Beam simulation of the SPS

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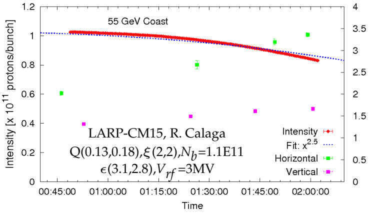
Motivation

- During 2010 machine study in the SPS, large transverse emittance growth rate ($\epsilon_x \simeq$140%/h, $\epsilon_y \simeq$60%/h) has been observed at 55 GeV.
  - Dipole voltage ripple, space-charge, tune modulation, RF phase noise, chromaticity, IBS, ... contribute to the emittance growth.



- Machine studies on low transition energy lattice in the SPS were done in 2011 to understand the source of emittance growth.
- We investigate the emittance growth in both nominal γ_T (22.90) and low γ_T (18.01) optics.
- SPS is a likey candidate for testing crab cavity for the HL-LHC.

Low transition gamma (γ_T)

parameter	symbol	nominal	low γ_T
transition energy		22.90	18.01
transverse tune	(ν_x, ν_y)	(26.13, 26.18)	(20.13, 20.18)
natural chromaticity		(-32.68, -32.74)	(-22.79, -22.83)
sextupole strength	m^{-2}	(0.063, -0.150)	(0.045, -0.041)
max. beta	(β_x, β_y)	(111, 109)	(109, 109)
max. dispersion	(η_x, η_y)	(4.9, 0)	(8.1, 0.0)
beam energy	GeV	55	
beam intensity		1×10^{11}	
chromaticity	(ξ_x, ξ_y)	(0, 0)	
transverse emittance	mm-mrad	3.5	
long. emittance, 4σ	eV s	0.24	
rf voltage	MV	3	

(Courtesy of I. Papaphilippou)

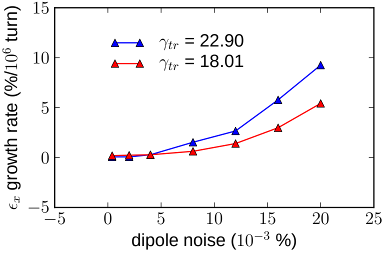
- Natural chromaticity and sextupole strength of nominal optics are 40% larger than those of low γ_T optics.
- β_{max} does not change much, but β_{min}=20m (nominal), β_{min}=34 (low γ_T).
- Weaker focusing has the consequence of increasing beta functions and dispersions - both of which increase the beam size.

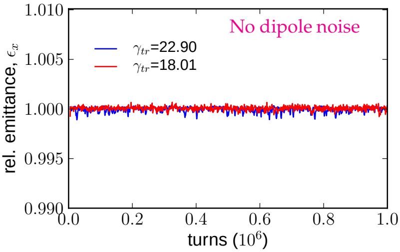
Beam-Beam Simulation (BBSIM) code

- 6D weak-strong tacking code.
- Linear transfer matrices btwn nonlinear elements + nonlinear kicks at the nonlinear elements (thin lens approximation: dipole, quadrupole, sextupole, multitpole, etc.).
- Space charge: 2-D and 3-D Poisson solvers using (1) Conjugate Gradient and (2) FFT.
- Beam-beam force: (1) Gaussian beam profile and (2) Poisson solver with FFT.
- Multiple-slice model for finite bunch length effects.
- Lorentz boost to handle crossing angle collisions.
- Modules: crab crossing, wire and electron lens compensation, etc.
- Fully parallelized with MPI.
- Diagnostics: Beam loss, emittance growth, beam profiles, BTFs, dynamic aperture, tune footprints.
- Simulations agree well with measurements in the Tevatron, RHIC. Also applied to wire compensation in the SPS, LHC.
- http://www-ap.fnal.gov/~hjkim

Emittance growth vs dipole noise

- Large emittance growth in the SPS (MD2010). Expect that dipole noise contributes to the emittance growth.
- Gaussian distribution with 10000 particles, 10^6 turns (23 seconds).
- Model: sextupole + dipole voltage ripple (white noise)
- Emittance growth only in horizontal plane.
- Vertical and longitudinal emittance growth is insignificant.
- Emittance growth is (2 times) less in low γ_T.
- Sextupole strength of nominal is 40% larger than low γ_T.
- Voltage ripple of LHC after active filtering is 2.5×10^{-3}%.



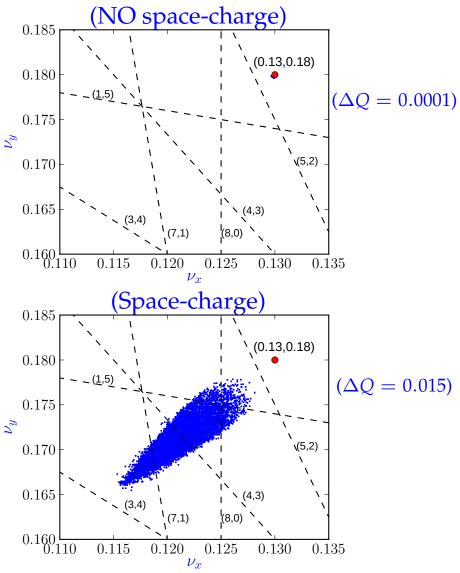


Tune footprint (space charge)

- Apply 2.5-D space-charge kicks
  - Transverse electric field (fast 2D Poisson solver, $\vec{E} = -\nabla\phi$)
  - longitudinal electric field (ρ_L line density)
- Apply space-charge kicks at quadrupole locations.
  - 72 kicks/turn is chosen.
- Tune shift for bunched beam due to space-charge

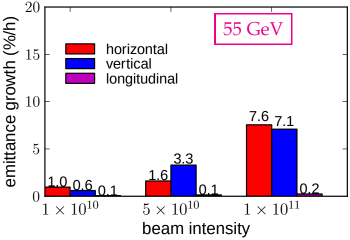
$$\Delta Q = -\frac{N_b r_p}{4\pi B\beta\gamma^2\epsilon_N}$$

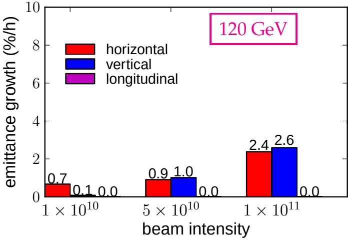
- Tune shift of particles with small betatron amplitude (55GeV, $N_b = 10^{11}$, ϵ_N=3.5μm, $\sigma_z = 0.18m$), $\Delta Q = 0.015$.
- 6-th, 7-th, and 8-th resonance lines are spanned.



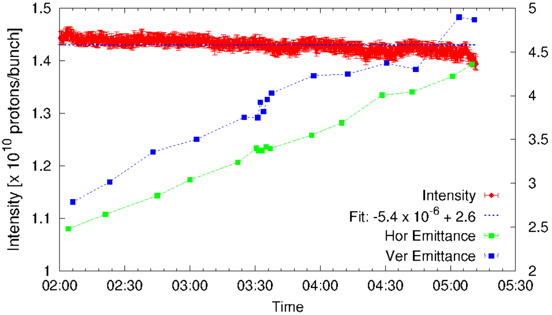
Emittance growth (space-charge)

- low γ_T (18.01) lattice.
- Gaussian distribution in (x, y, z) with 60,000 particles.
- No noises are added in the model.
- Space charge kicks at 72 locations.
- Space-charge induces 7%/hr emittance growth in both horizontal and vertical planes. 0.2%/hr emittance growth in longitudinal plane.
- γ_T (22.90) has the same growth.
- Space-charge is expected to have a small contribution to observed emittance growth in MD2010 .
- (Emittance growth due to space-charge + dipole noise equals the sum of emittance growth due to space-charge and dipole noise respectively.)





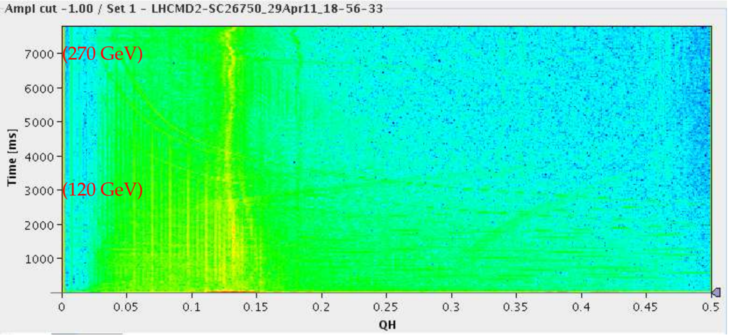
SPS machine study (120 GeV, May 25, 2011)



(Courtesy of R. Calaga)

- Beam energy is 120 GeV. Low transition energy (18.01).
- Tunes are nominal (0.13, 0.18). Chromaticity is close to 0.5 units.
- Observed about 25%/hr emittance growth in both horizontal and vertical planes even at small intensity.
- Tunes are scanned up to 0.35, but there is no effect on the slope of the emittance.

Tune modulation



(Courtesy of R. Calaga)

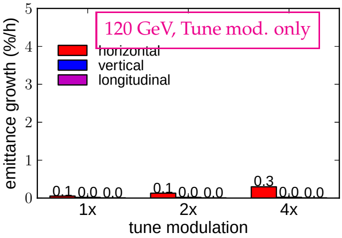
- Horizontal tune spectra during the energy ramp 26-450 GeV. (Vertical tune spectra is similar to horizontal one.)
- Beam energy 120 GeV is around 3084 ms. Horizontal nominal tune is $\nu_x = 0.13$.
- One can see a lot of harmonics with relative amplitudes below 4000 ms. In order to see the effect of tune modulation, an experiment was done at 270 GeV in July, 2011. We see about 20%/hr emittance growth as well.

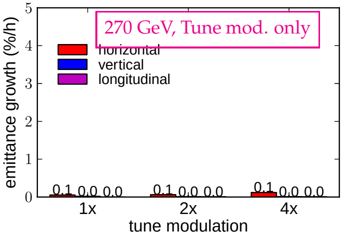
Emittance growth (tune modulation)

k	Ω_k	ϵ_k $\left(\times 10^{-4}\right)$
1	$2\pi/867.5$	1.000
2	$2\Omega_1$	0.218
3	$3\Omega_1$	0.708
4	$6\Omega_1$	0.254
5	$7\Omega_1$	0.100
6	$10\Omega_1$	0.078
7	$12\Omega_1$	0.218

(M. Giovannozzi, Phy. Rev. E, 1998)

- These lines are taken as one sample, but may not be updated.
- Considered one main frequency Ω_1 and six harmonics with relative amplitudes.
- Tune modulation due to the observed ripple in the quadrupole of the SPS.
- No emittance growth due to tune modulation in both 120 GeV and 270 GeV.
- 1x - amplitude of tune modulation as shown in previous table, 2x - double amplitude of 1x.





Emittance growth due to multiple Coulomb scattering

The multiple elastic Coulomb scattering will cause the transverse emittance of the beam to grow. The rate of the normalized emittance growth is expressed as

$$\frac{d\epsilon_{(x,y),N}}{dt} \simeq \frac{1}{2}\gamma \left\langle \beta_{(x,y)} \right\rangle \dot{\Theta}^2_{(x,y),rms} \propto \frac{1}{\gamma}\frac{1}{L_{rad}},$$

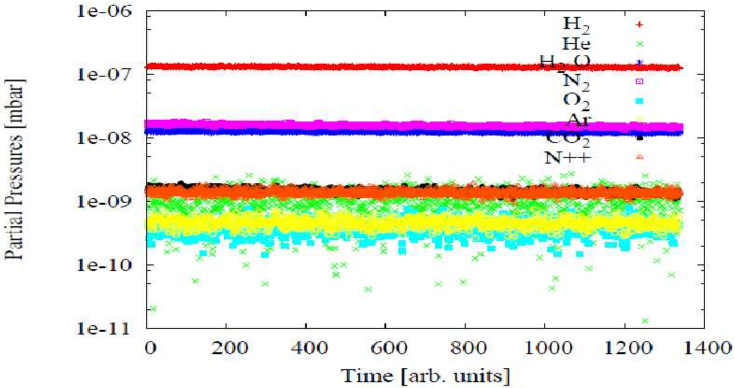
The average rate of change of the transverse scattering angle $\dot{\Theta}^2_{rms}$ is

$$\dot{\Theta}^2_{(x,y),rms} = \left(\frac{13.6MeV}{m_p c^2 \gamma}\right)^2 \frac{Z_P^2}{A_P^2}\frac{c}{L_{rad}},$$

The radiation length L_{rad} can be written as

$$\begin{aligned}\frac{1}{L_{rad}} &= \sum_i^{N_{species}} \frac{\rho_i\,(g/cm^3)}{X_{0,i}\,(g/cm^2)} \\ &= \sum_i^{N_{species}} \frac{10^3 \cdot p_i\,(mbar) \cdot M_i\,(g/mol)}{R\,(erg/K\,mol) \cdot T\,(K) \cdot X_{0,i}\,(g/cm^2)}.\end{aligned}$$

Pressure without beam



(Courtesy of Holger Neupert)

- SPS MD May 10, 2011.
- Figure shows the (corrected) pressure profile without beam, in between beam cycles.
- SPS MDs for beam emittance used "low intensity" beams.
- These pressures are used for estimates.

Radiation lengths L_{rad}

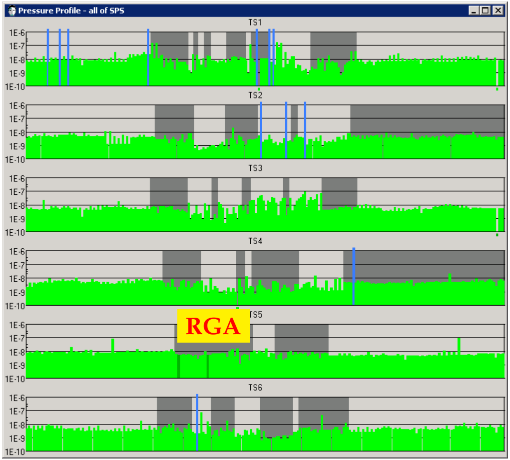
Gas	Average static pressure [mbar]	Density* [g/cm^3]	Radiation length* L_{rad} [cm]
N_2	1.54×10^{-8}	1.77×10^{-14}	2.1×10^{15}
Water vapor	1.24×10^{-8}	9.15×10^{-15}	3.9×10^{15}
H_2	1.28×10^{-7}	1.04×10^{-14}	5.8×10^{15}
CO_2	1.45×10^{-9}	2.62×10^{-15}	1.4×10^{16}
Ar	4.58×10^{-10}	7.52×10^{-16}	2.1×10^{16}
N^{++}	1.35×10^{-9}	7.81×10^{-16}	4.9×10^{16}
O_2	3.97×10^{-10}	5.21×10^{-16}	6.6×10^{16}
He	9.46×10^{-10}	1.55×10^{-16}	6.1×10^{17}

* Gas temperature is assumed to be 300 K.

Emittance growth rate

- Beam energy is assumed 120 GeV.
- With static pressures (without beam):
  - $L_{rad} = 9.53 \times 10^{14}$ cm
  - $\frac{d\epsilon_N}{dt} = 3.9$ mm-mrad/hr
- Nitrogen is the dominant contributor to emittance growth.
- Measured $\frac{d\epsilon_N}{dt} \sim 0.75$ mm-mrad/hr at 120 GeV [SPS MD, May 25, 2011].

Caveats



Pressure distribution around SPS with LHC type beam (Courtesy: H. Neupert)

- Pressure distribution varies by more than 1 order of magnitude around the ring
- Pressure readings at RGA may be higher than "average" pressure around the ring.
- Pressure profile used (static pressure) may not be representative of test beam at 120 GeV.
- Ionization cross-section depend on the energy

Summary of SPS studies

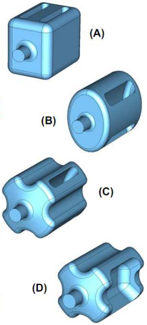
- Source of emittance growth in the SPS

Sources	Comment	Likelihood
Dipole noise	Needs large coupling for vertical emittance growth	weak
Tune modulation	Simulations do not show much effect	?
Intra-beam scattering	Needs large coupling for vertical emittance growth Intensity dependent	weak
Space charge	Simulations do not show much effect Intensity dependent	weak
Vacuum	Estimates show large effect Explains independence of growth on intensity, tunes and transeverse plane	strong

Compact crab cavity

Cavity Properties – 400 MHz Designs

Parameter	(A)	(B)	(C)	(D)	Unit
Frequency of π mode	400.0	400.0	400.0	400.0	MHz
λ/2 of π mode	374.7	374.7	374.7	374.7	mm
Frequency of 0 mode	411.0	687.0	541.5	665.9	MHz
Nearest mode to π mode	411.0	611.6	541.5	619.6	MHz
Cavity reference length	444.7	445.0	525.0	525.0	mm
Cavity width / diameter	300.0	290.0	404.5	373.0	mm
Cavity height	383.2	408.6	404.5	373.0	mm
Bars length	330.0	330.0	330.0	330.0	mm
Bars width	55.0	60.0	60.0	-	mm
Aperture diameter	84.0	84.0	84.0	84.0	mm
Deflecting voltage (V_T^*)	0.375	0.375	0.375	0.375	MV
Peak electric field (E_P^*)	2.2	3.4	3.3	3.7	MV/m
Peak magnetic field (B_P^*)	7.9	7.71	8.2	8.3	mT
B_P^* / E_P^*	3.6	2.27	2.45	2.24	mT / (MV/m)
Geometrical factor ($G = QR_S$)	74.1	109.4	81.3	88.4	Ω
$[R/Q]_T$	413.34	255.68	372.83	285.25	Ω
$R_T R_S$	3.1×10^4	2.8×10^4	3.0×10^4	2.5×10^4	$Ω^2$
At E_T^* = 1 MV/m					



(J. Delayen, LARP CM16)

- Proposed compact cavities which are compatible with LHC local option, such as parallel bar design (JLab & ODU), 400MHz ridged waveguide (SLAC), etc.
- Develop simulation model of parallel bar design of JLab & ODU, because the design fits both horizontal and vertical crabbing schemes, the deflecting mode is the lowest frequency mode, and the EM fields inside the cavity are provided.

TEM-mode crab cavity (Simulation model)

- Electromagnetic fields inside the cavity are available numerically at grid points (No analytic formula).
- The electric and magnetic field of actual cavity design contains all vector components, i.e., $\mathbf{E} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}$ and $\mathbf{H} = H_x\hat{x} + H_y\hat{y} + H_z\hat{z}$.
- Electric and magnetic fields are

$$\tilde{\mathbf{E}}\left(x, y, z, t\right) = \mathbf{E}\left(x, y, t\right) \sin\left(\omega\left(t - \frac{z}{\beta c}\right)\right),$$
$$\tilde{\mathbf{H}}\left(x, y, z, t\right) = \mathbf{H}\left(x, y, t\right) \cos\left(\omega\left(t - \frac{z}{\beta c}\right)\right).$$

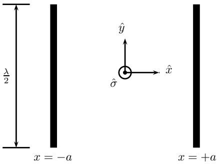
- Equation of motion becomes

$$\frac{dp_x}{dt} = \frac{q}{p_0}E_x \sin\left(\omega\left(t - \frac{z}{\beta c}\right)\right) - \frac{q\beta c}{p_0}\frac{1}{\mu_0}H_y \cos\left(\omega\left(t - \frac{z}{\beta c}\right)\right),$$
$$\frac{dp_y}{dt} = \frac{q}{p_0}E_y \sin\left(\omega\left(t - \frac{z}{\beta c}\right)\right) + \frac{q\beta c}{p_0}\frac{1}{\mu_0}H_x \cos\left(\omega\left(t - \frac{z}{\beta c}\right)\right),$$
$$\frac{dp_z}{dt} = \frac{q}{p_0}E_\sigma \sin\left(\omega\left(t - \frac{z}{\beta c}\right)\right).$$

- Using thin-lens approximation, the kicks are calculated at grid points. Interpolation scheme is applied to get the kick at any (x, y, z).

Benchmark of analytic model

- Simplified parallel wire cavity is considered to benchmark the simulation model of TEM-mode cavity. (J. Delayen, PRSTAB 12, 062002)



TEM wave propagates along the y-direction. Beam direction is along $\hat{\sigma}$.

- Electric and magnetic fields are

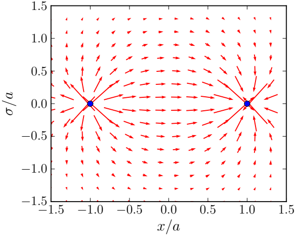
$$E_x(x,\sigma) = -\frac{aq}{\pi\epsilon_0}\left[\frac{x^2 - a^2 - \sigma^2}{r_-^2 r_+^2}\right],$$

$$E_\sigma(x,\sigma) = -\frac{aq}{\pi\epsilon_0}\left[\frac{2x\sigma}{r_-^2 r_+^2}\right],$$

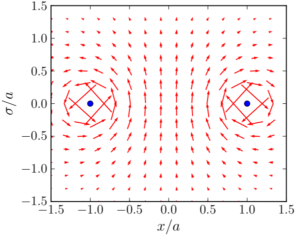
$$B_x(x,\sigma) = -\frac{1}{c}E_\sigma(x,\sigma), \quad B_\sigma(x,\sigma) = \frac{1}{c}E_x(x,\sigma).$$

$$r_\pm = (x \pm a)^2 + \sigma^2.$$

(Electric field)



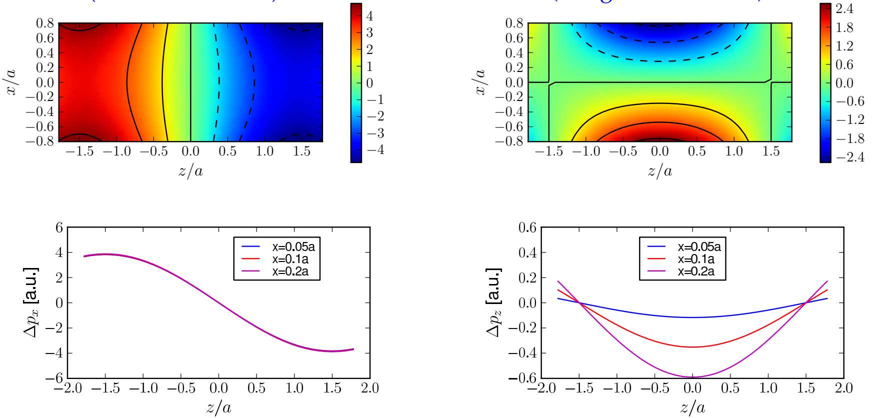
(Magnetic field)



Benchmark of analytic model

- Kicks due to the TEM-mode cavity are calculated. Vertical kicks are negligible.

(Horizontal kicks) (Longitudinal kicks)

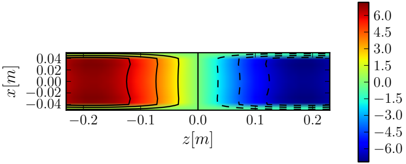


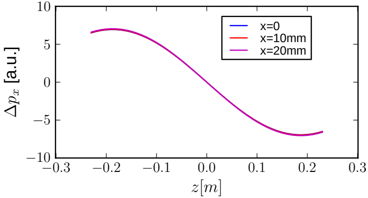
- In the model, $a = 125$ mm. At the (proposed) crab cavity location of SPS, the rms beam size is $\sigma_x = 0.91$ mm.

Cavity kicks (Parallel bar design at JLab & ODU)

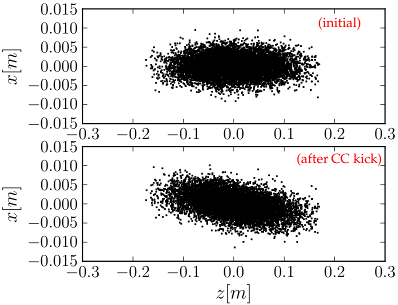
- Kicks due to the TEM-mode cavity are calculated using the electromagnetic field provided by J. Delayen and S. De Silva.
- Actual EM fields are available only numerically at grid points. This example uses the number of grid points, $(n_x, n_y, n_z) = (11, 11, 47)$.

(Horizontal kicks)





(Deflecting angle)



Note) applied large cavity voltage to see a tilt

Summary

- Dipole noise, tune modulation, intra-beam scattering and space-charge may have a small contribution to emittance growth in SPS.
- Multiple elastic Coulomb scattering is expected to have a significant contribution to emittance growth in SPS due its high background gas pressure. Simulation in progress.

- Proposed crab cavity simulation model using electromagnetic fields of crab cavity from JLab & ODU.
- The model is implemented in BBSIM, and long term simulations of emittance growth are ongoing.

Thank you for your attention!