Dynamical Electroweak Symmetry Breaking with a Fourth Generation

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Outline

Fourth Generation

Motivation

High masses for new quarks $m_{u_4,d_4} \gtrsim 400$ GeV imply strong Yukawa couplings. So, it is natural to expect that this fourth generation could play a special role in the electroweak symmetry breaking.

Our Model

Dynamical electroweak symmetry breaking induced by a fourth generation in a SM-like scenario with a genuine scalar sector without self-interactions at classical level (perturbative approach) [a].

[a] D. Delepine, M. Napsuciale, C. A. Vaquera-Araujo, Phys. Rev. D 84, 033008 (2011)

SM4 DSB

Assumptions

- SM is an effective theory (valid only below a certain cut-off Λ)
- Trivial classical potential ($\mu^2 > 0$, $\lambda = 0$)

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\mu^2}{2}\phi^2 + \sum_a \left[\bar{\psi}^a i\gamma^\mu\partial_\mu\psi^a - \frac{g_a}{\sqrt{2}}\phi\bar{\psi}^a\psi^a\right] \quad (1)$$

where ϕ is the neutral component of the standard Higgs doublet and ψ^a is the corresponding fermion field with $a = t, u_4, d_4, \ell_4, \nu_4$.

One loop corrections to the classical potential $V^0 = \frac{1}{2}\mu^2\phi^2$ are

$$V^{(1)} = \frac{\mu^2}{2}\phi^2 - \sum_a \frac{4N_c^a}{32\pi^2}\int_0^{\Lambda^2} dk_E^2 k_E^2 \ln\left[\frac{k_E^2 + m_a^2(\phi)}{k_E^2 + m_a^2(0)}\right]. \qquad (2)$$

where N_c^a is the number of colors of the field labeled by a and $m_a^2(\phi) = g_a^2\phi^2/2$.

Including one-loop gauge boson contributions to this potential is straightforward and yields

$$V^{(1)} = \frac{\mu^2\phi^2}{2} + \sum_a \frac{n_a}{32\pi^2}\int_0^{\Lambda^2} dk_E^2 k_E^2 \ln\left[\frac{k_E^2 + m_a^2(\phi)}{k_E^2 + m_a^2(0)}\right] \qquad (3)$$

where now $a = t, u_4, d_4, \ell_4, \nu_4, W, Z$, with $m_W^2(\phi) = g_2^2\phi^2/4$ and $m_Z^2(\phi) = (g_1^2 + g_2^2)\phi^2/4$. The degeneracies per particle are the following $n_W = 6$, $n_Z = 3$, $n_t = n_{u_4} = n_{d_4} = -12$ y $n_{\ell_4} = n_{\nu_4} = -4$.

Solving the integral

$$V^{(1)} = \frac{\mu^2\phi^2}{2} + \sum_a \frac{n_a}{64\pi^2}\left\{ \left[m_a^2(\phi) - m_a^2(0)\right]\Lambda^2 + \Lambda^4 \ln\left[\frac{\Lambda^2 + m_a^2(\phi)}{\Lambda^2 + m_a^2(0)}\right] - m_a^4(\phi)\ln\left[1 + \frac{\Lambda^2}{m_a^2(\phi)}\right] + m_a^4(0)\ln\left[1 + \frac{\Lambda^2}{m_a^2(0)}\right]\right\} \quad (4)$$

the classical minimum $\langle\phi\rangle = 0$ can be turned into a local maximum by the one-loop corrections. A new minimum appears then at $\langle\phi\rangle = v \neq 0$ and all particles in the model acquire a mass $m_a = m_a(v)$. The only non-trivial solution to $\partial V^{(1)}/\partial\phi|_{\phi=v} = 0$ is

$$\mu^2 = -\sum_a \frac{n_a m_a^4}{16\pi^2 v^2}\left[\frac{\Lambda^2}{m_a^2} - \ln\left(1 + \frac{\Lambda^2}{m_a^2}\right)\right], \quad (5)$$

Higgs boson mass

$$\left.\frac{\partial^2 V^{(1)}}{\partial\phi^2}\right|_{\phi=v} = m_H^2 = -\sum_a \frac{n_a m_a^4}{8\pi^2 v^2}\left[\ln\left(1+\frac{\Lambda^2}{m_a^2}\right) - \frac{\Lambda^2}{m_a^2+\Lambda^2}\right] \quad (6)$$

fourth derivative of the effective potential

$$\left.\frac{\partial^4 V^{(1)}}{\partial\phi^4}\right|_{\phi=v} = -\sum_a \frac{3 n_a m_a^4}{8\pi^2 v^4}\left[\ln\left(1+\frac{\Lambda^2}{m_a^2}\right) + 9\frac{m_a^2}{m_a^2+\Lambda^2} - 8\frac{m_a^4}{(m_a^2+\Lambda^2)^2} + \frac{8}{3}\frac{m_a^6}{(m_a^2+\Lambda^2)^3} - \frac{11}{3}\right]. \quad (7)$$

Limit case: Vanishing of Higgs self-coupling at v

Assuming that Higgs self-interaction vanishes at the symmetry breaking scale $v = 246$ GeV

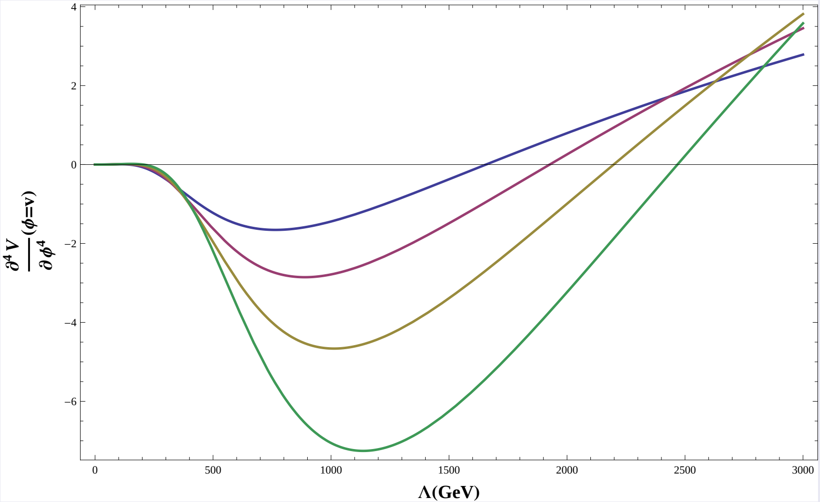
$$\left.\frac{\partial^4 V^{(1)}}{\partial \phi^4}\right|_{\phi=v} = 0. \tag{8}$$

one can extract the value of the cutoff Λ as a function of the masses of the involved fields.

For the present analysis we take $350\ \text{GeV} < m_{u_4} < 500\ \text{GeV}$ with $m_{b_4} = m_{u_4} - 60\ \text{GeV}$ and $100\ \text{GeV} < m_{\ell_4} < 400\ \text{GeV}$ with $m_{\nu_4} = m_{\ell_4} - 45\ \text{GeV}$. The solution for Λ lies in the following range:

$$1700\ \text{GeV} < \Lambda < 2500\ \text{GeV}. \tag{9}$$

Fourth derivative of the effective potential



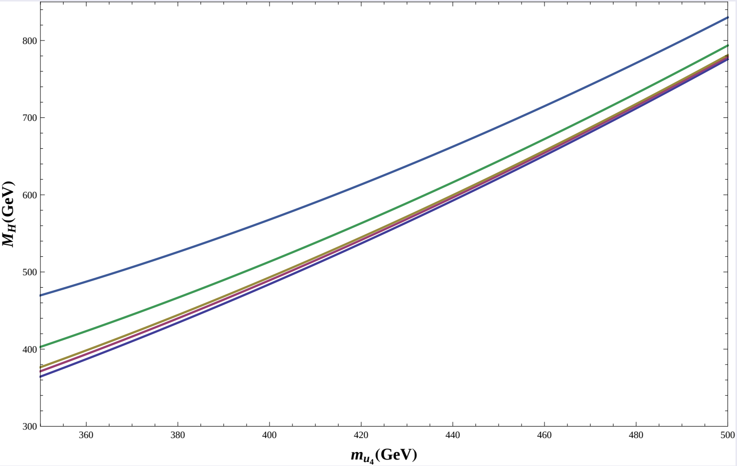
m_{u_4} =350, 400, 450 and 500 GeV with $m_{\ell_4} = 200$ GeV.

Once we have fixed the cutoff Λ, we obtain the corresponding Higgs mass as a function of m_{u_4} y m_{ℓ_4}. In general, Higgs mass is a smooth function of m_{u_4} and m_{ℓ_4} and has a more pronounced dependence on m_{u_4}.

The resulting Higgs mass is

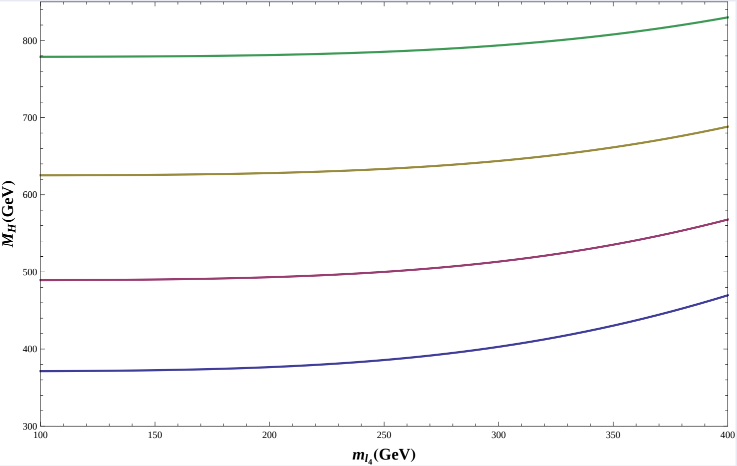
$$350 \text{ GeV} < M_H < 800 \text{ GeV}. \tag{10}$$

Higgs mass as a function of m_{u_4}



for m_{ℓ_4} =100, 200, 300 and 400 GeV. Lower curve contains only u_4 and d_4 contribution.

Higgs mass as a function of m_{ℓ_4}



for $m_{u_4} = 350,\ 400,\ 450,\ 500$ GeV from bottom to top.

From this analysis one can conclude that

- Gauge boson and top contributions to Higgs mass are negligible compared to that of the fourth generation
- If fourth generation lepton masses are moderately light (100 GeV $< m_{\ell_4}, m_{\nu_4} < 250$ GeV), the dominant contribution comes from u_4 and d_4. A simple and good approximation for the Higgs mass in this case is

$$m_H \approx \frac{1.89}{\pi v}\sqrt{m_{u_4}^4 + m_{d_4}^4}, \tag{11}$$

with $\Lambda \approx 5 m_{u_4}$.

RG Improved Model

Effective model

$$\mathcal{L}^{(\Lambda)} = \frac{1}{2}\partial^{\mu}\phi\partial_{\mu}\phi - V^{(0)}\left(\phi^2;\Lambda\right) + \sum_{a=u_4,d_4}\left[\bar{\psi}^a i\gamma^{\mu}\partial_{\mu}\psi^a - \frac{g_a(\Lambda)}{\sqrt{2}}\phi\bar{\psi}^a\psi^a\right]$$

Tree-level potential

$$V^{(0)}\left(\phi^2;\Lambda\right) = \frac{1}{2}\mu^2(\Lambda)\phi^2 + \frac{\lambda(\Lambda)}{4!}\phi^4 \,. \tag{12}$$

One-loop level potential

$$V^{(1)}\left(\phi^2;\Lambda\right) = V^{(0)}\left(\phi^2;\Lambda\right) - \sum_{a=u_4,d_4}\frac{4N_c^a}{32\pi^2}\int_0^{\Lambda^2} dk_E^2 k_E^2 \ln\left[1 + \frac{g_a^2(\Lambda)\phi^2}{2k_E^2}\right]. \tag{13}$$

Minimum condition

Again, if we insist in a dynamical SB triggered by fourth generation quarks and we set $\lambda(\Lambda) = 0$, the only non-trivial solution to $\partial V^{(1)}/\partial\phi|_{\phi=\langle\phi\rangle_1} = 0$ is

$$\mu^2(\Lambda) = \sum_{a=u_4,d_4} \frac{g_a^2(\Lambda)N_c}{8\pi^2}\left[\Lambda^2 - m_a^{(0)2}(\Lambda)\ln\left(\frac{\Lambda^2}{m_a^{(0)2}(\Lambda)} + 1\right)\right], \qquad (14)$$

with $\mu^2(\Lambda) > 0$ for the inputs of the analysis and

$$m_a^{(0)}(\Lambda) = \frac{g_a(\Lambda)\langle\phi\rangle_1}{\sqrt{2}} \qquad (15)$$

with $\langle\phi\rangle_1$ as the “bare” vacuum expectation value.

Physical quantities

In order to obtain predictions on physical quantities, we must make an adequate choice of Λ taking special care in the preservation of the perturbative expansion.

Physical (pole) mass M_H of the scalar:

$$\begin{aligned} M_H^2 &= \left.\frac{d^2 V^{(0)}}{d\phi^2}\right|_{\phi=\langle\phi\rangle_1} + \Sigma_{HH}\left(q^2 = M_H^2\right) \\ &= \left.\frac{d^2 V^{(1)}}{d\phi^2}\right|_{\phi=\langle\phi\rangle_1} - \Sigma_{HH}\left(q^2 = 0\right) + \Sigma_{HH}\left(q^2 = M_H^2\right), \end{aligned} \quad (16)$$

where $\Sigma_{HH}(q^2)$ stands for the scalar self energy

$$\Sigma_{HH}(q^2) = -\sum_{a=u_4,d_4} \frac{g_a^2(\Lambda)N_c}{8\pi^2} \left\{ \Lambda^2 + \left[\frac{q^2}{2} - 3m_a^{(0)2}(\Lambda)\right] \ln\left(\frac{\Lambda^2}{m_a^{(0)2}(\Lambda)}\right) \right.$$
$$+ 2m_a^{(0)2}(\Lambda) - \frac{7}{12}q^2 + \frac{m_a^{(0)2}(\Lambda)}{\Lambda^2}\left[\frac{q^2}{2} - 5m_a^{(0)2}(\Lambda)\right]$$
$$+ \frac{m_a^{(0)4}(\Lambda)}{\Lambda^4}\left[q^2 + \frac{7}{2}m_a^{(0)2}(\Lambda)\right]$$
$$\left. + \mathcal{O}\left((q^2; m_a^{(0)2}(\Lambda))\frac{m_a^{(0)6}(\Lambda)}{\Lambda^6}\right)\right\}. \tag{17}$$

Renormalized VEV

$$Z_\phi \langle \phi \rangle_1^2 = v^2, \tag{18}$$

where

$$\begin{aligned} Z_\phi =& \left[1 - \frac{d\Sigma_{HH}(q^2)}{dq^2} \Big|_{q^2 = M_H^2} \right] \\ =& 1 + \sum_{a = u_4, d_4} \frac{4 g_a^2(\Lambda) N_c}{64\pi^2} \left\{ \ln \left(\frac{\Lambda^2}{m_a^{(0)2}(\Lambda)} \right) - \frac{7}{6} \right. \\ & \left. + \frac{m_a^{(0)2}(\Lambda)}{\Lambda^2} + 2 \frac{m_a^{(0)4}(\Lambda)}{\Lambda^4} + \mathcal{O}\left(\frac{m_a^{(0)6}(\Lambda)}{\Lambda^6} \right) \right\}. \end{aligned} \tag{19}$$

Yukawa coupling running

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}g_{u_4} = \frac{9}{2}g_{u_4}^3 + \frac{3}{2}g_{u_4}g_{d_4}^2 \tag{20}$$

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}g_{d_4} = \frac{9}{2}g_{d_4}^3 + \frac{3}{2}g_{d_4}g_{u_4}^2 . \tag{21}$$

In the approximation $g_{u_4} \approx g_{d_4}$, defining $g_{u_4} - g_{d_4} \equiv \Delta g$:

$$g_{u_4}(\mu) \approx \left[\frac{1}{g_{u_4}^2(\mu_0)} - \frac{6}{16\pi^2}\ln\left(\frac{\mu^2}{\mu_0^2}\right)\right]^{-1/2} \tag{22}$$

$$\Delta g(\mu) \approx \Delta g(\mu_0)\left[1 - \frac{3g_{u_4}^2(\mu_0)}{8\pi^2}\ln\left(\frac{\mu^2}{\mu_0^2}\right)\right]^{-1} . \tag{23}$$

Physical mass of the heaviest quark

$$m_{u_4} \equiv \frac{g_{u_4}(m_{u_4})v}{\sqrt{2}}. \tag{24}$$

The running of Yukawa couplings from $E = m_{u_4}$ to $E' = \Lambda$ is given by

$$g_{u_4}(\Lambda) \approx \left[\frac{1}{g^2_{u_4}(m_{u_4})} - \frac{6}{16\pi^2}\ln\left(\frac{\Lambda^2}{m^2_{u_4}}\right)\right]^{-1/2} \tag{25}$$

$$\Delta g(\Lambda) \approx \Delta g(m_{u_4})\left[1 - \frac{3g^2_{u_4}(m_{u_4})}{8\pi^2}\ln\left(\frac{\Lambda^2}{m^2_{u_4}}\right)\right]^{-1}. \tag{26}$$

Squared scalar pole mass

$$M_H^2 = \sum_{a=u_4,d_4} \frac{8g_a^2(\Lambda)N_c Z_\phi^{-2} v^2}{64\pi^2}\left[\ln\left(\frac{\Lambda^2}{m_a^{(0)2}(\Lambda)}+1\right) - \frac{\Lambda^2}{\Lambda^2 + m_a^{(0)2}(\Lambda)}\right]. \quad (27)$$

Maximum perturbative cut-off

$$\frac{g_{u_4}^2(\Lambda_{max})}{4\pi} = 1, \quad (28)$$

which can be solved to yield

$$\Lambda_{max} = m_{u_4} e^{\frac{2\pi^2 v^2}{3m_{u_4}^2}\left(1-\frac{m_{u_4}^2}{2\pi v^2}\right)} \quad (29)$$

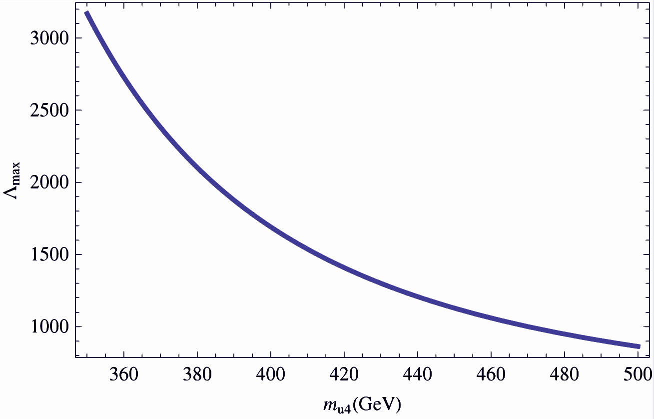


Figure: Λ_{max} as a function of the physical mass of the heaviest quark m_{u_4}.

Comparing with the previous section we can see that perturbative effects indeed appear at a lower scale than the naive scale for new physics. Still, taking the worst case, *e.g.*, $\Lambda = 2m_{u_4}$, the predicted Higgs mass lies in the same range as before and the perturbative expansion is valid up to $m_{u_4} \approx 480$GeV, where $\Lambda \approx \Lambda_{\max}$. Thus, the predictions of the previous model are not strongly modified by the RG Yukawa couplings, but the interpretation of the cutoff scale is different.

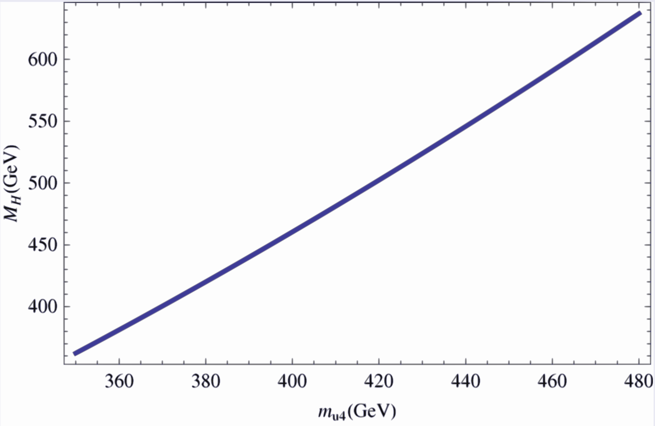


Figure: Higgs (pole) mass as a function of m_{u_4} with $\Lambda = 2m_{u_4}$ for $\Delta m = m_{u_4} - m_{d_4} = 60$ GeV.

Conclusions

- 4G Yukawa couplings are strong enough to drive electroweak symmetry breaking. We isolate the 4G effects by taking a vanishing scalar self-coupling at the classical level and maintaining this condition valid at one-loop level at the EWSB scale. Here EWS is broken by radiative corrections due mainly to the 4G and Higgs masses of the order of a few hundreds of GeV are consistent with electroweak precision data. Furthermore, the theory is valid only up to $\Lambda \sim 1 - 2$ TeV, a scale measurable at the LHC.
- We use the renormalization group equation to study the impact of the running of the Yukawa couplings in our results. The predictions are not strongly modified by the running of the Yukawa couplings, but a slightly lower cut-off related to the breaking of the perturbative regime is expected in this case.

Teşekkürler!