Next Generation of Parton Shower Models and Validation

In collaboration with David Kosower, Peter Skands

- Introduction
- A new parton shower approach, why?
- Subtraction based parton showers
- Matching to LO & NLO Fixed Order
- Conclusions

Introduction

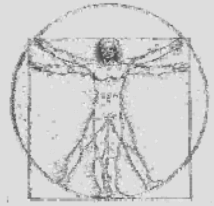
There are several fully developed event generators which contain a traditional approach to parton showers. These generators continue to evolve:

- **HERWIG++: complete reimplementation**
  - Improved parton shower and decay algorithms
  - Eventually to include CKKW-style matching (?)
  - B.R. Webber; S. Gieseke, D. Grellscheid, A. Ribon, P. Richardson, M. Seymour, P. Stephens, . . .
- **SHERPA: complete implementation, has CKKW**
  - ME generator + wrappers to / adaptations of PYTHIA, HERWIG parton showers, underlying event, hadronization
  - F. Krauss; T. Fischer, T. Gleisberg, S. Hoeche, T. Laubrich, A. Schaelicke, S. Schumann, C. Semmling, J. Winter
- **PYTHIA8: selective reimplementation**
  - Improved parton shower and underlying event, limited number of hard subprocesses
  - Many obsolete features not carried over → simpler, less parameters
  - T. Sjöstrand, S. Mrenna, P. Skands

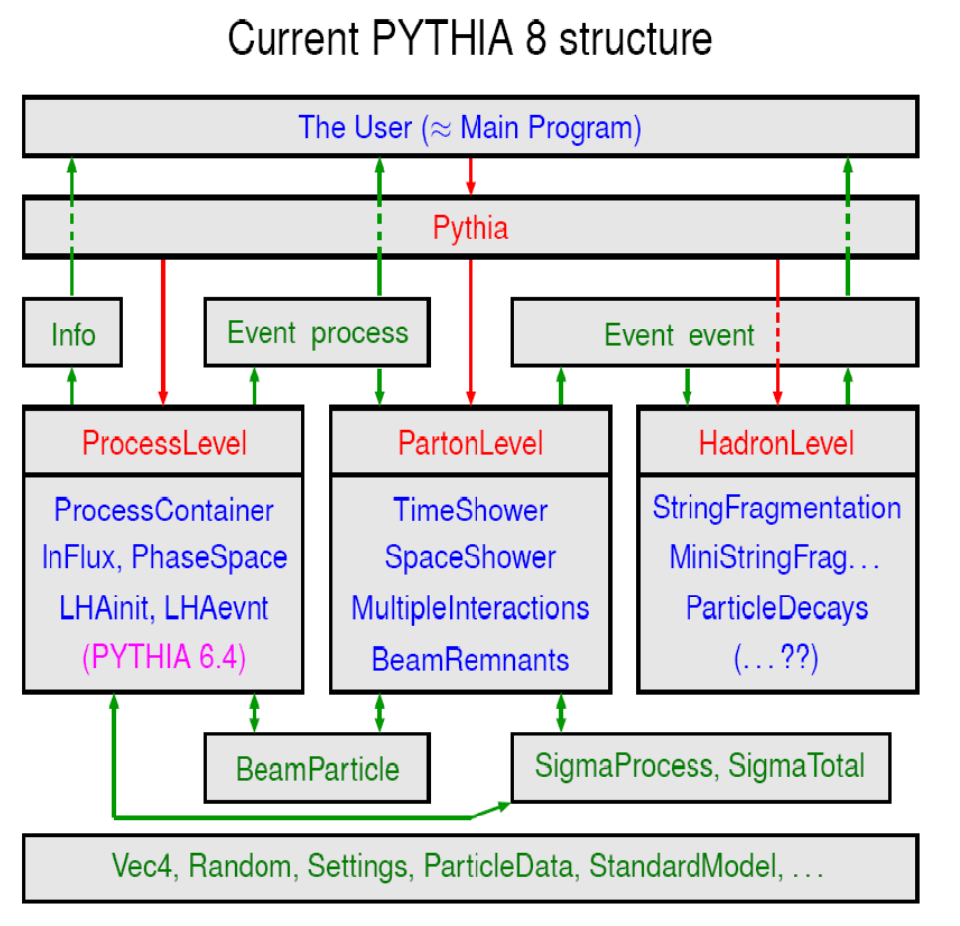
Introduction

- Here I will only talk about an new approach to the parton shower part of event generators.
- The parton shower is still modeled by perturbative QCD (in the strong ordering limit, i.e. an expansion in $\alpha_S \log^2\left(Q_R^2\right)$). As such it is a calculable contribution.
- The parton shower "evolves" the event from the high momentum transfer hard scattering process (described by fixed order calculations) down to the soft physics where hadronization ensues.
- The parton shower is the interface of the event generator with the hard scattering matrix elements and controls the "matching conditions".
- Driven by matching to LO, NLO, NNLO,… matrix elements new approaches to parton showers are being explored:
    - Interfacing to "traditional" parton showers in existing event generators:
        - Refinements of MC@NLO (Nason)
        - CKKW-style at NLO + "Quantum Monte Carlo" (Nagy, Soper)
    - New parton showers:
        - SCET approach (based on SCET – Bauer, Tackmann; Alwall, Mrenna, Schwarz)
        - VINCIA (based on QCD antennae – Kosower, Skands, WG)

Introduction



- Here I will talk about the approach we are developing in VINCIA (VIrtual Numerical Collider Interactions with Antennae).
- A new parton shower does not mean a new event generator:
  - PYTHIA 8 is a C++ program.
  - This means we can simply replace the TimeShower and SpaceShower modules and replace them with the VinciaShower module without altering the PYTHIA 8 code.
  - Also the hard scattering generators will be replaced as they are affected by the implemented parton shower through matching.
  - This leaves everything else unaltered (possibly re-tuning of haronization model is needed)



A new parton shower approach, why?

- Traditional parton showers are constructed for use with a limited set of born cross sections.
- The hard multi-parton scattering has to be modeled by the parton shower, leading to “enhanced” parton showers.
- Nowadays we can efficiently calculate LO multi-parton scattering amplitudes, and our ability to calculate NLO multi-parton scattering amplitudes is rapidly increasing
- This has lead to a paradigm shift: we want the hard physics be described by the fixed order matrix element calculations and have the parton shower evolve the event further down to the hadronization scale (no longer a complementarity between the parton shower approach and matrix element calculations, but a merger)
- This is accomplished by matching: up to now the merging is being performed with the “traditional/enhanced” showers unnecessarily complicating the matching procedures.

A new parton shower approach

- This leads us to the conclusion that we want a simpler shower which is build upon the concept of matching to fixed order.
- This will simplify/systematize the merging procedures.
- We will need an extended set of LO/NLO/… matrix elements for realistic phenomenology.
- The advantage of this approach is that the hard physics phenomenology is determined by the deterministic matrix element calculations.
- Because of the parton shower simplicity we can identify the matched fixed order matrix elements with the commonly used slicing/subtraction schemes used in NLO calculations.
- This allows us to construct an “automated” matching to any set of mixed LO and NLO matrix element calculations (e.g. matched MCFM and beyond).
- With the ability to insert this in existing event generators by replacing the perturbative part of the event generator makes the integration of these developments into experimental analysis straightforward (just “throwing a switch”).

Subtraction based parton showers

- We need a formal description of the parton shower as we need to derive matching and other properties of the shower.
- To this end we generalize the operator I_{MC} introduced by Frixione and Webber in MC@NLO
- In a fixed order calculation the measurement function for an observable is a distribution, leading to some pathological behavior:

$$\frac{d\sigma}{d\mathrm{O}} = \sum_n \int d\mathrm{PS}_n \left| m_n \right|^2 \delta\left(\mathrm{O} - \mathrm{O}_n\right)$$

- The parton shower replaces this distribution by a function (i.e. it “smears” the distribution by further branchings):

$$\frac{d\sigma}{d\mathrm{O}} = \sum_n \int d\mathrm{PS}_n \left| \tilde{m}_n \right|^2 S_n\left(\left\{p_i\right\}_{i=1}^n ; Q_R^2 ; q_{cut}^2 \mid \mathrm{O}\right)$$

Subtraction based parton showers

- The shower operator is numerically implemented through a Markov chain algorithm.
- The following Markov chain definition of the showering operator exactly matches the numerical algorithm:

$$S_n\left(\{p_i\}_{i=1}^n; Q_R^2; q_{cut}^2 \mid \mathrm{O}\right) = \delta\left(\mathrm{O} - \mathrm{O}\left(\{p_i\}_{i=1}^n\right)\right) \Delta_{\text{event}}\left(\{p_i\}_{i=1}^n; 0, Q_R^2; q_{cut}^2\right)$$
$$+ \int_0^{Q_R^2} dt \left.\frac{\partial \Delta_{\text{event}}\left(\{p_i\}_{i=1}^n; t_b, Q_R^2; q_{cut}^2\right)}{\partial t_b}\right|_{t_b = t} \otimes S_{n+1}\left(\{\hat{p}_i\}_{i=1}^{n+1}; t; q_{cut}^2 \mid \mathrm{O}\right)$$

- Q_R^2 is the resolution scale at which we look at the event.
- By lowering the resolution scale we can resolve more “clusters”.
- Eventually, once the resolution scale reaches the hadronization scale q_{cut}^2 we can no longer resolve further clusters.
- The probability of not resolving new clusters when lowering the resolution scale from t_1 to t_2 is given by $\Delta_{\text{event}}\left(\{p_i\}_{i=1}^n; t_2, t_1; q_{cut}^2\right)$
- With this definition we can derive any property of the shower, including matching equations.

Subtraction based parton showers

- What remains is the definition of the no-branching probability: i.e. the sudakov function.
- We use dipole factorization of the event sudakov:

$$\Delta_{\text{event}}\left(\{p_i\}_{i=1}^{n};t_2,t_1;Q_H^2\right)=\prod_{\text{i}\in\text{dipoles}}\Delta^{(i)}_{p_a p_b\to\hat{p}_a\hat{p}_1\hat{p}_b}\left(t_2,t_1;Q_H^2\right)$$

  That is the event sudakov is the product of the individual dipole sudakov's (which give the likelyhood of not resolving an additional parton in a 2-parton dipole).
- These dipole sudakov's can contain all possible branching (even into EW or if needed SUSY particles).
- Right now only the $gg\to ggg$ included to setup the parton shower algorithm. To include other branching will be only a minor addition.

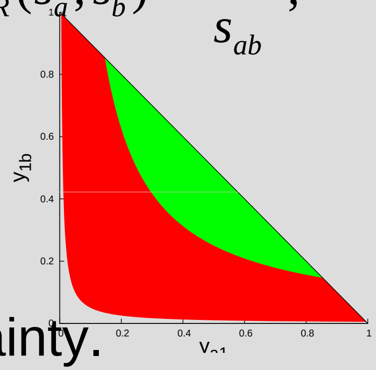
Subtraction based parton showers

- The dipole sudakov is given by

$$\Delta_{ab}^{(i)}\left(t_2, t_1; q_{\text{cut}}^2\right) = \exp\left(-\int ds_a \int ds_b \int \frac{d\phi}{2\pi} \frac{\alpha_S N_C}{2\pi} \text{Ant}(s_a, s_b, \phi) \times \text{Veto}(s_a, s_b, \phi)\right)$$

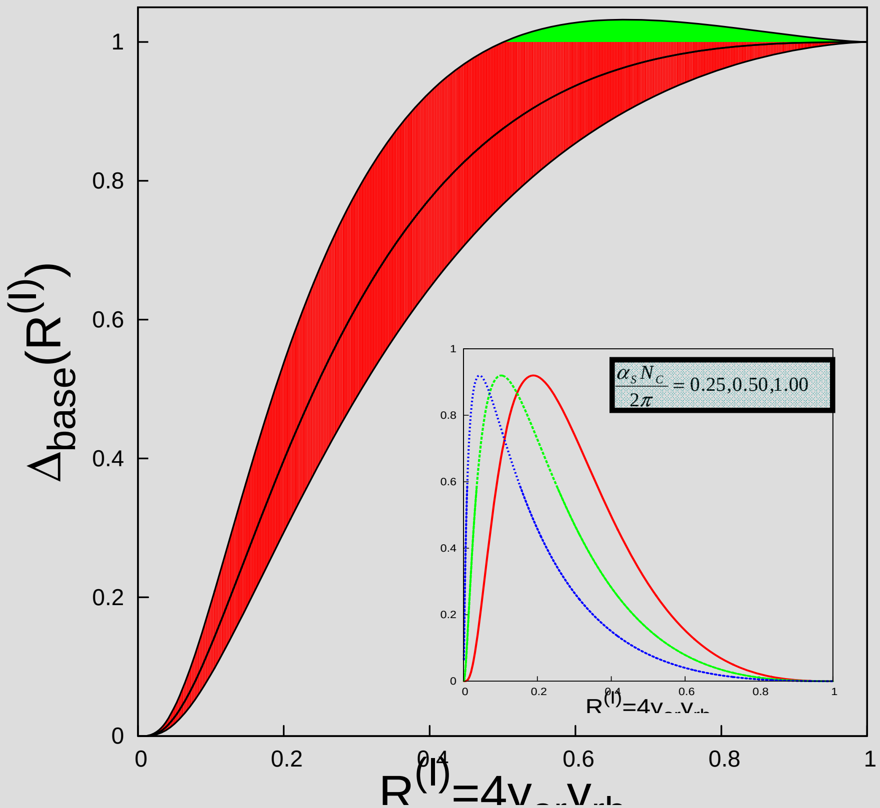
$$\text{Veto}(s_a, s_b, \phi) = \Theta\left(Q_R^2(s_a, s_b) - t_2\right)\Theta\left(t_1 - Q_R^2(s_a, s_b)\right)\Theta\left(s_{ab} - s_a - s_b\right)\Theta\left(Q_H^2(s_a, s_b) - q_{\text{cut}}^2\right)$$

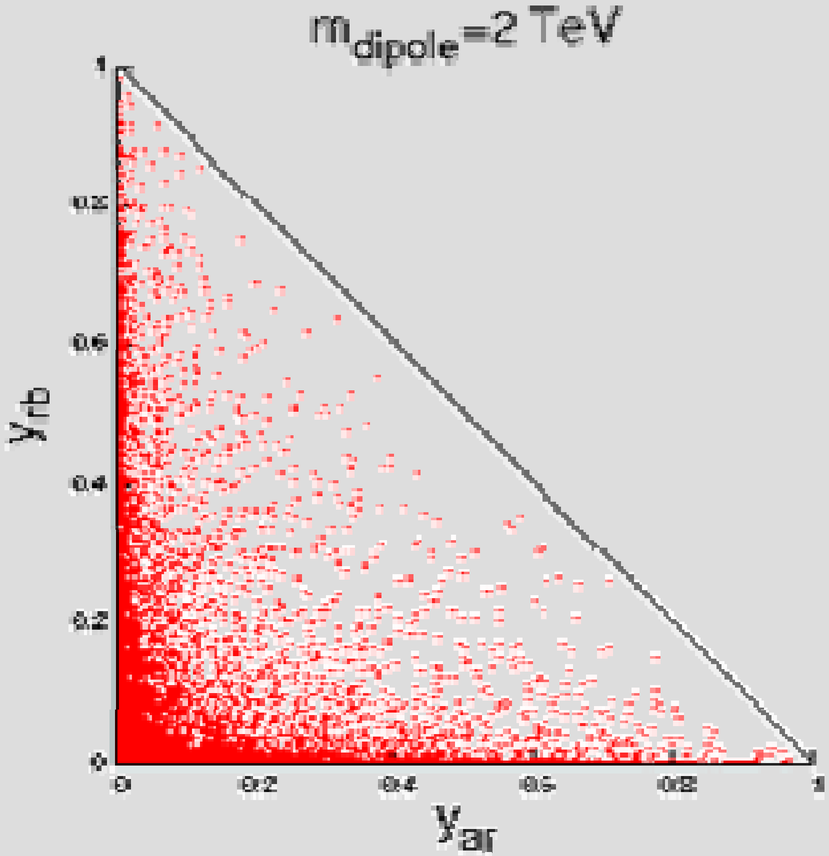
- There are choices to be made: different choices will give different showers (this is an inherent uncertainty within the leading log parton shower):
  - Evolution variable (choice of function $Q_R^2(s_a, s_b) = \min(s_a, s_b); Q_R^2(s_a, s_b) = \frac{s_a s_b}{s_{ab}}; \cdots$
  - Radiation function (only its soft-collinear behavior is fixed)
  - Kinematic map ($\{p_i\}_{i=1}^{n} \rightarrow \left\{\hat{p}_i\left(\{p_j\}_{j=1}^{n}, s_a, s_b, \phi\right)\right\}_{i=1}^{n+1}$)
  - Renormalization variable ($\alpha_S = \alpha_S\left(\mu(s_a, s_b, \phi)\right)$)
  - Infrared cutoff contour (hadronization cutoff $Q_H^2\left(s_a, s_b\right)$)



- Appropriate variation in these variables probes the uncertainty.
- Matching with fixed order matrix elements will greatly reduce this uncertainty. (Instead of “tuning” these quantities to reproduce hard physics.)

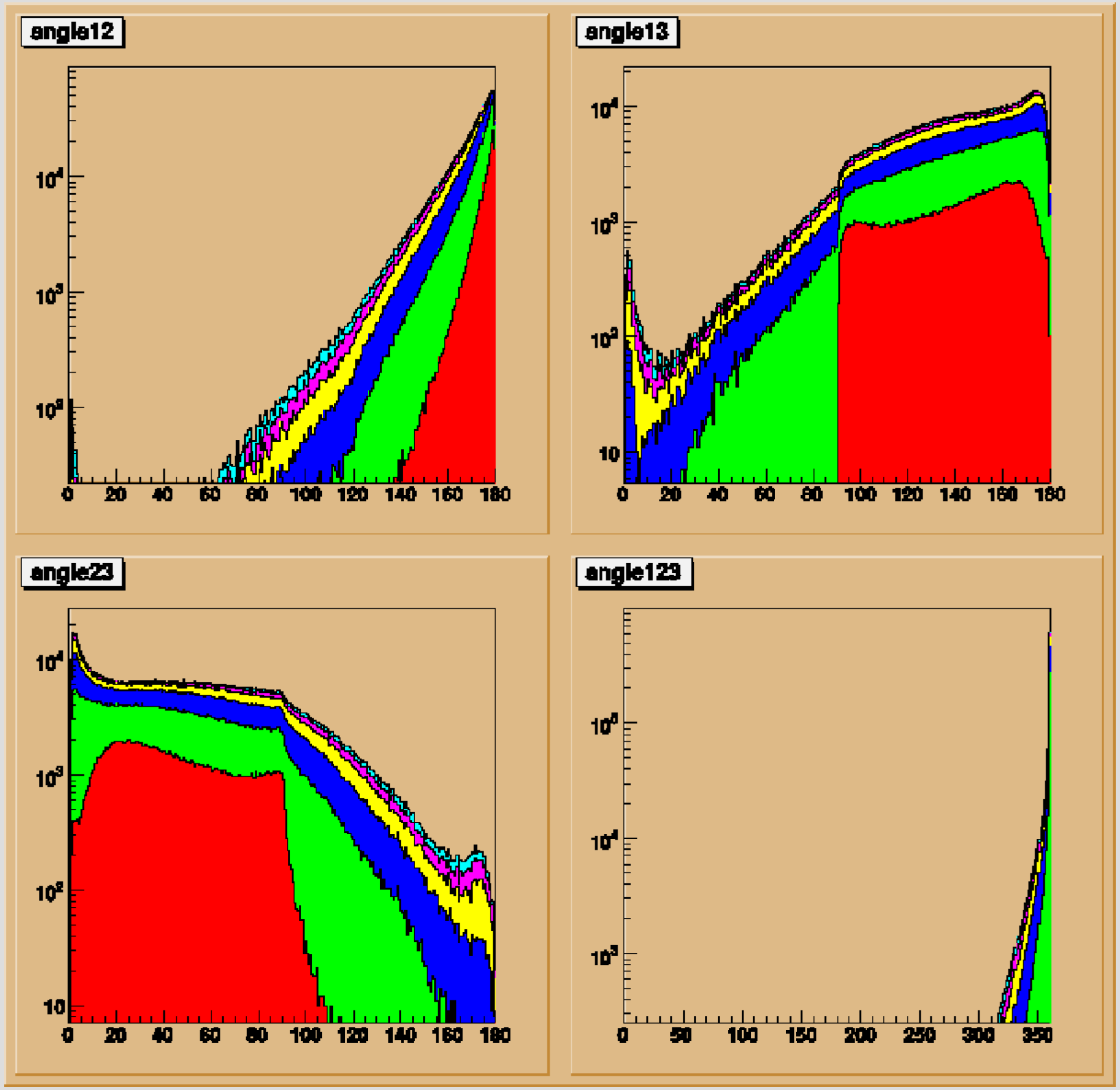
Dipole-Antenna Sudakov's





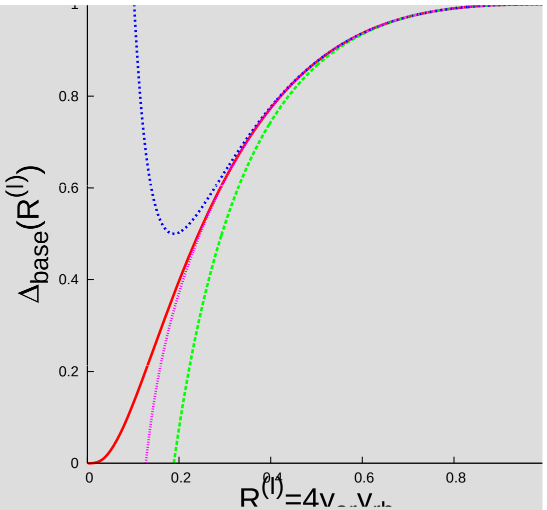
In the shower MC the Sudakov is calculated at initialization phase from the numerically implemented antenna function and resolution scale.

The dipole shower



- Angular distributions between the 3 leading jets (angle(j1,j2), angle(j1,j3), angle(j2,j3)) in 3,4,5,6,7,8 exclusive jet events.
- Distribution of the sum of the 3 angles.
- Kt-jet algorithm used with Yr=0.001; M=500 GeV
- 1,000,000 showered events.
- (stacked histograms)
- (logarithmic vertical scale)
- Distributions rich in structure (which are all explainable…)

Matching to Fixed Order



- First we need to expand the shower operator in α_S
- The expanded event sudakov is given by

$$\Delta_{\text{event}}(Q_1^2;Q_2^2) = \prod_{i\in dipoles} \exp(-\alpha_S \int_{Q_1^2}^{Q_2^2} \text{Ant}_i) = 1 - \alpha_S \sum_{i\in dipoles} \int_{Q_1^2}^{Q_2^2} \text{Ant}_i + \text{O}(\alpha^2)$$

- Now we can expand out the shower function:

$$S_n(\{p_i\}_{i=1}^n;Q_R^2) = \delta(O-O_n) + \alpha_S \sum_{i\in dipoles} \int_{Q_0^2}^{Q_m^2} \text{Ant}_i \left(\delta(O-O_{n+1}) - \delta(O-O_n)\right) + \text{O}(\alpha_S^2)$$

- Note that the expanded shower operator is preserves the infra-red safety of the observable.
- Using the expanded shower operator leads to the matching equations.

Matching to Fixed Order

- The matching equations for LO/NLO matrix element generators simply follow from expanding out the shower operator and equating the result to the regular perturbative expansion.
- The matching condition is given by:

$$\frac{d\sigma}{d\mathrm{O}} = \sum_n \int d\mathrm{PS}_n |m_n|^2 \delta(\mathrm{O}-\mathrm{O}_n) \equiv \sum_n \int d\mathrm{PS}_n |\tilde{m}_n|^2 S_n\left(\{p_i\}_{i=1}^n; Q_R^2; q_{cut}^2 \mid \mathrm{O}\right)$$

$$= \sum_n \int d\mathrm{PS}_n |\tilde{m}_n|^2 \left(\delta(O-O_n) + \alpha_S \sum_{i\in dipoles} \int_{Q_0^2}^{Q_n^2} \mathrm{Ant}_i \left(\delta(O-O_{n+1}) - \delta(O-O_n)\right) \right)$$

- This leads to the following result:

$$\left|\tilde{m}_n^{(0)}\left(\{p_i\}_{i=1}^n\right)\right|^2 = \left|m_n^{(0)}\left(\{p_i\}_{i=1}^n\right)\right|^2 - \sum_{i\in dipoles} \mathrm{Ant}_i \left|m_{n-1}^{(0)}\left(\{\hat{p}_i\}_{i=1}^{n-1}\right)\right|^2$$

$$"\left|\tilde{m}_n^{(1)}\left(\{p_i\}_{i=1}^n\right)\right|^2" = "\left|m_n^{(1)}\left(\{p_i\}_{i=1}^n\right)\right|^2" + \sum_{i\in dipoles} \int_0^{Q_n^2} \mathrm{Ant}_i \left|m_n^{(0)}\left(\{p_i\}_{i=1}^n\right)\right|^2 + \mathrm{O}\left(\frac{q_{\mathrm{cut}}^2}{Q_n^2}\right)$$

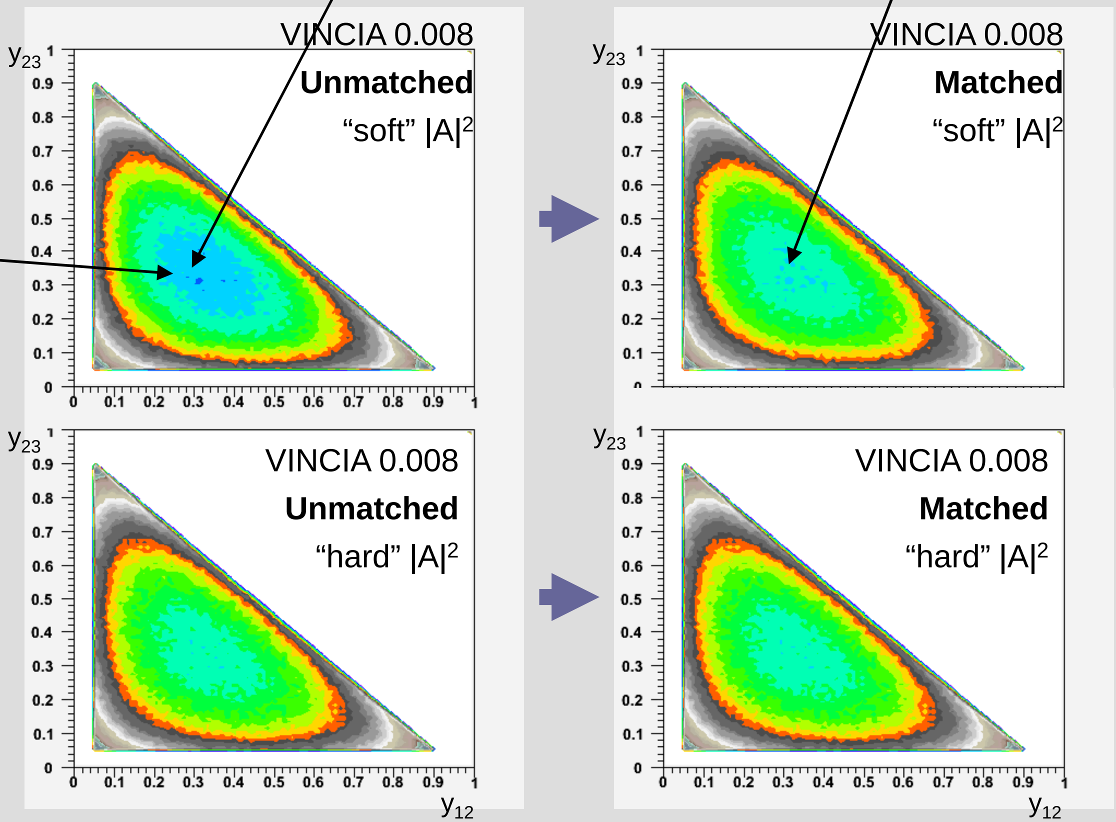
- This is the standard subtraction scheme… almost all NLO calculations are done in this manner.

VINCIA Example: H → gg → ggg

- First Branching ~ first order in perturbation theory
- Unmatched shower varied from "soft" to "hard" :

soft shower has "radiation hole". Filled in by matching.

radiation hole in high-p_T region



Outlook:

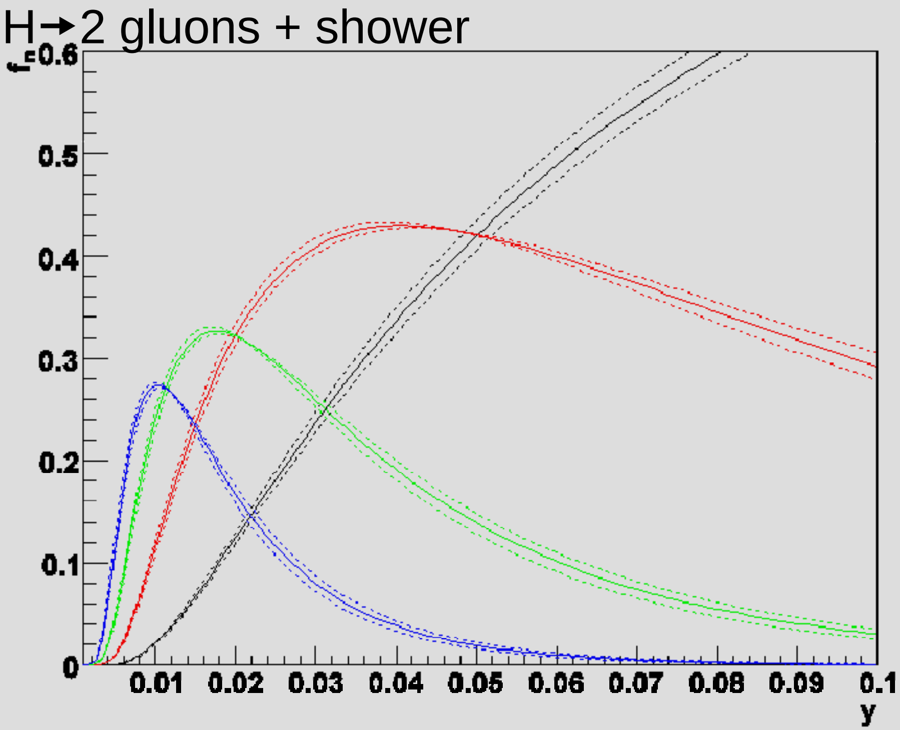
Immediate Future:

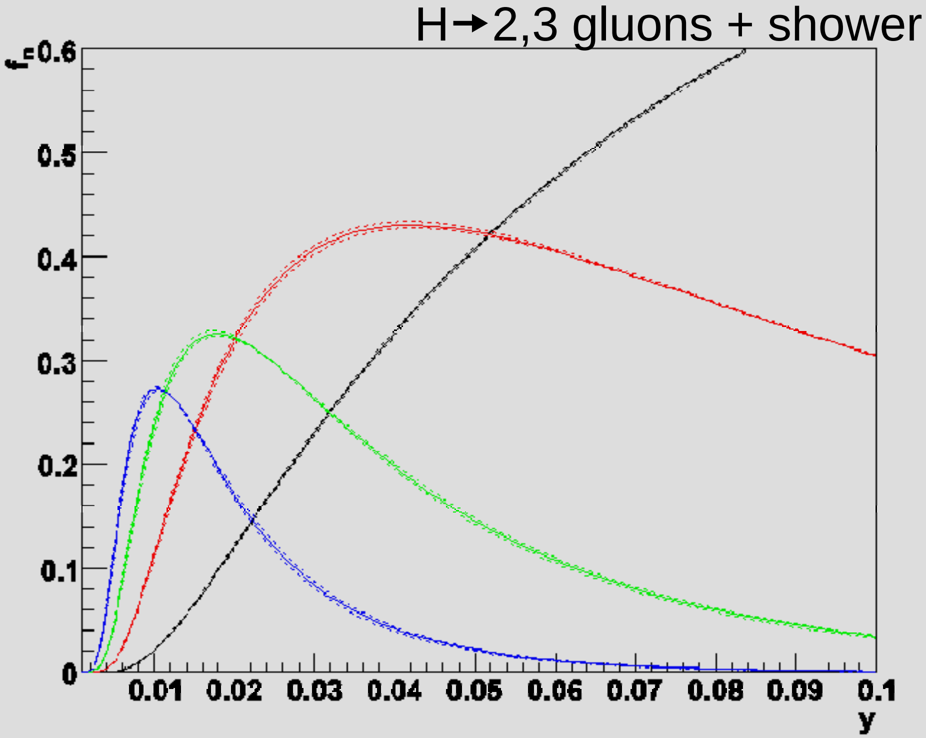
- Paper about gluon shower
- Include quarks → Z decays
- Matching

Then:

- Initial State Radiation
- Hadron collider applications

VINCIA Example: H → gg → ggg





- 2, 3,… exclusive jet fractions as a function of the Kt-jet resolution parameter.
- Matching shower with fixed order strongly reduces the dependence on the (arbitrary) hard part (non soft/collinear) of the antenna function.
- Being able to change the shower hardness we can see the importance of matching
- We can also estimate the residual uncertainties ***within*** the leading log approximation

Conclusions

- Our purpose is to construct a parton shower made for matching with hard matrix element, not to construct a new event generator.
- The parton shower can easily be inserted into event generators (we use PYTHIA 8)
- Because the parton shower is designed for matching, we get a straightforward matching scheme (which is closely related to NLO fixed order calculations).
- We also took care to allow variation of all undetermined "parameters" (which reflect the uncertainty within the leading log ($\alpha_s^n \log^{2n}$ & $\alpha_s^n \log^{2n-1}$) approximation.
- Including hard matrix elements reduce these uncertainies greatly (instead of "tuning" them to model the hard physics).
- Near future: inclusion of quarks (including masses) & EW particles in the shower; initial state partons
- Further off: beyond leading log parton showers for NNLO matching; more systematic approach towards hadronization modeling.