Non-Linearity of Deflecting Field in LHC crab-cavities

A.Grudiev, CERN, BE-RF

15.11.2011

LHC-CC11, 14-15.11.2011, CERN

Acknowledgements

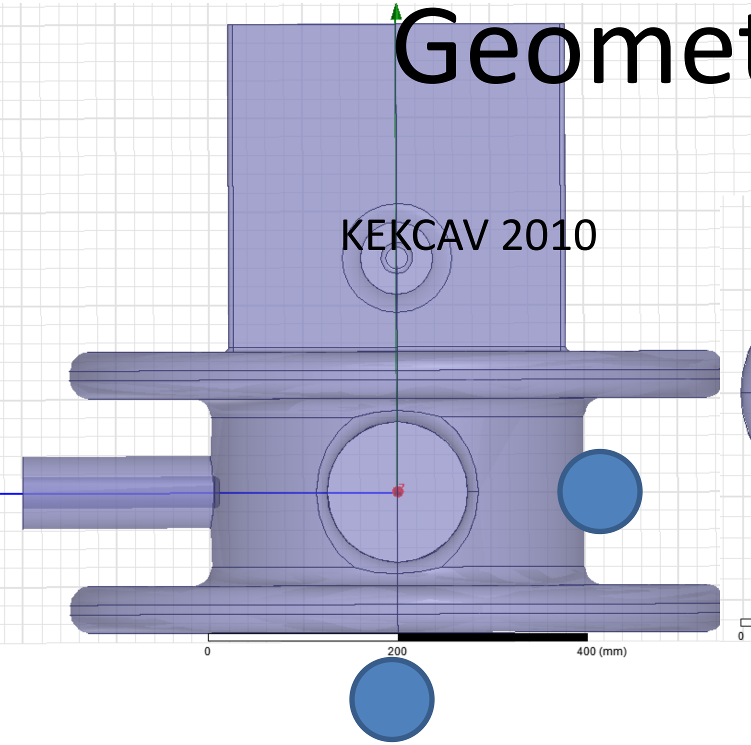
- Many thanks to Rama Calaga for stimulating discussions and introduction to the LHC crab cavity. Also for providing 3D models of the ODUCAV, KEKCAV, UKCAV, SRHW, QWAVE
- And to Riccardo de Maria for introducing to LHC bunch-crossing schemes
- 3D HFSS simulations were performed on the CLIC study workstation for HFSS simulations (128GB, 24CPU)

Outline

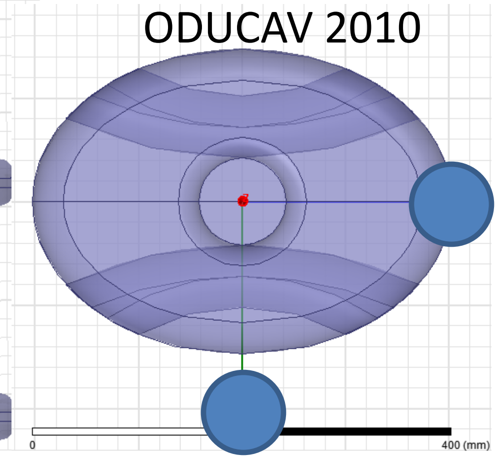
- Cavity geometry and mesh
- Non-linearity of the deflecting field treatment in terms of multipoles
- An example (KEKCAV)
- Comparison of RF multipoles to typical magnetic multipoles
- Conclusion
- If time permit, yet another compact crab cavity design

Geometry of the cavities

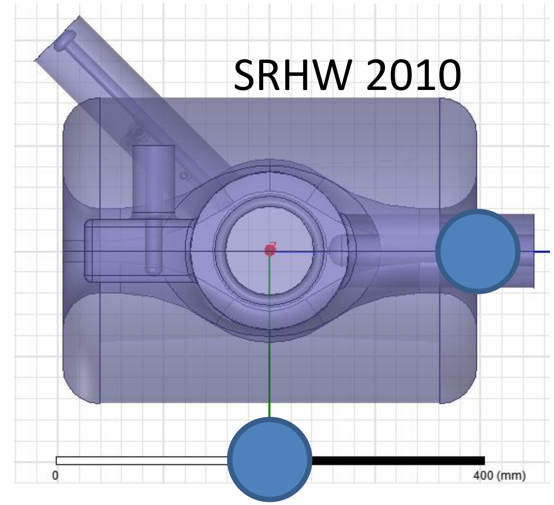
KEKCAV 2010



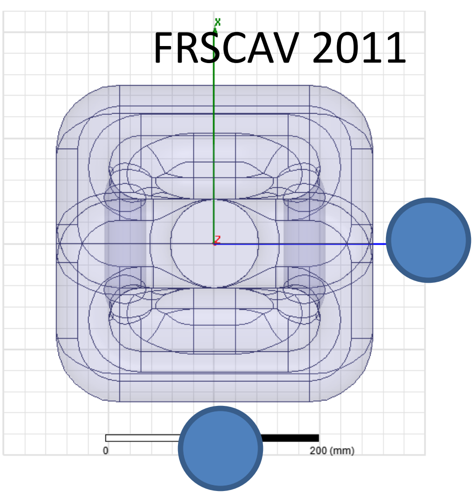
ODUCAV 2010



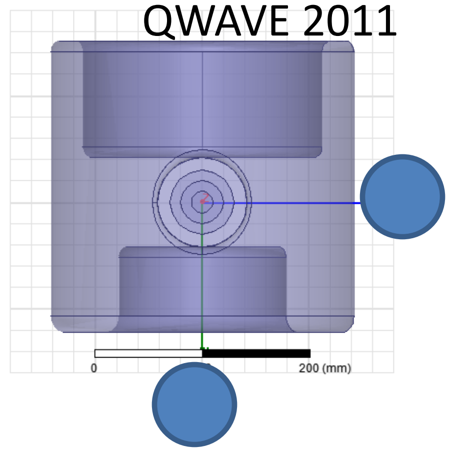
SRHW 2010



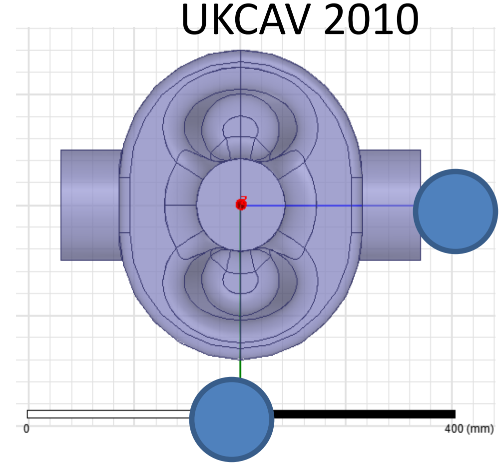
FRSCAV 2011



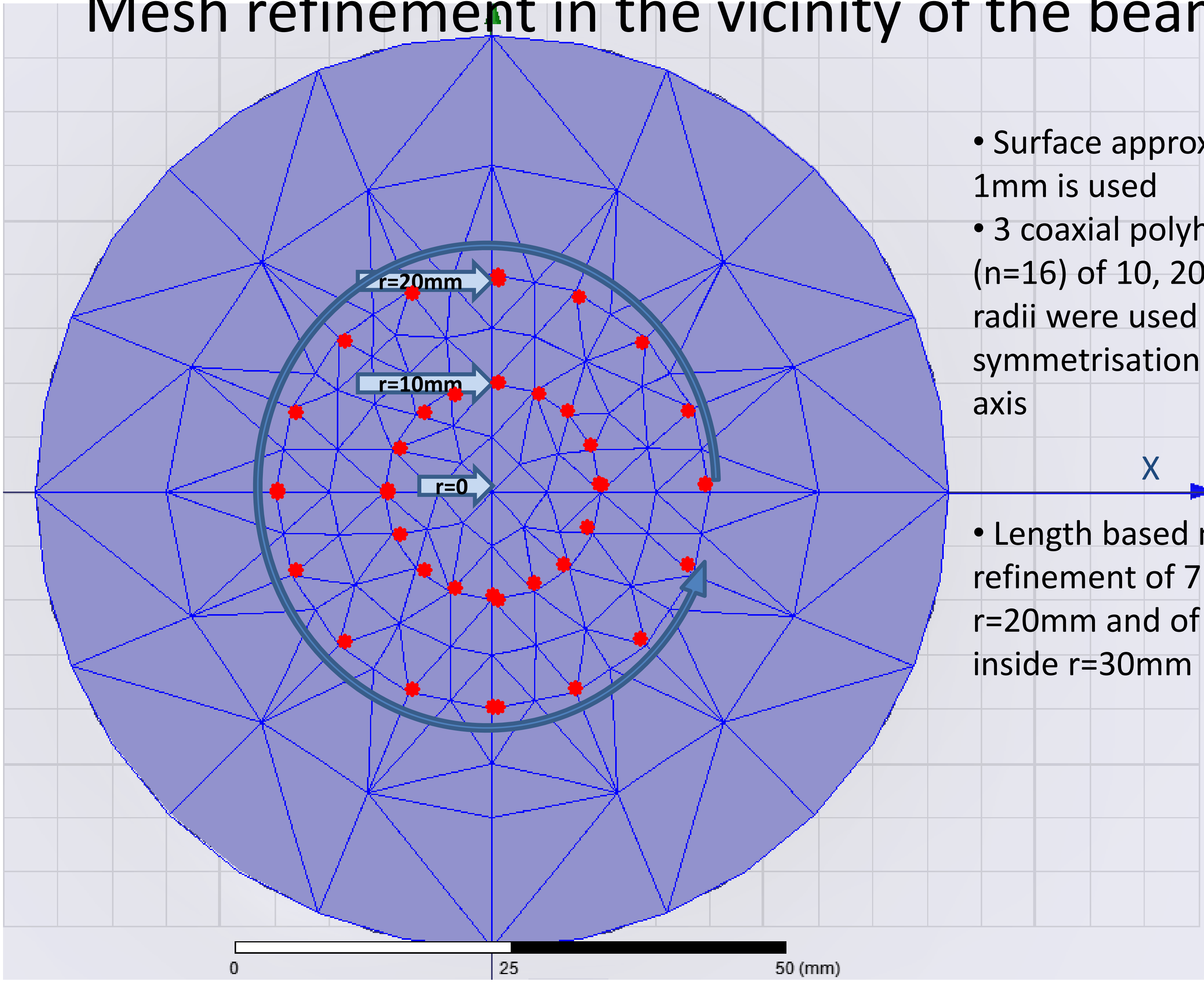
QWAVE 2011



UKCAV 2010



Mesh refinement in the vicinity of the beam axis



- Surface approximation of 1mm is used
- 3 coaxial polyhedrones (n=16) of 10, 20, 30 mm radii were used for mesh symmetrisation near the axis

- Length based mesh refinement of 7 mm inside r=20mm and of 15 mm inside r=30mm

Multipole expansion of E_z

Skew components = 0 due to the symmetry

Accelerating gradient: $E_{acc}(r,\varphi,z) = E_z(r,\varphi,z)\cdot e^{j\frac{\omega}{c}z}$

Multipole expansion: $E_{acc}(r,\varphi,z) = \sum_n E_{acc}^{(n)} r^n e^{jn\varphi} = \sum_n E_{acc}^{(n)} r^n \cos(n\varphi)$

Panofsky-Wenzel (PW) theorem:

$$p_\perp(r,\varphi) = \frac{+je}{\omega}\int_0^L dz\left[\nabla_\perp E_{acc}(r,\varphi,z)\right]; \quad \text{for} \quad E \sim e^{+j\omega t}$$

$$\text{where: } \nabla_\perp = \vec{u}_r\frac{\partial}{\partial r} + \vec{u}_\varphi \frac{1}{r}\frac{\partial}{\partial \varphi}$$

Gives an expression for multipolar RF kicks:

$$p_\perp^{(n)}(r,\varphi) = \frac{je}{\omega} n r^{n-1}\left[\vec{u}_r\cos(n\varphi) + \vec{u}_\varphi \sin(n\varphi)\right]\int_0^L E_{acc}^{(n)}(z)dz$$

Lorenz Force (LF):
Gives an expression for kick directly from the RF EM fields:

$$F_\perp = e\left[E_\perp + v_z \times B_\perp\right] =\big|_{v_z=c} e\left[E_\perp + Z_0 \vec{u}_z \times H_\perp\right]$$

$$p_\perp(r,\varphi) = \int_0^L \frac{F_\perp}{v_z}dz =\big|_{v_z=c} \frac{e}{c}\int_0^L \left[E_\perp + Z_0\vec{u}_z \times H_\perp\right]dz$$

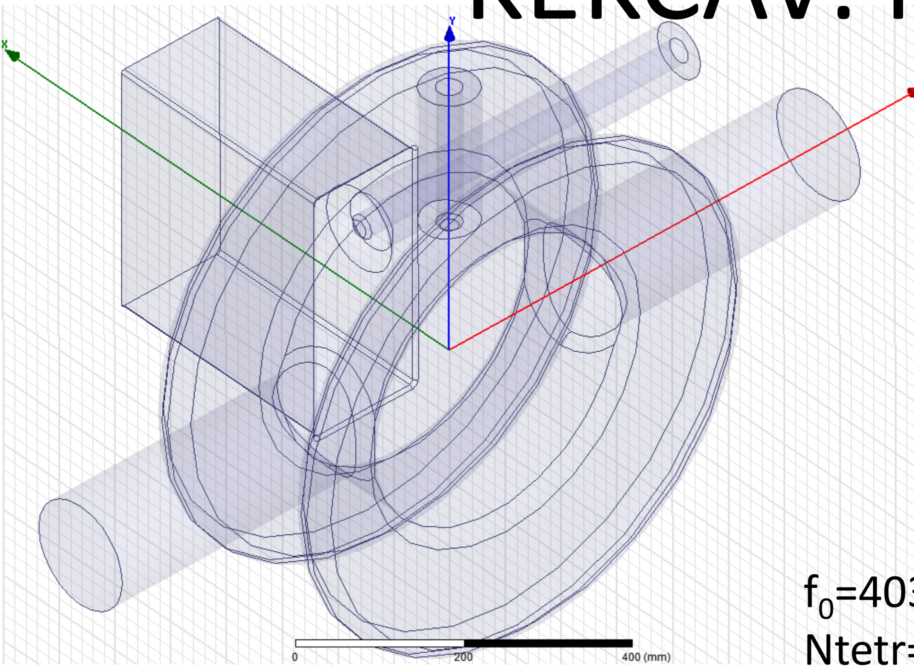
Which can be decomposed into multipoles:

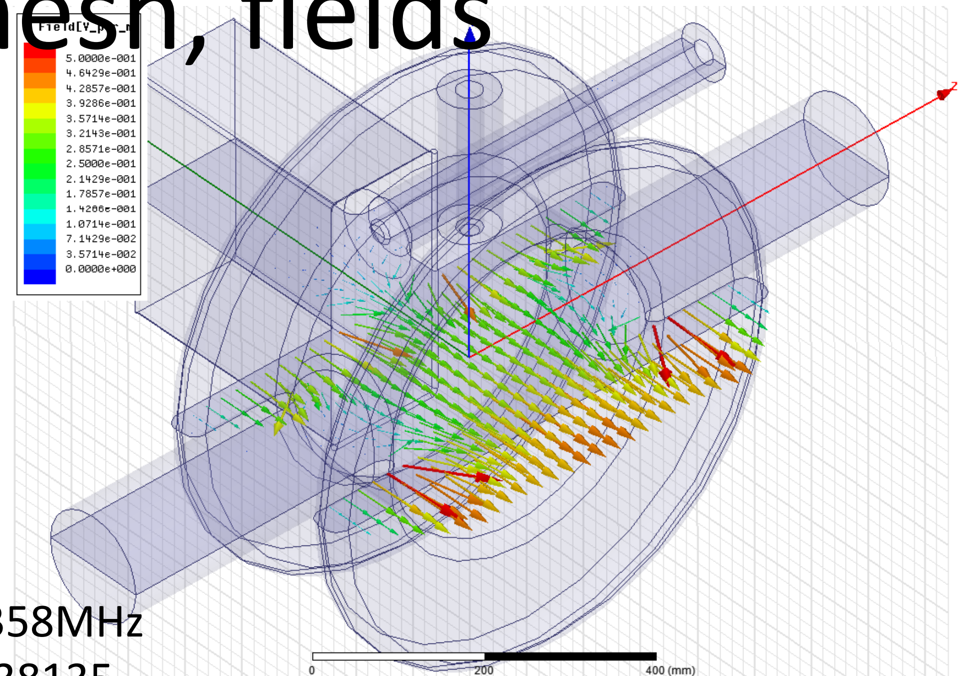
$$p_\perp^{(n)}(r,\varphi) = \frac{1}{c} r^{n-1}\left[\vec{u}_r\cos(n\varphi) + \vec{u}_\varphi\sin(n\varphi)\right]\int_0^L F_\perp^{(n)}dz$$

Equating the RF and magnetic kicks, RF kick strength can be expressed in magnetic units:

$$B_\perp^{(n)} = \frac{1}{ec}F_\perp^{(n)} = \frac{nj}{\omega}E_{acc}^{(n)} \quad [Tm/m^n]$$

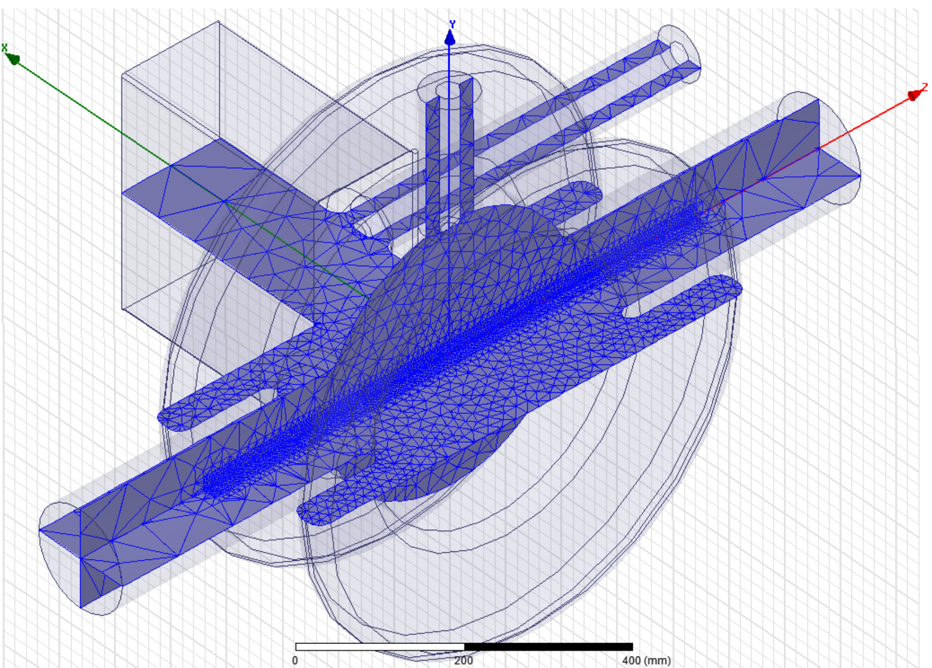
KEKCAV: mesh, fields

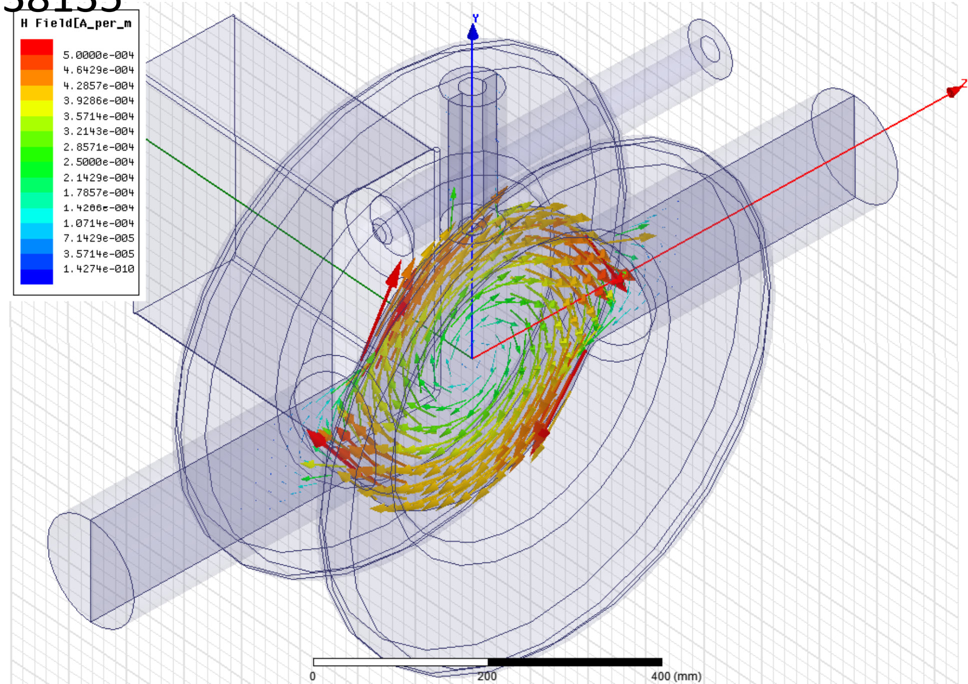




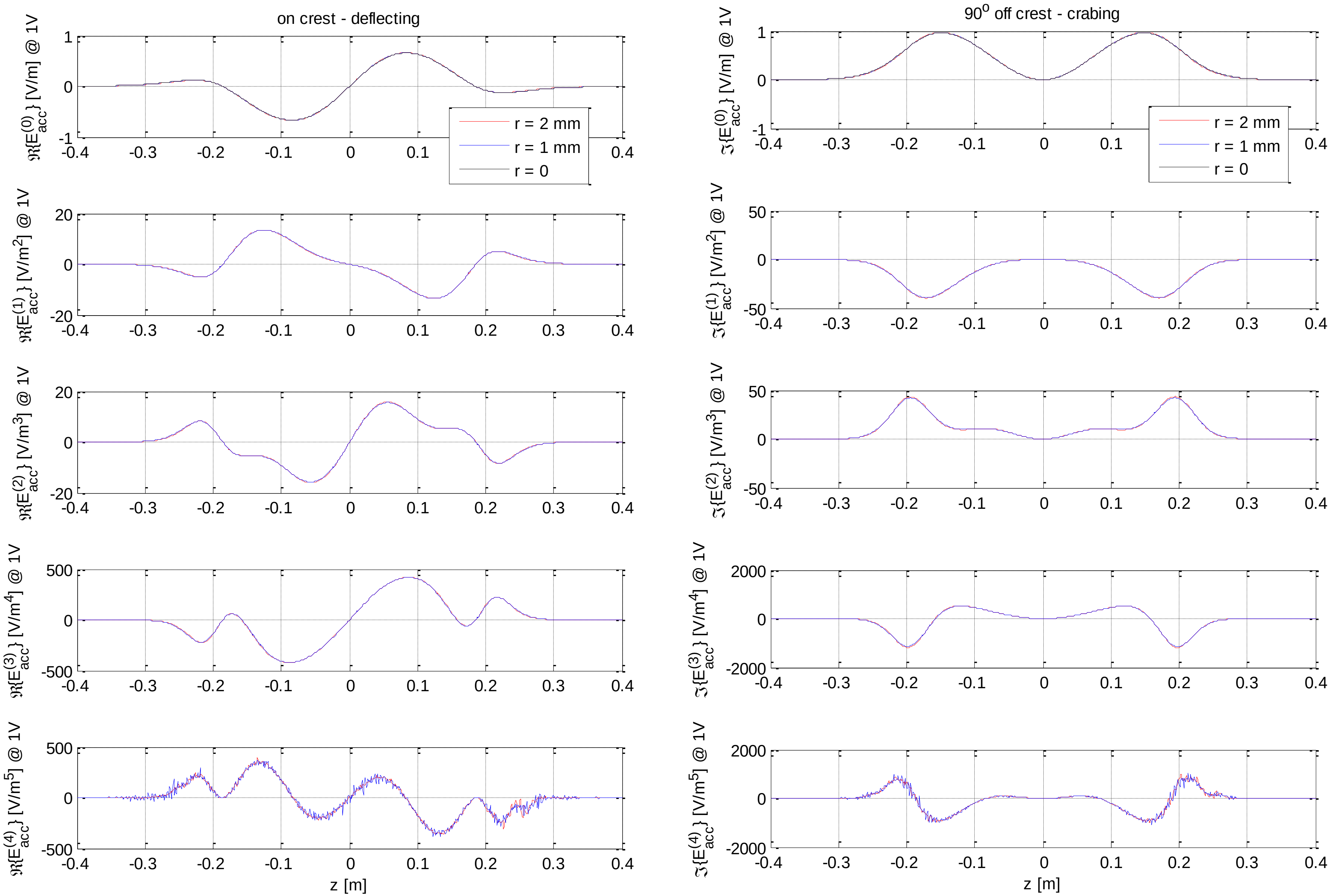
f_0=403.358MHz

Ntetr=338135

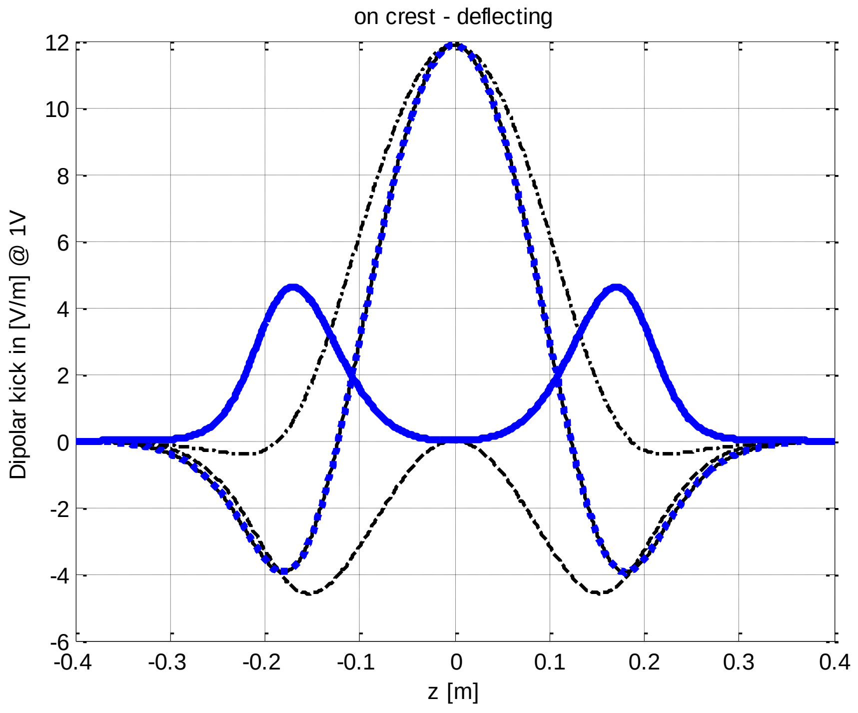


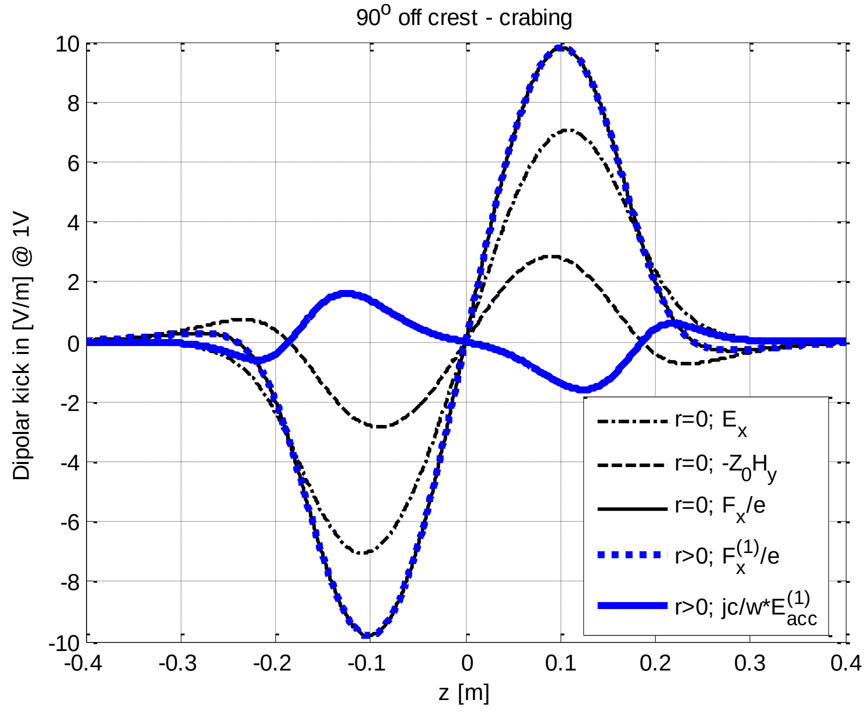


KEKCAV: multipoles of E_{acc} at V_x=1V



KEKCAV: dipolar kick, PW versus LF

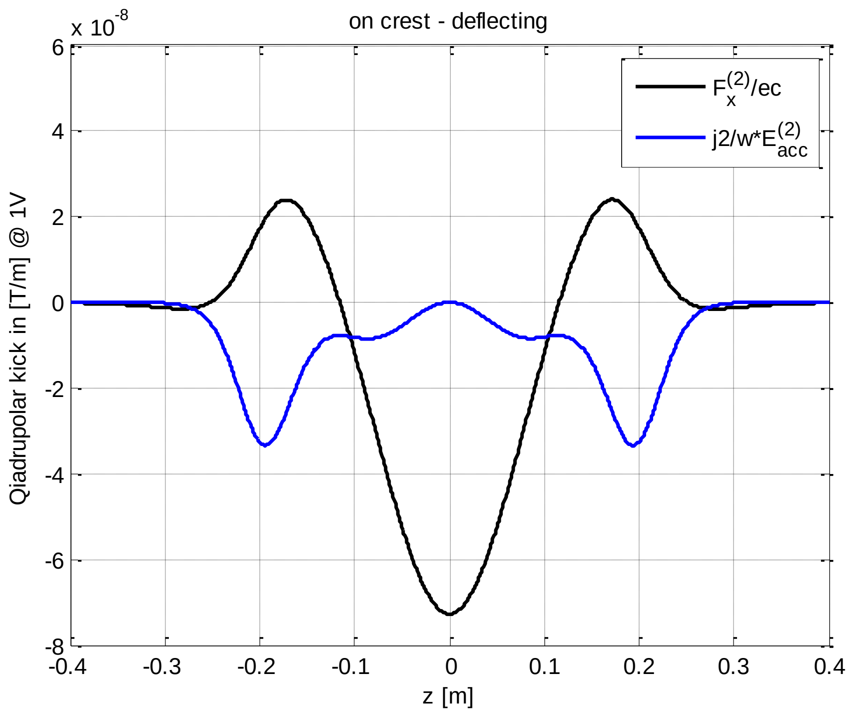


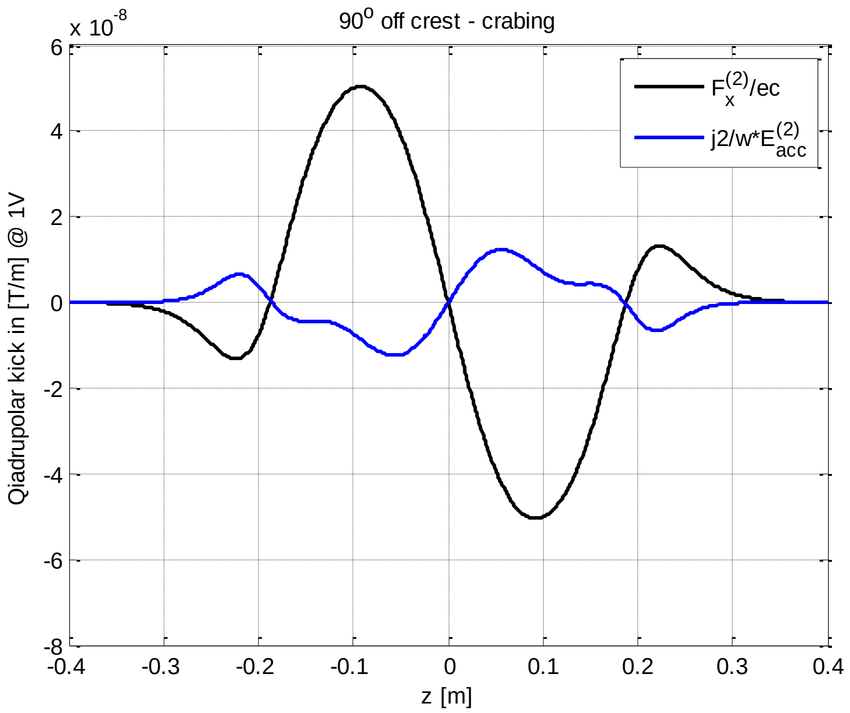


- Dipolar kick strength F_x and corresponding multipole of $E_{acc}^{(1)}$ have very different dependence along the beam axis but the integrals are equal.
- Dipolar kick is mainly electric, but is reduced significantly by the magnetic part

Comparison V_x
LF: 1.0000 + 0.0000i [V]
PW: 1.0018 + 0.0000i [V]

KEKCAV: quadrupolar kick, PW versus LF





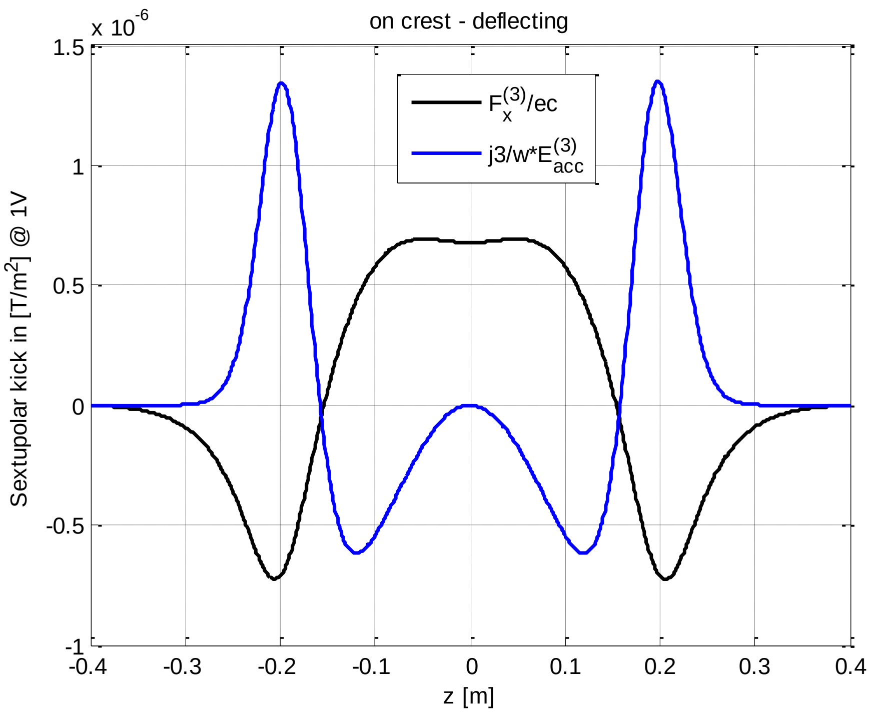
Quadrupolar kick strength $F_x^{(2)}$ and corresponding multipole of $E_{acc}^{(2)}$ have very different dependence along the beam axis but the integrals are equal.

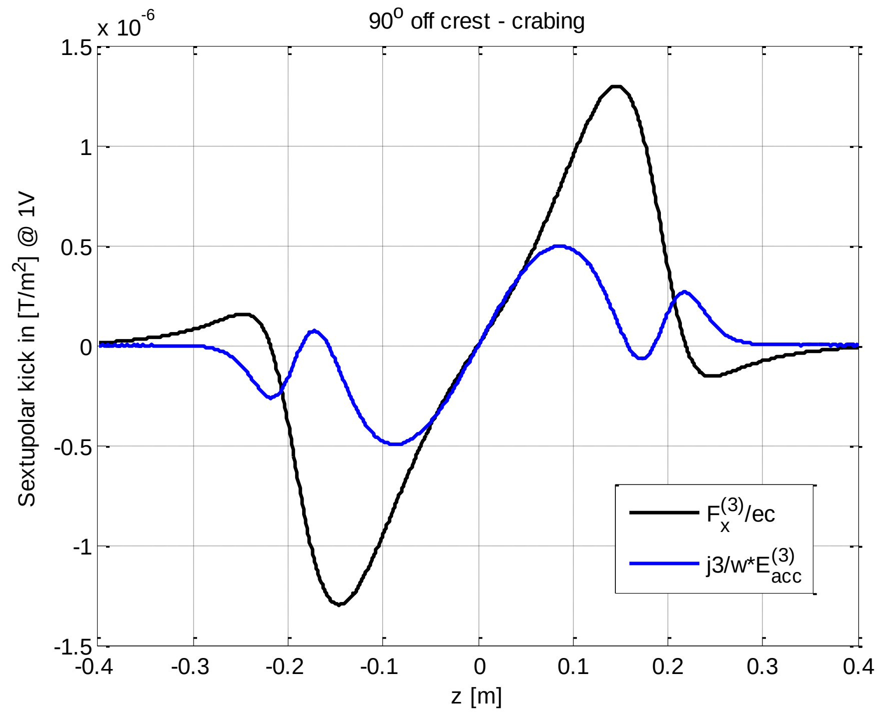
Comparison $\int B^{(2)}\,dz$ @V_x=1V

LF: -6.53 -0.02i [nTm/m]

PW: -6.54 -0.02i [nTm/m]

KEKCAV: sextupolar kick, PW versus LF

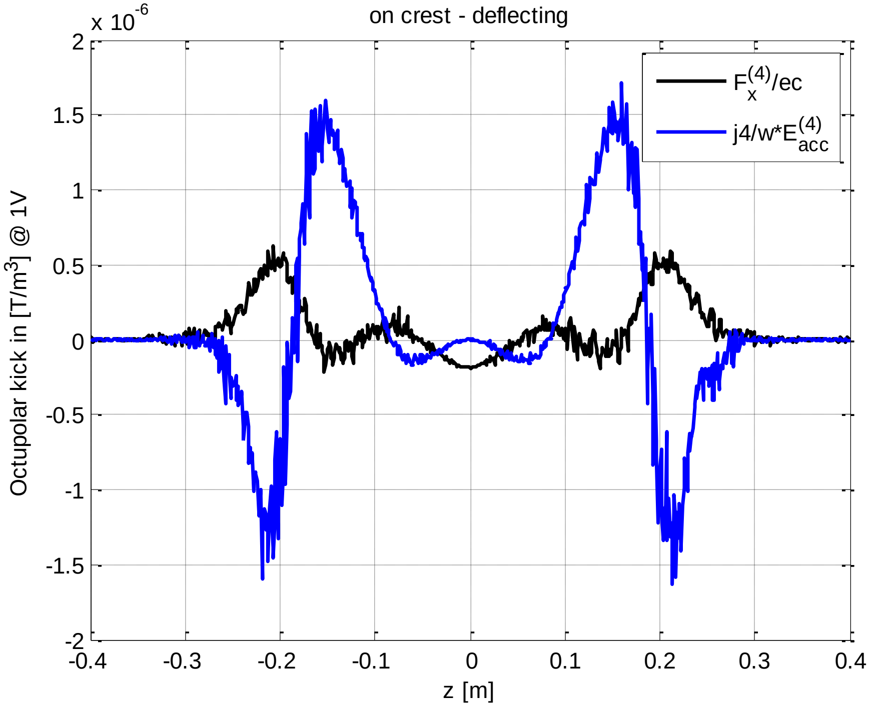


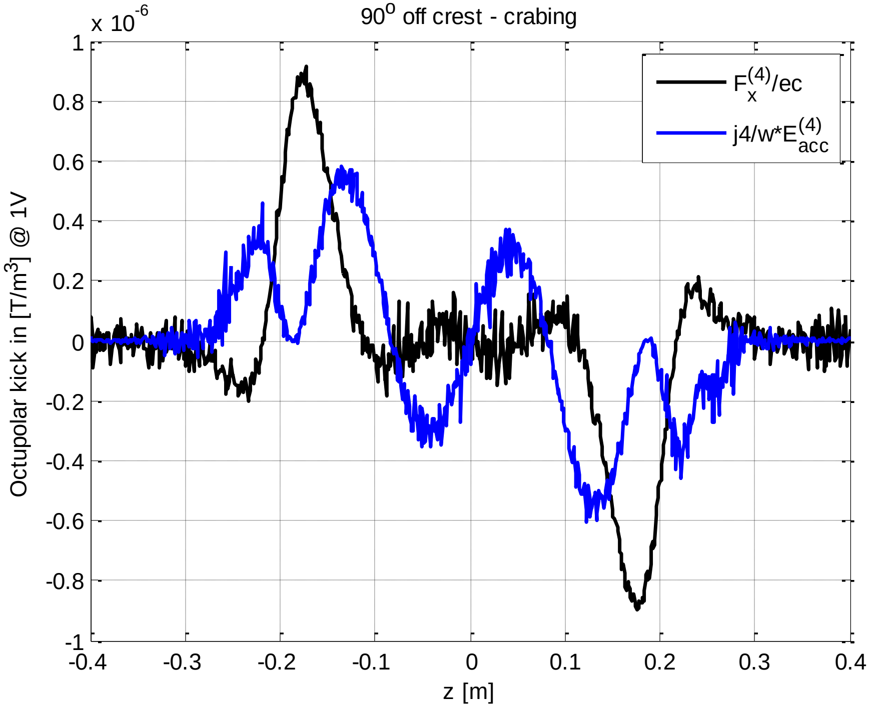


Sextupolar kick strength $F_x^{(3)}$ and corresponding multipole of $E_{acc}^{(3)}$ have very different dependence along the beam axis but the integrals are equal.

Comparison $\int B^{(3)}\,dz$ @V_x=1V
LF: 49.9 -0.08i [nTm/m^2]
PW: 50.2 -0.10i [nTm/m^2]

KEKCAV: octupolar kick, PW versus LF





Octupolar kick strength $F_x^{(4)}$ and corresponding multipole of $E_{acc}^{(4)}$ have very different dependence along the beam axis but the integrals are equal.

Comparison $\int B^{(4)}\,dz$ @V_x=1V

LF: 53.1 -0.9i [nTm/m^3]

PW: 56.2 -1.4i [nTm/m^3]

Summary table for V_x = 5MV

Values are calculated from Lorenz force

	ODUCAV	SRHW	KEKCAV	UKCAV	QWAVER	FRSCAV
Vz(x=0) [kV]	0.0	-2.1 - 2.5i	-4 +**1378**i	0.0	0 +85.7i	-0.1 -0.2i
Vx [MV]	5	5	5	5	5	5
$B^{(2)}$ [mTm/m]	0	0 -0.04i	-32.7 - 0.1i	0.02 + 0i	25 + 0i	0 +**108**i
$B^{(3)}$ [mTm/m^2]	1250 + 0i	229 + 0i	250 - 0i	**2452** - 0.5i	464 + 0i	-233 +1i
$B^{(4)}$ [mTm/m^3]	0	0	266 - 5i	0	540 +0i	-189 -**14209**i

NB: $B_y^{(n)}(y=0,x=a) = B^{(n)}a^{n-1}$. This is not MAD convention for multipolar strength.

There is the following dependences of the multipolar kick on the RF phase, where $\delta\phi_{crab}$ is the deviation of the (macro)particle RF phase from the crabbing phase

$$p_{\perp}^{(n)}(r,\varphi,\delta\phi_{crab}) = er^{n-1}\left[\vec{u}_r\cos(n\varphi)+\vec{u}_\varphi\sin(n\varphi)\right]\Im\left\{B^{(n)}e^{j\delta\phi_{crab}}\right\}$$

Multipole errors in 1e-4 units at R=17mm

	ODUCAV		SRHW		KEKCAV		UKCAV		QWAVER		FRSCAV	
	real	imag	real	imag	real	imag	real	imag	real	imag	real	imag
Vx [MV]	5	0	5	0	5	0	5	0	5	0	5	0
$B^{(2)}$ [mTm/m]	0	0	0	-0.04	-32.7	-0.1	0.02	0	25	0	0	**108**
$B^{(3)}$ [mTm/m^2]	1250	0	229	0	250	0	**2452**	-0.5	464	0	-233	1
$B^{(4)}$ [mTm/m^3]	0	0	0	0	266	-5	0	0	540	0	-189	-**14209**
multipole error relative to dipole kick of the crab cavity itself												
Vx [MV] => B1*L [mTm]	16.68	0	16.68	0	16.68	0	16.68	0	16.68	0	16.68	0
b2[1e-4@R17mm]	0	0	0	-0.41	-333.3	-1.02	0.204	0	254.8	0	0	**1100.8**
b3[1e-4@R17mm]	216.6	0	39.68	0	43.32	0	**424.9**	-0.09	80.40	0	-40.37	0.173
b4[1e-4@R17mm]	0	0	0	0	0.784	-0.01	0	0	1.591	0	-0.557	-**41.86**
multipole error relative to the LHC main dipole at injection 0.45 TeV												
B1*L [Tm]	7.728	7.728	7.728	7.728	7.728	7.728	7.728	7.728	7.728	7.728	7.728	7.728
b2[1e-4@R17mm]	0	0	0	-0	-0.719	-0	4E-04	0	0.550	0	0	**2.376**
b3[1e-4@R17mm]	0.467	0	0.086	0	0.093	0	**0.917**	-0	0.174	0	-0.0871	0.0004
b4[1e-4@R17mm]	0	0	0	0	0.002	-0	0	0	0.003	0	-0.0012	-**0.090**
multipole error relative to the LHC main dipole at 7TeV												
B1*L [Tm]	119.2	119.2	119.2	119.2	119.2	119.2	119.2	119.2	119.2	119.2	119.2	119.2
b2[1e-4@R17mm]	0	0	0	-0	-0.047	-0	3E-05	0	0.036	0	0	**0.154**
b3[1e-4@R17mm]	0.03	0	0.006	0	0.006	0	**0.059**	-0	0.011	0	-0.006	2E-05
b4[1e-4@R17mm]	0	0	0	0	1E-04	-0	0	0	0.0002	0	-8E-05	-**0.006**

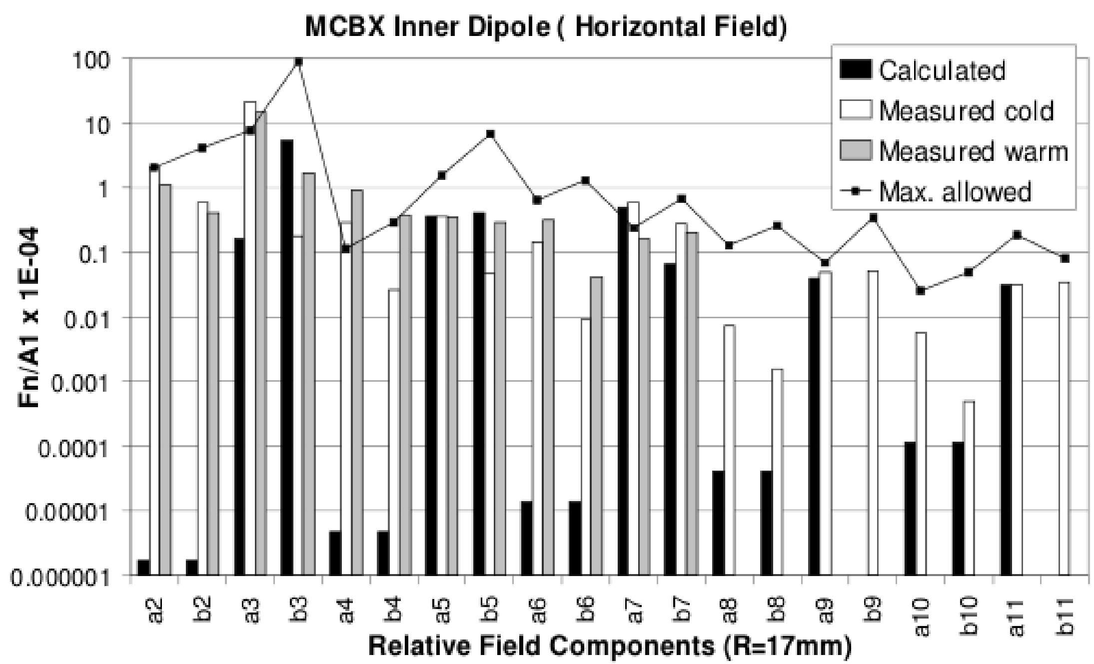
Multipole errors relative to the LHC main dipole at 7TeV is very small

For example, LHC sextupole corrector (MCS) parameters from the Blue Book: field strength: 1630 T/m2, magnetic length: 110 mm, integrated kick: 179300 mTm/m^2

Multipole errors in 1e-4 units at R=17mm

	ODUCAV		SRHW		KEKCAV		UKCAV		QWAVER		FRSCAV	
	real	imag	real	imag	real	imag	real	imag	real	imag	real	imag
Vx [MV]	5	0	5	0	5	0	5	0	5	0	5	0
$B^{(2)}$ [mTm/m]	0	0	0	-0.04	-32.7	-0.1	0.02	0	25	0	0	**108**
$B^{(3)}$ [mTm/m^2]	1250	0	229	0	250	0	2452	-0.5	464	0	-233	1
$B^{(4)}$ [mTm/m^3]	0	0	0	0	266	-5	0	0	540	0	-189	-14209
multipole error relative to the LHC dipole corrector MCBXH												
B1*L [Tm]	1.508	1.508	1.508	1.508	1.508	1.508	1.508	1.508	1.508	1.508	1.508	1.508
b2[1e-4@R17mm]	0	0	0	-0	-3.688	-0.01	0.002	0	2.8192	0	0	**12.179**
b3[1e-4@R17mm]	2.396	0	0.439	0	0.479	0	**4.701**	-0	0.8895	0	-0.4467	0.00192
b4[1e-4@R17mm]	0	0	0	0	0.009	-0	0	0	0.018	0	-0.0062	**-0.4631**

Multipole errors relative to the LHC dipole correctors MCBXH are comparable



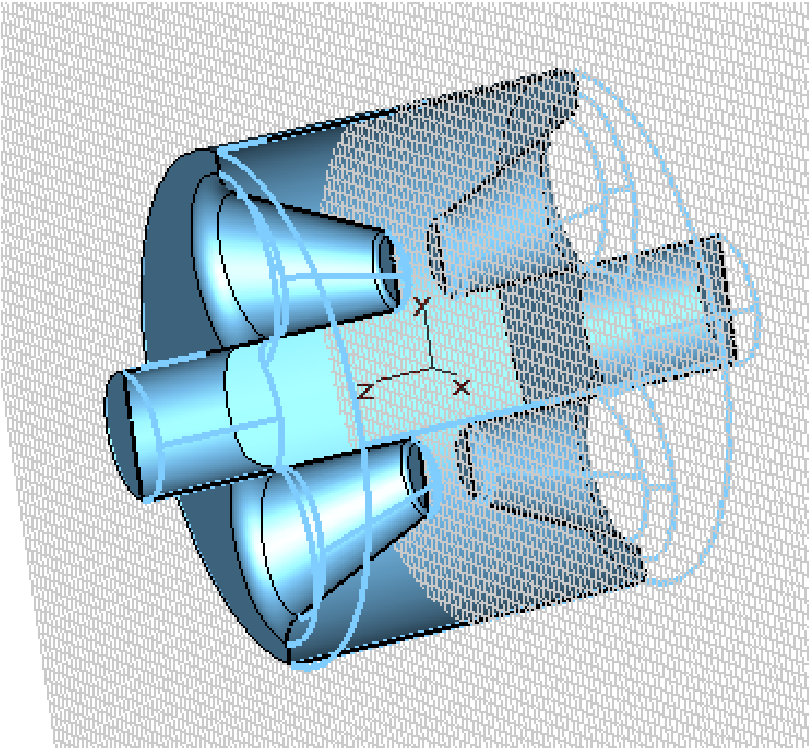
Conclusions

- Strength of the multipolar RF kicks associated with the EM field of deflecting mode has been calculated for ultra relativistic particles using HFSS code
- The strength has been expressed in terms of equivalent magnetic multipole strength giving the same transverse kick
- Whereas magnetic multipoles are constant in time the RF multipoles have harmonic dependence on the RF phase which has been introduced
- The calculated strength has been compared to the magnetic field strength/quality of different magnetic elements of LHC. The effect seems to be small. BUT those are static magnetic multipoles and not harmonic RF multipoles

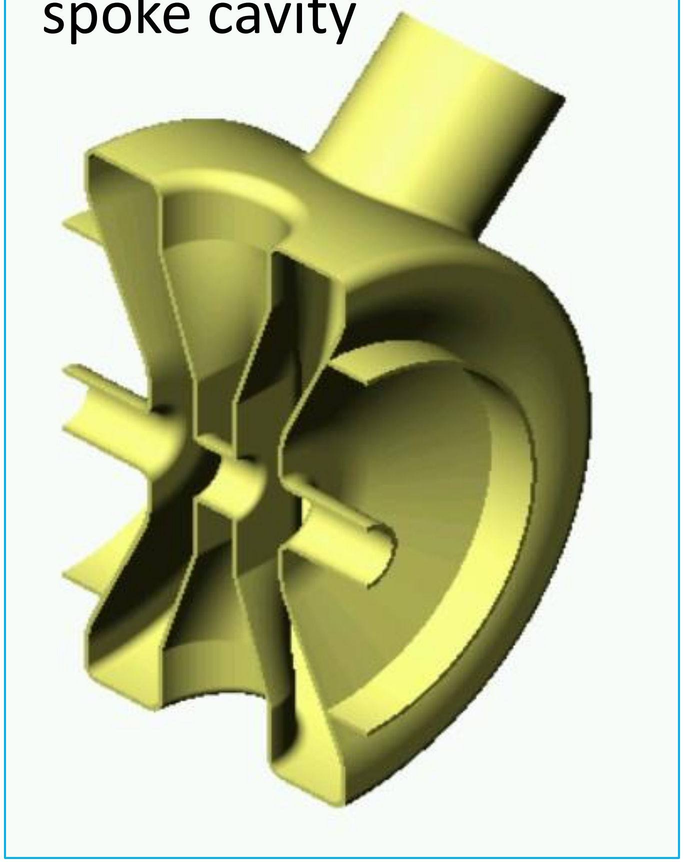
Flat Rod Spoke cavity

Idea

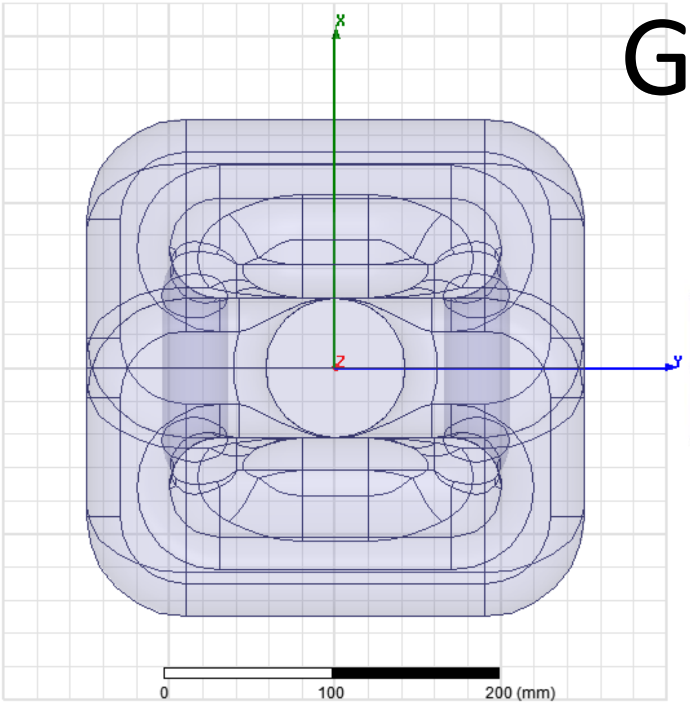
4-rod cavity = 4 x QW-resonators

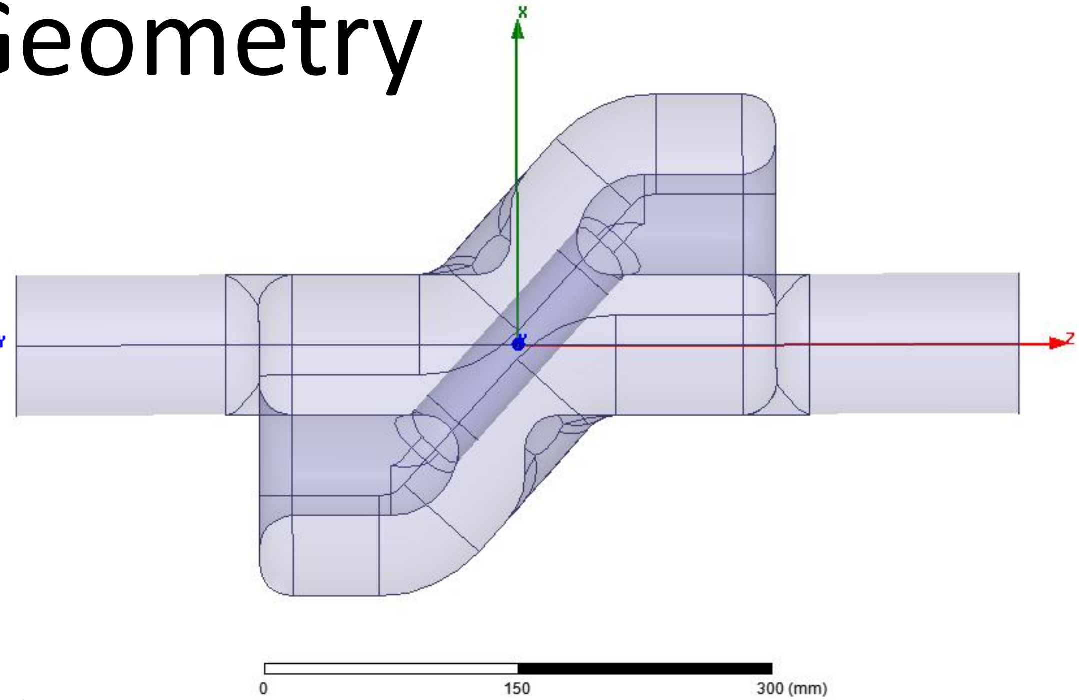


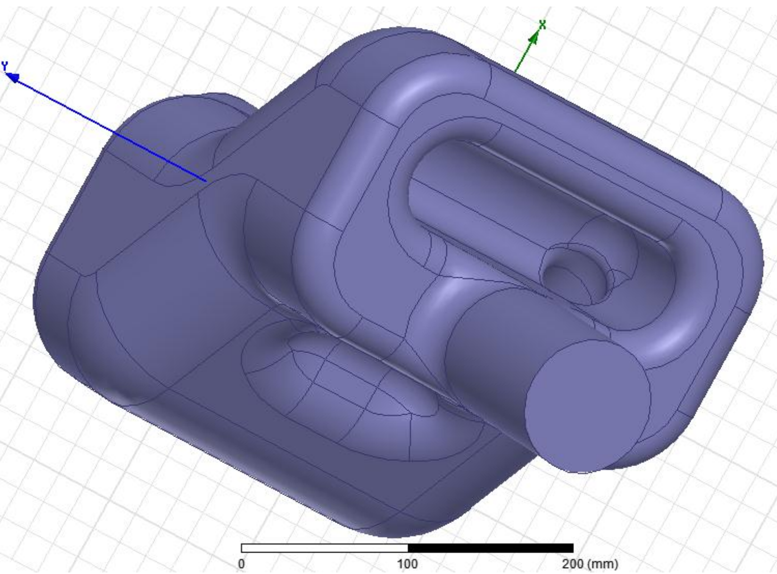
spoke cavity

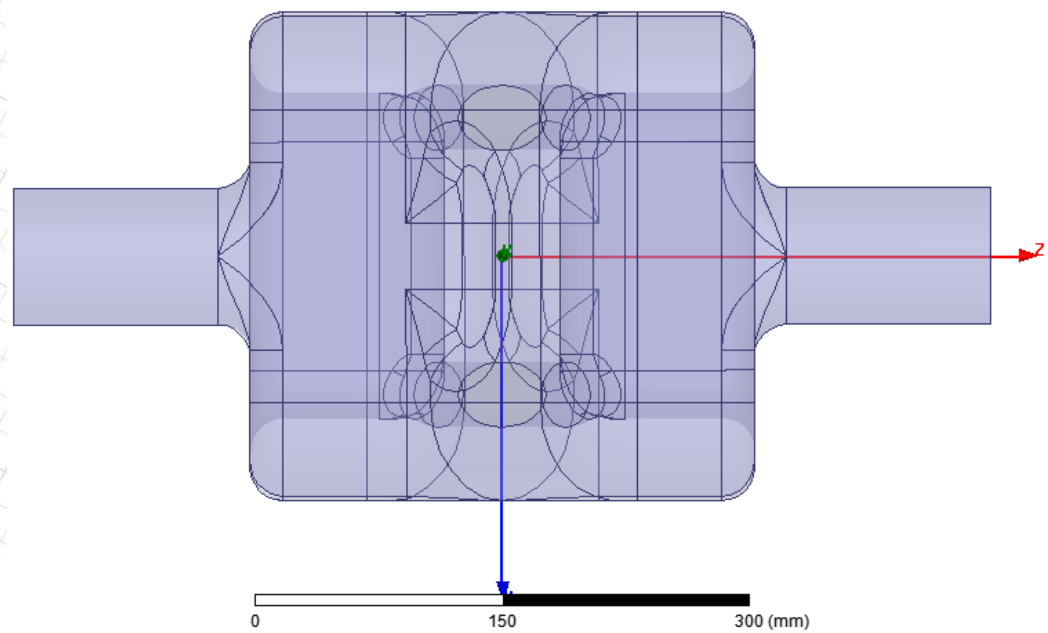


Geometry

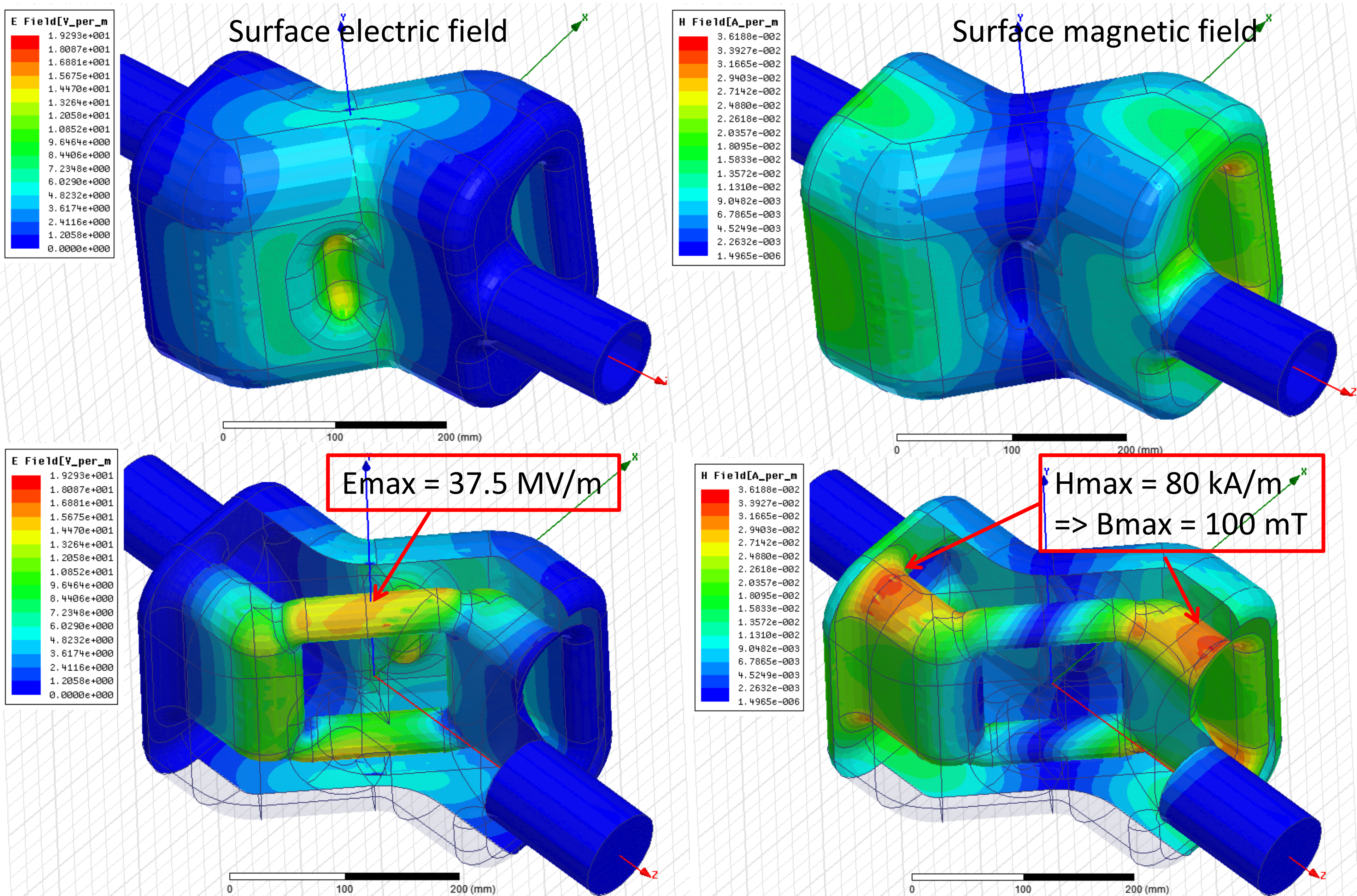








Surface EM fields for dipole kick: Vx=2.5MV



R upon Q of the main mode and HOMs

$$\frac{R_x}{Q}=\frac{V_xV_x^*}{\omega W};\quad \text{where}\quad V_x=\int\left[E_x-Z_0H_y\right]e^{jkz}dz=\frac{jc}{\omega}\frac{dV_z}{dx}$$

$$\frac{R_y}{Q}=\frac{V_yV_y^*}{\omega W};\quad \text{where}\quad V_y=\int\left[E_y+Z_0H_x\right]e^{jkz}dz=\frac{jc}{\omega}\frac{dV_z}{dy}$$

$$\frac{R_z}{Q}=\frac{V_zV_z^*}{\omega W};\quad \text{where}\quad V_z=\int E_z e^{jk_z z}dz$$