

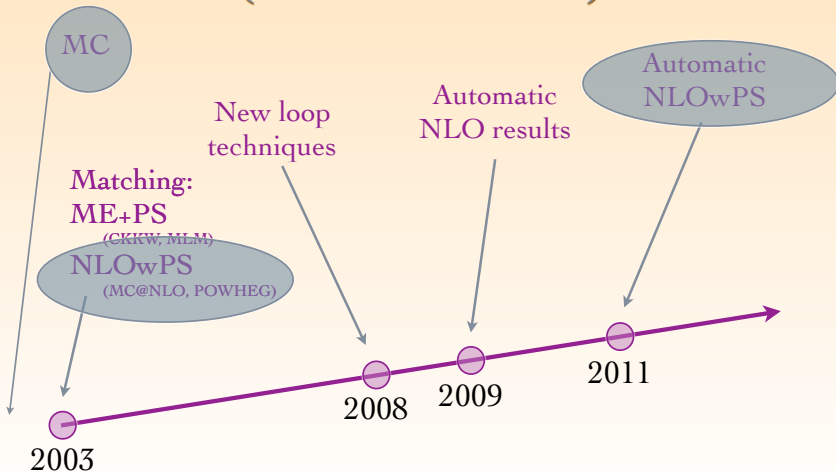
Interfacing NLO with Parton Showers

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ThinkTank on Physics @ LHC

QCD AND MC PROGRESS (SIMPLIFIED)



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Summary

- ▶ Parton showers
- ▶ Matching parton showers and NLO computations
- ▶ MC@NLO
- ▶ Automatic MC@NLO
- ▶ POWHEG

Generalities

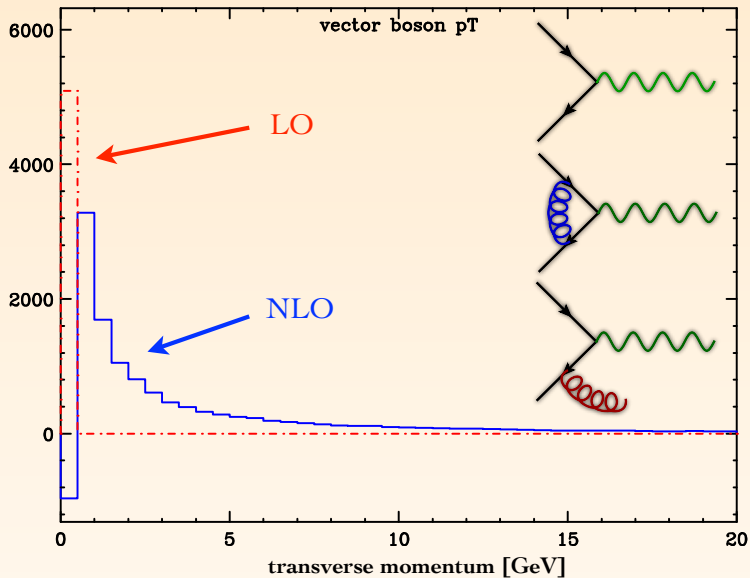
Preamble

- ▶ Perturbation theory formally offers a systematic way to approximate the prediction for any physical observable. But one should not emphasize the word "systematic", since the behavior of perturbation theory depends crucially on the observable one is considering.
- ▶ There are lots of observables that are perfectly well-behaved in this perturbative approach, i.e. that show a good convergence behavior. In particular, sufficiently inclusive observables are well described and for them, neglecting higher orders, is really a small correction.

Preamble

- ▶ But more exclusive observables will in general be poorly described in perturbation theory.

Preamble

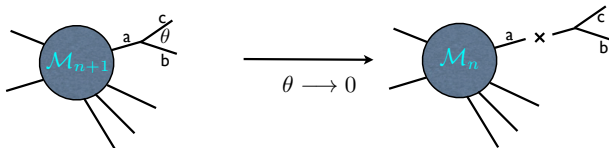


Preamble

- ▶ But more exclusive observables will in general be poorly described in perturbation theory.
- ▶ One could take the conservative attitude of considering only perturbatively well-behaved observables. But thus one would miss an extremely rich variety of observables which may play important roles in experimental analyses.
- ▶ If perturbation theory breaks down for an observable, this does NOT mean that observable is useless / unimportant: it is just that one is not using the good tools to describe it.
- ▶ It is better to try and find a way to reorganize the computation in order to take into account emissions in the singular regions of the phase space, to all orders in perturbation theory.
- ▶ As a not so trivial truth, this can be done in a systematic way.

Parton Showers

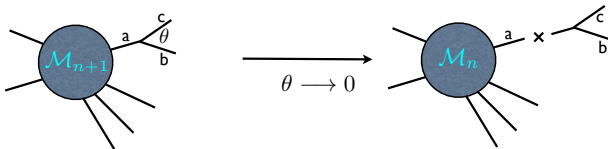
Collinear factorization



Consider a massless particle that splits into a pair of massless particles separated by a small angle θ .

- ▶ In the limit of $\theta \rightarrow 0$ the parent particle goes on shell: its branching is thus related to time scales which are very long with respect to the core interaction (hard subprocess).
- ▶ The inclusion of such a branching can not completely change the pictures set up by the hard process: the whole emission process should be writeable in this limit as the basic one times some branching probability.
- ▶ The first task of Monte Carlo physics is actually to make this statement quantitative.

Collinear factorization



Cross section factorization in the collinear limit (universal !!):

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z).$$

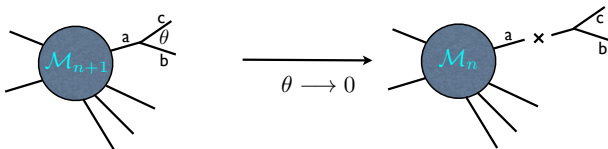
Notice that what has been roughly called "branching probability" is actually a singular factor, so one will need to make sense precisely of this definition.

This is the leading contribution to the $n + 1$ -body cross section (fixed the energies involved).

► $P_{a \rightarrow bc}(z)$ = Altarelli-Parisi splitting kernel:

$$\begin{aligned} P_{g \rightarrow qq}(z) &= T_R \left[z^2 + (1-z)^2 \right], & P_{g \rightarrow gg}(z) &= C_A \left[z(1-z) + \frac{z}{1-z} + \frac{1-z}{z} \right], \\ P_{q \rightarrow qg}(z) &= C_F \left[\frac{1+z^2}{1-z} \right], & P_{q \rightarrow gq}(z) &= C_F \left[\frac{1+(1-z)^2}{z} \right]. \end{aligned}$$

Collinear factorization

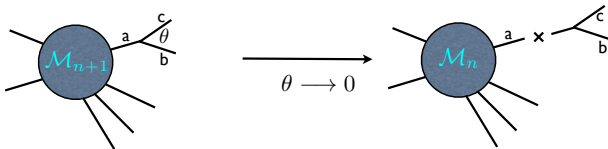


Cross section factorization in the collinear limit (universal !!):

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z).$$

- ▶ t can be called the "evolution variable" (will become clearer later): it can be the virtuality m^2 of particle a (i.e. p_a^2), its p_\perp or $\tilde{t} = E_a^2 \theta^2$...
It represents the hardness of the branching and tends to 0 in the collinear limit.
- ▶ Indeed in the collinear limit one has $m^2 = z(1-z)\theta^2 E_a^2$, $p_T^2 = zm^2$, so that the factorization takes place for all these definitions: $d\theta^2/\theta^2 = dm^2/m^2 = dp_T^2/p_T^2$.

Collinear factorization

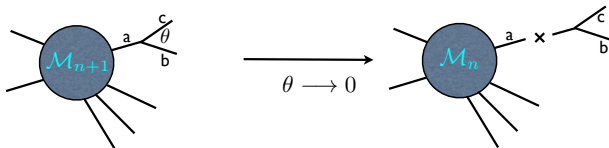


Cross section factorization in the collinear limit:

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z).$$

- ▶ z is the "energy variable": it can be defined as the relative energy of b , i.e. E_b/E_a , ...
It represents the energy sharing between b and c and tends to 1 in the soft limit (daughter c going soft).
- ▶ ϕ = azimuthal angle between the polarization of a and the plane of branching.

Collinear factorization

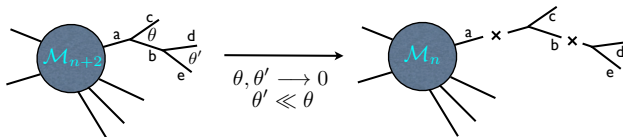


Cross section factorization in the collinear limit:

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z).$$

- Why isn't there a t^2 in the denominator? This is the square of an amplitude that explicitly features a $1/t$.
- Take for example the splitting $q \rightarrow qg$: helicity is conserved for the quarks, so the final state spin differs by one unity with respect to the initial one. The scattering happens in a p -wave (orbital angular momentum = 1), so it is suppressed as $t \rightarrow 0$.
- Indeed a factor $p_b \cdot p_c$ always appears at the numerator if one performs the explicit computation.

Multiple emission

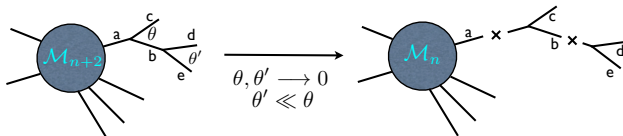


Now consider \mathcal{M}_{n+1} as the new core process and use the recipe we used for the first emission in order to get the dominant contribution to the $n+2$ -body cross section: add a new branching at angle much smaller than the previous one:

$$|\mathcal{M}_{n+2}|^2 d\Phi_{n+2} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(z) \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi} \frac{\alpha_S}{2\pi} P_{b \rightarrow de}(z').$$

This can be done for an arbitrary number of emissions. The recipe to get the leading collinear singularity is thus cast in the form of an iterative sequence of emissions whose probability does not depend on the past history of the system, so a Markov chain.

Multiple emission



The dominant contribution to the cross section comes from the region where the subsequently emitted partons satisfy the strong ordering requirement: $\theta \gg \theta' \gg \theta'' \dots$. Indeed the rate for multiple emission in the branching sequence is

$$\sigma_{n+k} \propto \alpha_s^k \int_{Q_0^2}^{Q^2} \frac{dt}{t} \int_{Q_0^2}^t \frac{dt'}{t'} \dots \int_{Q_0^2}^{t^{(k-2)}} \frac{dt^{(k-1)}}{t^{(k-1)}} \propto \sigma_n \left(\frac{\alpha_s}{2\pi} \right)^k \log^k(Q^2/Q_0^2),$$

where Q is a typical hard scale and Q_0 is a small infrared cutoff that separates perturbative from non perturbative regimes.

The logarithm can easily be large. It is thus clear that perturbation theory breaks down since the effective coupling is $\alpha_s \log(Q^2/Q_0^2)$ instead of just α_s .

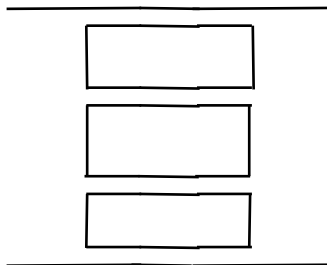
The previous formula shows that Monte Carlo simulations know about the leading logarithmic collinear approximation of the total rate.

Absence of interference

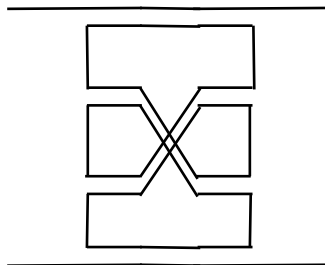
- ▶ The branching sequence from a given leg, the parton shower, is the description of the history of that leg starting from the hard subprocess all the way down to the non perturbative region.
- ▶ Suppose you want to describe two such histories, the showers from two different legs that are present at the hard subprocess level: then these two showers are treated in a completely uncorrelated way. And even within the same history, subsequent emissions are uncorrelated.
- ▶ The parton shower misses all the variety of interference effects between the various legs: the single branching just knows about the kinematics and the identity of the parent particle, so the extreme simplicity comes with the price of quantum inaccuracy.
- ▶ Nevertheless, this captures the leading singularities, so it gives the amazing possibility of having a good description of an arbitrary number of emissions.
- ▶ It is a resummed computation, half the way between perturbation theory and non perturbative approach.

Leading color

- ▶ Since the Monte Carlo is missing interference effects in the multiple particle emission chain, also the color flow between the various QCD particles emitted is only approximately described.
- ▶ In particular, interference effects are always suppressed by some power of the color number N_c , so, avoiding the description of interference effects implies the color flow description is correct only in the limit $N_c \rightarrow \infty$.



$$N_c^3$$



$$N_c$$

Emission probability and Sudakov form factor

Try and give a meaning to the word "branching probability" used before.

Differential probability for the branching $a \rightarrow bc$ between scales t and $t + dt$ knowing that no emission occurred before:

$$dp(t) = \sum_{bc} \frac{dt}{t} \int dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z).$$

Starting from a scale Q^2 , the probability that the parent parton has not splitted at a smaller scale t it is the product of the probabilities that it did not split in any interval dt_k between Q^2 and t .

Probability that particle a does not emit between scales Q^2 and t :

$$\Delta(Q^2, t) = \prod_k \left[1 - \sum_{bc} \frac{dt_k}{t_k} \int dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z) \right] = \\ \exp \left[- \sum_{bc} \int_t^{Q^2} \frac{dt'}{t'} \int dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z) \right] = \exp \left[- \int_t^{Q^2} dp(t') \right].$$

- ▶ $\Delta(Q^2, t)$ is the Sudakov form factor.
- ▶ Property: $\Delta(A, B) = \Delta(A, C)\Delta(C, B)$.

- ▶ This is actually similar to what one gets in considering a radioactive decay of a nucleus: there one has that the number of survived nuclei at time t changes as

$$\frac{dN(t)}{dt} = -c(t)N(t),$$

so that the differential emission probability at time t is

$$dP(t) = dN(t)/N(0) = -c(t) \exp\left(-\int_0^t c(t')dt'\right).$$

- ▶ In the branching case, one has that the role of the decay time is played by the virtuality (or similar) of the parent particle, and the full ensemble of events has a distribution in this variable.

Unitarity

Define dP_k as the probability for n ordered splittings from leg a at given scales:

$$\begin{aligned}dP_1(t_1) &= \Delta(Q^2, t_1) dp(t_1) \Delta(t_1, Q_0^2), \\dP_2(t_1, t_2) &= \Delta(Q^2, t_1) dp(t_1) \Delta(t_1, t_2) dp(t_2) \Delta(t_2, Q_0^2) \Theta(t_1 - t_2), \\&\dots = \dots \\dP_k(t_1, \dots, t_k) &= \Delta(Q^2, Q_0^2) \prod_{l=1}^k dp(t_l) \Theta(t_{l-1} - t_l).\end{aligned}$$

Integrate: probability for k splittings:

$$P_k \equiv \int dP_k(t_1, \dots, t_k) = \Delta(Q^2, Q_0^2) \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k, \quad \forall k = 0, 1, \dots$$

Sum of probabilities:

$$\sum_{k=0}^{\infty} P_k = \Delta(Q^2, Q_0^2) \sum_{k=0}^{\infty} \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k = \Delta(Q^2, Q_0^2) \exp \left[\int_{Q_0^2}^{Q^2} dp(t) \right] = 1.$$

Unitarity

Cross section for 0 or 1 emissions from leg a in the parton shower:

$$\frac{d\sigma}{\sigma_n} = \Delta(Q^2, Q_0^2) + \Delta(Q^2, Q_0^2) \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(z).$$

Expand at first order in α_S :

$$\frac{d\sigma}{\sigma_n} \simeq 1 - \sum_{bc} \int_{Q_0^2}^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(z) + \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(z).$$

- ▶ Same structure of the two latter terms, with opposite signs: cancellation of divergences between the approximate virtual and approximate real emission cross sections.
- ▶ The probabilistic interpretation of the shower ensures that infrared divergences will cancel for each emission.
The cancellation of infinities comes simply out as the basic statement that $P(\text{emission}) + P(\text{no emission}) = 1$, without any computational efforts.
- ▶ As in $e^+e^- \rightarrow \text{hadrons}$, one can define jets with different algorithms, and the jet separation will play the role of the regulator Q_0 . Unitarity is implemented by $\sigma_{\text{NLO}} = \sigma_2 + \sigma_3$, and in that case one can perfectly define probabilities for jet multiplicity i as $\sigma_i / \sigma_{\text{NLO}}$.

Azimuthal kernels

- ▶ Recall the factorization formula

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z).$$

This formula is actually cheating in one point: the integration on the azimuthal angle.

- ▶ One can prove that if one performs the integration on $d\phi$, the formula, azimuthally averaged, is valid.
- ▶ But: if one wants a description exclusive in the angle, then, depending on the parent particle, another term may arise, so that the completely correct formula is actually:

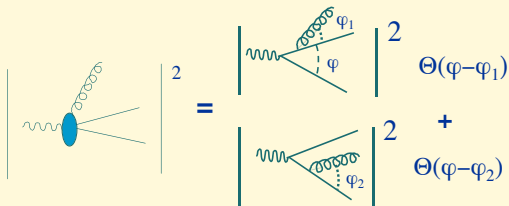
$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} (P_{a \rightarrow bc}(z) |\mathcal{M}_n|^2 + Q_{a \rightarrow bc}(z) |\tilde{\mathcal{M}}_n|^2).$$

- ▶ Q is called azimuthal kernel, and it arises from the interference of parent particles with different polarizations, so it is $\neq 0$ just if the parent parton is a gluon.
- ▶ If the parent is a quark, helicity conservation implies no contribution from different helicity configurations.
- ▶ Normally the term in Q is ignored in a Monte Carlo simulation, but one has to keep in mind they are there (they will play a role in MC@NLO).

Angular ordering

(slide by M. Mangano)

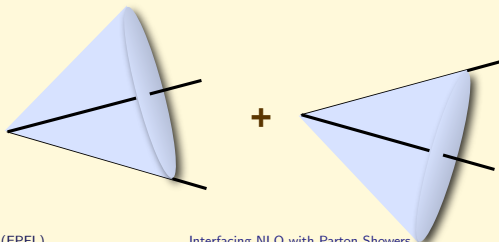
Angular ordering



The diagram shows an equation for the squared amplitude of a process. On the left, a vertical line (incoming particle) enters a blue oval (interaction region). From the oval, a wavy line (radiation) and two straight lines (outgoing particles) emerge. The wavy line is at an angle φ relative to the vertical. The entire left side is enclosed in large vertical bars with a superscript 2. This is followed by an equals sign. On the right, there are two terms added together. The first term shows a vertical line entering a triangle (interaction region). From the top vertex, a wavy line emerges at an angle φ_1 . From the bottom vertex, two straight lines emerge at an angle φ relative to the vertical. The triangle is enclosed in large vertical bars with a superscript 2. To the right of this term is the expression $\Theta(\varphi - \varphi_1)$. The second term is identical to the first, but the wavy line emerges at an angle φ_2 and the expression is $\Theta(\varphi - \varphi_2)$. A plus sign is between the two terms.

$$\left| \text{Diagram} \right|^2 = \left| \text{Diagram}_1 \right|^2 \Theta(\varphi - \varphi_1) + \left| \text{Diagram}_2 \right|^2 \Theta(\varphi - \varphi_2)$$

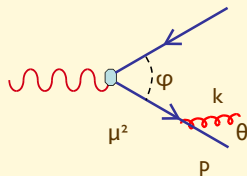
Radiation inside the cones is allowed, and described by the eikonal probability, radiation outside the cones is suppressed and averages to 0 when integrated over the full azimuth



Angular ordering

(slide by M. Mangano)

An intuitive explanation of angular ordering



Lifetime of the virtual intermediate state:

$$\tau < \gamma/\mu = E/\mu^2 = 1/(k_0 \theta^2) = 1/(k_\perp \theta)$$

$$\begin{aligned}\mu^2 &= (p+k)^2 = 2E k_0 (1-\cos\theta) \\ &\sim E k_0 \theta^2 \sim E k_\perp \theta\end{aligned}$$

Distance between q and \bar{q} after τ :

$$d = \varphi \tau = (\varphi/\theta) 1/k_\perp$$

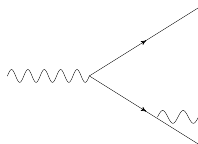
If the transverse wavelength of the emitted gluon is longer than the separation between q and \bar{q} , the gluon emission is suppressed, because the $q \bar{q}$ system will appear as colour neutral (\Rightarrow dipole-like emission, suppressed)

Therefore $d > 1/k_\perp$, which implies

$$\theta < \varphi$$

Angular ordering in a shower

- ▶ In the soft limit, gluon emission is ordered in angle: a gluon emitted at an angle larger than the previous cannot resolve the color charge of the dipole which has emitted it.



- ▶ In terms of the evolution variable $\zeta = 1 - \cos \theta$ (θ = branching angle), the soft limit of the cross section (after azimuthal integration) becomes

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{d\zeta}{\zeta} \frac{dz}{z} \frac{\alpha_s}{2\pi} (-2C).$$

- ▶ Improved by replacing $\frac{1}{z} \rightarrow P_{a \rightarrow bc}(z)$ to get the correct collinear non-soft limit.
- ▶ The fact that one can emit only on a given cone is a genuine quantum interference effect: so it is not true that all interferences are neglected in a parton shower algorithm. An angular ordered algorithm include a certain class of interference effects.

This is a very convenient way of including a quantum effect in a classical language (it is still a Markov chain).

Angular ordering in a shower

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{d\zeta}{\zeta} dz P(z) \frac{\alpha_s}{2\pi} (-2C).$$

- ▶ Since some interference effects are there, one should expect the presence of subdominant contributions to the extra emission cross section (recall the picture of the first slides).
- ▶ Indeed it can be shown that the angular ordered algorithm correctly reproduces the leading and next-to leading collinear logarithms in the soft limit.
- ▶ To summarize.
Ordinary approach = leading collinear logarithm $\forall z$, so leading collinear logarithm also in the soft limit.
Angular ordered approach (improved by the inclusion of the Altarelli-Parisi) = leading collinear logarithm \forall non-soft z ; in the soft limit it is leading + next to leading collinear logarithm.

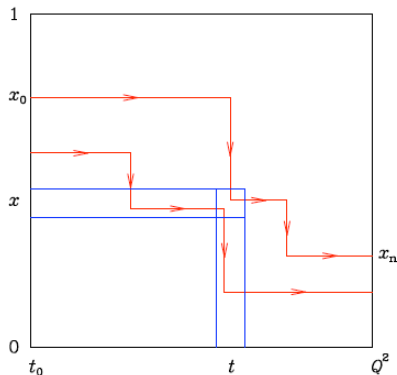
Initial state radiation

Up to now we have explicitly dealt with final state radiation: what about initial state radiation?

- ▶ For final state radiation one starts from the hard subprocess and evolves forward in time, towards the final state particles.
- ▶ If one had to evolve forward in time also for initial state radiation, this would very rarely lead to the wanted hard process kinematical configuration: tremendous inefficiency.
- ▶ Backwards evolution: start from the hard subprocess even for initial state radiation, and evolve back to the incoming colliding hadrons.
- ▶ Use the so called DGLAP equation to determine the parton evolution backwards in time.

DGLAP equation

Establish the scale dependence of a parton distribution function.



- ▶ Sequence of branchings as a path in the (t, z) plane (warning: z is called x in the plot). Each branching is a step downwards in z at a scale t .
- ▶ Change in the parton density $f_b(z, t)$ when t is increased from t to $t + dt$ is the number of paths entering the small square $dt dz$ minus the number of paths going out, divided by dz .

DGLAP equation

- ▶ Number of paths going in (or better: probability that a path goes in) = probability to have a parent particle a at scale t and fraction $z' > z$, times the probability for it to branch to b in the interval between t and $t + dt$, summed over all possible starting values z' :

$$\begin{aligned} df_b^{(IN)}(z, t) &= \frac{dt}{t} \sum_{ac} \int_z^1 dz' \int_0^1 dw \frac{\alpha_S}{2\pi} f_a(z', t) P_{a \rightarrow bc}(w) \delta(z - wz') \\ &= \frac{dt}{t} \sum_{ac} \int_0^1 \frac{dw}{w} \frac{\alpha_S}{2\pi} f_a\left(\frac{z}{w}, t\right) P_{a \rightarrow bc}(w). \end{aligned}$$

- ▶ Number of paths going out (or better: probability that a path goes out) = probability to have a parton a at scale t and fraction z , times the probability for it to branch to b , summed over all the possible arriving values $z' < z$:

$$\begin{aligned} df_b^{(OUT)}(z, t) &= \sum_{ac} \frac{dt}{t} f_a(z, t) \int_0^z dz' \int_0^1 dw \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(w) \delta(z' - wz) \\ &= \frac{dt}{t} \sum_{ac} f_a(z, t) \int_0^1 dw \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(w). \end{aligned}$$

DGLAP equation

Infinitesimal change in $f_b(z, t)$:

$$\begin{aligned}df_b(z, t) &= df_b^{(IN)}(z, t) - df_b^{(OUT)}(z, t) \\&= \frac{dt}{t} \sum_{ac} \int_0^1 dw \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(w) \left[\frac{1}{w} f_a\left(\frac{z}{w}, t\right) - f_a(z, t) \right] \\&= \frac{dt}{t} \sum_{ac} \int_0^1 \frac{dw}{w} \frac{\alpha_S}{2\pi} [P_{a \rightarrow bc}(w)]_+ f_a\left(\frac{z}{w}, t\right),\end{aligned}$$

where the "+" prescription is defined as usual as

$$\int_0^1 dx [g(x)]_+ f(x) \equiv \int_0^1 dx g(x) [f(x) - f(1)].$$

Initial state radiation

Infinitesimal change in $f_b(z, t)$:

$$df_b(z, t) = \frac{dt}{t} \sum_{ac} \int_0^1 \frac{dw}{w} \frac{\alpha_S}{2\pi} [P_{a \rightarrow bc}(w)]_+ f_a\left(\frac{z}{w}, t\right).$$

Differential emission probability in backwards evolution = infinitesimal change in $f_b(z, t)$ normalized to $f_b(z, t)$:

$$d\hat{p}(t) = \frac{df_b(z, t)}{f_b(z, t)} = \sum_{ac} \frac{dt}{t} \int \frac{dw}{w} \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(w) \frac{f_a(z/w, t)}{f_b(z, t)},$$

as opposed to the final state radiation probability (averaged over azimuth)

$$dp(t) = \sum_{bc} \frac{dt}{t} \int dz \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(z).$$

Thus, the Sudakov form factor for initial state radiation is

$$\hat{\Delta}(Q^2, t) = \exp \left[- \int_t^{Q^2} d\hat{p}(t') \right].$$

Initial state radiation: comments

Differential emission probability in backwards evolution:

$$d\hat{p}(t) = \frac{df_b(z, t)}{f_b(z, t)} = \sum_{ac} \frac{dt}{t} \int \frac{dw}{w} \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(w) \frac{f_a(z/w, t)}{f_b(z, t)}.$$

- ▶ Directly proportional to f_a : the more the partons a in the hadron, the easier to create a parton b out of one of them (given that a is allowed to split to b).
- ▶ Inversely proportional to f_b : the more partons b have already been produced, the less probable to produce new ones.
- ▶ The presence of f_a ensures that the parton composition of the hadron is correctly reflected in the branching sequence.

Argument for the coupling constant

Each choice of argument for α_s is equally acceptable at the leading-logarithmic accuracy. However, there is a choice that allows one to resum certain classes of subleading logarithms.

- ▶ Consider the one loop running coupling (for definiteness: $t =$ virtuality here):

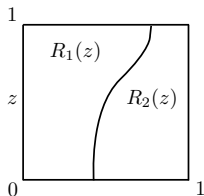
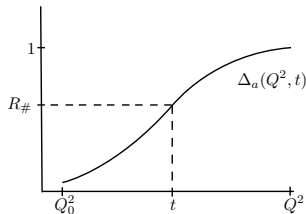
$$\alpha_s(t) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)b \log \frac{t}{\mu^2}} \sim \alpha_s(\mu^2) \left(1 - \alpha_s(\mu^2)b \log \frac{t}{\mu^2} \right).$$

- ▶ It can be shown that higher order corrections in the DGLAP equation imply the Altarelli-Parisi kernels to be modified to $P_{a \rightarrow bc}(z) \rightarrow P_{a \rightarrow bc}(z) + \alpha_s P'_{a \rightarrow bc}(z)$.
- ▶ $P'_{a \rightarrow bc}(z)$ diverges as $-b \log z(1-z)P_{a \rightarrow bc}(z)$ for $g \rightarrow gg$ in the soft gluon limit (just z or $1-z$ if a quark is there).
- ▶ Thus, one can simply take into account these higher order corrections by choosing $z(1-z)t \sim p_\perp^2$ as argument of the coupling. Indeed, the kernel $\alpha_s P_{a \rightarrow bc}(z)$ becomes

$$\begin{aligned} \alpha_s[z(1-z)t]P_{a \rightarrow bc}(z) &\sim \alpha_s(t)(1 - \alpha_s(t)b \log z(1-z))P_{a \rightarrow bc}(z) \\ &= \alpha_s(t)(P_{a \rightarrow bc}(z) + \alpha_s(t)P'_{a \rightarrow bc}(z)). \end{aligned}$$

Implementation

- ▶ Extract the evolution variable t of the branching by solving the equation $\Delta(Q^2, t) = R_{\#}$, with $R_{\#}$ a flat random number between 0 and 1. This correctly reproduces the probability distribution since the probability of extracting a splitting scale t between t_1 and t_2 is $\Delta(Q^2, t_2) - \Delta(Q^2, t_1)$.



- ▶ Extract the energy sharing z and the daughter identities b and c according to $P_{a \rightarrow bc}(z)$. For two possible branchings $P_1(z)$ and $P_2(z)$ one can call $R_i(z) = P_i(z)/(P_1(z) + P_2(z))$, and choose z and parton identities by extracting a random point in the plane.

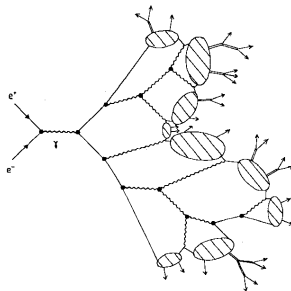
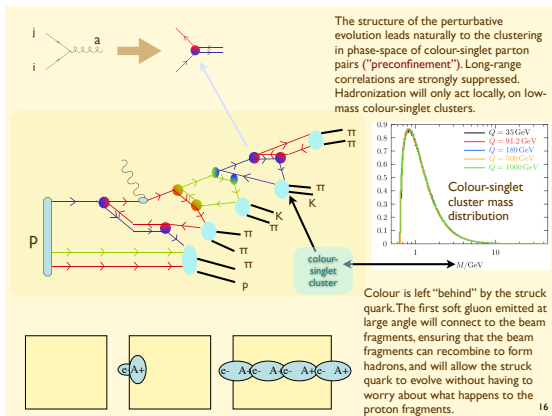
- ▶ Extract ϕ (flat).
- ▶ Reiterate (updating the maximum scale for the Sudakov) until all the 'external' partons are characterized by a scale smaller than a threshold $Q_0^2 \sim 1 \text{ GeV}$.
- ▶ Put partons on shell and hadronize.

Hadronization

- ▶ The pure shower stops when all "external" partons are characterized by a scale below a certain infrared cut-off $Q_0 \sim 1\text{GeV}$, and at that moment they are put on their mass-shell.
- ▶ But what one physically observes in a detector are colorless hadrons.
- ▶ Need to formulate a model for passing from partons to hadrons: this is a delicate part since there is not a strong theoretical understanding of the phenomenon.
- ▶ However the formulation of such models can be guided by some physical and phenomenological considerations.

Color preconfinement and cluster formation

Color is left behind by the quark during its evolution thus color partners are close in phase space (strongly suppressed long-distance correlation: color "preconfinement"). The most so for an angular-ordered shower. Formation of small-mass colorless clusters to be decayed into physical hadrons.



Quark antiquark color potential and string model

From lattice QCD one sees that the color confinement potential of a quark-antiquark grows linearly with their distance: $V(r) \sim kr$, with $k \sim 0.2 \text{ GeV}^2$. This is modeled with a string with uniform tension (energy per unit length) k that gets stretched between the $q\bar{q}$ pair.

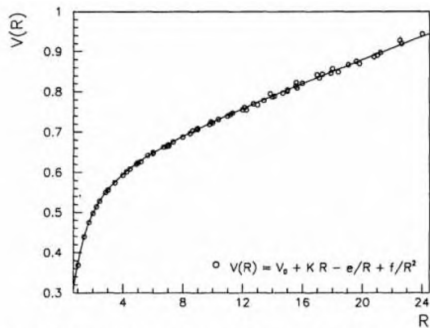
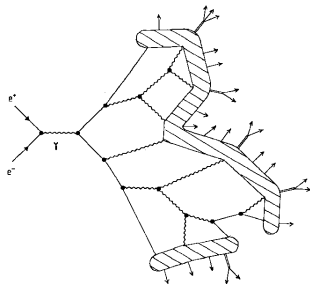


Fig. 2.9. QCD potential vs. R (in lattice units) from lattice QCD. Figure from ref. [23].



At a certain point it becomes energetically favorable to break the string in two by creating a new $q\bar{q}$ pair in the middle of the string.

Main Monte Carlos available on the market: HERWIG

All HERWIG versions (Fortran and C++) implement the angular-ordering: subsequent emissions are characterized by smaller and smaller angles.

► HERWIG 6: $t = \frac{p_b \cdot p_c}{E_b E_c} \simeq 1 - \cos \theta.$

► Herwig++: $t = \frac{(p_{b\perp})^2}{z^2(1-z)^2} = t(\theta).$

Implementing angular ordering, the parton shower (without matrix element corrections) cannot populate the full phase space (without matrix element corrections): empty regions of the phase space, called "dead zones", will arise.

Note. It may seem that the presence of dead zones is a weakness, but it is not so: they implement correctly the collinear approximation, in the sense that they constrain the shower to live uniquely in the region where it is reliable.

► Hadronization: cluster model.

Main Monte Carlos available on the market: PYTHIA

Choice of evolution variables for Fortran and C++ versions:

- ▶ PYTHIA 6: $t = (p_b + p_c)^2 \sim z(1-z)\theta^2 E_a^2$.
- ▶ Pythia 8: $t = (p_b)_\perp^2$.

Simpler variables, but decreasing angles not guaranteed: PYTHIA has to reject the events that don't respect the angular ordering (though this is not completely equivalent to ordering in angle).

Not implementing directly angular ordering, the phase space can be filled entirely, even without matrix element corrections, so one can have the so called "power shower" (use with a certain care).

- ▶ Hadronization: string model.

Note. Usually PYTHIA is faster than HERWIG.

Main Monte Carlos available on the market: SHERPA

- ▶ A new and completely different kind of shower not based on the collinear $1 \rightarrow 2$ branching, but on more complex $2 \rightarrow 3$ elementary process: emission of the daughter off a color dipole.
- ▶ The real emission matrix element squared is decomposed into a sum of terms $D_{ij,k}$ (dipoles) that capture the soft and collinear singularities in the limits i collinear to j , i soft (k is the spectator), and a factorization formula is deduced in the leading color approximation:

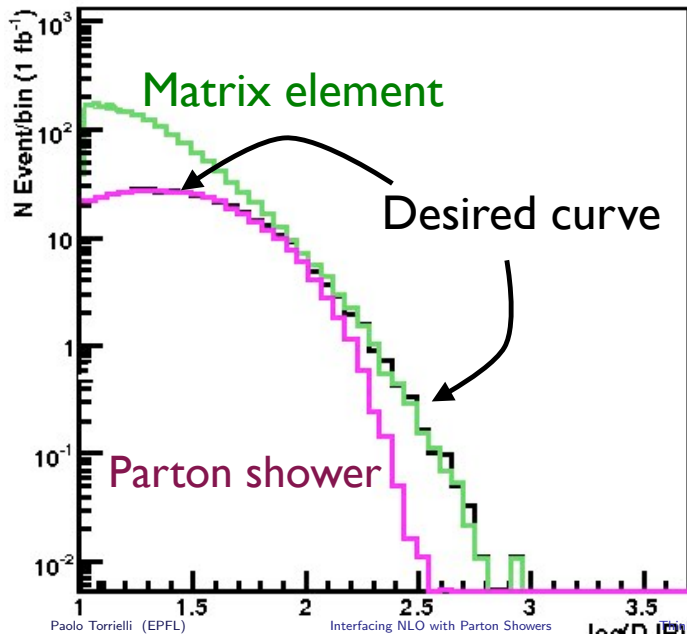
$$D_{ij,k} \rightarrow B \frac{\alpha_S}{p_i \cdot p_j} K_{ij,k}.$$

- ▶ The shower is developed from a Sudakov form factor

$$\Delta = \exp \left(- \int \frac{dt}{t} \int dz \alpha_S K_{ij,k} \right).$$

- ▶ It treats correctly the soft gluon emission off a color dipole, so angular ordering is built in.
- ▶ Hadronization: cluster model.

Beyond the collinear approximation in the shower



Matrix element corrections

- ▶ Pure parton shower approach developed near the boundaries of the phase space, where the cross section is singular: far from there the parton shower is not trustable. Try and include real matrix element information to better describe the tails.

PYTHIA: Matrix element reweighting.

- ▶ For many simple processes, the real emission matrix element ($d\sigma_{ME}^1$) is smaller than the corresponding first-emission parton shower prediction ($d\sigma_{MC}^1$).
- ▶ The phase space allowed for the shower is maximally extended and the first parton shower emissions are rejected with ratio $d\sigma_{ME}^1/d\sigma_{MC}^1$, which ensures a correct hard-emission spectrum.

HERWIG.

- ▶ The allowed region for the parton shower is kept limited, but in the dead zones is generated radiation according to the correct first emission matrix element distribution.

Matching at the NLO

Why matching at the NLO

- ▶ We have seen that the pure parton shower approach captures the most singular behavior of the cross section near the boundaries of the phase space and far from there it is not reliable.
- ▶ Matrix element corrections partially remedy this inefficiency, but are just an improved leading order matching: in particular **they completely miss the information on the finite piece of the virtual corrections.**
Also: not available for arbitrary processes.
- ▶ Nevertheless, the parton shower is an excellent approach to collider physics, since they provide, even if approximately, a realistic simulation of the real collision events, taking into account interactions beyond fixed-order, hadronization phase, multiple interactions, distributions of partons into the protons, pile-up effects, ...
- ▶ Conversely, a next to leading order Feynman diagram computation is a "theoretical exercise" (no ideas about non-perturbative effects), but well predicts observables in the part of the phase space far from the singular boundaries.

Why matching at the NLO

- ▶ NLO and parton showers are thus complementary approaches, the former good for hard emissions, the latter for soft / collinear ones.
- ▶ A good strategy is to formulate a method able to retain the virtues of the two while discarding their weaknesses: give a prediction which comes from the pure parton shower where the resummation of large logarithms is needed (soft and collinear region), and coincides with the NLO for hard radiation.

Special attention to be put on

- ▶ Avoiding double counting: there must be a way to assign a kinematics either to the shower part or to the NLO part.
- ▶ Achieving a smooth transition between the two different régimes.
- ▶ Attaining full NLO precision.

MC@NLO

Naive matching at the NLO

Consider leading order (= Born: suppose n -bodies), next to leading order, and parton shower differential cross sections:

$$\begin{aligned}d\sigma_{\text{LO}} &= d\Phi_B B, \\d\sigma_{\text{NLO}} &= d\Phi_B (B + V + d\Phi_{(+1)} R), \\d\sigma_{\text{MC}} &= d\Phi_B B I_{\text{MC}}^{(n)}(O) dO.\end{aligned}$$

- ▶ B and V have Born-like kinematics (n -bodies), while R has real-like kinematics ($n+1$ -bodies). V and R have implicitly one power of α_s more than B .
- ▶ $I_{\text{MC}}^{(k)}(O)$: parton shower spectrum for observable O , showering from a k -body initial condition (for example, a Les Houches file with events with k particles each).

Remember slide 22: $\int d\Phi_B B$ is what we had called σ_n , so that it easy to recognize $I_{\text{MC}}^{(n)}(O) dO$ in the case of 0 or 1 emission to be

$$I_{\text{MC}}^{(n)}(O) dO = \Delta(Q^2, Q_0^2) + \Delta(Q^2, Q_0^2) \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z).$$

Note. In the following, when talking about Monte Carlo's, it is understood the absence of matrix element corrections.

Naive matching at the NLO

Naive matching definition

$$\frac{d\sigma^{\text{"MC@NLO"}}}{dO} = [d\Phi_B(B + V)] I_{\text{MC}}^{(n)}(O) + [d\Phi_B d\Phi_{(+1)} R] I_{\text{MC}}^{(n+1)}(O).$$

This simple approach does NOT work:

- ▶ **Instability:** weights associated to $I_{\text{MC}}^{(n)}(O)$ and $I_{\text{MC}}^{(n+1)}(O)$ are separately divergent. Remember from the KLN theorem that only the sum of V and $\int d\Phi_{(+1)} R$ is finite, so it is hopeless to treat n - body and $n + 1$ -body configurations separately. One could regulate the divergence by means of some cut-off but then
 - ▶ one should prove the independence upon this cutoff,
 - ▶ the unweighting of real-like configurations would be highly inefficient (hard to extract events from a singular function).
- ▶ **Double counting:** this $d\sigma^{\text{"MC@NLO"}}$, expanded at the NLO does not coincide with the NLO differential rate.

One must indeed avoid overcounting the exact virtual with the approximated contribution from the Sudakov, otherwise some Born-like configurations, to be passed to the shower are spurious (not there in the exact NLO computation). Some configurations are thus accounted for by both the parton shower and the NLO.

Modified subtraction

Modify the naive formula:

$$\frac{d\sigma_{\text{MC@NLO}}}{dO} = \left[d\Phi_B(B + V + \int d\Phi_{(+1)} \text{MC}) \right] I_{\text{MC}}^{(n)}(O) + [d\Phi_B d\Phi_{(+1)} (R - \text{MC})] I_{\text{MC}}^{(n+1)}(O),$$

as opposed to the naive

$$\frac{d\sigma_{\text{"MC@NLO"}}}{dO} = [d\Phi_B(B + V)] I_{\text{MC}}^{(n)}(O) + [d\Phi_B d\Phi_{(+1)} R] I_{\text{MC}}^{(n+1)}(O).$$

The term MC is called the Monte Carlo counterterm, and its rough structure is

$$\text{MC} = \left| \frac{\partial(t^{\text{MC}}, z^{\text{MC}}, \phi)}{\partial\Phi_{(+1)}} \right| \frac{1}{t^{\text{MC}}} \frac{\alpha_s}{2\pi} \frac{1}{2\pi} P(z^{\text{MC}}) B.$$

- ▶ It is the cross section for the first emission in the parton shower: it is the Born matrix-element squared times the differential emission probability in a given point of the extra-parton phase space.
- ▶ It has the same collinear and soft singularities as the real and virtual emission amplitudes squared, so it acts as a local counterterm for them (subtlety on soft poles, see below).
- ▶ It is basically process independent.
- ▶ It essentially depends on the Monte Carlo one is interfacing to.

Properties

Good features of the modified subtraction with respect to the naive one (to be shown explicitly in the next slides):

- ▶ **Stability**: weights associated to different multiplicities (i.e. to $I_{\text{MC}}^{(n+1)}(O)$ and $I_{\text{MC}}^{(n+1)}(O)$) are now separately finite, because the Monte Carlo counterterm has the same collinear and soft poles as the real and virtual emission amplitude squared. Unweighting possible!
- ▶ **Double counting avoided**: the rate $d\sigma_{\text{MC@NLO}}$, expanded at the NLO, coincides with the total NLO cross section.
- ▶ **Smooth matching**: spectra coming from MC@NLO coincide in shape with the pure parton shower in the soft / collinear region (where the shower is actually reliable), and coincide both in shape and in normalization with the pure NLO for hard emission.
- ▶ **Normalization**: the MC@NLO cross section is naturally normalized to the total NLO cross section.

Properties

Avoiding of double counting:

$$\frac{d\sigma_{\text{MC@NLO}}}{dO} = \left[d\Phi_B(B + V + \int d\Phi_{(+1)} \text{MC}) \right] I_{\text{MC}}^{(n)}(O) + [d\Phi_B d\Phi_{(+1)} (R - \text{MC})] I_{\text{MC}}^{(n+1)}(O)$$

where, recalling slides 48 and 50, and using a simple notation for the Sudakov form factor Δ , we have

$$I_{\text{MC}}^{(k)}(O) dO = \Delta + \Delta d\Phi_{(+1)} \frac{MC}{B} + \dots ,$$

$$\Delta = \exp \left(- \int d\Phi_{(+1)} \frac{MC}{B} \right) .$$

Expand at the NLO (recall that MC , R and V have one α_s more than B):

$$I_{\text{MC}}^{(k)}(O) dO = 1 - \int d\Phi_{(+1)} \frac{MC}{B} + d\Phi_{(+1)} \frac{MC}{B} + \dots$$

$$\begin{aligned} d\sigma_{\text{MC@NLO}} &= \left[d\Phi_B(B + V + \int d\Phi_{(+1)} \text{MC}) \right] \left[1 - \int d\Phi_{(+1)} \frac{MC}{B} + d\Phi_{(+1)} \frac{MC}{B} + \dots \right] \\ &+ [d\Phi_B d\Phi_{(+1)} (R - \text{MC})] [1 + \dots] \\ &= d\Phi_B(B + V + d\Phi_{(+1)} R) = d\sigma_{\text{NLO}} \end{aligned}$$

Properties

$$\frac{d\sigma_{\text{MC@NLO}}}{dO} = \left[d\Phi_B(B + V + \int d\Phi_{(+1)} \text{MC}) \right] I_{\text{MC}}^{(n)}(O) + [d\Phi_B d\Phi_{(+1)} (R - \text{MC})] I_{\text{MC}}^{(n+1)}(O)$$

Smooth matching:

- ▶ In the soft / collinear region $R - \text{MC} \sim 0$ and one gets

$$d\sigma_{\text{MC@NLO}} \propto I_{\text{MC}}^{(n)}(O) dO.$$

The shape of the spectrum is identical to the underlying Monte Carlo, while the normalization in that region takes into account real and virtual emission, at variance with the pure parton shower. This is a way to include all the aspects of an NLO computation consistently in a parton shower.

- ▶ In the hard region, pure shower effects are suppressed (recall that matrix element corrections are excluded, so the maximum scale imposed on the shower implies MC to be zero far from the singular boundaries): $\text{MC} \sim 0$, $I_{\text{MC}}^{(n)}(O) dO \sim 1$, $B = V = 0$, and then

$$d\sigma_{\text{MC@NLO}} \sim d\Phi_B d\Phi_{(+1)} R.$$

Normalization:

- ▶ Based on the unitarity property of the parton shower

$$\int I_{\text{MC}}^{(k)}(O) dO = 1 \implies \int d\sigma_{\text{MC@NLO}} = \int d\sigma_{\text{NLO}}.$$

Subtleties: G-function and FKS subtraction

- ▶ The Monte Carlo counterterm is the collinear limit of the real matrix element squared, but what about soft poles?
- ▶ One includes the soft poles by modifying $MC \rightarrow MCG + (1 - G)R$, where G is a function that is 0 in the soft limit and goes smoothly to 1 immediately outside the limit.
One can show that this replacement does not spoil the properties of the matching. No dependence on the choice of G function has ever shown.
- ▶ It is too complicated to integrate analytically $\int d\Phi_{(+1)} MC$, so one cannot get the $1/\epsilon^2$ and $1/\epsilon$ poles to cancel the ones from V , even if the sum is effectively finite.
- ▶ Need for a subtraction method that analytically cancels the poles: FKS subtraction.

Implementation

$$\frac{d\sigma_{\text{MC@NLO}}}{dO} = \left[d\Phi_B(B + V + \int d\Phi_{(+1)} \text{MC}) \right] I_{\text{MC}}^{(n)}(O) + [d\Phi_B d\Phi_{(+1)} (R - \text{MC})] I_{\text{MC}}^{(n+1)}(O)$$

Integrands associated with n - and $n + 1$ - parton multiplicities are called \mathbb{S} (for "standard Monte Carlo") and \mathbb{H} (for "hard").

They can be negative somewhere: MC@NLO is not positive definite (more on this later).

- ▶ Compute the integrals of \mathbb{S} - and \mathbb{H} - integrands ($I_{\mathbb{S}}$, and $I_{\mathbb{H}}$) and the integrals of the absolute values of \mathbb{S} - and \mathbb{H} -integrands ($J_{\mathbb{S}}$ and $J_{\mathbb{H}}$).
- ▶ Generate \mathbb{S} - or \mathbb{H} - kinematical configurations (events) distributed according to $J_{\mathbb{S}}$ and $J_{\mathbb{H}}$ (probability distributions are positive definite), but assign them a weight with sign \pm depending on the sign of the \mathbb{S} - and \mathbb{H} - integrand in that particular configuration (unweighting up to a sign).

Negative weights

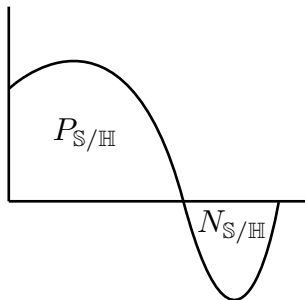
- ▶ Calling $P_{S/H}$ and $N_{S/H}$ the absolute values of the areas of the positive and negative regions, then

$$I_{S/H} = P_{S/H} - N_{S/H},$$

$$J_{S/H} = P_{S/H} + N_{S/H},$$

and the fraction of negative weights is

$$f_{S/H} = \frac{N_{S/H}}{P_{S/H} + N_{S/H}} = \frac{1}{2} \left(1 - \frac{I_{S/H}}{J_{S/H}} \right).$$



- ▶ The fraction of negative weights is expected to be reasonably small, since the Born piece is positive-definite and perturbatively dominant.

But, basic question: is it a conceptual problem to have negative weights?

- ▶ **No**: after showering, MC@NLO distributions are positive definite (for sufficiently high statistics) and physical: this can be easily understood because MC@NLO interpolates smoothly between two positive-definite contributions.
- ▶ Fraction of negative weights just affects the "efficiency", i.e. the number total events needed to get smooth histograms (the less the negative weights the smoother the spectrum).

MC@NLO: old limitations

Lack of a systematic approach:

- ▶ One code per process / simple processes only.
- ▶ Necessary slowness in including new processes.
- ▶ Necessary slowness in adding a new PSMC.

HERWIG 6, Herwig++: $\mathcal{O}(30)$ processes. PYTHIA 6: 2 processes.

IPROC	IV	IL ₁	IL ₂	Spin	Process
-1350-IL				✓	$H_1 H_2 \rightarrow (Z/\gamma^* \rightarrow) l_{\text{IL}} l_{\text{IL}} + X$
-1360-IL				✓	$H_1 H_2 \rightarrow (Z \rightarrow) l_{\text{IL}} l_{\text{IL}} + X$
-1370-IL				✓	$H_1 H_2 \rightarrow (\gamma^* \rightarrow) l_{\text{IL}} l_{\text{IL}} + X$
-1460-IL				✓	$H_1 H_2 \rightarrow (W^+ \rightarrow) l_{\text{IL}}^+ \nu_{\text{IL}} + X$
-1470-IL				✓	$H_1 H_2 \rightarrow (W^- \rightarrow) l_{\text{IL}}^- \bar{\nu}_{\text{IL}} + X$
-1396				×	$H_1 H_2 \rightarrow \gamma^* (\rightarrow \sum_i f_i f_i) + X$
-1397				×	$H_1 H_2 \rightarrow Z^0 + X$
-1497				×	$H_1 H_2 \rightarrow W^+ + X$
-1498				×	$H_1 H_2 \rightarrow W^- + X$
-1600-ID					$H_1 H_2 \rightarrow H^0 + X$
-1705					$H_1 H_2 \rightarrow b\bar{b} + X$
-1706	7	7		×	$H_1 H_2 \rightarrow t\bar{t} + X$
-2000-IC	7			×	$H_1 H_2 \rightarrow t/\bar{t} + X$
-2001-IC	7			×	$H_1 H_2 \rightarrow t + X$
-2004-IC	7			×	$H_1 H_2 \rightarrow t + X$
-2030	7	7		×	$H_1 H_2 \rightarrow tW^-/\bar{t}W^+ + X$
-2031	7	7		×	$H_1 H_2 \rightarrow tW^+ + X$
-2034	7	7		×	$H_1 H_2 \rightarrow tW^- + X$
-2040	7	7		×	$H_1 H_2 \rightarrow tH^-/\bar{t}H^+ + X$
-2041	7	7		×	$H_1 H_2 \rightarrow tH^+ + X$
-2044	7	7		×	$H_1 H_2 \rightarrow tH^- + X$
-2600-ID	1	7		×	$H_1 H_2 \rightarrow H^0 W^+ + X$

IPROC	IV	IL ₁	IL ₂	Spin	Process
-1706		i	j	✓	$H_1 H_2 \rightarrow (t \rightarrow) b_k f_i f'_j (t \rightarrow) b_l f'_i f'_j + X$
-2000-IC		i		✓	$H_1 H_2 \rightarrow (t \rightarrow) b_k f_i f'_j / (t \rightarrow) b_k f_i f'_j + X$
-2001-IC		i		✓	$H_1 H_2 \rightarrow (t \rightarrow) b_k f_i f'_j + X$
-2004-IC		i		✓	$H_1 H_2 \rightarrow (t \rightarrow) b_k f_i f'_j + X$
-2030		i	j	✓	$H_1 H_2 \rightarrow (t \rightarrow) b_k f_i f'_j (W^- \rightarrow) f_j f'_j /$ $(\bar{t} \rightarrow) \bar{b}_k f_i f'_j (W^+ \rightarrow) f_j f'_j + X$
-2031		i	j	✓	$H_1 H_2 \rightarrow (t \rightarrow) b_k f_i f'_j (W^+ \rightarrow) f_j f'_j + X$
-2034		i	j	✓	$H_1 H_2 \rightarrow (t \rightarrow) b_k f_i f'_j (W^- \rightarrow) f_j f'_j + X$
-2040		i		✓	$H_1 H_2 \rightarrow (t \rightarrow) b_k f_i f'_j H^- /$ $(\bar{t} \rightarrow) \bar{b}_k f_i f'_j H^+ + X$
-2041		i		✓	$H_1 H_2 \rightarrow (\bar{t} \rightarrow) b_k f_i f'_j H^+ + X$
-2044		i		✓	$H_1 H_2 \rightarrow (t \rightarrow) b_k f_i f'_j H^- + X$
-2850		i	j	✓	$H_1 H_2 \rightarrow (W^+ \rightarrow) l_i^+ \nu_i (W^- \rightarrow) l_j^- \bar{\nu}_j + X$
-2870		i	j	✓	$H_1 H_2 \rightarrow (W^+ \rightarrow) l_i^+ \nu_i (Z^0 \rightarrow) l_j^0 \bar{\nu}_j + X$
-2880		i	j	✓	$H_1 H_2 \rightarrow (W^+ \rightarrow) l_i^+ \nu_i (Z^0 \rightarrow) l_j^0 \bar{\nu}_j + X$
-2600-ID	1	i		✓	$H_1 H_2 \rightarrow H^0 (W^+ \rightarrow) l_i^+ \nu_i + X$
-2600-ID	-1	7		×	$H_1 H_2 \rightarrow H^0 W^- + X$
-2600-ID	-1	i		✓	$H_1 H_2 \rightarrow H^0 (W^- \rightarrow) l_i^- \bar{\nu}_i + X$
-2700-ID	0	7		×	$H_1 H_2 \rightarrow H^0 Z + X$
-2700-ID	0	i		✓	$H_1 H_2 \rightarrow H^0 (Z \rightarrow) l_i l_i + X$
-2850		7	7	×	$H_1 H_2 \rightarrow W^+ W^- + X$
-2860		7	7	×	$H_1 H_2 \rightarrow Z^0 Z^0 + X$
-2870		7	7	×	$H_1 H_2 \rightarrow W^+ Z^0 + X$
-2880		7	7	×	$H_1 H_2 \rightarrow W^- Z^0 + X$

POWHEG

Naive definition

Recall the Monte Carlo cross section for up to 1 emission, in the simplified notation of slide 52:

$$d\sigma = d\Phi_B B \left[\Delta + \Delta d\Phi_{(+1)} \frac{MC}{B} \right].$$

Try first to attain NLO precision by replacing the Born contribution with the NLO cross section integrated in the extra parton phase space:

$$d\sigma^{\text{"POWHEG"}} = d\Phi_B \left[B + V + \int d\Phi_{(+1)} R \right] \left[\Delta + \Delta d\Phi_{(+1)} \frac{MC}{B} \right].$$

This naive definition does not work, since if one expands it at the NLO, it does not coincide with the differential NLO cross section (**double counting**). The integral implicit in the Sudakov form factor definition does not contain any R term to cancel the one in the first parenthesis, and the piece in $d\Phi_{(+1)}$ is not R/B .

Modified Sudakov form factor

The previous considerations suggest that in order to avoid double counting, one should replace the definition of the Sudakov form factor with the following:

$$\tilde{\Delta}(Q^2, Q_0^2) = \exp \left[- \int_{Q_0^2}^{Q^2} d\Phi_{(+1)} \frac{R}{B} \right],$$

corresponding to a modified differential branching probability $d\tilde{p} = d\Phi_{(+1)} R/B$. It is thus tempting to define the POWHEG cross section as

$$d\sigma_{\text{"POWHEG"}} = d\Phi_B \left[B + V + \int d\Phi_{(+1)} R \right] \left[\tilde{\Delta}(Q^2, Q_0^2) + \tilde{\Delta}(Q^2, Q_0^2) d\Phi_{(+1)} \frac{R}{B} \right].$$

This time the double counting is avoided, but the integral of the cross section is different from the total NLO, since the second parenthesis does not integrate to 1. The above formula has thus to be modified to

$$d\sigma_{\text{POWHEG}} = d\Phi_B \left[B + V + \int d\Phi_{(+1)} R \right] \left[\tilde{\Delta}(Q^2, Q_0^2) + \tilde{\Delta}(Q^2, t) d\Phi_{(+1)} \frac{R}{B} \right],$$

where t is the scale at which the branching probability R/B in the second parenthesis is evaluated.

Properties

$$d\sigma_{\text{POWHEG}} = d\Phi_B \left[B + V + \int d\Phi_{(+1)} R \right] \left[\tilde{\Delta}(Q^2, Q_0^2) + \tilde{\Delta}(Q^2, t) d\Phi_{(+1)} \frac{R}{B} \right].$$

- ▶ The second piece in the second parenthesis is $d\tilde{\Delta}(Q^2, t)$, so its integral over the extra parton phase space (here between scales Q_0^2 and Q^2) is $\tilde{\Delta}(Q^2, Q^2) - \tilde{\Delta}(Q^2, Q_0^2) = 1 - \tilde{\Delta}(Q^2, Q_0^2)$, so the parenthesis integrates to 1 (this can also be understood as unitarity of the shower below scale t).
So the POWHEG **cross section is normalized at the NLO**.

- ▶ Expand at the first-emission level:

$$d\sigma_{\text{POWHEG}} = d\Phi_B \left[B + V + \int d\Phi_{(+1)} R \right] \left[1 - \int d\Phi_{(+1)} \frac{R}{B} + d\Phi_{(+1)} \frac{R}{B} \right] = d\sigma_{\text{NLO}},$$

so **double counting is avoided**.

- ▶ Its structure is identical an ordinary shower, with just normalization rescaled by a global k - factor and a different Sudakov: **no negative weights** are involved.

The first two items are defining properties of both MC@NLO and POWHEG, and indeed show how they are formally equivalent at the NLO level. Nevertheless, there are many practical differences between the two.

MC@NLO vs POWHEG

- ▶ \uparrow MC@NLO, \downarrow POWHEG: MC@NLO does not exponentiate of the non-singular part of the real emission amplitude.
- ▶ \uparrow MC@NLO, \downarrow POWHEG: MC@NLO does not require tricks for treating Born zeros ($MC \propto B$).
- ▶ \uparrow POWHEG, \downarrow MC@NLO: POWHEG is independent from the parton shower one is interfacing the computation to.
- ▶ \uparrow POWHEG, \downarrow MC@NLO: POWHEG has not negative weights. A slightly smaller statistics is required to POWHEG than to MC@NLO in order to get equally smooth plots.

Many complicated processes implemented in POWHEG (for example: $Wb\bar{b}$, WW , Wj , di-jets, ...), but not fully automatic: the implementation of new processes requires some dedicated code.

What about MC@NLO?

State of the art, automatic NLO+PS matching: aMC@NLO

From MC@NLO to aMC@NLO

- ▶ MC@NLO framework solid and mature.
- ▶ Limitations only in the lack of a systematic implementation, not in the method.

Then: build a framework that automates the computation of all the steps needed for the matching at the NLO, and also makes it easy the inclusion of a new parton shower.

- ▶ MadGraph / MadFKS: Born contribution, poles subtraction and finite part of the Real.
- ▶ MadLoop: finite part of the Virtual.
- ▶ Compute automatically MC counterterms.
- ▶ Brand new: compute automatically scale and PDF uncertainties without rerunning the code!

aMC@NLO: detailed structure of the MC counterterm

$$MC = \sum_{pq,c,l \in c} \frac{\delta_{p \in l}}{N_p} \frac{\alpha_S}{(2\pi)^2} \left| \frac{\partial(t_p^{(l)}, z_p^{(l)}, \phi)}{\partial \Phi_{(+1)}} \right| \Theta(DZ) d\Phi_B \frac{P_{p \rightarrow qr}(z_p^{(l)}) |\overline{\mathcal{M}}_c|_B^2 + Q_{p \rightarrow qr}(z_p^{(l)}) |\overline{\mathcal{M}}_c|^2}{t_p^{(l)}}$$

- ▶ c, l = color flow / color line: shower variables and scales may depend on it.
- ▶ $|\overline{\mathcal{M}}_c|_B^2 \equiv B |\mathcal{M}_c|_B^2 / \sum_{c'} |\mathcal{M}_{c'}|_B^2$ = 'barred' Born amplitude squared, to recover the full Born summing only on leading color.
- ▶ $Q_{p \rightarrow qr}(z_p^{(l)})$ = azimuthal kernel.
- ▶ $|\overline{\mathcal{M}}_c|^2$ = 'barred' azimuthal amplitude squared.
- ▶ $\Theta(DZ)$ = dead zone (built-in for HERWIG, imposed to PYTHIA).

Steps performed in a fully automatic and process-independent way:

- ▶ Assignment of color flow and color partner.
- ▶ Assignment of the splitting type (initial state radiation from leg 1 or 2, final state radiation from massive or massless leg).
- ▶ Shower variables definitions and computation of the jacobian.
- ▶ Computation of barred amplitudes and Altarelli-Parisi kernels.

aMC@NLO: checks and validation

Process-independent checks:

- ▶ Infrared limits of the Monte Carlo counterterm have to coincide with the ones of the real emission contribution: integrals \mathbb{S} and \mathbb{H} have to be separately finite.
- ▶ The total cross section has to be the NLO one.

Validation:

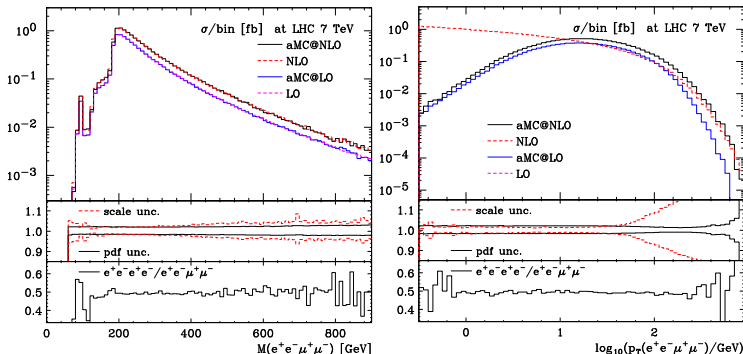
- ▶ Fixed process and parameters, all spectra have to coincide with MC@NLO.

New results

new results with aMC@NLO

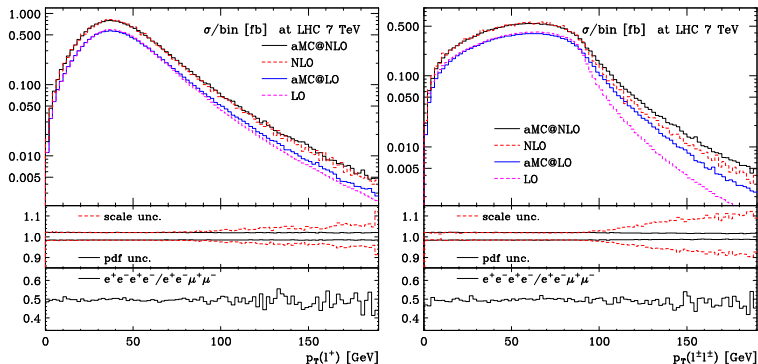
- ☼ Recently published results (all this year) using the aMC@NLO code:
 - ☼ (pseudo-)scalar Higgs production in association with a top-antitop pair
[RE, Frixione, Hirschi, Maltoni, Pittau & Torrielli, arXiv:1104.5615]
 - ☼ Vector boson production in association with a bottom-antibottom pair
[RE, Frixione, Hirschi, Maltoni, Pittau & Torrielli, arXiv:1106.6019]
 - ☼ Four charged lepton production at hadron colliders
[RE, Frixione, Hirschi, Maltoni, Pittau & Torrielli, arXiv:1110.4758]
 - ☼ Wjj at the Tevatron
[RE, Frixione, Hirschi, Maltoni, Pittau & Torrielli, arXiv:1110.5502]

Four-lepton production



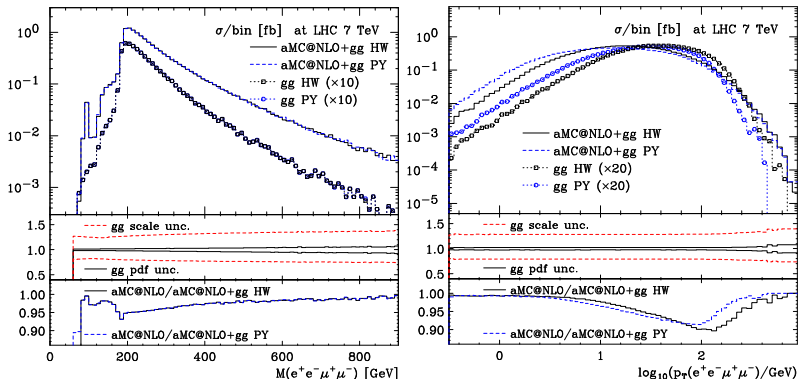
- 4-lepton invariant mass is almost insensitive to parton shower effects.
- 4-lepton transverse moment is extremely sensitive
- Including scale uncertainties

Four-lepton production



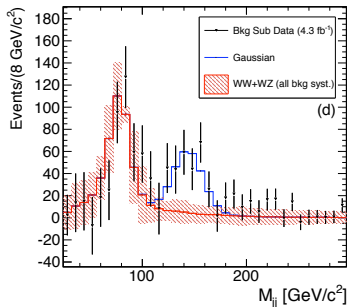
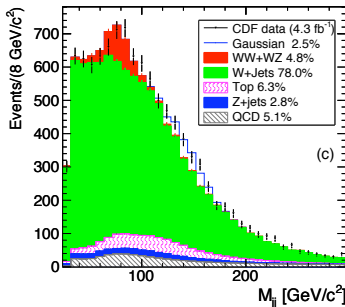
- For observables that are sensitive to radiation, corrections from LO \rightarrow NLO \rightarrow aMC@NLO can be sizable
- Same feature in scale uncertainty

Four-lepton production



- ✱ Differences between Herwig (black) and Pythia (blue) showers large in the Sudakov suppressed region
- ✱ Contributions from gg initial state (formally NNLO) are of 5-10%

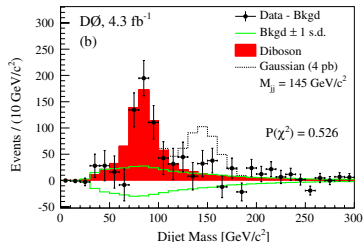
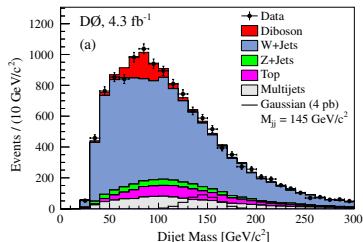
Wjj at CDF



- ✿ In April CDF reported an excess of events with 3.2 standard deviation significance in the dijet invariant mass distribution (with invariant mass 130-160 GeV) for Wjj events
- ✿ The update in June (using 7.3 fb^{-1} of data) increased significance of the excess to 4.1 standard deviations

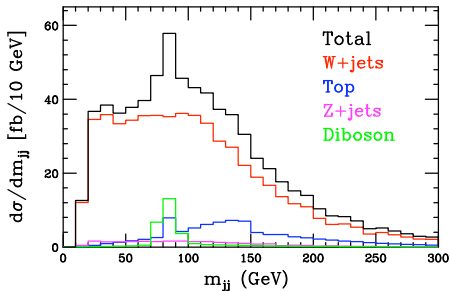
Response...

- By now ~100 papers have appeared trying to explain this excess by introducing BSM physics
- 2 papers tried to explain the results within the SM (by addressing issues in the top quark sector)
- CDF's results are not confirmed by DØ



NLO effects

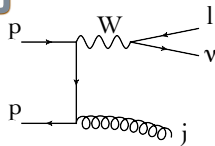
- Both CDF and DØ estimates their backgrounds using LO SMC programs (Alpgen+Pythia & Sherpa) normalized to (N)NLO or to the data
- J. Campbell, A. Martin & C. Williams have looked at the same distribution at parton level to study the impact of NLO corrections on differential distributions
- Using the newly developed tool, **amc@NLO**, we would like to address the main background, W+2j, at the NLOwPS level to see how well LOwPS or fixed order NLO describe this distribution



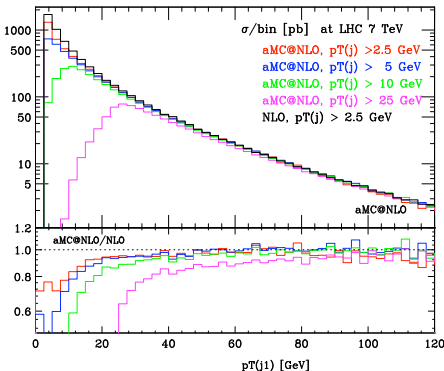
Computational challenge

- ✿ This is the first time that such a process with so many scales and possible (IR) divergences is matched to a parton shower at NLO accuracy
- ✿ Start with $W+1j$ production to validate processes which need cuts at the matrix-element level
- ✿ To check the insensitivity to this cut:
 - ✿ generate a couple of event samples with different cuts and show that the distributions after analysis cuts are statistically equivalent

pp \rightarrow Wj



- For W+1j the easiest cut would be in on the p_T of the W boson
- However, for validation purposes it is more appropriate to apply this cut on the jet instead (because that is what we'll be doing in W+2j). Same at LO, but different at NLO
- Different cuts at generation level yield the same distributions at analysis level if the analysis level cut is 3-4 times larger

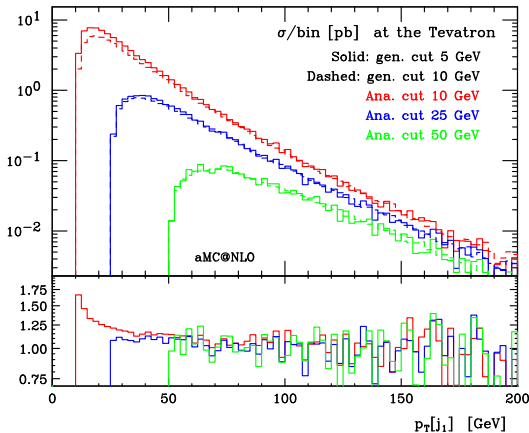


pp \rightarrow Wjj

- Two event samples with 5 GeV and 10 GeV p_T cuts on the jets at generation level, respectively, each with 10 million unweighted events
- Renormalization and factorization scales equal to $\mu_R = \mu_F = H_T/2$
$$2\mu_R = 2\mu_F = H_T = \sqrt{(p_{T,l}^2 + m_l^2) + \sum |p_{T,i}|}$$
where sum is over the 2 or 3 partons (and the matrix element level)
- Jets are defined with anti- k_T and $R=0.4$
- MSTW2008(N)LO PDF set for the (N)LO predictions (with $\alpha_s(m_Z)$ from PDF set using (2)1-loop running)
- $m_W = 80.419$ GeV,
 $G_F = 1.16639 \cdot 10^{-5}$ GeV⁻²,
 $\alpha^{-1} = 132.507$,
 $\Gamma_W = 2.0476$ GeV

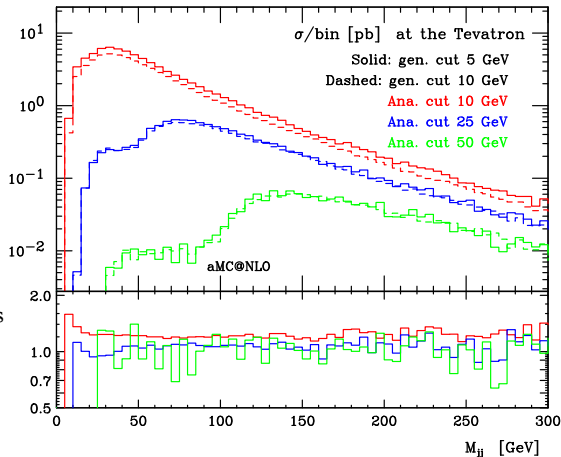
$pp \rightarrow Wjj$

- ✱ The two generation level cuts agree for high enough momenta (or harder analysis cuts)
- ✱ Middle plot shows ratio of NLO (solid), LO (dotted) and LOwPS (dashed) over aMC@NLO
- ✱ Good agreement with (N) LO, slight difference in shape
- ✱ Tails have low statistics, in particular for the 5 GeV generation cuts

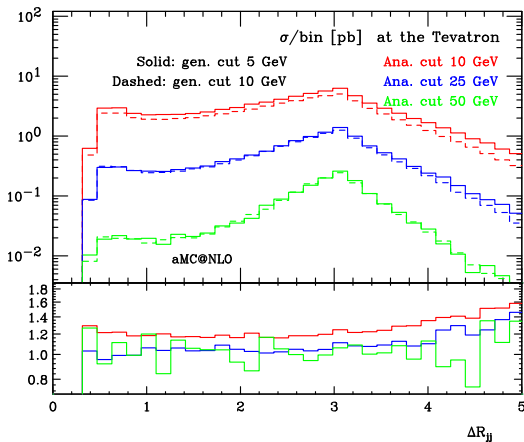


pp \rightarrow Wjj

- ✱ Dijet invariant mass
- ✱ For analysis cuts larger than 25 GeV the two event samples coincide (except for the very low mass region)
- ✱ For smaller analysis cuts the bias is flat in this distribution



pp \rightarrow Wjj



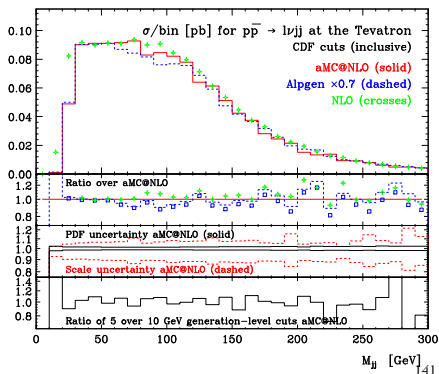
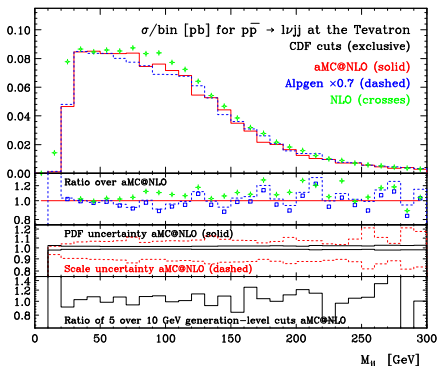
- Distance between the jets
- A small bias remains at 25 GeV analysis in the tail of the distribution, but reduced a lot from lower cuts analysis cuts
- 5 GeV sample probably ok, 10 GeV gen. cut is a bit too hard
- Of all distributions we have looked at, this one shows the largest bias due to generation cut

pp \rightarrow Wjj

- minimal transverse energy for the lepton: $E_T(l) > 20$ GeV;
 - maximal pseudo rapidity for the lepton: $|\eta(l)| < 1$;
 - minimal missing transverse energy: $\cancel{E}_T > 25$ GeV;
 - minimal transverse W-boson mass: $M_T(l\nu_l) > 30$ GeV;
 - jet definition: JetClu algorithm with 0.75 overlap and $R = 0.4$;
 - minimal transverse jet energy: $E_T(j) > 30$ GeV;
 - maximal jet pseudo rapidity: $|\eta(j)| < 2.4$;
 - minimal jet pair transverse momentum: $p_T(j_1 j_2) > 40$ GeV;
 - minimal jet-lepton separation: $\Delta R(lj) > 0.52$;
 - minimal jet-missing energy separation: $\Delta\phi(\cancel{E}_T j) > 0.4$;
 - hardest jets close in pseudorapidity: $|\Delta\eta(j_1 j_2)| < 2.5$;
 - jet veto: no third jet with $E_T(j) > 30$ GeV and $|\eta(j)| < 2.4$;
 - lepton isolation: transverse hadronic energy smaller than 10% of the lepton transverse energy in a cone of $R = 0.4$ around the lepton.
- ☼ To slightly simplify the analysis, the MC truth is used to assign the lepton to the W-boson decay
 - ☼ Only W^+ events (simply a factor 2)
 - ☼ No underlying event

pp \rightarrow Wjj

- Dijet invariant mass with/without jet veto
- This is the distribution in which CDF found an excess of events around 130-160 GeV
- No differences in shape between the 5 and 10 GeV generation level cuts
- No sign of enhancement over (N)LO or LOwPS in the mass range 130-160 GeV



<http://amcatnlo.cern.ch>