Beam Stability in LHC with Crab Cavities

Alexey Burov

LHC-CC11 talk, Nov 2011

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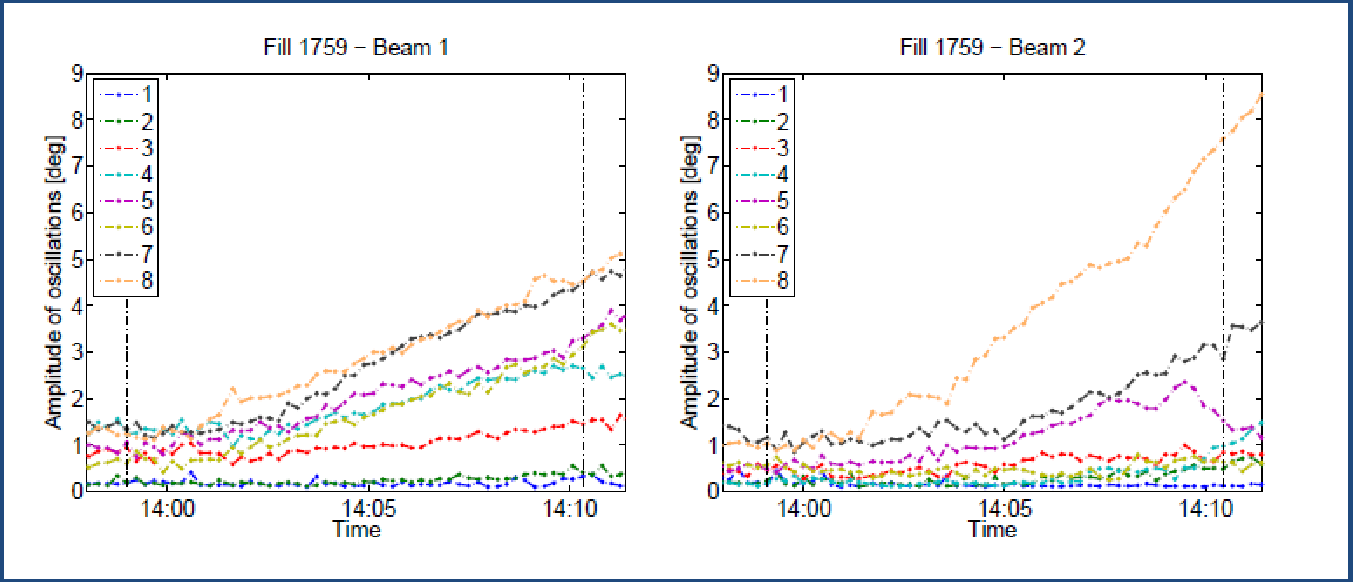
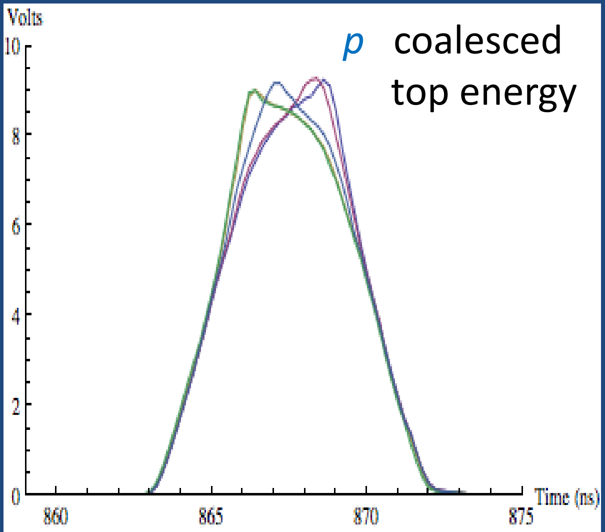
Loss of Landau Damping (LLD): Observations

LHC, *E. Shaposhnikova et al.,IPAC'11*

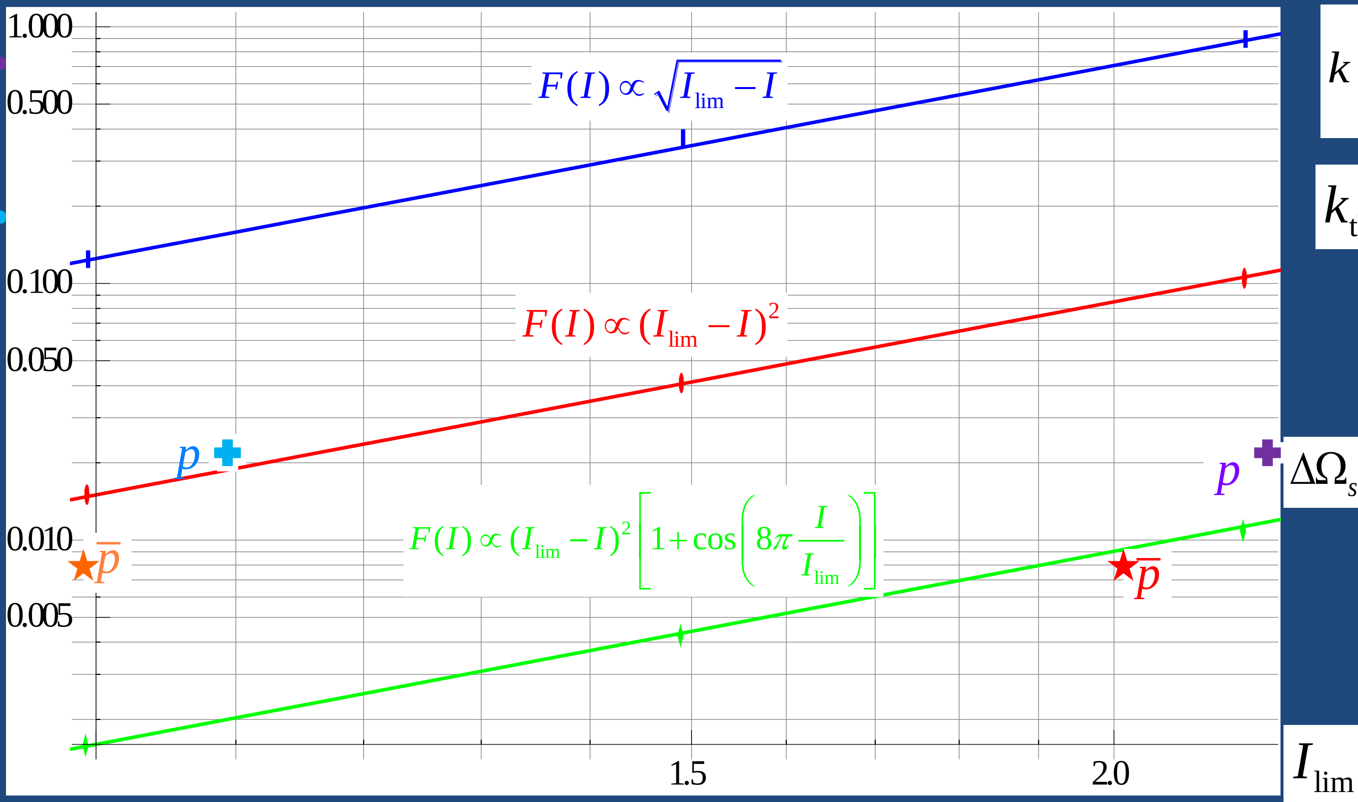


Tevatron, C. Y. Tan & A. Burov



$$|\Delta\Omega_0|/\Omega_0 \approx (1-2)\%$$

LLD thresholds, Tevatron



$$k = -\frac{2 N r_0 \eta \omega_{rf}^3}{\gamma c \Omega_0^2} \frac{\operatorname{Im} Z_{\|}}{n Z_0} .$$

$$k_{th} \propto I_{lim}^{5/2}$$

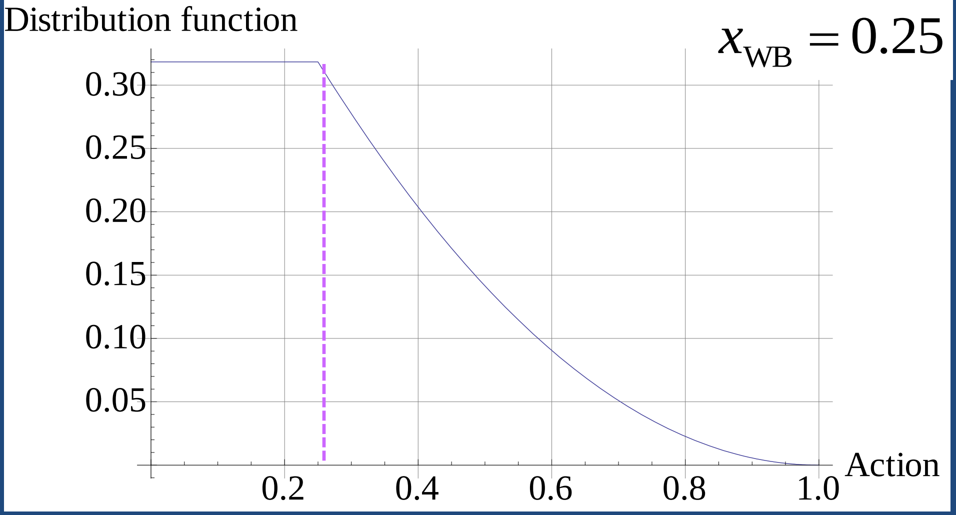
- protons, top energy
- pbars, top energy

- protons, injection
- pbars, injection

For Gaussian, $F(I) \propto \exp(-2I / I_{lim})$, k_{th} is 35% higher then for $F(I) \propto (I_{lim} - I)^2$

LLD for partial water-bag distribution (PWB)

$$k_{\text{th}} = C\left(I_{\lim} / I_{\text{bkt}}\right)^{5/2}$$



Threshold form-factor

x_{WB}	$C(x_{\text{WB}})$
0	0.15
0.25	2.6
0.5	6.8

Equivalent to 3 times of emittance blow-up

Equivalent to 4.5 times of emittance blow-up

Thus, even a relatively small water-bagging increases the threshold 20 times!

How to make PWB

Let the RF phase be modulated near the synchrotron frequency.
Then, equation of motion is:

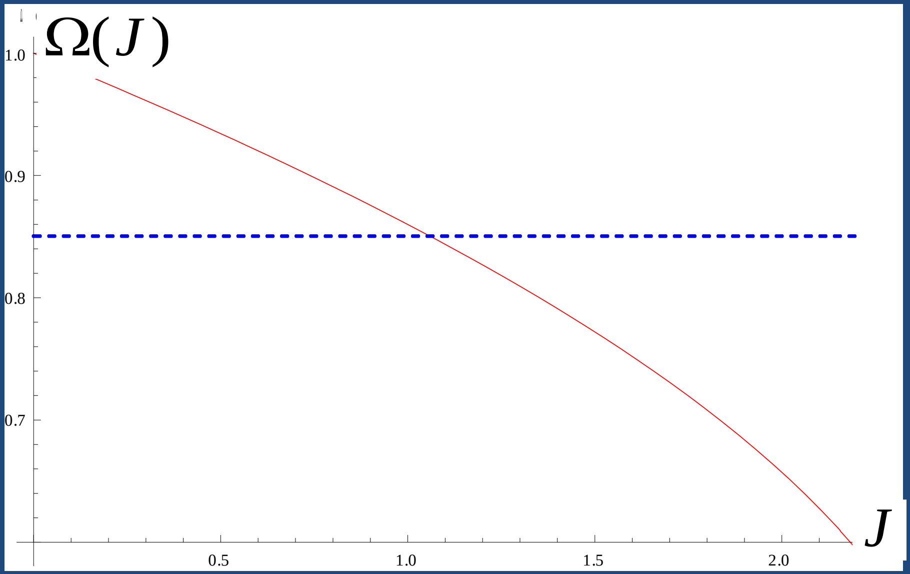
$$\ddot{z} + \sin\left[z - a(t)\sin\left((1-\varepsilon)t\right)\right] = 0$$

Slowly changed amplitude — $a(t)$

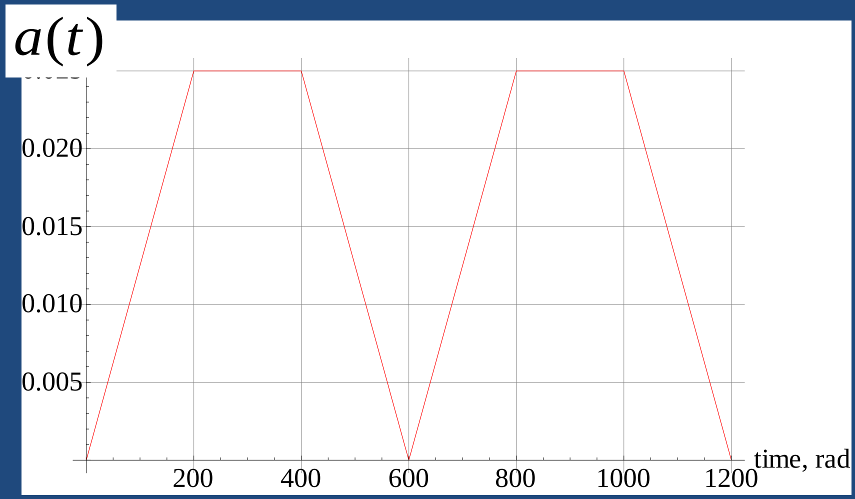
Detuning — ε

$$\Omega(J) \approx 1 - \frac{J}{\pi J_{\text{bucket}}}$$

$$J_{\text{bucket}} = 8/\pi$$



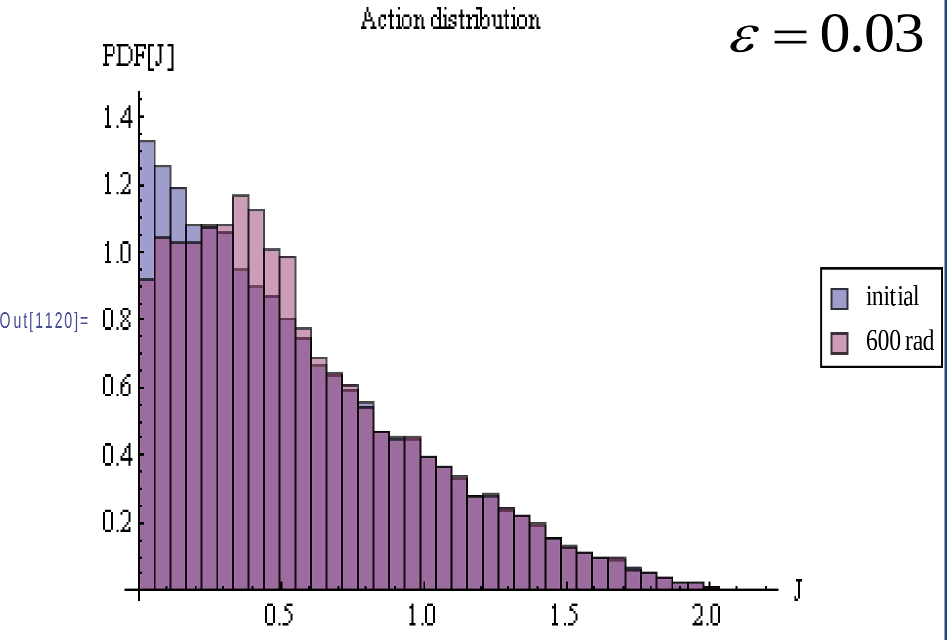
Distribution function is changed:

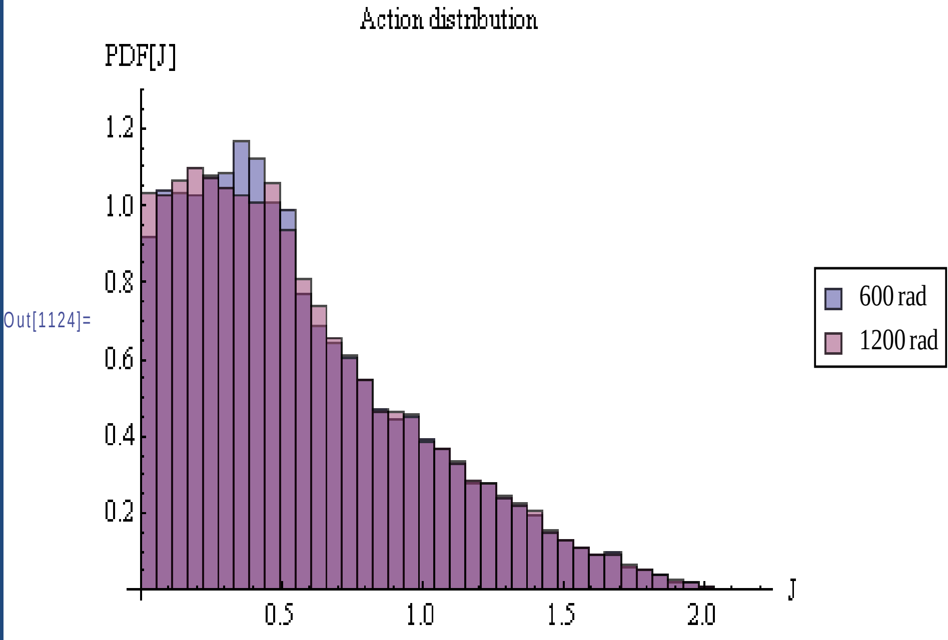


The affected area

$$J_{\text{diff}} / J_{\text{bucket}} \approx 6\varepsilon$$

$$a_{\max} \geq 3\varepsilon^{3/2}$$

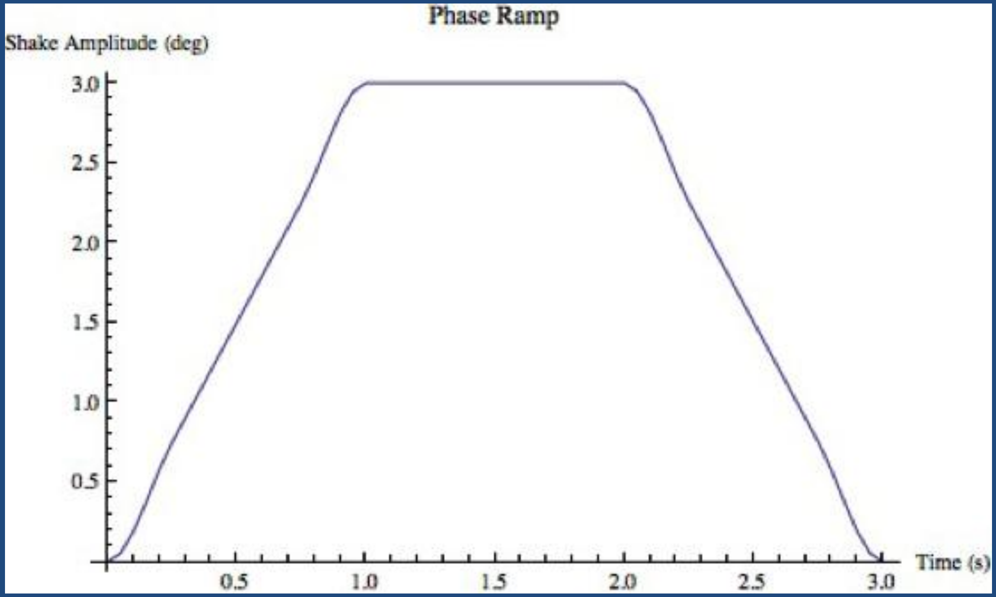




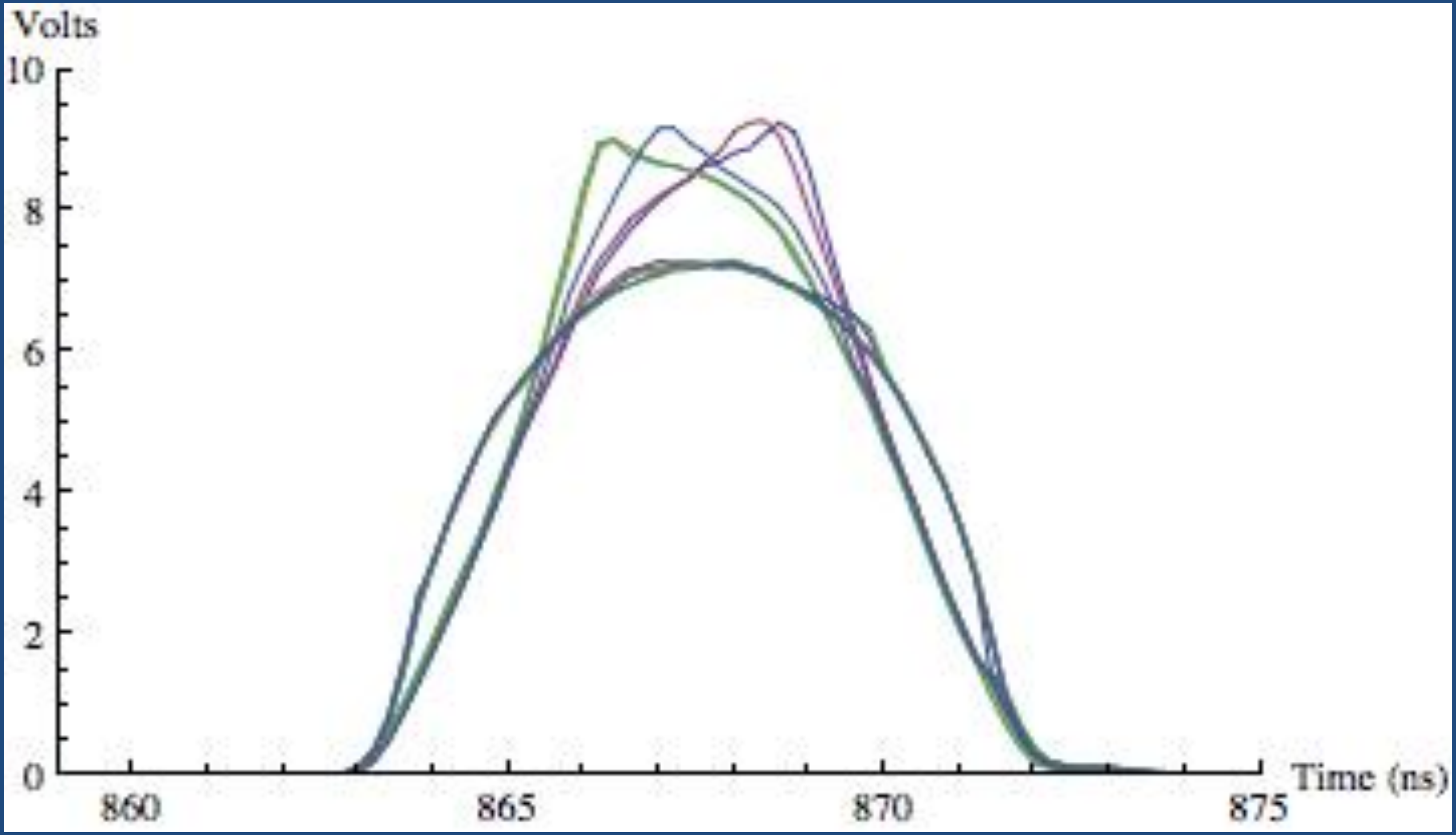
Tevatron experiment (A. Burov, C. Y. Tan, 2011)

- Two coalesced bunches are accelerated to 980 GeV. The experiment is done at flattop energy because the bucket area is much larger than beam longitudinal beam size and so any emittance growth can be observed without beam loss.

- Beam is shaken by phase modulating the Tevatron RF at the synchrotron frequency 34 Hz. The amplitude of the shake is 3° peak. To prevent excitation at the tails, the amplitude is ramped adiabatically:



Before versus after



Possible issues

- This scheme is sensitive to the detuning from the maximal incoherent synchrotron frequency. Accuracy of the detuning should be at the level of few % .
- Calculated optimal detuning may differ from the actual due to the wake-caused potential well distortion and beam loading.
- As a consequence, different bunches result with somewhat different PWB step width.
- To see importance of these and may be some other issues, MD studies (similar to Tevatron) are needed. PS? SPS? LHC?

Narrow-band Impedance, Sacherer theory

$$1+\Delta\Omega_{\text{c}}\int_{0}^{J_{\max}}\frac{F'(J)JdJ}{\omega-\Omega(J)+i0}=0;$$

$$\Delta\Omega_{\text{c}}=i\Omega_{s}\frac{N_{\text{t}}r_{0}\eta}{\gamma C\nu_{s}^{2}}\frac{Z(\omega_{pm})\omega_{pm}\rho^{2}(\omega_{pm})}{Z_{0}\omega_{0}};$$

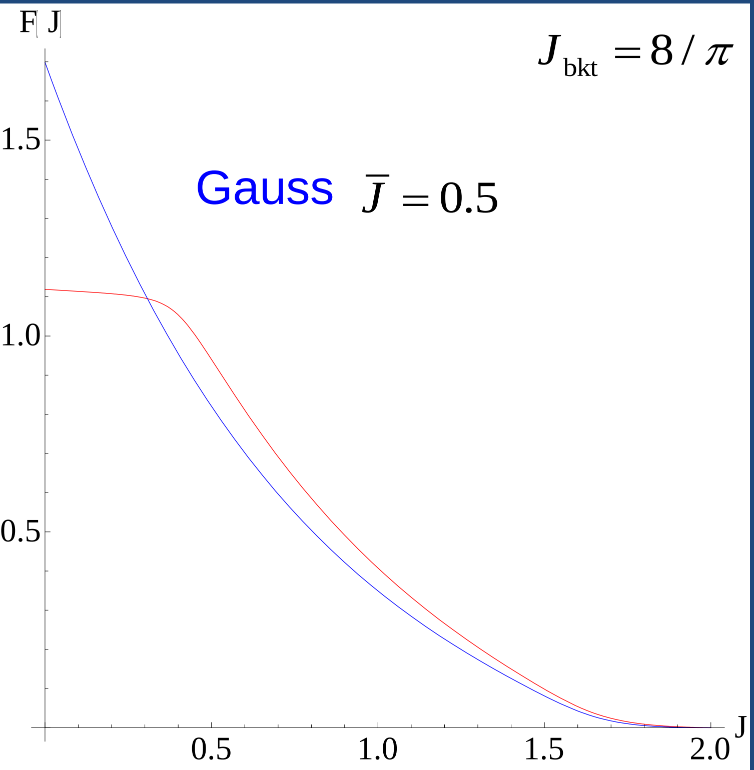
$$\omega_{pm}=pM\omega_{0}+m\omega_{0}+\Omega_{s};\quad \Omega_{s}\equiv\Omega(0);$$

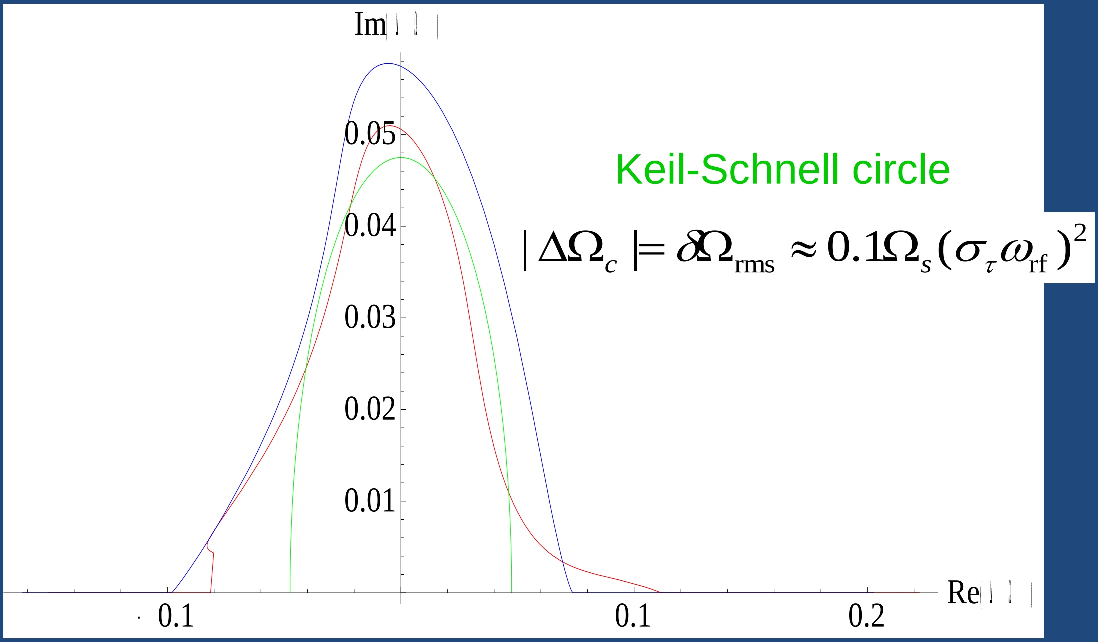
$$\rho(\omega)=\int_{-\infty}^{\infty}\exp(-i\omega\tau)\rho(\tau)d\tau;$$

$$\text{water-bag}:\quad \rho(\omega)=\frac{2J_{1}(\omega\hat{\tau})}{\omega\hat{\tau}}$$

$$\text{Gaussian}:\quad \rho(\omega)=\exp(-\omega^{2}\sigma_{\tau}^{2}/2)$$

Stability Diagrams





No big difference from the PWB for the narrow band impedance…

Keil-Schnell circle:

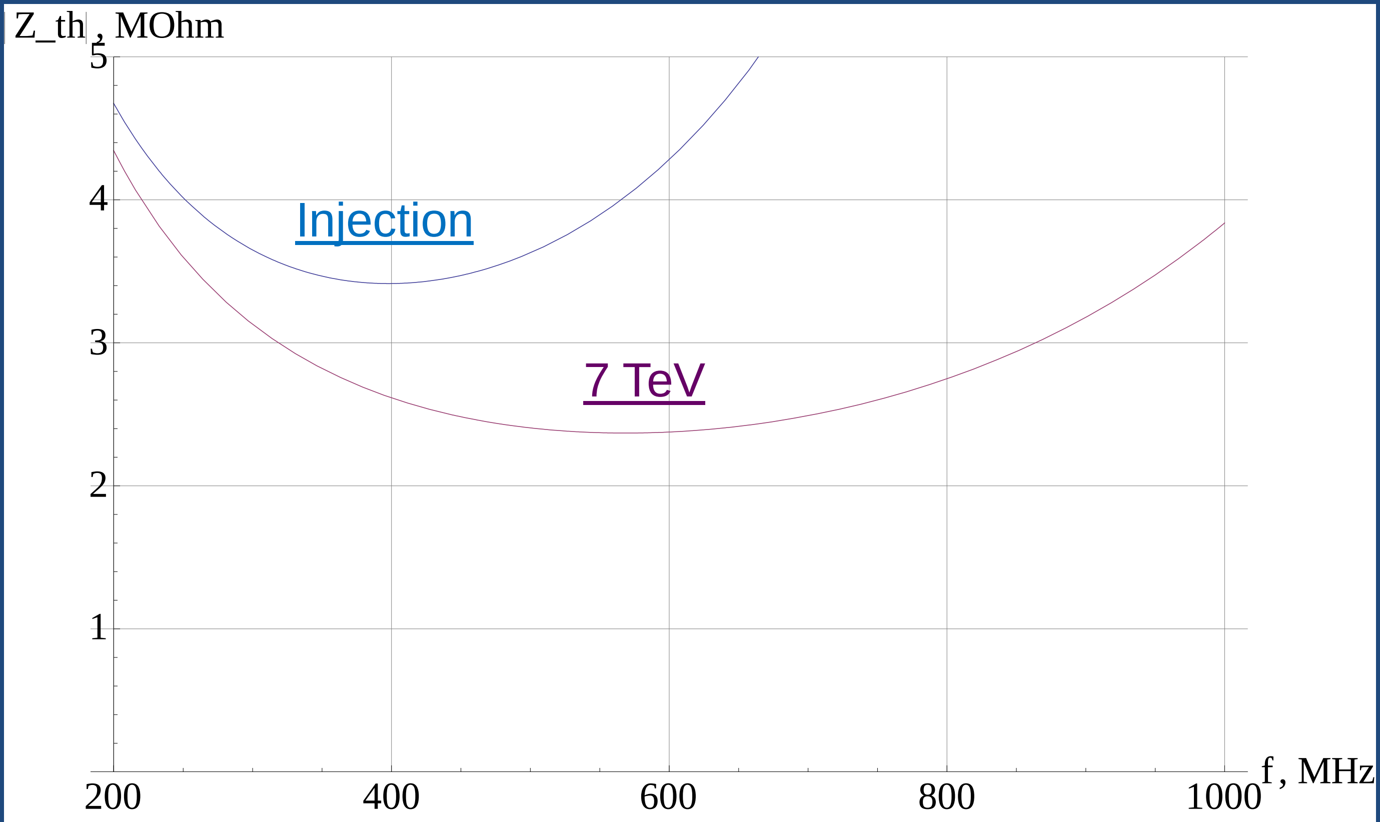
$$|Z(\omega_{\text{pm}})|_{\text{th}} \cong 0.1 \frac{Z_0}{\rho^2(\omega_{\text{pm}})} \frac{\omega_0}{\omega_{\text{pm}}} \frac{\gamma C_0 \nu_s^2}{N_t r_0 |\eta|} (\sigma_\tau \omega_{\text{rf}})^2$$

LHC CC ultimate parameters

From E. Shaposhnikova, LHC-CC 2010:

Energy		TeV	0.45	7.0
RF frequency	$\omega_{\mathrm{rf}} / (2\pi)$	MHz	400.8	400.8
RF voltage		MV	8.0	16.0
synchrotron frequency	$\Omega_s / (2\pi)$	Hz	66.08	23.86
revolution frequency	$\omega_0 / (2\pi)$	kHz	11.245	11.245
betatron tunes, H/V	ν_x / ν_y		59.3/64.28	59.3/64.31
rms bunch length	σ_τ	ns	0.4	0.275
Number of particles	$N_t = M \cdot N_b$		2808 ·1.7E11	2808 ·1.7E11

HOM threshold impedance



Thus, the HOM impedance may not exceed 2.4/4=0.6 Mohm per cavity at the synchrotron sidebands.

$$Q_{\|} < 2\pi Z_{max} / Z_0 \cong 10^4$$ - sufficient condition

Transverse limits

Assuming e-cloud removed with efficient scrubbing,

Assuming the broad-band impedance be below TMCI limit,

Assuming the chromaticity is negative and sufficiently small by value,

The only problem could be the narrow-band impedance at the betatron sidebands.

Then, the stability is guaranteed, as soon as:

$$-\mathrm{Re}\, Z_{x,y}(\omega_{pm})\beta_{x,y} \le Z_0 \frac{\gamma c T_0^{\ 2}}{2\pi\tau_d N_t r_0}$$

$$\omega_{pm} = \omega_0(pM + m + \nu_{x,y})$$

τ_d - damping time

Threshold impedance

For $\tau_d = 60\ \mathrm{ms}$, it yields, per a cavity:

Energy $\gamma m_{\mathrm{p}} c^2$	Beta-function $\beta_{x,y}$	Impedance $-\mathrm{Re}\, Z_{\mathrm{th}}$
450 GeV	150 m	2.7 MOhm/m
7 TeV	4 km	1.5 Mohm/m

$$Q_\perp < \lambda Z_{\max} / Z_0 \cong 3 \cdot 10^3$$

At zero chromaticity, and below TMCI,
all the head-tail modes are stable against single-bunch instabilities.

Chromaticity allows redistribution of damping/growth rates between 0 mode and other HT modes.

Octupoles add some Landau damping.

Thus, the single-bunch stability depends on the accuracy of the chromaticity setting.

Conclusions

- Partial water-bagging allows secure Landau damping even for short bunches.

- This can be done by means of anomalous diffusion at resonance shaking. Studies are needed to see how important are beam loading and potential well distortion.

- Sacherer theory: PWB does not change much stability against pure narrow-band impedance.

- For ultimate LHC-CC parameters, the longitudinal HOM impedance may not exceed 0.6 MOhm per cavity at the revolution harmonics.

- The transverse HOM may not exceed 1.5 Mohm/m per cavity.

Acknowledgements

- Many thanks to Elena Shaposhnikova, Elias Metral and Slava Yakovlev for clarifying discussions.

Thank you all for your attention!