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TRANSVERSE MOMENTUM DECORRELATION AND THE WINNER-TAKES-ALL AXIS

Rudi Rahn

17-02-25,
QCD Seminar, CERN

Based on [2005.12279],[2205.05104], [2412.05358] with
Yang-Ting Chien (Georgia State), Rong-Jun Fu, Ding Yu Shao (Fudan),
Solange Schrijnder van Velzen, Wouter Waalewijn (Nikhef/UvA), Bin Wu (USC)

HISTORY

- [1905.01335]: (Chien, Shao, Wu)
  q_T decorrelation in V+jet using Standard Jet axis

- [2005.12279] & [2205.05104]: (Chien, Schrijnder van Velzen, RR, Shao, Waalewijn, Wu)
  Azimuthal decorrelation using Winner-Takes-All axis

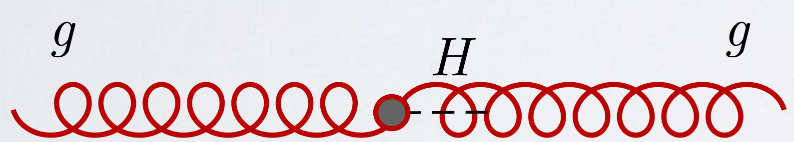
- [2412.05358]: (Fu, RR, Shao, Waalewijn, Wu)
  q_T decorrelation using WTA axis for slicing

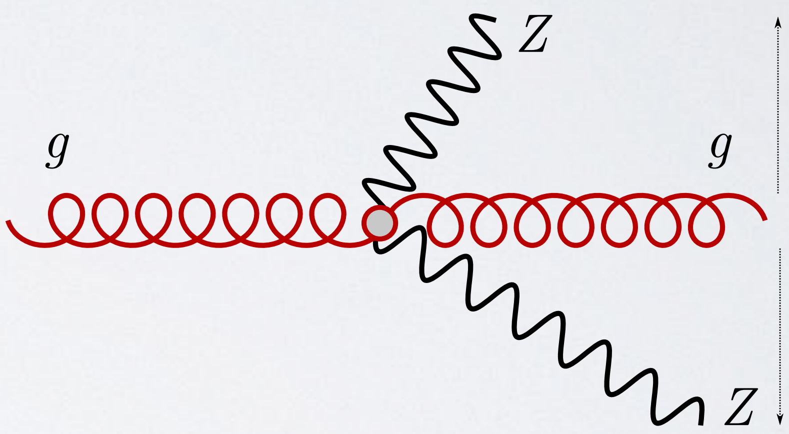
OUTLINE

- Transverse momentum decorrelation
- Soft-Collinear Effective Theory
- Standard jet axis vs Winner-Takes-All axis
- Azimuthal vs radial decorrelation
- Slicing

TRANSVERSE MOMENTUM CORRELATION

- pp collisions: no control over longitudinal momenta, but final state momenta correlated





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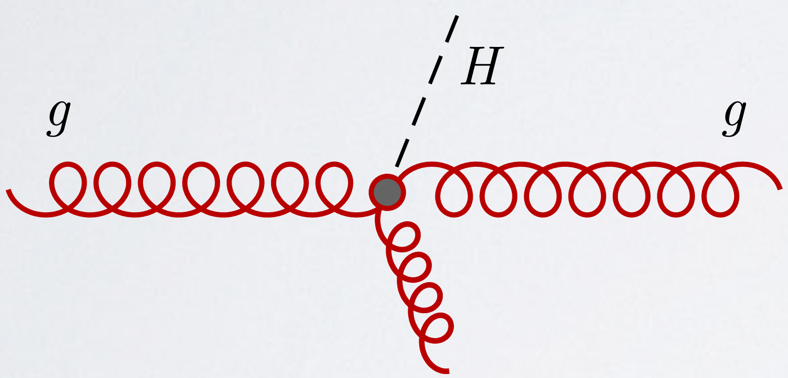
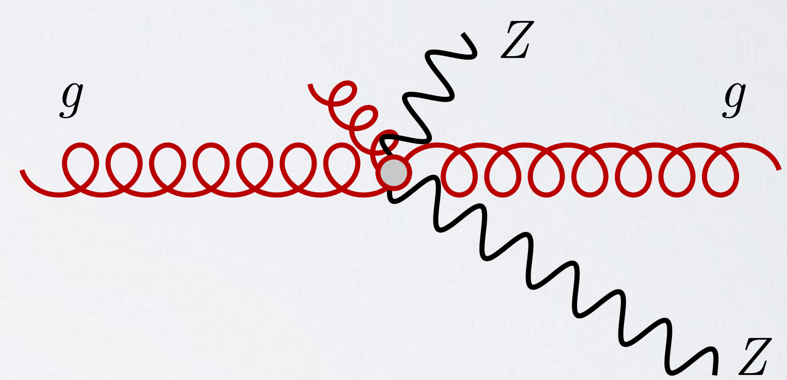




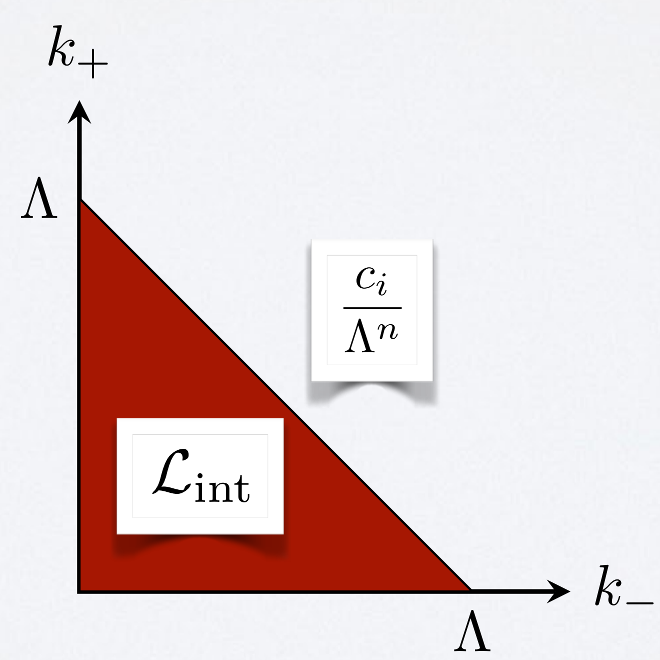
- For small decorrelation: soft and collinear emissions

- Appearance of large logarithms, e.g. $\alpha_s^n \ln^{2n} \frac{q_T}{m_H}$

SOFT-COLLINEAR EFFECTIVE THEORY

[Bauer, Fleming, Pirjol, Stewart, Rothstein, '01/'02]
[Beneke, Chapovsky, Diehl, Feldmann, '02]

- Starting point: "Wilsonian" Effective Theory



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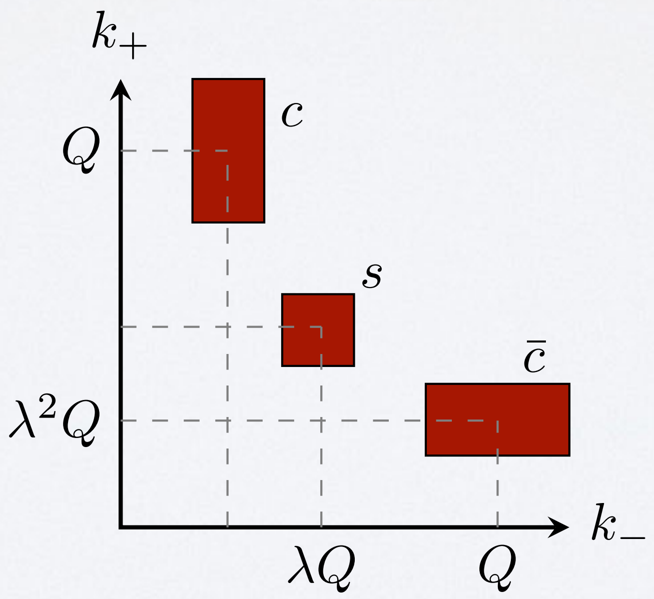

For Higgs @ small q_T:

$$\lambda = \frac{q_T}{m_H}\,, \; Q = m_H$$

SOFT-COLLINEAR EFFECTIVE THEORY

[Bauer, Fleming, Pirjol, Stewart, Rothstein, '01/'02]
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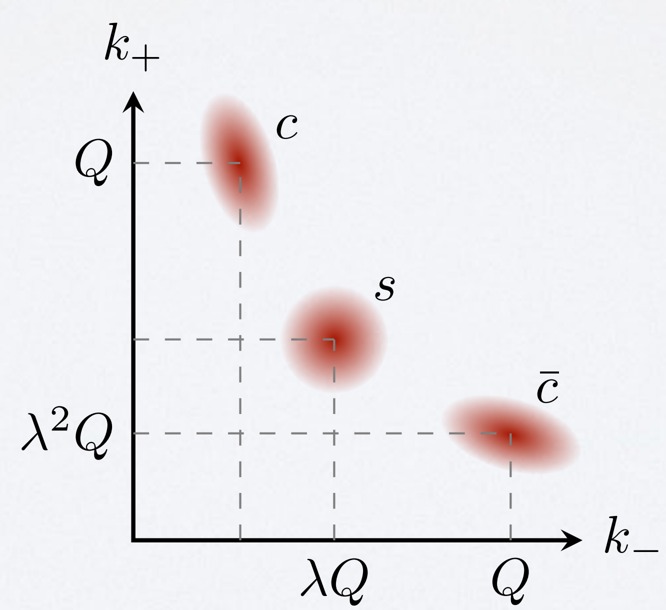
- Starting point: "Wilsonian" Effective Theory
- With complex selection criteria
- We don't like cutoffs → "continuum" EFT



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SOFT-COLLINEAR EFFECTIVE THEORY

[Bauer, Fleming, Pirjol, Stewart, Rothstein, '01/'02]
[Beneke, Chapovsky, Diehl, Feldmann, '02]

- Calculation factorises
- One multi-scale problem → multiple single-scale problems

$$\mathrm{d}\sigma = H \times B_a \otimes B_b \otimes S$$

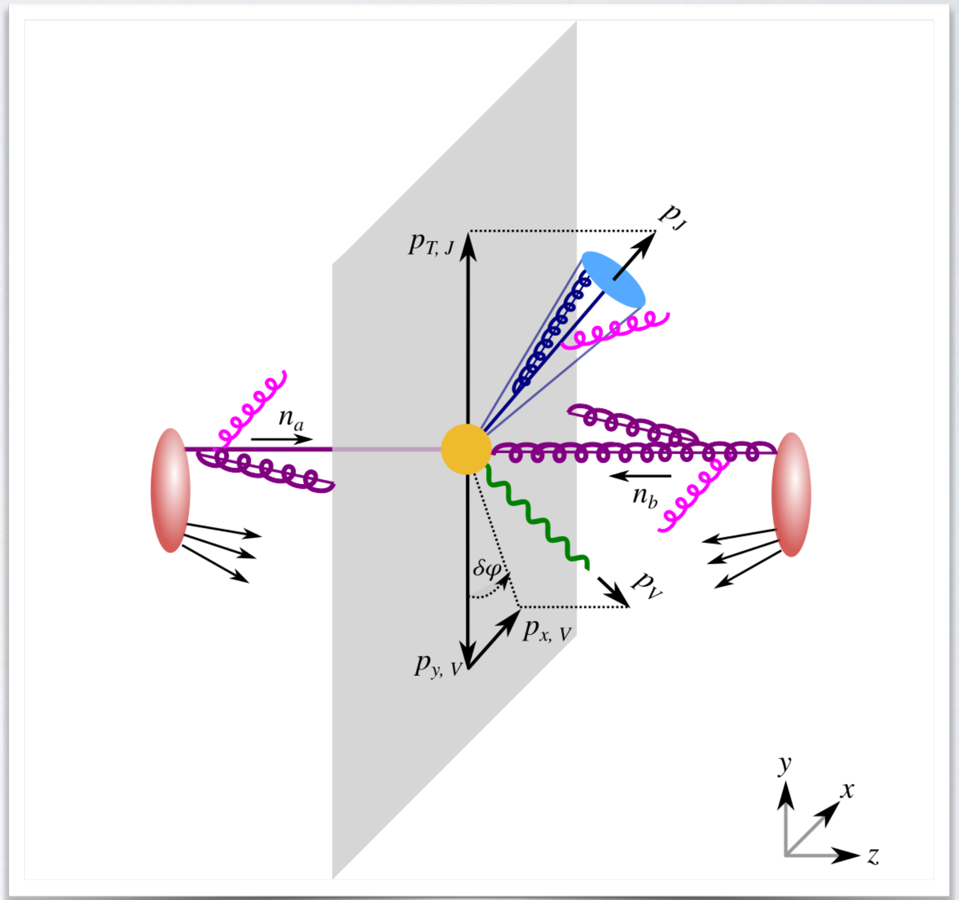
$$\mu_H \sim m_H$$
$$\mu_B = \mu_S \sim m_H \lambda = q_T$$

- Hierarchies between momentum scalings can be exploited
- Resummation via Renormalisation Group Equations

TRANSVERSE MOMENTUM DECORRELATION

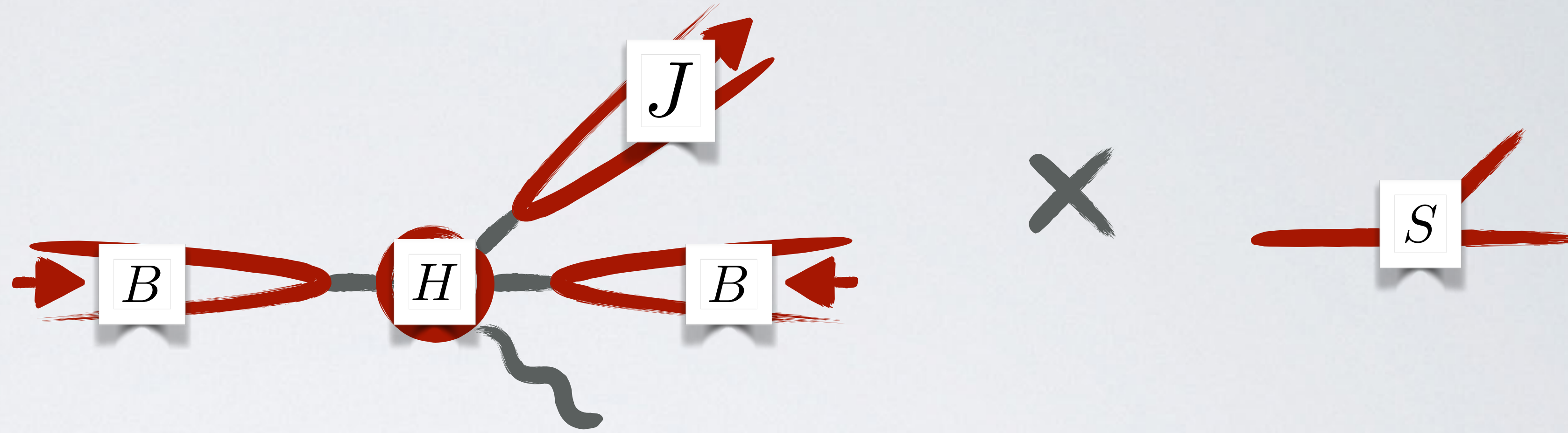
- Laboratory: V+jet, $R \gg \delta\varphi$
- Leading partonic: planar
- Soft/Collinear emissions:

$$\vec{q}_T = \vec{p}_{T,V} + \vec{p}_{T,J} \neq 0$$



GENERAL SETUP

- Expectation:

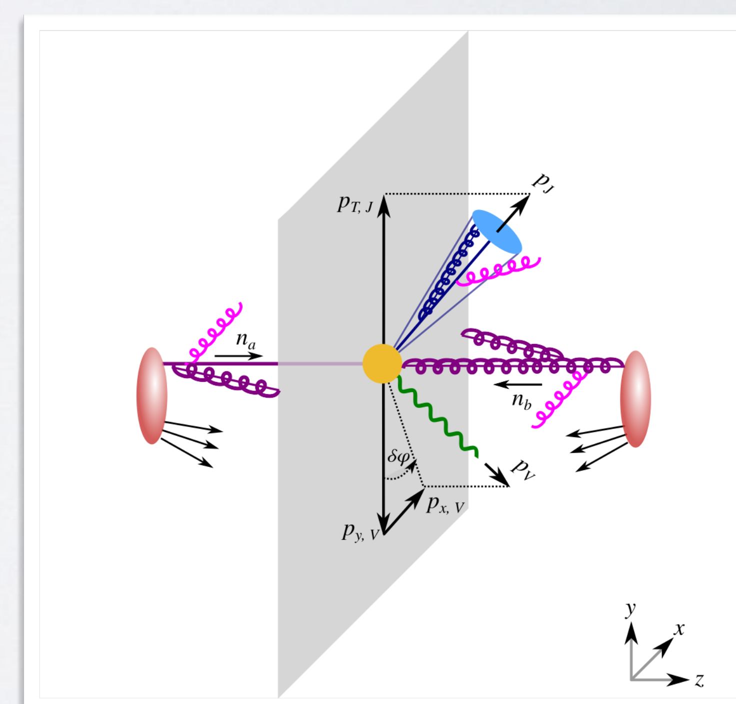


$$d\sigma = H \otimes B_a \otimes B_b \otimes J \otimes S$$

- How do we represent the jet?

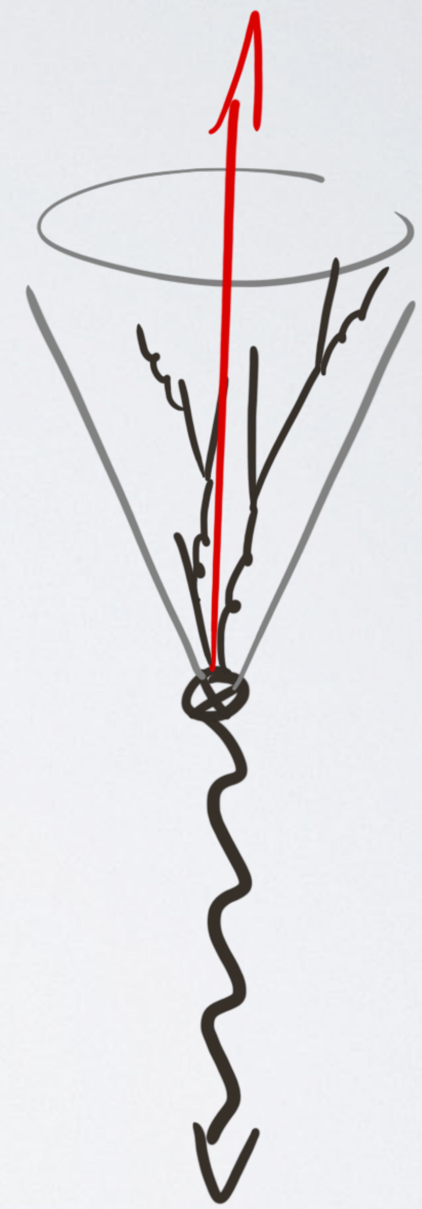
  - Standard jet axis: [Chen, Qin, Wang, Wei, Zhao, (Zhang)² '18] [Buonocore, Grazzini, Haag, Rottoli '21] [Sun, Yan, (Yuan)² '18] [Hatta, Yuan, Xiao, Zhou '21] [Chien, Shao, Wu '19]

  - Winner-Takes-All Axis: this talk



STANDARD JET

- Standard Jet axis: $\vec{p}_{T,J} = \sum_{i \in \text{jet}} \vec{p}_{T,i}$



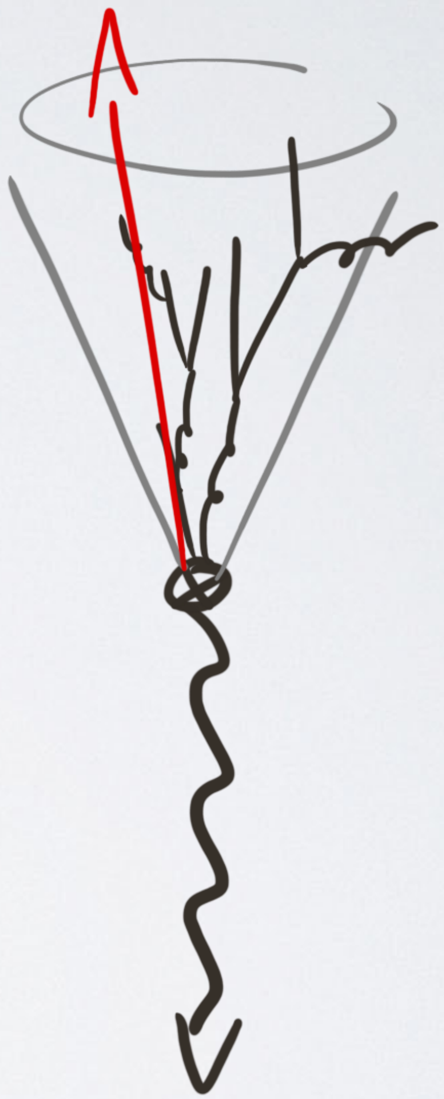
STANDARD JET

- Standard Jet axis: $\vec{p}_{T,J} = \sum_{i \in \mathrm{jet}} \vec{p}_{T,i}$

- Contribution to $\vec{q}_T$ requires out-of-jet radiation

  ➞ Non-global [Dasgupta, Salam, '01]

- Swiss treatment required [Chien, Shao, Wu, '19]



THE WINNER-TAKES-ALL AXIS

[Salam, unpublished]
[Bertolini, Chan, Thaler, '14]

- Endpoint of a reclustering sequence, based on some distance measure

- Clustering step: closest emission pair i,j with $|\vec{p}_{T,i}|$, $\hat{n}_i$ & $|\vec{p}_{T,j}|$, $\hat{n}_j$

$$|\vec{p}_{T,i+j}| = |\vec{p}_{T,i}| + |\vec{p}_{T,j}|$$

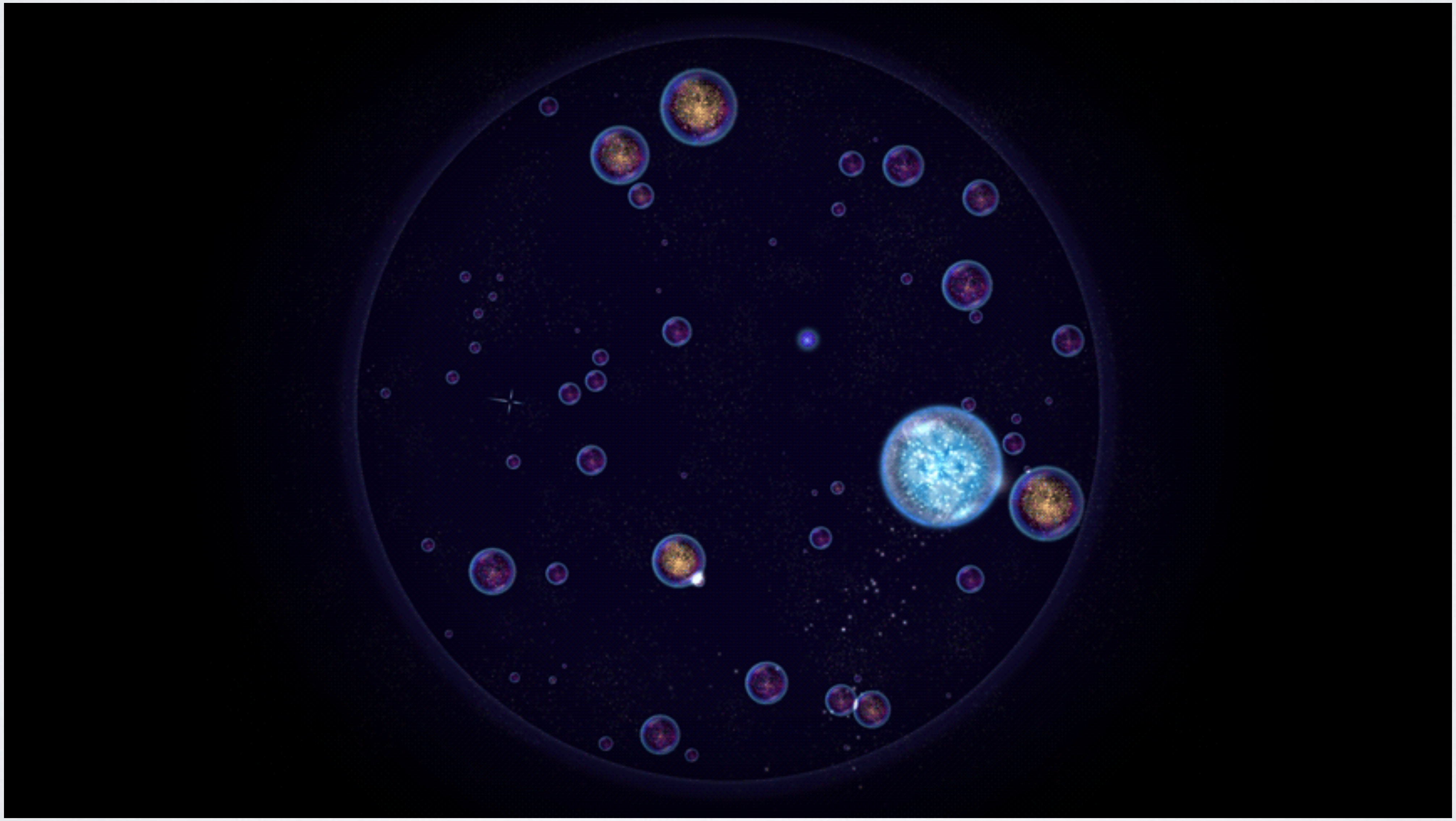
$$\hat{n}_{i+j} = \begin{cases} \hat{n}_i & \text{if } |\vec{p}_{T,i}| > |\vec{p}_{T,j}| \\ \hat{n}_j & \text{if } |\vec{p}_{T,i}| < |\vec{p}_{T,j}| \end{cases}$$

- NOT the "highest energetic particle", clustering ensures IRC safety!

WTA FOR BACTERIA

- Osmos

[Hemisphere Games, '09]



THE WINNER-TAKES-ALL AXIS

[Salam, unpublished]
[Bertolini, Chan, Thaler, '14]

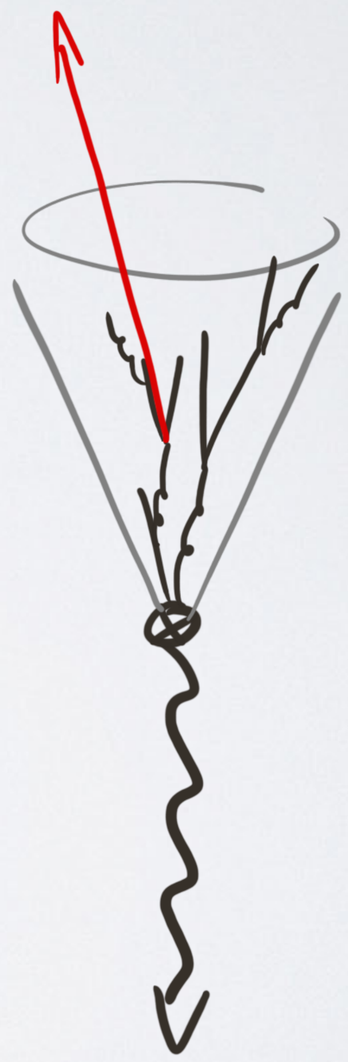
- Powerful when combined with SCET: $p_{T,c} \sim Q, \; p_{T,S} \sim Q\lambda$
  - Direction purely a collinear affair: $\max(p_{T,c}, p_{T,S}) = p_{T,c}$
  - Soft subleading in magnitude: $p_{T,c} + p_{T,S,\text{in-jet}} \approx p_{T,c}$

$\Rightarrow$ No non-global logarithms*
[Larkoski, Neill, Thaler, '14]

WTA AXIS IN ACTION

- Even if all radiation is inside the jet:

$$\vec{p}_{T,J} \neq \sum_{i \in \text{jet}} \vec{p}_{T,i}$$



WTA AXIS IN ACTION

- Even if all radiation is inside the jet:

$$\vec{p}_{T,J} \neq \sum_{i \in \text{jet}} \vec{p}_{T,i}$$

- In- and out-of-jet soft radiation contribute through recoil

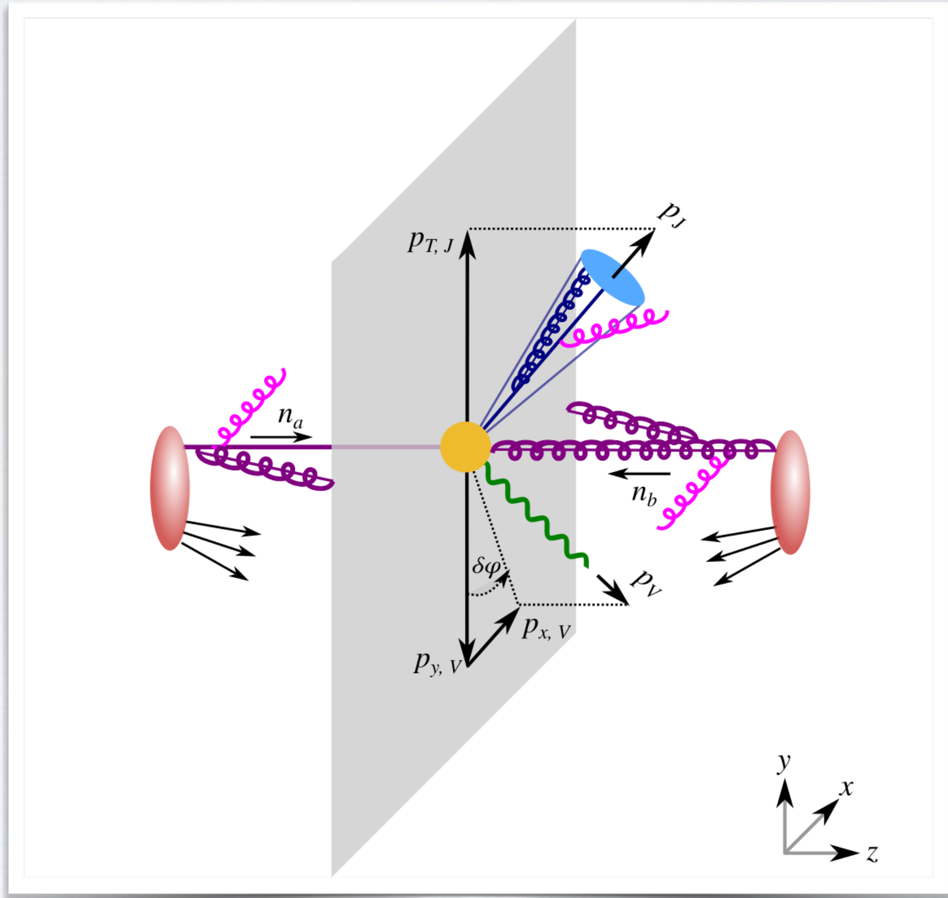
- Only in-jet soft radiation contributes to $|\vec{p}_{T,J}|$: power suppressed

TRANSVERSE MOMENTUM DECORRELATION

- Two-component decorrelation:

$$\vec{q}_T = \vec{p}_{T,V} + \vec{p}_{T,J}$$

- Align y-axis with jet axis:
  - Azimuthal decorrelation $q_x = p_{x,V}$
  - Radial decorrelation $q_y = p_{y,V} + |\vec{p}_{T,J}|$



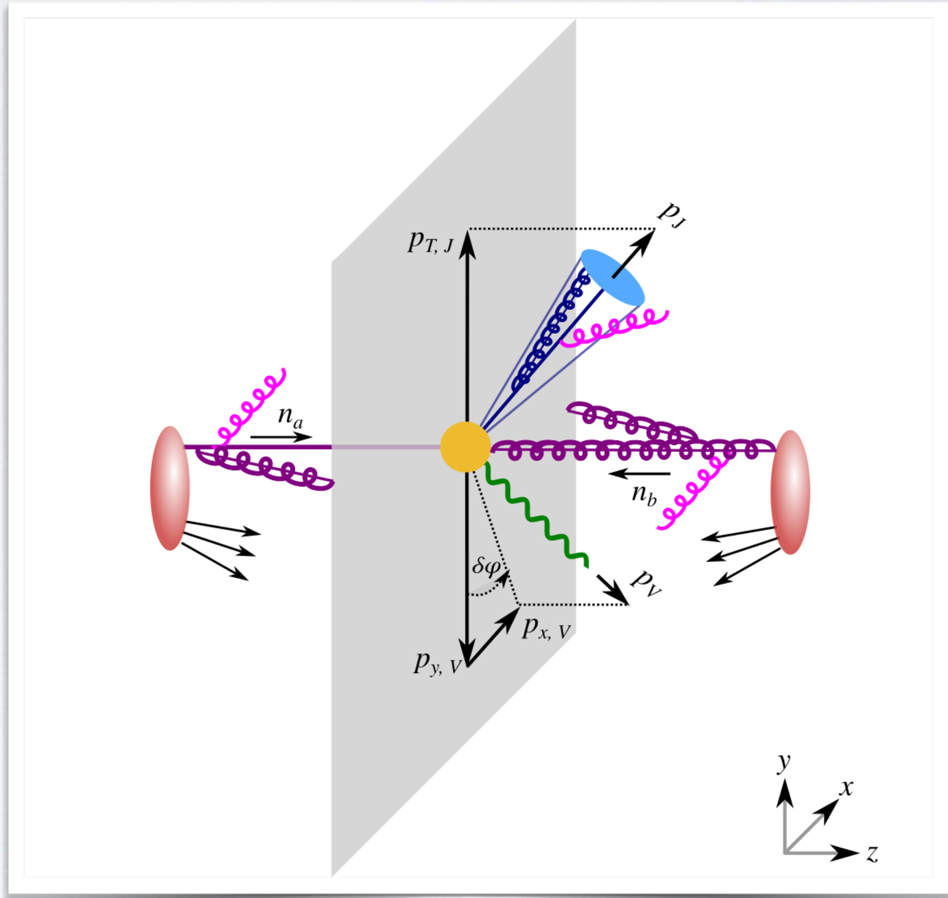
AZIMUTHAL DECORRELATION

- From momentum conservation:

$$q_x = p_{x,V} = -p_{x,c} - p_{x,a} - p_{x,b} - p_{x,S}$$

$$= p_{T,V} \sin \delta\varphi$$

- Factorisation is very simple:
  - TMD Beam function
  - TMD Soft function (+boost)
  - WTA jet function



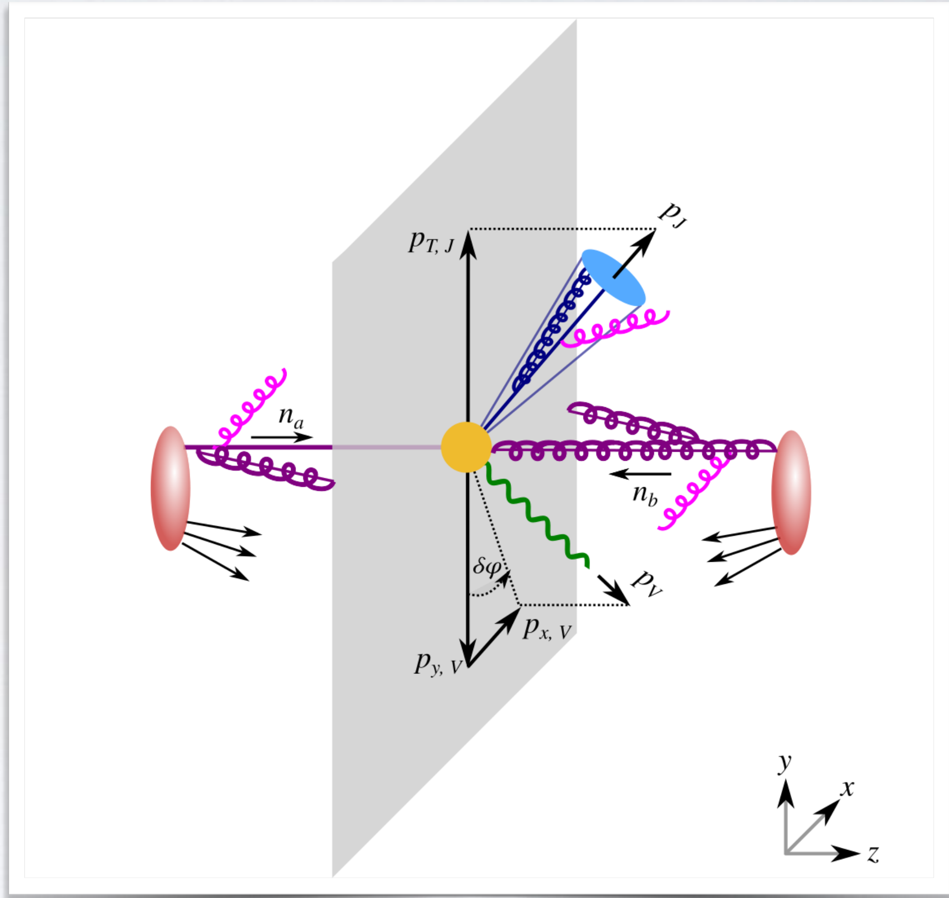
RADIAL DECORRELATION

- From momentum conservation:

$$\begin{aligned} q_y &= |\vec{p}_{T,J}| + p_{y,V} \\ &= |\vec{p}_{T,J}| - |\vec{p}_{T,V}| \cos\delta\varphi \end{aligned}$$

$$\approx |\vec{p}_{T,J}| - |\vec{p}_{T,V}| + \frac{1}{2}|\vec{p}_{T,V}|\delta\varphi^2$$

$$\text{“}\mathcal{O}(Q) - \mathcal{O}(Q) + \mathcal{O}(Q\lambda)\text{”}$$



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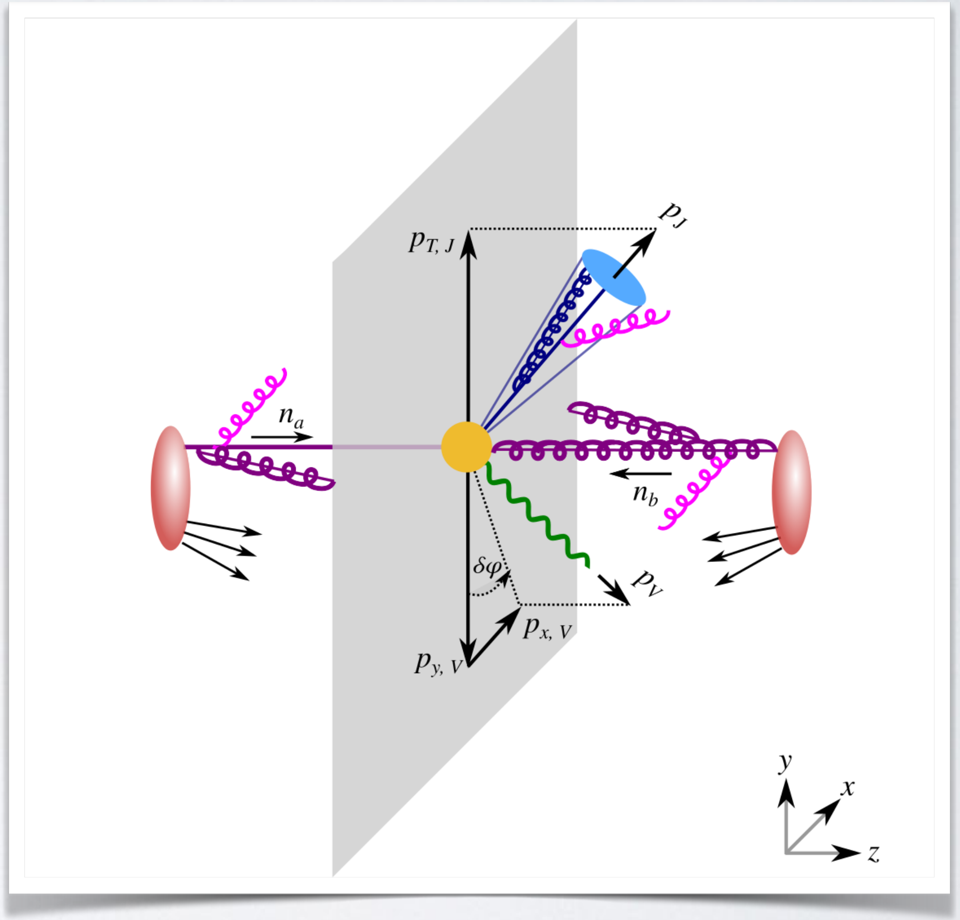
\end{aligned}
$$

$$
\approx |\vec{p}_{T,J}| - |\vec{p}_{T,V}| + \frac{1}{2}|\vec{p}_{T,V}|\delta\varphi^2
$$

Are these the same?



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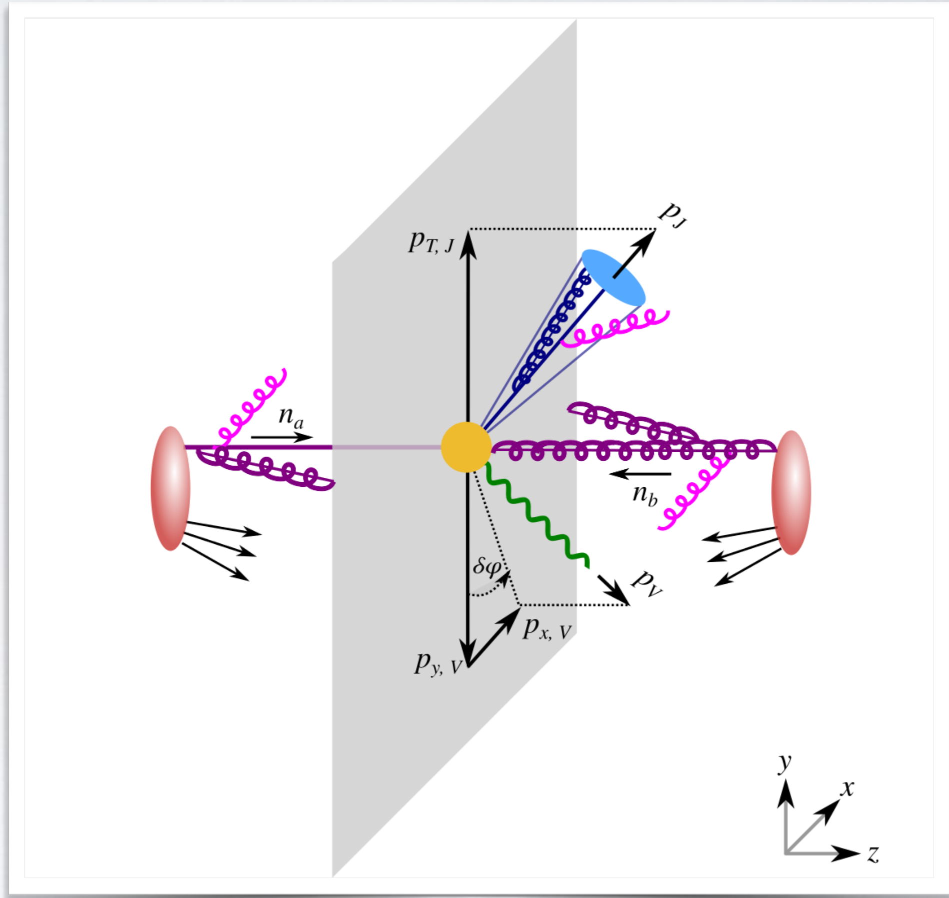
&\approx |\vec{p}_{T,J}| - |\vec{p}_{T,V}| + \frac{1}{2}|\vec{p}_{T,V}|\delta\varphi^2
\end{aligned}
$$

Are these the same?

No!



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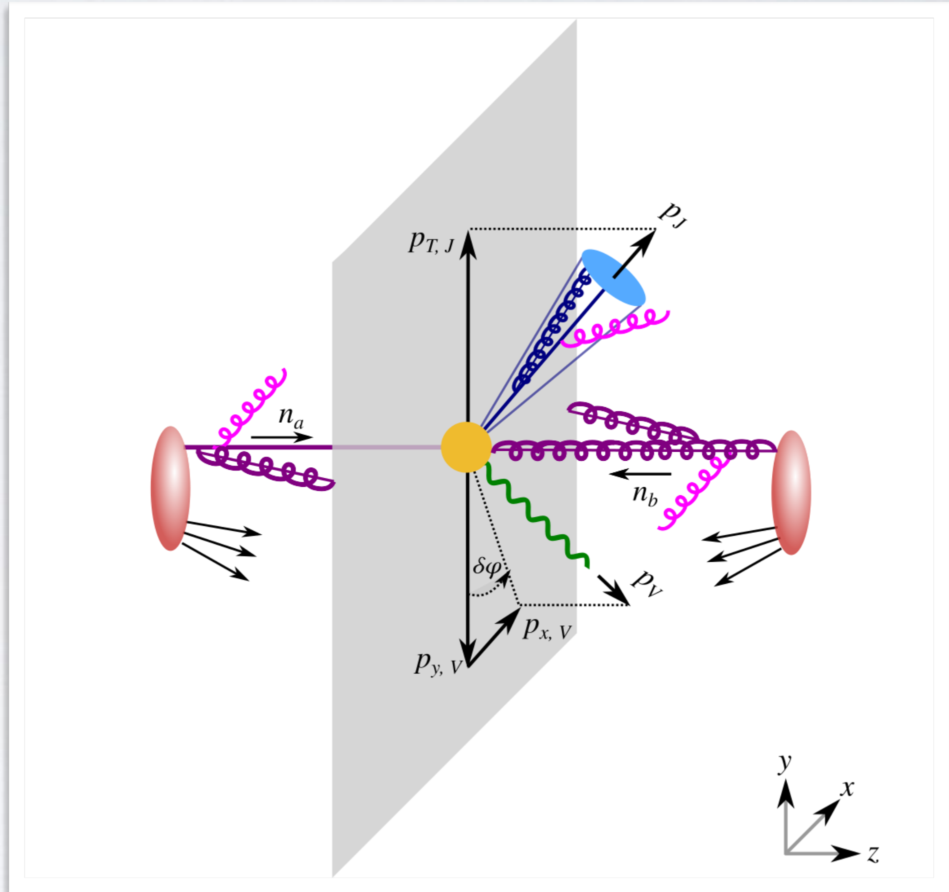
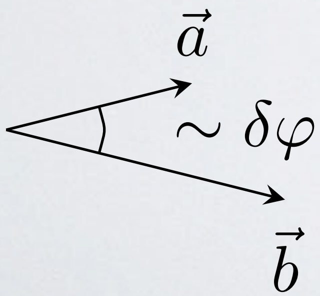


$$|\vec{p}_{T,J}| = \sum_{i\in\text{jet}} |\vec{p}_{T,i}| \qquad |\vec{p}_{T,V}| = \left| - \sum_{i\neq V} \vec{p}_{T,i} \right|$$

I) scalar sum vs. vector sum



$$\Rightarrow \quad (|\vec{a}| + |\vec{b}|) - |(\vec{a} + \vec{b})| \sim \delta\varphi^2$$

RADIAL DECORRELATION

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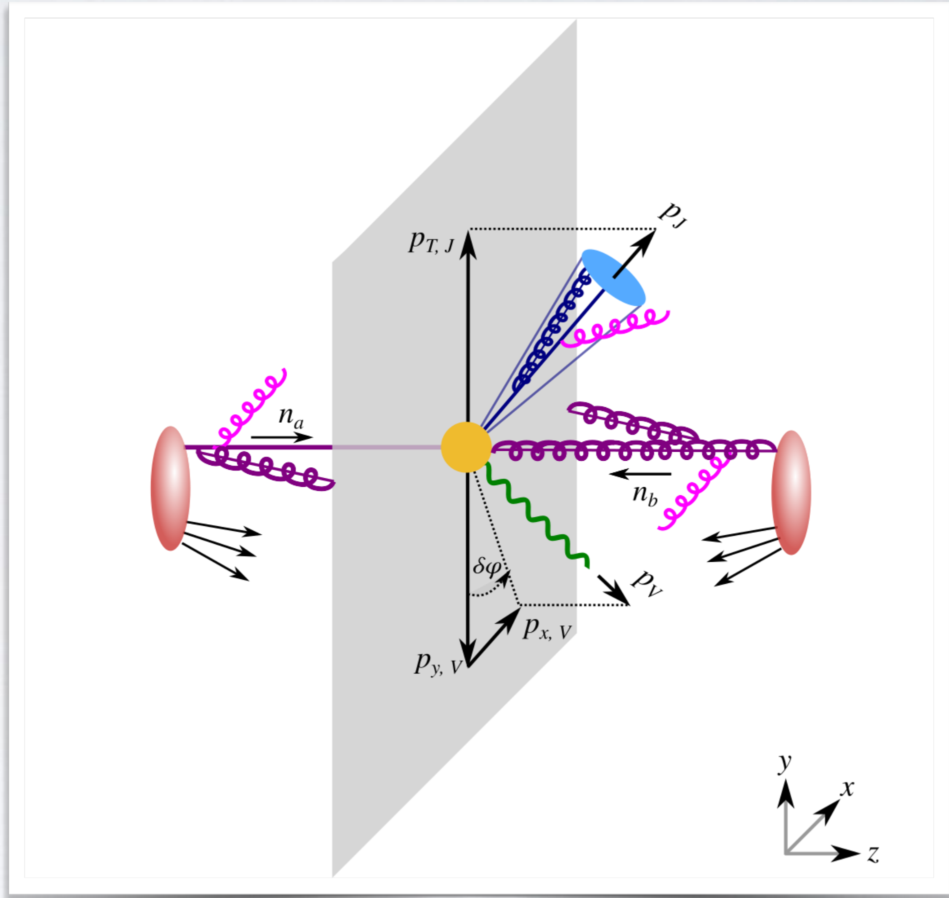

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I) scalar sum vs. vector sum

II) soft out-of-jet emissions

$$\text{``}\ (p_{T,c} + p_{T,S,\text{in-jet}}) - (p_{T,c} + p_{T,S}) = -p_{T,S,\text{out-of-jet}}\ \text{''}$$

$p_{T,J} : \mathcal{O}(Q) + \mathcal{O}(Q\lambda)$ $\quad p_{T,V} : \mathcal{O}(Q) + \mathcal{O}(Q\lambda)$ $\quad \mathcal{O}(Q\lambda)$

RADIAL DECORRELATION

- From momentum conservation:

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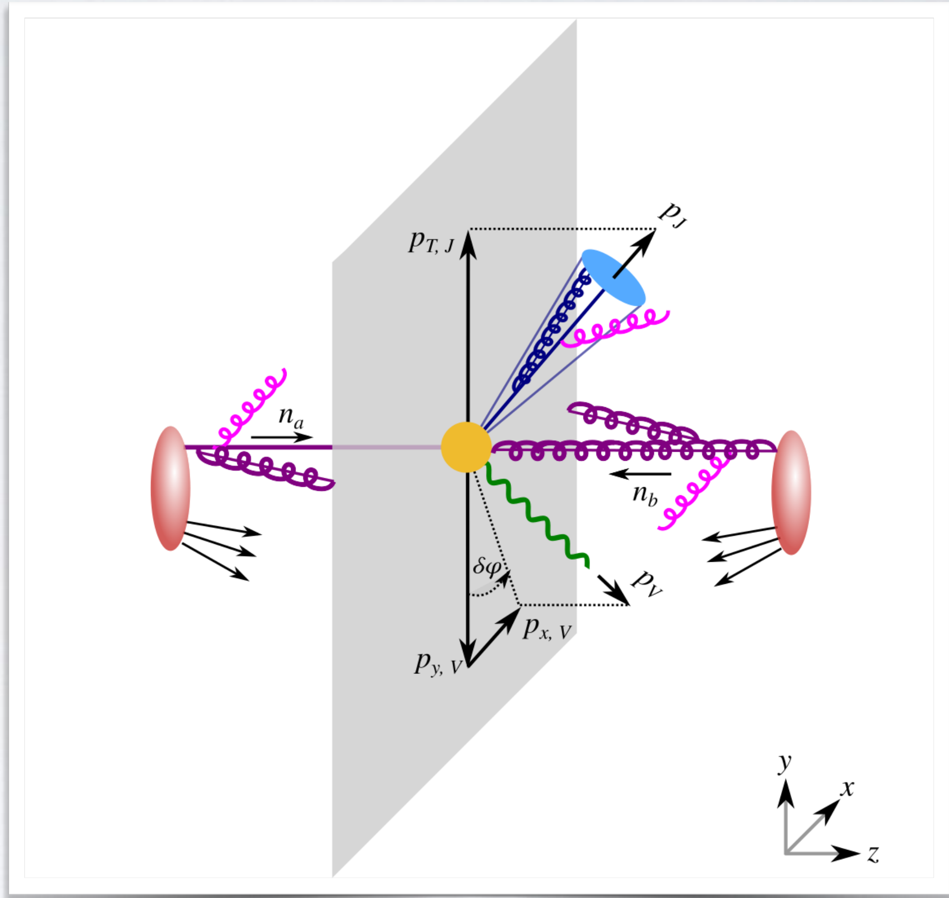


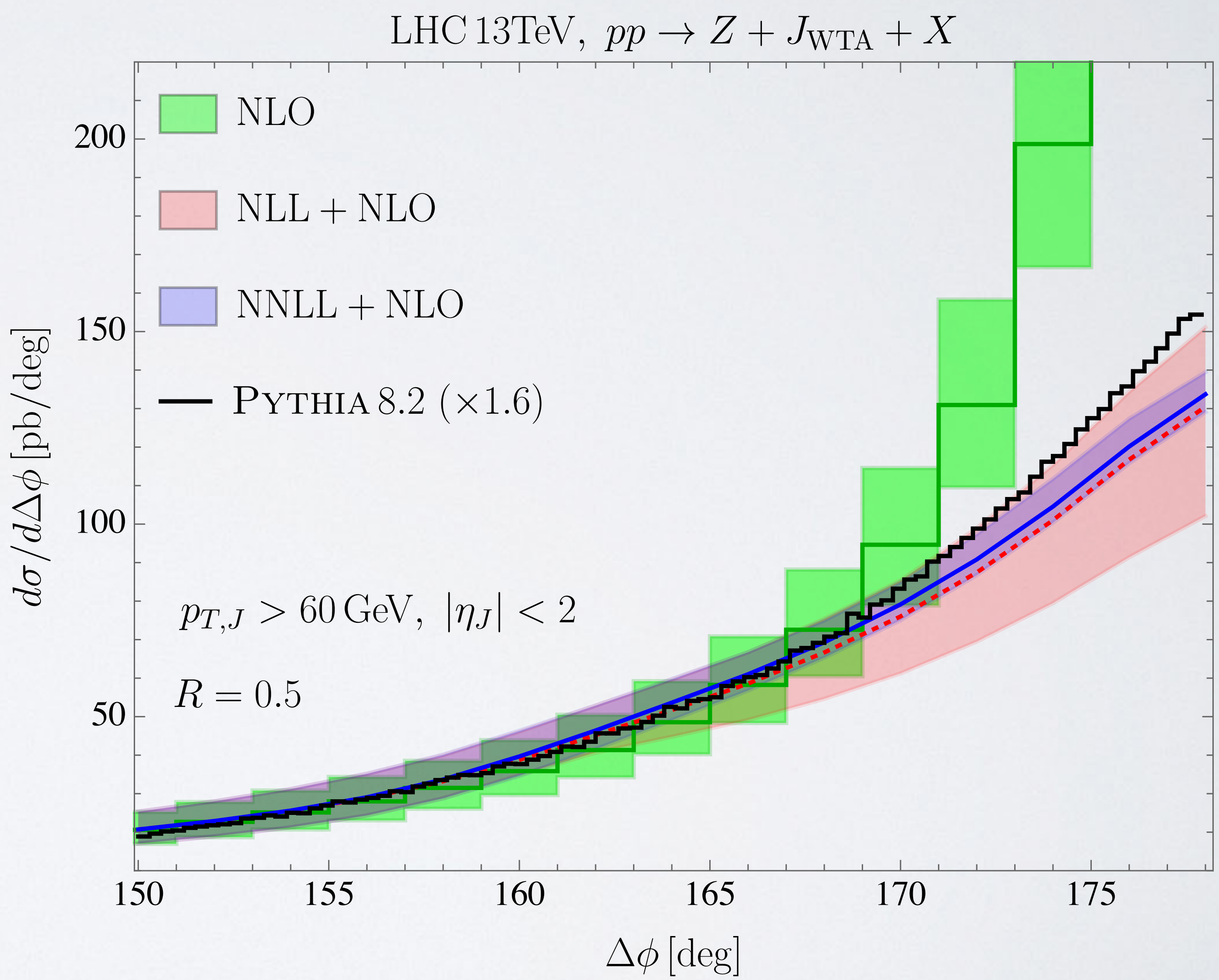
I) scalar sum vs. vector sum

II) soft out-of-jet emissions

$\Rightarrow$ Complicated, and non-global

CALCULATIONS AND TRIVIA

- Resummation of azimuthal decorrelation to NNLL
- Linearly polarised NNLO beam and jet functions
  [Catani, Grazzini, '11]
- "Giant K-factors" from EW
  [Rubin, Salam, Sapeta, '10]
- Track-based measurements and p_T-weighted jet axes



SLICING

- How to access higher-order calculations?

$$\sigma = \sigma_V + \sigma_R$$

Individually divergent, different phase spaces

- Soft and collinear emissions difficult to observe

$\Rightarrow$ Split unobserved reals and add to virtuals

- Find resolution variable, e.g. q_T: [Catani, Grazzini, '07]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} = \int_0^\delta \mathrm{d}q_T \, \frac{\mathrm{d}\sigma_{\mathrm{SCET}}}{\mathrm{d}X\,\mathrm{d}q_T}[1 + \mathcal{O}(\delta^p)] + \int_\delta^\infty \mathrm{d}q_T \, \frac{\mathrm{d}\sigma_{\mathrm{QCD}}}{\mathrm{d}X\,\mathrm{d}q_T}$$

q_{wTa} -SLICING

- q_T -slicing vs. Jets: SJA blind to in-jet dynamics

  $\Rightarrow$ N-jettiness [Boughezal, Focke, Liu, Petriello, '15] [Gaunt, Stahlhofen, Tackmann, '15]

  (Kinematic dependence? Power corrections?)

  [Bell, Dehnadi, Mohrmann, RR, '23] [Campbell, Ellis, Seth, '22]

  [Agarwal, Melnikov, Pedron, '24]

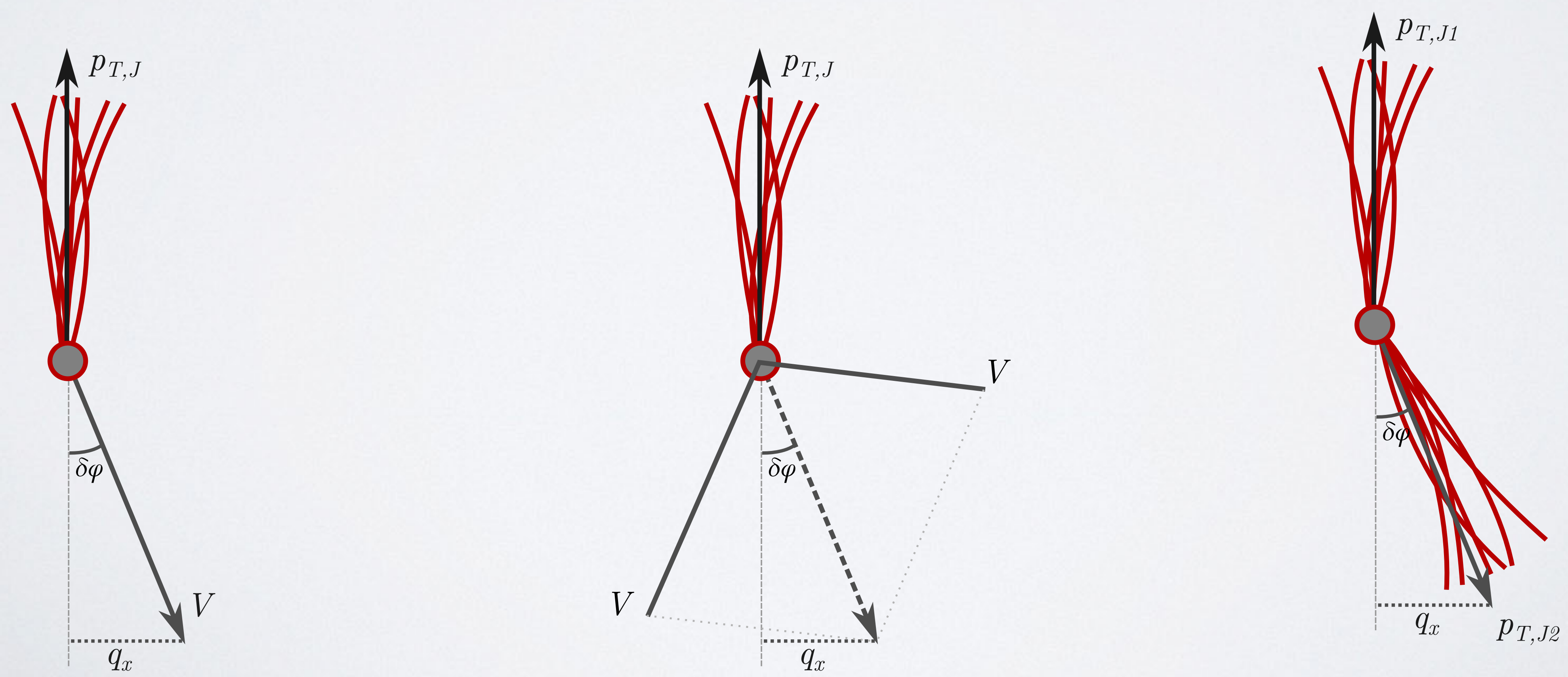
  $\Rightarrow$ k_T-ness [Buonocore, Grazzini, Haag, Rottoli, Savoini '22]

  (Higher orders?)

- Winner-Takes-All axis isn't blind!

q_{wTa}-IMBALANCE

- Azimuthal decorrelation is much nicer than radial
  - $\Rightarrow$ Can we avoid using the radial decorrelation?
- For planar Born processes azimuthal suffices:



UNRESOLVED PIECE

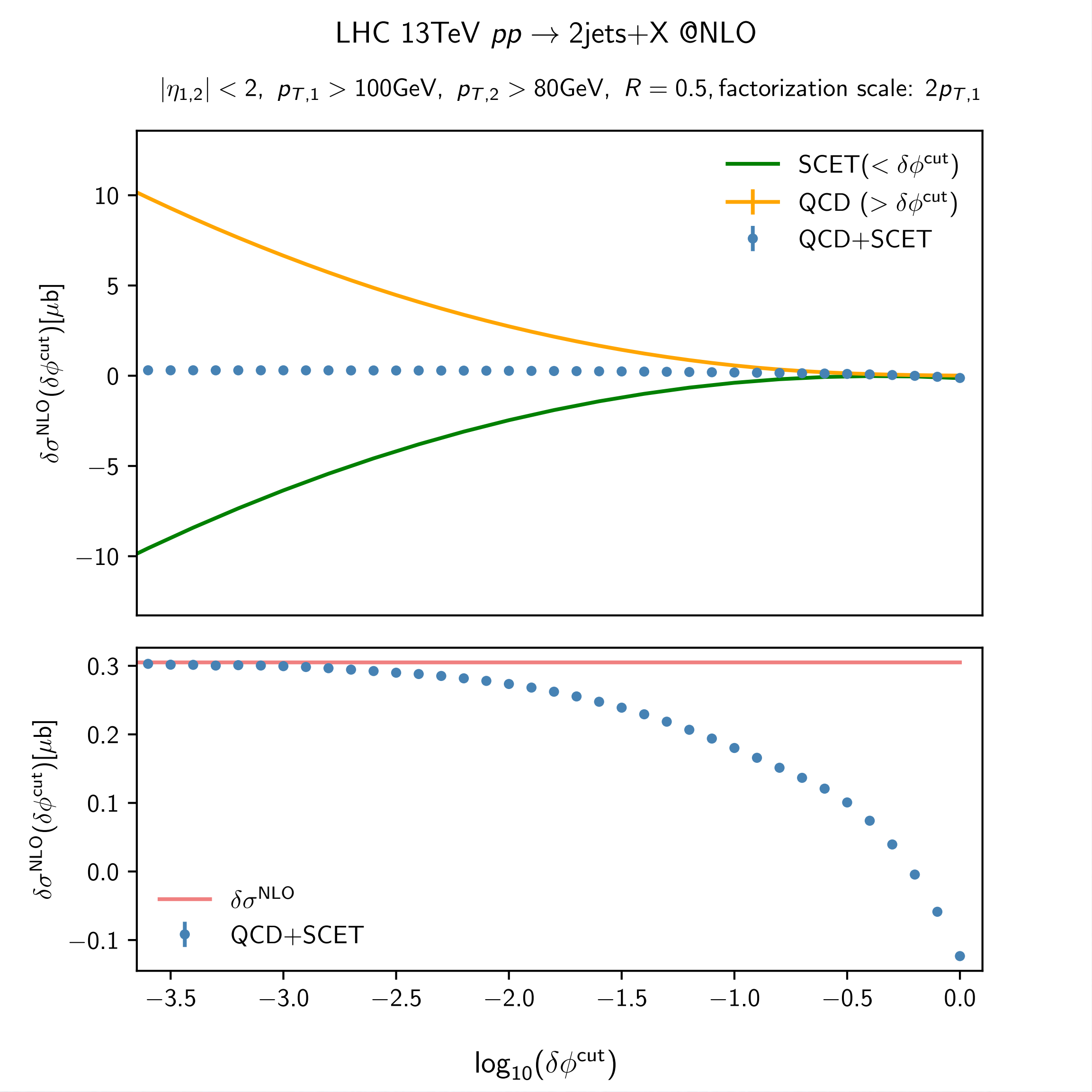
- Factorisation:

$$\frac{\mathrm{d}\sigma_{\mathrm{SCET}}}{\mathrm{d}p_{T,1}\,\mathrm{d}\eta_1\,\mathrm{d}\eta_2\,\mathrm{d}q_x} = \int \frac{\mathrm{d}b_x}{2\pi}\, e^{\mathrm{i}q_x b_x} \sum_{i,j,k,\ell} B_i(x_a, b_x)\, B_j(x_b, b_x)\, \mathscr{J}_k(b_x)\, \mathscr{J}_\ell(b_x) \times \mathrm{tr}\big[\hat{\mathcal{H}}_{ij\to k\ell}(p_{T,1}, \eta_1 - \eta_2)\, \hat{S}_{ijk\ell}(b_x, \eta_1, \eta_2)\big]$$

- Involves:
  - Standard TMD beam function
  - Standard TMD soft function
  - WTA TMD jet function
- Beyond NLO: linear polarisations in beam/jet

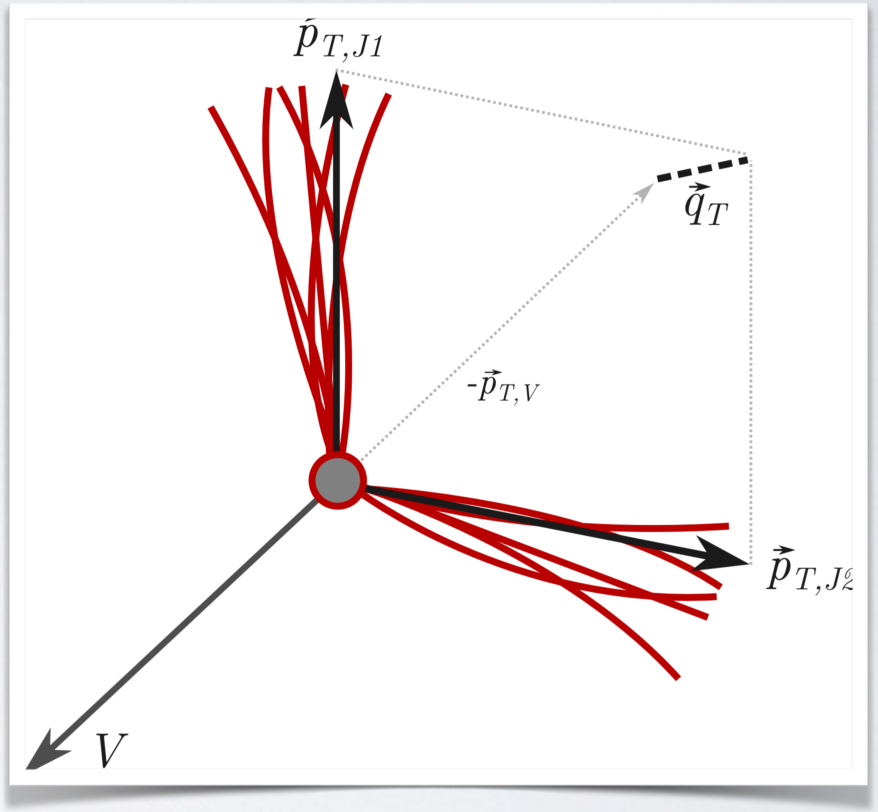
NLO VERIFICATION

- Missing for NNLO: gluon jet function



NON-PLANAR CASES

- Radial decorrelation required
- A priori SCET+ framework: [Procura, Waalewijn, Zeune, '15, '18]
  - Complicated mode structure
  - Multiple hierarchies $(q_x \sim q_y, q_x \sim \sqrt{Q q_y}, \ldots)$
- But: $|\vec{q}_T| = \sqrt{q_x^2 + q_y^2}$ suffices!



DIJET AGAIN

- Modes inherited from azimuthal decorrelation

$$q_x = -p_{x,c} - p_{x,a} - p_{x,b} - p_{x,S} \qquad \sim \mathcal{O}(Q\delta\varphi)$$

$$q_y = p_{y,J} - |\vec{p}_{T,c}| \cos\phi_c - p_{y,S} - p_{y,a} - p_{y,b}$$

$$= \underbrace{-p_{y,a} - p_{y,b}}_{\text{beam functions}} + \underbrace{\sum_{i\in(s\in J)} |p_{T,i}| - p_{y,S}}_{\text{soft function}} + \underbrace{\sum_{i\in c} |\vec{p}_{T,i}| \frac{\phi_c^2}{2} + \frac{\sum_{(i<j)\in c} |\vec{p}_{T,i}||\vec{p}_{T,j}|\phi_{ij}^2}{2\sum_{i\in c} |\vec{p}_{T,i}|}}_{\text{jet function}}$$

$$= -p_{y,S,\text{out-of-jet}} \qquad \sim \mathcal{O}(Q\delta\varphi^2)$$

- q_y behaves like Standard Jet axis case!

q_{WTa}-DIJET

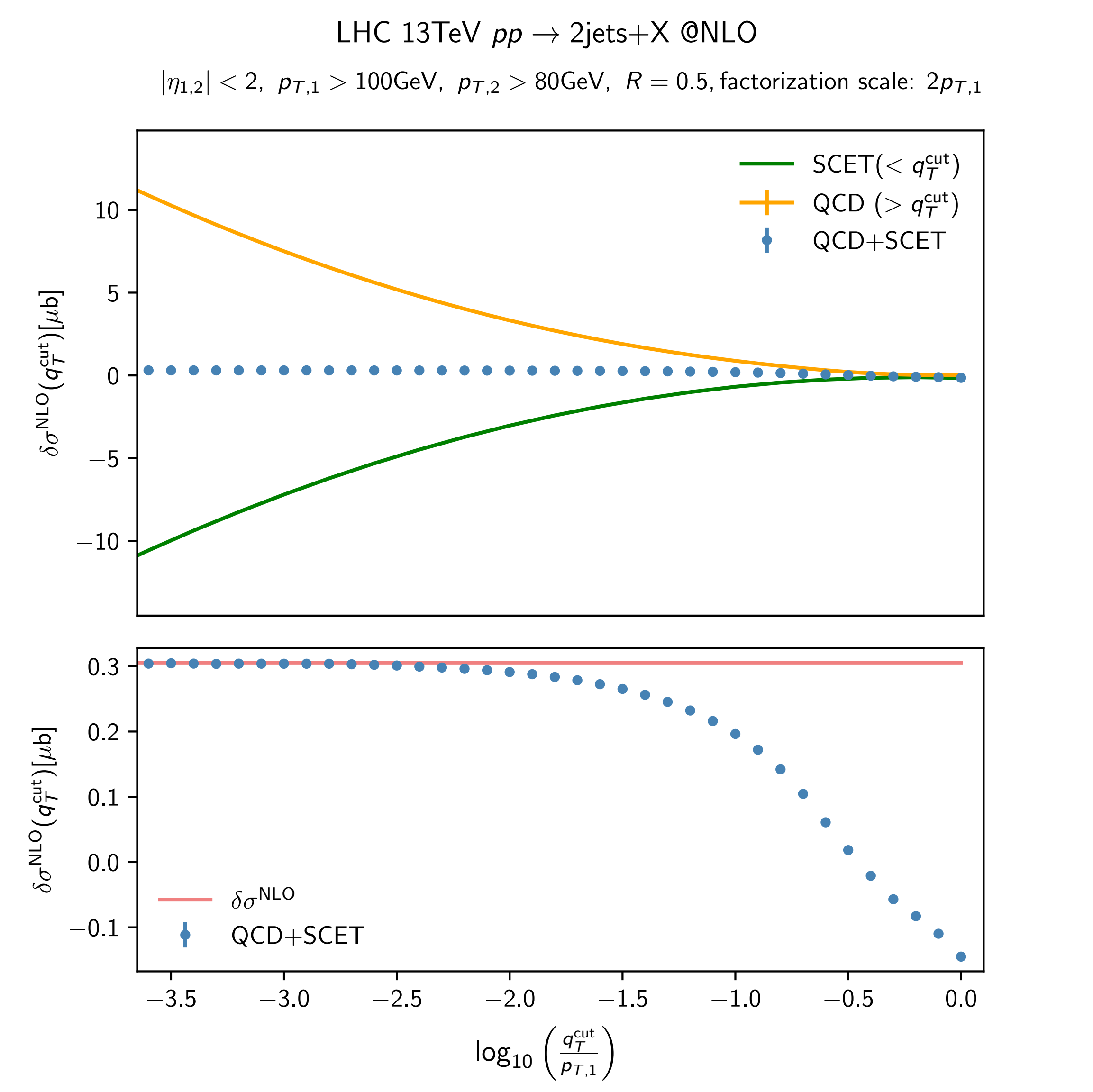
- Factorisation:

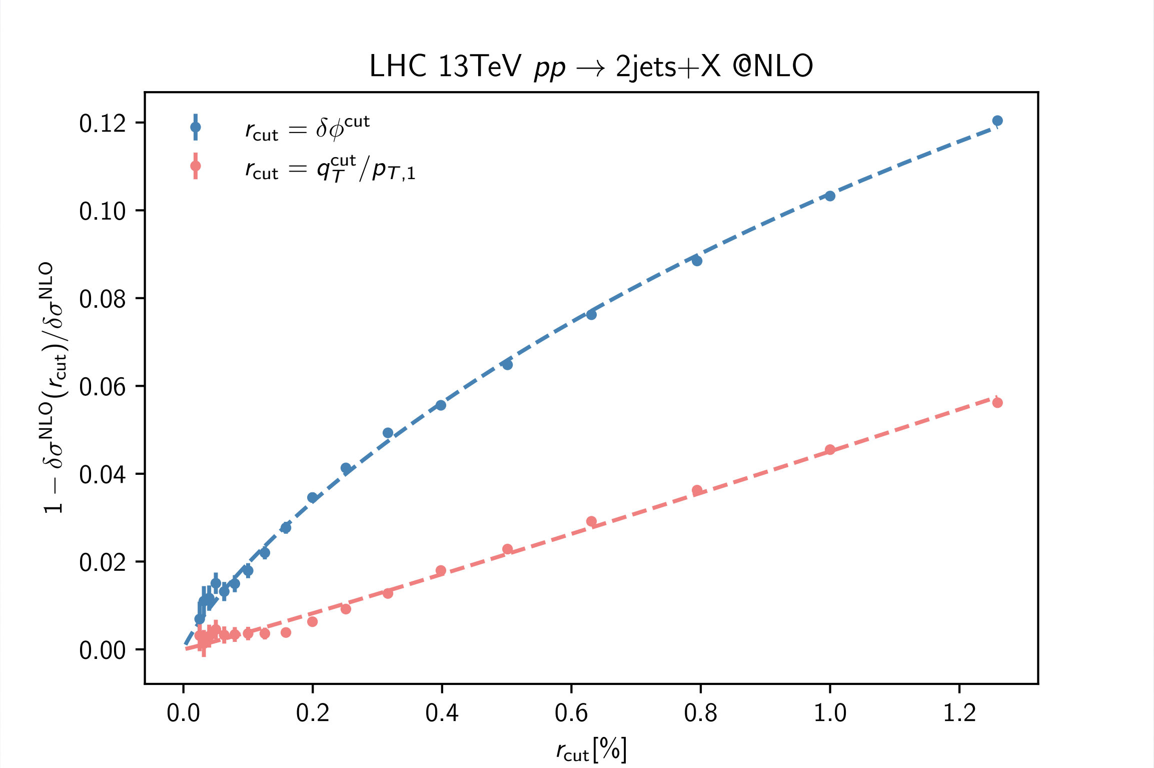
$$\frac{\mathrm{d}\sigma_{\mathrm{SCET}}}{\mathrm{d}p_{T,1}\,\mathrm{d}\eta_1\,\mathrm{d}\eta_2\,\mathrm{d}q_T} = q_T \int \frac{\mathrm{d}^2\vec{b}_T}{2\pi}\, J_0(q_T|\vec{b}_T|) \sum_{i,j,k,\ell} B_i(x_a,\vec{b}_T)\, B_j(x_b,\vec{b}_T)\, \mathscr{J}_k(b_x)\, \mathscr{J}_\ell(b_x) \times \mathrm{tr}\big[\hat{\mathcal{H}}_{ij\to k\ell}(p_{T,1},\eta_1-\eta_2)\, \hat{S}_{ijk\ell}(\vec{b}_T,\eta_1,\eta_2,R)\big]$$

- Involves:
  - Standard TMD beam function
  - Dedicated soft function
  - WTA TMD jet function

- Soft function:
  - Outside jet: $\vec{q}_T = \begin{pmatrix} q_x \\ q_y \end{pmatrix}$
  - Inside jet: $\begin{pmatrix} q_x \\ 0 \end{pmatrix}$

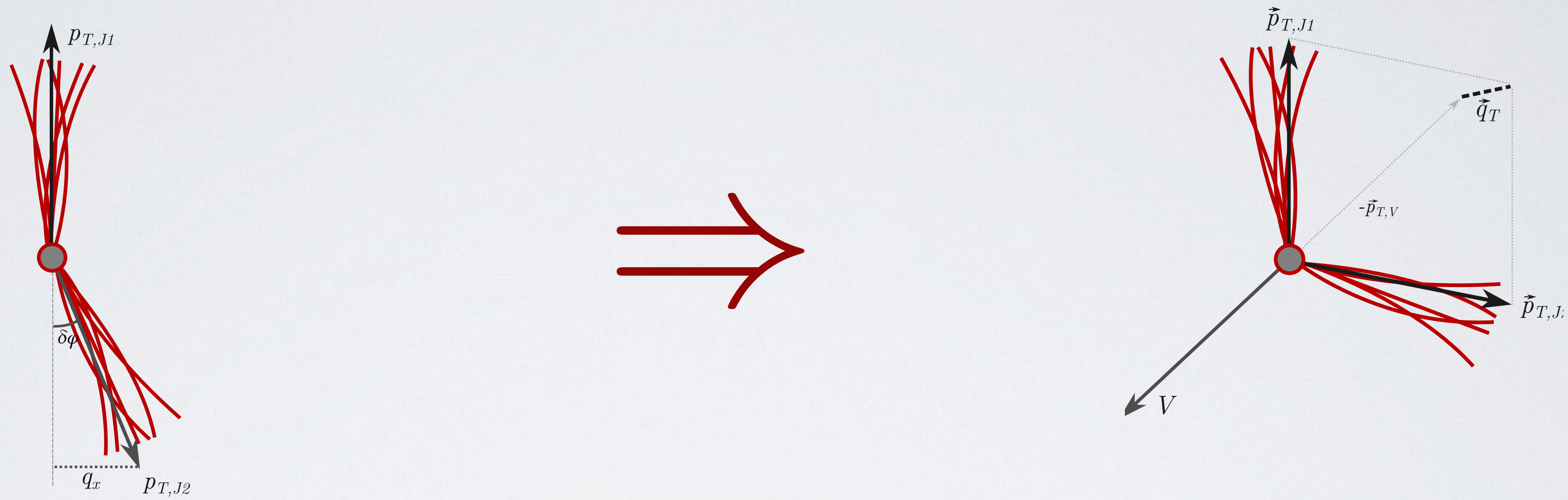
COMPARE





- Full q_T performs better

MULTIJET CASES



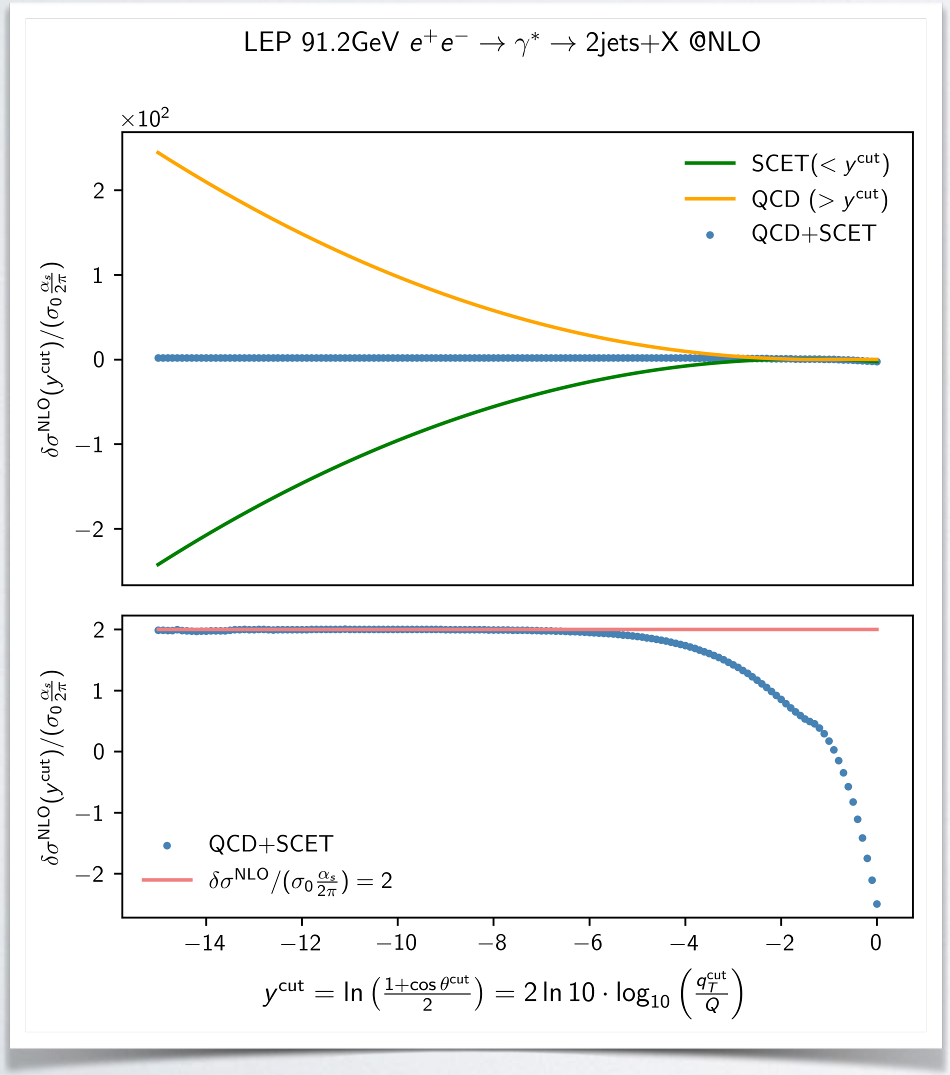
- x/y now means perpendicular/parallel to a jet
- Jet function: $\mathscr{J}(b_x) \to \mathscr{J}(b_{\perp,J})$
- Soft function:
  - Outside jet: $\vec{q}_T$
  - Inside jet J: $\vec{q}_T - \hat{n}_J(\hat{n}_J \cdot \vec{q}_T)$

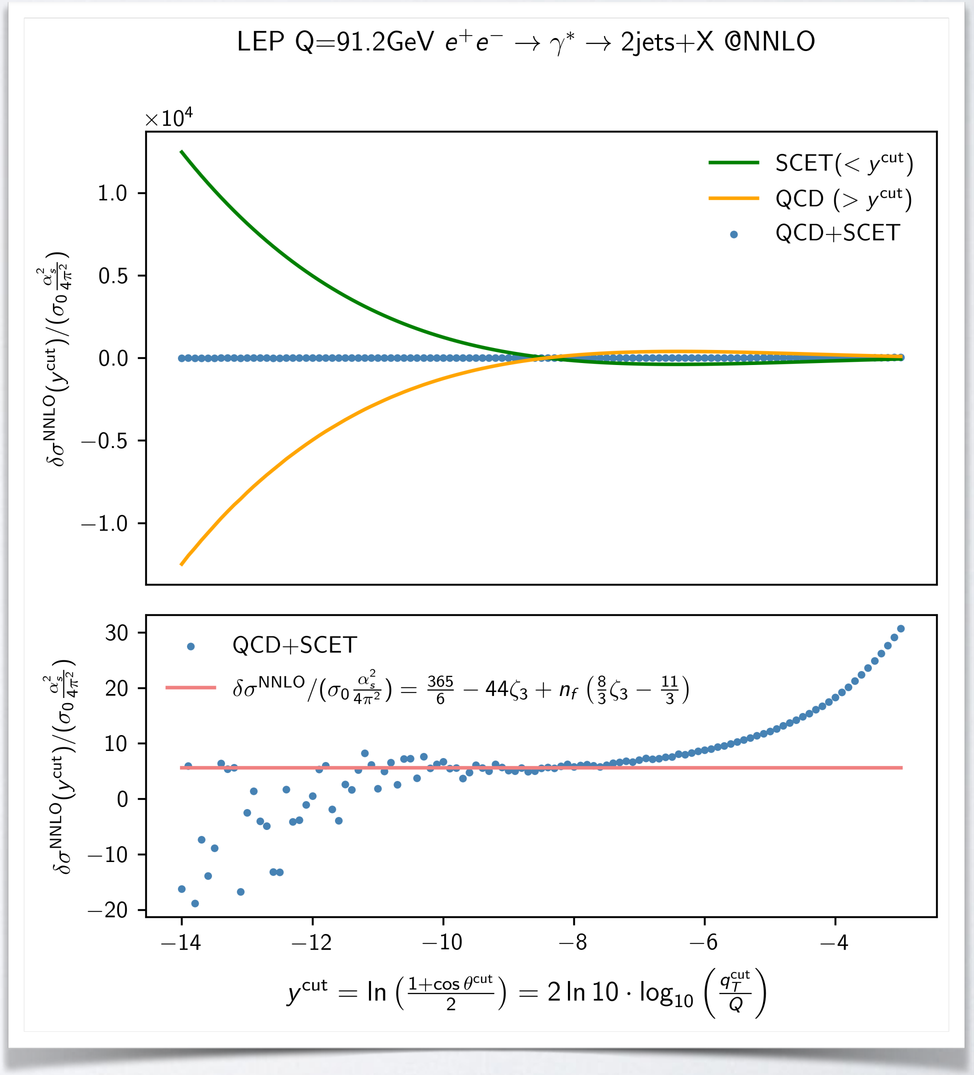
SIMPLIFICATIONS

- Soft function can be refactorised
- Purpose is not to resum, but to simplify
- Global soft function only depends on dipoles
- CS function only depends on dipole and jet kinematics
- $R \sim \mathcal{O}(1)$ restored by including subleading terms

LEPTON NNLO

- No gluon jet function needed for lepton collisions





CONCLUSION

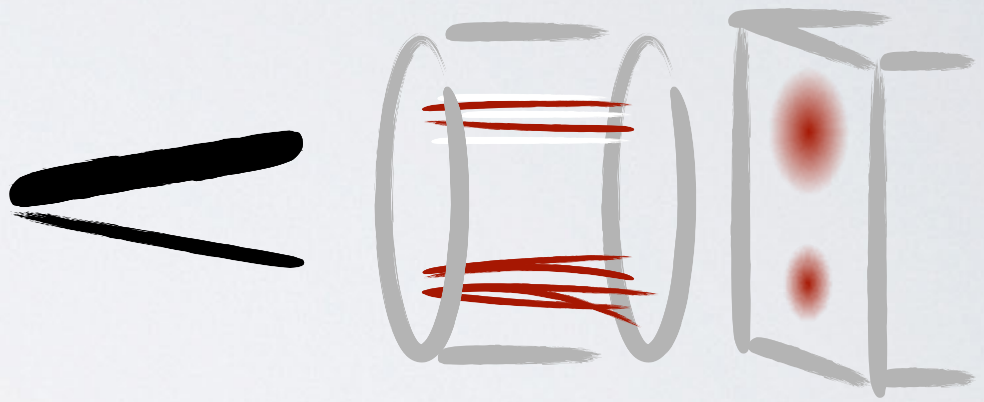
- Azimuthal decorrelation is a really nice observable
- Radial decorrelation isn't
- The WTA axis salvages q_T-slicing for jets
- Planar slicing is simple, non-planar less so

THANK YOU!

TRACKS

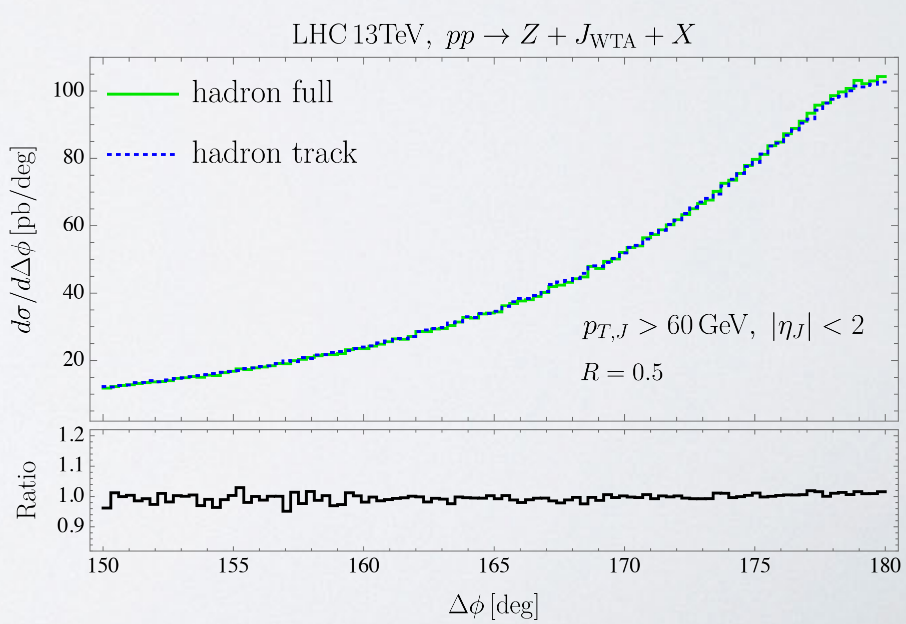
- Angular resolution of calorimetry could be a problem

$\Rightarrow$ Use charged particle tracks



- Only affects Jet function finite term

$$\bar{\mathscr{J}}_q^{(1)} = \mathscr{J}_q^{(1)} + 4C_F \int_0^1 \mathrm{d}x \, \frac{1+x^2}{1-x} \ln \frac{x}{1-x} \int_0^1 \mathrm{d}z_1 \, T_q(z_1,\mu) \times \int_0^1 \mathrm{d}z_2 \, T_g(z_2,\mu) [\theta(z_1 x - z_2(1-x)) - \theta(x - \tfrac{1}{2})]$$



- Track function T, energy fraction z

[Chang, Procura, Thaler, Waalewijn, '13]

CALCULATION

$$\frac{d\sigma}{dp_{x,V}\,dp_{T,V}\,dy_V\,d\eta_J} = \int \frac{db_x}{2\pi}\, e^{ip_{x,V} b_x} \sum_{i,j,k} B_i(x_a, b_x) B_j(x_b, b_x) S_{ijk}(b_x, \eta_J) \times \mathcal{H}_{ij\to Vk}(p_{T,V}, y_V - \eta_J)\, \mathscr{J}_k(b_x) \Big[1 + \mathcal{O}\Big(\frac{p_{x,V}^2}{p_{T,V}^2}\Big)\Big]$$

- NNLL resummation
  - 3-loop $\Gamma_{\rm cusp}$, 2-loop γ_i, 1-loop finite H,J,S,B

[Arnold, Reno, '89]
[Becher, Lorentzen, Schwartz, '12]
[Moch, Vermaseren, Vogt, '04/'05]
[Becher, Neubert, '09]
[Gehrmann, Luebbert, Yang, '14]
[Echevarria, Scimemi, Vladimirov, '16]
[Luebbert, Oredsson, Stahlhofen, '16]

- Most known, only minor recalculations:
  - S at NLO via boost (non-)invariance:
    See also [Gao, Li, Moult, Zhu, '19]

$$S_{ij}^{(1)}(b_x, \eta_J, \mu, \nu) = -\sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j S^{(1)}\Big(b_x, \mu, \nu\sqrt{n_i \cdot n_j/2}\Big)$$

  - TMD jet function with η-regulator and large jet radius
    [Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi, '18/'19]
    [Chiu, Jain, Neill, Rothstein, '12]

BACKUP TRIVIA

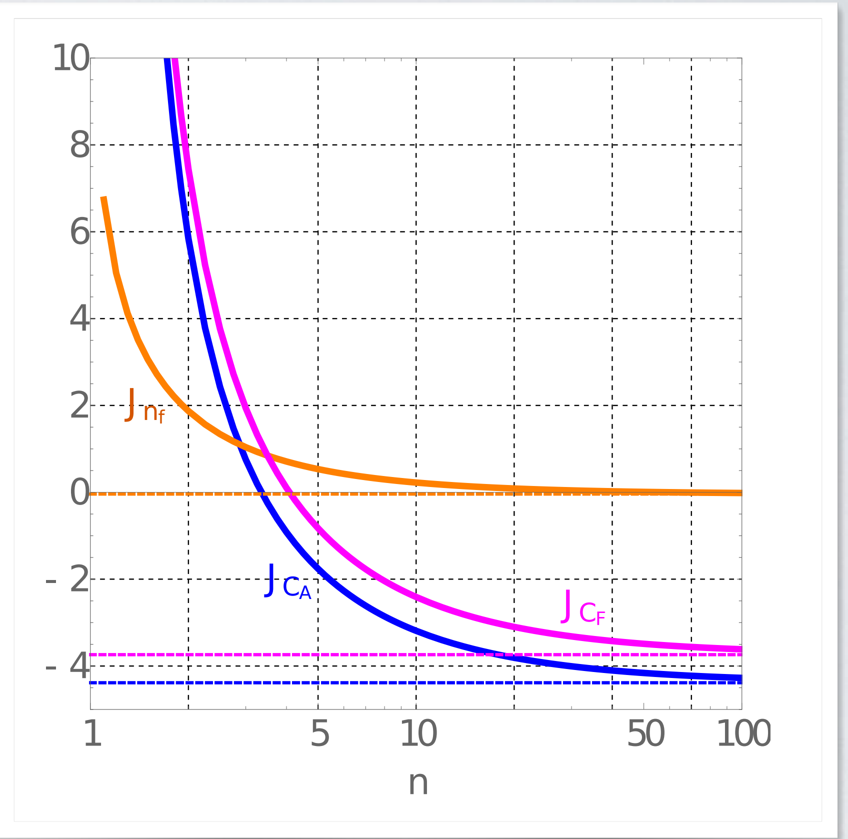
- p_T-weighted jet recombination:

$$p_{T,r} = p_{T,i} + p_{T,j},$$
$$\phi_r = (p^n_{T,i}\phi_i + p^n_{T,j}\phi_j)/(p^n_{T,i} + p^n_{T,j}),$$
$$y_r = (p^n_{T,i}y_i + p^n_{T,j}y_j)/(p^n_{T,i} + p^n_{T,j}),$$

- Reproduces WTA as $n \to \infty$

- Linear polarisation:

$$B_g^{\mu\nu}(\vec{b}_\perp,\mu,\nu) = \frac{-g_\perp^{\mu\nu}}{d-2} B_g^{(1)}(\vec{b}_\perp,\mu,\nu) + \left(\frac{g_\perp^{\mu\nu}}{d-2} + \frac{b_\perp^\mu b_\perp^\nu}{b_T^2}\right) B_g^{(2)}(\vec{b}_\perp,\mu,\nu)$$



$$\mathscr{J}_g^T\big|_{L_b=0} = 1 + \frac{\alpha_s}{4\pi}(C_A J_{C_A} + T_F n_f J_{n_f})$$

[Catani, Grazzini, '11]

GIANT K-FACTORS

- Large matching corrections for high-p_T jets
- Electroweak corrections to dijet decorrelation