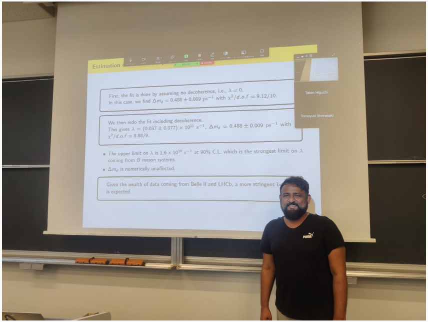
Experimental limits on Quantum decoherence from B meson systems

Based on arXiv: 2501.03136 and JHEP 05 (2024) 124

By Prof. S. Uma Sankar (IIT Bombay)
on behalf of Authors

Late Prof. Ashutosh Kumar Alok (IIT Jodhpur)
Prof. Subhashish Banerjee (IIT Jodhpur)
Dr. Jitendra Kumar (IIT Jodhpur)
Dr. Neetu Chundawat (IIT Jodhpur, now at IHEP Beijing)
Saurabh Rai (IIT Jodhpur)



Prof. Ashutosh delivered a talk on "Investigating Quantum Decoherence at Belle II and LHCb" at Kavli IPMU, Tokyo, just 15 days before his untimely passing.

Plan of talk

1. Theoretical Motivation
2. Effect of decoherence on the determination of Δm_d and Δm_s
3. Effect of decoherence on the determination of $\sin 2\beta$ and $\sin 2\beta_s$
4. Results from arXiv:2501.03136
5. A method to obtain the best limit on decoherence parameter.
6. Conclusions

The talk is based on our recent works *"Experimental limits on quantum decoherence from B-meson systems"*, **arXiv:2501.03136** and previous work *" Probing quantum decoherence at Belle II and LHCb "* published in **JHEP 05, 124 (2024)**.

Theoretical Motivation

- Any system of particles interacts with its environment.
- The formalism of Open Quantum Systems is developed to describe this interaction.
- Density matrix formalism is used to describe the time evolution of the system.
- If the system consists of a single unstable particle, this formalism just yields the radioactive decay law.
- However, if the system consists of two particles which can oscillate into each other, then the formalism necessarily introduces a parameter which leads to quantum decoherence in the two state system.
  P. Caban *et. al.* Unstable particles as open quantum systems,
  Phys. Rev. A**72** (2005) 032106.
- This decoherence arises due to the system-environment interactions.

Possible environment

System-environment interactions may arise at a fundamental level, such as the fluctuations in a quantum gravity space-time background [S.W. Hawking (1982); J. R. Ellis et. al. (1984); Huet-Peskin (1995)].

Neutral B mesons as an Open Quantum Systems

- We use an effective description which is phenomenological in nature. It is independent of the details of the actual dynamics between the system and environment.
- Within the framework of open quantum systems, the neutral B mesons are described as subsystems in interaction with an environment.
- The evolution of the complete system is given by the standard unitary operator.
- The dynamics of a B meson alone is obtained by a suitable integration over the environment degrees of freedom.
- Assuming the interaction between the B meson and the environment to be weak, the dynamics of the B meson subsystem can be described by quantum dynamical semigroups satisfying the condition of complete positivity.

Open time evolution of B mesons

- We are interested in the decays of B^0 and $\bar{B}^0$ mesons as well as $B^0 \leftrightarrow \bar{B}^0$ oscillations.

- To describe the time evolution of all these transitions, we need a basis of three states: $\left|B^0\right\rangle$, $\left|\bar{B}^0\right\rangle$ and $|0\rangle$, where $|0\rangle$ represents a state with no B meson. It characterizes the decayed state.

- We use the density matrix formalism to represent the time evolution of the B^0 system: $\rho_{B^0}(0)$ is the initial density matrix for the state which starts out as B^0. Similarly $\rho_{\bar{B}^0}(0)$ is for $\bar{B}^0$.

- The time evolution of these matrices is governed by the Kraus operators $K_i(t)$ as operator-sum form $\rho(t) = \sum_i K_i(t)\rho(0)K_i^\dagger(t)$. These operators are constructed taking into account the decoherence in the system which occurs due to the evolution under the influence of the environment.

- The Kraus operators, initially developed for the K meson system, have been utilized in our analysis to explore the B meson systems.
  P. Caban *et. al.* An Open Quantum Systems Approach to the Evolution of Entanglement in $K^0 - \bar{K}^0$ systems. Phys. Lett. A **363** (2007) 389.

Open time evolution of B mesons

- A meson initially in state $\rho_{B^0}(0) = \left|B^0\right\rangle \left\langle B^0\right|$ evolves in time to $\rho_{B^0}(t)$.
- The time evolution is implemented through the Kraus Operators $K_i(t)$ mentioned in the previous slide.
- There is a similar relation between $\rho_{\bar{B}^0}(0) = \left|\bar{B}^0\right\rangle \left\langle \bar{B}^0\right|$ and $\rho_{\bar{B}^0}(t)$.

Time dependent density matrices

$$\frac{\rho_{B^0}(t)}{\frac{1}{2}e^{-\Gamma t}} = \begin{pmatrix} a_{ch} + e^{-\lambda t}a_c & (\frac{q}{p})^*(-a_{sh} - ie^{-\lambda t}a_s) & 0 \\ (\frac{q}{p})(-a_{sh} + ie^{-\lambda t}a_s) & |\frac{q}{p}|^2(a_{ch} - e^{-\lambda t}a_c) & 0 \\ 0 & 0 & \rho_{33}(t) \end{pmatrix}$$

$$\frac{\rho_{\bar{B}^0}(t)}{\frac{1}{2}e^{-\Gamma t}} = \begin{pmatrix} |\frac{p}{q}|^2(a_{ch} - e^{-\lambda t}a_c) & (\frac{p}{q})(-a_{sh} + ie^{-\lambda t}a_s) & 0 \\ (\frac{p}{q})^*(-a_{sh} - ie^{-\lambda t}a_s) & a_{ch} + e^{-\lambda t}a_c & 0 \\ 0 & 0 & \rho'_{33}(t) \end{pmatrix}$$

Open time evolution of B mesons

- The mixing of B^0 and $\bar{B}^0$ forms two mass eigenstates $B_L^0 = pB^0 + q\bar{B}^0$ and $B_H^0 = pB^0 - q\bar{B}^0$ (light and heavy states of B mesons) with complex coefficients p and q, satisfying the condition $|p|^2 + |q|^2 = 1$.

- These states have masses m_L and m_H and decay widths Γ_L and Γ_H respectively. We also define $\Gamma = (\Gamma_L + \Gamma_H)/2$, $\Delta\Gamma = \Gamma_L - \Gamma_H$ and $\Delta m = m_H - m_L$.

- The quantities in the boxed equations of the previous slide are

$$a_{ch} = \cosh\left(\frac{\Delta\Gamma\, t}{2}\right),\ a_{sh} = \sinh\left(\frac{\Delta\Gamma\, t}{2}\right),\ a_c = \cos\left(\Delta m\, t\right) \text{ and } a_s = \sin\left(\Delta m\, t\right).$$

- λ is the decoherence parameter, arising due to the interaction of the B meson with the environment.

- Non-zero value of λ leads to the loss of the perfect coherence, usually assumed, between B^0 and $\bar{B}^0$ in the time evolution of the mass eigenstates B_L and B_H.

- $\rho_{33}(t)$ and $\rho'_{33}(t)$ are some functions of parameters defined above, $2(e^{\Gamma t} - a_{ch})$ in the limit $p/q \to 1$. However, they do not contribute to the present analysis.

Open time evolution of B mesons

- In the formalism of density matrices, any physical observable of the neutral B-meson system is described by a suitable hermitian operator $\mathcal{O}$.
- Its evolution in time can be obtained by taking its trace with the density matrix $\rho(t)$.

Of particular interest are those observables $\mathcal{O}_f$ that are associated with the decay of a B-meson into final states 'f'. In the $\left|B^0\right\rangle$, $\left|\bar{B^0}\right\rangle$, and $|0\rangle$ basis, $\mathcal{O}_f$ is represented by

$$\mathcal{O}_f = \begin{pmatrix} |A(B^0 \to f)|^2 & A(B^0 \to f)^* A(\bar{B^0} \to f) & 0 \\ A(B^0 \to f) A(\bar{B^0} \to f)^* & |A(\bar{B^0} \to f)|^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Here the entries are written in terms of the two independent decay amplitudes $A(B^0 \to f) \equiv A_f$ and $A(\bar{B^0} \to f) \equiv \bar{A}_f$.

The Probability that an initial B^0 meson decays, at time t, into a given state f is given by $P_f(B^0; t) = \mathrm{Tr}\left[\mathcal{O}_{\mathrm{f}}\, \rho_{\mathrm{B^0}}(\mathrm{t})\right]$. Similarly, $P_f(\bar{B^0}; t) = \mathrm{Tr}\left[\mathcal{O}_f\, \rho_{\bar{B}^0}(t)\right]$.

We are now equipped with with all the pieces required for the calculation of effects of decoherence on various important B physics observables.

Effect of decoherence on meson anti-meson mixing

Measurement of Δm

- Consider a final state f such that it occurs in the decay of only B^0 meson. Charged current semi-leptonic decays are a good example of such states.
- If the B meson at production (time $t = 0$) is tagged as B^0, it can decay into f only if it survived as B^0 at the time of decay t.
- On the other hand, if the B meson at $t = 0$ is tagged as a $\bar{B}^0$, then it can decay into f only if it oscillated into B^0 at the time of decay t.
- This naturally leads to the definitions of survival and oscillation probabilities as

Survival and Oscillation Probabilities

$$
\begin{aligned}
P_{\rm sur}(t) &= \frac{e^{-\Gamma t}}{2}\left[\cosh(\Delta\Gamma t/2) + \cos(\Delta m\, t)\right], \\
P_{\rm osc}(t) &= \frac{e^{-\Gamma t}}{2}\left|\frac{q}{p}\right|^2\left[\cosh(\Delta\Gamma t/2) - \cos(\Delta m\, t)\right].
\end{aligned}
$$

Decoherence in the Measurement of Δm_d

- **LHCb**, **CDF** and **D0** experiments determine Δm_d by measuring $P_{\rm sur}(t)$ and $P_{\rm osc}(t)$ as function of proper decay time t.

- In the presence of decoherence, these probabilities are modified to

Survival and Oscillation Probabilities

$$
\begin{aligned}
P_{\rm sur}(t,\lambda) &= \frac{e^{-\Gamma t}}{2}\left[\cosh(\Delta\Gamma_d t/2) + e^{-\lambda t}\cos(\Delta m_d t)\right], \\
P_{\rm osc}(t,\lambda) &= \frac{e^{-\Gamma t}}{2}\left|\frac{q}{p}\right|^2\left[\cosh(\Delta\Gamma_d t/2) - e^{-\lambda t}\cos(\Delta m_d t)\right].
\end{aligned}
$$

- Note that the decay width Γ multiplies the whole expression whereas the decoherence term λ multiplies only the oscillating term, which depends on Δm_d.

Decoherence in the Measurement of Δm_d

- For the B_d system, the Standard Model (SM) predicts $|q/p|$ to be very close to 1 and $\Delta\Gamma_d$ is negligibly small.
- The value of Δm_d is determined from the survival/oscillation probabilities by assuming perfect coherence ($\lambda = 0$), in addition to the above two SM based assumptions.
- However, the time evolution from open quantum systems point of view shows that these probabilities depend on the decoherence parameter λ.
- One must do a two parameter fit with Δm_d and λ of the above expressions, rather than a one parameter fit with just Δm_d.

The true value of Δm_d is modified when the decoherence parameter λ is included in the fit.

- Three years ago, the LHCb Collaboration published the most precise determination of $B_s - \bar{B}_s$ oscillation frequency. Nature Phys. 18, no.1, 1-5 (2022).

- In addition to $P_{\rm sur}(t)$ and $P_{\rm osc}(t)$, they have also used the mixing asymmetry

Time dependent mixing asymmetry

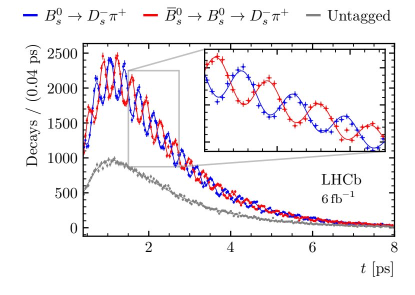
$$A_{\rm mix}(t) = \frac{P_{\rm sur}(t) - P_{\rm osc}(t)}{P_{\rm sur}(t) + P_{\rm osc}(t)} = \frac{\cos(\Delta m_s t) + \delta_B \cosh(\Delta\Gamma_s t/2)}{\cosh(\Delta\Gamma_s t/2) + \delta_B \cos(\Delta m_s t)}.$$

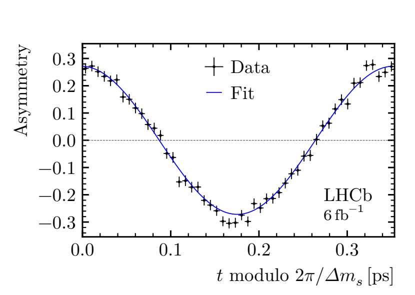
where

$$\delta_B = \frac{1 - \left|\frac{q}{p}\right|^2}{1 + \left|\frac{q}{p}\right|^2}.$$

Measurement of Δm_s

In the plot below, the left panel shows $P_{\mathrm{sur}}(t)$ (in blue) and $P_{\mathrm{osc}}(t)$ (in red) as a function of the proper decay time t. The right panel shows the mixing asymmetry as a function of t modulo $2\pi/\Delta m_s$.





We note that the fit in the right panel is essentially a cosine function. The effect of $\Delta\Gamma_s$ and δ_B, in the fit to $A_{\mathrm{mix}}(t)$, is not very significant.

Decoherence in the Measurement of Δm_s

- In the presence of decoherence, the survival and the oscillation probabilities get modified in exactly the same way they got modified for the B_d system.

- The mixing asymmetry now has a more complicated form given by

$$A_{\rm mix}(t,\lambda) = \frac{P_{\rm sur}(t,\lambda) - P_{\rm osc}(t,\lambda)}{P_{\rm sur}(t,\lambda) + P_{\rm osc}(t,\lambda)} = \frac{e^{-\lambda t}\cos(\Delta m_s t) + \delta_B \cosh(\Delta\Gamma_s t/2)}{\cosh(\Delta\Gamma_s t/2) + \delta_B e^{-\lambda t}\cos(\Delta m_s t)}.$$

- The net effect is to replace $\cos(\Delta m_s t)$ in the mixing asymmetry by $e^{-\lambda t}\cos(\Delta m_s t)$.

- Fitting the survival/oscillation probabilities or $A_{\rm mix}$, including decoherence, will be a complicated process because the effects of $\Delta\Gamma_s$ and δ_B can not be neglected.

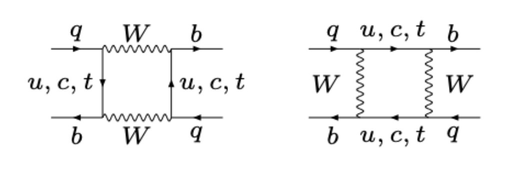
Effect of decoherence on determination of CP violation parameters

CP violation in B decays

The CKM phase in 3×3 quark mixing matrix manifests in the B meson systems in three different ways:

1. CP violating in mixing: $P(B^0 \to \bar{B}^0) \neq P(\bar{B}^0 \to B^0)$

Box diagram enables $B^0 \leftrightarrow \bar{B}^0$ oscillation:



- B_d^0 oscillations are fast! 2 million times a second
- And B_s^0 are even faster, 35 times faster
- Mass and flavour eigenstates not the same:

$$|B_H^0\rangle = p|B^0\rangle - q|\bar{B}^0\rangle \quad |B_L^0\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$$

- CP violation in mixing occurs if $|q/p| \neq 1$

CP violation in B decays

2. CP violation in decay: $P(B \to f) \neq P(\bar{B} \to \bar{f})$

CP violation in decay occurs if

$$\left| \frac{\bar{A}_{\bar{f}}}{A_f} \right| \neq 1,$$

where

$$A_f \equiv \langle f | H | P \rangle \quad \text{and} \quad \bar{A}_{\bar{f}} \equiv \langle \bar{f} | H | \bar{P} \rangle .$$

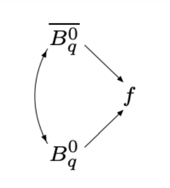
In charged meson decays, no mixing is involved. In such a situation, an observable CP violating quantity is

$$A_{f\pm} \equiv \frac{\Gamma(P^- \to f^-) - \Gamma(P^+ \to f^+)}{\Gamma(P^- \to f^-) + \Gamma(P^+ \to f^+)} .$$

A nonvanishing $A_{f\pm}$ is often termed “direct” CP violation.

CP violation in B decays

3. CP violation through mixing-decay interference: $P(B^0 \to \bar{B}^0 \to f) \neq P(\bar{B}^0 \to B^0 \to \bar{f})$



- This type of CP violation occurs if $\mathrm{Im}(\lambda_f) \neq 0$, where

$$\lambda_f \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f},$$

where $\bar{A}_f \equiv \langle f|H|\bar{P}\rangle$.

- The CP violating parameters, $\sin 2\beta$ and $\sin 2\beta_s$, are related to third type of CP violation. Their measurement requires construction of time-dependent CP-asymmetry.

Unitarity triangles

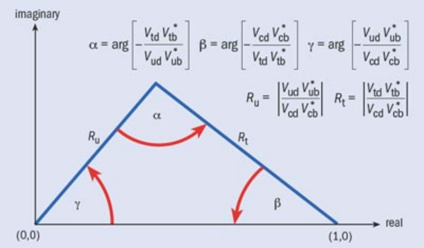
The unitarity of the CKM matrix implies the relation $V^\dagger V = 1$. This can be viewed as conditions on combinations of CKM elements in a complex plane.

The CKM matrix satisfies three *distinct* relations of the form $[V^\dagger V]_{ij} = 0$ $(i \neq j)$. These give rise to three different unitarity triangles.

For example,

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0.$$

This relation may be represented as a triangle in the complex plane, whose sides are the three complex quantities $V_{ub}^* V_{ud}$, $V_{cb}^* V_{cd}$, and $V_{tb}^* V_{td}$. This triangle is:

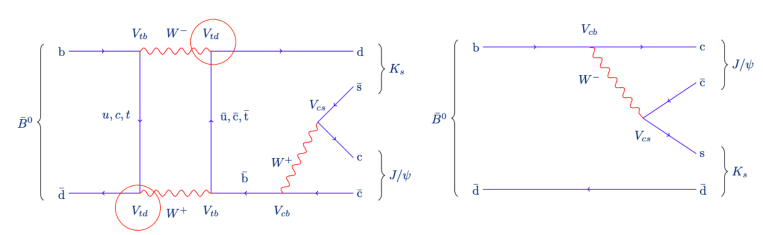


$\sin 2\beta$ is obtained through a time-dependent analysis of $B_d^0 \to J/\psi\, K_S$ decay.

Time dependent CP violation in $B_d^0 \to J/\psi\, K_S$

The "golden channel" $B_d/\bar{B}_d \to J/\psi K_S$ (induced by the quark level transition $b \to c\bar{c}s$), has given us a rather clean measurement of β, one of the angles of the unitarity triangle.

Mixing and decay diagrams interfere:



- $B^0 - \bar{B}^0$ oscillation is periodic in time $\Rightarrow$ CP violation is time dependent.
- The dominant contribution comes from one tree level decay diagram shown above, so $|\lambda_f| \approx 1$.
- As $\bar{A}_f/A_f \approx (V_{cb}V_{cs}^*)/(V_{cb}^*V_{cs}) \approx 1$ and as $q/p = \exp[-i\,\mathrm{Arg}(M_{12})]$ where $\mathrm{Arg}(M_{12}) = \mathrm{Arg}(V_{tb}^*V_{td}V_{tb}^*V_{td}) \approx -2\beta$, so $q/p \approx e^{2i\beta}$. Thus $\lambda_f \approx \sin 2\beta$.
- In the B_d system, the lifetime difference $\Delta\Gamma$ is extremely small: in the SM, $\Delta\Gamma/\Gamma \approx 0.5\%$.

Thus the "standard" time dependent CP asymmetry for decay turns out to be $\sin 2\beta\, \sin(\Delta m t)$.

Decoherence in CP asymmetry in $B^0/\bar{B^0} \to f_{CP}$ decays

Let us consider $B^0/\bar{B^0} \to f_{CP}$ decays where, f_{CP} can be $J/\psi K_S$ or D^+D^- final states for B_d^0 and $\psi\phi$ for B_s^0 meson.

The operator for these decay modes can be written as

$$\mathcal{O}_{f_{CP}} = |A_f|^2 \begin{pmatrix} 1 & (\frac{p}{q})\,\lambda_f & 0 \\ (\frac{p}{q})^* \lambda_f^* & |\frac{p}{q}|^2 |\lambda_f|^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} .$$

λ_f is the phase invariant quantity defined as:

$$\lambda_f = \frac{q}{p} \frac{\bar{A}_f \left(\equiv A(\bar{B^0} \to f_{CP})\right)}{A_f\,(\equiv A(B^0 \to f_{CP}))} .$$

Therefore the probability rate that an initial state $B^0/\bar{B^0}$ decays into final state f_{CP} is given by

$$\begin{aligned} \frac{P_{f_{CP}}(B^0;t)}{\frac{1}{2}e^{-\Gamma t}|A_f|^2} \quad = \quad & (1+|\lambda_f|^2)\cosh\left(\frac{\Delta\Gamma t}{2}\right) + (1-|\lambda_f|^2)\, e^{-\lambda t}\cos(\Delta m t) \\ & -2\mathrm{Re}(\lambda_f)\sinh\left(\frac{\Delta\Gamma t}{2}\right) - 2\mathrm{Im}(\lambda_f) e^{-\lambda t}\sin(\Delta m t) . \end{aligned}$$

Decoherence in CP asymmetry in $B^0/\bar{B}^0 \to f_{CP}$ decays

$$\frac{P_{f_{CP}}(\bar{B}^0;t)}{\frac{1}{2}e^{-\Gamma t}|A_f|^2|\frac{p}{q}|^2} = \left(1+|\lambda_f|^2\right)\cosh\left(\frac{\Delta\Gamma t}{2}\right) - \left(1-|\lambda_f|^2\right)e^{-\lambda t}\cos(\Delta m t)$$
$$-2\mathrm{Re}(\lambda_f)\sinh\left(\frac{\Delta\Gamma t}{2}\right) + 2\mathrm{Im}(\lambda_f)e^{-\lambda t}\sin(\Delta m t)\ .$$

CP asymmetry in the interference of mixing and decay

$$\mathcal{A}_{f_{CP}}(t) = \frac{P_{f_{CP}}(B^0;t) - P_{f_{CP}}(\bar{B}^0;t)}{P_{f_{CP}}(B^0;t) + P_{f_{CP}}(\bar{B}^0;t)}.$$

Neglecting CP violation in mixing (setting $|q/p|^2 = 1$), we get

$$\mathcal{A}_{f_{CP}}(t) = \frac{A_{\mathrm{CP}}^{\mathrm{dir,\,f_{CP}}}\cos(\Delta m t) + A_{\mathrm{CP}}^{\mathrm{mix},\,f_{CP}}\sin(\Delta m t)}{\cosh\left(\frac{\Delta\Gamma t}{2}\right) + A_{\Delta\Gamma}^{f_{CP}}\sinh\left(\frac{\Delta\Gamma t}{2}\right)}e^{-\lambda t}\ .$$

$$A_{\mathrm{CP}}^{\mathrm{dir,\,f_{CP}}} = \frac{1-|\lambda_{f_{CP}}|^2}{1+|\lambda_{f_{CP}}|^2},\quad A_{\Delta\Gamma}^{f_{CP}} = -\frac{2\mathrm{Re}(\lambda_{f_{CP}})}{1+|\lambda_{f_{CP}}|^2},\quad A_{\mathrm{CP}}^{\mathrm{mix},\,f_{CP}} = -\frac{2\mathrm{Im}(\lambda_{f_{CP}})}{1+|\lambda_{f_{CP}}|^2},\ .$$

Decoherence in CP asymmetry in $B^0/\bar{B}^0 \to f_{CP}$ decays

This time-dependent CP asymmetry with $\lambda = 0$ is used to determine quantities like $\sin 2\beta$ and $\sin 2\beta_s$ where $\beta \equiv \arg\left[-(V_{cd}V_{cb}^*)/(V_{td}V_{tb}^*)\right]$ and $\beta_s \equiv \arg\left[-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)\right]$ are the angles of the unitarity triangle.

Determination of $\sin 2\beta$

For e.g., the most statistically precise measurement of $\sin 2\beta$ is obtained through a time-dependent analysis of $B_d^0 \to J/\psi\, K_S$.

For this decay channel, $\Delta\Gamma_d \approx 0$, $|\lambda_f| \approx 1$ and $\mathrm{Im}(\lambda_f) \approx \sin 2\beta$ which gives

$$\mathcal{A}_{f_{CP}}(t) \approx \sin 2\beta \sin\left(\Delta m_d t\right).$$

In the presence of decoherence, this asymmetry becomes

$$\mathcal{A}_{f_{CP}}(t) \approx \left[e^{-\lambda t} \sin 2\beta\right] \sin\left(\Delta m_d t\right).$$

The coefficient of $\sin\left(\Delta m_d\, t\right)$ is $e^{-\lambda t}\, \sin 2\beta$ and not $\sin 2\beta$!

The measurement of $\sin 2\beta$ (and $\sin 2\beta_s$) is masked by the presence of decoherence.

Estimation of λ_d and λ_s from the B meson data of LHCb (arXiv: 2501.03136)

Mixing asymmetry in B_d system

- In order to obtain an estimate of the decoherence parameter λ, the data on survival/oscillation probabilities or the CP asymmetry, should be expressed as functions of the proper decay time t.
- For most of the published data, the independent variable is not t.
- The following two LHCb papers, the time-dependent mixing asymmetry in B_d system is indeed given as a function of t: arXiv:1604.03475 and arXiv:2309.09728. We used these data to obtain an estimate of λ_d
- Assuming $|q/p| = 1$, the time-dependent mixing asymmetry, with decoherence, is

$$A_{\rm mix}(t, \lambda_d) = \frac{\cos(\Delta m_d\, t)}{\cosh(\Delta\Gamma_d\, t/2)} e^{-\lambda_d\, t} \,.$$

Fitting this expression to the experimental data will yield an estimate of λ_d

Mixing asymmetry in B_d system

Validating the Parameter Estimation (arXiv:1604.03475)

- We set $|q/p| = 1$ and take the decay width difference $\Delta\Gamma_d$ to be zero.
- The measurement of the oscillation frequency of B_d-mesons are from the decays $B_d^0 \to D^{(*)-}\mu^+\nu_\mu X$ and $B_d^0 \to D^-\mu^+\nu_\mu X$.
- The results are reported for four different tagging efficiencies across 2011 and 2012 runs.
- We first validate our method by reproducing the reported results (only with the best tagging quality data) by putting the decoherence parameter to zero.
- We obtained the oscillation frequency $\Delta m_d = 0.494 \pm 0.007$ ps^{-1} against the reported result $\Delta m_d = 0.505 \pm 0.002 \pm 0.001$ ps^{-1}.

CP Asymmetry in B_d system

- In addition to mixing asymmetry, neutral mesons also exhibit CP-asymmetry where mesons can decay into states of definite CP.

- Neglecting CP violation in mixing, the time-dependent CP asymmetry in systems having decoherence is:

$$\mathcal{A}_{f_{CP}}(t, \lambda_d) = \frac{A_{\rm CP}^{\rm dir,\, f_{CP}} \cos(\Delta m_d t) + A_{\rm CP}^{\rm mix,\, f_{CP}} \sin(\Delta m_d t)}{\cosh(\Delta\Gamma_d t/2) + A_{\Delta\Gamma}^{f_{CP}} \sinh(\Delta\Gamma_d t/2)} e^{-\lambda_q t},$$

- where

$$A_{\rm CP}^{\rm dir,\, f_{CP}} = \frac{1 - |\lambda_{f_{CP}}|^2}{1 + |\lambda_{f_{CP}}|^2}, \quad A_{\Delta\Gamma}^{f_{CP}} = -\frac{2{\rm Re}(\lambda_{f_{CP}})}{1 + |\lambda_{f_{CP}}|^2}, \quad A_{\rm CP}^{\rm mix,\, f_{CP}} = -\frac{2{\rm Im}(\lambda_{f_{CP}})}{1 + |\lambda_{f_{CP}}|^2}.$$

- The denominator of the $\mathcal{A}_{f_{CP}}(t, \lambda_d)$ simplifies to 1 in the limit $\Delta\Gamma_d$ is neglected.

The procedure for extracting the decoherence parameter is the same as for mixing asymmetry.

CP asymmetry in B_d system

Validating the Parameter Estimation (arXiv:2309.09728)

- To this end, we used the LHCb measurement of CP-asymmetry in $B_d^0 \to \psi K_S^0$ decays.

- We first set the decoherence parameter to be zero

- We obtained the values of CP-violating parameters to be $A_{\mathrm{CP}}^{\mathrm{dir,\,f_{CP}}} = -0.005 \pm 0.012$ and, $A_{\mathrm{CP}}^{\mathrm{mix},\,f_{CP}} = 0.715 \pm 0.014$.
- These values were found to be in good agreement with the published results, $A_{\mathrm{CP}}^{\mathrm{dir,\,f_{CP}}} = -0.004 \pm 0.012$ and, $A_{\mathrm{CP}}^{\mathrm{mix},\,f_{CP}} = 0.724 \pm 0.014$.

Estimation of λ_d from a Combined Fit of Mixing and CP asymmetries

We performed a combined fit to the time-dependent mixing and CP-asymmetry data.

Results of the Combined Fit without decoherence

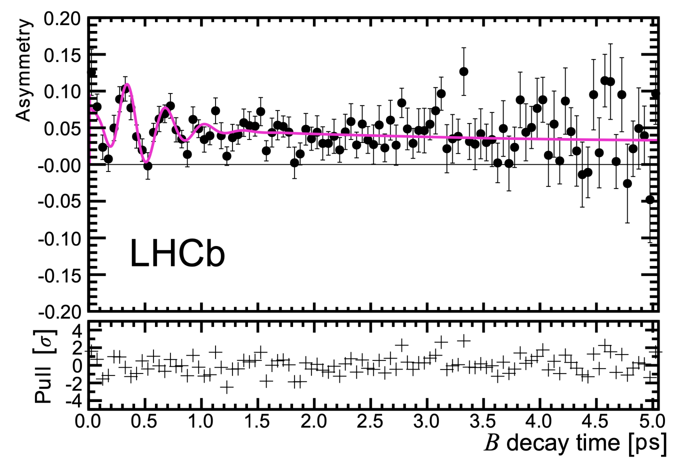
- The oscillation frequency was found to be $\Delta m_d = 0.494 \pm 0.007$.
- The CP-violating parameters were found to be $A_{\mathrm{CP}}^{\mathrm{dir},\, \mathrm{f_{CP}}} = -0.010 \pm 0.018$ and, $A_{\mathrm{CP}}^{\mathrm{mix},\, f_{CP}} = 0.711 \pm 0.020$.
- The value of χ^2/dof for this fit was found to be 2.84.

Results of the Combined Fit with decoherence

- The oscillation frequency was found to be $\Delta m_d = 0.469 \pm 0.005$ ps^{-1}.
- The CP-violating parameters were found to be $A_{\mathrm{CP}}^{\mathrm{dir},\, \mathrm{f_{CP}}} = -0.005 \pm 0.021$ and, $A_{\mathrm{CP}}^{\mathrm{mix},\, f_{CP}} = 0.836 \pm 0.038$.
- The decoherence parameter was found to be $\lambda_d = 0.055 \pm 0.009$ ps^{-1}. The χ^2/dof for the fit was 1.76, which indicates a better fit as compared to one with vanishing decoherence parameter.

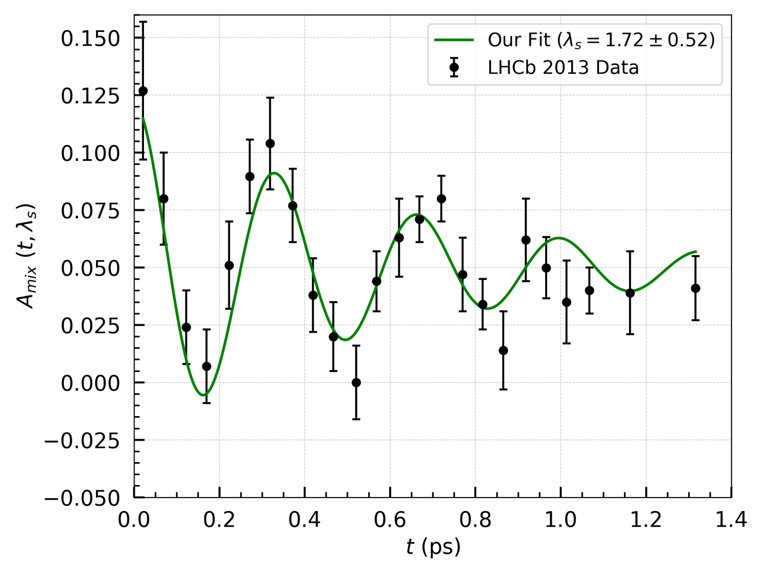
Estimation of λ_s from B_s-meson data

We used the data for mixing asymmetry from an old LHCb paper (arXiv:1308.1302, Eur. Phys. C (2013) 73:2655). The left panel of figure-7 of this paper is reproduced below.



Estimation of λ_s from B_s-meson data

From the figure in the previous slide, we generated the following figure:



Estimation of λ_s from B_s-meson data

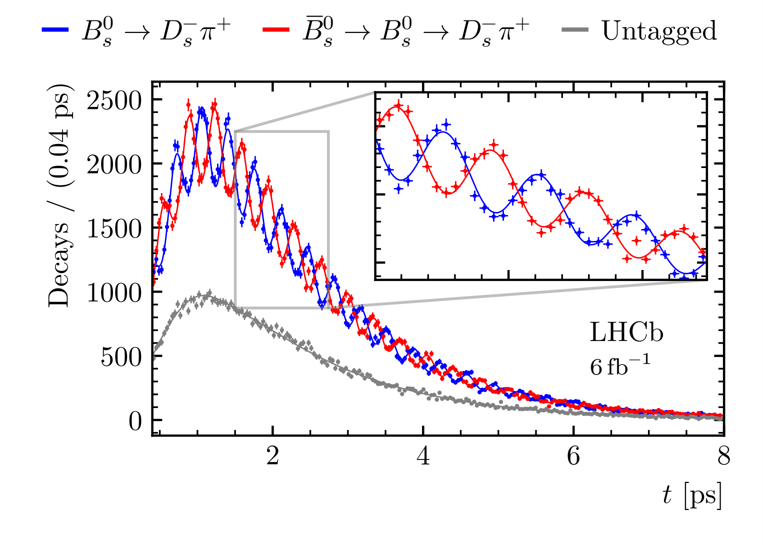
- As in the case of B_d meson, we assumed $|q/p| = 1$.
- Unlike in the case of B_d meson, the value of $\Delta\Gamma$ is not neglected.
- The decay width difference was kept constant at its current world average value of $\Delta\Gamma_s = 0.083 \pm 0.005$.

Results

- We first validated the applicability of our method by taking $\lambda_s = 0$. We obtained the oscillation frequency to be $\Delta m_s = 18.65 \pm 0.32$ with $\chi^2/dof = 1.74$.
- λ_s was now floated as a free parameter in the fit and we obtained $\Delta m_s = 18.85 \pm 0.33$ and $\lambda_s = 1.72 \pm 0.52$, with $\chi^2/dof = 1.02$.

Precision Measurement of B_s-meson Oscillation Frequency

Recently, the LHCb Collaboration published a paper (Nature Phys. **18** 1-5 (2022), arXiv:2104.04421) on the precise determination of $B_s - \bar{B}_s$ oscillation frequency.



Final Comment

- As we can see, the B_s oscillation data of LHCb from the 2021 paper has a far greater statistical weightage compared to that from 2013 paper.
- We were unable to convert the data from the 2021 paper into a form that can be used to make a fit.
- The data in the previous figure is directly in the form $P_{\rm sur}(t)$ and $P_{\rm osc}(t)$.
- The strongest constraint on λ_s can be obtained by fitting the above data to the formulae below:

Survival and Oscillation Probabilities

$$\begin{aligned} P_{\rm sur}(t,\lambda_s) &= \frac{e^{-\Gamma t}}{2}\left[\cosh(\Delta\Gamma_s t/2) + e^{-\lambda_s t}\cos(\Delta m_s t)\right], \\ P_{\rm osc}(t,\lambda_s) &= \frac{e^{-\Gamma t}}{2}\left|\frac{q}{p}\right|^2\left[\cosh(\Delta\Gamma_s t/2) - e^{-\lambda_s t}\cos(\Delta m_s t)\right]. \end{aligned}$$


Thank You

Back-up Slides

Semigroup

What is a Semigroup?

A **semigroup** is a concept from abstract algebra. It is a set equipped with an associative binary operation. Specifically, a semigroup must satisfy two properties:
1. Closure: For any two elements a and b in the set, the result of the operation on a and b must also be in the set. If the operation is denoted by $\circ$, then for all $a, b \in S$, we must have:

$$a \circ b \in S$$

This means the set is "closed" under the operation.
2. Associativity: The operation must be associative. That is, for all elements a, b, and c in the set, the operation satisfies:

$$(a \circ b) \circ c = a \circ (b \circ c)$$

This means that how you group the elements when performing the operation doesn't affect the result.

Example: Consider the set of non-negative integers $\mathbb{N}_0 = \{0, 1, 2, 3, \dots\}$ with the operation of addition. The pair $(\mathbb{N}_0, +)$ forms a semigroup because:

- **Closure:** The sum of any two non-negative integers is also a non-negative integer.
- **Associativity:** Addition is associative, i.e.,

$$(a + b) + c = a + (b + c).$$

Semigroups vs Groups

Semigroups:

- No requirement for an identity element or inverses.

Groups:

- Must have an identity element (neutral element for the operation).
- Must have inverses for every element (i.e., an element that "undoes" the operation).

In summary:

- A **semigroup** is a set with an associative operation.
- A **group** is a semigroup with an identity element and inverses.

In **open quantum systems**, semigroups describe the time evolution where the evolution is governed by a family of operators S_t (superoperators), forming a one-parameter semigroup:

- The time evolution operators S_t are associative.
- For any times $t_1, t_2 \geq 0$, the evolution satisfies:

$$S_{t_1+t_2} = S_{t_1} S_{t_2}.$$

This means the system's evolution from time t_1 to $t_1 + t_2$ can be broken down into two sequential steps.

Complete positivity

Complete Positivity

Complete positivity is a critical concept in quantum information theory and quantum mechanics, particularly when discussing the evolution of open quantum systems. It is a stronger version of positivity that ensures that the physical state of a quantum system remains valid even when the system is entangled with an external environment.

To understand complete positivity, let's first discuss **positivity**.

Positivity

In quantum mechanics, a **density matrix** ρ describes the state of a quantum system. The density matrix must satisfy certain conditions to be physically meaningful:

- It must be **positive semi-definite**, meaning all of its eigenvalues are non-negative. This guarantees that all probabilities derived from it (such as measurement outcomes) are valid.

A quantum operation or map Φ is **positive** if it takes any valid density matrix ρ (with non-negative eigenvalues) and transforms it into another valid density matrix $\Phi(\rho)$ that is also positive semi-definite.

However, positivity alone is not enough to describe the full behavior of quantum systems, particularly when we consider subsystems of a larger, entangled system with environment. This is where complete positivity comes in.

Complete Positivity

A quantum operation Φ is **completely positive** if, when it acts on part of a larger system (for example, a quantum system entangled with another system), it ensures that the entire system remains in a valid quantum state.

Why Complete Positivity is Important

When dealing with quantum systems that are not isolated (which is the case for real-world quantum systems), we need to consider the interactions between the system and its environment. A quantum operation must not only preserve the positivity of the system's state but also ensure that when the system is part of a larger entangled system, the overall state remains physically valid. This is crucial for ensuring consistency with quantum mechanics.

Imagine we have a quantum system A and an ancillary system B that may be entangled with A. If we apply a quantum operation to system A alone, but system A is entangled with B, the operation on A should not produce unphysical or invalid results when considering the entire system $A + B$.

Complete positivity guarantees that even when the quantum operation acts only on part of the system, the overall system's state remains valid, preserving the structure of quantum mechanics.

Completely Positive Maps and Kraus Operators

Choi matrix: The Choi matrix is a tool used to determine if a map is completely positive. A map is completely positive if and only if its corresponding Choi matrix is positive semi-definite.

A quantum operation Φ can be expressed using **Kraus operators**:

$$\Phi(\rho) = \sum_i K_i \rho K_i^\dagger,$$

where K_i are Kraus operators. For Φ to be completely positive, they must satisfy:

$$\sum_i K_i^\dagger K_i = I.$$

Summary

- **Positivity** ensures a quantum operation keeps a density matrix positive (physically valid) when acting on an isolated system.
- **Complete positivity** guarantees the operation also works correctly when the system is part of a larger entangled system.

Complete positivity is a key requirement for quantum channels and is essential for maintaining the physicality of quantum states in open quantum systems.