Description and Suppression of Beam Instabilities and Space Charge

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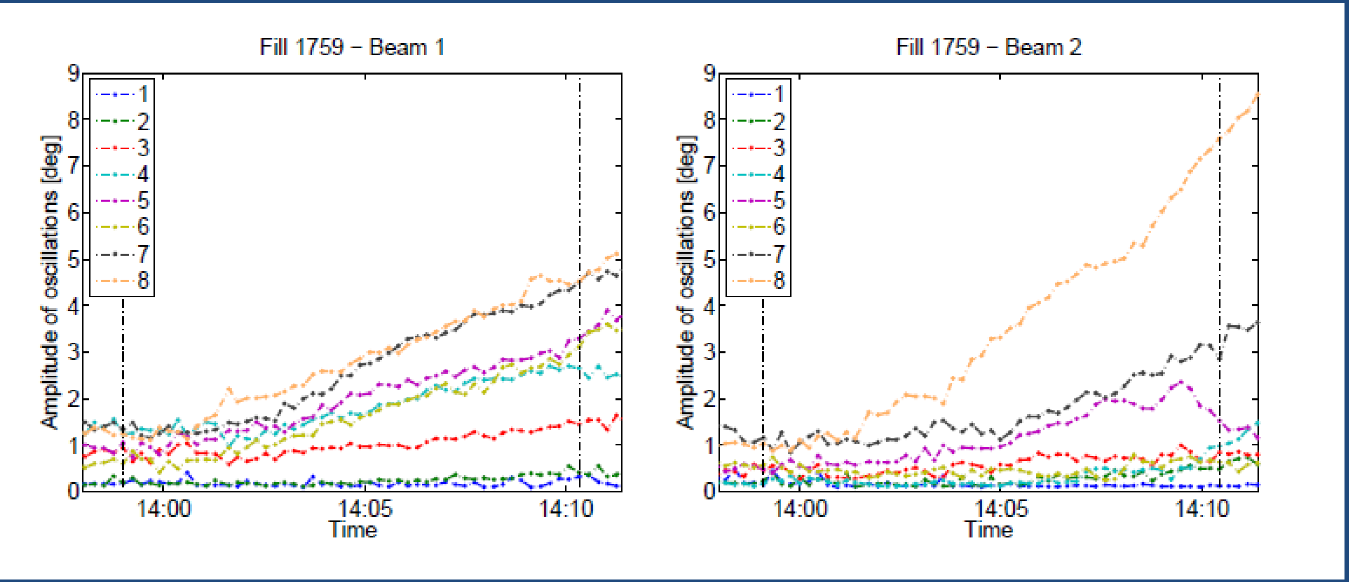
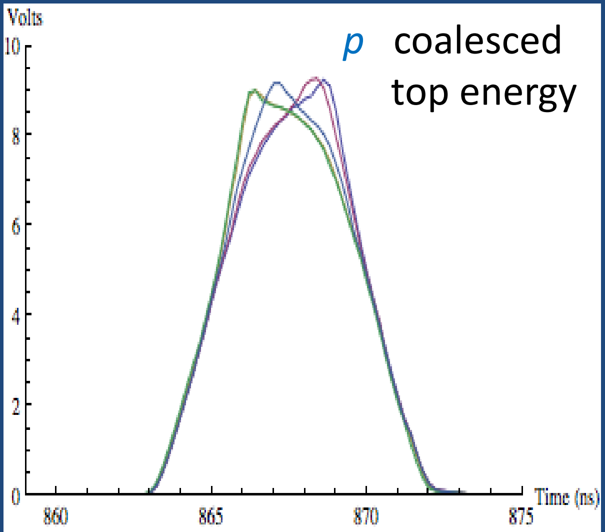
Longitudinal loss of Landau damping and its prevention for small emittances

Loss of Landau Damping (LLD): Observations

LHC, *E. Shaposhnikova et al.,IPAC'11*

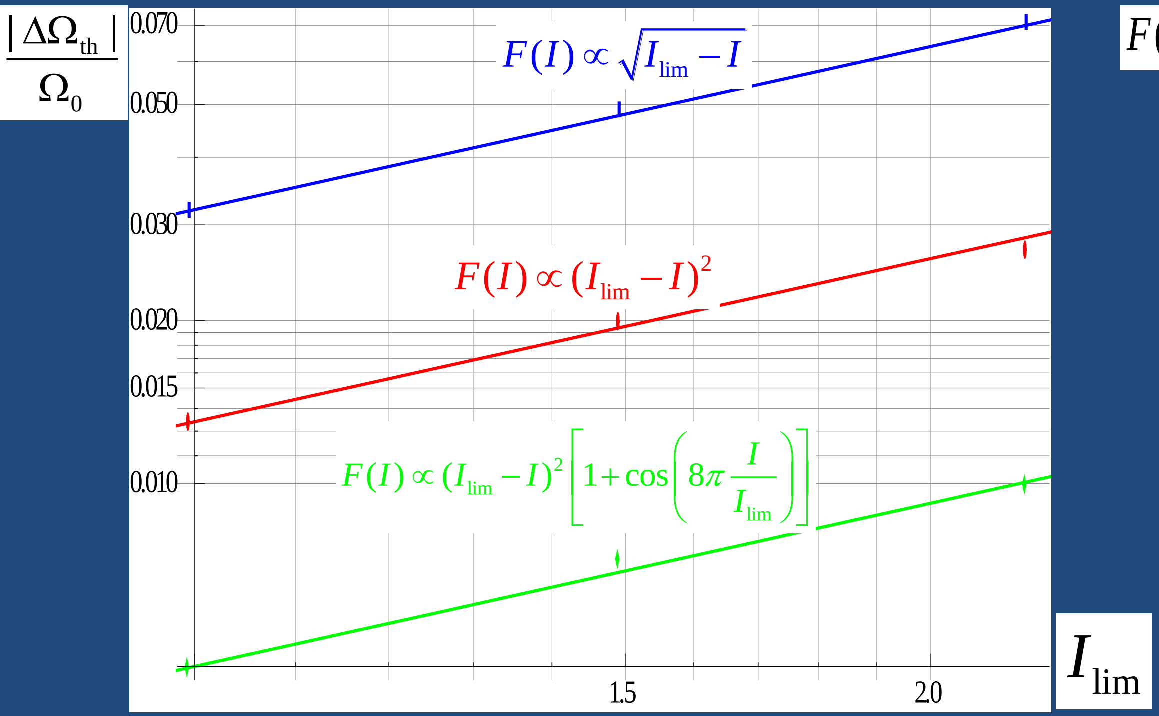


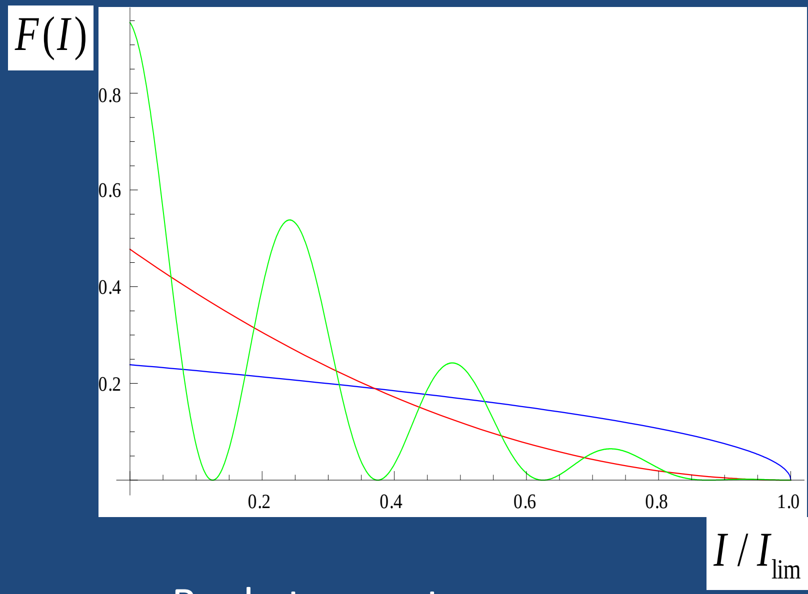
Tevatron, C. Y. Tan & A. Burov



$$|\Delta\Omega_0|/\Omega_0 \approx (1-2)\%$$

LLD thresholds, inductance above transition, $m=n=1$





Bucket acceptance

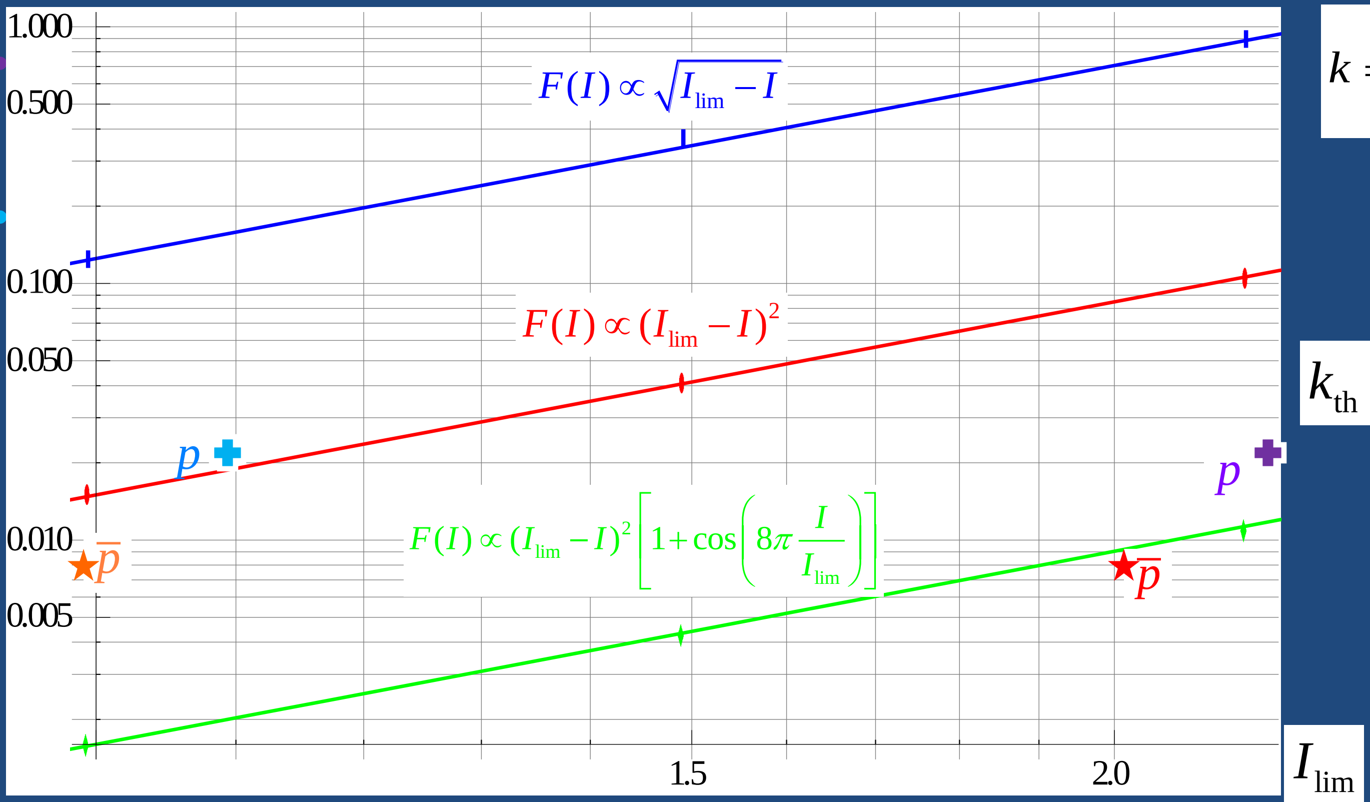
$I_{max} = 8/\pi \approx 2.5$

For the H-P distribution $F(I) \propto \sqrt{I_{lim} - I}$ the threshold is 3 times below rigid-bunch mode result.

For the coalesced bunch and full bucket the threshold is as low as $|\Delta\Omega_{th}|/\Omega_0 \approx 1\%$

Note how strong is dependence on the distribution function!

LLD thresholds, Tevatron



$$k = -\frac{2Nr_0\eta\omega_{rf}^3}{\gamma c\Omega_0^2} \frac{\operatorname{Im} Z_{\|}}{nZ_0} .$$

$$k_{th} \propto I_{lim}^{5/2}$$

✚ protons, top energy

★ pbars, top energy

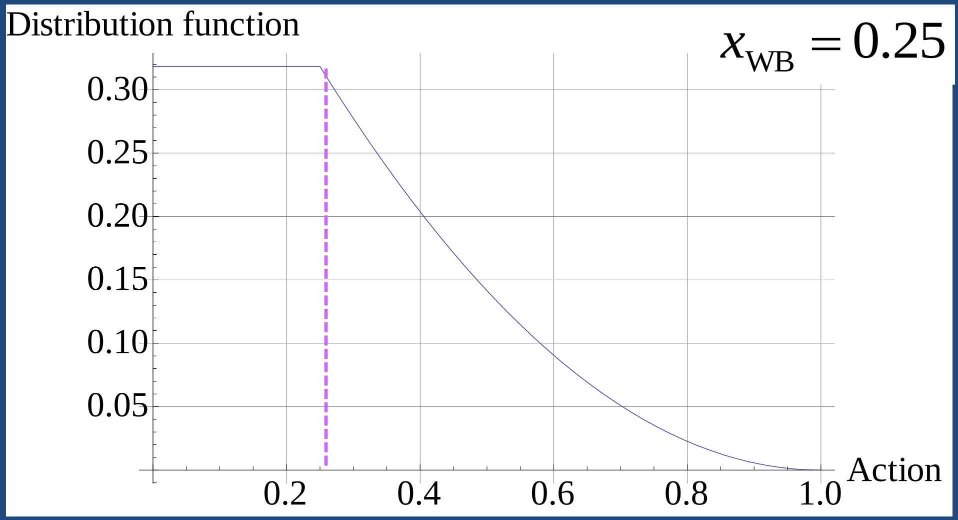
✚ protons, injection

★ pbars, injection

For Gaussian, $F(I) \propto \exp(-2I / I_{lim})$, k_{th} is 35% higher then for $F(I) \propto (I_{lim} - I)^2$

LLD for partial water-bag distribution (PWB)

$$k_{\text{th}} = C\left(I_{\text{lim}} / I_{\text{bkt}}\right)^{5/2}$$



Threshold form-factor

x_{WB}	$C(x_{\text{WB}})$
0	0.15
0.25	2.6
0.5	6.8

Equivalent to 3 times of emittance blow-up

Equivalent to 4.5 times of emittance blow-up

Thus, even a relatively small water-bagging increases the threshold 20 times!

How to make PWB

Let the RF phase be modulated near the synchrotron frequency. Then, equation of motion is:

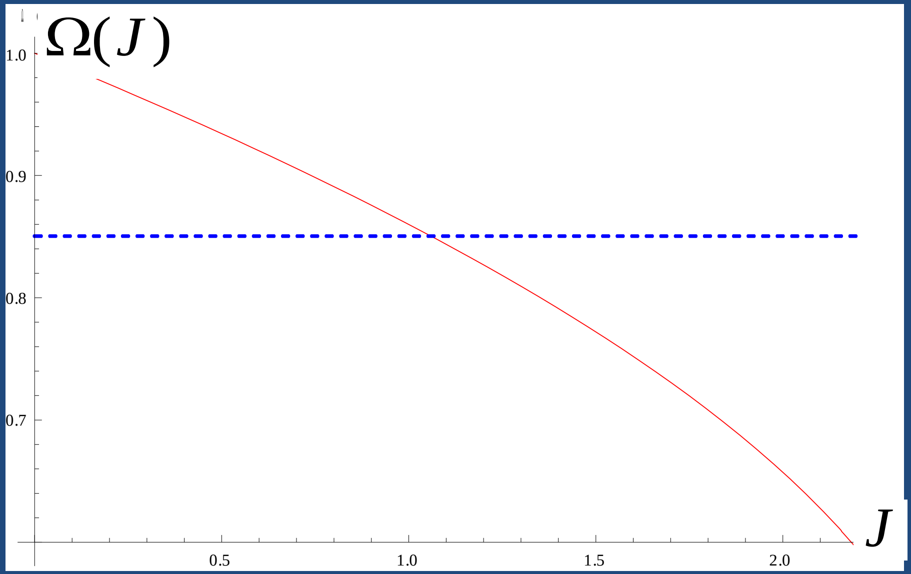
$$\ddot{z} + \sin\left[z - a(t)\sin\left((1-\varepsilon)t\right) \right] = 0$$

Slowly changed amplitude

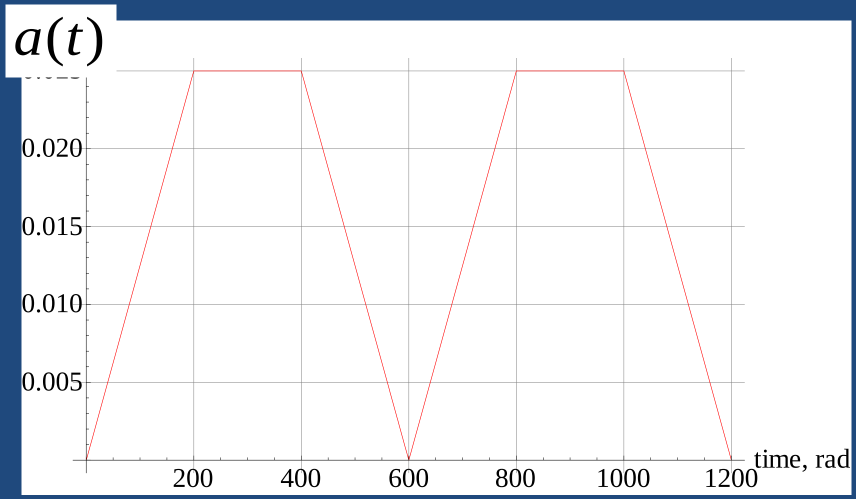
Detuning

$$\Omega(J) \approx 1 - \frac{J}{\pi J_{\text{bucket}}}$$

$$J_{\text{bucket}} = 8/\pi$$



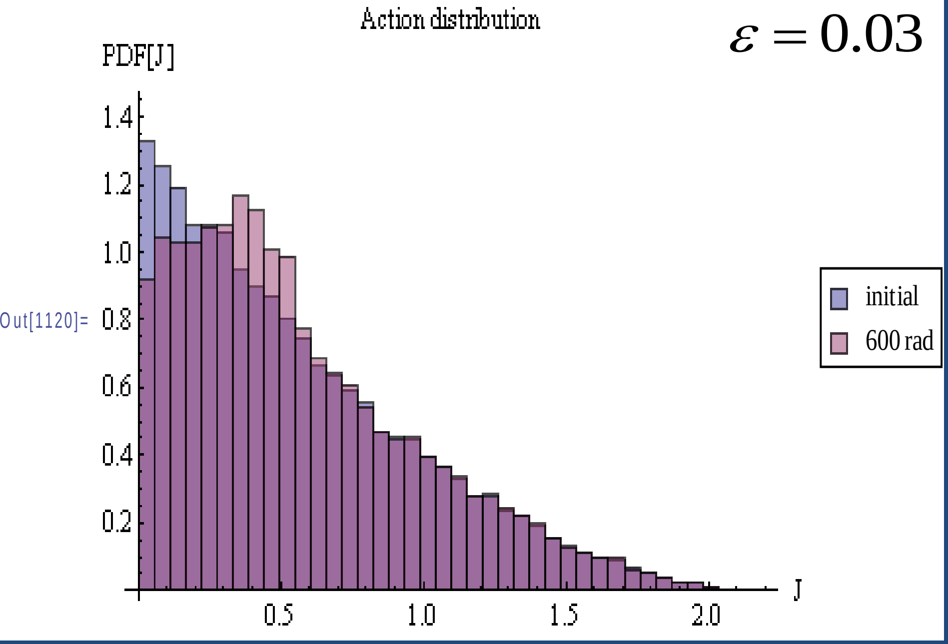
Distribution function is changed:

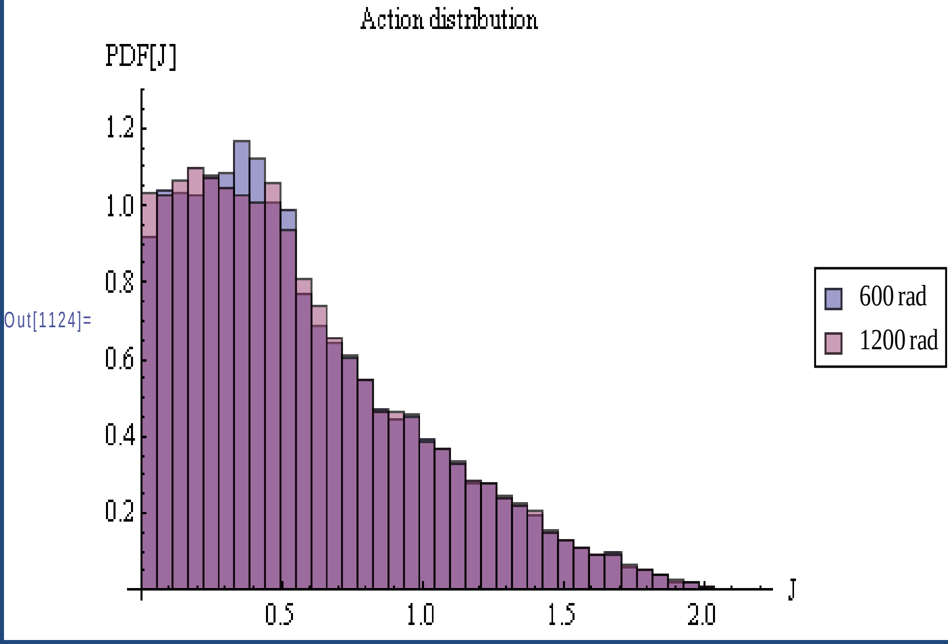


The affected area

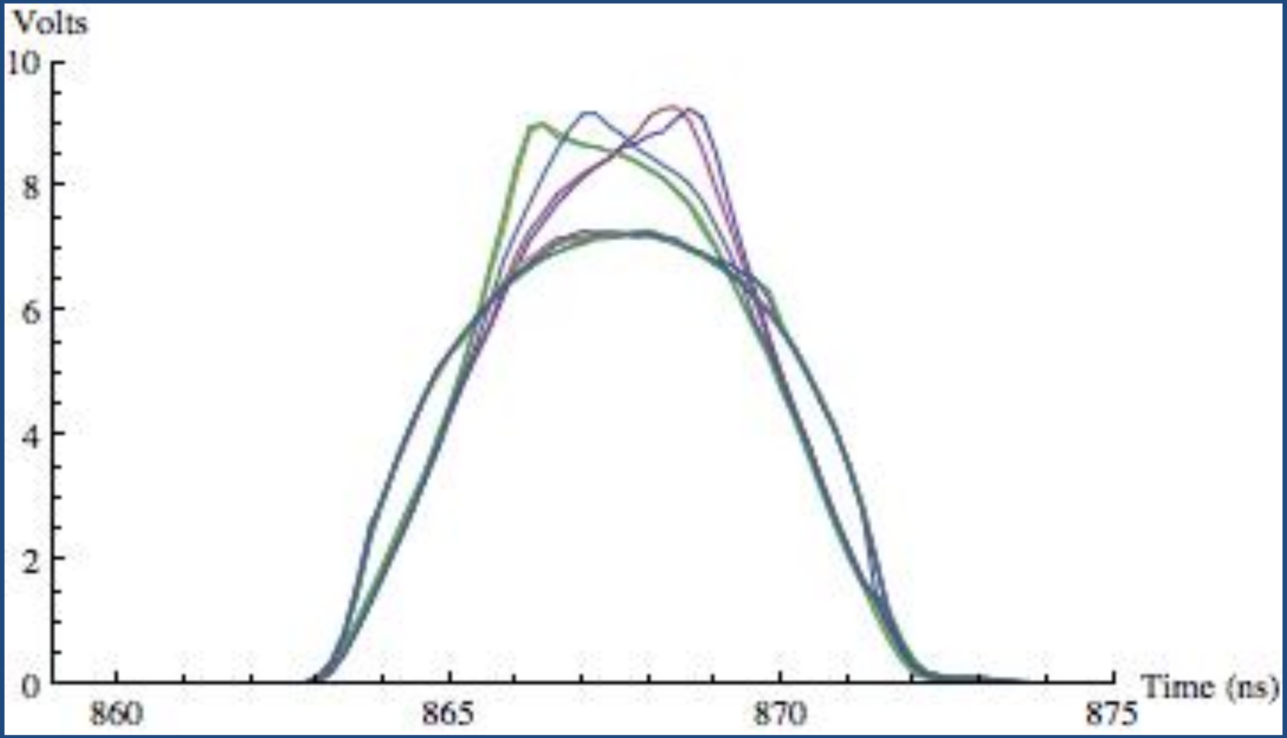
$$J_{\text{diff}} / J_{\text{bucket}} \approx 6\varepsilon$$

$$a_{\max} \geq 3\varepsilon^{3/2}$$





Tevatron: before and after RF shaking



Tevatron 2011, C.Y. Tan & A. Burov, *to be published in PRST-AB*

See more about that in my talks in Impedance and Collective Effects (ICE) section:

- Theory, observations and mitigation of dancing bunches (mtg 34 & 35)
- Anomalous Diffusion for Phase Space Density Modification (mtg 43)

Issues and plans

- This scheme is sensitive to the detuning from the maximal incoherent synchrotron frequency. Accuracy of the detuning should be at the level of ~1% or so.

- Calculated optimal detuning may differ from the actual due to the wake-caused potential well distortion and beam loading.

- As a consequence, different bunches result with somewhat different PWB step width.

- To see importance of these and may be some other issues, MD studies (similar to Tevatron) are needed. PS? SPS? LHC?

- Longitudinal simulations with Head-Tail (Kevin Lee)

- Analysis of the beam tomography at SPS and LHC and comparing observed LLD thresholds with the calculated. Checking the impedance model of the machines (Theodoros Argyropoulos).

Space charge and transverse instabilities

Space charge for transverse stability

- Recently new theoretical papers were published about transverse stability with strong space charge (A. Burov, 2009; V. Balbekov, 2011) with quantitative analysis of weak and strong head-tail cases. With certain contradiction between them, simulations are needed (Vladimir Kornilov, GSI; Chris Hansen, CERN).

- Space charge and impedance for coupled optics – reliable and efficient simulation tool is required: Head-Tail with Möhl-Schönauer space charge model.

Space charge suppression and flat beams in LHC

Space charge suppression for smaller emittance

- Conventional space charge tune shift:

$$\Delta Q_{x,y} = -\frac{\lambda r_0}{2\pi\gamma_0^3\beta_0^2}\oint\frac{\beta_{x,y}ds}{a_{x,y}(a_x + a_y)} \cong -\frac{\lambda r_0 C}{4\pi\gamma_0^2\beta_0\varepsilon_{x,y}} = -\frac{NBr_0}{4\pi\gamma_0^2\beta_0\varepsilon_{x,y}}$$

λ - line density

$\varepsilon_{x,y}$ - normalized rms emittances

B - bunching factor

Space charge tune shift leads to lifetime reduction and loss of Landau damping for transverse degrees of freedom.

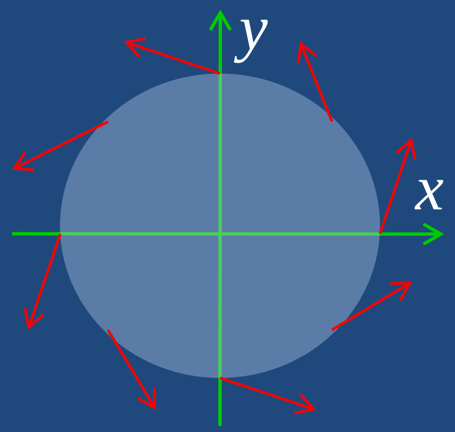
It appears that it can be reduced only by means of the proper reduction of the beam brightness. However, this statement is not correct!

Circular modes

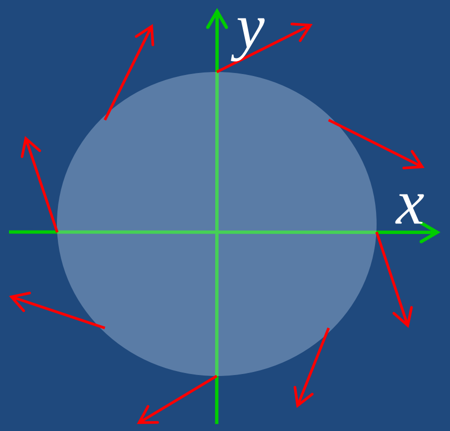
- The space charge is determined by the beam sizes $a_{1,2}$, and for conventional optical modes the sizes are determined by the 2 emittances $\varepsilon_{x,y}$.

- However, the family of the optical modes is much wider, than conventional x/y modes… For instance, circular modes are very different:

Counter-clockwise



Clockwise



General Solution

- Turn-by-turn particle positions and angles:

$$\mathbf{x} = \mathrm{Re}\left(\sqrt{2J_1}e^{-i\psi_1}\mathbf{v}_1 + \sqrt{2J_2}e^{-i\psi_2}\mathbf{v}_2\right) = \mathbf{V}\boldsymbol{\xi}$$

$$\boldsymbol{\xi} = \begin{pmatrix} \sqrt{2J_1}\cos\psi_1 \\ -\sqrt{2J_1}\sin\psi_1 \\ \sqrt{2J_2}\cos\psi_2 \\ -\sqrt{2J_2}\sin\psi_2 \end{pmatrix} = \begin{pmatrix} \xi_1 \\ \pi_1 \\ \xi_2 \\ \pi_2 \end{pmatrix}$$

- Transformation $\mathbf{x} \rightarrow \boldsymbol{\xi}$ is canonical,

$J_{1,2}$ - actions $\psi_{1,2}$ - phases

$\mu_{1,2} = 2\pi\nu_{1,2}$ - phase advances

$\langle J_{1,2} \rangle \equiv \varepsilon_{1,2}$ - rms emittances:

Circular eigenvectors

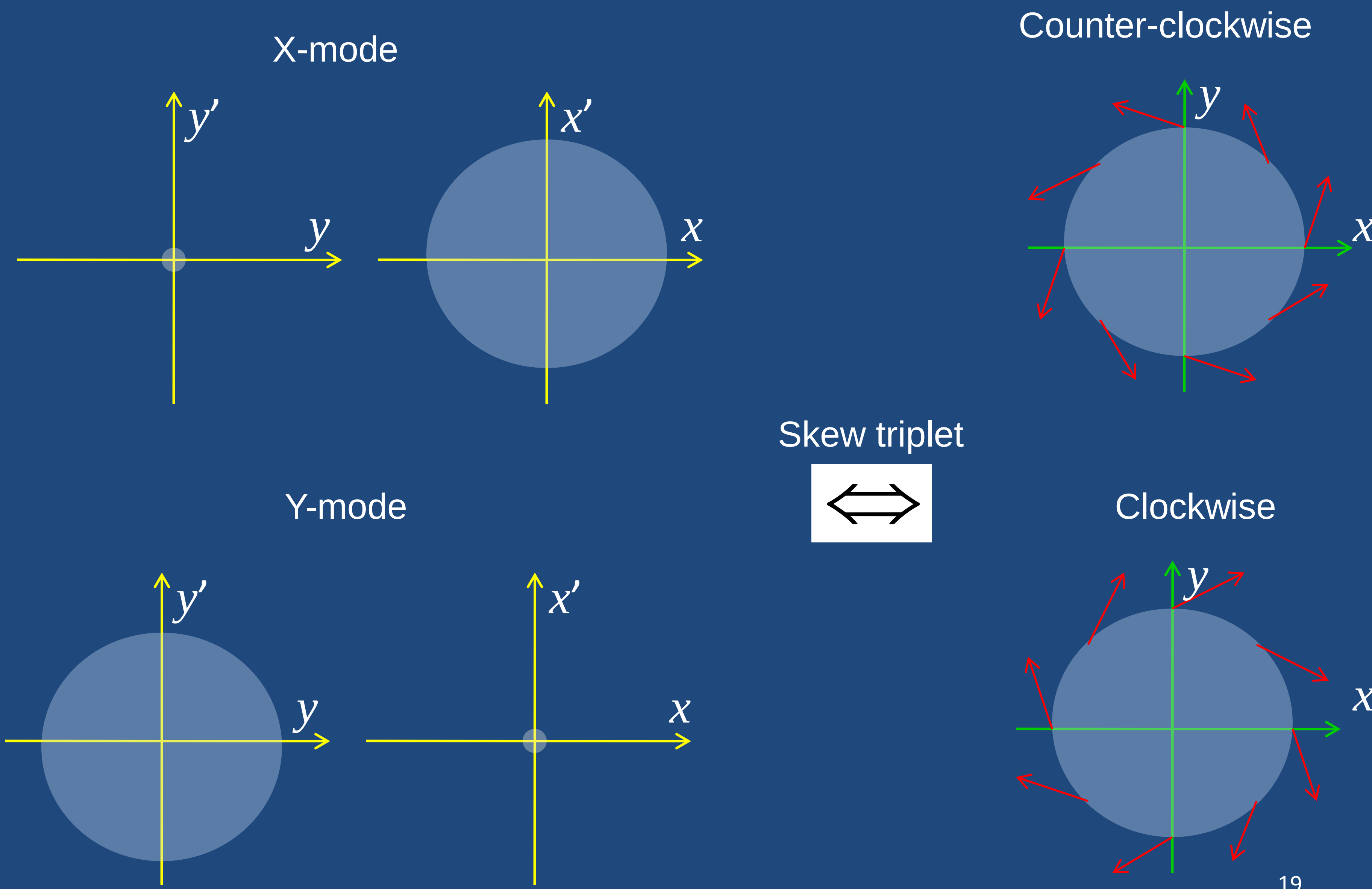
- With $\beta_{lx}=\beta_{ly}=\beta, \quad \alpha_{lx}=\alpha_{ly}=\alpha, \quad u=1/2,$ and $\nu_{1,2}=\pi/2$:

$$\mathbf{v}_1 = \left(\sqrt{\beta}, -\frac{i/2+\alpha}{\sqrt{\beta}}, i\sqrt{\beta}, -i\frac{i/2+\alpha}{\sqrt{\beta}} \right)^T ,$$
$$\mathbf{v}_2 = \left(i\sqrt{\beta}, -i\frac{i/2+\alpha}{\sqrt{\beta}}, \sqrt{\beta}, -\frac{i/2+\alpha}{\sqrt{\beta}} \right)^T .$$

- In a matched solenoid one of modes is a Larmor motion with center at the solenoid axis, and another one is a pure offset, $x, y = const$.

$$\varepsilon_1\varepsilon_2 = \varepsilon_{4D}; \quad \varepsilon_1 - \varepsilon_2 = \left\langle x\theta_y - y\theta_x \right\rangle$$

Planar-Circular mode transformation



Space charge suppression

- For a conventional uncoupled planar modes, the SC tune shifts:

$$\Delta Q_{1,2} = -\frac{\lambda r_0}{2\pi\gamma_0^3\beta_0^2}\oint\frac{\beta_{x,y}ds}{a_{1,2}(a_1+a_2)}\ ;\quad a_{1,2}=\sqrt{\varepsilon_{1,2}\beta_{1,2}}$$

- For $\varepsilon_1 \gg \varepsilon_2$, smooth approximation and equal betas:

$$\Delta Q_2\big|_{\text{planar}} = -\frac{\lambda r_0 C}{2\pi\gamma_0^3\beta_0^2\sqrt{\varepsilon_1\varepsilon_2}} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}\,\Delta Q_1\big|_{\text{planar}}\ .$$

- The same approximation for the circular optics yields

$$\Delta Q_2\big|_{\text{circular}} = \Delta Q_1\big|_{\text{circular}} = \frac{\lambda r_0 C}{2\pi\gamma_0^3\beta_0^2\varepsilon_1}$$

- For the circular optics, the tune shifts are finite even for $\varepsilon_2 = 0$!

$$\frac{\Delta Q\big|_{\text{circular}}}{\Delta Q_2\big|_{\text{planar}}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \ll 1$$

Flat beams at LHC?

- These "ε-flat" beams may have minor emittance as small as the linac has. The major emittance is filled with a multi-turn injection.

- Circular (elliptical) optics in the Booster and PS would remove the space charge limits for the injector chain. At the SPS the modes could be conventional planar ones.

- At the LHC, the luminosity gain for the flat beams

$$\mathcal{L} \propto \frac{1}{\sqrt{\varepsilon_2}}$$

Many thanks for everyone of you!