Recent developments in lattice strong-Yukawa models

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The collaboration

- Germany
  - NIC, DESY Zeuthen and Humboldt University Berlin
    John Bulava ($\rightarrow$ CERN), Philipp Gerhold, Karl Jansen, Attila Nagy.

- Japan
  - Kobayashi-Maskawa Institute, Nagoya University
    Kei-Ichi Nagai.

- Taiwan
  - National Chiao-Tung University, Hsinchu
    C.-J.David Lin, Kenji Ogawa.
  - National Taiwan University, Taipei
    George W.-S. Hou, Bastian Knippschild (Mainz $\rightarrow$), Brian Smigielski ($\rightarrow$ U.S.).

Motivation

Heavy fermions beyond SM3?

- Heavy extra generation of fermions may
  - enhance CP violation.

    G.W.S. Hou, 2008

  - offer an alternative way to break EW symmetry dynamically and induces bound states to unitarise WW scattering.

    B. Holdom, 2007

  - UV stablise the SM.

    P.Q. Hung, C. Xiong, 2009

- (Surprisingly) little is known for strong Yukawa theory.

Outline

- Goals, general issues and recent developments.
- Simultation setup.
- The phase structure.
- Exploratory numerical studies.
  - VEV.
  - Susceptibility and critical exponents.
- Future plan.

Targets for the bare strong-Yukawa regime

- The nature of the phase transitions.

  $\Rightarrow$ Connection to the continuum world (next slide).

- Possible bound states.

  $\Rightarrow$ Computation of the spectrum.

- Possible new mechanism for dynamical symmetry breaking.

  $\Rightarrow$ Heavy scalar with fermion condensate?

General issues and strategy

- The triviality problem.

  $\Rightarrow$ Cannot take the cut-off to ∞.

  $\Rightarrow$ Cannot take the lattice spacing to zero.

- Look for 2nd-order phase transitions via "scanning simulations".

  $\Rightarrow$ $\xi \to \infty$.

- Problem: Finite-volume effects.

  $\Rightarrow$ Severe near the critical points since $L = \widehat{L}a$.

  $\Rightarrow$ Phase transitions are washed out.

- Chiral fermions required. Challenging to simulate chiral gauge theories.

New ingredients in current work

- Previous studies (circa 1990):
  - Use fermions without exact chiral symmetry.

    $\Rightarrow$ Ambiguity in defining chiral fermions.
  - Small ($\sim 8^3 \times 16$) volumes and no $L \to \infty$ limit taken.
- Current new-generation simulations:
  - Use the overlap fermion (exact chiral symmetry).
  - Several large volumes and $L \to \infty$ limit taken.

    $\Rightarrow$ Test finite-size scaling behaviour.

    $\Rightarrow$ Determine the order of the phase transition.

    $\Rightarrow$ Look for continuum physics reliably.

Reminder: Notaion for scalar field theory

- The discretised scalar action ($a = 1$)

$$S_\varphi = -\sum_{x,\mu} \varphi_x^\alpha \varphi_{x+\hat{\mu}}^\alpha + \sum_x \left[\frac{1}{2}(2d + m_0^2)\varphi_x^\alpha \varphi_x^\alpha + \frac{1}{4}\lambda_0 (\varphi_x^\alpha \varphi_x^\alpha)^2 \right].$$

- $\varphi = \sqrt{2\kappa}\phi, \quad m_0^2 = \frac{1-2\hat{\lambda}}{\kappa}, \quad \lambda_0 = \frac{\hat{\lambda}}{\kappa^2}$

$$S_\phi = -2\kappa \sum_{x,\mu} \phi_x^\alpha \phi_{x+\hat{\mu}}^\alpha + \sum_x \left[\phi_x^\alpha \phi_x^\alpha + \hat{\lambda}(\phi_x^\alpha \phi_x^\alpha - 1)^2 \right],$$

$$Z_\phi = \int \prod_{x,\alpha} d\phi_x^\alpha \ \exp(-S_\phi) = \int \prod_{x,\alpha} d\mu(\phi_x^\alpha) \ \exp\left(2\kappa \sum_{x,\mu} \phi_x^\alpha \phi_{x+\hat{\mu}}^\alpha \right),$$

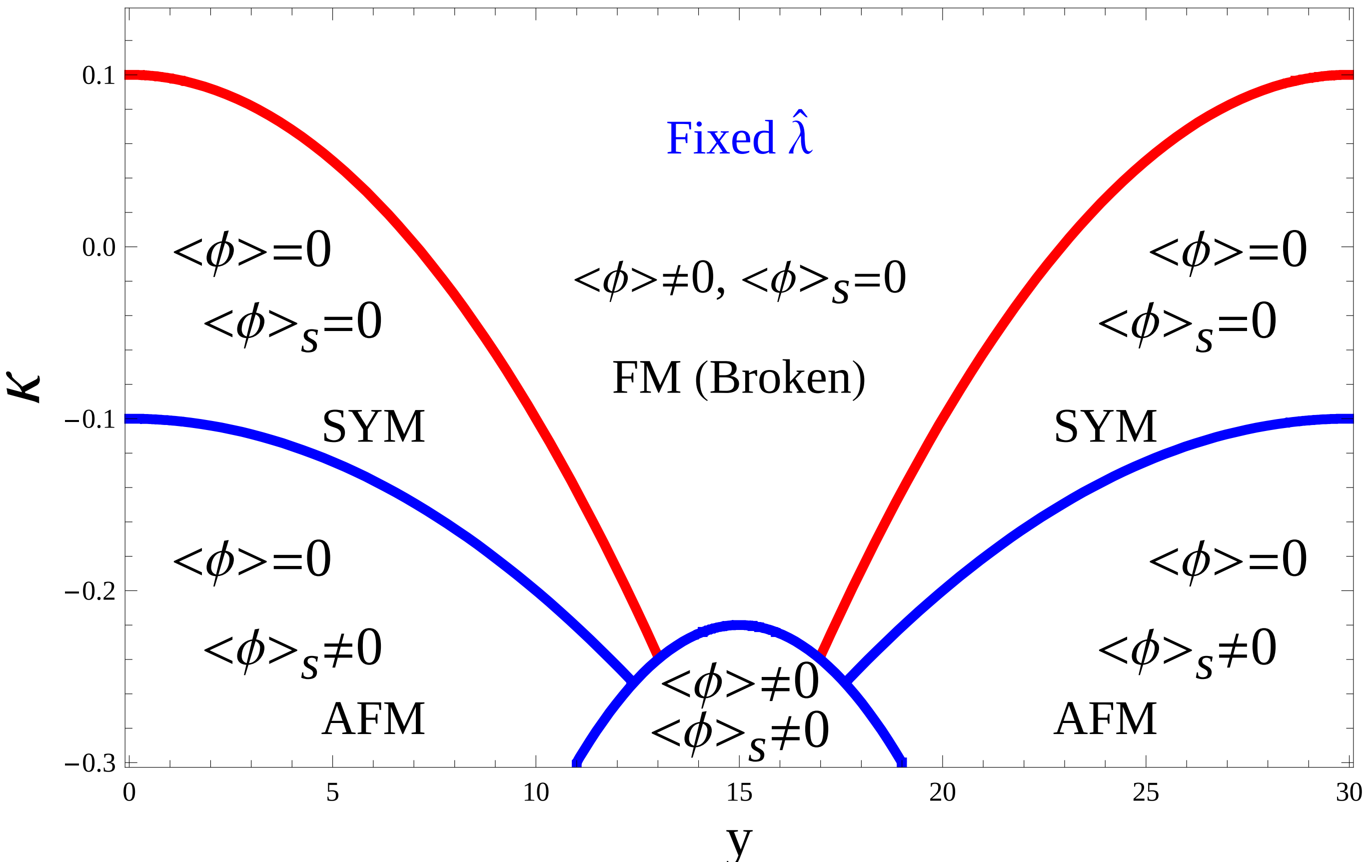
$$d\mu(\phi_x^\alpha) = d\phi_x^\alpha \ \exp\left[-\phi_x^\alpha \phi_x^\alpha - \hat{\lambda}(\phi_x^\alpha \phi_x^\alpha - 1)^2 \right].$$

- “staggered symmetry”: $\kappa \to -\kappa$ and $\phi_x^\alpha \to (-1)^{x_1 + x_2 + \ldots + x_d} \phi_x^\alpha$.

Fermions and the Yukawa couplings

- Use the overlap Dirac operator with exact lattice chiral symmetry.
- The Yukawa terms $S_{HY} = \sum_x y(\bar{t}_x, \bar{b}_x)_L \Phi_x b_{x,R} + y(\bar{t}_x, \bar{b}_x)_L \tilde{\Phi}_x t_{x,R} +$ h.c..
  - Φ is a complex scalar doublet and $\tilde{\Phi} = i\tau_2 \Phi^*$.
- Results presented in this talk are from $8^3 \times 16$, $12^3 \times 24$ and $16^3 \times 32$.

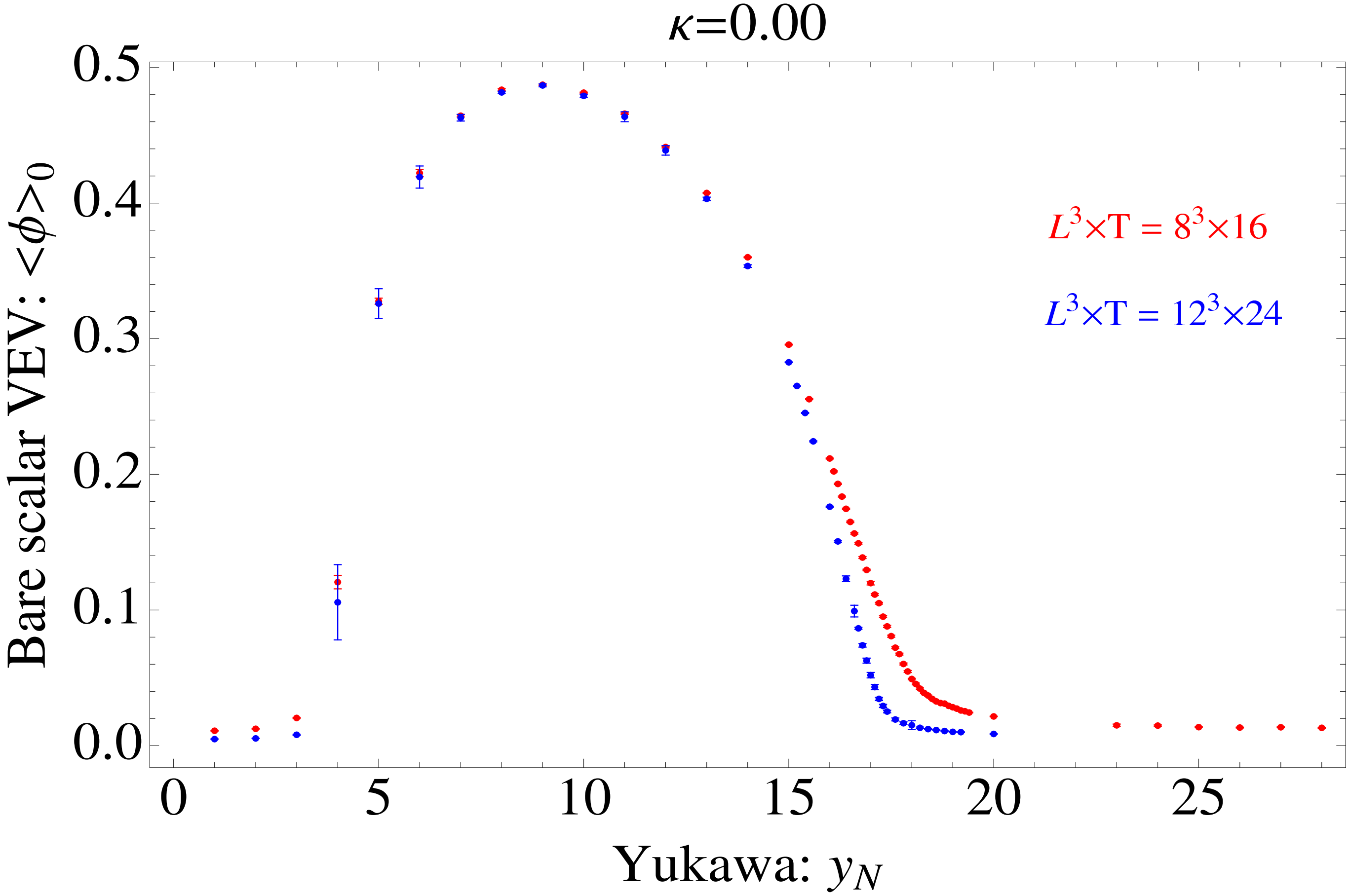
Phase diagram of the H-Y model (qualitative)



From earlier work using Wilson fermion.

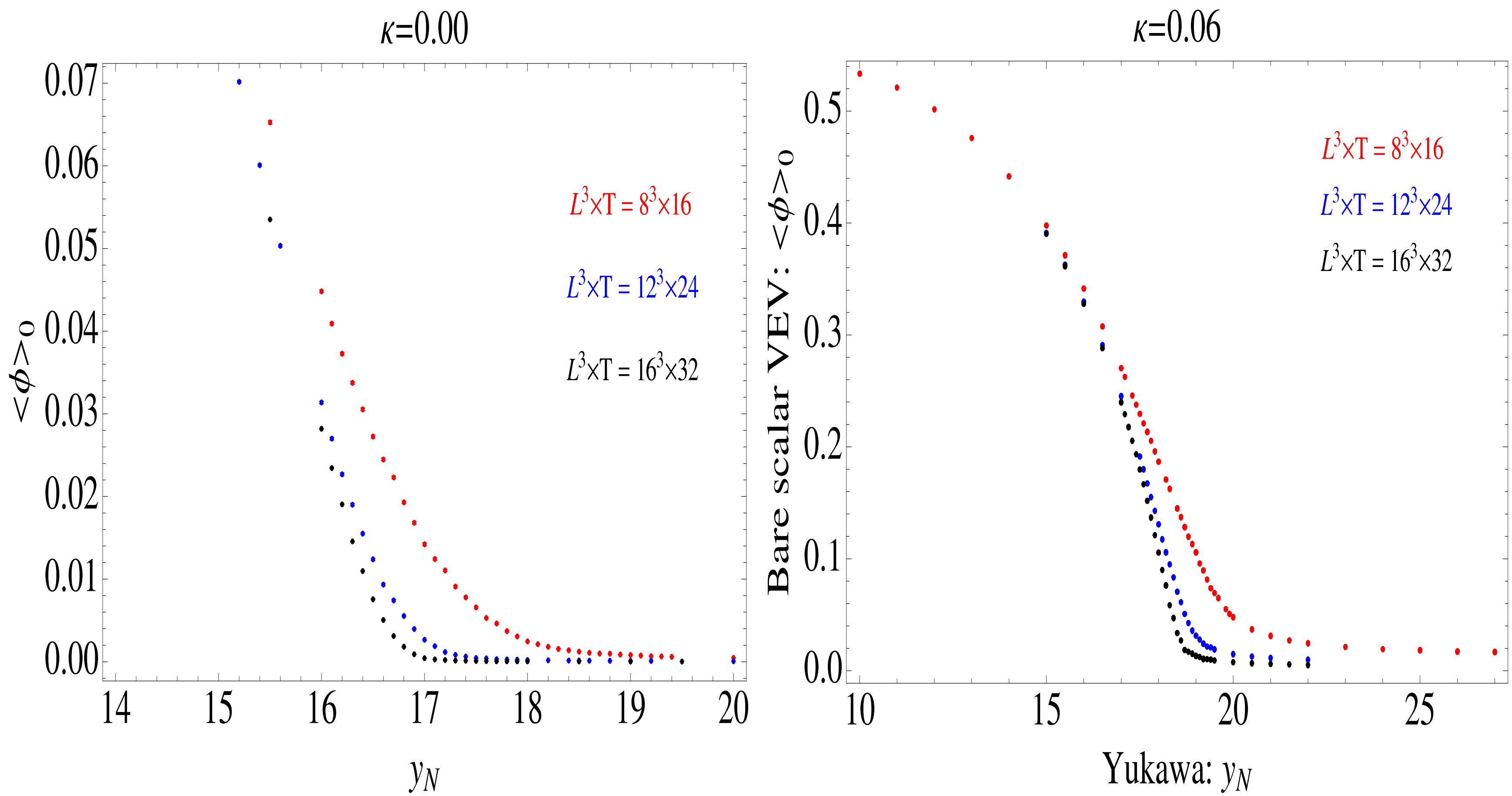
W. Bock et al., 1990.

Evidence of a symmetric phase at large y



Consistent with recent results in P. Gerhold and K. Jansen, 2007.

The bare scalar vev at large Y



Finite-size scaling of susceptibility

- Definition of susceptibility

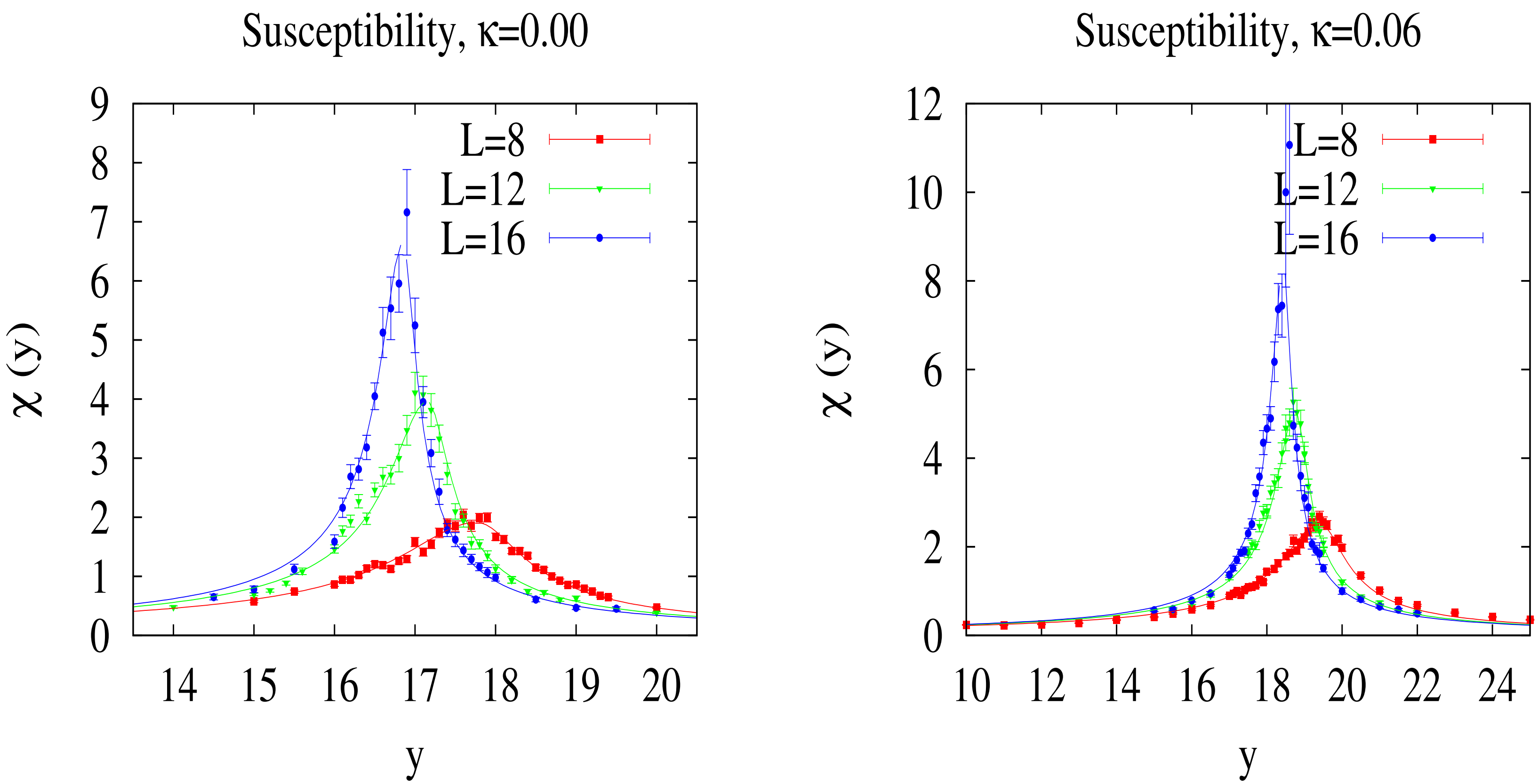
$$\chi = \int d^4x \left(\langle \phi^2 \rangle - \langle \phi \rangle \langle \phi \rangle \right).$$

- The scaling behaviour from solving the finite-volume RGE,
  - Universal scaling function for 2nd-order phase transitions.
  - $\chi L_s^{-\gamma/\nu} \sim g(t L_s^{1/\nu})$, where $t = (y/(y_{\text{crit}} - A_4/L_s) - 1)$.
  - critical exponents γ and ν.
  - Fit all the data to the (partly empirical) function at fixed κ

K. Jansen and P. Seuferling, 1990

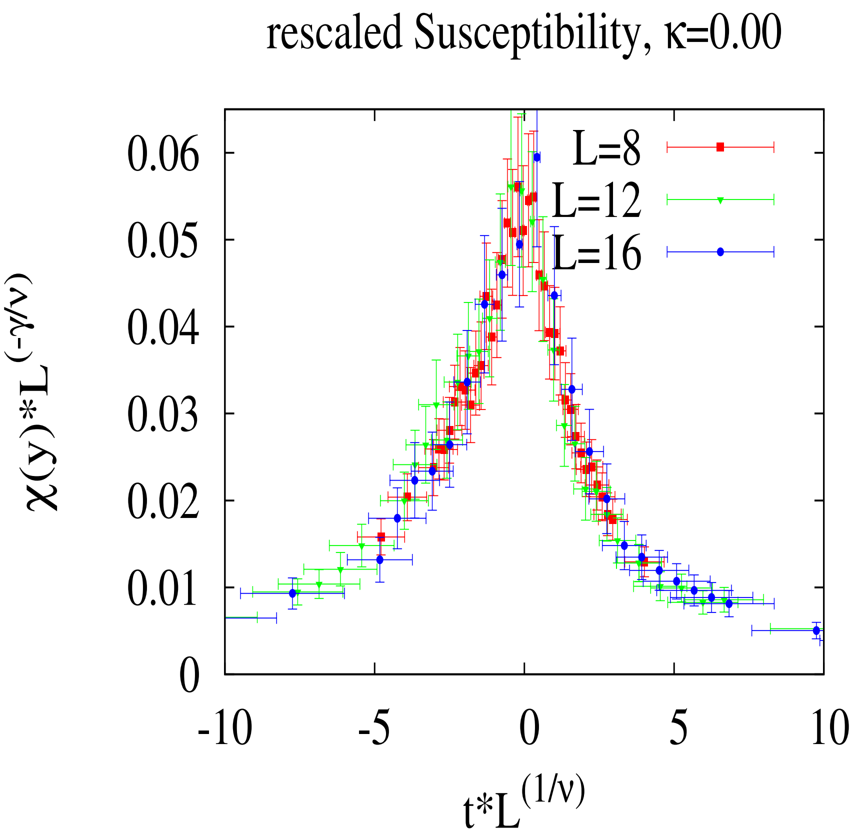
$$\chi \;\; = \;\; A_1 \left\{ L_s^{-2/\nu} + A_{2,3} \left(y - y_{\text{crit}} - A_4/L_s \right)^2 \right\}^{-\gamma/2}.$$

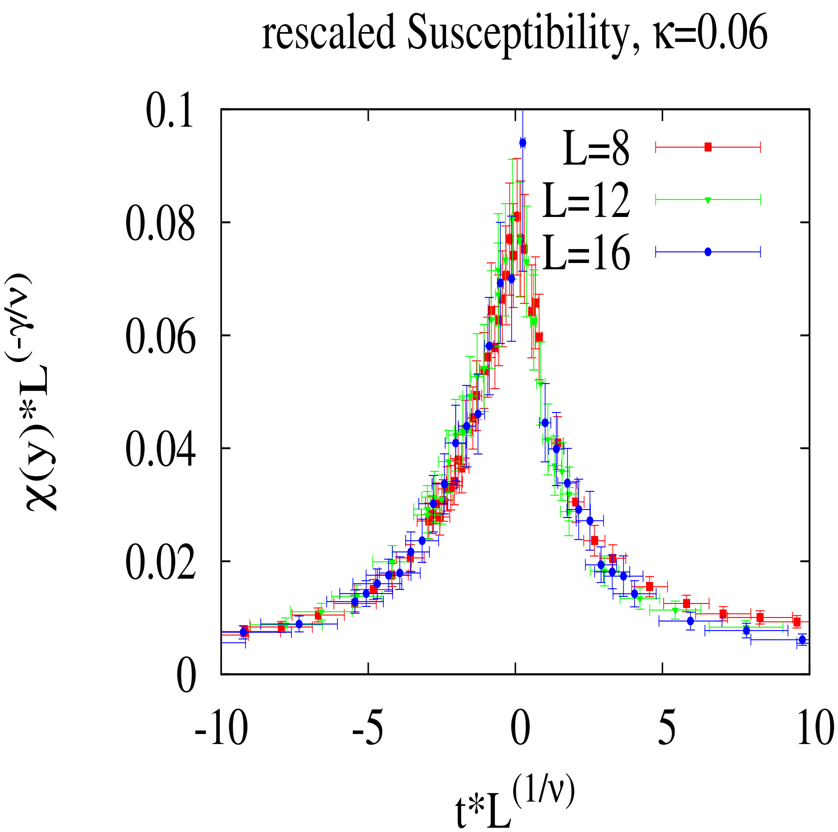
Finite-size fit of susceptibility



Fit range: $y = 15.5 \sim 18.5$ $y = 16.5 \sim 20.5$

Finite-size scaling of susceptibility





Probing the phase structure using susceptibility

	$\kappa = 0.00$	$\kappa = 0.06$	O(4) scalar model
y_{crit}	15.96 ± 0.02	17.46 ± 0.03	N/A
γ	1.02 ± 0.03	1.04 ± 0.02	1
ν	0.57 ± 0.03	0.62 ± 0.03	0.5

- Quoted errors are statistical, from uncorrelated fits with $\chi^2/\text{dof} \sim 0.001$.

- Estimate systematics by changing the fit range in y.

- Systematic effects
  - y_{crit} is very stable.
  - γ can change by $\sim 2\%$.
  - ν can vary by $\sim 8\%$. $\Rightarrow$ Different from O(4) scalar model?

Outlook

- Improving results by
  - running at large lattices.
  - using more sophisticated fit ansatz, e.g., log corrections.
  - studying the scaling behaviour of Binder's cummulant.
- More information:
  - Compute three renormalised couplings to "trade" with κ, $\widehat{\lambda}$ and y.
  - Study the spectrum in the strong Yukawa regime.

A lot more to do and to understand.