

# Heavy Fermions

beyond the 3g Standard Model

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We shall concentrate our analysis on extensions of the SM with vector-like isosinglet quarks of  $Q = -1/3$  or  $Q = 2/3$ .

Question : Why study them ?  
 What can vector-like quarks do for us ?

- Provide a simple self-consistent framework with naturally small violations of  $3 \times 3$  unitarity of  $V^{\text{CKM}}$ 
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- Lead to naturally small Flavour Changing Neutral Currents (FCNC) in the down and/or up sector (mediated by  $Z$ )

- Provide the simplest framework to have spontaneous CP violation which generates a non-trivial CKM phase. This is an important requirement since there is experimental evidence of a complex  $V_{CKM}$  even if one allows for the presence of New Physics
- Provide New Physics contributions to  $B_d - \bar{B}_d$  and  $B_s - \bar{B}_s$  mixings.
- Provide a simple solution to the Strong CP-problem, which does not require Axions

- Provide a framework where there is a common origin of all CP violations:
  - CP violation in the quark sector
  - CP violation in the lepton sector, detectable through neutrino oscillations
  - CP violation needed to generate the Baryon Asymmetry of the Universe (BAU) through Leptogenesis

All these "manifestations" of CP Violation originate in a single phase of the vev of a complex singlet neutral scalar  $S$ :

$$\langle S \rangle = V e^{i\alpha}$$

## A Minimal Model

Consider an extension of the SM where the following fields are added to the SM:

- A vectorial quark  $D^\circ$ :  
Both  $D_L^\circ$  and  $D_R^\circ$  are  $SU(2)_L$  singlets, with charge  $Q = -1/3$ ; (Could be "up type")
- Three right-handed neutrinos  $\nu_{Rj}^\circ$
- A neutral complex singlet  $S$
- Impose CP invariance at the Lagrangian level
- Introduce a  $Z_4$  symmetry on the Lagrangian, under which:

$$\psi_l^\circ \rightarrow i\psi_l^\circ \quad e_{Rj}^\circ \rightarrow ie_{Rj}^\circ \quad \nu_{Rj}^\circ \rightarrow i\nu_{Rj}^\circ$$

$$D^\circ \rightarrow -D^\circ ; \quad S \rightarrow -S$$

All other fields are invariant under  $Z_4$

Since we impose CP invariance at the Lagrangian level, all couplings are real. CP is spontaneously broken by the vacuum :

The Scalar potential contains various terms which do not have phase dependence, but there are terms with phase dependence:

$$V_{\text{phase}} = (u^2 + \lambda_1 S^* S + \lambda_2 \phi^+ \phi) (S^2 + S^{*2}) + \\ + \lambda_3 (S^4 + S^{*4})$$

There is a range of the parameters of the Higgs potential, where the minimum is:

$$\langle \phi^0 \rangle = \frac{v}{\sqrt{2}} \quad ; \quad \langle S \rangle = \frac{V}{\sqrt{2}} e^{i\theta}$$

Most general  $SU(2) \times U(1) \times Z_4$  invariant 7

Yukawa couplings in the quark sector:

$$\mathcal{L}_Y = -(\bar{u}^0 \bar{d}^0)_L [g_{ij} d_R^j + h_{ij} \tilde{\phi} u_R^j] - \bar{M} (\bar{D}_L^0 D_R^0) \\ - (f_i S + f'_i S^*) \bar{D}_L^i d_R^i + h.c.$$

Quark mass-matrix for down-type quarks:

$$\left[ \begin{array}{cccc} \bar{d}_{1L}^0 & \bar{d}_{2L}^0 & \bar{d}_{3L}^0 & \bar{D}_L^0 \end{array} \right] \left[ \begin{array}{c|c} \text{3x3, real} & \\ \hline m_d & 0 \\ \hline M_1 & M_2 & M_3 & \bar{M} \end{array} \right] \left[ \begin{array}{c} d_{1R}^0 \\ d_{2R}^0 \\ d_{3R}^0 \\ D_R^0 \end{array} \right]$$

$M \rightarrow 4 \times 4$

$$M_j = f_j V e^{i\theta} + f'_j V e^{-i\theta}$$

$$U_L^+ (M M^+) U_L = \begin{bmatrix} d^2 \\ D^2 \end{bmatrix}$$

$$U = \begin{bmatrix} K & R \\ S & T \end{bmatrix}$$

$$K^{-1} \left[ m_d m_d^+ - \frac{m_d M^+ M^- m_d^+}{M M^+ + \bar{M}^2} \right] K = d^2$$

A remarkable feature of the Model :

The phase  $\Theta$  arising from  $\langle S \rangle$  generates a non-trivial CKM phase, provided  $|M_j|$  and  $|\bar{M}|$  are of the same order of magnitude.

$$K^{-1} (m_{\text{eff}}^+ m_{\text{eff}}^-) K = \text{diag.}(m_d^2, m_s^2, m_b^2)$$

$$m_{\text{eff}}^+ m_{\text{eff}}^- = m_d^+ m_d^- - \frac{m_d^+ M^+ M^- m_d^-}{M^+ M^- + \bar{M}^2}$$

$$M_j = (f_i V e^{i\theta} + f'_j V e^{-i\theta})$$

$|M_j|$ ,  $|\bar{M}|$  can naturally be of the same order of magnitude

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Violating  $3 \times 3$  Unitarity :  
 Suppose that one drops the requirement  
 of  $3 \times 3$  unitarity.

How many independent parameters are there  
 in  $V_{CKM}$  ?

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} & \dots \\ V_{cd} & V_{cs} & V_{cb} & \dots \\ V_{td} & V_{ts} & V_{tb} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

9 moduli + 4 rephasing invariant phases  
 $= 13$  parameters

$$\beta \equiv \arg(-V_{cd} V_{tb} V_{cb}^* V_{td}^*)$$

$$\gamma \equiv \arg(-V_{ud} V_{cb} V_{ub}^* V_{cd}^*)$$

$$\chi \equiv \arg(-V_{cb} V_{ts} V_{cs}^* V_{tb}^*)$$

$$\chi' \equiv \arg(-V_{us} V_{cd} V_{ud}^* V_{cs}^*)$$

The SM with 3 generations predicts a series of exact relations among these 13 measurable (in principle) quantities. Violation of any of these exact relations signals the presence of New Physics which may involve deviations of  $3 \times 3$  unitarity or not.

The presence of New Physics contributions to  $B_d - \bar{B}_d$  and  $B_s - \bar{B}_s$  mixings affects the extraction of  $|V_{td}|$ ,  $|V_{ts}|$  from the data, even in the framework of New Physics which respects  $3 \times 3$  unitarity.

Example :

SUSY extensions of the SM

In many of the extensions of the SM  
 the dominant effect of New Physics  
 arises from New contributions to  
 $B_d - \bar{B}_d$  and  $B_s - \bar{B}_s$  mixings, which is  
 convenient to parametrize as :

$$M_{12}^q = (M_{12}^q)^{SM} r_q^2 e^{2i\theta_q}$$

$q = d, s$

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$$\Delta M_{B_d} = r_d^2 (\Delta M_{B_d})^{SM} \rightarrow \begin{array}{l} \text{Affects the extraction} \\ \text{of } |V_{cd}| \text{ from} \\ \Delta M_{B_d} \end{array}$$

$$\Delta M_{B_s} = r_s^2 (\Delta M_{B_s})^{SM} \rightarrow \begin{array}{l} \text{Affects the extraction} \\ \text{of } |V_{ts}| \text{ from} \\ \Delta M_{B_s} \end{array}$$

$$S_{J/\psi K_S} = \sin(2\beta + 2\theta_d) = \sin(2\bar{\beta})$$

$$S_{\gamma^+\gamma^-} = \sin(2\alpha - 2\theta_d) = \sin(2\bar{\alpha})$$

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## How to detect the presence of New Physics?

Answer: Use the exact relations predicted by the SM.

$$(db) \quad |V_{ub}| = \frac{|V_{cd}||V_{cb}|}{|V_{ud}|} \frac{\sin\beta}{\sin(\delta+\beta)} \quad \xrightarrow{\text{extraction of } \theta_d}$$

$$(sb) \quad \sin\chi = \frac{|V_{us}||V_{ub}|}{|V_{cs}||V_{cb}|} \sin(\delta-\chi+\chi') \quad \xrightarrow{\text{extraction of } \theta_s}$$

$$\sin\chi = \frac{|V_{us}||V_{cd}||V_{cb}|}{|V_{ts}||V_{tb}||V_{ud}|} \frac{\sin\beta \sin(\delta+\chi')}{\sin(\delta+\beta)}$$

well approximated by :

$$\boxed{\sin\chi \approx \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{\sin\beta \sin\delta}{\sin(\delta+\beta)}}$$

Silva, Wolfenstein

$$\boxed{\sin\chi = \frac{|V_{td}|}{|V_{ts}|} \frac{|V_{cd}|}{|V_{cs}|} \sin\beta}$$

$\Rightarrow$  If either  $(\delta, \chi)$  or  $\left(\left(\frac{\Delta M_{Bd}}{\Delta M_{Bs}}\right), \chi\right)$

are measured with some precision, one has novel stringent tests of the SM where contribution of New Physics can be large

Extraction of  $\theta_d$  :

$$\tan \theta_d = \frac{R_u \sin(\delta + \bar{\beta}) - \sin \bar{\beta}}{\cos \bar{\beta} - R_u \cos(\delta + \bar{\beta})}$$

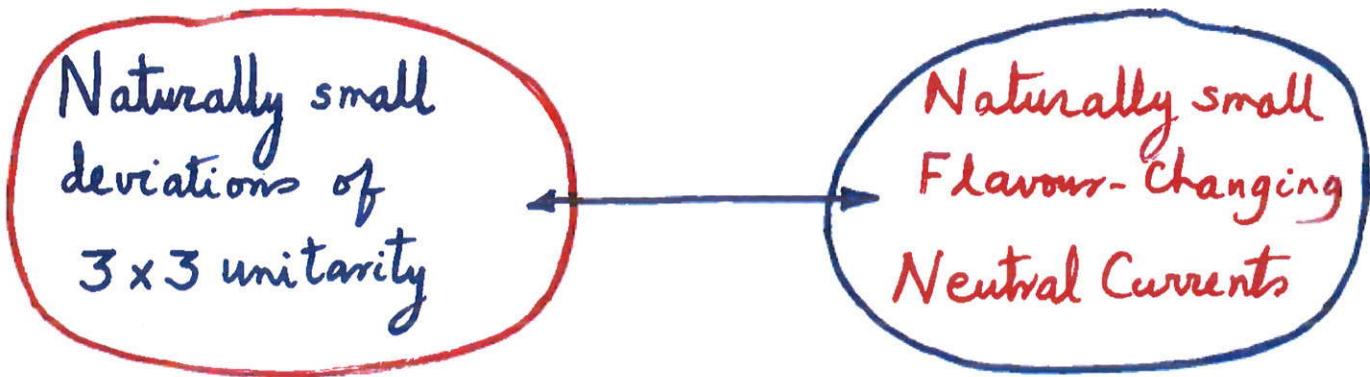
where

$$R_u = \frac{|V_{ud}| |V_{ub}|}{|V_{cd}| |V_{cb}|}$$

Extraction of  $\theta_s$  :

$$\tan \theta_s = \frac{\sin \bar{\chi} - C \sin(\delta - \bar{\chi})}{C \cos(\delta - \bar{\chi}) + \cos \bar{\chi}}$$

$$C = \frac{|V_{us}| |V_{ub}|}{|V_{cs}| |V_{cb}|}$$



For definiteness consider the case of one isosinglet  $Q = -1/3$  quark.

$(V_{CKM})_{3 \times 3}$

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} (\bar{u} \bar{c} \bar{t})_L \gamma^\mu \begin{bmatrix} K & R \\ R^+ & R^+ \end{bmatrix} \begin{bmatrix} d \\ s \\ b \\ D \end{bmatrix} W_\mu^+$$

$$\mathcal{L}_Z = -\frac{g}{2 \cos \theta_W} \left\{ (\bar{u} \bar{c} \bar{t})_L \gamma^\mu \begin{bmatrix} u \\ c \\ t \end{bmatrix}_L - \right.$$

$$\left. - [\bar{d} \bar{s} \bar{b} \bar{D}] \begin{bmatrix} K^+ K & K^+ R \\ R^+ K & R^+ R \end{bmatrix} \begin{bmatrix} d \\ s \\ b \\ D \end{bmatrix} - \sin^2 \theta_W J_{dm}^\mu \right\} Z,$$

From unitarity of  $U_L \equiv \begin{bmatrix} K & R \\ S & T \end{bmatrix}$  one obtains:

$$K^+ K + S^+ S = 0$$

but  $S \approx -\frac{M' m_d K}{M^2} \rightarrow O(\frac{m}{M})$ ;  $K^+ K = 0 - O(\frac{m^2}{M^2})$

Similar analysis can be done for  
 $Q = 2/3$  vector like quarks

Without loss of generality, one may choose to work in a WB where the down quarks mass matrix is diagonal. Let  $U$  be the  $4 \times 4$  unitary basis which diagonalizes the up-quark mass matrix. The quark mixing matrix consists of the first 3 columns of  $U$

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ V_{Td} & V_{Ts} & V_{Tb} \end{bmatrix}; \quad \text{From orthogonality of the 2nd and 3rd column, one obtains:}$$

$$\sin \chi = \frac{V_{ub} V_{us}}{V_{cb} V_{cs}} \sin (\gamma - \chi + \chi') +$$

$$+ \frac{|V_{Tb}| |V_{Ts}|}{|V_{cb}| |V_{cs}|} \sin (\sigma - \chi)$$

$$\chi \equiv \beta_s$$

$$\sigma = \arg (V_{Ts} V_{cb} V_{Tb}^* V_{cs}^*)$$

$$\text{If } V_{Tb} \approx O(1) \\ V_{Ts} \approx O(1^2) \\ \sigma \approx O(1); \boxed{\chi = \lambda}$$

## Leptonic Sector

Recall that the relevant fields transform under  $\mathbb{Z}_4$  as :

$$\psi_L^o \rightarrow i\psi_L^o ; e_R^o \rightarrow ie_R^o ; \nu_R^o \rightarrow i\nu_R^o$$

Leptonic Yukawa terms :

$$\begin{aligned} \mathcal{L}_L = & \overline{\psi}_L^o G_L \phi e_R^o + \overline{\psi}_L^o G_\nu \tilde{\phi} e_R^o + \\ & + \frac{1}{2} \nu_R^{oT} C (f_\nu S + f'_\nu S^*) \nu_R^o + h.c. \end{aligned}$$

Leptonic mass matrices :

$$M_\nu = \begin{bmatrix} 0 & m \\ m^T & M \end{bmatrix} ; \quad m_L = \frac{v}{\sqrt{2}} G_L$$

$$m = \frac{v}{\sqrt{2}} G_\nu$$

$$M = \frac{v}{\sqrt{2}} f_\nu^+ \cos \alpha + i f_\nu^- \sin \alpha$$

$$f_\nu^\pm = f_\nu \pm f'_\nu$$

# Leptonic Mixing

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In the weak-basis where  $m_L$  is diagonal, real, the light neutrino masses and the low energy leptonic mixing are obtained from the diagonalization of the effective neutrino mass matrix:

$$m_{\text{eff}} = -m \frac{1}{M} m^T$$

$$-K^T m \frac{1}{M} K^* = d_\nu$$

$m$  is real, but since  $M$  is a generic complex matrix,  $m_{\text{eff}}$  is also a generic complex matrix. Therefore  $K$  will have three complex phases, one Dirac-type, two Majorana-type. It can be shown that one has viable leptogenesis.

# Conclusions

Vector-like quarks provide a very interesting scenario for New Physics.

- 1) They provide a consistent framework where there are naturally small violations of  $3 \times 3$  unitarity in  $V_{CKM}$ , leading to naturally small FCNC.
- 2) They provide a framework for having a common origin for all CP violations.
- 3) They play a crucial rôle in the simplest model of spontaneous CP violation where a complex  $V_{CKM}$  is generated
- 4) They provide a simple solution to the strong CP problem.