_{-0.02}$	$0.78 \pm 0.04^{+0.09}_{-0.16}$	—	49%
3-jet	$0.42 \pm 0.01^{+0.02}_{-0.04}$	$0.98 \pm 0.04^{+0.55}_{-0.14}$	—	10%
3-jet	$0.44 \pm$ fixed	0.87 ± 0.01	—	3%

- consistent values of α
- $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$ to 0.5σ for 2-jet and to 1.5σ for 3-jet
- Simplified τ-model works well

fitR fitRR

- R seems to be larger for 3-jet than for 2-jet events

simp3j

Fit Results Simplified τ-model

parameter	two-jet	three-jet
λ	$0.63 \pm 0.03^{+0.08}_{-0.35}$	$0.92 \pm 0.05^{+0.06}_{-0.48}$
α	$0.41 \pm 0.02^{+0.04}_{-0.06}$	$0.35 \pm 0.01^{+0.03}_{-0.04}$
R (fm)	$0.79 \pm 0.04^{+0.09}_{-0.19}$	$1.06 \pm 0.05^{+0.59}_{-0.31}$
R_a (fm)	$0.69 \pm 0.04^{+0.21}_{-0.09}$	$0.85 \pm 0.04^{+0.15}_{-0.05}$
ϵ (GeV^{-1})	$0.001 \pm 0.002^{+0.005}_{-0.008}$	$0.000 \pm 0.002^{+0.001}_{-0.007}$
γ	$0.988 \pm 0.005^{+0.026}_{-0.012}$	$0.997 \pm 0.005^{+0.019}_{-0.002}$
χ^2/DoF	91/94	84/94
confidence level	57%	76%

Fit Results Simplified τ-model

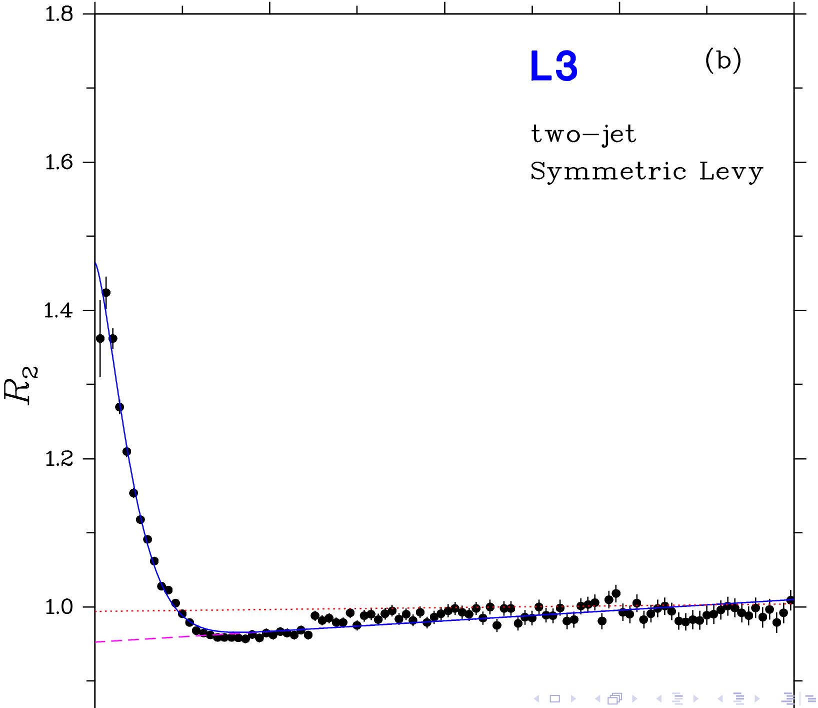
parameter	two-jet	three-jet
λ	$0.61 \pm 0.03^{+0.08}_{-0.26}$	$0.84 \pm 0.04^{+0.04}_{-0.37}$
α	$0.44 \pm 0.01^{+0.05}_{-0.02}$	$0.42 \pm 0.01^{+0.02}_{-0.04}$
R (fm)	$0.78 \pm 0.04^{+0.09}_{-0.16}$	$0.98 \pm 0.04^{+0.55}_{-0.14}$
ϵ (GeV^{-1})	$0.005 \pm 0.001 \pm 0.003$	$0.008 \pm 0.001 \pm 0.005$
γ	$0.979 \pm 0.002^{+0.009}_{-0.003}$	$0.977 \pm 0.001^{+0.013}_{-0.008}$
χ^2/DoF	95/95	113/95
confidence level	49%	10%

Fit Results Full τ-model for 2-jet events

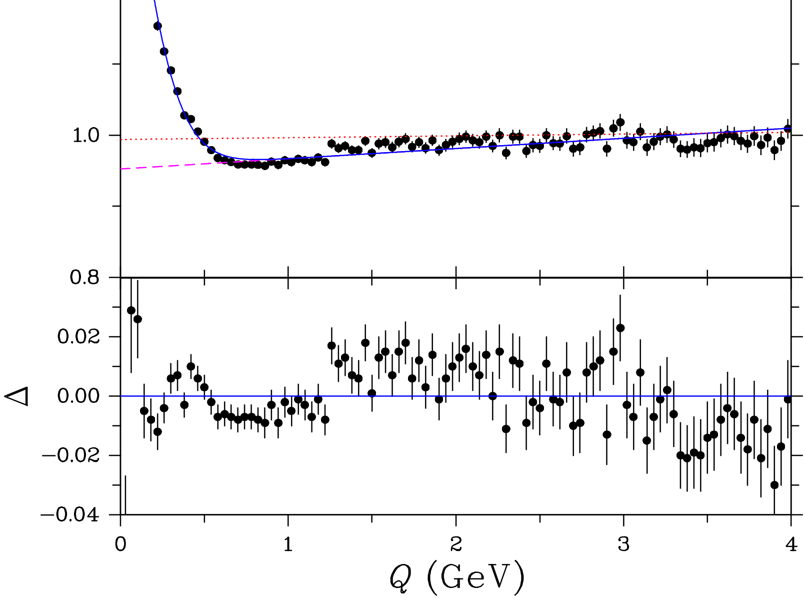
m_t regions (GeV) m_{t1}	m_{t2}	average m_t (GeV) $Q < 0.4$	all	confidence level (%)	λ
0.14 – 0.26	0.14 – 0.22	0.19	0.19	10	0.39 ± 0.02
0.14 – 0.34	0.22 – 0.30	0.27	0.27	48	0.76 ± 0.03
0.14 – 0.46	0.30 – 0.42	0.37	0.37	74	0.83 ± 0.03
0.14 – 0.66	0.42 – 4.14	0.52	0.52	13	0.97 ± 0.04
0.26 – 0.42	0.14 – 0.22	0.25	0.26	22	0.53 ± 0.02
0.34 – 0.46	0.22 – 0.30	0.32	0.33	33	0.80 ± 0.03
0.46 – 0.58	0.30 – 0.42	0.43	0.44	34	0.91 ± 0.04
0.66 – 0.86	0.42 – 4.14	0.65	0.65	66	1.01 ± 0.05
0.42 – 0.62	0.14 – 0.22	0.34	0.34	17	0.41 ± 0.03
0.46 – 0.70	0.22 – 0.30	0.41	0.41	55	0.64 ± 0.03
0.58 – 0.82	0.30 – 0.42	0.52	0.52	59	0.70 ± 0.04
0.86 – 1.22	0.42 – 4.14	0.80	0.81	24	0.66 ± 0.05
0.70 – 4.14	0.22 – 0.30	0.59	0.65	4	0.37 ± 0.04
0.82 – 4.14	0.30 – 0.42	0.71	0.76	11	0.56 ± 0.05

Fit Result $R_2(Q, m_{t1}, m_{t2})$

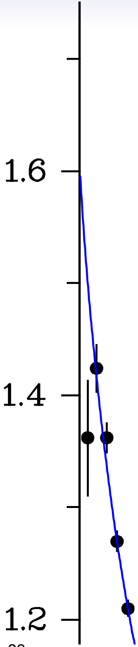
parameter	
λ	$0.58 \pm 0.03^{+0.08}_{-0.24}$
α	$0.47 \pm 0.01^{+0.04}_{-0.02}$
$\Delta\tau$ (fm)	$1.56 \pm 0.12^{+0.32}_{-0.45}$
ϵ (GeV^{-1})	$0.001 \pm 0.001 \pm 0.003$
γ	$0.988 \pm 0.002^{+0.006}_{-0.002}$
χ^2/DoF	90/95
confidence level	62%



1.8
1.6
1.4
1.2
1.0
R_2
L3
(b)
two−jet
Symmetric Levy

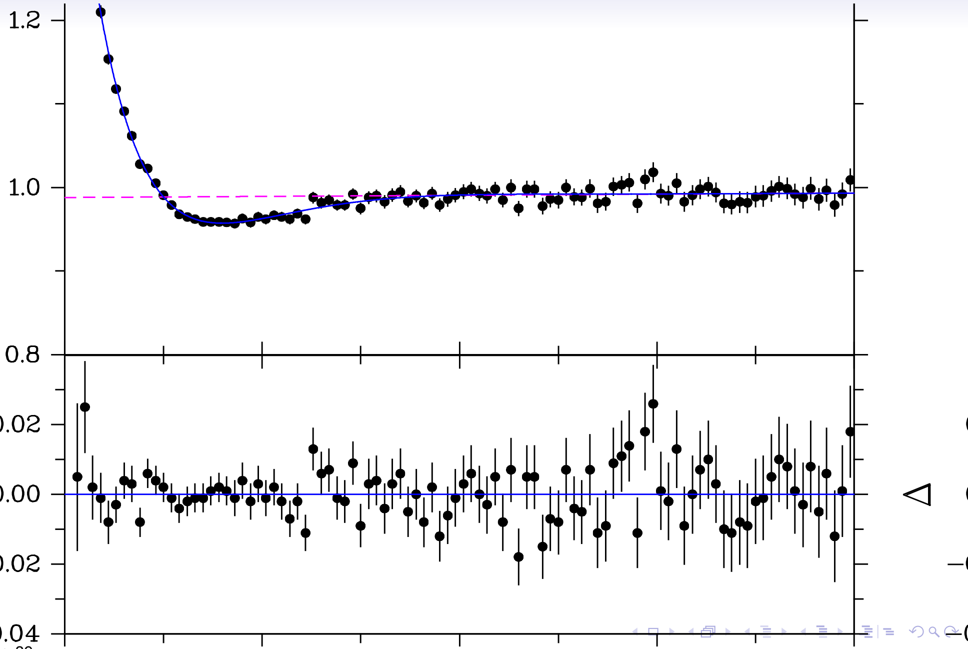


	Gauss	Edgew	Lévy
CL:	10^{-15}	10^{-5}	10^{-8}



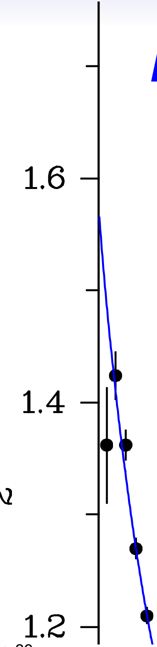
R_a free

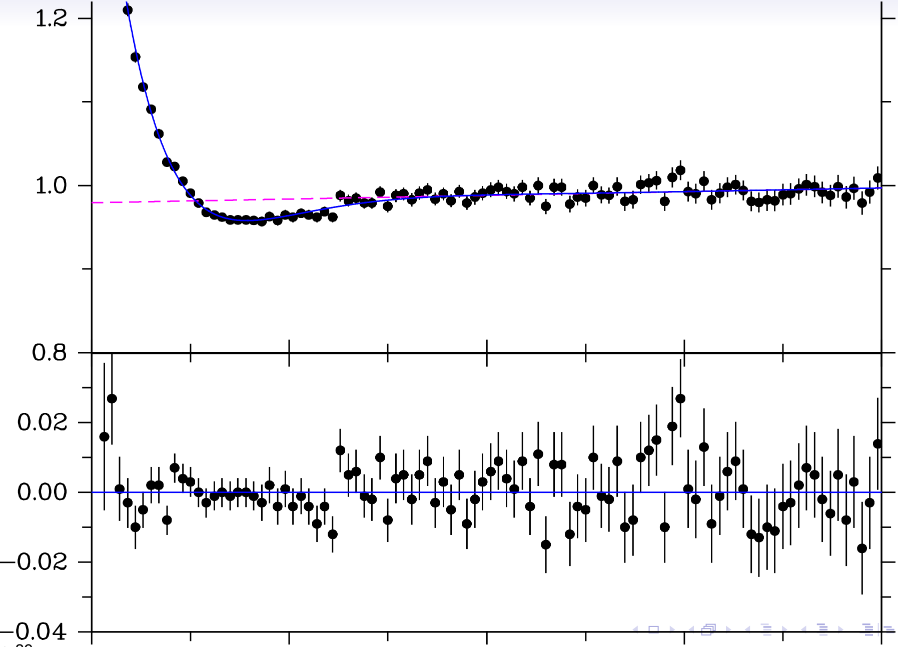
$$\chi^2/\text{dof} = 91/94$$

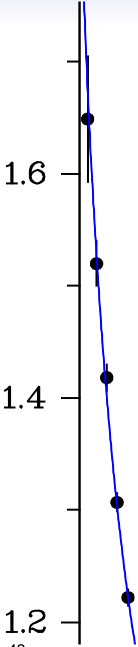

1.2
1.0
0.8
0.02
0.00
0.02
0.04
Δ

$$R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$$

$$\chi^2/\text{dof} = 95/95$$

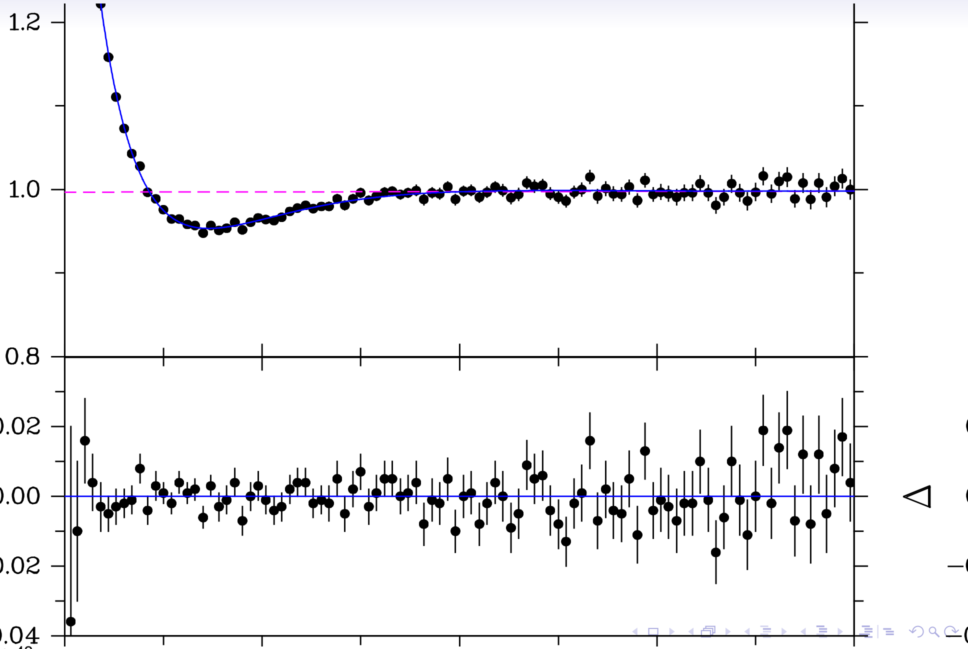


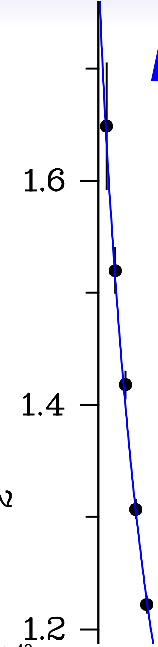

1.2
1.0
0.8
0.02
0.00
−0.02
−0.04



R_a free

χ^2/dof = 84/94

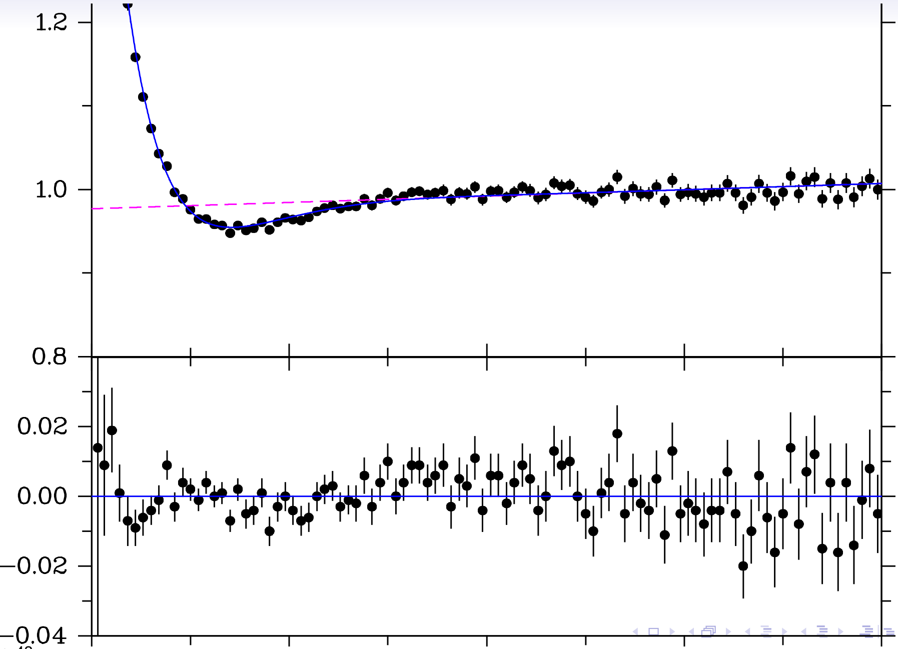

1.2
1.0
0.8
0.02
0.00
0.02
0.04
Δ

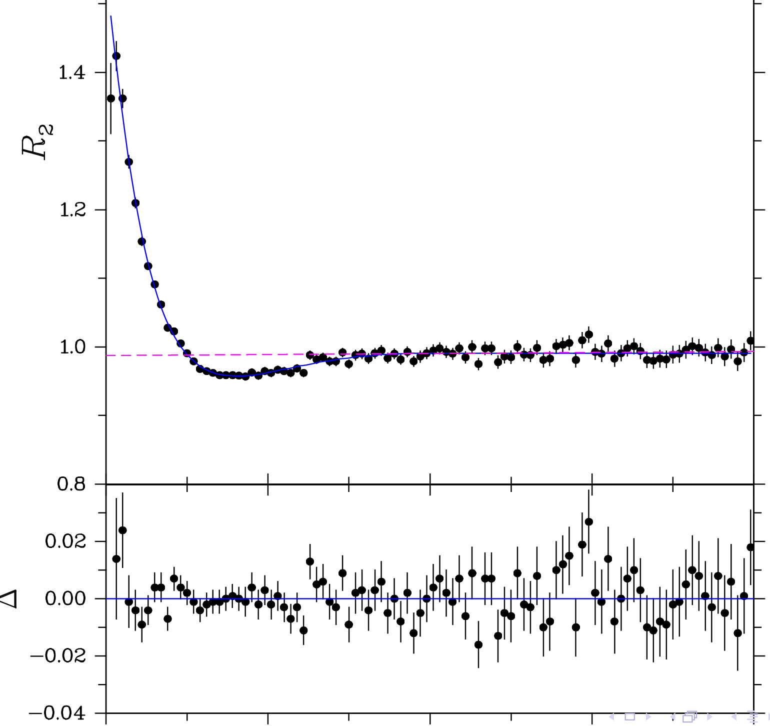


$$R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$$

$$\chi^2/\mathrm{dof} = 113/95$$

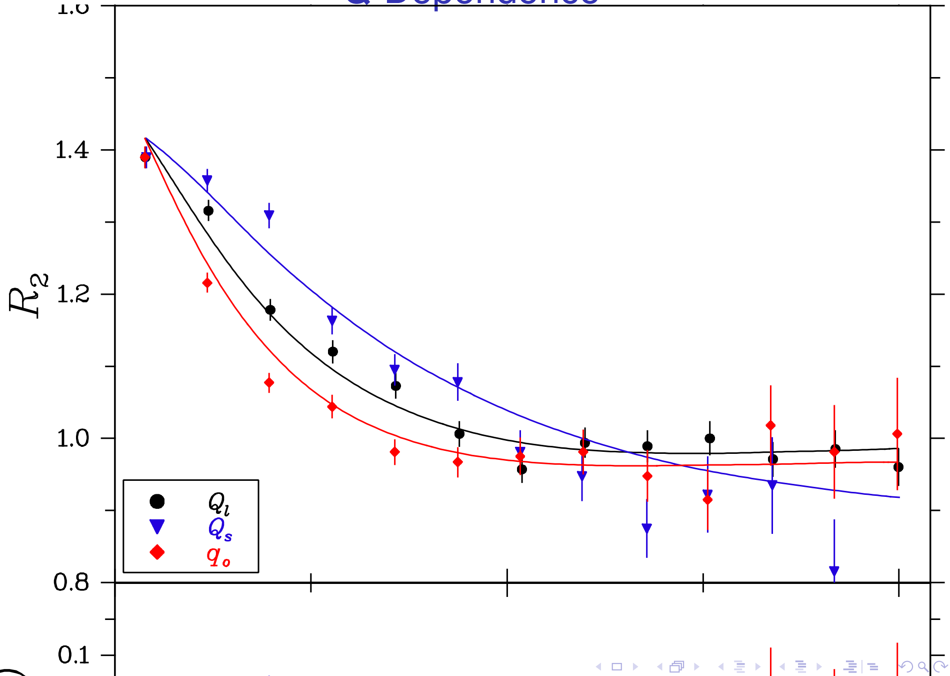
$$\mathrm{CL} = 10\%$$


1.2
1.0
0.8
0.02
0.00
−0.02
−0.04

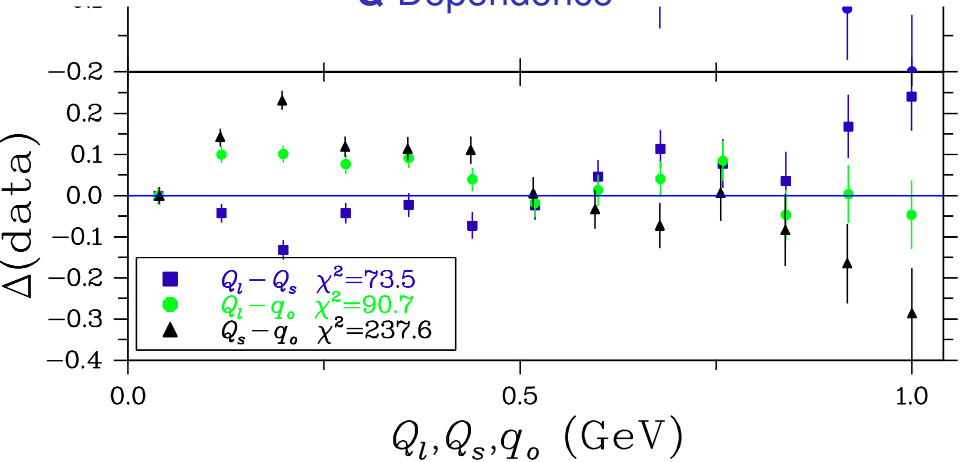


1.4
1.2
1.0
0.8
R_2
0.02
0.00
−0.02
−0.04
Δ

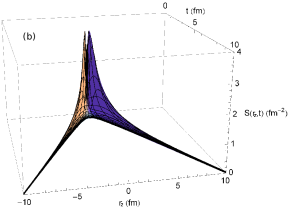
$\boldsymbol{Q}$ Dependence



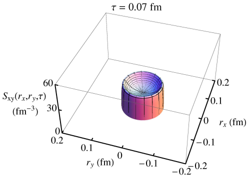
Q Dependence

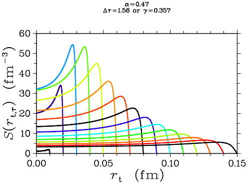


Integrating over *r*,



Integrating over *z*,

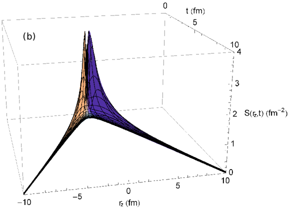




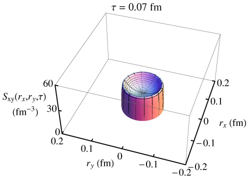
"Boomerang" shape

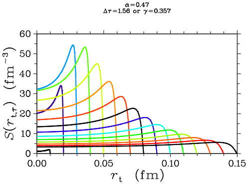
Particle production is close to the light-cone

Integrating over r,



Integrating over z,

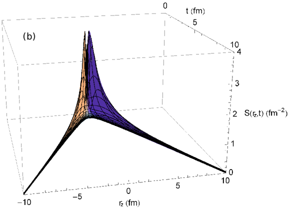




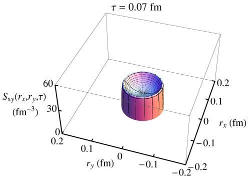
"Boomerang" shape

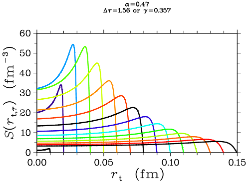
Particle production is close to the light-cone

Integrating over r,



Integrating over z,

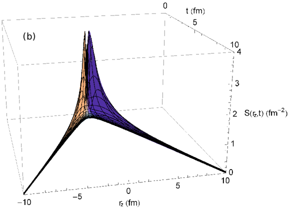




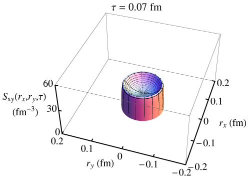
"Boomerang" shape

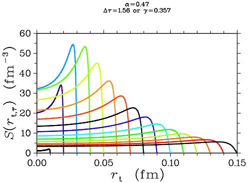
Particle production is close to the light-cone

Integrating over *r*,



Integrating over *z*,

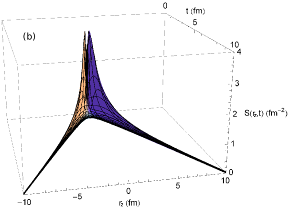




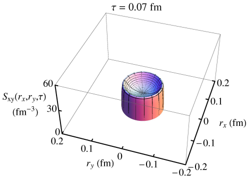
"Boomerang" shape

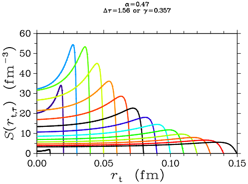
Particle production is close to the light-cone

Integrating over *r*,



Integrating over *z*,

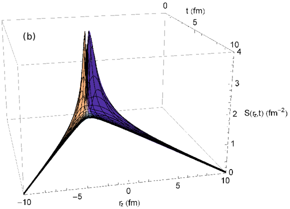




"Boomerang" shape

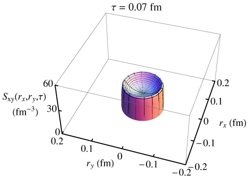
Particle production is close to the light-cone

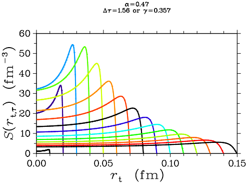
Integrating over *r*,



"Boomerang" shape

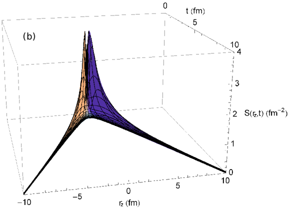
Integrating over *z*,



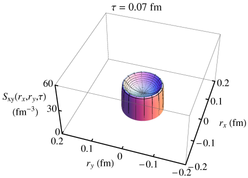


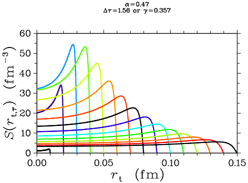
Particle production is close to the light-cone

Integrating over r,



Integrating over z,

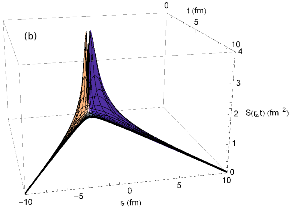




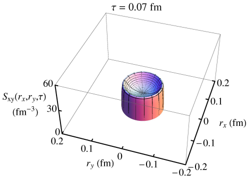
"Boomerang" shape

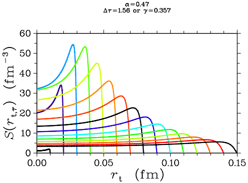
Particle production is close to the light-cone

Integrating over r,



Integrating over z,

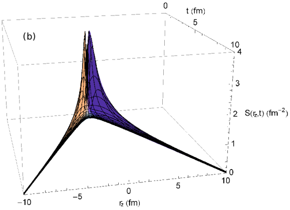




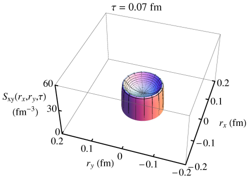
"Boomerang" shape

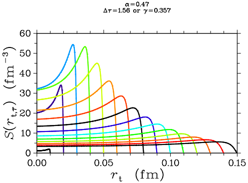
Particle production is close to the light-cone

Integrating over *r*,



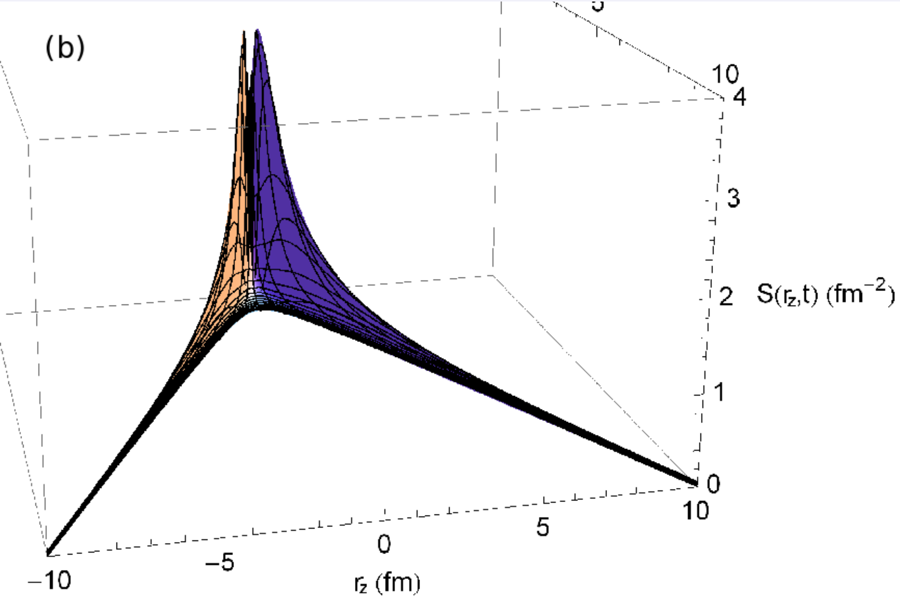
Integrating over *z*,

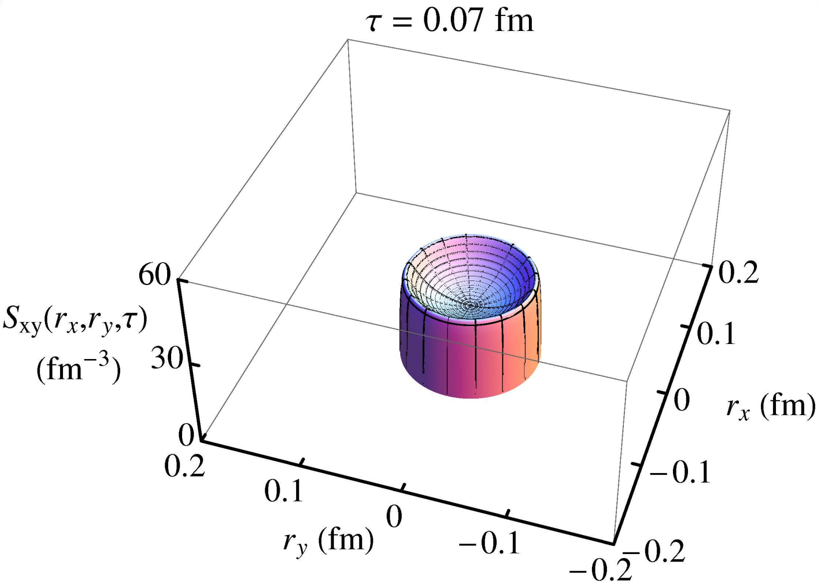




"Boomerang" shape

Particle production is close to the light-cone


(b)
S(rz,t) (fm−2)
rz (fm)
−10
−5
0
5
10
0
1
2
3
4


τ = 0.07 fm
$S_{xy}(r_x,r_y,\tau)$
(fm^{-3})
60
30
0
0.2
0.1
0
−0.1
−0.2
r_y (fm)
r_x (fm)

$\alpha = 0.47$
$\Delta\tau = 1.56$ or $\gamma = 0.357$