Analytic Models of Final Cooling with Genetic Algorithms

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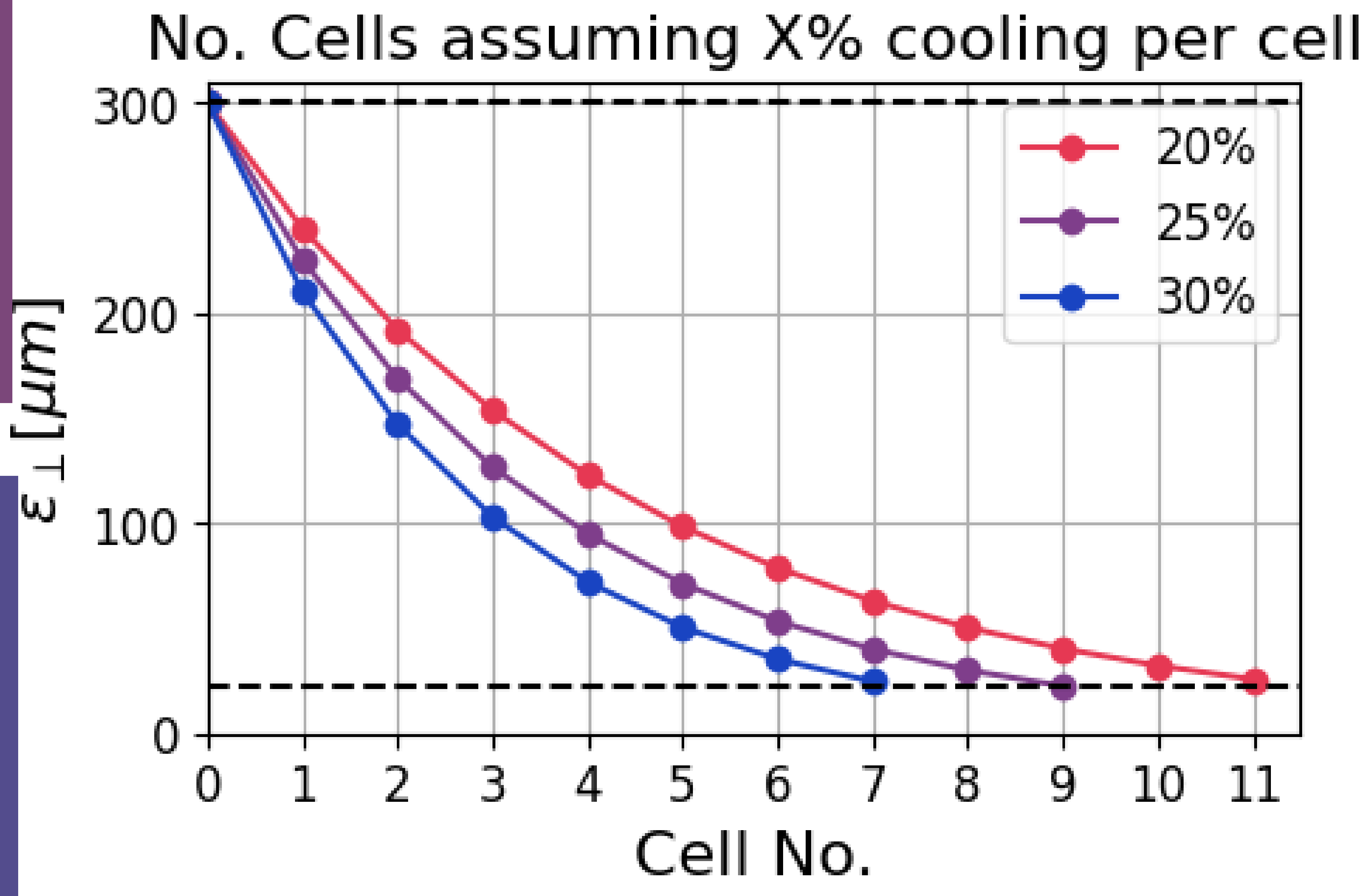
Lattice Goals:

Starting from ~MAP Parameters, to prevent high cost & length of 6D lattice

εT = 300 um
εL = 1.3 mm
Ekin ~ 120 MeV

Arriving at ~MAP targets, with room to improve, without impacting transmission

εT = 22.5 um
εL = 65 mm
Ekin ~ 5 MeV



Previous Work

Cell no.	ε_T µm	ε_L mm	ε_{6D} µm	Cumulative transmission %
Start	300	1.5		100
1	275.2	2.7	586.1	97.5
2	212.7	5.9	645.4	94.1
3	170.4	6.8	582.8	88.9
4	138	12.4	617.5	81.9
5	102.5	20.6	600	74.4
6	81.3	25	548.8	61.1
7	59.5	32.7	486.9	53.1
8	50.8	43.6	482.8	46.9
9	41.2	48.4	434.2	37
10	32.9	66.1	414.6	31.7
11	29.5	82	414.5	28.5

Cell no.	Solenoid length m	Stage length m	Max. B_z on-axis T	Low B_z on-axis T	Absorbe length m
1	1.48	1.48	44.63	4.63	0.85
2	1.75	4.57	44.63	4.63	0.47
3	1.00	6.61	44.63	4.63	0.47
4	1.00	7.75	44.63	4.63	0.40
5	1.00	5.09	44.63	4.63	0.30
6	1.11	6.86	44.63	4.63	0.25
7	1.33	7.06	42.00	2.00	0.30
8	0.80	6.70	42.00	2.00	0.10
9	1.48	8.37	41.00	1.00	0.17
10	0.95	6.76	40.80	0.80	0.08
11	0.95	7.60	40.80	0.80	0.05

Parameters which were optimised: **ldrift, number rot cavities, number acc cavities, absorber length, rotation phase, solenoid length**. Freq from sigma_t, grad from sqrt(freq).
Performed cell by cell.

Stechauner Scattering Model

For constant absorber length and a given energy spread, find a parabolic distribution between initial kinetic energy and cooling ratio.

Steps of the analytical model:

Runge-Kutta stepwise **semi-gaussian** scattering through an absorber of a given material/density. Heating and cooling terms.

Longitudinal straggling of energy distribution

Longitudinal rotation due to drift

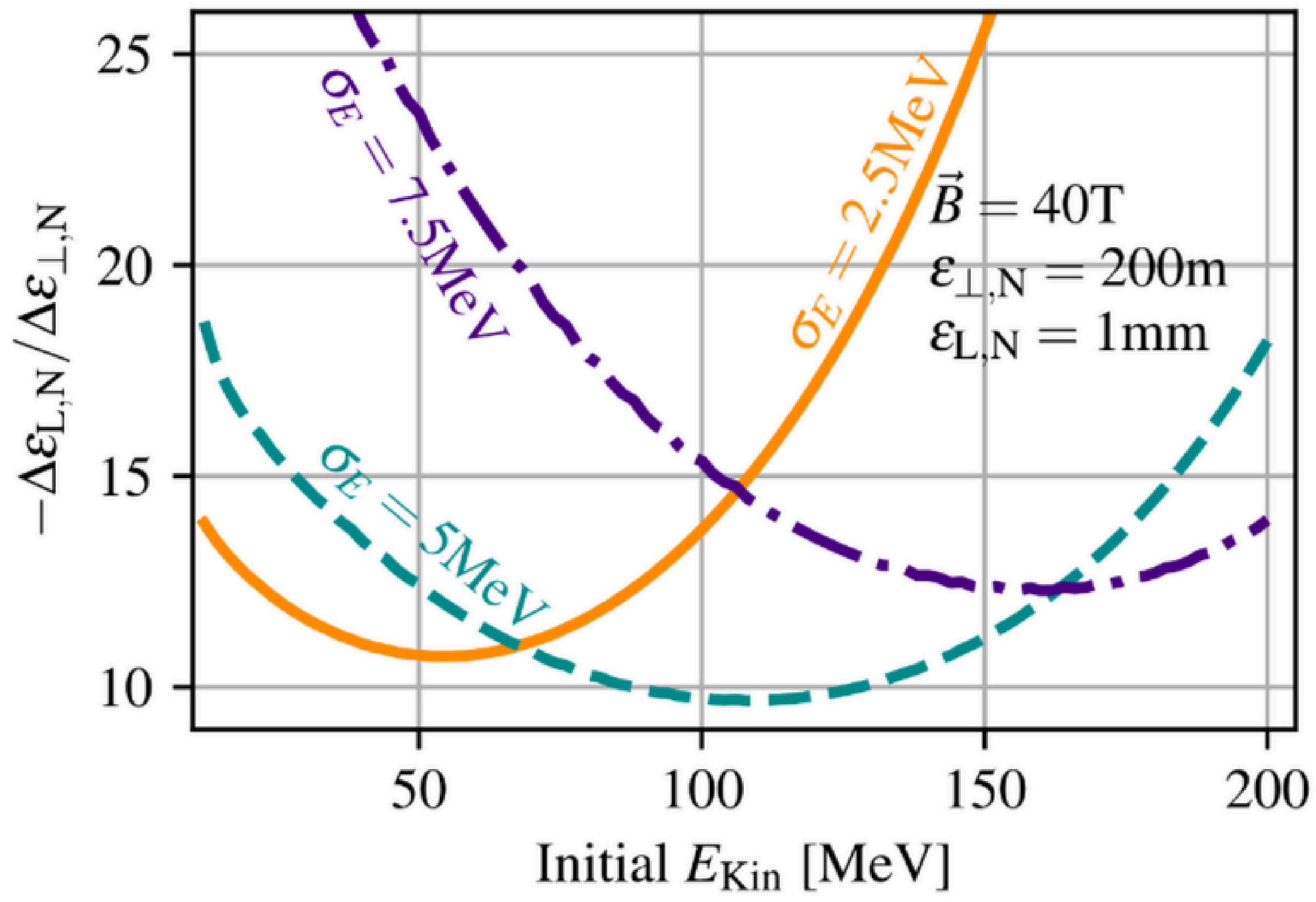


Figure 2: For given beam and machine parameters the best initial beam energy can be estimated by observing the minimum of the trade-off function $-\Delta\varepsilon_{L,N}/\Delta\varepsilon_{\perp,N}$.

"SEARCHING FOR THE BEST INITIAL BEAM PARAMETERS FOR EFFICIENT MUON IONIZATION COOLING" **B. Stechauner *2024***

Identifying crucial parameters

Absorber Length

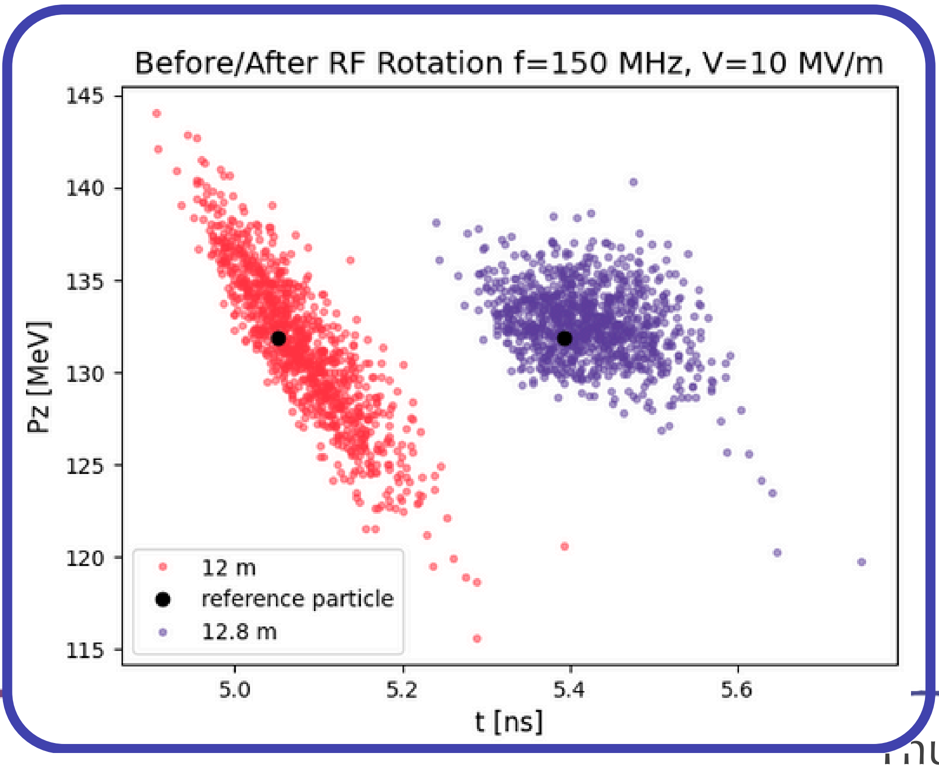
Modelling as length for **constant density of liquid hydrogen**.
(In reality, either both will change depending on hydrogen pressure.)

Initial Energy

RF Cavities between cells can accelerate to required energy prior to each absorber.
Lower energies provides **faster cooling** reduction, but **more losses** within absorber

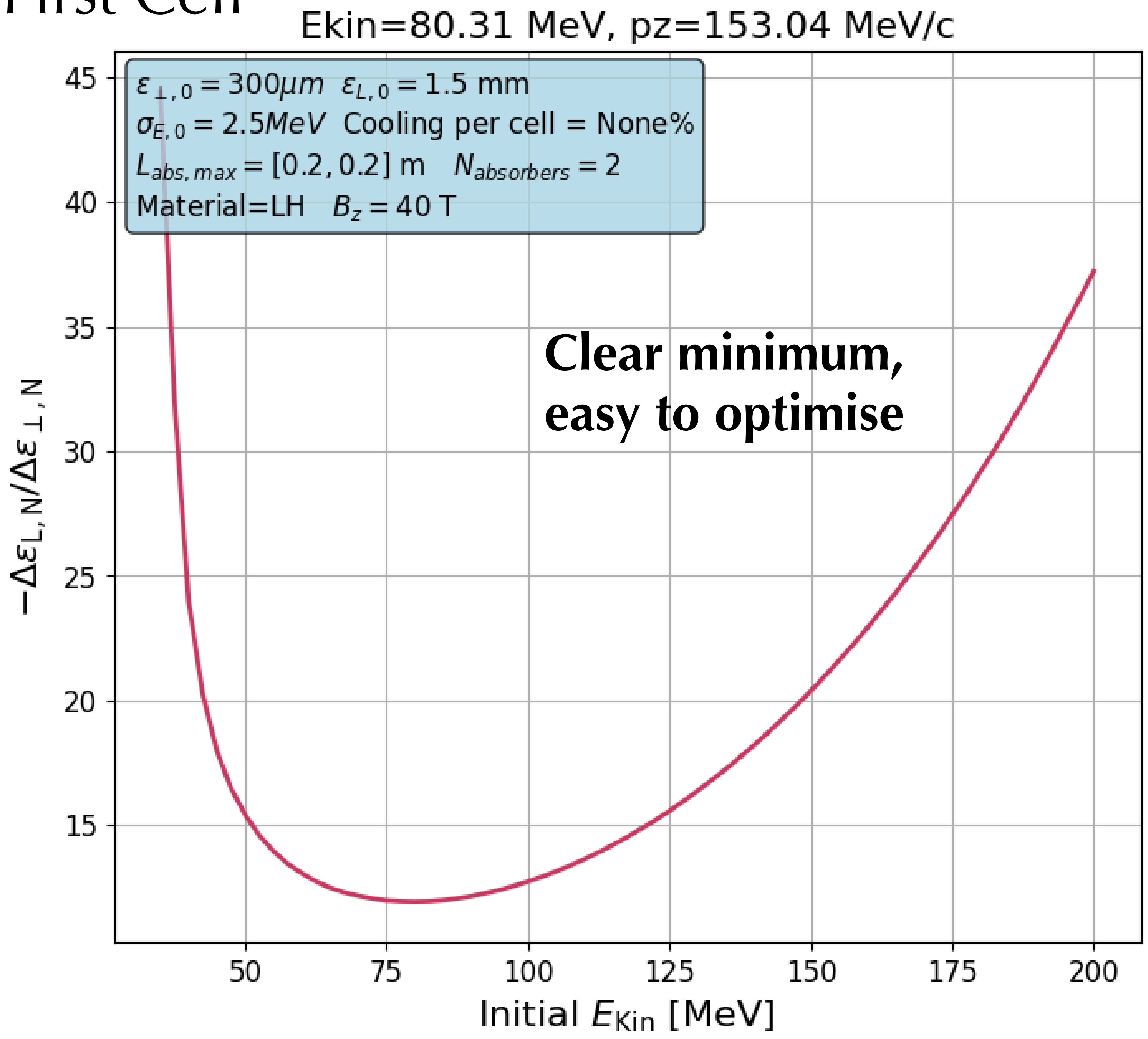
Initial Energy Spread

RF rotation converts bunch length to energy spread.
Larger energy spread gives larger losses

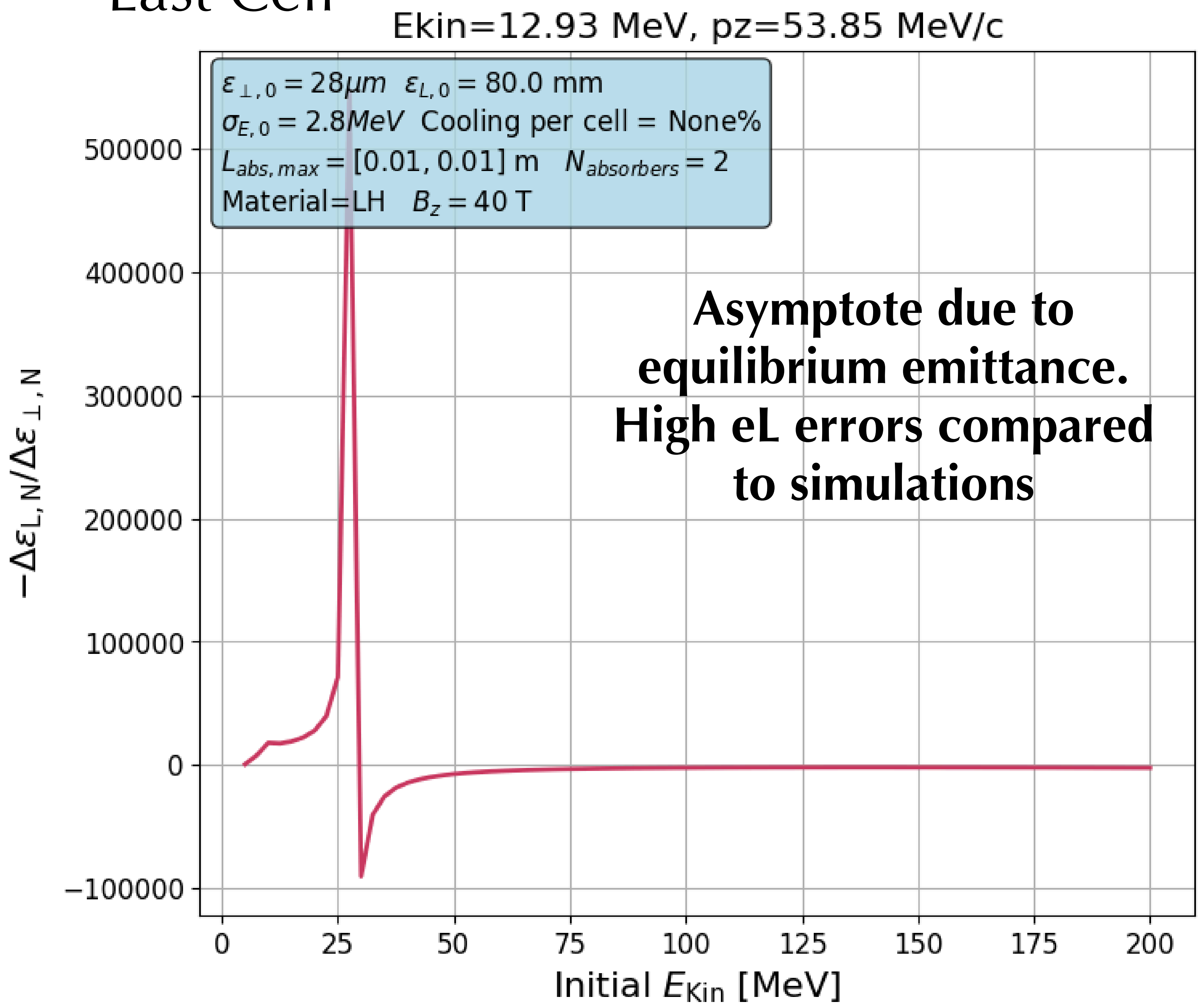


1D Analytical Model
Constant length and energy spread

First Cell



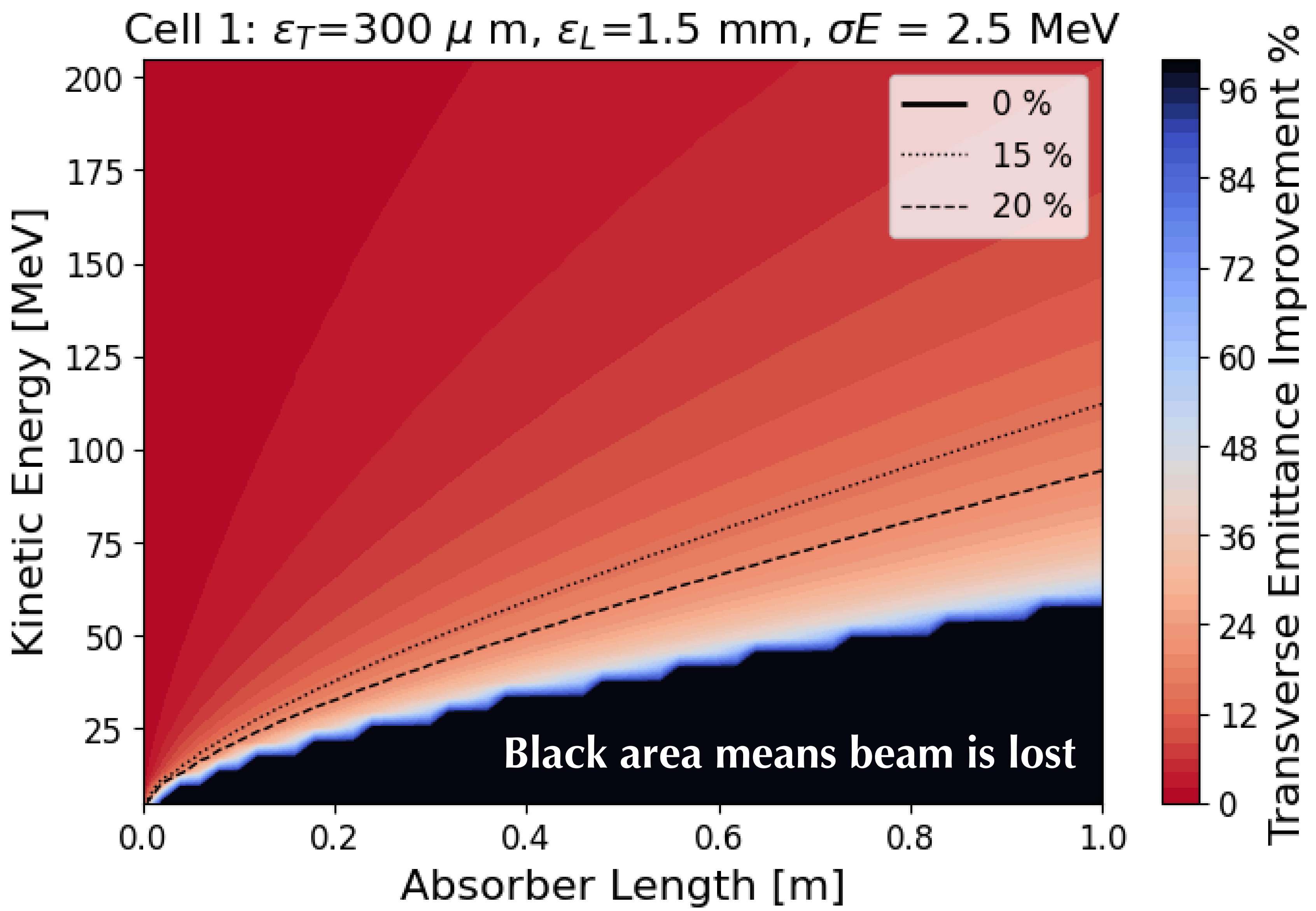
Last Cell



2D Analytical Model - **Transverse**
Constant energy spread

eT / eL is non-optimal figure of merit as it does not consider absolute improvements.
Instead plot as 2D contour plot. Find that KE and L(absorber) are coupled.

First Cell



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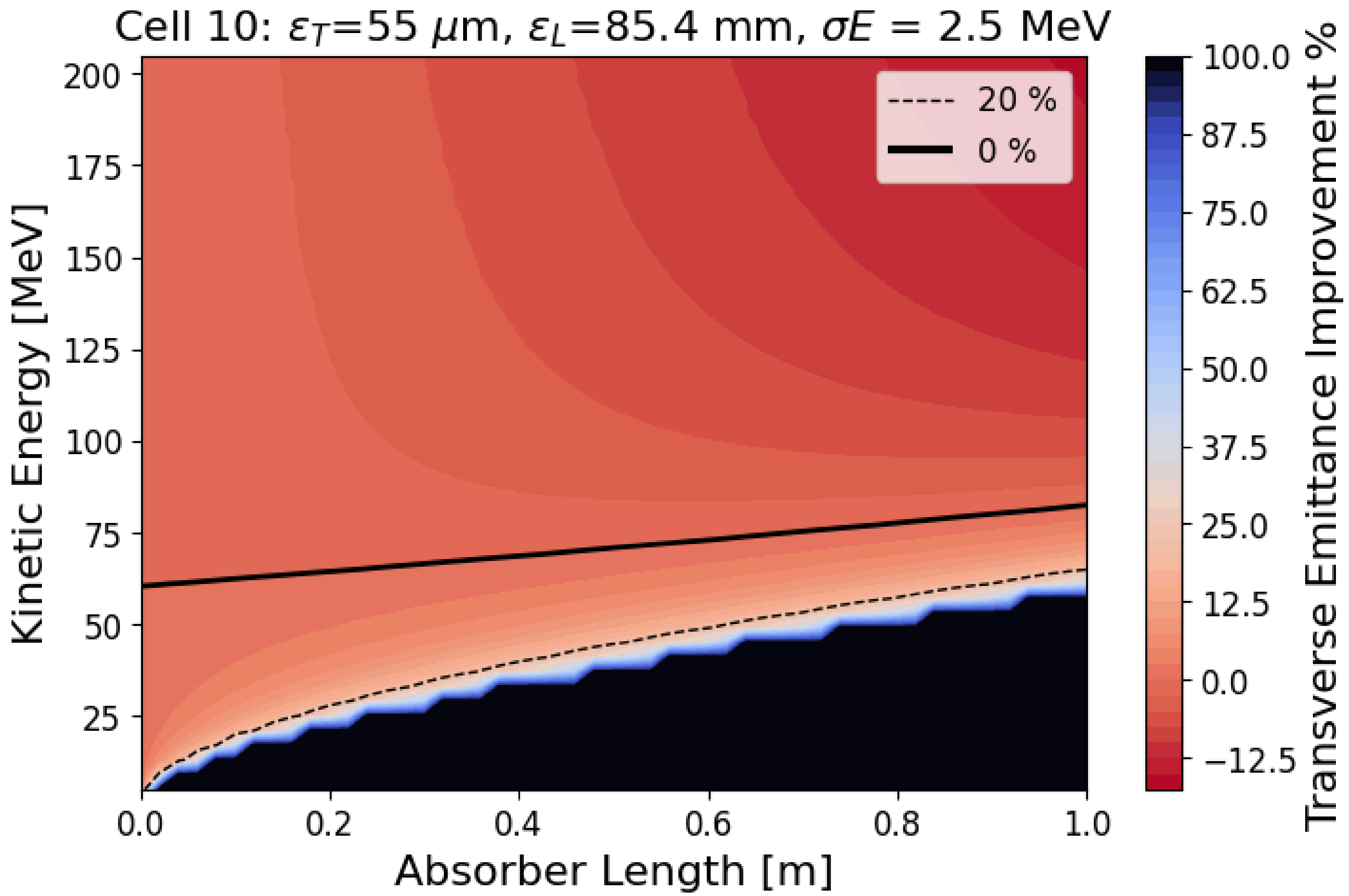
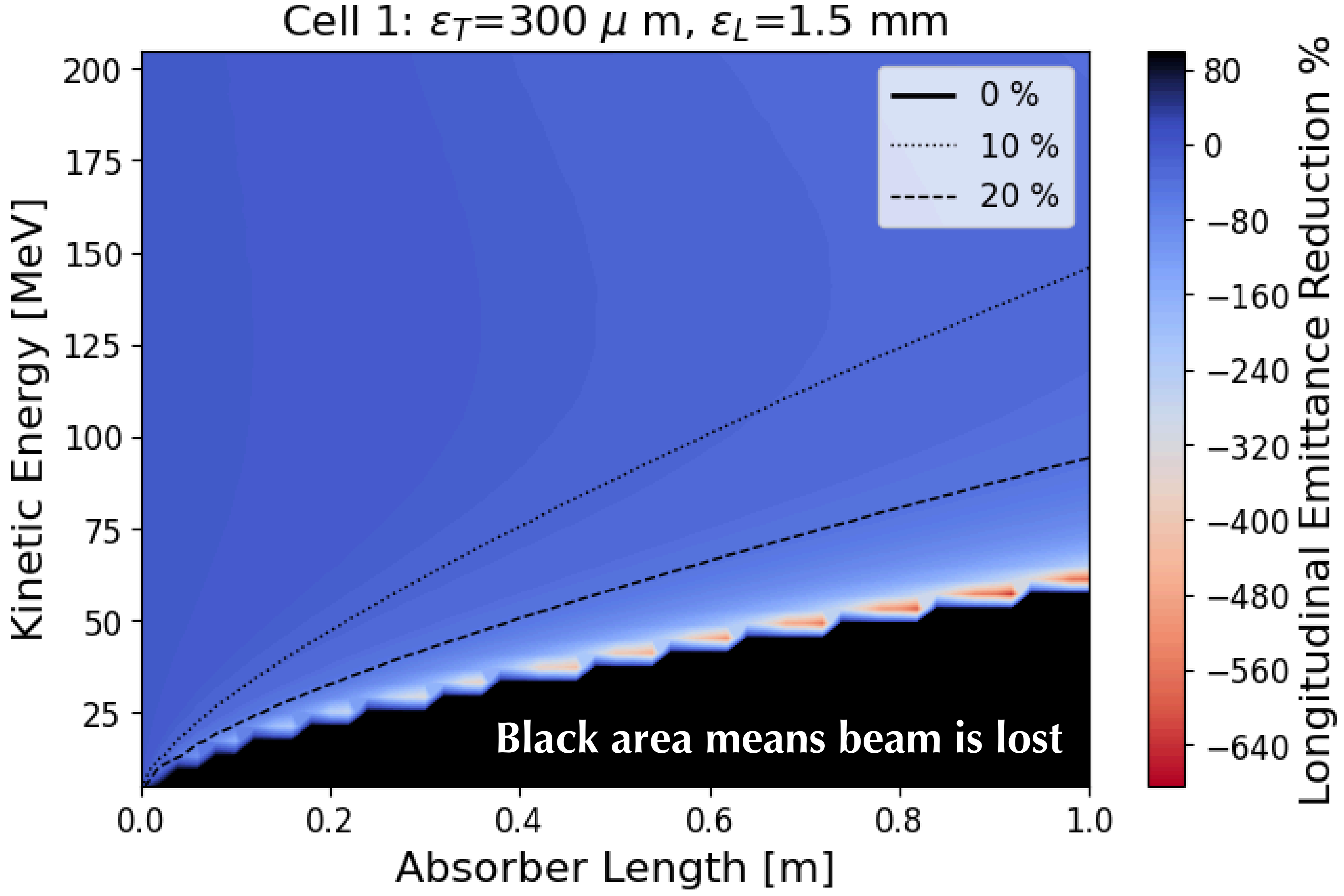


2D Analytical Model - **Longitudinal**
Constant energy spread

Find that KE and L(absorber) are coupled....but smaller absorber lengths are preferred...

First Cell



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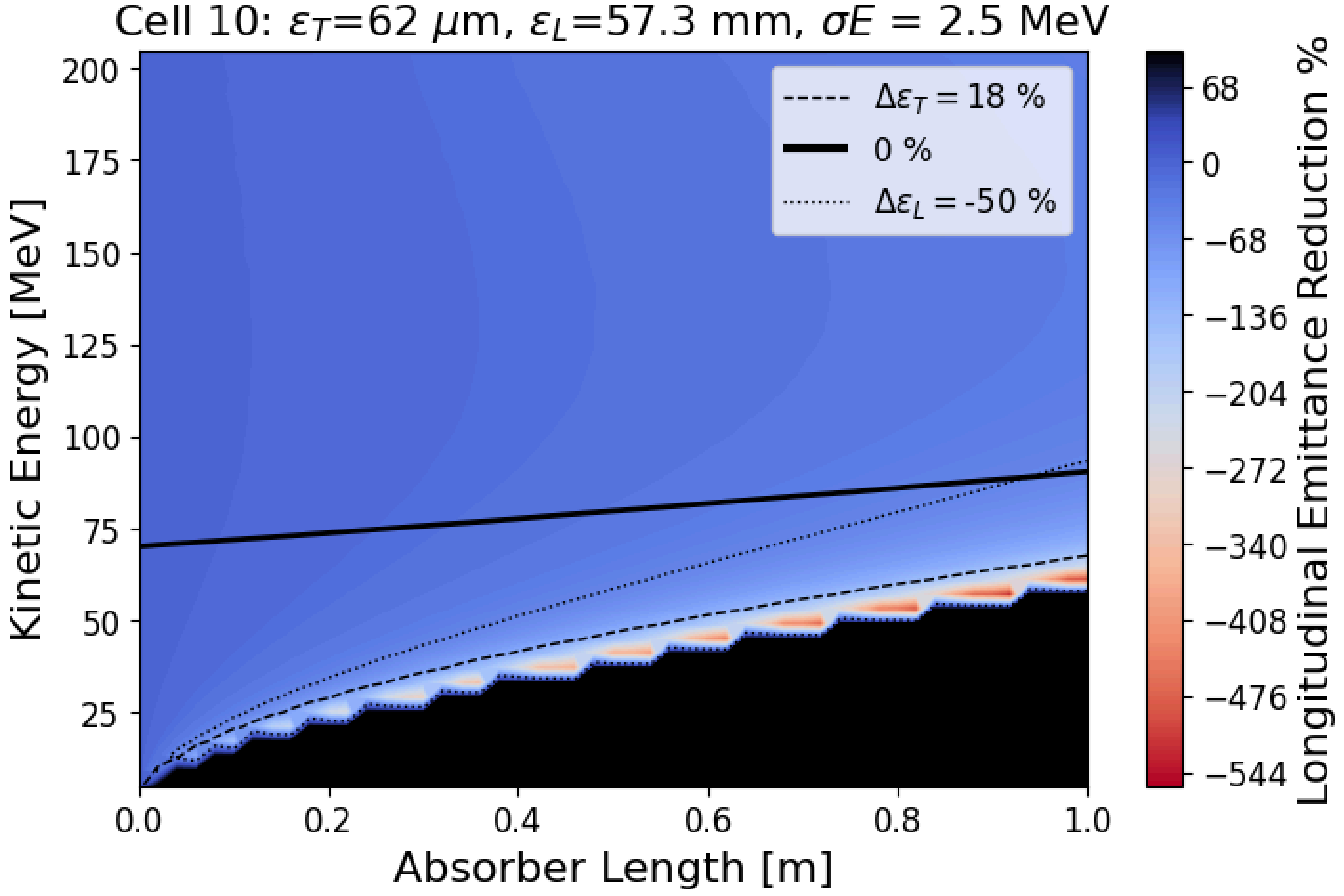
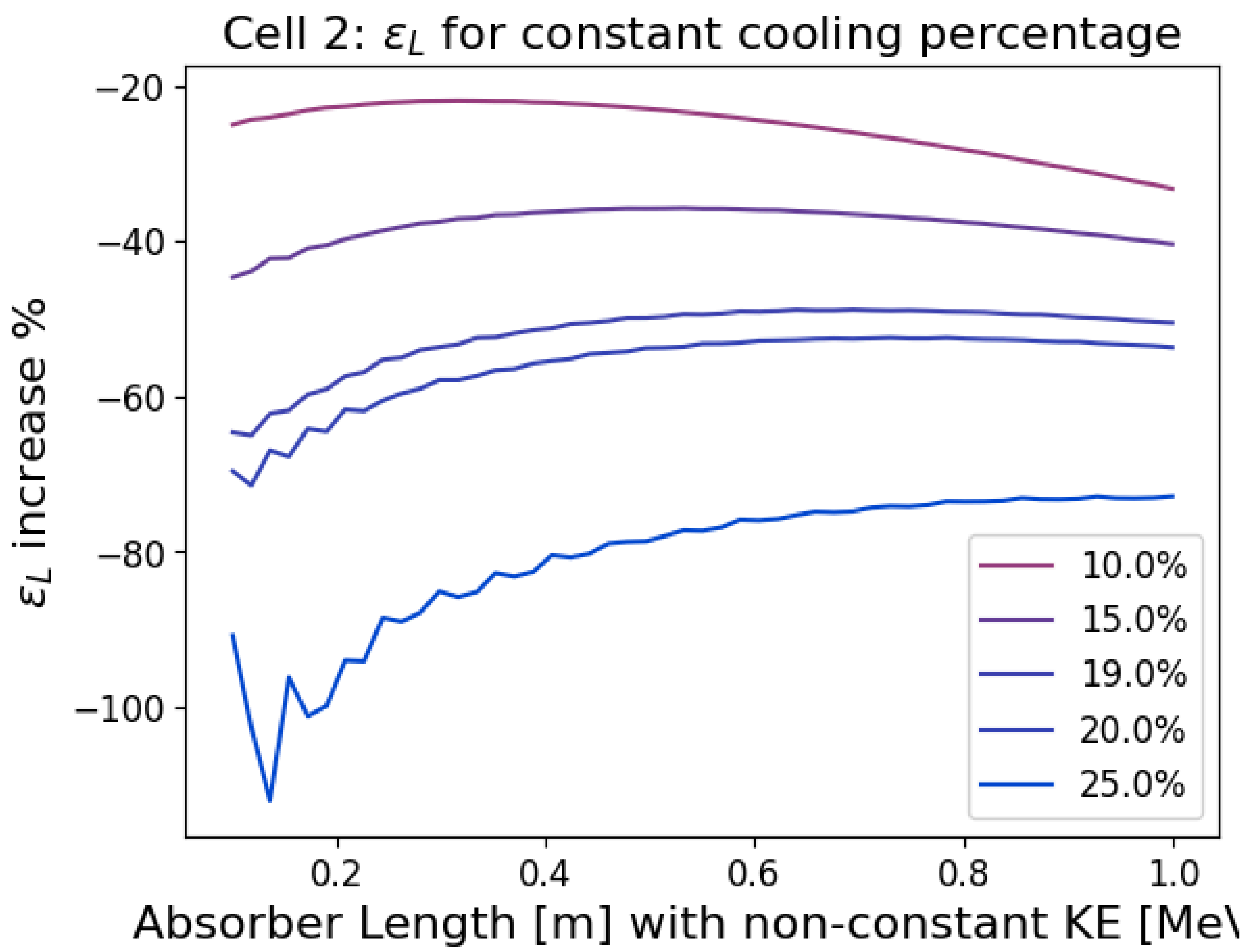


2D Analytical Model - **Longitudinal**
Constant energy spread

Optional: Can find the minimum longitudinal emittance value for a given percentage improvement of transverse cooling.
Find contour line of transverse and interpolate the value of longitudinal along that line.

First Cell



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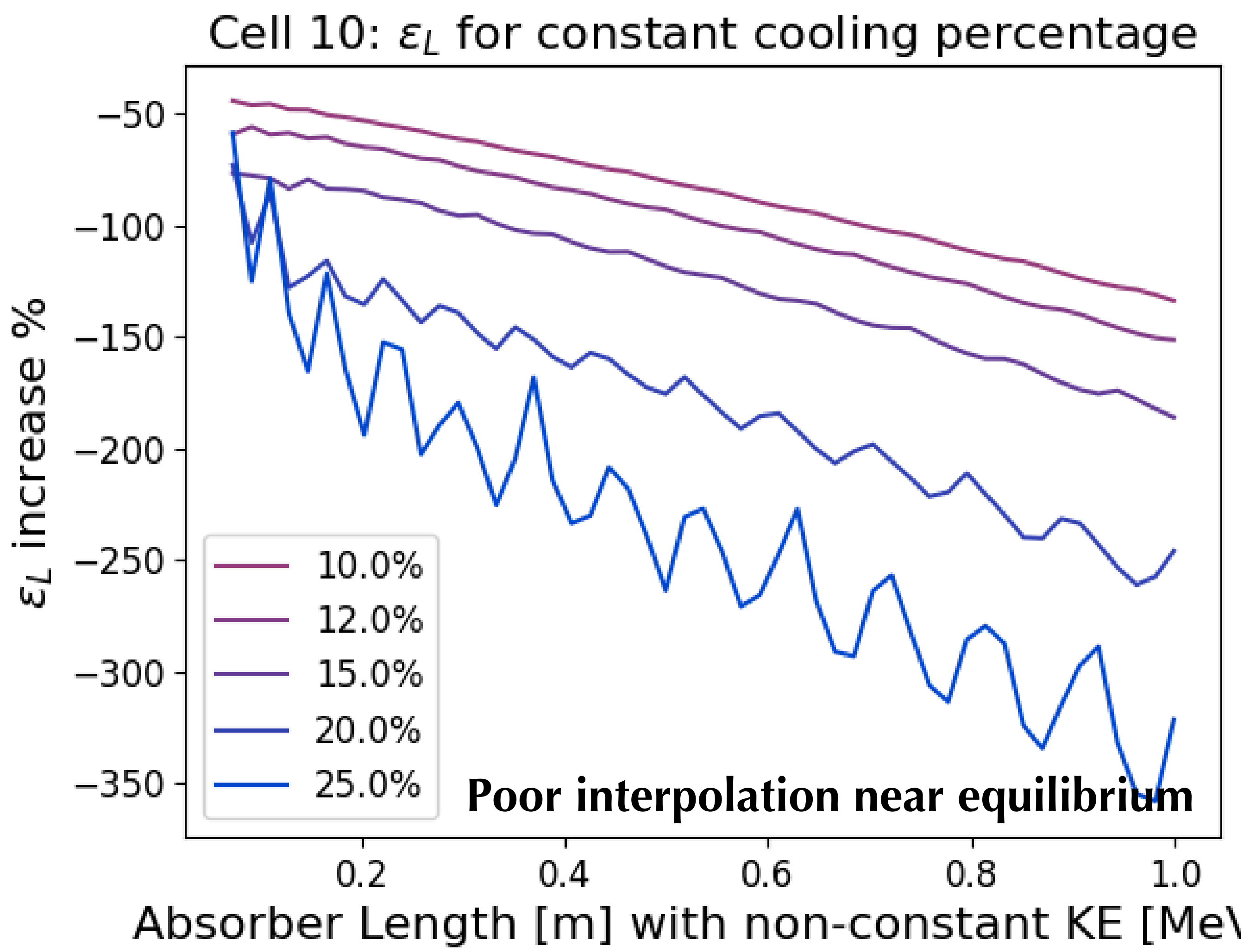


Poor interpolation near equilibrium

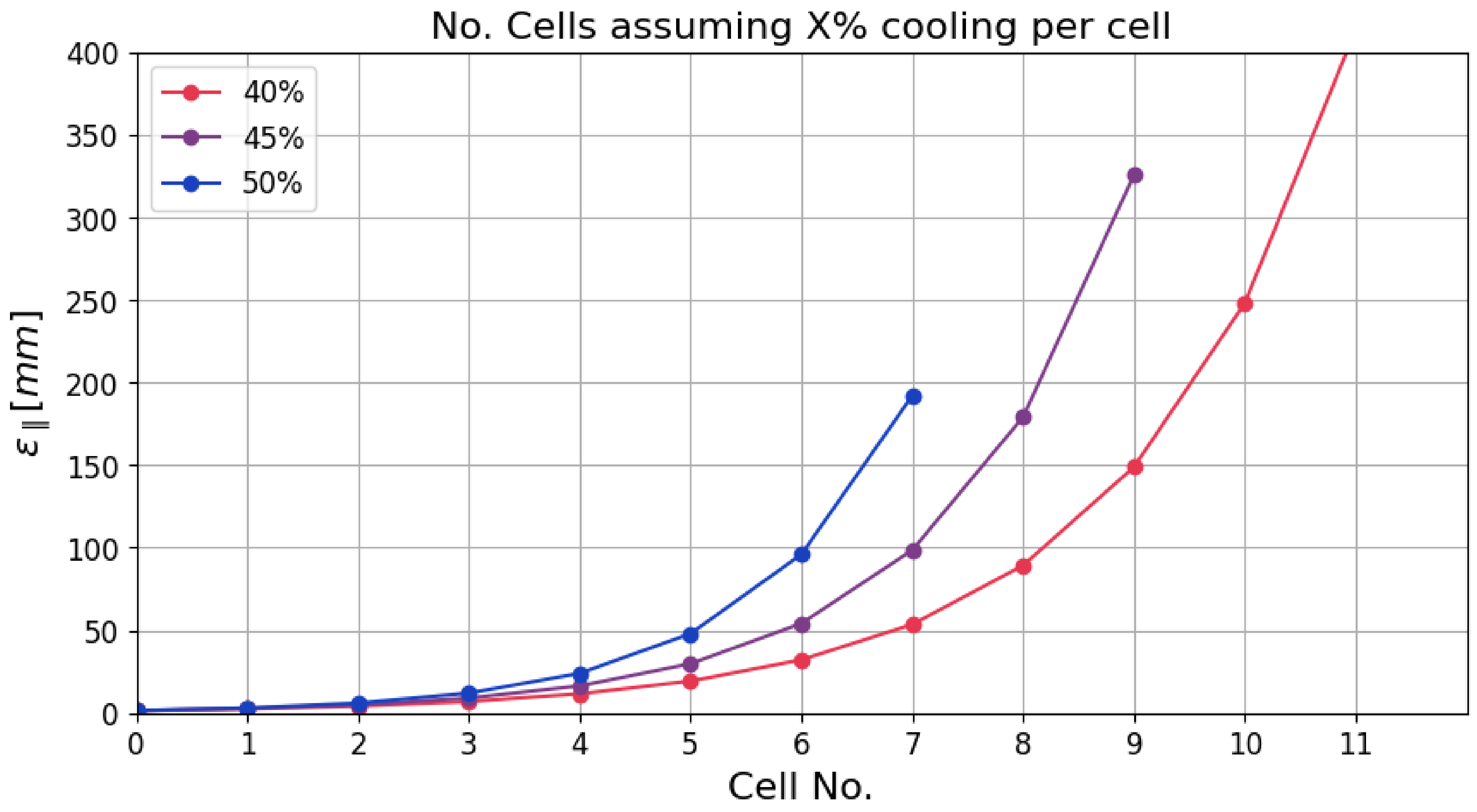
Optimisation with analytical algorithm

Optimising cell-by-cell may not guarantee a global minimum.
Often find that for a ~20% improvement in transverse emittance, longitudinal emittance increases by ~40-50%.

Small changes in early cells can have an exponential impact on the resulting longitudinal emittance.

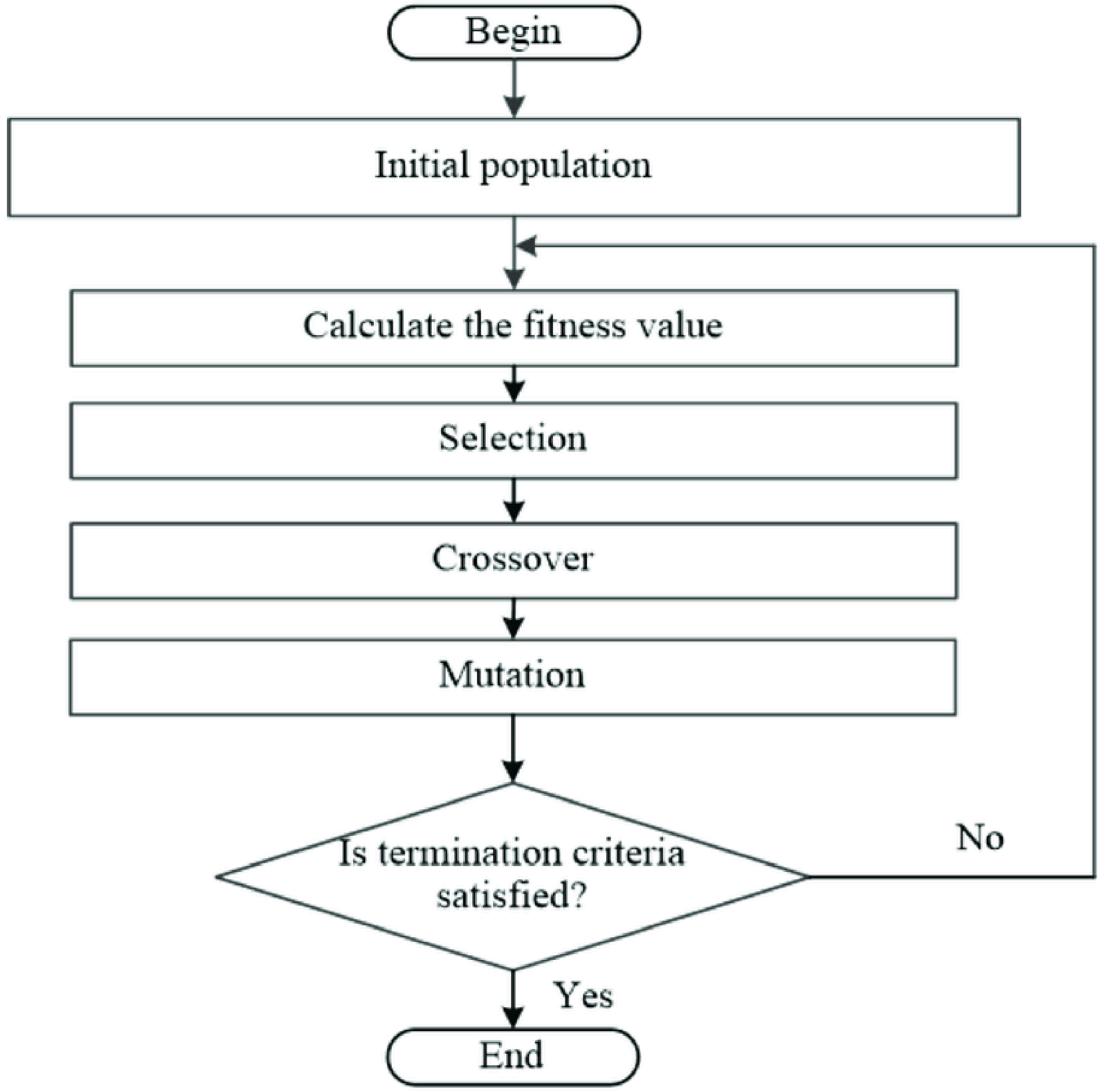
Highly recommend a **global optimiser** to improve the parameters of the whole lattice, rather than optimising cell-by-cell.

For this exercise, used a **Genetic Algorithm**, but any will do.
(Next steps: try multi-optimisation algorithms)



Genetic Algorithms

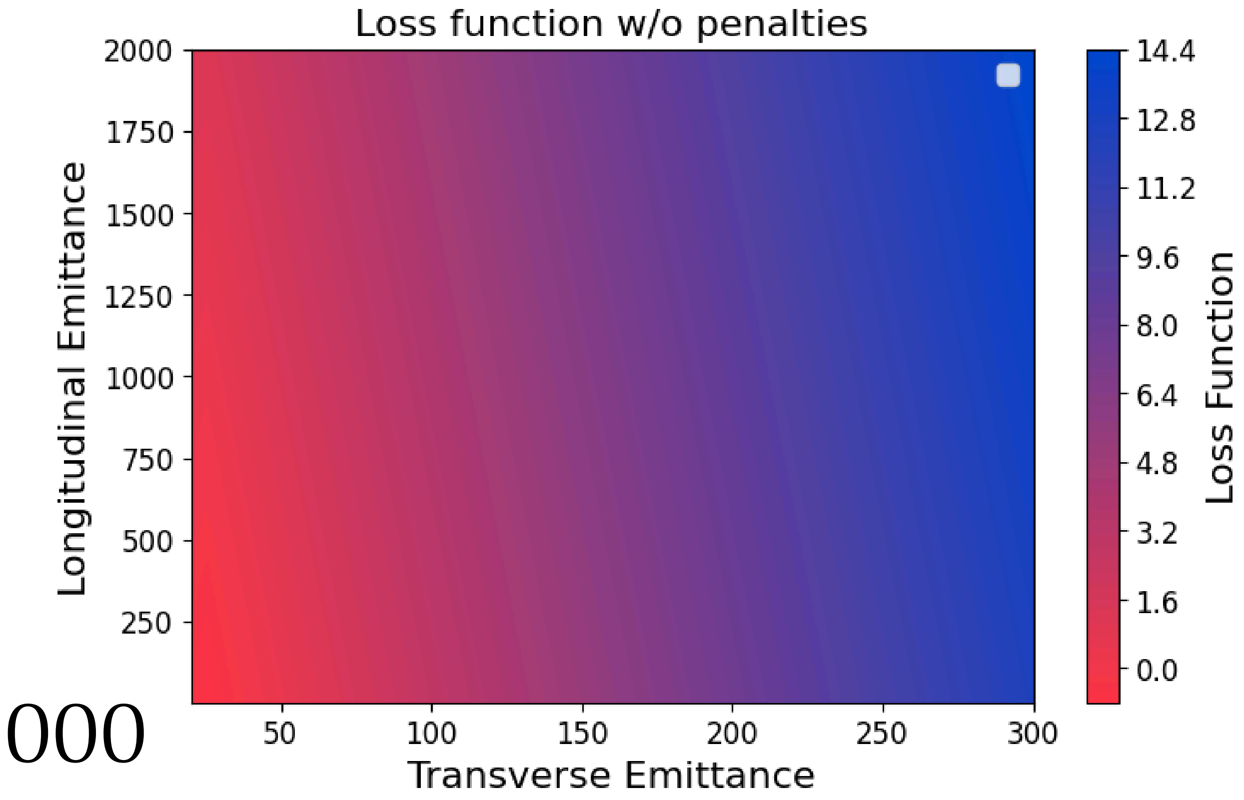
30 parameters to optimize. (Kinetic Energy, Absorber Length, Energy Spread) for 10 cells.
Start by picking random population of 100. *Works best if a few existing solutions are included*.
Mutates for more successful results according to a given **loss function**. Repeat for ~500 generations.



Decided loss function as:

$$\left|\varepsilon_T - \varepsilon_{T,target}\right| \times W + \left|\varepsilon_L - \varepsilon_{L,target}\right| + \sum\left(N_{E_{kin}<5\text{MeV}}\right)$$

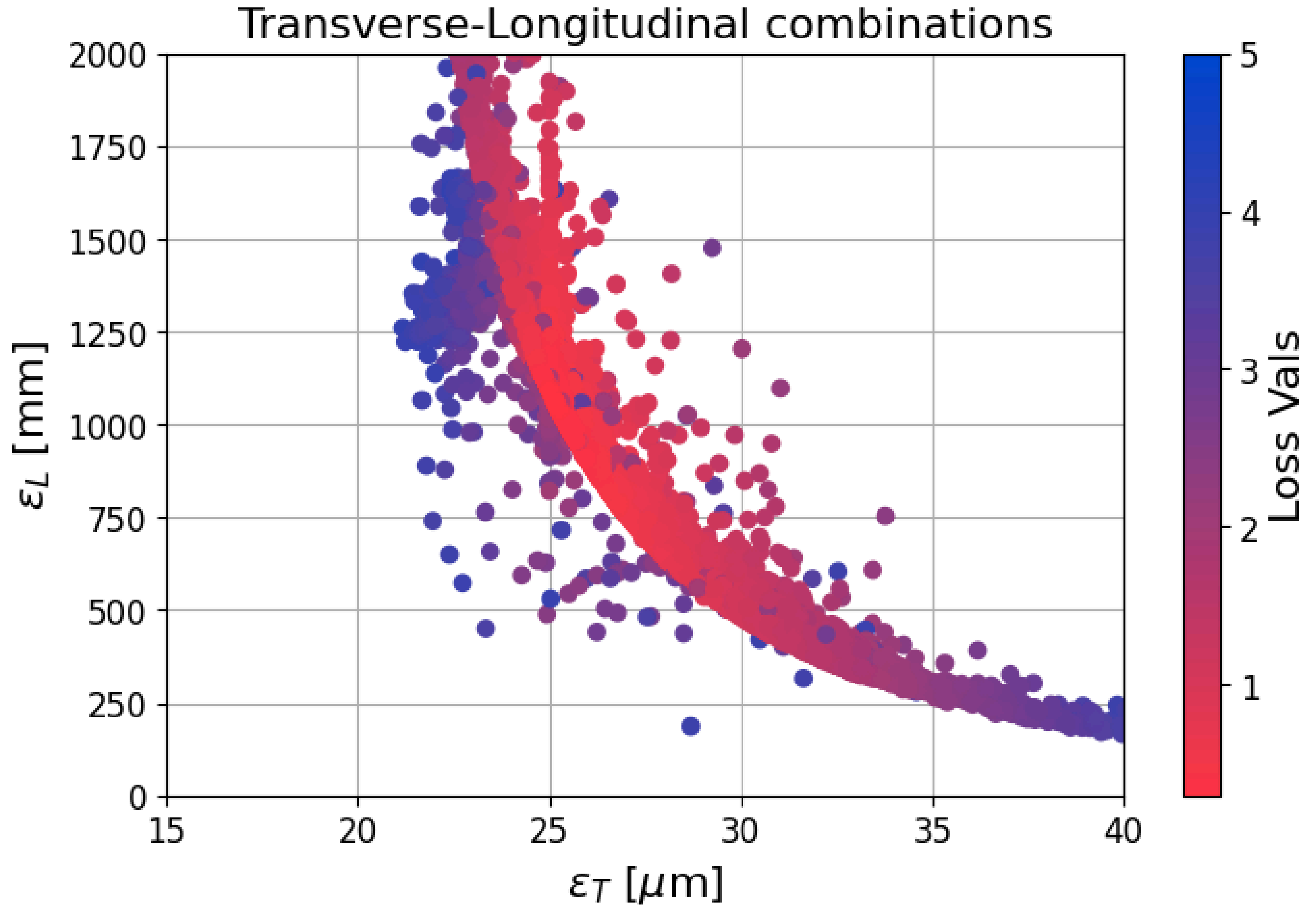
e.g. if weighting is 48000



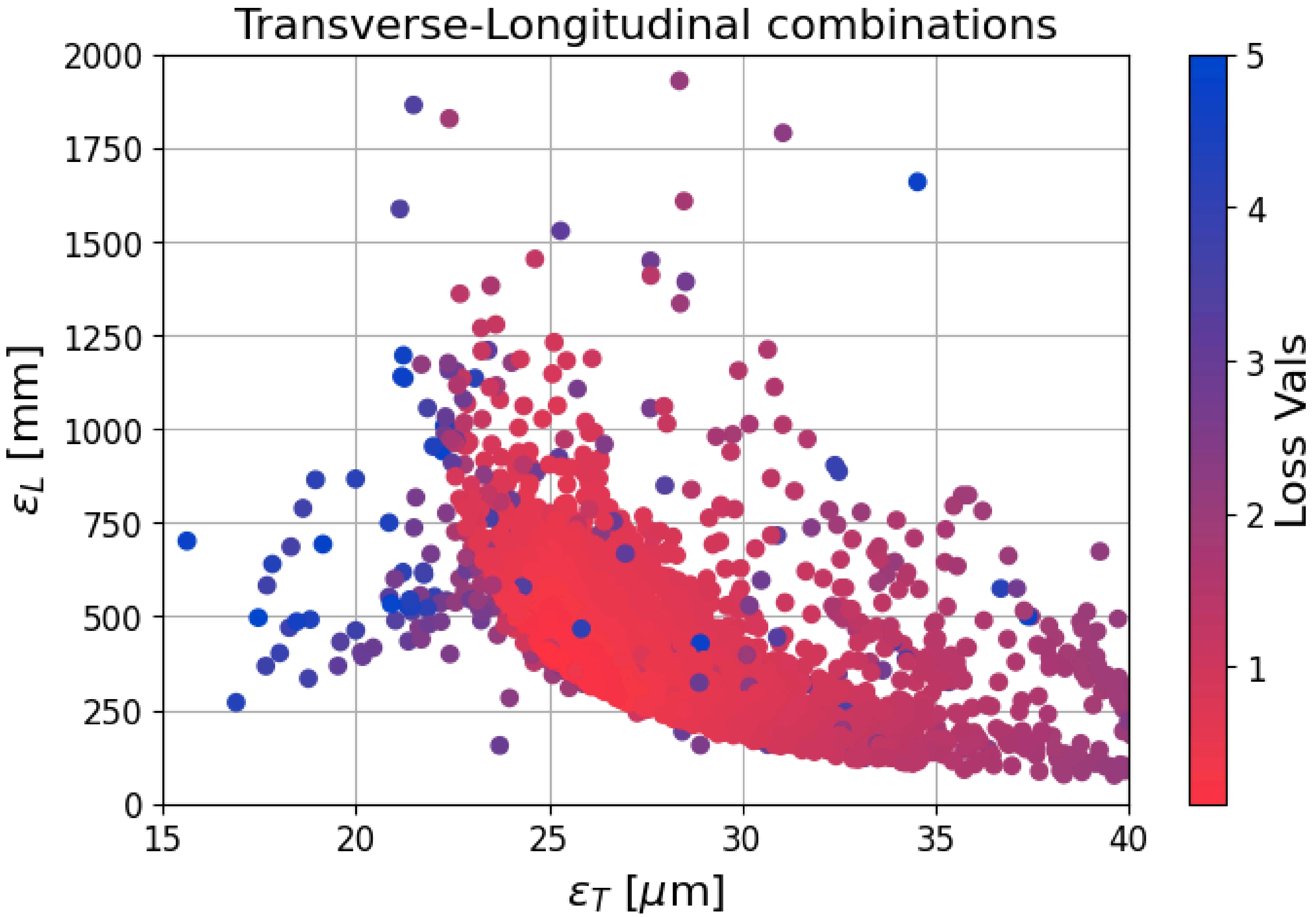
Genetic Algorithm Solutions

Each point is parameters set with a final (eT, eL). Can select result with lowest loss value, or from saved progress. Higher loss value means Energy < 5 MeV, so beam lost in absorber.

10 Cells from eT = 300 um



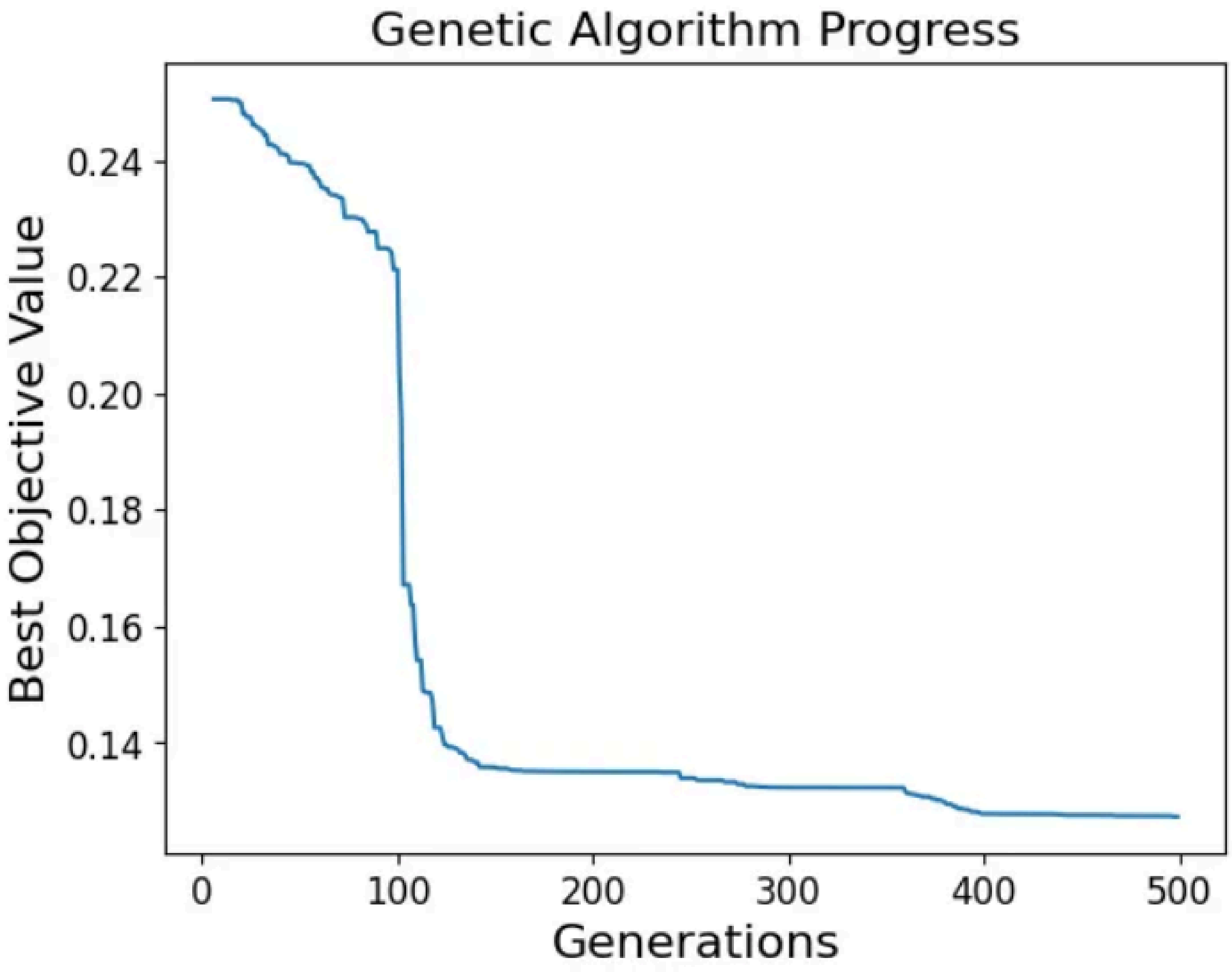
8 Cells from eT = 140 um



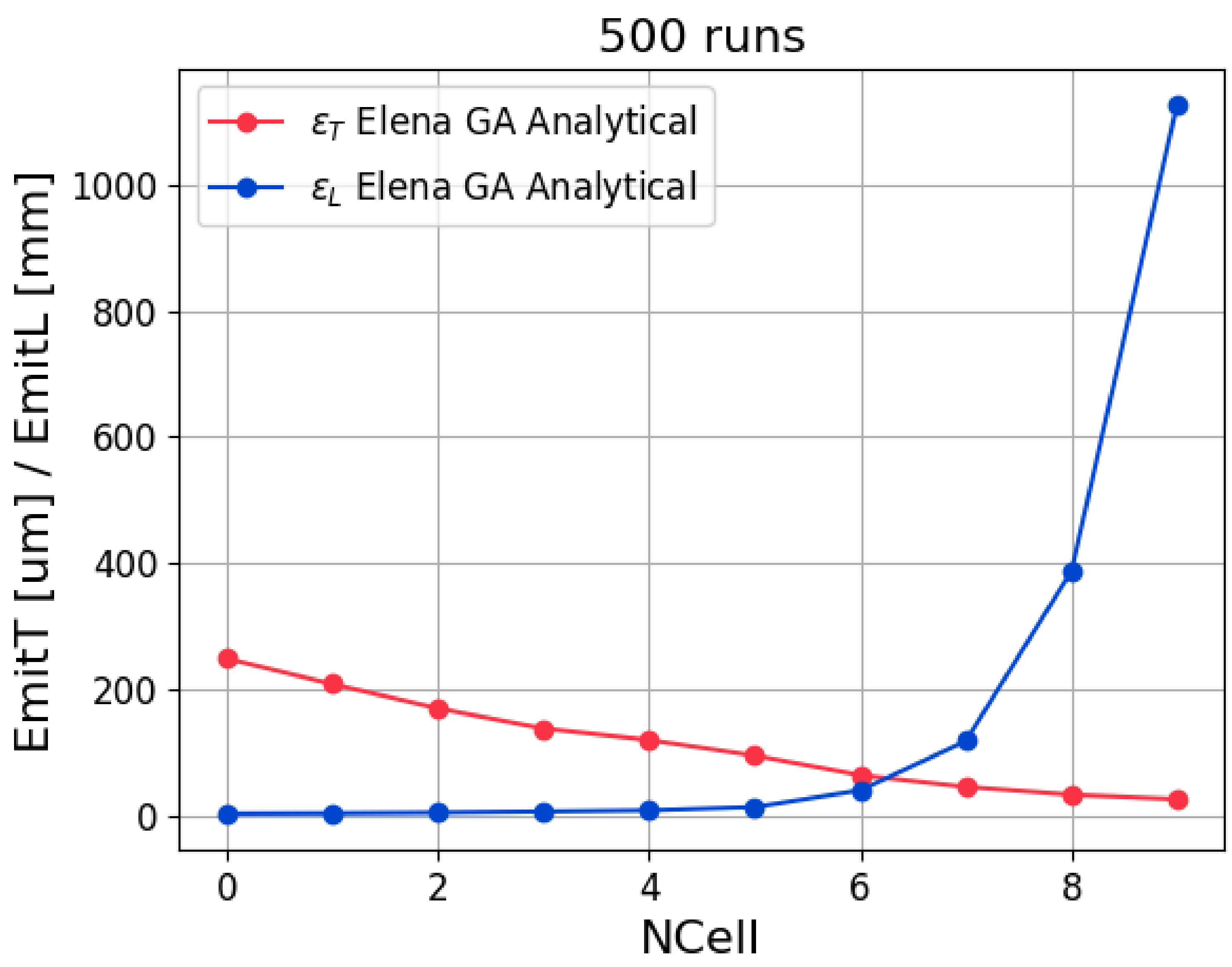
Pareto front: The solution in which one of the objectives cannot be improved without worsening another objective.

Flaw of the algorithm to find further solutions, or hard-limit of physics?

Algorithm process



Solution chosen after 500 iterations



How to verify simulation has the same?

How to compare analytical solution with simulation

1. Create full lattice with RF cavities and matched solenoids

   ✓ - approximate time, 2 months

2. Create simple lattice with RF cavities and high/low solenoids

   x - will get emittance blow up

3. Create simpler lattice with RF cavities and constant high solenoid

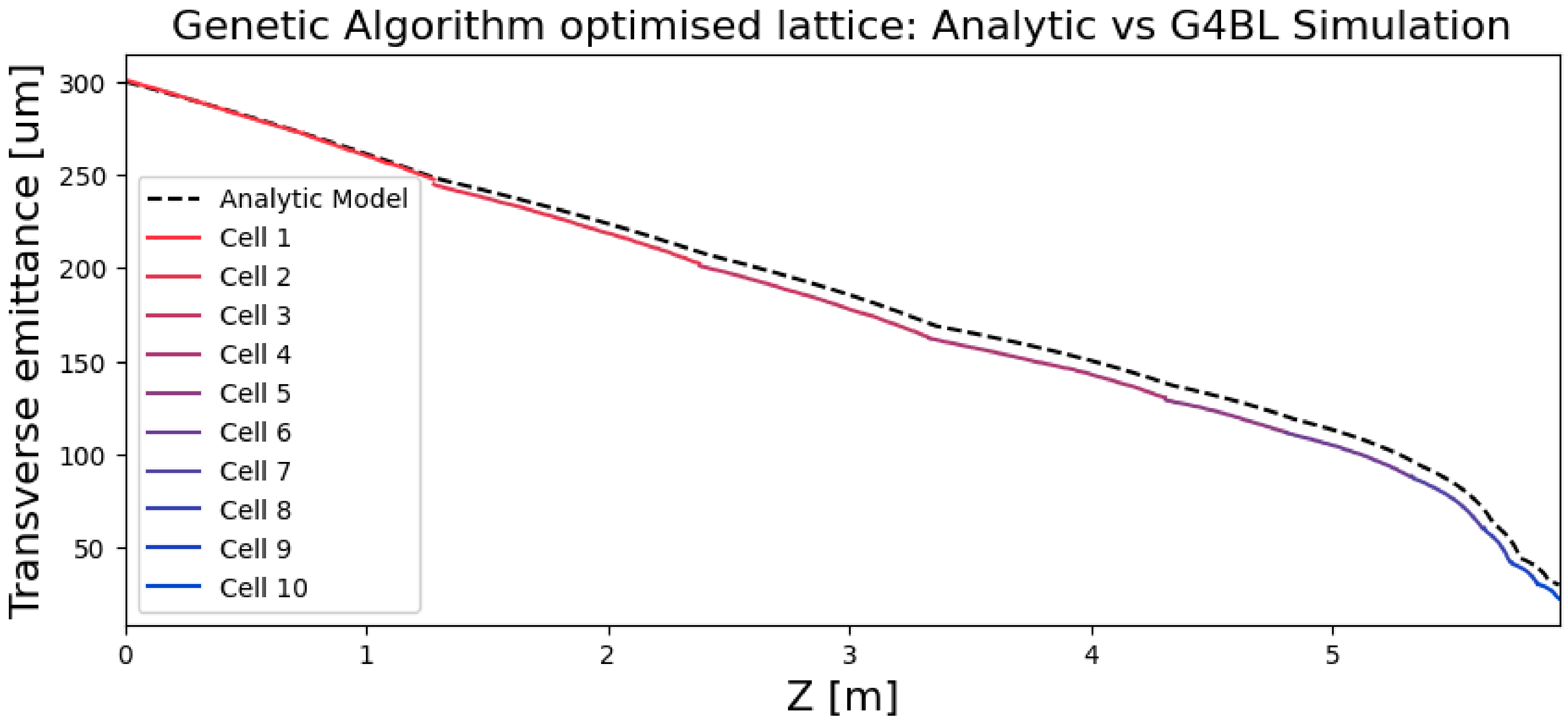
   x - will blow up longitudinal emittance

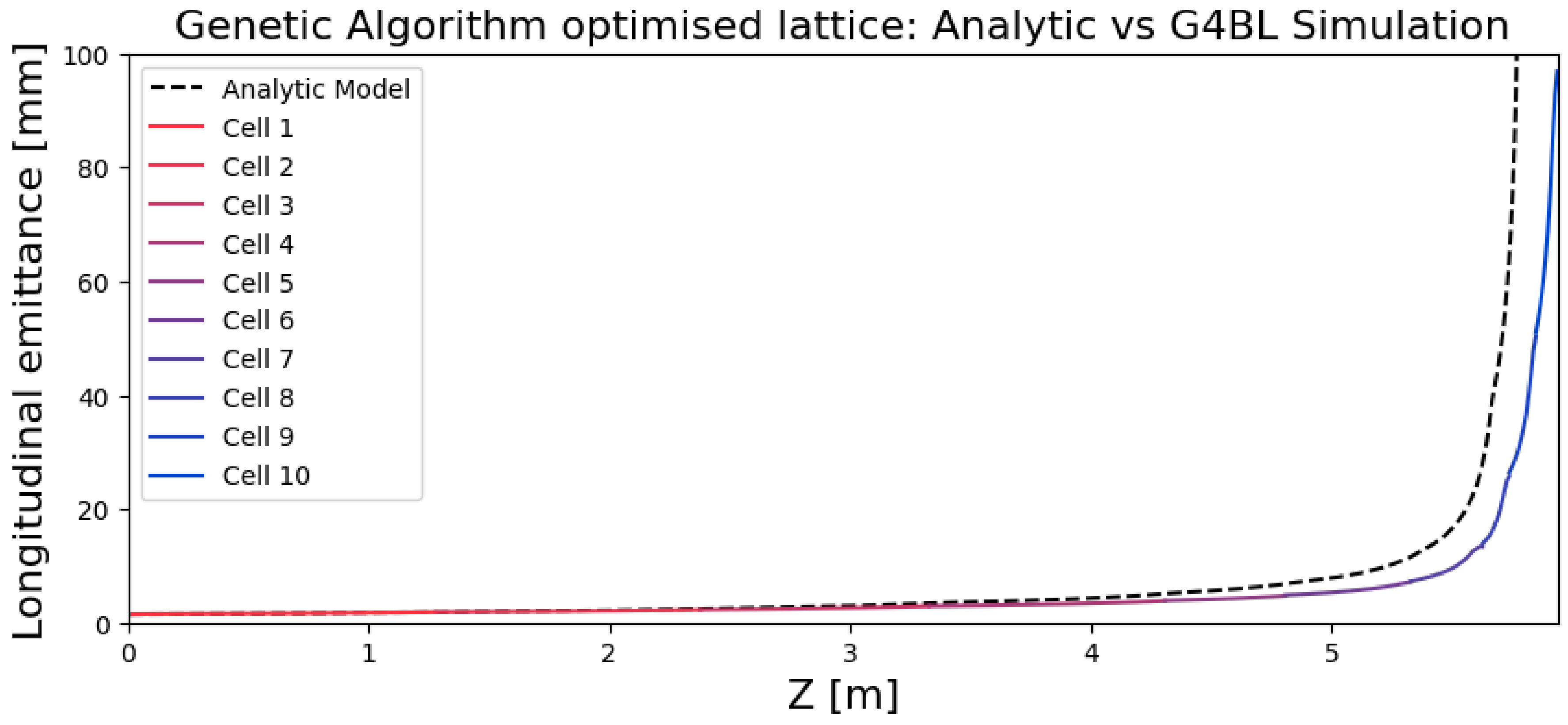
4. Run 1 file for each cell, only the absorber in high field, new beam each time

   ✓ - approximate time, 10 minutes

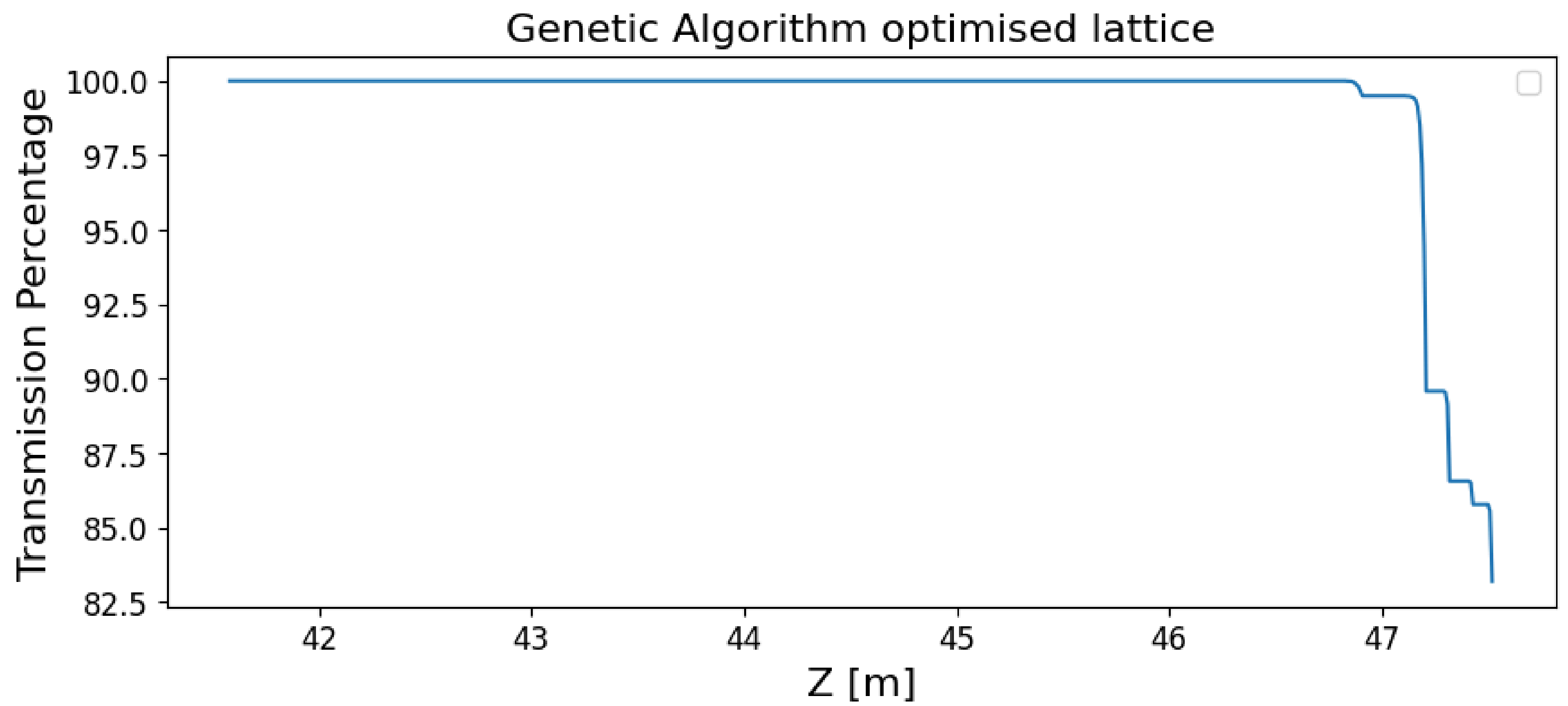
Assumes perfect beam throughout
i.e. reduced beam loss, no RF buckets
Can be considered 'maximum achievable setting'

How to compare analytical solution with simulation

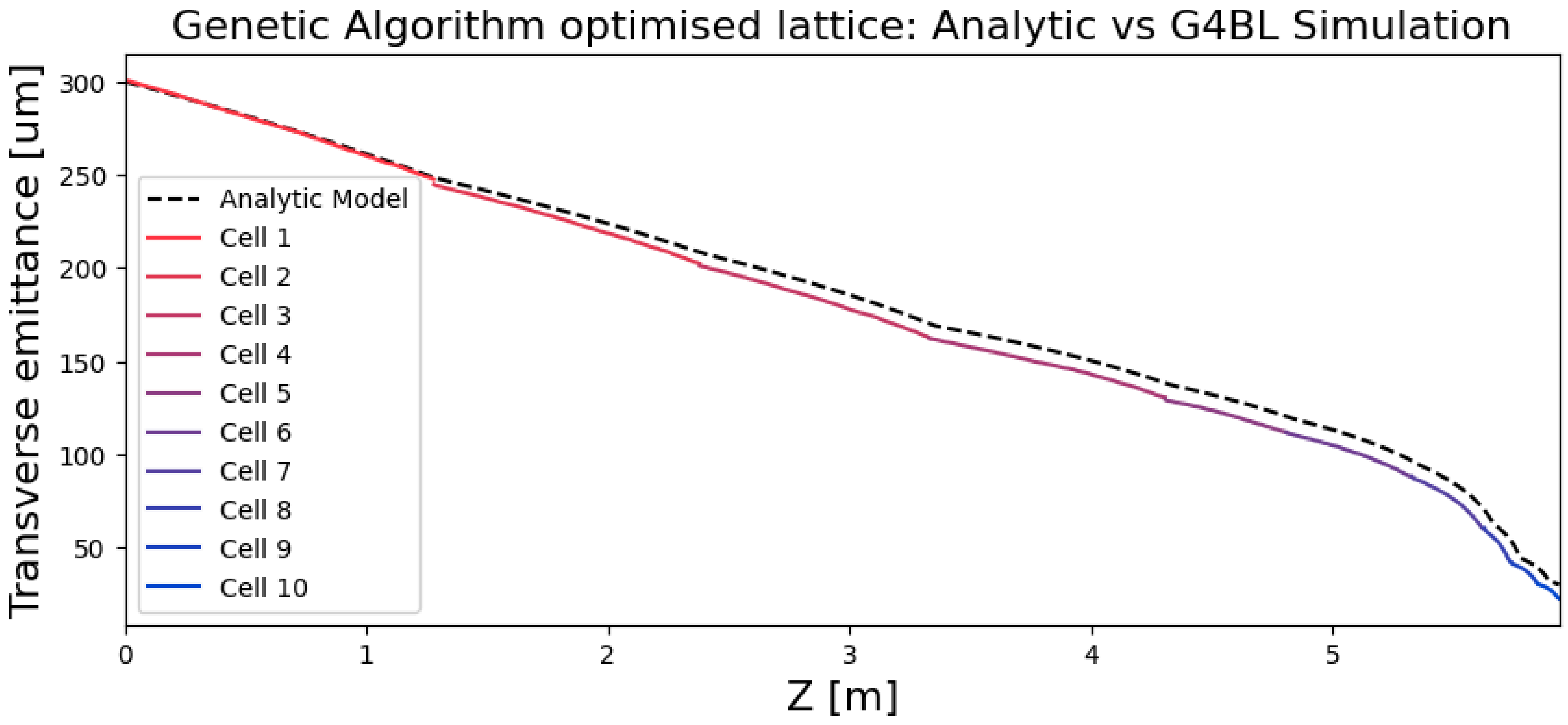


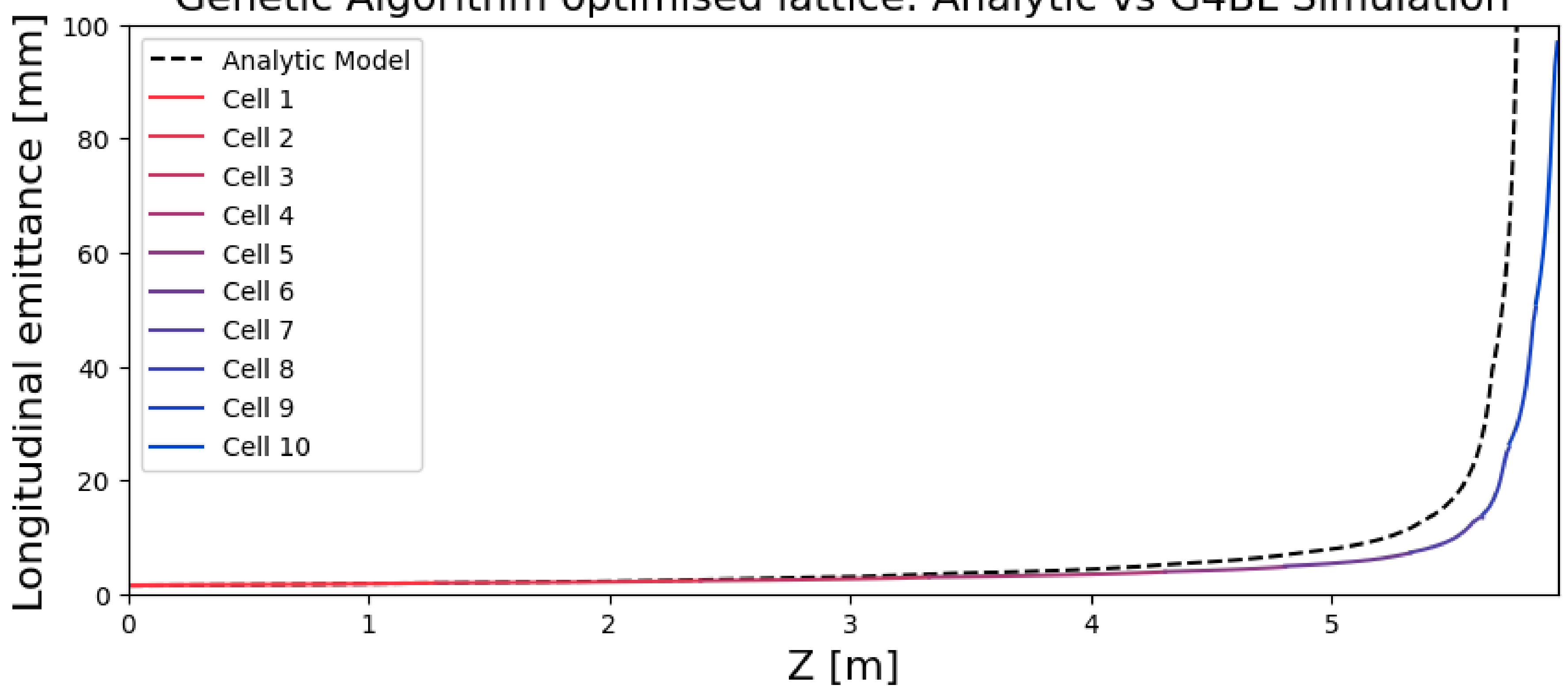


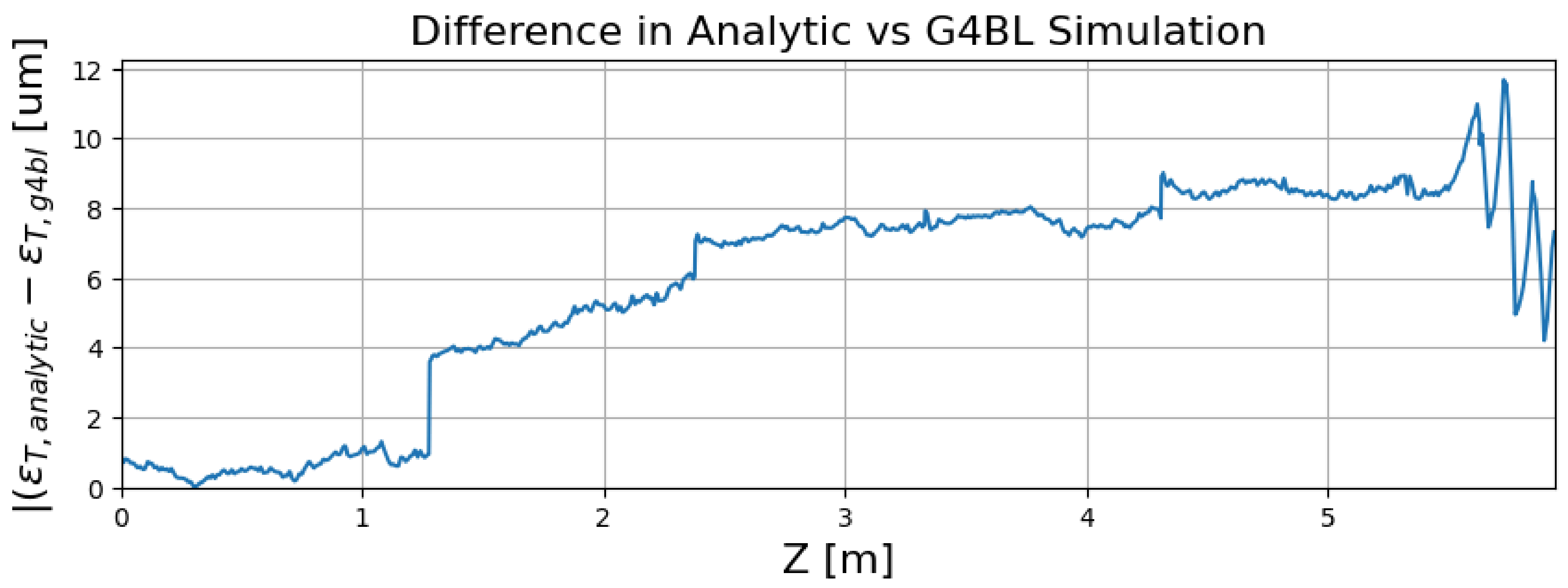
G4Beamline simulations performed cell-by-cell, starting with initalised beam each time.
Results in small jumps between cells.

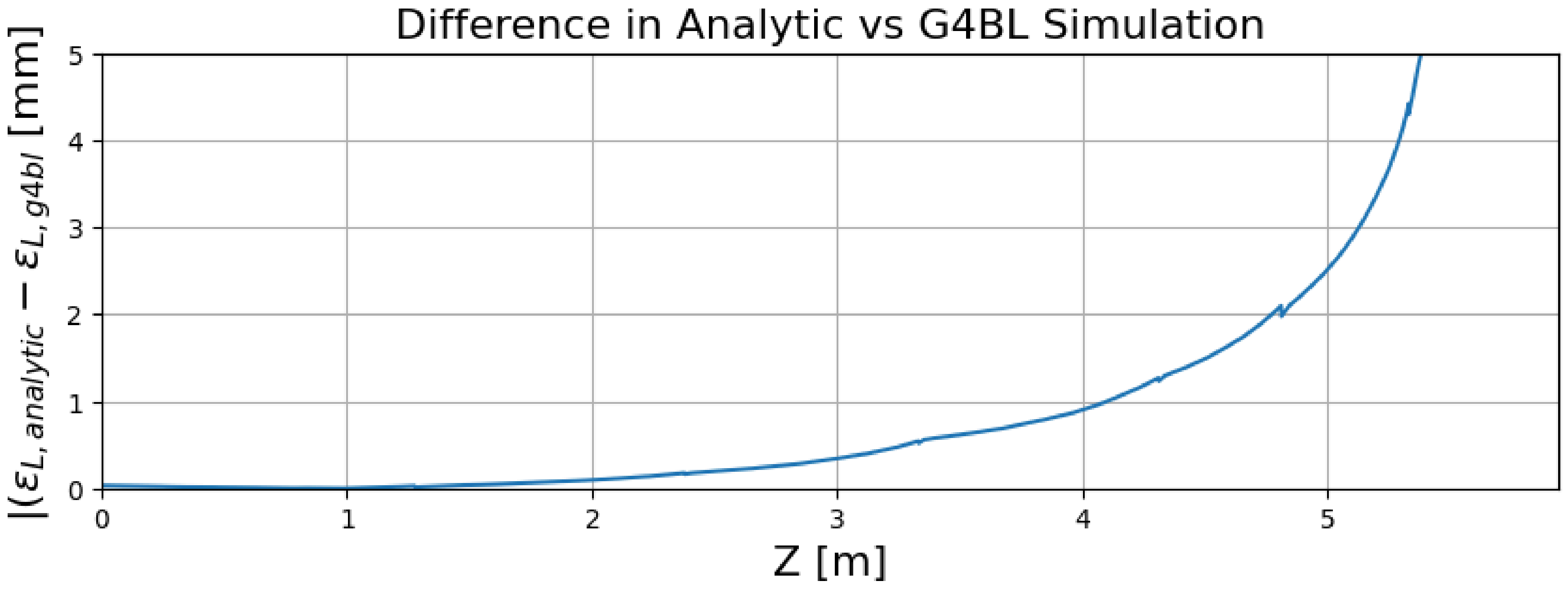


How to compare errors: analytical solution with simulation

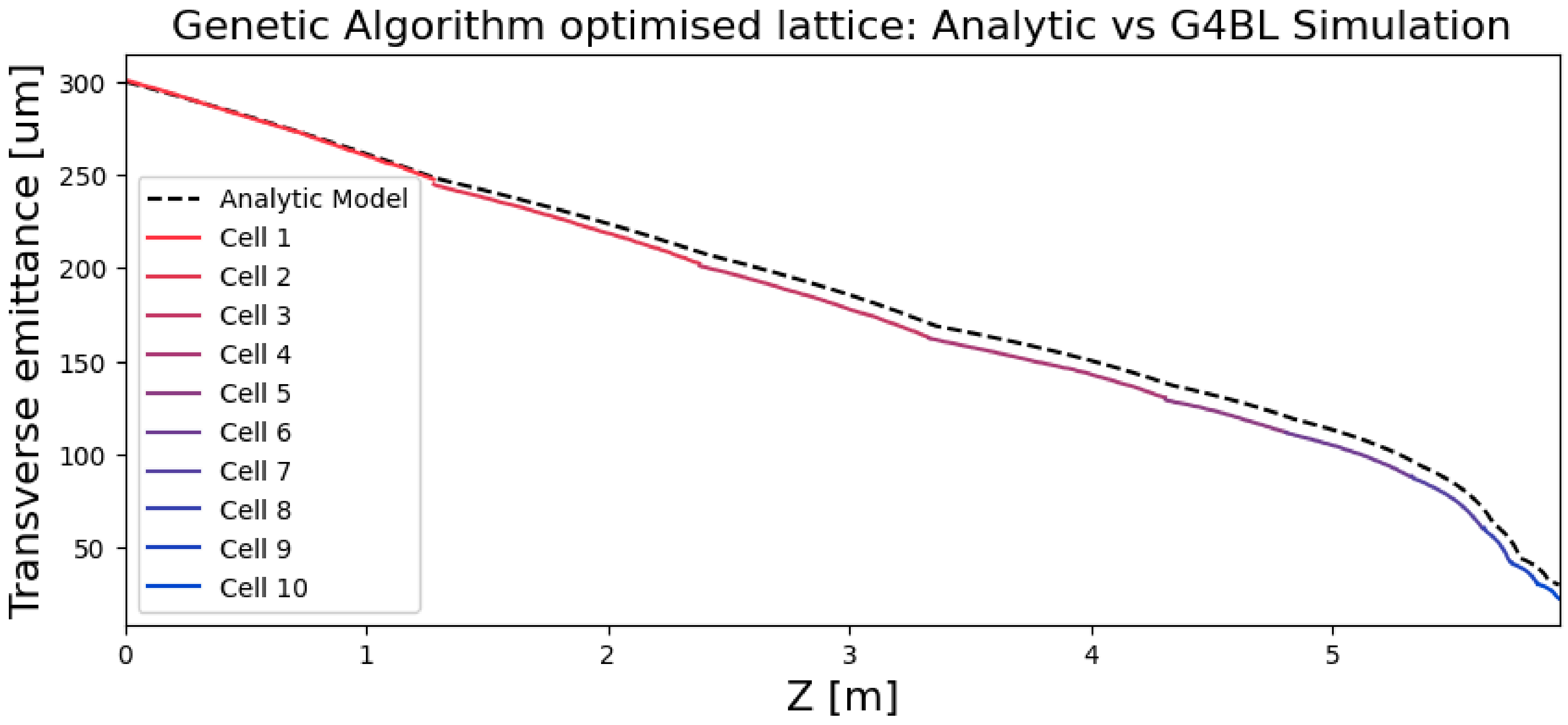


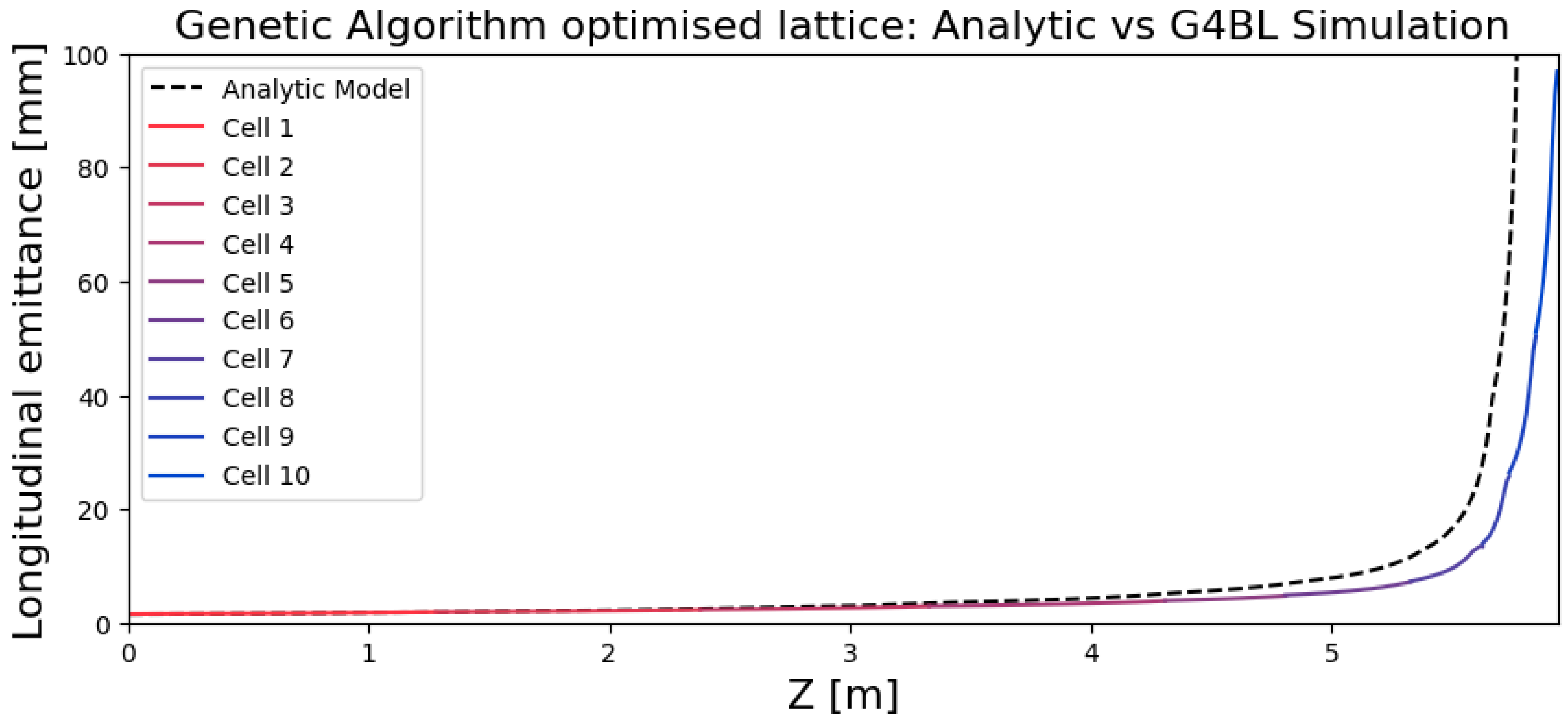


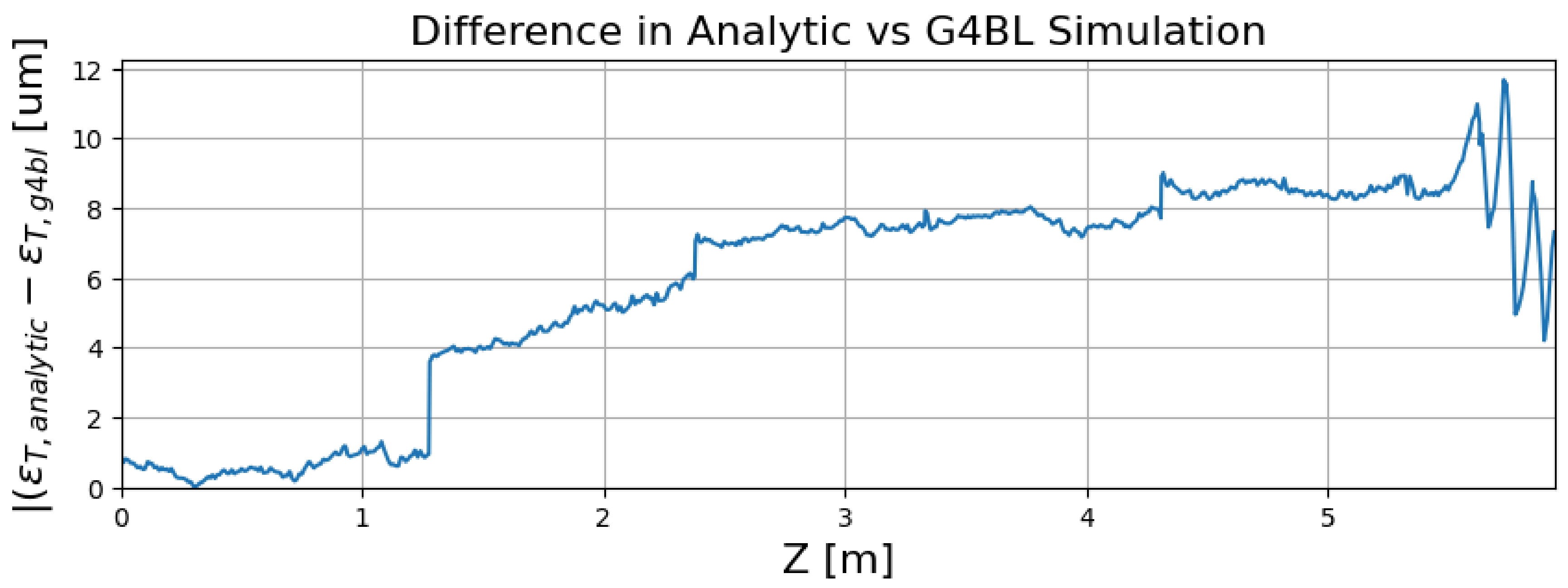


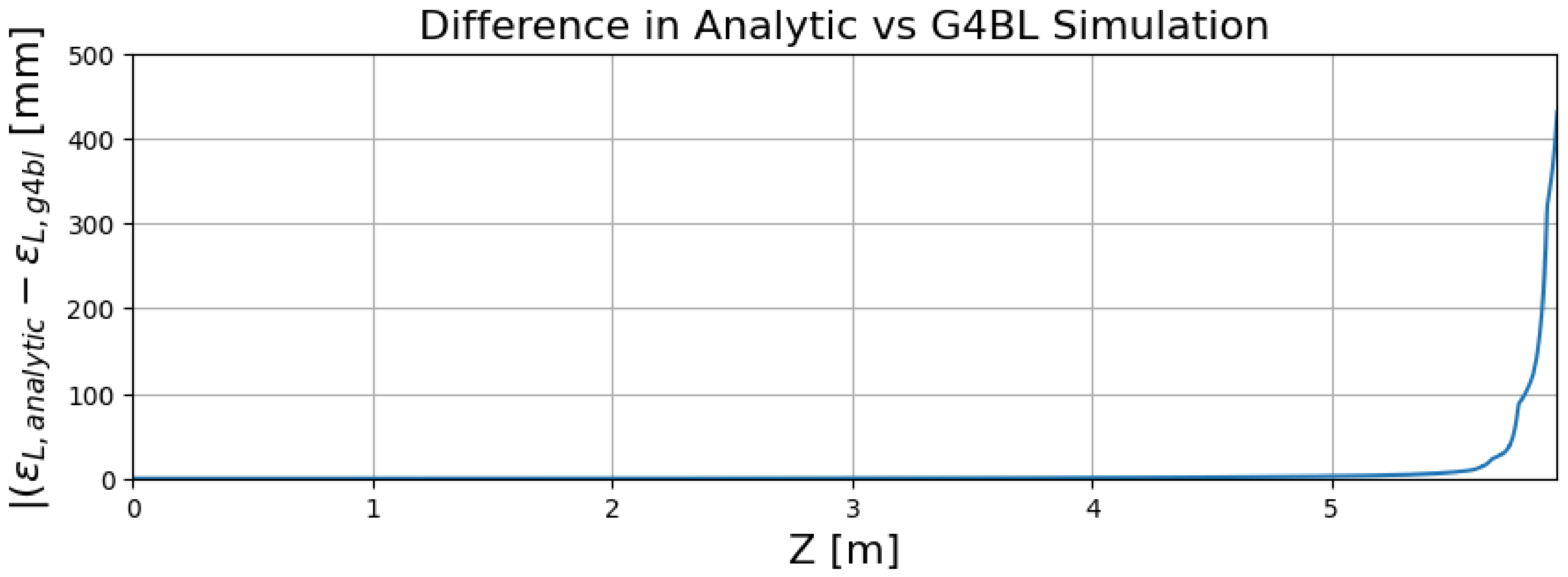


How to compare errors: analytical solution with simulation









Results

No solenoid matching, no RF cavities
Only absorbers in 40 T, new beam for each cell
Capacity to explore for further results with algorithm

From short-rectilinear cooling (eT = 300 um)

Ncell	Transverse um	Longitudinal mm
1	300	1.5
2	209.3	2.3
3	175.2	2.8
4	144.2	3.5
5	117.9	4.5
6	104.4	5.6
7	81.5	8.2
8	56.5	15.6
9	41.5	29.1
10	31.7	54.3
End	24.5	100.8

83% transmission not inc. decays

From long-rectilinear cooling (eT = 140 um)

Ncell	Transverse um	Longitudinal mm
1	140	1.5
2	134.9	1.6
3	131.2	1.6
4	121.3	1.8
5	88.6	3.1
6	73.6	4.4
7	49.7	9.5
8	35.4	21.5
	25.8	46.9

Found another with 21.4 um and 50 mm

86% transmission not inc. decays

Next Steps

Find relation between analytical eL and simulated eL?

Find cause of Pareto front - algorithm limit or hard limit

Explore other solutions

Start to build full lattice with chosen solution!

Thank you

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