Bootstrapping black holes

Henry Lin, Stanford University

February 5, 2025

This talk is based on:
HL, 2302.04416
2410.14647 w/ **Zechuan Zheng**
25??.????? w/ **Zechuan Zheng**

see also:

[Anderson & Kruczenski, 1612.08140],
[**HL**, 2002.08387]
[Han, Harnoll, Kruthoff, 2004.10212]
[Kazakov & Zheng, 2108.04830]
[Cho, Gabai, Sandor, Yin, 2410.04262]

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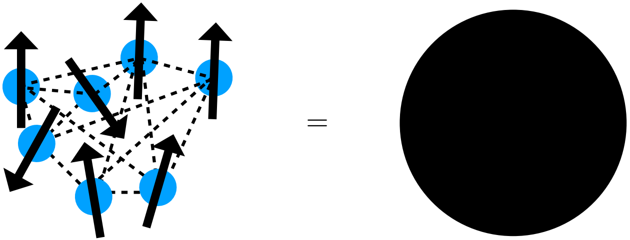

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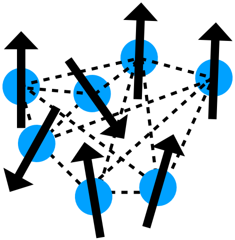

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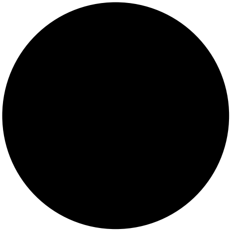

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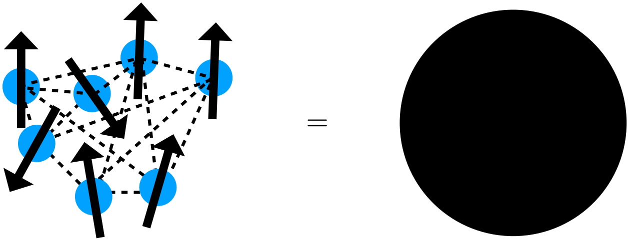

It would be nice to solve the quantum system. This means, e.g., computing correlators like $\frac{1}{Z}\,\mathrm{tr}\left(e^{-\beta H}\mathcal{O}\right)$ on the LHS.

For certain special black holes, we know explicitly what the quantum system is (large N super Yang-Mills gauge theories)

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So why haven't we solved these black holes yet?

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These features make solving the dual quantum mechanics hard.

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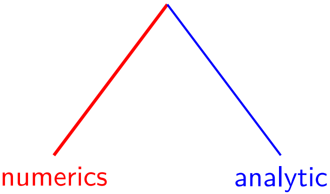

- numerics is hard at large N
- analytic methods are sparse due to strong coupling

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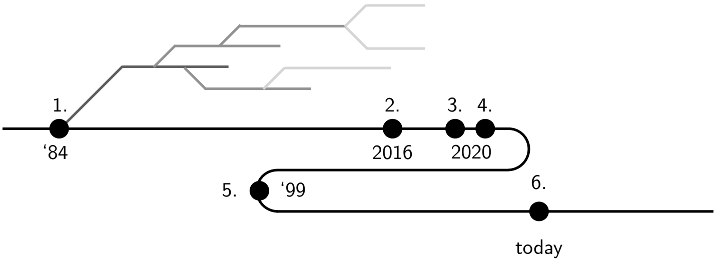
Bootstrap: a timeline



1. CFT bootstrap [Ferrara '73], [Polyakov '74], [Belavin, Polyakov, Zamolodchikov '84]
2. Lattice Yang Mills bootstrap [Anderson & Kruczenski '16, Kazakov & Zheng '22]
3. Matrix bootstrap [HL '20]
4. Quantum mechanical bootstrap [Han, Hartnoll, Kruthoff '20]
5. Virial bound [Polchinski '99]
6. BFSS [today]

Outline

	$d = 0$ (stat mech)	$d = 1$
solvable	1-matrix integral	1-matrix quantum mech $c = 1$ matrix model
unsolvable	multi-matrix integral	D0-brane quantum mech BFSS matrix theory

D0-brane theory = simplest known system dual to a certain black hole = dimensional reduction of $\mathcal{N} = 4$ SYM to 0+1d.

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1-Matrix model

Probability distribution over $N \times N$ Hermitian matrix M_{ij}:

$$p(M) = \frac{1}{\mathcal{Z}} e^{-N^2 \operatorname{tr} V(M)}, \quad V(M) = \frac{1}{2}M^2 + \frac{g}{4}M^4$$

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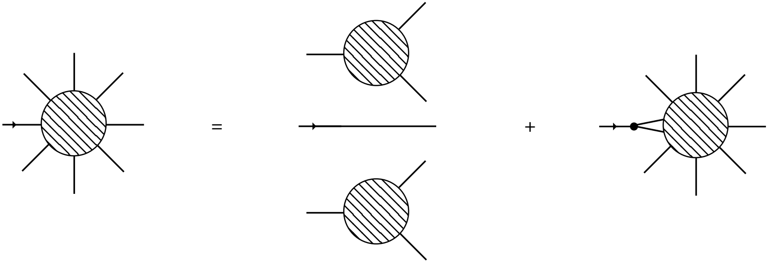
Goal: compute moments $\langle \operatorname{tr} M^k \rangle$ as a function of g.

$$\langle \operatorname{tr} M^2 \rangle = \lim_{N \to \infty} \frac{1}{\mathcal{Z}} \int \mathrm{d}M \, e^{-N^2 \operatorname{tr} V(M)} \operatorname{tr} M^2$$

Bootstrapping matrices

1. Guess the value of some simple correlator, e.g. $\langle \operatorname{tr} M^2 \rangle$
2. Feed it through the loop eqns to generate more correlators
3. Demand that $\langle \operatorname{tr} \mathcal{O}^\dagger \mathcal{O} \rangle \geq 0$.

Loop (Schwinger-Dyson) equations



- relates lower-pt correlators to higher-pt correlators
- uses large N factorization ('t Hooft)

$$\langle \mathrm{tr}\, M^k \rangle = \sum_{\ell=0}^{k-1} \langle \mathrm{tr}\, M^\ell \rangle \langle \mathrm{tr}\, M^{k-\ell-2} \rangle + g \langle \mathrm{tr}\, M^{k+2} \rangle$$

Positivity

Naive algorithm: starting with some guess for $\langle \mathrm{tr}\, M^2 \rangle$, generate moments $\langle \mathrm{tr}\, M^4 \rangle, \langle \mathrm{tr}\, M^6 \rangle, \langle \mathrm{tr}\, M^8 \rangle, \cdots$.

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If any even moment is negative, **rule out the guess.**

Positivity

More systematically, we can consider a general polynomial in the matrix M:

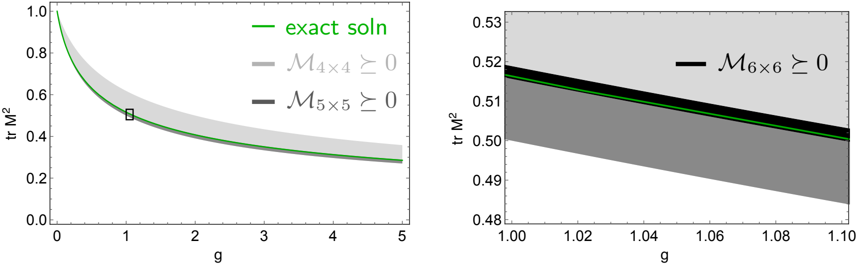
$$\mathcal{O} = \sum \alpha_k M^k \Rightarrow \operatorname{tr} \mathcal{O}^\dagger \mathcal{O} \geq 0.$$

This implies that $\alpha_i^* \mathcal{M}_{ij} \alpha_j \geq 0$ for all coefficients α, where we have assembled all the correlators into a big matrix $\mathcal{M}_{ij} = \left\langle \operatorname{tr} M^{i+j} \right\rangle$:

$$\mathcal{M} = \begin{pmatrix} 1 & \langle \operatorname{tr} M \rangle & \langle \operatorname{tr} M^2 \rangle \\ \langle \operatorname{tr} M \rangle & \langle \operatorname{tr} M^2 \rangle & \langle \operatorname{tr} M^3 \rangle \\ \langle \operatorname{tr} M^2 \rangle & \langle \operatorname{tr} M^3 \rangle & \langle \operatorname{tr} M^4 \rangle \end{pmatrix} \succeq 0$$

Here $\mathcal{M}_{ij} = \left\langle \operatorname{tr} M^{i+j} \right\rangle$.

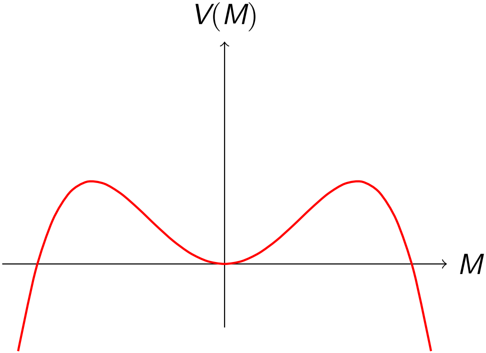
Review of the matrix bootstrap



As the size of $\mathcal{M}$ increases, rapid convergence to the exact solution.

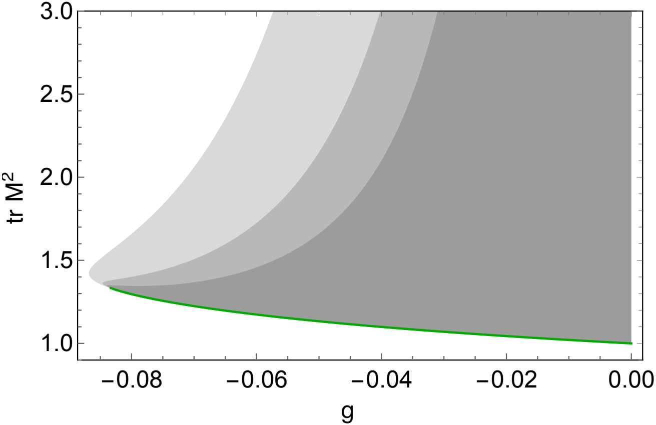
Metastability

To address the issue of metastability, consider $g < 0$. The potential is unbounded from below:



In the large N limit, tunneling is suppressed.

Metastability



For $-g_* < g < 0$ the model still makes sense at $N = \infty$

	$d = 0$ (stat mech)	$d = 1$
solvable	1-matrix integral	1-matrix quantum mech $c = 1$ matrix model
unsolvable	**multi-matrix integral**	D0-brane quantum mech BFSS matrix theory

Multi-matrix integrals

Main challenge: exponentially many correlators for a given length L, e.g., for $L = 7$:

$$\langle \mathrm{Tr}\, ABABBBA \rangle \,, \langle \mathrm{Tr}\, BBBABAB \rangle \,, \cdots$$

Also more loop equations and more positivity constraints:

$$\mathcal{M} = \begin{pmatrix} 1 & \mathrm{Tr}\, A & \mathrm{Tr}\, B & \cdots \\ \mathrm{Tr}\, A & \mathrm{Tr}\, A^2 & \mathrm{Tr}\, AB & \\ \mathrm{Tr}\, B & \mathrm{Tr}\, BA & \mathrm{Tr}\, B^2 & \\ \vdots & & & \ddots \end{pmatrix}$$

Despite these challenges, the bootstrap gives strong results for multi-matrix integrals [HL '20], e.g.,

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$$\mathcal{Z} = \int \mathrm{d}A \, \mathrm{d}B \, e^{-N^2 \operatorname{tr} V(A,B)}$$

$$V(X,Y) = -\frac{1}{2}[A,B]^2 + v(A) + v(B),$$

$$v(X) = \frac{1}{2}X^2 + \frac{1}{4}X^4$$

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Using non-linear relaxation, one can convert it to a standard semi-definite programming problem [Kazakov & Zheng '22].

$$0.4217836 \leq \langle \operatorname{tr} A^2 \rangle \leq 0.4217847$$

$$0.3333413 \leq \langle \operatorname{tr} A^4 \rangle \leq 0.3333421$$

~ 6 decimal digits on a laptop!

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1-matrix QM

N^2 non-relativistic particles arranged in a matrix.

$$\mathrm{i}[X_{ij}, P_{kl}] = \delta_{il}\delta_{jk}.$$

Hamiltonian:

$$H = N\left(\frac{1}{2}\,\mathrm{Tr}\,P^2 + \frac{m^2}{2}\,\mathrm{Tr}\,X^2 + \frac{g}{4}\,\mathrm{Tr}\,X^4\right).$$

U(N) gauge constraint:

$$J_{ik} = \mathrm{i}(X_{ij}P_{jk} - P_{ij}X_{jk}) + N\delta_{ik} = 0$$

[for a review, see Klebanov hep-th/9108019] [Brezin, Itzykson, Parisi, Zuber, Douglas, Klebanov, Kutasov, Maldacena, Martinec, Takayangi, Toumbas, Verlinde, $\cdots$]

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known as $c = 1$ or $\hat{c} = 1$ matrix model[1].

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[1] in the double scaling limit

Review of the quantum mechanical bootstrap

1. Replace loop eqns with $O' = [O, H]$. In energy eigenstates $\langle E| O' |E\rangle = \langle E|O|E\rangle E - E \langle E|O|E\rangle = 0$.

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   example: $0 = \langle [\mathrm{tr}\, XP, H] \rangle = -\mathrm{tr}\, P^2 + \mathrm{tr}\, X^2 + g\, \mathrm{tr}\, X^4$

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$$\langle E| \operatorname{tr} \mathcal{O}^\dagger \mathcal{O} |E\rangle \geq 0 \Rightarrow \mathcal{M}_{ij} = \langle E| \operatorname{tr} \mathcal{O}_i^\dagger \mathcal{O}_j |E\rangle \geq 0$$
3. Optional: ground state bootstrap positivity:
$$\langle O^\dagger [H, O] \rangle_{\text{gs}} = \langle O^\dagger H O \rangle_{\text{gs}} - E_{\text{gs}} \langle O^\dagger O \rangle_{\text{gs}} \geq 0$$

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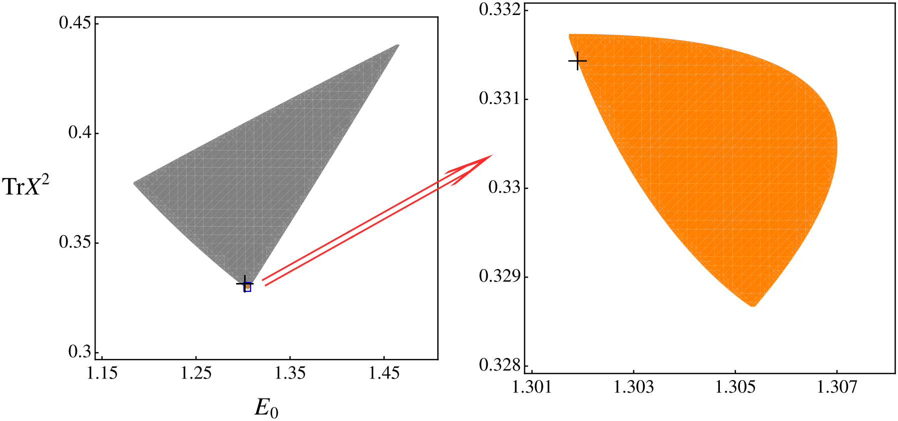
Ground state bootstrap



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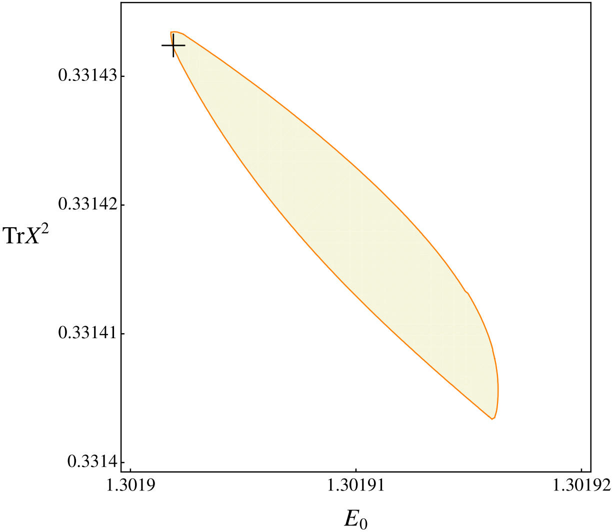

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D0-brane quantum mechanics

Hilbert space: 9 bosonic matrices and 16 fermionic matrices.
Transform as a vector and spinor of $SO(9)$.

$$H = \frac{1}{2}\mathrm{Tr}\left(g^2 P_I^2 - \frac{1}{2g^2}\left[X_I, X_J\right]^2 - \psi_\alpha \gamma^I_{\alpha\beta}\left[X_I, \psi_\beta\right]\right)$$

[Banks, Fischler, Shenker, Susskind '97]

Most of what we know on the matrix side is due to heroic Monte Carlo simulations [Kabat *et al.*, Anagnostopoulos *et al.*, Hanada *et al.*, ..., Berkowitz *et al.*, Pateloudis *et al.*]

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In the strongly coupled regime $\lambda\beta^3 \gg 1$, dual to a metastable black hole in Type IIA [Itzhaki, Maldacena, Sonneschein, Yankielowicz]:

$$\frac{\mathrm{d}s^2}{\alpha'} = -f(r) r_c^2 \,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r) r_c^2} + \left(\frac{r}{r_c}\right)^{-3/2} \mathrm{d}\Omega_8^2$$

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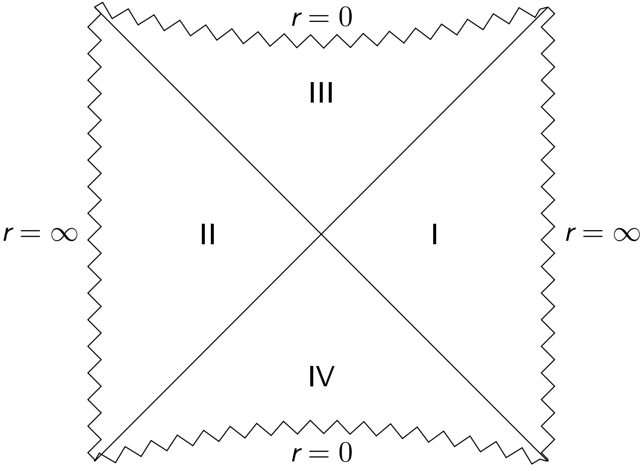
S_8 *shrinks* with r. At $r \sim \lambda^{1/3} \Rightarrow$ string scale curvature.



At $E/N^2 \gtrsim \lambda^{1/3}$ geometry is nowhere reliable.

Kinematic constraints

Less trivial example:

$$\mathcal{M}_{\alpha\beta\gamma\eta} = \left\langle \mathrm{tr}\Big(\psi^{\alpha}\psi^{\beta}\psi^{\delta}\psi^{\eta}\Big)\right\rangle$$

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Viewed as a matrix in the $\{\alpha, \beta\}$ and $\{\gamma, \eta\}$ indices, positivity requires $\mathcal{M} \succeq 0$.

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Using the "addition of SO(9) angular momentum" rules:

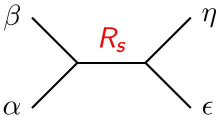
$$(\mathbf{16})^4 = (\mathbf{16}\times\mathbf{16})^2 = (\mathbf{1}+\mathbf{9}+\mathbf{36}+\mathbf{84}+\mathbf{128})^2$$
$$= 5(\mathbf{1}) + \text{non-singlets}$$

Thus group theory determines this $16^4 = 65536$ to just 5 unknowns.

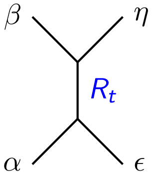
$$\mathcal{M}_{\alpha\beta\gamma\eta} = \delta_{\alpha\beta}\delta_{\eta\epsilon} a_{\mathbf{1}} + \gamma^{I}_{\alpha\beta}\gamma^{I}_{\eta\epsilon} a_{\mathbf{9}} + \gamma^{IJ}_{\alpha\beta}\gamma^{IJ}_{\eta\epsilon} a_{\mathbf{36}} + \gamma^{IJK}_{\alpha\beta}\gamma^{IJK}_{\eta\epsilon} a_{\mathbf{84}}$$
$$+ \gamma^{IJKL}_{\alpha\beta}\gamma^{IJKL}_{\eta\epsilon} a_{\mathbf{128}}$$

Cyclicity and the fermion anti-commutation relations cuts this further to just 2 unknowns.

Expand *s*-channel block in terms of *t*-channel blocks:



$$= \sum_{R_t} \mathbb{F}_{R_s, R_t} \begin{bmatrix} \mathbf{16} & \mathbf{16} \\ \mathbf{16} & \mathbf{16} \end{bmatrix}$$



⇒ 6j symbol. At higher levels, need higher-pt crossing kernels.

Kinematics determined $\mathcal{M}_{\alpha\beta\gamma\eta}$ in terms of 2 unknowns. We still need to impose positivity of a large matrix $\mathcal{M}_{\alpha\beta,\gamma\eta}$. By decomposing $\psi_\alpha\psi_\beta$ into irreps, one can easily diagonalize $\mathcal{M}$.

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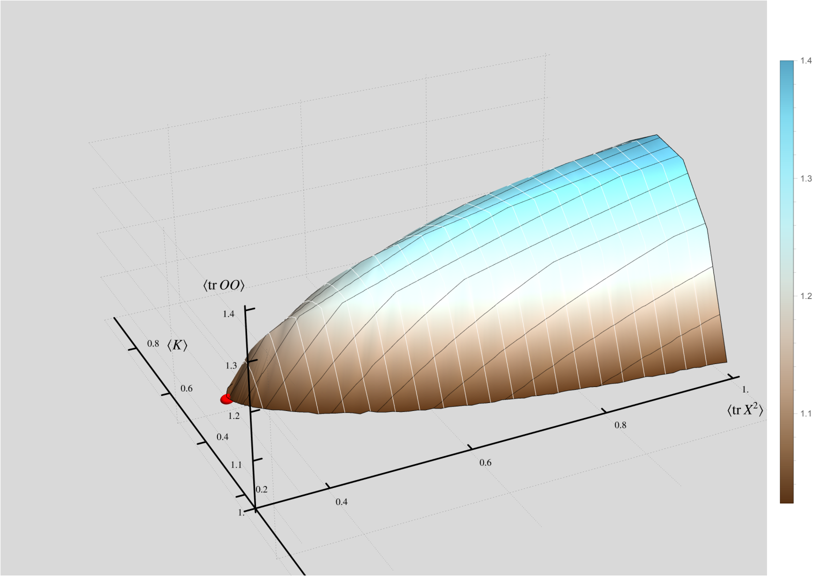
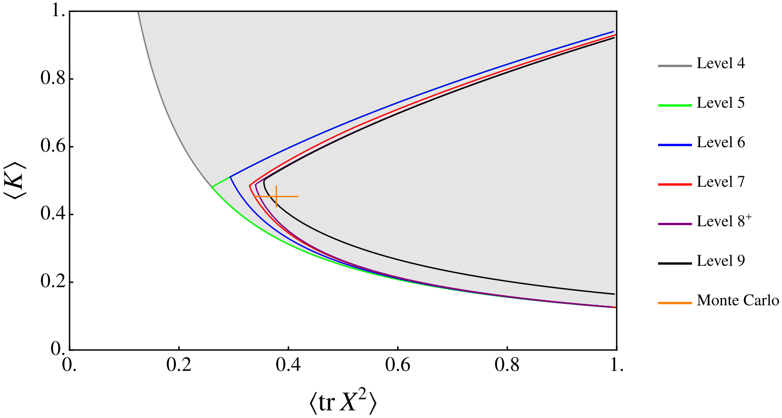
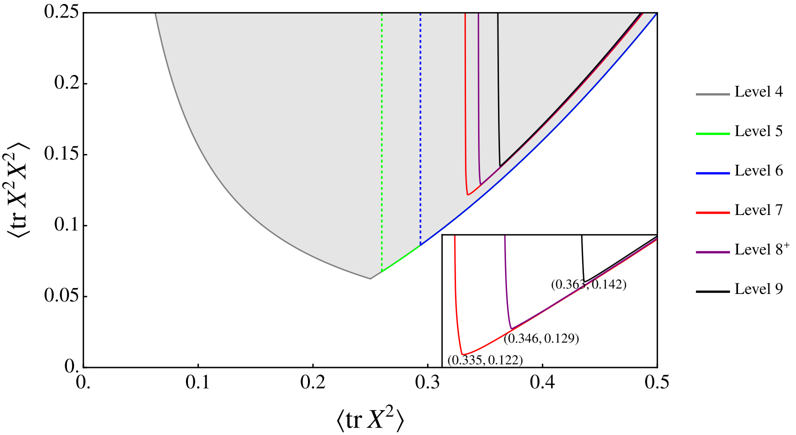
The upshot is that by leveraging the symmetries of the model, the D0-brane bootstrap is practical. ☺


⟨tr OO⟩
1.4
1.3
1.2
1.1
1.
⟨K⟩
0.8
0.6
0.4
0.2
0.4
0.6
0.8
1.
⟨tr X²⟩
1.4
1.3
1.2
1.1



Cross + is the Monte Carlo result* of [Berkowitz *et al.*'16].



The lower bound on $\langle \mathrm{tr}\, X^2 X^2 \rangle$ was derived (up to some factors) in [Polchinski '99]. It can also be improved to finite energy [HL '23].

	method	$\langle \mathrm{tr}\, X^2 \rangle$
	Monte Carlo [Pateloudis *et al.*'22]	$\approx 0.37 \pm 0.05$
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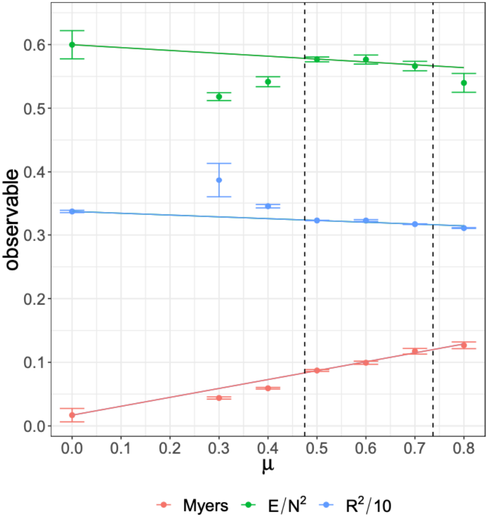
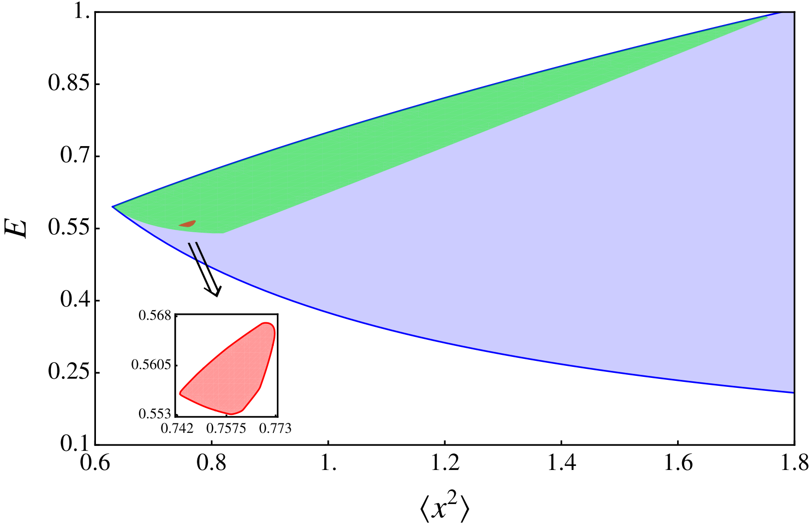
$\sim 90\%$ of the MC value with just level 7:
19 variable SDP, ~ 170 EoMs, matrices of size $\lesssim 20 \times 20$.

Metastability in Monte Carlo



Monte Carlo results [Pateloudis *et al.* '22]

Toy supermembrane problem



In a simpler toy problem, we see a similar-looking peninsula at low levels, but an island at higher levels.

I have presented some evidence that the bootstrap could yield precision data on some correlators like $\langle \mathrm{tr}\, X^2 \rangle$.

A high precision measurement of these and/or related correlators could give the *leading* corrections to the semiclassical black hole background. [A nice candidate is $\langle O_{\mathrm{BPS}} \rangle \sim T^{\Delta+\delta}$.]

In principle, we could use this to constrain unknown $O(\alpha'^3)$ corrections to the IIa effective action. [See Hanada, Berkowitz, Pateloudis, ... for similar discussions involving BH thermodynamics. Similar in spirit to the CFT bootstrap program by e.g. Binder, Chester, Pufu, Wang, ...]

Future directions

- Islands?
- Constraints on the bound state?
- Finite energy/temperature
- Large N lattice systems, especially those with sign problems? [Anderson & Kruczenski, Kazakov & Zheng, ...]

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Thanks!

Sign problem?

[WIP w/ Gauri Batra and Haifeng Tang]

Motivated by the agreement with the bootstrap (at least $\ell \leq 9$), we wish to study the sign problem. Integrating out the fermions,

$$\int D\psi Dc\, e^{-S} = \mathrm{Pf}(A, X^I) \to |\mathrm{Pf}(A, X^I)|$$

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$$\int D\psi Dc\, e^{-S} = \mathrm{Pf}(A, X^I) \to |\mathrm{Pf}(A, X^I)|$$

Calculate in perturbation theory:

$$|\mathrm{Pf}(A, X^I)|^{2n} = \mathrm{Pf}(A, X^I)^n \mathrm{Pf}(A^*, X^I)^n$$

High temperature expansion $\rightarrow$ reduce to a matrix integral + massive modes. So far: agreement at 1-loop...

I have presented some evidence that the bootstrap could yield precision data on some correlators like $\langle \mathrm{tr}\, X^2 \rangle$.

In the remainder of the talk, I will comment on:
What could we hope to learn by measuring $\langle \mathrm{tr}\, X^2 \rangle$ precisely?

Suppose that one day we have high precision measurements of 1-pt functions like $\langle \mathrm{tr}\, X^n \rangle$. What can we learn?

The semiclassical BH geometry and its stringy corrections

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In principle, this includes properties that are currently inaccessible by worldsheet methods.

[See Hanada *et al.*, Berkowitz *et al.*, Pateloudis, *et al.*for similar discussions involving the BH thermodynamics.]

In principle, a theory of quantum gravity should predict the higher-derivative corrections to Einstein gravity, e.g.,

$$\mathcal{L} \sim R + \#\alpha'^3 R^4 + \#\alpha'^3 R^3 F^2 + \cdots .$$

For charged black holes (with Ramond-Ramond gauge fields), the *leading* correction is unknown.

A precision measurement of certain correlators will give us information about these corrections. Similar program in the CFT bootstrap; e.g., [Binder, Chester, Pufu, Wang '19]

A clear target is the only SO(9) singlet field in this background χ.

χ has scaling dimension $\Delta = 28/5$ [Sekino & Yoneya '00, Biggs & Maldacena '23]. The leading α'^3 correction breaks the scaling symmetry and gives rise to a non-trivial 1-pt function:

$$S_{\text{eff}} \supset \frac{(\alpha')^3}{G_N} \int \sqrt{g} e^{-2\phi} \chi \left(\#_1 R^4 + \#_2 e^{2\phi} R^3 F^2 + \cdots + \right)$$

$$\langle \mathcal{O}_\chi \rangle \propto T^{\Delta+\delta} = T^{28/5}$$

On the matrix side, the operator $\mathcal{O}_\chi$ is known [Van Raamsdonk and Taylor '98] :

$$\mathcal{O}_\chi \sim \mathrm{Tr}\, P^I P^I P^J P^J + \mathrm{Tr}[X_I, X_J][X_J, X_K] P^K P^I + \cdots + \text{fermions}$$

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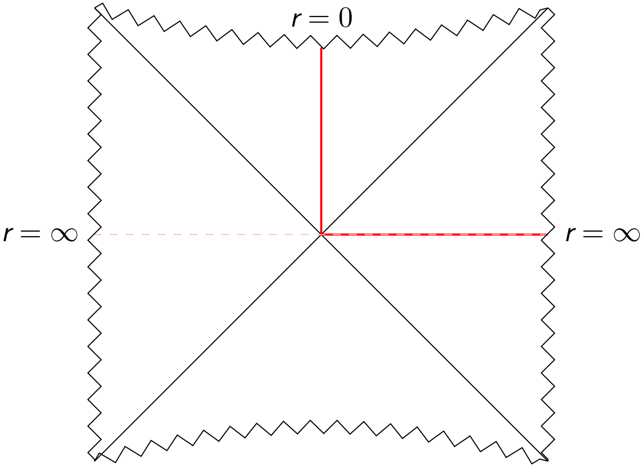
χ is also expected to contribute to a generic SO(9) singlet due to operator mixing, e.g.,

$$\left\langle \mathrm{tr}\, X^2 \right\rangle \sim \#_1 + \#_H T^{14/5} + \#_{H'} T^{23/5} + \#_\chi T^{28/5} + \cdots$$

More generally, thermal 1-pt functions in black hole backgrounds probe regions of high curvature, e.g., the black hole singularity [Grinberg & Maldacena].



Summary

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2. for "unsolvable" models like BFSS, bootstrap gives us some non-trivial bounds. Old results from the matrix side [Polchinski '99] can be reformulated and improved as a bootstrap result.

Summary

1. solvable matrix models can also be solved by bootstrap
2. for "unsolvable" models like BFSS, bootstrap gives us some non-trivial bounds. Old results from the matrix side [Polchinski '99] can be reformulated and improved as a bootstrap result.
3. In principle, we could learn about stringy black holes using the bootstrap. We are in the process of putting this into practice.

Future directions I.

Bootstrapping the thermal entropy, e.g.,

$$S = A/(4G_N) + \text{corrections.}$$

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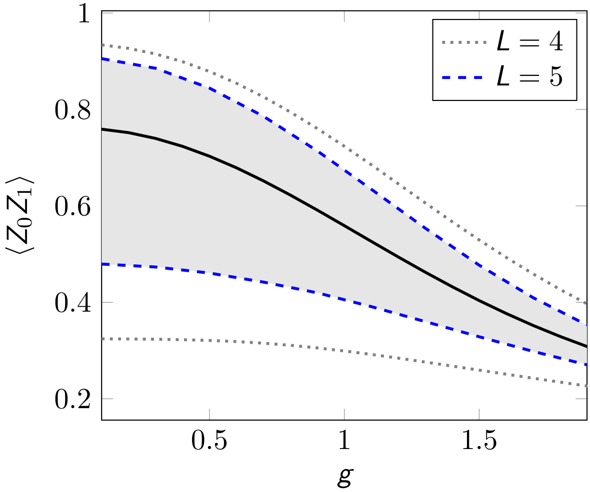
Recent progress [Fawzi, Fawzi & Scalet '23] in inputting the KMS condition into the bootstrap (in the Hamiltonian approach). Uses a non-linear relaxation of the relative entropy.

Can be applied to large N matrix quantum mechanics [Cho, Sandor, & Yin, WIP]



Transverse-field Ising Model at $\beta = 1$ [from Fawzi, Fawzi & Scalet].

$$-H = \sum_{i=-\infty}^{\infty} Z_i Z_{i+1} + g X_i$$

Future directions II.

	$d = 0$	$d = 1$	$d \geq 2$
	1-matrix integral	1-matrix model $c = 1$ matrix model	
	multi-matrix integral	D0-brane BFSS	

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	$d = 0$	$d = 1$	$d \geq 2$
	1-matrix integral	1-matrix model $c = 1$ matrix model	't Hooft model, ...
	multi-matrix integral	D0-brane BFSS	large N Yang Mills large N QCD

Already some interesting progress...

[Anderson & Kruczenski '16] [Kazakov & Zheng '22] [Kazakov & Zheng, WIP]

Many other strongly-coupled lattice systems seem possible...