Dynamical Electroweak Symmetry Breaking with a Heavy 4th Generation

3rd Workshop on Beyond 3 Generation Standard Model
— Under the light of the initial LHC results

Oct 24th, 2011, Bogazici University, Istanbul

Chi Xiong

A heavy 4th generation

- drives the Yukawa couplings to the strong region $\Rightarrow$ binding force for condensates
- brings the cutoff down to $\Lambda \sim \text{TeV} \Rightarrow$ hierarchy problem
- leads to a quasi-fixed point around Λ at two-loop level $\Rightarrow$ Landau pole, triviality
- suggests the restoration of scale invariance above $\Lambda \Rightarrow$ new conformal theories?
- arXiv:0911.3890, 0911.3892, 1012.4479 (P.Q. Hung, CX)

RGE in SM4

- We study the evolutions of gauge and Higgs couplings (quartic and Yukawa) in SM4
- At two-loop level ($\overline{MS}$ scheme), $16\pi^2 \frac{dy_i}{dt} = \beta_{y_i}$
- e.g. the Higgs quartic coupling

$$\begin{aligned}\beta_\lambda = \quad & 24\lambda^2 + 4\lambda(3g_t^2 + 6g_q^2 + 2g_l^2 - 2.25g_2^2 - 0.45g_1^2) \\ & -12(3g_t^4 + 6g_q^4 + 2g_l^4) + (16\pi^2)^{-1}[180g_t^6 \\ & +288g_q^6 + 96g_l^6 - (3g_t^4 + 6g_q^4 + 2g_l^4 - 80g_3^2(g_t^2 \\ & +2g_q^2))\lambda - 6\lambda^2(24g_t^2 + 48g_q^2 + 16g_l^2) - 312\lambda^3 \\ & -192g_3^2(g_t^4 + 2g_q^4)] + ...\end{aligned}$$

- other couplings $\beta_{y_q}, \beta_{y_l}, \beta_{y_t}, \beta_{g_i, i=1,2,3}$ ... (Machacek and Vaughn, 1983)

Zeros of β-functions

These RGEs can be integrated numerically, but first we search for roots of $\beta_{y_i} = 0$ with fixed gauge couplings.

g_3^2	g_2^2	g_1^2		λ	g_t^2	g_q^2	g_l^2
1.478	0.425	0.213		17.561	31.407	52.298	56.583
1.225	0.413	0.217		17.457	31.200	52.185	55.664
1.003	0.404	0.223		17.376	31.073	52.147	54.934
0.902	0.396	0.226		17.339	31.014	52.126	54.604
0.815	0.386	0.230		17.308	30.963	52.107	54.321
0.652	0.366	0.239		17.249	30.866	52.066	53.792
0.565	0.354	0.245		17.218	30.814	52.042	53.511
0.304	0.284	0.304		17.125	30.655	51.966	52.661
0.999	0.666	0.333		17.339	31.039	52.089	54.817
0.500	0.500	0.500		17.164	30.754	51.990	53.152
0.000	0.000	0.000		17.059	30.488	51.902	51.902

Zeros of β-functions

The fixed point values of the quartic and Yukawa couplings are approximately

$$\lambda^*/(4\pi)^2 \approx 0.11,\ g_t^{2*}/(4\pi)^2 \approx 0.2,\ g_q^{2*}/(4\pi)^2 \approx 0.33,\ g_l^{2*}/(4\pi)^2 \approx 0.34$$

The gauge couplings contribute only small fluctuations. These correspond to naive $\overline{MS}$ masses (using $\overline{m}_H = v\sqrt{2\lambda}, \overline{m}_f = v g_f/\sqrt{2},\ v = 246$ GeV)

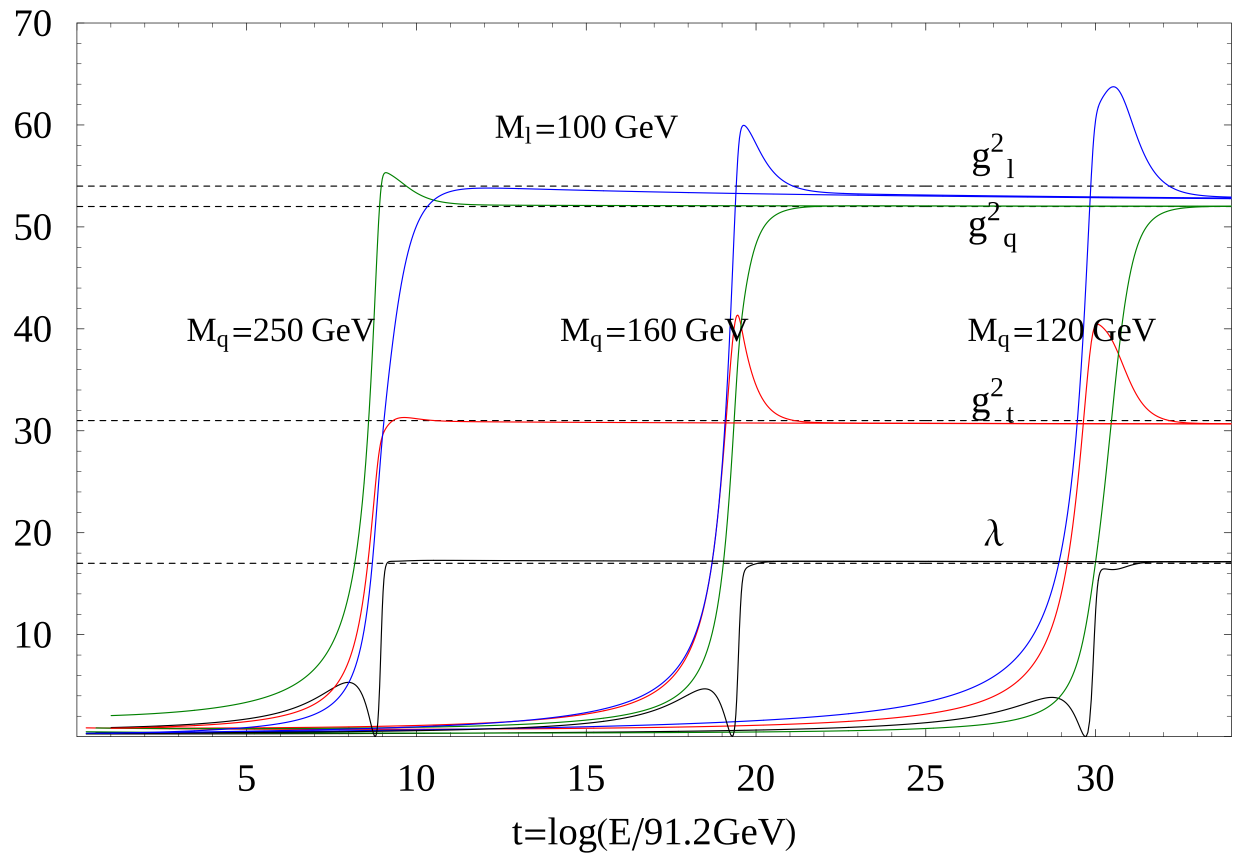
$$\overline{m}_H^* = 1.44 \text{ TeV},\ \overline{m}_t^* = 0.97 \text{ TeV},\ \overline{m}_q^* = 1.26 \text{ TeV},\ \overline{m}_l^* = 1.28 \text{ TeV}.$$

Questions:

- Can this (quasi)fixed point be reached?
- If yes, at what energy scale?

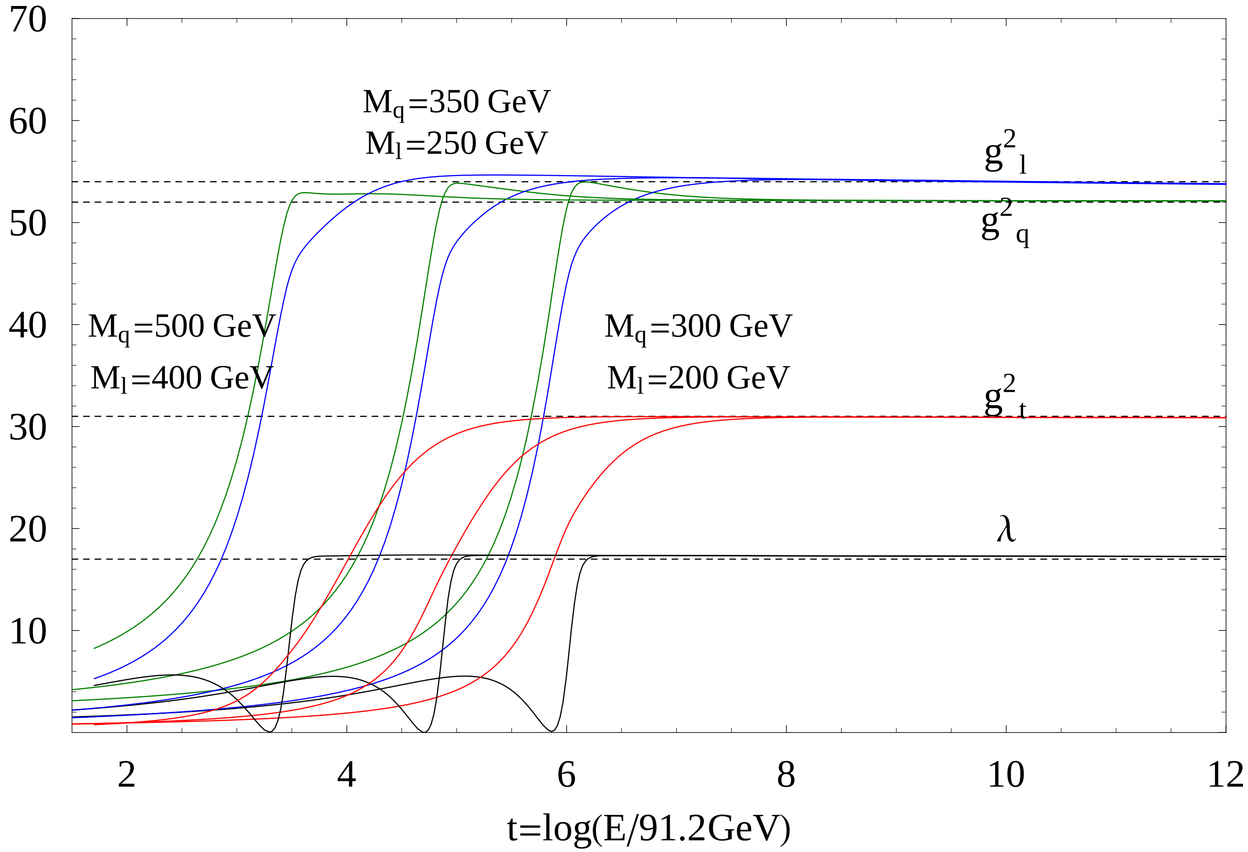
RG running of Higgs couplings

The RG running of Higgs couplings (quartic and Yukawa), light mass cases ($M_q = 120GeV \sim 250GeV$)



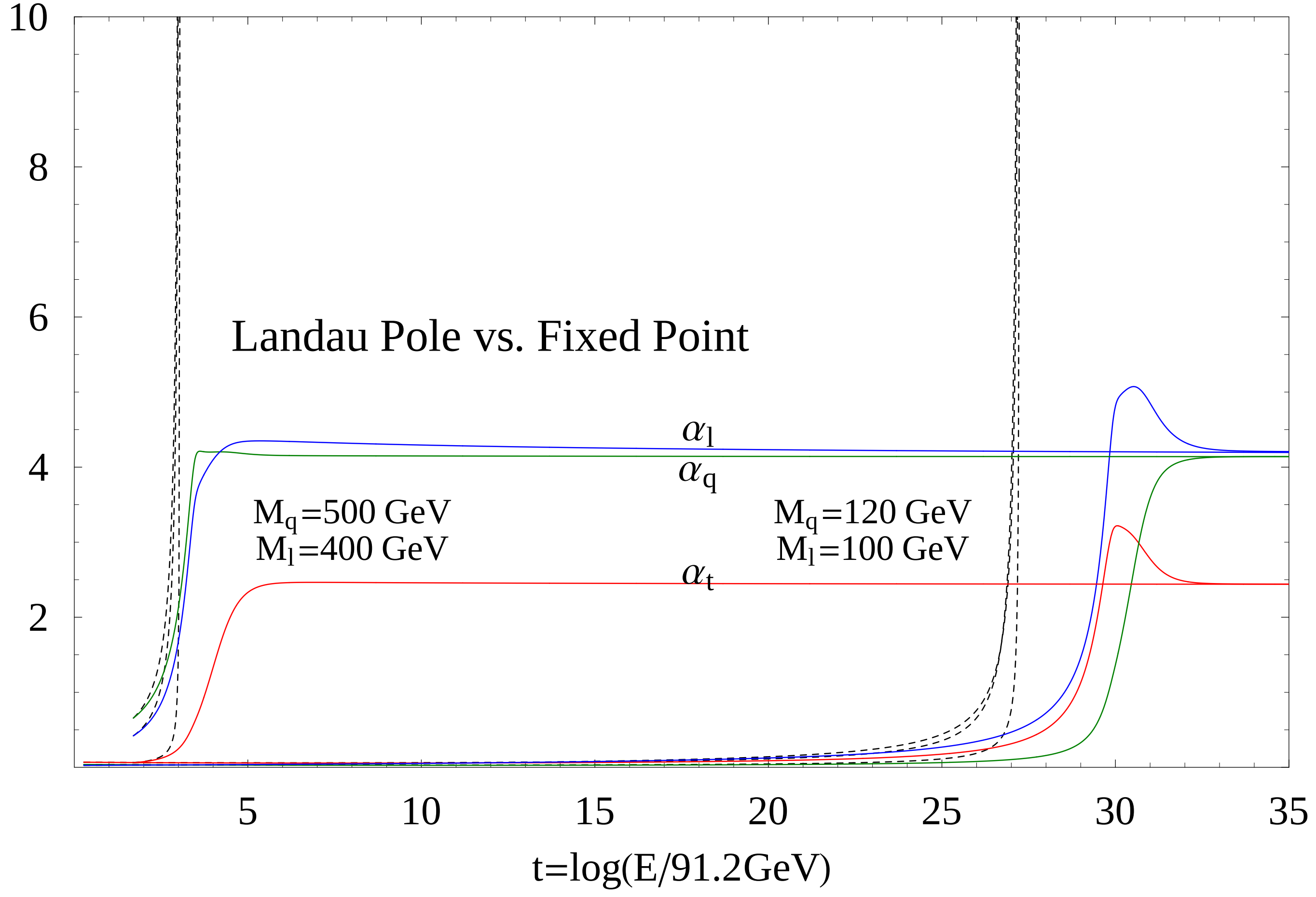
RG running of Higgs couplings

The RG running of Higgs couplings (quartic and Yukawa), heavy mass case ($M_q = 300GeV \sim 500GeV$)



Landau Pole vs. Fixed Point

Compare 1-loop and 2-loop results



RG running of Higgs couplings

From the numerical calculations, we see that

- The evolution of Higgs couplings run into a quasi-fixed point at some scale Λ_{FP}
- Λ_{FP} decreases when the mass the 4th generation increases

An interesting thing is, by increasing their masses just about 3 times, the 4th generation brings Λ_{FP} from $\approx 10^{16}$ GeV down to few TeV.

The existence of a quasi-fixed point $\Rightarrow$ the triviality problem; The physical consequences of shifting the scale Λ_{FP} down to TeV level $\Rightarrow$ provide an alternative solution to the hierarchy problem.

Comments

- For the RGEs, the expansion parameters are $g_t^{2*}/16\pi^2 \approx 0.2,\ \ g_q^{2*}/16\pi^2 \approx 0.33,\ \ g_q^{2*}/16\pi^2 \approx 0.34,\ \ \lambda^*/16\pi^2 \approx 0.11$.
- These represent strong quartic and Yukawa couplings. The exact location of Λ_{FP} and the values of Higgs couplings at Λ_{FP} should be studied non-perturbatively, e.g. lattice methods
- $\beta^{2-loop}(g^*) = 0$ may give a clue in finding $\beta(g^*) = 0$ where the scale invariance is restored at some energy
- $\frac{\alpha}{\pi}$ or $\frac{\alpha}{4\pi}$? An $\mathcal{O}(1)$ expansion parameter? We have seen similar situations before –Wilson-Fisher ϵ-expansion, $g_4 = 16\pi^2\epsilon/3$.

Wilson-Fisher ϵ-expansion

$$\mu \frac{d}{d\mu} g_4(\mu) = -\epsilon\, g_4(\mu) + \frac{3g_4^2(\mu)}{16\pi^2} + \mathcal{O}(g_4^3(\mu))$$
$$\mu \frac{d}{d\mu} g_2(\mu) = g_2(\mu)[-2 + \frac{g_4(\mu)}{16\pi^2} + \mathcal{O}(g_4(\mu))]$$

where $\epsilon = 4 - d$ and g_2 and g_4 come from terms $g_2\phi^2/2, g_4\phi^4/4!$ respectively.
Solving $\beta(g_4), \beta(g_2) = 0$ equations, one finds a non-trivial fixed point at

$$g_4^* = \frac{16\pi^2\epsilon}{3}, \quad g_2^* = 0.$$

For d=3, it corresponds to $g_4^* = 16\pi^2/3 \approx 52.64$ or $g_4^*/16\pi^2 = 1/3$

Wilson-Fisher ϵ-expansion

The critical exponent ν is then given by the ϵ-expansion

$$\nu = 1/2 + \epsilon/12 + 7\epsilon^2/162 - 0.01904\epsilon^3 + \mathcal{O}(\epsilon^4)$$

At 1-loop level, $\nu = 0.58$, at 2-loop level, $\nu = 0.63$, at 3-loop level, $\nu = 0.61$ while the experimental value is $\nu = 0.63$

–“ It is fortunate though still somewhat mysterious that an expansion in powers of 1 should work so well.” –(Weinberg, QFT II)

We do not expect such a precise calculation, but hopefully inclusion of higher order terms will not shift the location and the values of the quasi-fixed point by an order of magnitude.

Bound States/Condensates

The Yukawa couplings become strong around Λ_{FP}.
Can the 4th generation fermions form bound states or condensates by the Yukawa coupling?
We perform a non-relativistic analysis at quantum mechanics level, with a Higgs-exchange potential

$$V(r) = -\alpha_Y(r)\frac{e^{-m_H(r)r}}{r}$$

where $\alpha_Y = m_1 m_2/4\pi v^2$.
The possibility of forming bound states is characterized by

$$K_f = \frac{g_f^3}{16\pi\sqrt{\lambda}}.$$

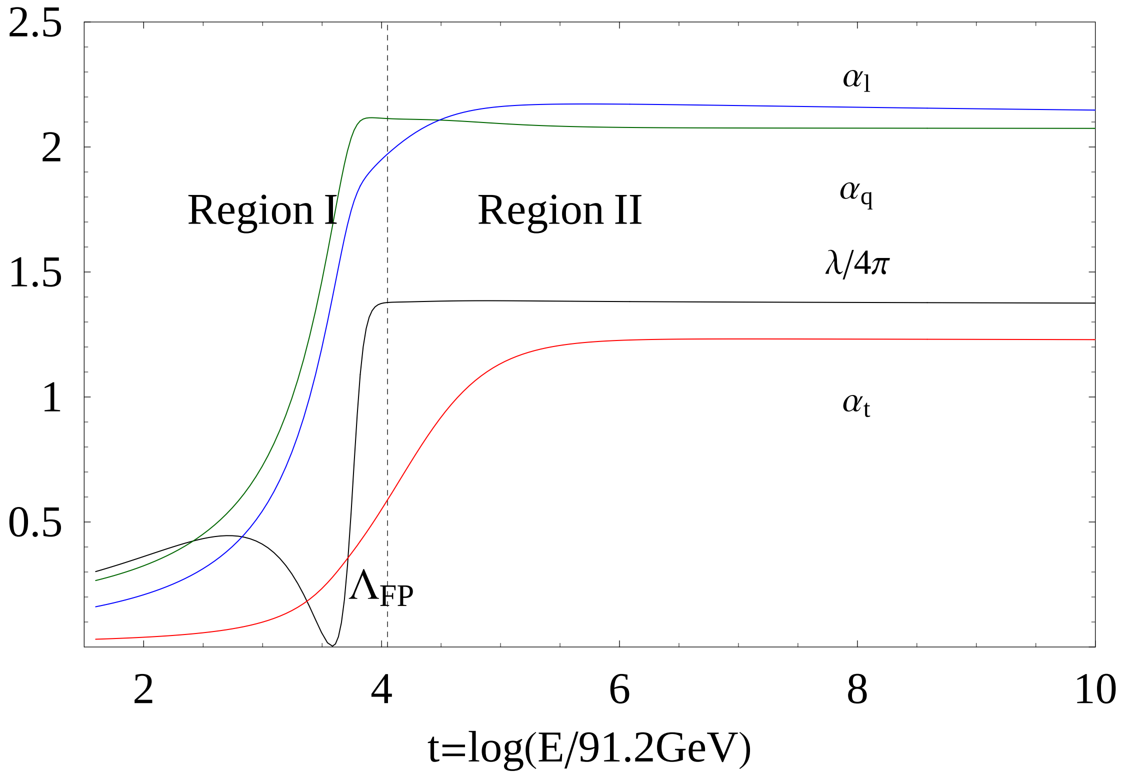
Bound States/Condensates

The criteria is

- $K_f > 2$ (variational method)
- $\boxed{K_f > 1.68}$ (numerical method)

If we use the fixed-point values of the quartic and Yukawa couplings, we find that the 4th generation may form loosely bound state, while the top quark cannot. $K_q = 1.82, \;\; K_l = 1.92, \;\; K_t = 0.82$
An interesting region is around the "dip", i.e. $\lambda \approx 0$, where the Yukawa potential becomes a strong Coulomb-like potential. $\longrightarrow$ formation of condensates (Rafelski, Fulcher and Klein, 1978).

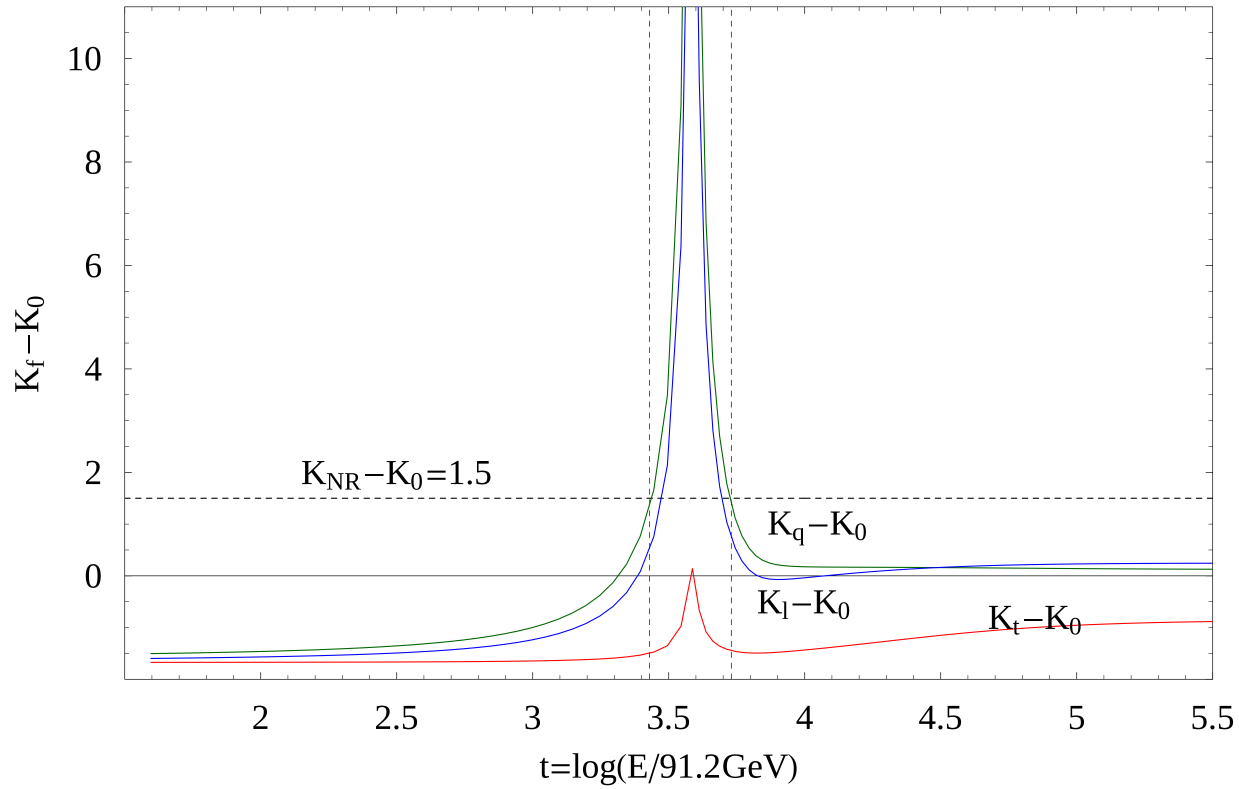
Bound States/Condensates



Region I: Condensates v.s. Region II: Fixed Point

Note: Neither technicolor nor other unkown interactions are introduced for condensates.

Bound States/Condensates



($m_q = 450$ GeV and $m_l = 350$ GeV) $K_f - K_0$ with $K_f = g_f^3/16\pi\sqrt{\lambda}$ and $K_0 = 1.68$. The horizontal dotted line indicates an estimate of K_f where the non-relativistic method is still applicable and the vertical dotted lines enclose the region where a fully relativistic approach is needed.

Schwinger-Dyson Equation

- In SM4 the Yukawa couplings become strong around TeV
- The perturbative approach becomes less reliable when approaching Λ_{FP}
- Lattice? Nielson-Ninomiya no-go theorem (chiral gauge theory) mirror fermion
- dispersion relation? $\pi - N$ system
- We use Schwinger-Dyson approach (Gap Equation, mean field theory, Hartree-Fock approximation,...)

Schwinger-Dyson Equation

Consider Yukawa couplings in SM4 (truncated to only the 4th generation)

$$\mathcal{L}_Y = -g_{b'}\ \bar{q}_L \Phi\, b'_R - g_{t'}\ \bar{q}_L \widetilde{\Phi}\, t'_R + h.c.$$

$\widetilde{\Phi} = i\tau_2 \Phi^*$, $q_L = (t', b')_L$ as usually defined in the SM.

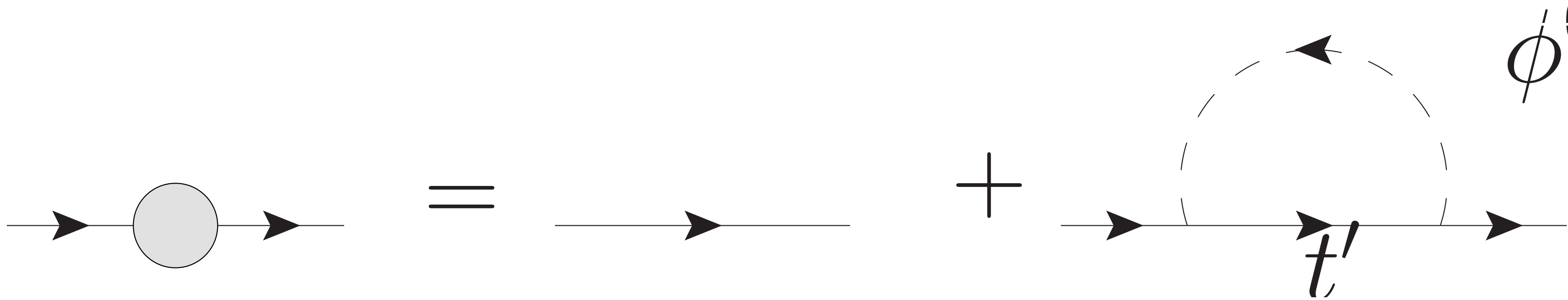


Figure 1: Graphic representation of the Schwinger-Dyson equation for the quark self-energy (quenched approximation)

Gap Equation

- For simplicity we only consider the 4th generation quarks. From the SDE the quark self energy satisfies

$$\Sigma_{4Q}(p) = \frac{+2g^2}{(2\pi)^4} \int d^4q \frac{1}{(p-q)^2} \frac{\Sigma_{4Q}(q)}{q^2 + \Sigma_{4Q}^2(q)}$$

- which can be converted to a differential equation

$$\Box \Sigma_{4Q}(p) = -(\frac{\alpha}{\alpha_c}) \frac{\Sigma_{4Q}(q)}{q^2 + \Sigma_{4Q}^2(q)}$$

where $\alpha_c = \pi/2$

- compared with $\alpha_c = \pi/3$ in strong QED (Fukuda & Kugo, Bardeen, Leung & Love)

Gap Equation

- Early work: K. Johnson, M. Baker and R. Willey. PR136(1964), 163(1967)
- Numerical analysis: Fukuda and Kugo, Nucl. Phys. B117(1976)
- Analytic analysis: C.Leung, S. Love and W. Bardeem, Nucl. Phys. B273(1986)
- The SDEs are similar, but the boundary conditions are different

$$\lim_{p\to 0} p^4 \frac{d\Sigma}{dp^2} = 0$$

$$\lim_{p\to \Lambda} p^2 \frac{d\Sigma}{dp^2} + \Sigma(p) = 0$$

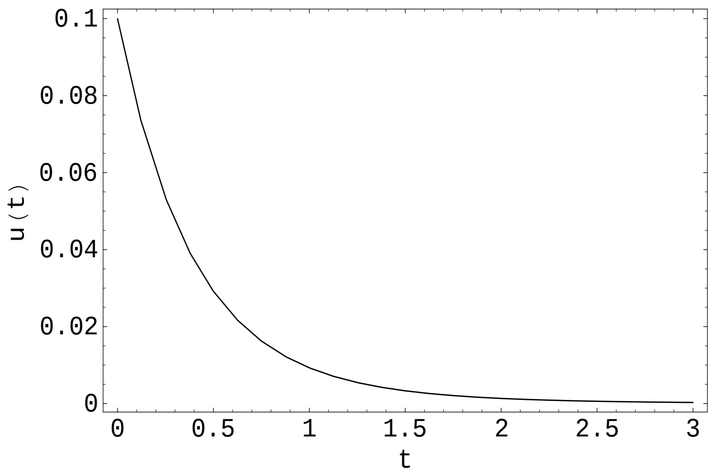
Solutions

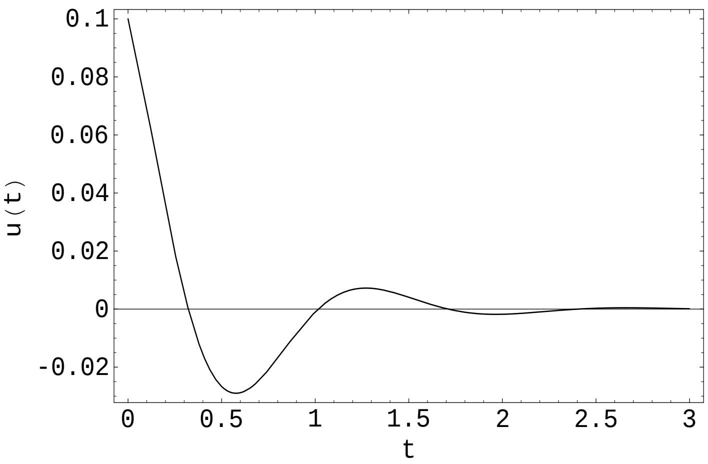
- asymptotic solutions in the weak and strong coupling regions:

$$\Sigma_{4Q}(p) \sim p^{-1+\sqrt{1-\frac{\alpha}{\alpha_c}}}, \qquad \text{for } \alpha \leq \alpha_c$$

$$\Sigma_{4Q}(p) \sim p^{-1} \sin[\sqrt{\frac{\alpha}{\alpha_c} - 1}(\ln p + \delta)], \quad \text{for } \alpha > \alpha_c$$

- Numerical Solutions





Condensates

- One can compute the condensates

$$\langle \bar{t'}_L t'_R \rangle = \langle \bar{b'}_L b'_R \rangle = -\frac{3}{4\pi^4} \int d^4q \frac{\Sigma_{4Q}(q)}{q^2 + \Sigma_{4Q}^2(q)}$$

- Self energy, condensates and induced scalar mass depend on the cutoff and the Yukawa couplings as

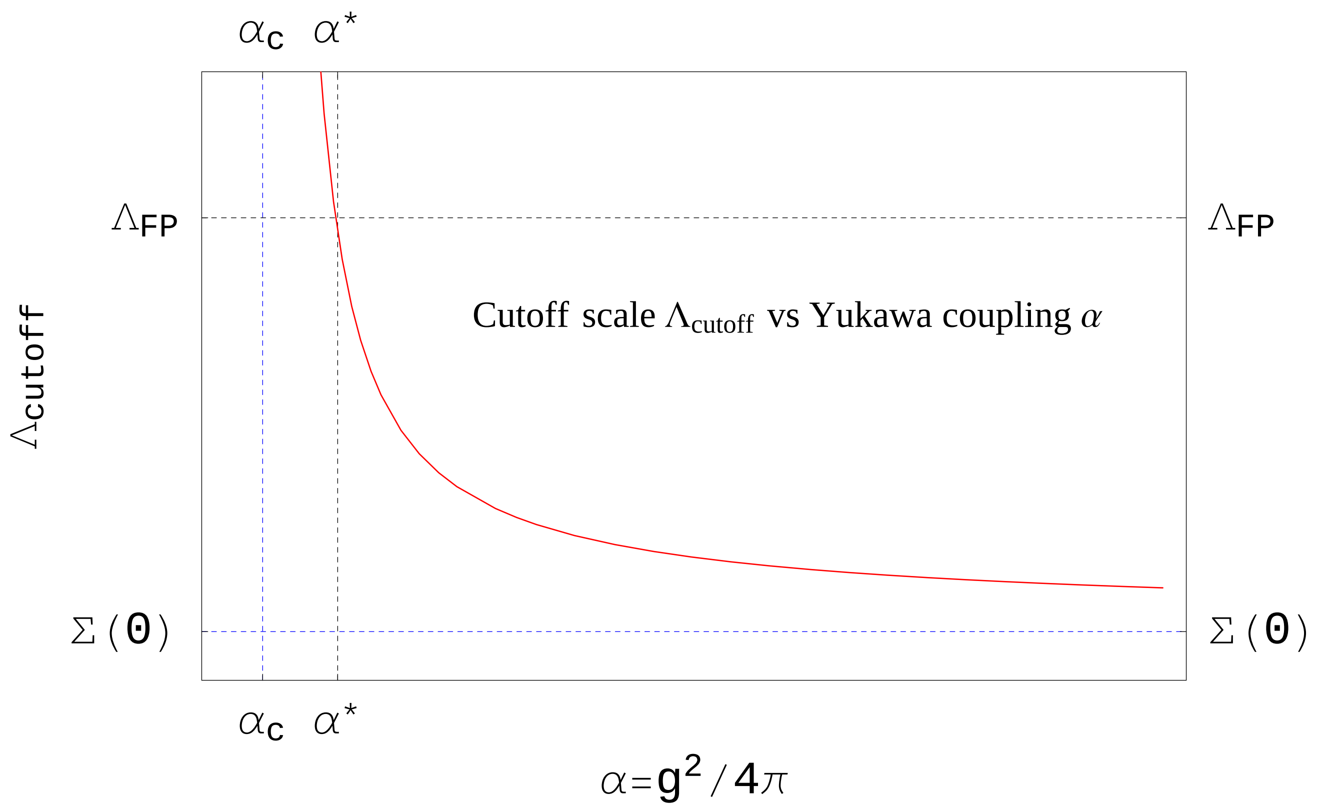
$$\Sigma(0) \sim \Lambda\, e^{\frac{-\pi}{\sqrt{\alpha/\alpha_c - 1}} + 1}, \quad \langle \bar{t}' t' \rangle \sim -\Lambda^3\, e^{\frac{-2\pi}{\sqrt{\alpha/\alpha_c - 1}}}, \quad \delta m_\phi^2 \sim -\Lambda^2\, e^{\frac{-\pi}{\sqrt{\alpha/\alpha_c -}}}$$

- The hierarchy problem appears in a different form. To avoid fine-tuning, one has to choose a cutoff at TeV scale.

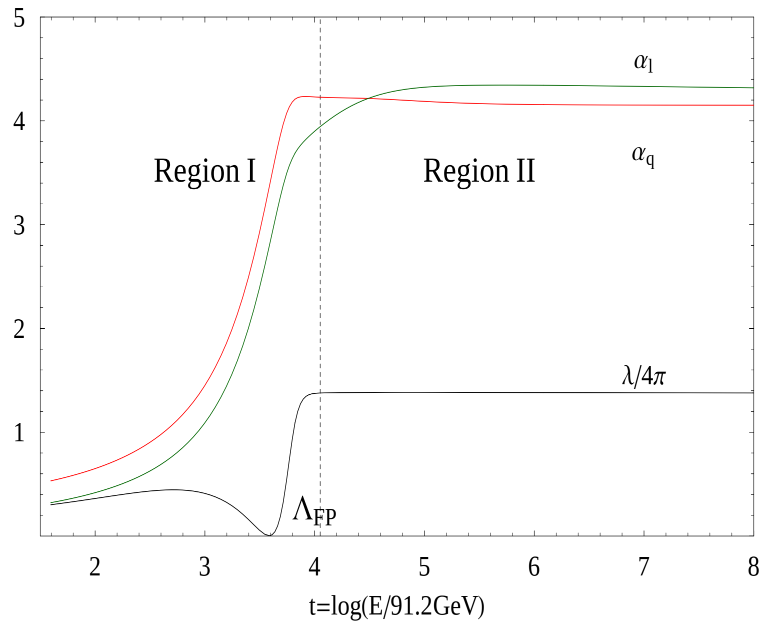
Cutoff vs. Yukawa couplings

$\frac{\alpha}{\alpha_c}$	1.0	1.1	1.2	1.4	1.8	2.2	2.6	3.0	3.4
$\frac{\Lambda}{\Sigma(0)}$	∞	7590	414	52.8	12.3	6.47	4.41	3.39	2.80
Λ (GeV)	∞	10^6	10^5	10^4	6167	3237	2205	1696	1398

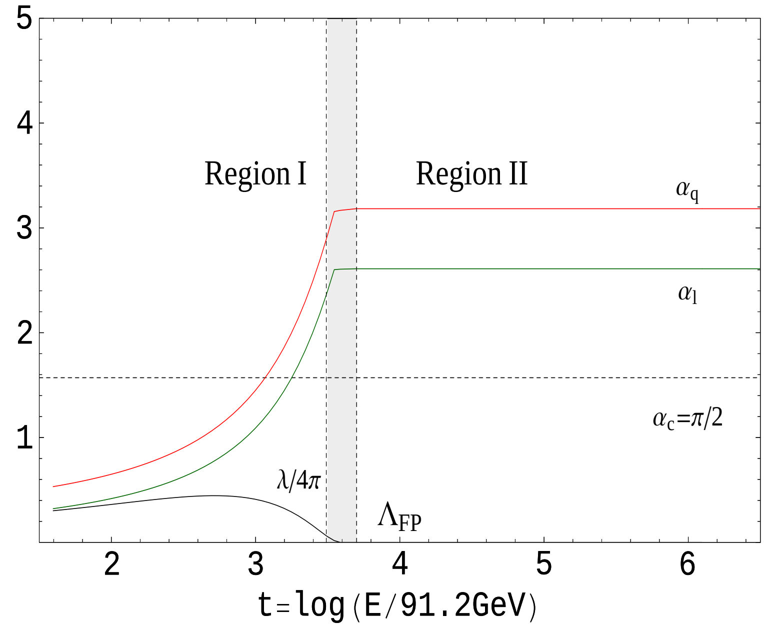
Table 1: The relation between the cutoff scale and the Yukawa coupling.



RGE + SDE



α_l
α_q
Region I
Region II
$\lambda/4\pi$
Λ_{FP}
t=log(E/91.2GeV)



Region I
Region II
α_q
α_l
$\alpha_c=\pi/2$
$\lambda/4\pi$
Λ_{FP}
t=log(E/91.2GeV)

Multiple Higgs doublets

- We might have three Higgs doublets: One fundamental, two composite

$$H_1 = (\pi^+, \pi^-, \pi^0, \sigma)$$

$$H_2 = (\bar{b}' t', \bar{t}' b', \bar{t}' t' - \bar{b}' b', \bar{t}' t' + \bar{b}' b')$$

$$H_3 = (\bar{\tau}' \nu'_\tau, \bar{\nu}'_\tau \tau', \bar{\nu}'_\tau \nu'_\tau - \bar{\tau}' \tau', \bar{\nu}'_\tau \nu'_\tau + \bar{\tau}' \tau')$$

- Their effective mass terms are described by

$$(H_1^\dagger, H_2^\dagger, H_3^\dagger)\, \mathcal{M}\, (H_1, H_2, H_3)^T$$

- The existence of the Nambu-Goldstone bosons leads to $\det \mathcal{M}^{+,-,0} = 0$ –modified gap equation

Summary

- At 2-loop level, the Yukawa sector of SM4 has a non-trivial quasi- fixed point
- A heavy 4th generation ($m_q > 400$ GeV) can set Λ_{FP} to be order of TeV
- A heavy 4th generation also drives Yukawa couplings become strong at TeV scale
- Bound states/ condensates of the 4th generation can be formed by exchanging Higgs bosons
- The perturbative (RGE) and non-perturbative (SDE) approaches lead to consistent results.