

Lepton flavor violation with four generations

P R E S E N T A T I O N

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mu-e Conversion with Four generations

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Outline

- **Lepton flavor violation (LFV)**
- **LFV in SM3 and in SM4**
 - $\mu \rightarrow e\gamma$
 - $\mu \rightarrow 3e$
 - $\mu \rightarrow e$ conversion in nuclei
 - Numerical results
- **Summary**

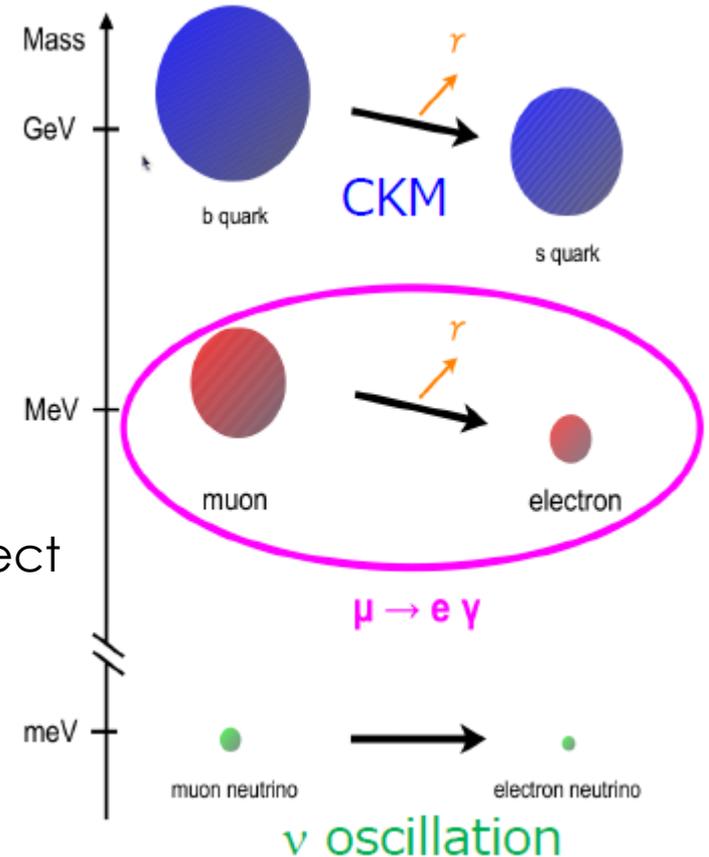
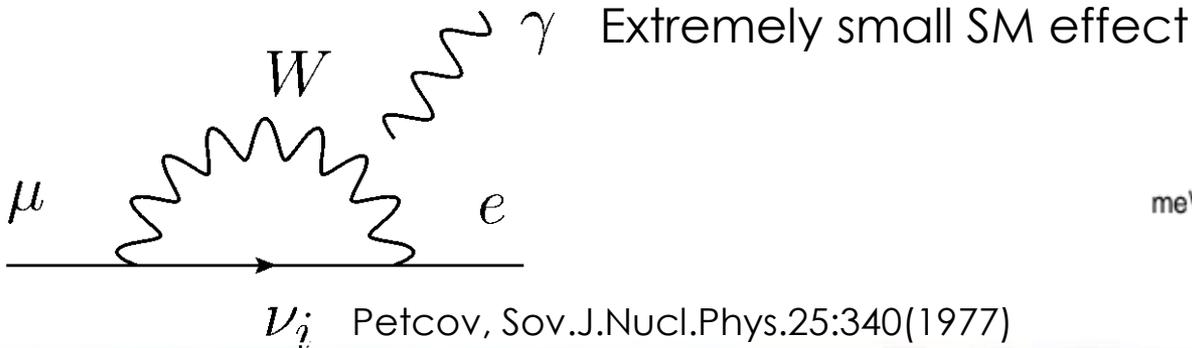
Motivations

- **Quark** and **neutrino** flavor violations has been observed !!
- **Neutrinos are members of lepton doublet**
 - charged LFV can be induced.

□ Clear evidence of Beyond SM

- In SM3 + massive ν

$$B(\mu \rightarrow e\gamma) \approx \frac{3\alpha}{32\pi} \left| U_{ei} \frac{m_{\nu i}^2}{m_W^2} U_{\mu i}^* \right|^2 < 10^{-45}$$



Current experimental limit

□ Muon LFV decays are highly constrained !!

- Order of 10 improvement on **muon LFV** in 60 years

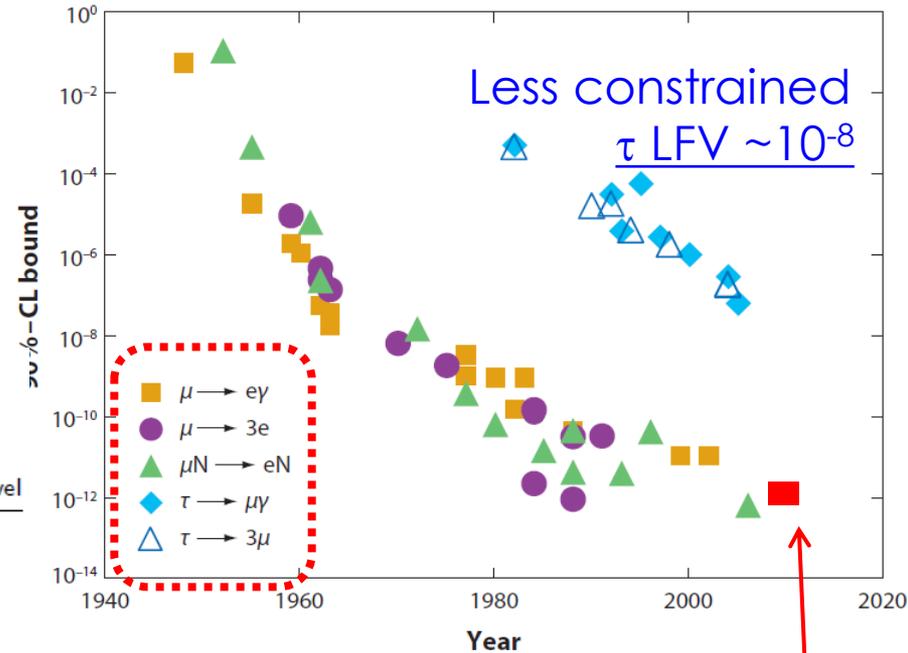
μ^- DECAY MODES

μ^+ modes are charge conjugates of the modes below.

Mode	Fraction (Γ_i/Γ)	Confidence level
Γ_1 $e^- \bar{\nu}_e \nu_\mu$	$\approx 100\%$	
Γ_2 $e^- \bar{\nu}_e \nu_\mu \gamma$	[a] $(1.4 \pm 0.4)\%$	
Γ_3 $e^- \bar{\nu}_e \nu_\mu e^+ e^-$	[b] $(3.4 \pm 0.4) \times 10^{-5}$	

Lepton Family number (LF) violating modes

Γ_i	Mode	LF	[c]	%	90%
Γ_4	$e^- \nu_e \bar{\nu}_\mu$	LF	< 1.2	%	90%
Γ_5	$e^- \gamma$	LF	< 1.2	$\times 10^{-11}$	90%
Γ_6	$e^- e^+ e^-$	LF	< 1.0	$\times 10^{-12}$	90%
Γ_7	$e^- 2\gamma$	LF	< 7.2	$\times 10^{-11}$	90%



New $\mu \rightarrow e\gamma$ limit
 $< 2.4 \times 10^{-12}$ since EPS2011 by MEG
 It will be improved to 10^{-13}

Current upper limit of $B(\mu \rightarrow e\gamma)$ and $BR(\mu \rightarrow 3e)$ in $\sim 10^{-12}$

Current experimental limit

□ Muon LFV decays are highly constrained !!

□ μ -e conversion in nuclei

LIMIT ON $\mu^- \rightarrow e^-$ CONVERSION

Forbidden by lepton family number conservation.

$\sigma(\mu^- {}^{32}\text{S} \rightarrow e^- {}^{32}\text{S}) / \sigma(\mu^- {}^{32}\text{S} \rightarrow \nu_\mu {}^{32}\text{P}^*)$

VALUE	CL%	DOCUMENT ID	TECN	COMMENT
$<7 \times 10^{-11}$	90	BADERT...	80	STRC SIN
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$<4 \times 10^{-10}$	90	BADERT...	77	STRC SIN

$\sigma(\mu^- \text{Cu} \rightarrow e^- \text{Cu}) / \sigma(\mu^- \text{Cu} \rightarrow \text{capture})$

VALUE	CL%	DOCUMENT ID	TECN	COMMENT
$<1.6 \times 10^{-8}$	90	BRYMAN	72	SPEC

$\sigma(\mu^- \text{Ti} \rightarrow e^- \text{Ti}) / \sigma(\mu^- \text{Ti} \rightarrow \text{capture})$

VALUE	CL%	DOCUMENT ID	TECN	COMMENT
$<4.3 \times 10^{-12}$	90	²¹ DOHMEN	93	SPEC SINDRUM II
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$<4.6 \times 10^{-12}$	90	AHMAD	88	TPC TRIUMF
$<1.6 \times 10^{-11}$	90	BRYMAN	85	TPC TRIUMF

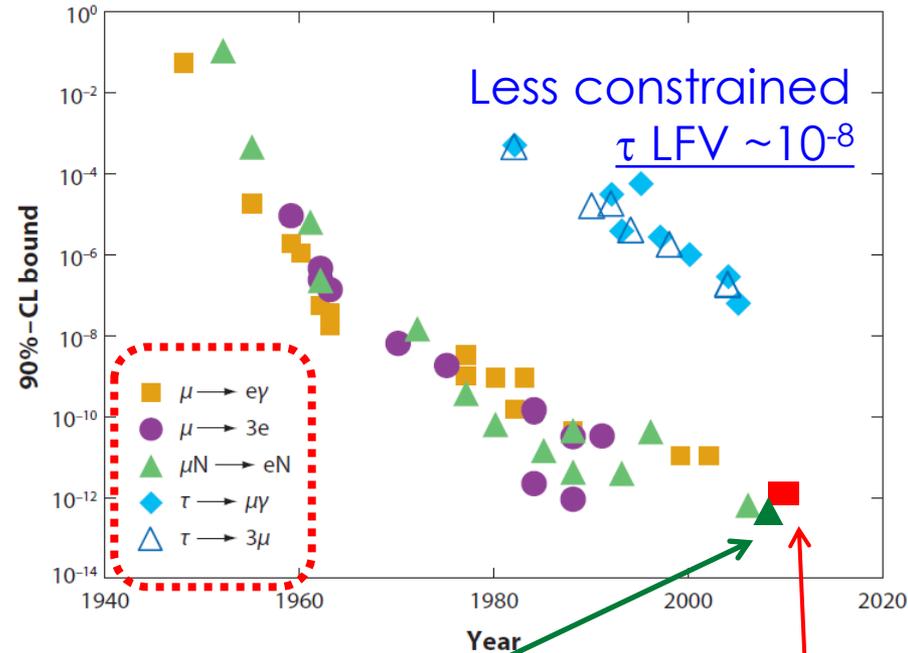
²¹DOHMEN 93 assumes $\mu^- \rightarrow e^-$ conversion leaves the nucleus in its ground state, a process enhanced by coherence and expected to dominate.

$\sigma(\mu^- \text{Pb} \rightarrow e^- \text{Pb}) / \sigma(\mu^- \text{Pb} \rightarrow \text{capture})$

VALUE	CL%	DOCUMENT ID	TECN	COMMENT
$<4.6 \times 10^{-11}$	90	HONECKER	96	SPEC SINDRUM II
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$<4.9 \times 10^{-10}$	90	AHMAD	88	TPC TRIUMF

$\sigma(\mu^- \text{Au} \rightarrow e^- \text{Au}) / \sigma(\mu^- \text{Au} \rightarrow \text{capture})$

VALUE	CL%	DOCUMENT ID	TECN	CHG	COMMENT
$<7 \times 10^{-13}$	90	BERTL	06	SPEC	- SINDRUM II



New $\mu \rightarrow e\gamma$ limit

Slightly better than $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$
 It will be improved to 10^{-18} for Ti !!
 (much better future prospects)

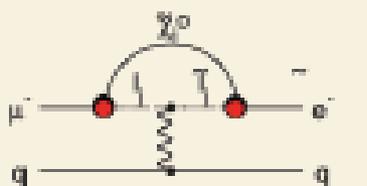
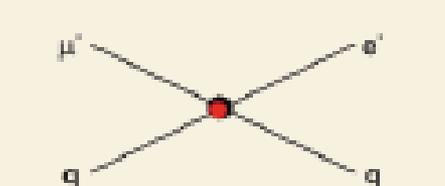
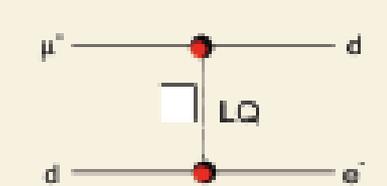
Beyond the SM

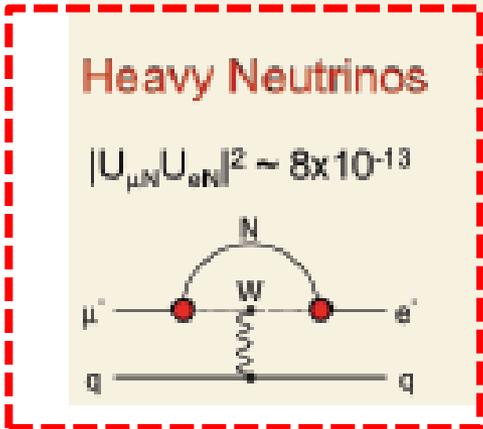
□ Possible new physics contributions

Hisano et.al., PRD53:2442(1996)

Nakagawa, Takasu, PTP59,548(1977)

Shanker, NPB206,253 (1982)

<p>Supersymmetry</p> <p>rate $\sim 10^{-15}$</p> 	<p>Compositeness</p> <p>$\Lambda_c \sim 3000 \text{ TeV}$</p> 	<p>Leptoquark</p> <p>$M_{LQ} = 3000 (\lambda_{\mu d} \lambda_{e d})^{1/2} \text{ TeV}/c^2$</p> 
<p>Heavy Neutrinos</p> <p>$U_{\mu N} U_{e N} ^2 \sim 8 \times 10^{-13}$</p> 	<p>Second Higgs Doublet</p> <p>$g(H_{\mu e}) \sim 10^{-4} g(H_{\mu \mu})$</p> 	<p>Heavy Z'</p> <p>Anomal. Z Coupling</p> <p>$M_{Z'} = 3000 \text{ TeV}/c^2$</p> 



Fourth generation neutrino !!

Chang, Ng, Ng, PRD50,4589(1994)

X.-G. He et.al. (1990)

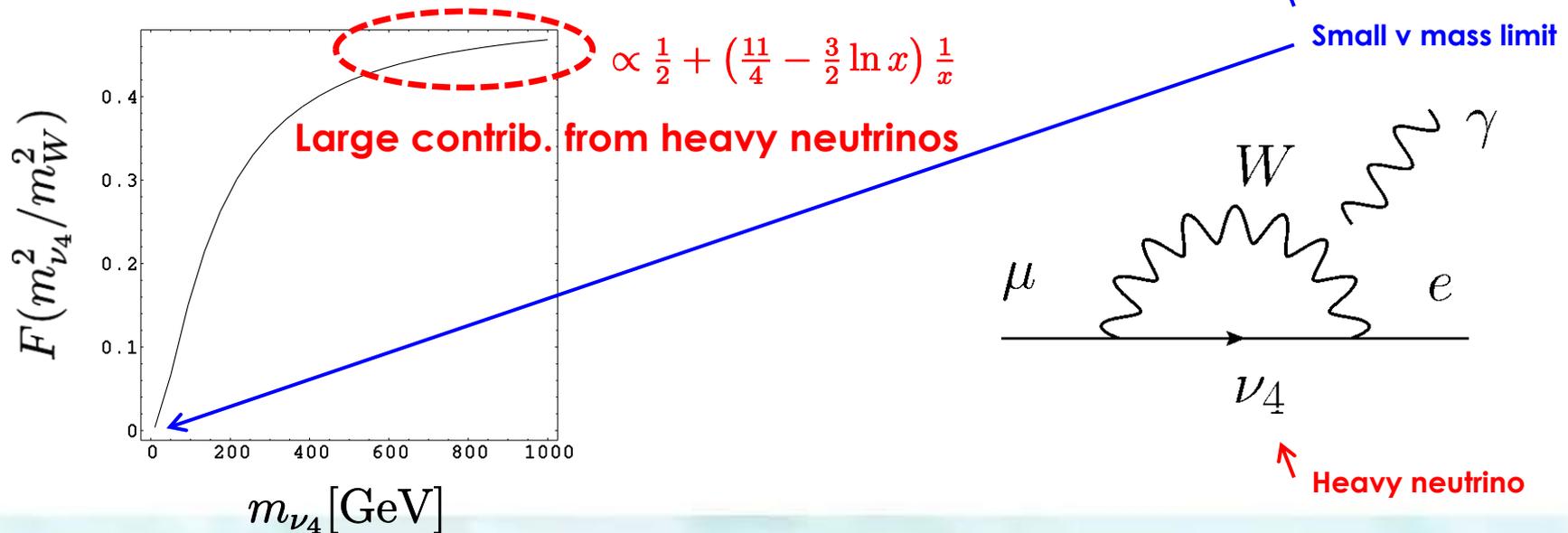
□ Here after, we assume Dirac neutrino for definiteness.

(But, it is true for Majorana neutrino from Type-I seesaw if MR is large.)

$$\mathcal{B}(\mu \rightarrow e\gamma)^{\text{SM3}} = \frac{3\alpha}{2\pi} \left| U_{ei} U_{\mu i}^* F_W \left(\frac{m_i^2}{m_W^2} \right) \right|^2 \approx \frac{3\alpha}{32\pi} \left| U_{ei} U_{\mu i}^* \frac{m_i^2}{m_W^2} \right|^2$$

More precise loop function

$$F_W(x) = -\frac{x(12+x-7x^2)}{12(x-1)^3} + \frac{x^2(12-10x+x^2)}{6(x-1)^4} \ln x \approx \frac{1}{4}x$$



4th generation neutrino

□ Mass bound for 4th generation neutrinos

□ Lower bound from Direct search

———— Heavy Neutral Lepton MASS LIMITS ————

Limits apply only to heavy lepton type given in comment at right of data Listings.

See the "Quark and Lepton Compositeness, Searches for" Listings for limits on radiatively decaying excited neutral leptons, *i.e.* $\nu^* \rightarrow \nu\gamma$.

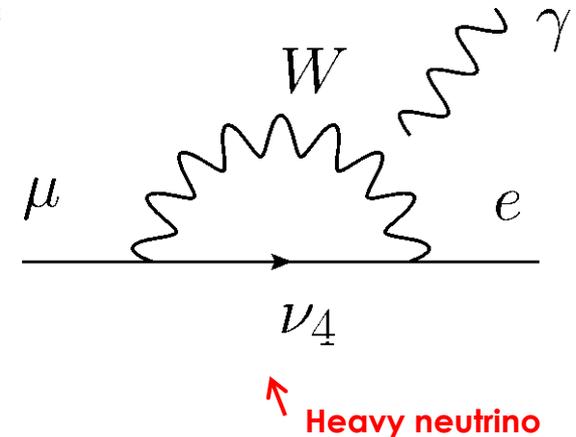
VALUE (GeV)	CL%	DOCUMENT ID	TECN	COMMENT
>101.3	95	ACHARD	01B L3	Dirac coupling to e
>101.5	95	ACHARD	01B L3	Dirac coupling to μ
>90.3	95	ACHARD	01B L3	Dirac coupling to τ
>89.5	95	ACHARD	01B L3	Majorana coupling to e
>90.7	95	ACHARD	01B L3	Majorana coupling to μ
>80.5	95	ACHARD	01B L3	Majorana coupling to τ

□ Upper bound from unitarity

1.2 TeV for heavy leptons

Chanowitz, PLB78,285(1978)

→ ν_4 can induce observable $\mu \rightarrow e\gamma$

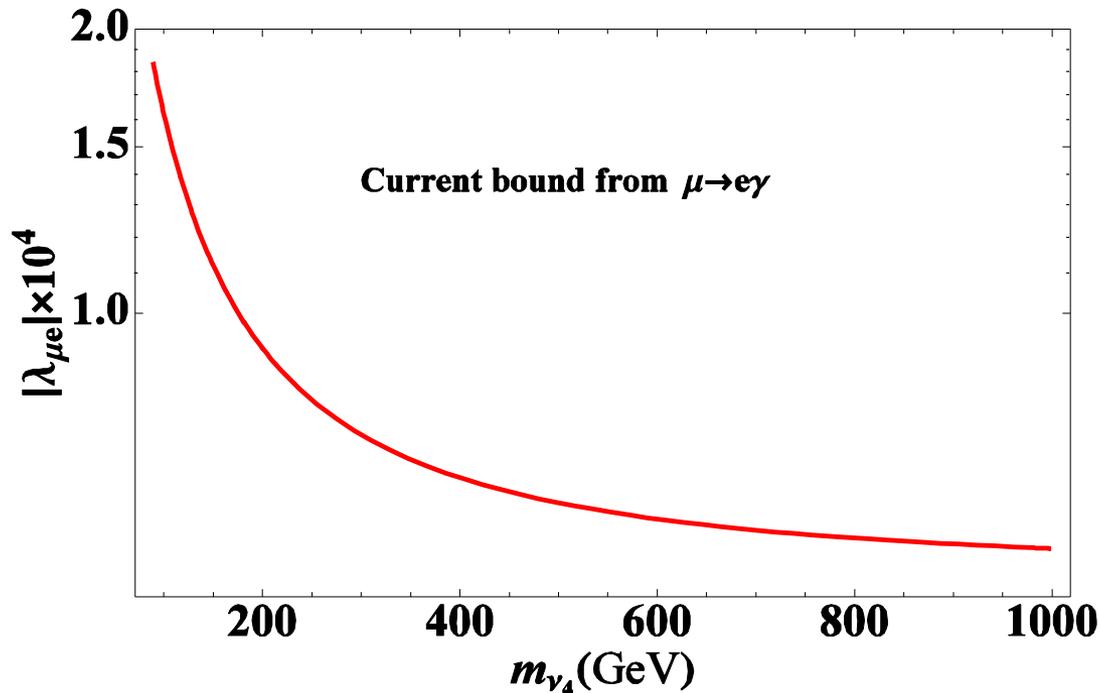


$\mu \rightarrow e\gamma$ in SM4

- Large BR possible if neutrino mixing is large

$$\mathcal{B}(\mu \rightarrow e\gamma)^{\text{SM4}} \approx \frac{3\alpha}{2\pi} \left| \lambda_{\mu e} F_W \left(\frac{m_{\nu_4}^2}{m_W^2} \right) \right|^2 \quad \text{where } \lambda_{\mu e} = U_{e4} U_{\mu 4}^*$$

→ current experimental bound can be converted to bound on $\lambda_{\mu e}$



$$\lambda_{\mu e} \lesssim 0.6 \times 10^{-4}$$

Effective Lagrangian

$$\mathcal{L}^{\mu \rightarrow e\bar{e}e} = -\frac{G_F}{\sqrt{2}} \frac{e^2}{4\pi^2} \lambda_{\mu e} \left[F_W(x_4) \bar{e} \gamma_\mu e \frac{q_\nu}{q^2} \bar{e} i \sigma^{\mu\nu} m_\mu R \mu + \bar{e} \gamma^\mu (a_L(x_4) L + a_R(x_4) R) e \bar{e} \gamma_\mu L \mu \right]$$

On-shell photon penguin

$$a_L(x) = F_{W^*}(x) + \frac{1}{s_W^2} \left(-\frac{1}{2} + s_W^2 \right) F_Z(x) - \frac{1}{2s_W^2} F_B(x), \quad a_R(x) = F_{W^*}(x) + F_Z(x)$$

Off-shell photon

Z photon

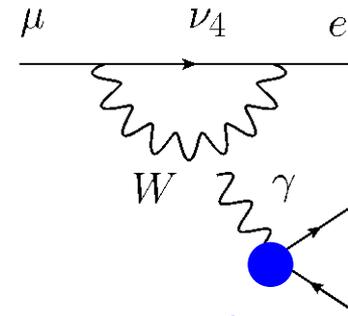
Box diagrams

On-shell photon penguin dominant case (in general)

$$\frac{\mathcal{B}(\mu \rightarrow 3e)}{\mathcal{B}(\mu \rightarrow e\gamma)} = \frac{\alpha}{3\pi} \left(\frac{m_\mu^2}{m_e^2} - \frac{11}{4} \right) \approx 0.006$$

Hisano, Nomura, PRD59,116005(1999)

2-3 orders magnitude smaller



Additional QED suppression

$\mu \rightarrow 3e$ in SM4

Effective Lagrangian

$$\mathcal{L}^{\mu \rightarrow e\bar{e}e} = -\frac{G_F}{\sqrt{2}} \frac{e^2}{4\pi^2} \lambda_{\mu e} \left[F_W(x_4) \bar{e} \gamma_\mu e \frac{q_\nu}{q^2} \bar{e} i \sigma^{\mu\nu} m_\mu R \mu + \bar{e} \gamma^\mu (a_L(x_4) L + a_R(x_4) R) e \bar{e} \gamma_\mu L \mu \right]$$

On-shell photon penguin

$$a_L(x) = F_{W^*}(x) + \frac{1}{s_W^2} \left(-\frac{1}{2} + s_W^2 \right) F_Z(x) - \frac{1}{2s_W^2} F_B(x), \quad a_R(x) = F_{W^*}(x) + F_Z(x)$$

Off-shell photon

Z photon

Box diagrams

Z penguin becomes important in SM4

$$F_{W^*}(x) = \frac{x(12 + x - 7x^2)}{12(x-1)^3} - \frac{x^2(12 - 10x + x^2)}{6(x-1)^4} \ln x$$

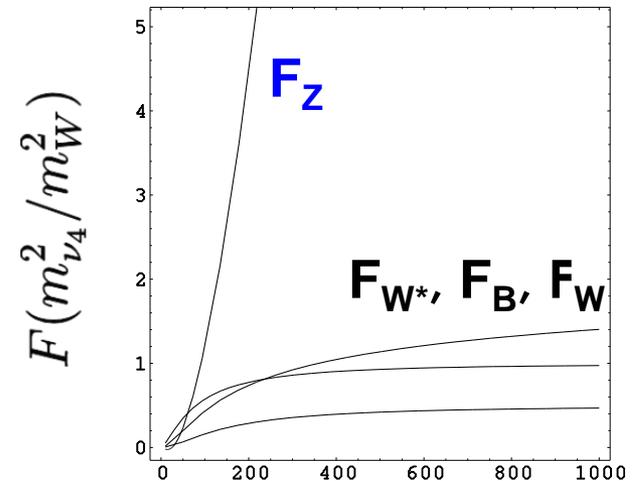
$$F_W(x) = \frac{x(1 - 5x - 2x^2)}{4(x-1)^3} + \frac{3x^3}{2(x-1)^4} \ln x$$

$$F_Z(x) = \frac{x(x^2 - 7x + 6)}{4(x-1)^2} + \frac{x(2 + 3x)}{4(x-1)^2} \ln x$$

$$F_B(x) = \frac{x}{x-1} - \frac{x}{(x-1)^2} \ln x$$

Increase for large ν_4 mass

m_{ν_4} [GeV]

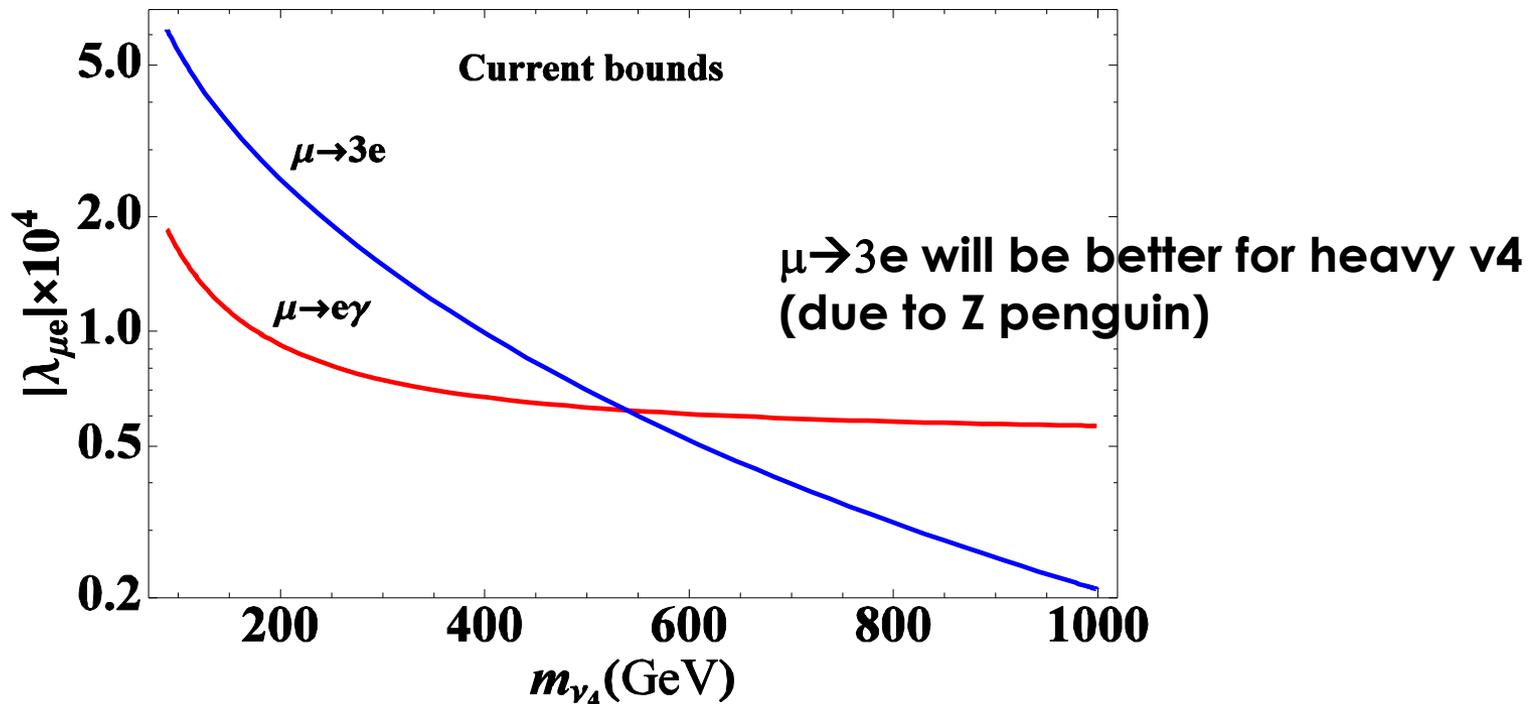


$\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma$ in SM4

□ Compare two results:

$$\mathcal{B}(\mu \rightarrow e\bar{e}e) = \frac{\alpha^2}{16\pi^2} \left[a_R^2(x_4) + 2a_L^2(x_4) - 4F_W(x_4)(a_R(x_4) + 2a_L(x_4)) + 4F_W^2(x_4) \left(4 \ln \frac{m_\mu}{m_e} - \frac{11}{2} \right) \right]$$

On-shell photon penguin



Effective Lagrangian

$$\mathcal{L}^{\mu \rightarrow e} = -\frac{G_F}{\sqrt{2}} \frac{e^2}{4\pi^2} \lambda_{\mu e} \left[F_W(x_4) \bar{q} \gamma_\mu q \frac{q_\nu}{q^2} \bar{e} i \sigma^{\mu\nu} m_\mu R_\mu + \bar{q} \gamma^\mu (V_q(x_4) + A_q(x_4) \gamma_5) q \bar{e} \gamma_\mu L_\mu \right]$$

On-shell photon penguin

$$V_q(x) = -Q^q F_{W^*}(x) + \frac{1}{s_W^2} (I_3^q - Q^q s_W^2) F_Z(x) - \frac{1}{4s_W^2} F_B(x), \quad A_q(x) = -\frac{1}{s_W^2} I_3^q F_Z(x) + \frac{1}{4s_W^2} F_B(x)$$

Off-shell photon

Z photon

Box diagrams

$\mu \rightarrow e$ conversion rate for nucleus A:

$$\frac{\mathcal{B}_{\mu \rightarrow e}^A}{\mathcal{B}(\mu \rightarrow e\gamma)} = R_{\mu \rightarrow e}^0 \left| 1 + \frac{8\pi e}{F_W(x_4) D(A)} \left[(2V_u + V_d) V_A^{(p)} + (V_u + 2V_d) V_A^{(n)} \right] \right|^2$$

Nucleus dependent parts were calculated several methods.

Kitano, Koike, Okada, PRD66,096002(2002)

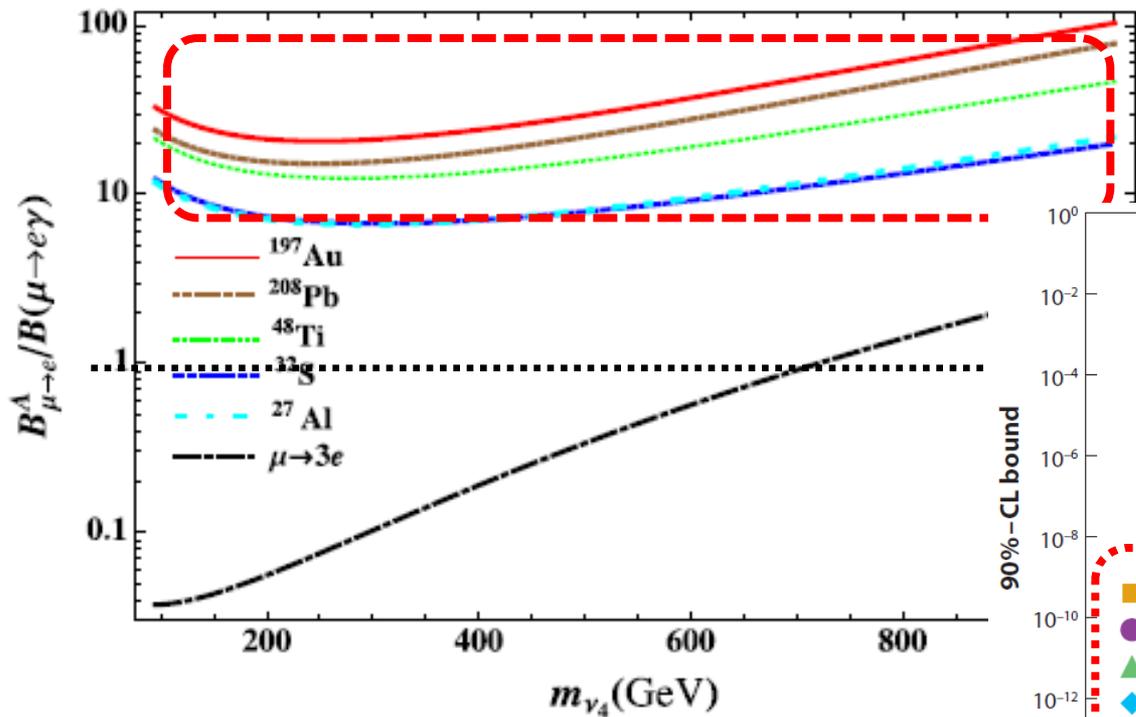
Weinberg, Feinberg, PRL3,111(1959)

Shanker, PRD20,1608(1979)

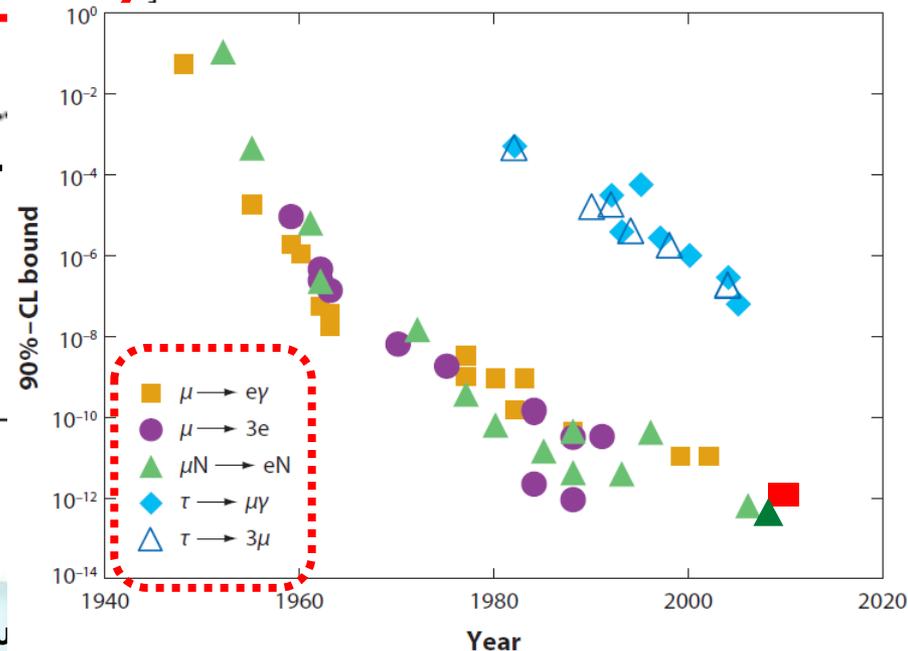
A	D(A)	V ^(p) (A)	V ⁽ⁿ⁾ (A)	R _{μ→e} ⁰ (A)
²⁷ ₁₃ Al	0.0362	0.0161	0.0173	0.0026
³² ₁₆ S	0.0524	0.0236	0.0236	0.0028
⁴⁸ ₂₂ Ti	0.0864	0.0396	0.0468	0.0041
¹⁹⁷ ₇₉ Au	0.189	0.0974	0.146	0.0039
²⁰⁸ ₈₂ Pb	0.161	0.0834	0.128	0.0027

LFV in SM4

- Loop induced LFV always proportional to $\lambda_{\mu e} = U_{e4}U_{\mu 4}^*$
- Compare LFV processes by taking ratio (cancel $\lambda_{\mu e}$)



$\mu \rightarrow e$ conv. would be better if exp. limit are the same level

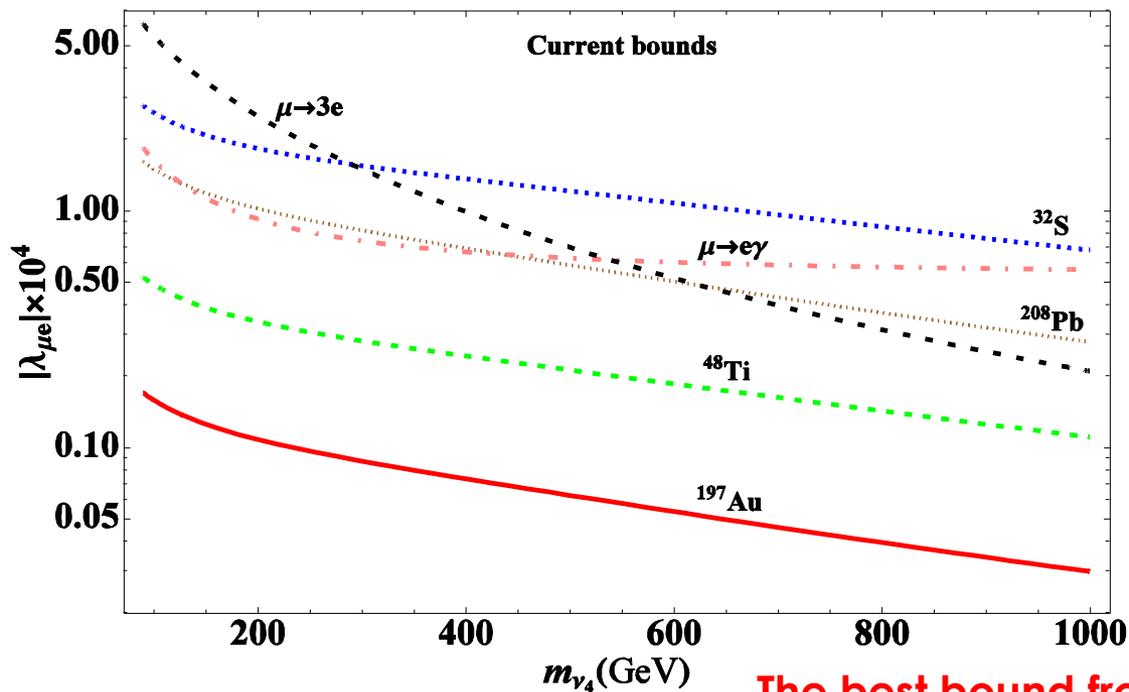


LFV in SM4

- Large BR possible if neutrino mixing is large

$$\mathcal{B}(\mu \rightarrow e\gamma)^{\text{SM4}} \approx \frac{3\alpha}{2\pi} \left| \lambda_{\mu e} F_W \left(\frac{m_{\nu_4}^2}{m_W^2} \right) \right|^2 \quad \text{where } \lambda_{\mu e} = U_{e4} U_{\mu 4}^*$$

→ current experimental bound can be converted to bound on $\lambda_{\mu e}$



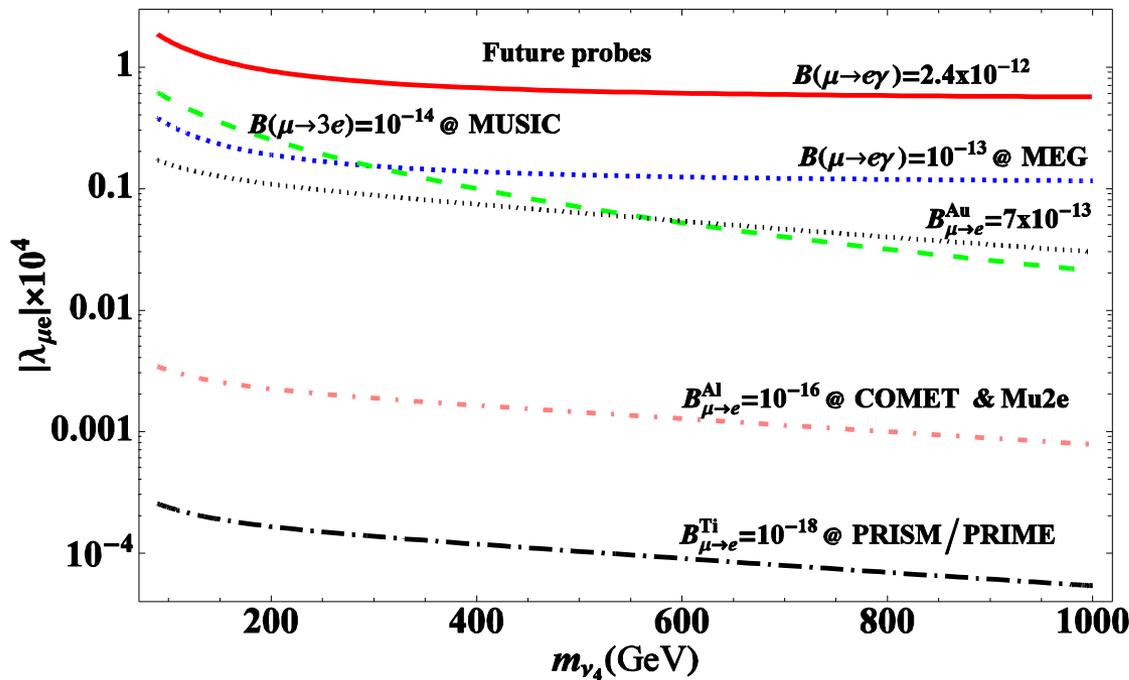
The best bound from $\mu \rightarrow e$ conv. in Au.

LFV in SM4 (Prospects)

- Large BR possible if neutrino mixing is large

$$\mathcal{B}(\mu \rightarrow e\gamma)^{\text{SM4}} \approx \frac{3\alpha}{2\pi} \left| \lambda_{\mu e} F_W \left(\frac{m_{\nu_4}^2}{m_W^2} \right) \right|^2 \quad \text{where } \lambda_{\mu e} = U_{e4} U_{\mu 4}^*$$

→ current experimental bound can be converted to bound on $\lambda_{\mu e}$



Several projects are proposed.

Current bound from $\mu \rightarrow e$ in Au is comparable level with projected $\mu \rightarrow e \gamma$ and $\mu \rightarrow 3e$ exp in SM4.

$\mu \rightarrow e$ conv. can improve $\lambda_{\mu e}$ by 3 orders of magnitude in future.

Summary

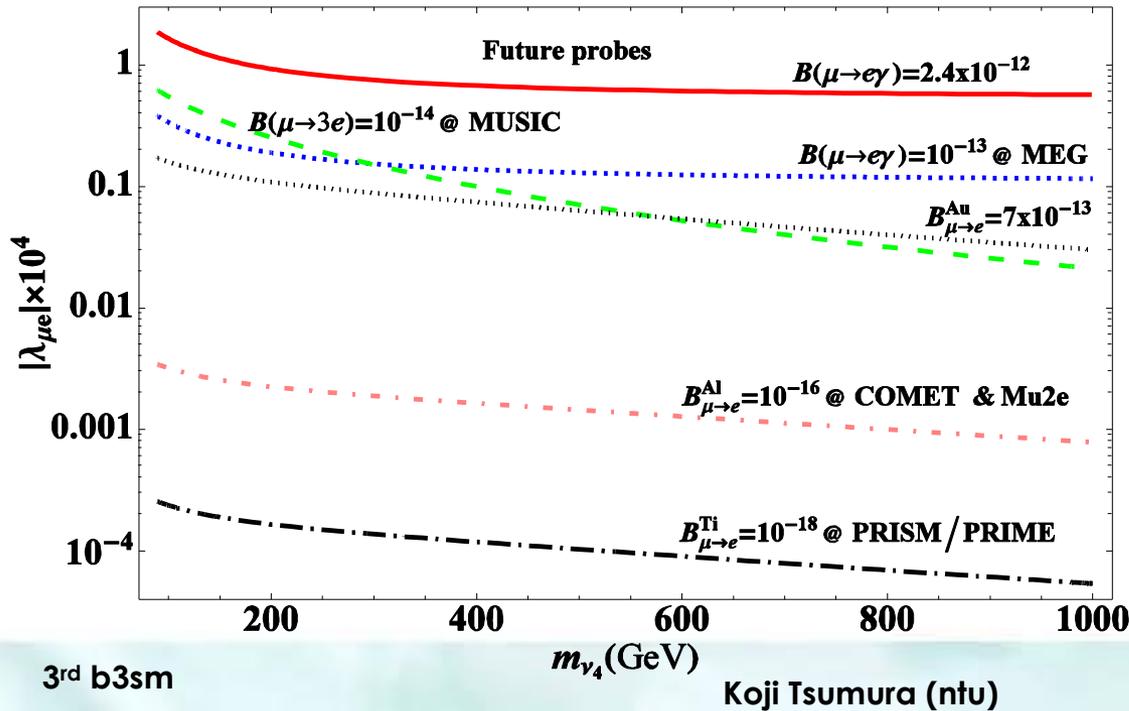
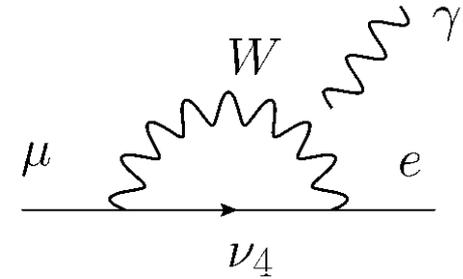
□ LFV in SM4

□ GIM suppression in SM3

□ ν_4 enhance LFV rate significantly

□ $\mu \rightarrow 3e$ is better than $\mu \rightarrow e\gamma$ for heavy ν_4 (due to Z penguin)

□ $\mu \rightarrow e$ conversion in Au gives the best bound on $\lambda_{\mu e}$



$\mu \rightarrow e$ conv. can improve $\lambda_{\mu e}$ by 3 orders of magnitude in future.