Accelerator & Technology Sector
Beams Department
Accelerator Beam Physics Group

Particle Accelerators and Beam Dynamics

Foteini Asvesta

Summer Student Lectures 2025

Disclaimer

Based on:

- Y. Papaphilippou : "Introduction to Accelerators"
- Summer student lectures:
  - B. Holzer, V. Kain, and M. Schaumann
- CERN accelerator school (CAS):
  - F. Tecker: *"Longitudinal beam dynamics"*
  - S. Sheehy: *"Applications of accelerators"*
- Joint Universities Accelerator School (JUAS):
  - F. Antoniou, H. Bartosik and Y. Papaphilippou*: "Linear imperfections"* and *"nonlinear dynamics"*
- Books:
  - K. Wille: *"The Physics of Particle Accelerators"*
  - S.Y. Lee: *"Accelerator Physics"*
  - A. Wolski: *"Beam Dynamics in High Energy Particle Accelerators"*

Images: cds.cern.ch

Overview

I. Introduction to Accelerators
  - Applications
  - Accelerator types (historic overview)
  - Accelerator performance indicators & examples
  - Synchrotrons

II. Accelerator beam dynamics

III. CERN accelerator complex

Why accelerators?

World wide about ~30,000 particle accelerators are in operation with a large variety of applications.

Industry

- Material studies and processing
- ***Food sterilization***
- Ion implantation

'Cold pasteurization' – before packaging

The **large majority** is used in **industry** and **medicine:**

- Industrial applications: ~20,000*
- Medical applications: ~10,000*

*Sources:
A. W. Chao, World Scientific Revies of Accelerator Science and Technology
A. Faus-Golfe, The brave new world of accelerator application
APAE report, Applications of particle accelerators in Europe
S. Sheehy, Applications of accelerators, CAS 2014

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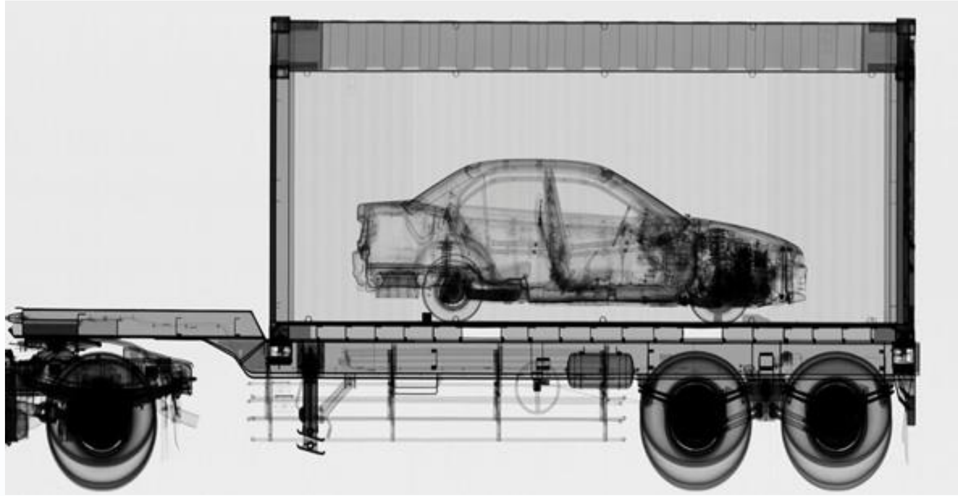
Security

- ***Airports & boarders***
- Nuclear security
- Imaging



- Cargo containers scanned at ports and border crossings.
- Accelerator-based sources of X-Rays can be far more penetrating (6MV) than Co-60 sources.
- Container must be scanned in 30 seconds.

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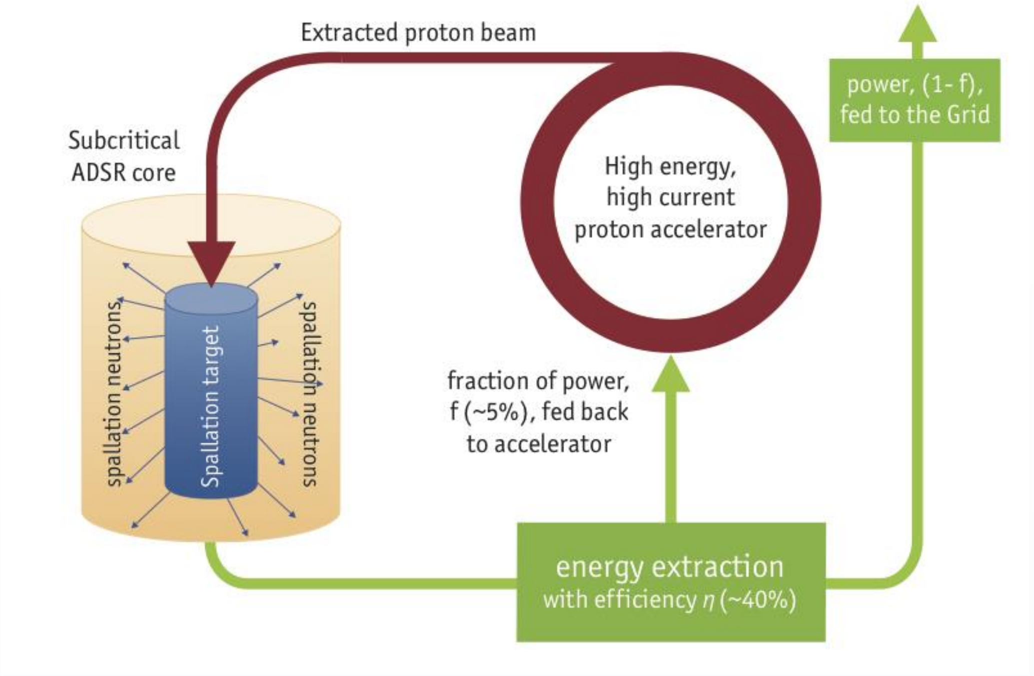
Energy

- Destroying radioactive waste
- ***Energy production***
- Nuclear fusion
- Thorium fuel amplifier



Accelerator Driven System (ADS)
Transmutation of nuclear waste isotopes or energy generation

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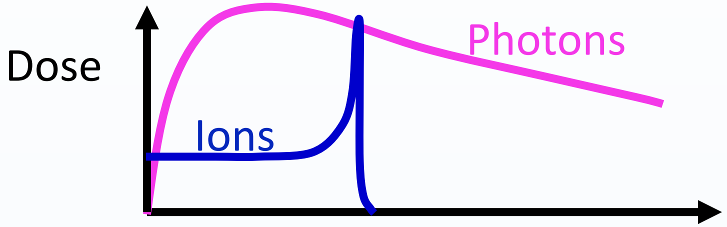
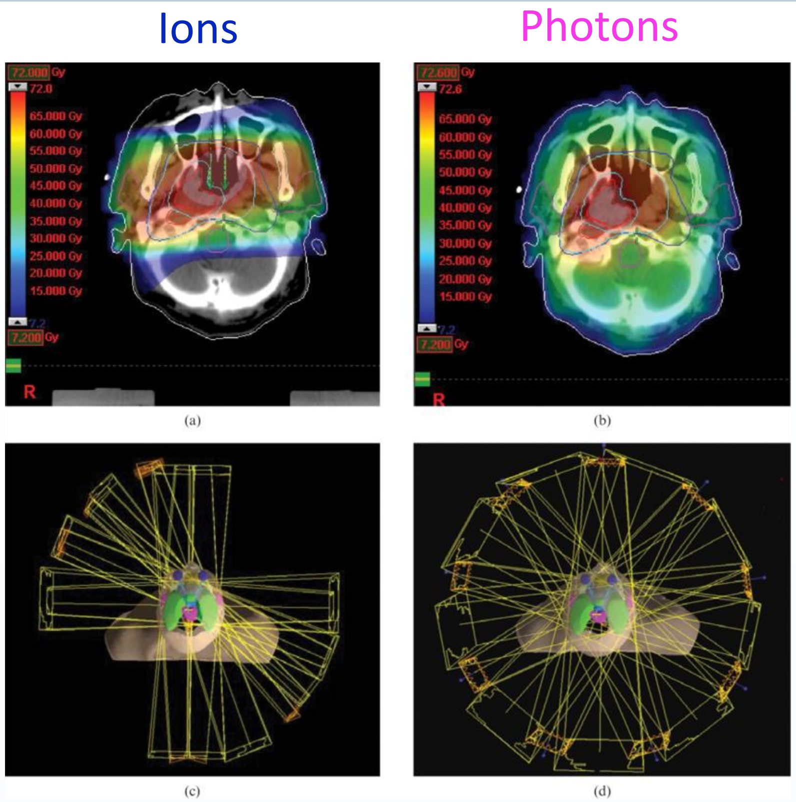
Health

- Diagnostic and imaging
- X-rays
- ***Cancer therapy***
- Radioisotope production





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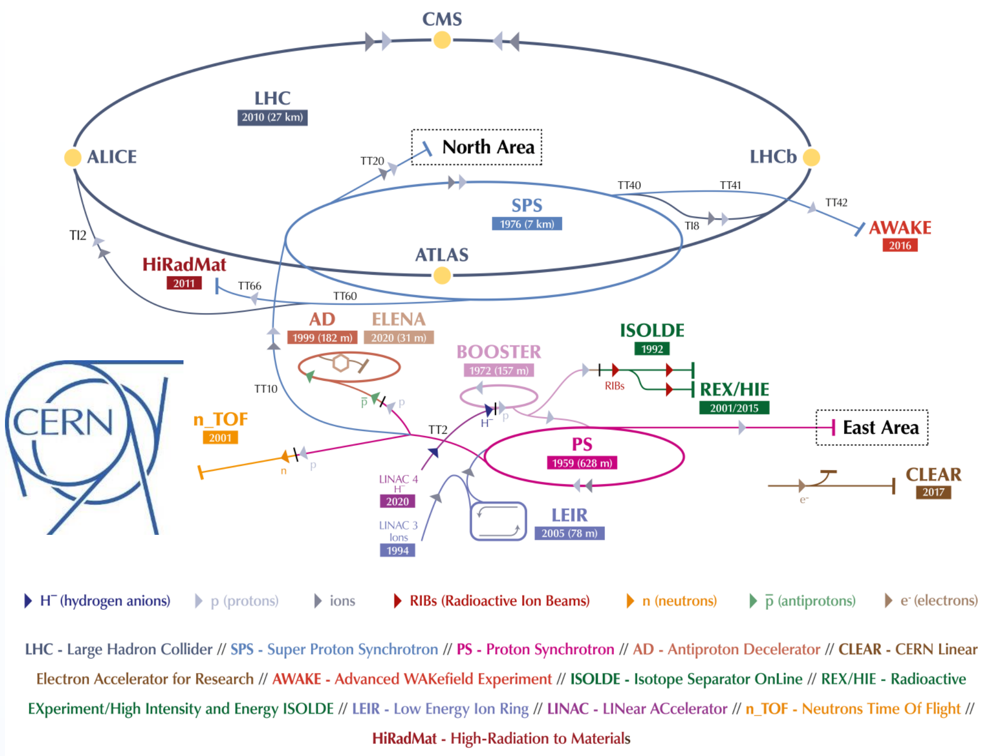
Why accelerators?

World wide about ~30,000 particle accelerators are in operation with a large variety of applications.

Less than a fraction of a percent is used for **Research!**

- **Particle Physics**
- Storage rings & Colliders
- Material science
- Light sources
- R&D



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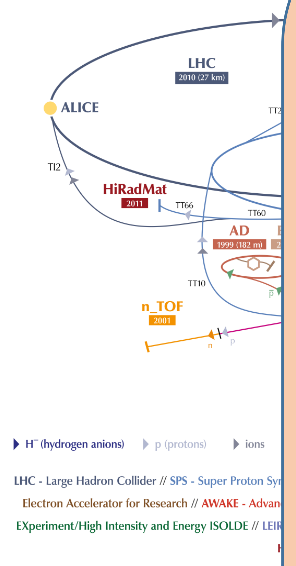
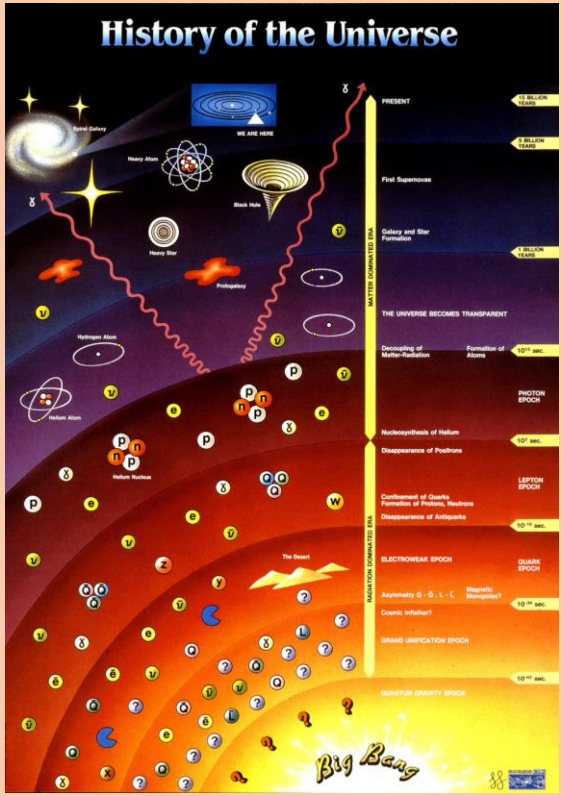
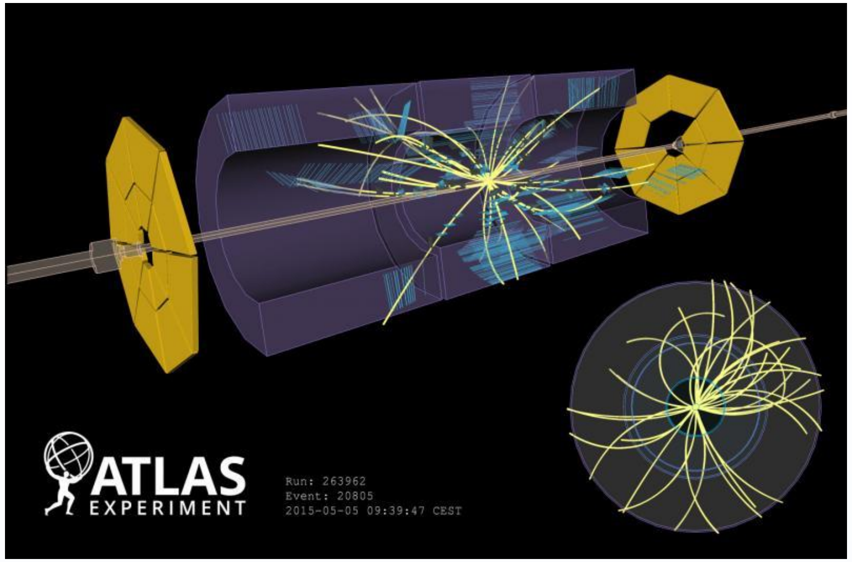
Accelerators at CERN

Full complex of Accelerators to give energy to particles





Understand the laws of physics and
Reveal the history of the universe



Particles produced in the collisions are observed in the detectors

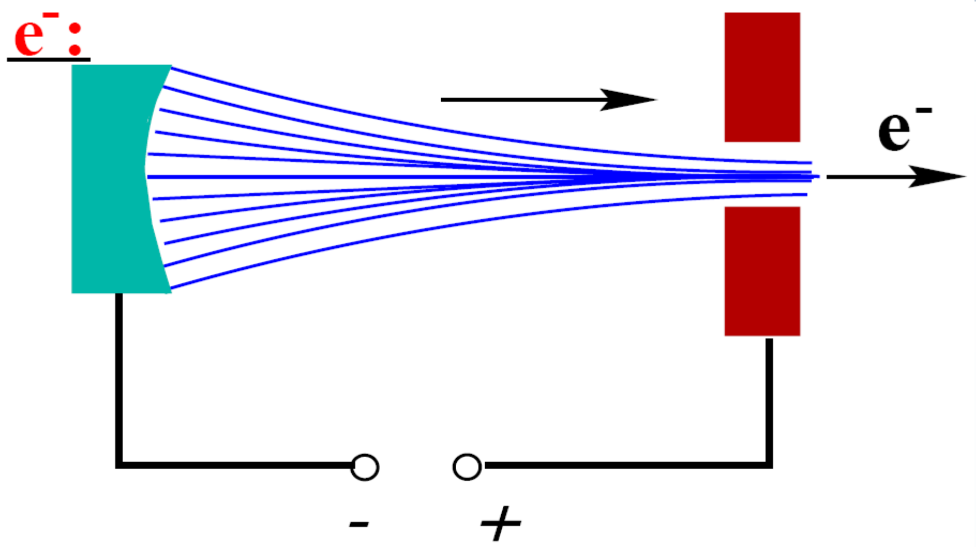
History of Accelerators

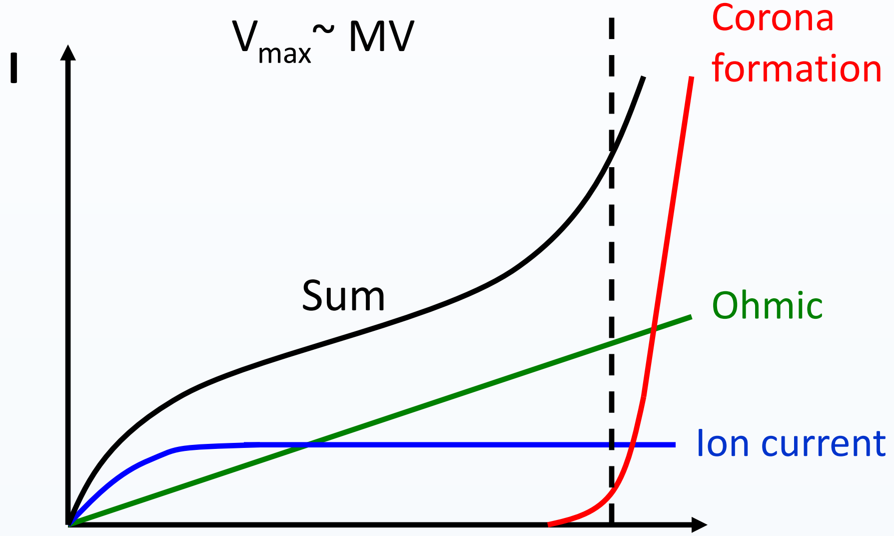
Race for higher energies

Electrostatic Accelerators

The simplest of Accelerators!
(cathode ray tubes – screens...)

- Particle source – **blue** electrode, acceleration in an electric fiels, exit – **red** electrode.
- Achieved energies depend on the applied voltage.
- Current increases exponentially for large voltages creating arcs and discharge
(*Corona formation*)



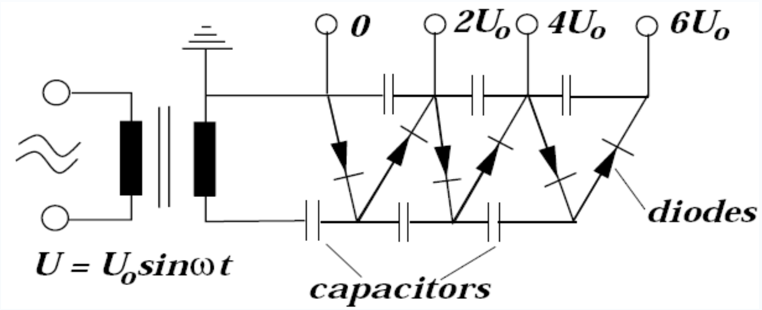


Voltage multipliers

Problem*:* Achieve higher voltages to push to higher energies

- **Cockcroft** and **Walton**(1932) developed a cascade generator based on multiple rectifiers
- Operating principle – **Greinacker circuit**
    - AC power supply
    - 2N diods (one-way current "switch") so that the maximum voltage on each couple of capacitors goes to $2V_0$, $4V_0$, $6V_0$, …,$2NV_0$
    - Voltages ~MV can be achieved for beams of ~100s of mA
- **Cockcroft** and **Walton** used such an accelerator to split lithium nuclei producing helium nuclei. (Nobel prize 1951)

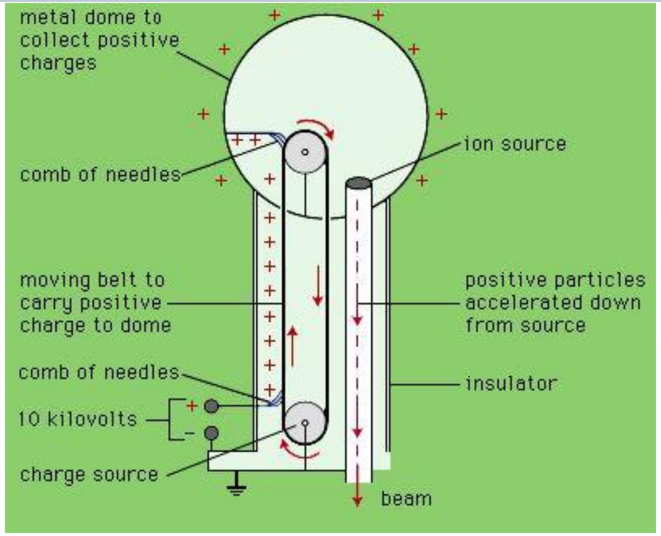
Greinacker Circuit

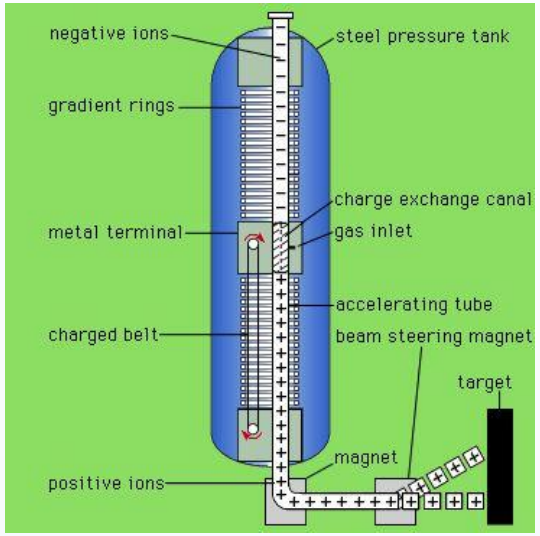


Fermilab cascade generator

Van de Graaff Generator (1930)

- Charges are accumulated through a moving belt charging the dome.
- Higher voltages can be achieved within a pressure tank – Paschen's law: Break down voltage depends on gas pressure & gap
- Possibility to double the voltage (Tandem)
  - Negative charge ions accelerated from 0 to V
  - Electrons absorbed from a gas and are accelerated again (from V to 0)

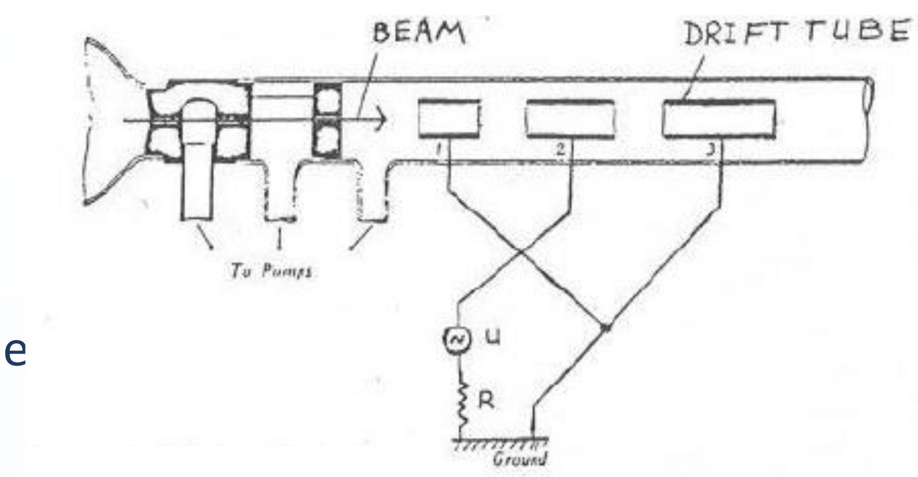





ROBERT VAN DE GRAAF DEMONSTRATES HIS FIRST GENERATOR TO KARL COMPTON

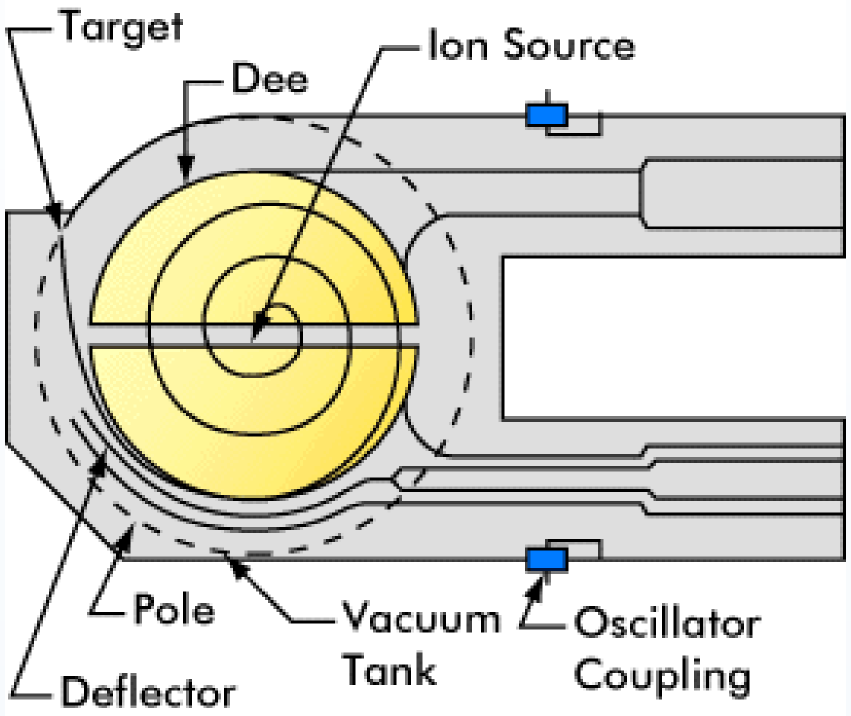
Linear accelerators (LINAC)

- **Ising's** Original idea (1924), first built by **Wideröe** (1928) and first high energy linac (1.3MeV) built by **Sloan** and **Lawrence** (1931)
- Line of drift tubes alternatingly connected to high frequency (RF) power supplies
- Particles accelerated in the gaps, but insulated in the tubes (no field – act as a Faraday cage)
- As the voltage changes sign, the particles are accelerated every time they enter a gap
- The length of the tubes, increases with acceleration for a given/constant frequency up to the relativistic limit
- Synchronization to the field is achieved via ***phase focusing***
- **Beams** (1933) first linac with waveguides. **Hansen** and **Varian** brothers (1937) invented the **klystron** (up to 10GHz)



Cyclotron

- **Lawrence's** and **Edelfsen's** original idea (1930), first built by **Lawrence** and **Livingston**(1932)
- Constant ***magnetic field B*** from an H-shaped magnet with a cyclotron frequency and a radius that increases with velocity, for non-relativistic particles: $\omega_c = qB/m$ (*spiral orbits*)
- The accelerating voltage is synchronous to the particles crossing the gap: $\omega_{RF} = (2n+1)\omega_c$
- Heavy particles accelerated up to ~20 MeV
- For higher energies (relativistic particles) the frequency reduces with the mass.
- **Synchro-cyclotron principle** (**McMillan** and **Veksle**r, 1945): $\omega_{RF} \propto 1/\gamma$ → different frequencies for different particle species | compensation for relativistic effects
- **Isochronous cyclotron principle**: $\omega_{RF} \propto \mathrm{B}/\gamma$ → Magnetic field increases with radius | Energies up to 600 MeV – prone to losses (field errors)

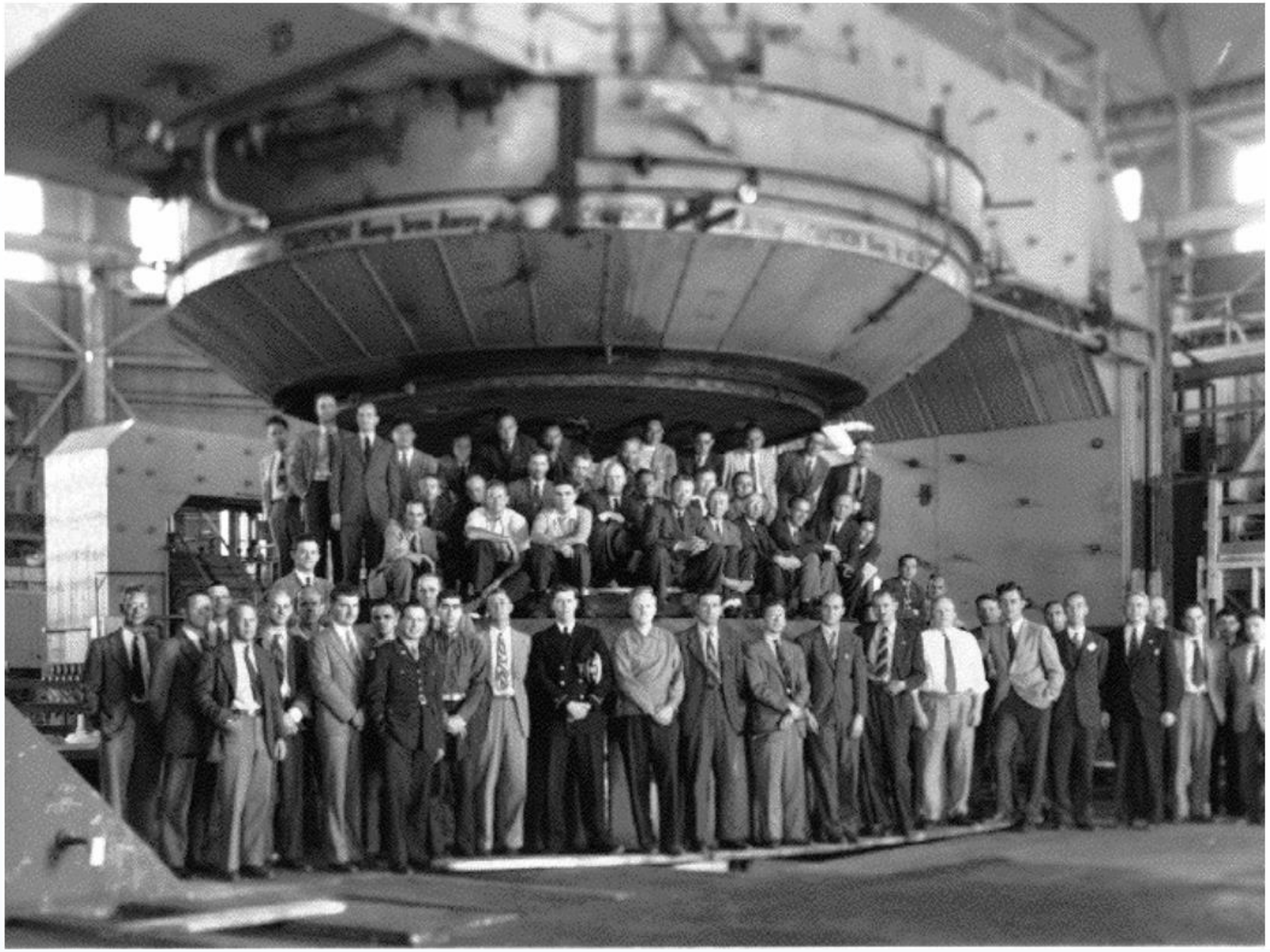


Cyclotron

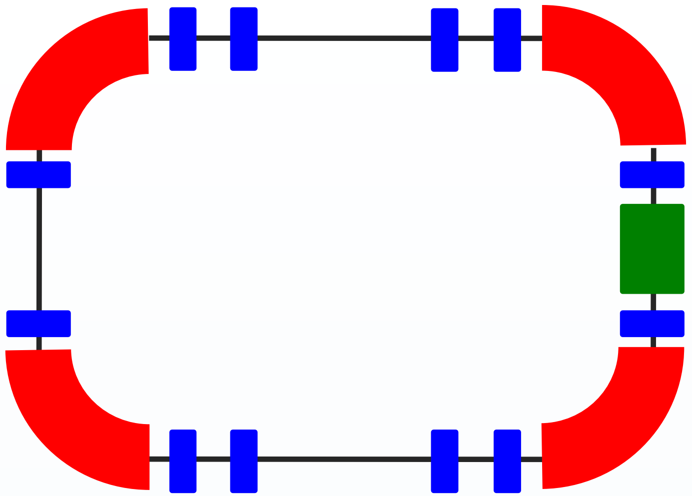
184-inch cyclotron:

1 single dipole with 467 cm diameter

Berkeley campus, 1942



Synchrotron



Could we further push the energy?

Colliders

- Two beams circulating at the Synchrotron's energy
- The beams are brought to collision

✓ energy at the *centre of mass* gets **double**

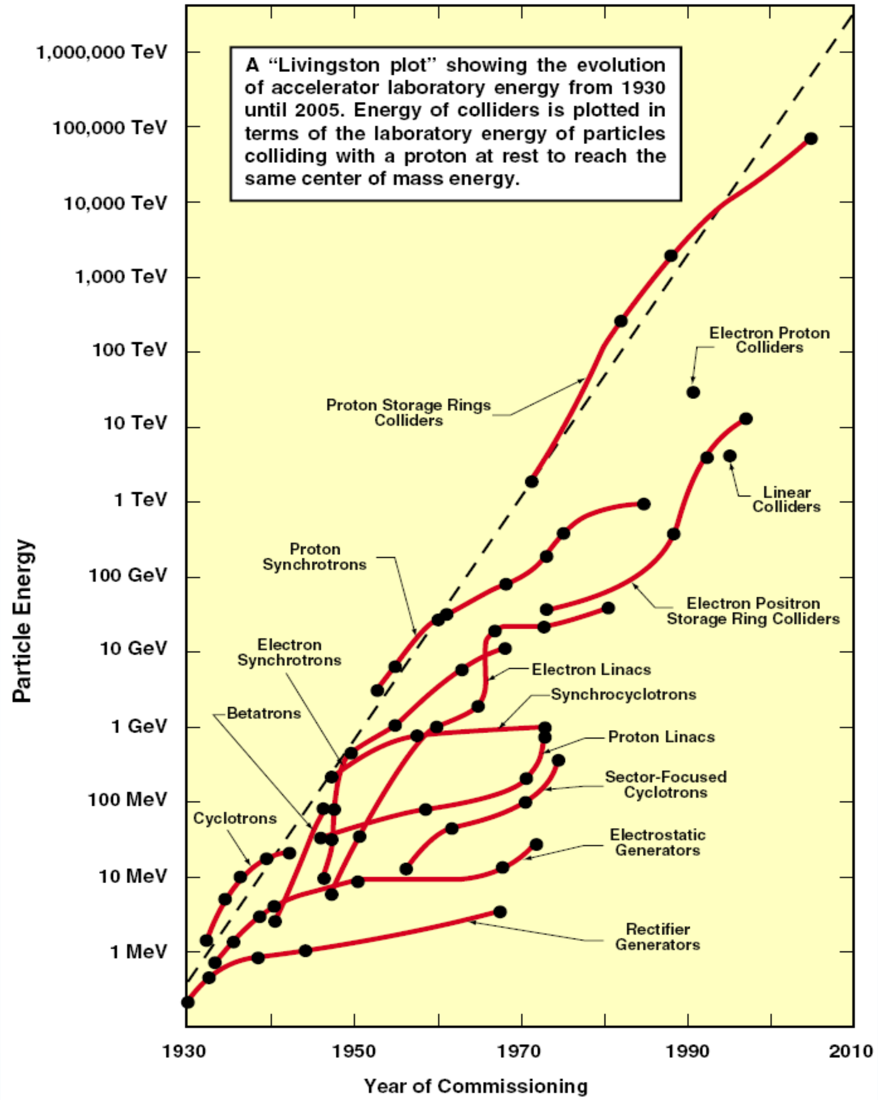
Developed in the 1950s | The CERN PS built in 1959 is still in operation!

Fixed trajectory: R = constant

Magnets only in the vicinity of the beam

- Electric fields used to **accelerate** and magnetic fields to steer the beam (**bending** & **focusing**)
- Magnetic field increases synchronously with the beam energy keeping the radius fixed!
- Beam rigidity: $B\rho = \frac{p}{q}$

Livingston Plot – evolution of energy reach



- The **Livingston** plot shows an exponential increase of energy with time
- Energy is increased by one order of magnitude each 6-10 years
- New technologies replace the old ones to achieve higher energies, until saturation. By then new technological advancements allow replacing the existing ones
- *And the process continues…*

- Energy is not the only relevant figure of merit:
  - Beam intensity
  - Beam emittance (size)

Accelerators and performance indicators

The design of an accelerator focuses on ***high performance***

- **Colliders – high** energy physics
  - **Luminosity:** *indicatory of the event production rate*
    - N_b # of particles per bunch
    - k_b # of bunches
    - $\gamma = E/(m_0 c^2)$ Lorentz factor
    - ε_n normalized emittance
    - β* betatron amplitude at interaction point
- **Spallation sources** – target experiments
  - **Average beam power**
    - $\bar{I}$ average current
    - **E** energy
    - f_n repetition rate
    - **N** # of particles per pulse
- **Synchrotron radiation sources** – spectroscopy
  - **Brightness:** *photon density*
    - N_b # of photons
    - $\varepsilon_{x,y}$ horizontal and vertical emittance

Energy

$$L = \frac{N_b^2 k_b \gamma}{4\pi\epsilon_n \beta^*}$$

Intensity

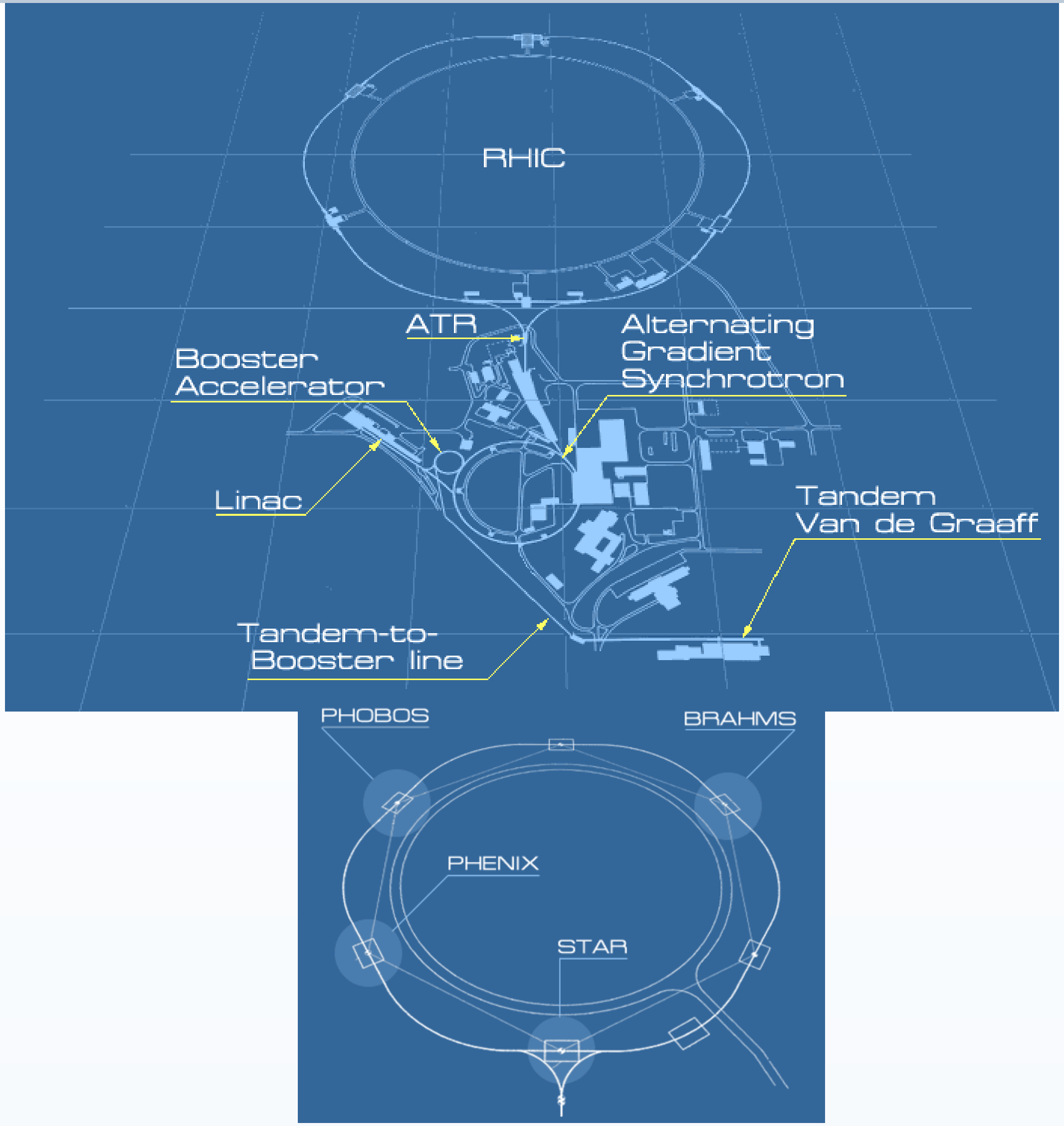
$$\bar{P} = \bar{I}E = f_N N e E$$

Beam size

$$B = \frac{N_p}{4\pi^2 \epsilon_x \epsilon_y}$$

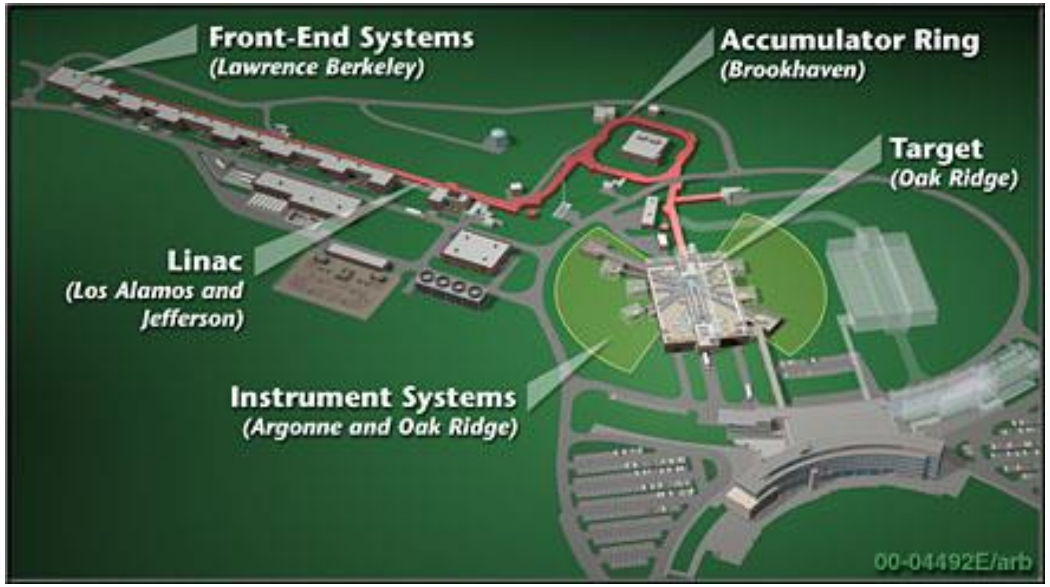
Relativistic Heavy Ion Collider (RHIC - BNL)

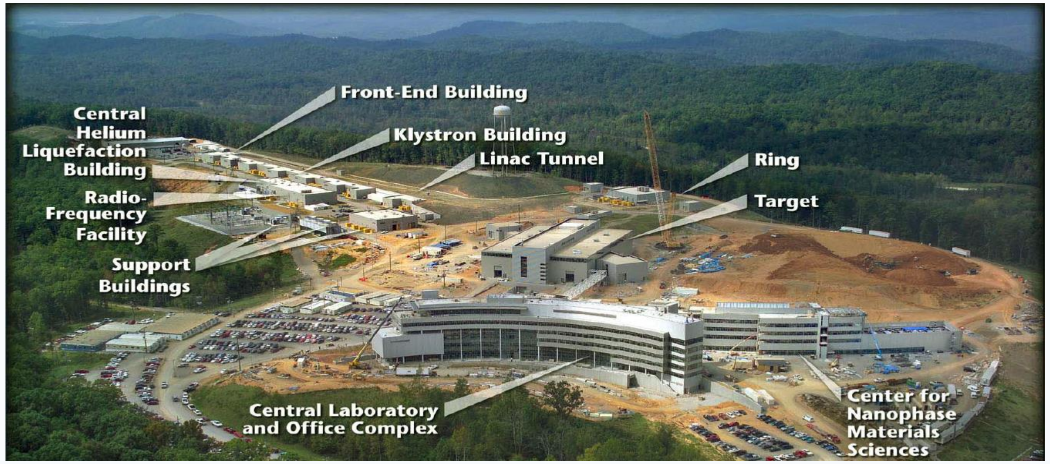
- **Ion collider** (gold, copper and polarized protons) with energies up to 100 GeV/u
- The beams are counter-rotated in a 2.4 mile (**~4km**) storage ring driven by 1740 superconducting dipoles
- The beams collide at 6 points in 4 of which the detectors of the **4 main experiments** (BRAHMS, PHENIX, PHOBOS, STAR) are placed
- The main purpose of the accelerator is the **production, detection and study of quark - gluon plasma**



Spallation Neutron Source (SNS - ORNL)

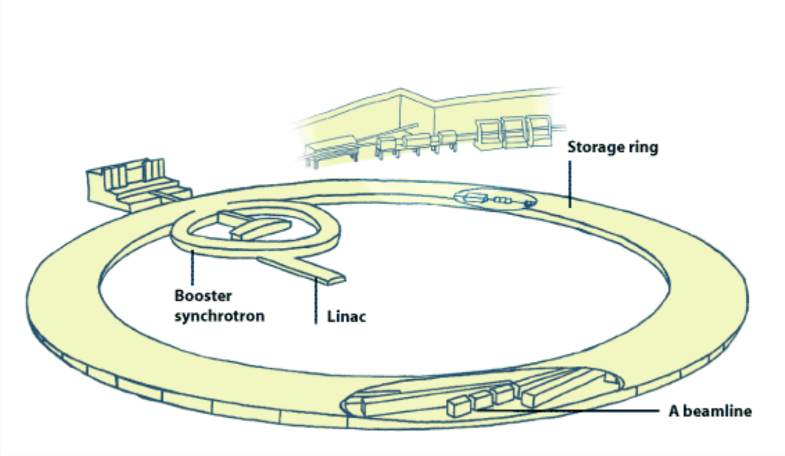
- Collaboration project of **6 laboratories** (LBNL, LANL, JLAB, BNL, ANL, ORNL)
- Spallation Neutron Source with a power of 1.4 MW
- The complex includes an **H^- source**, a 300m **linear accelerator**, with superconducting RF cavities, a proton **accumulator ring** with a perimeter of 248m and a liquid mercury **target** for the production of neutrons.
- The main purpose is neutron scattering **spectroscopy experiments at 24 stations** (magnetic structure of materials, nanotechnology, etc.)



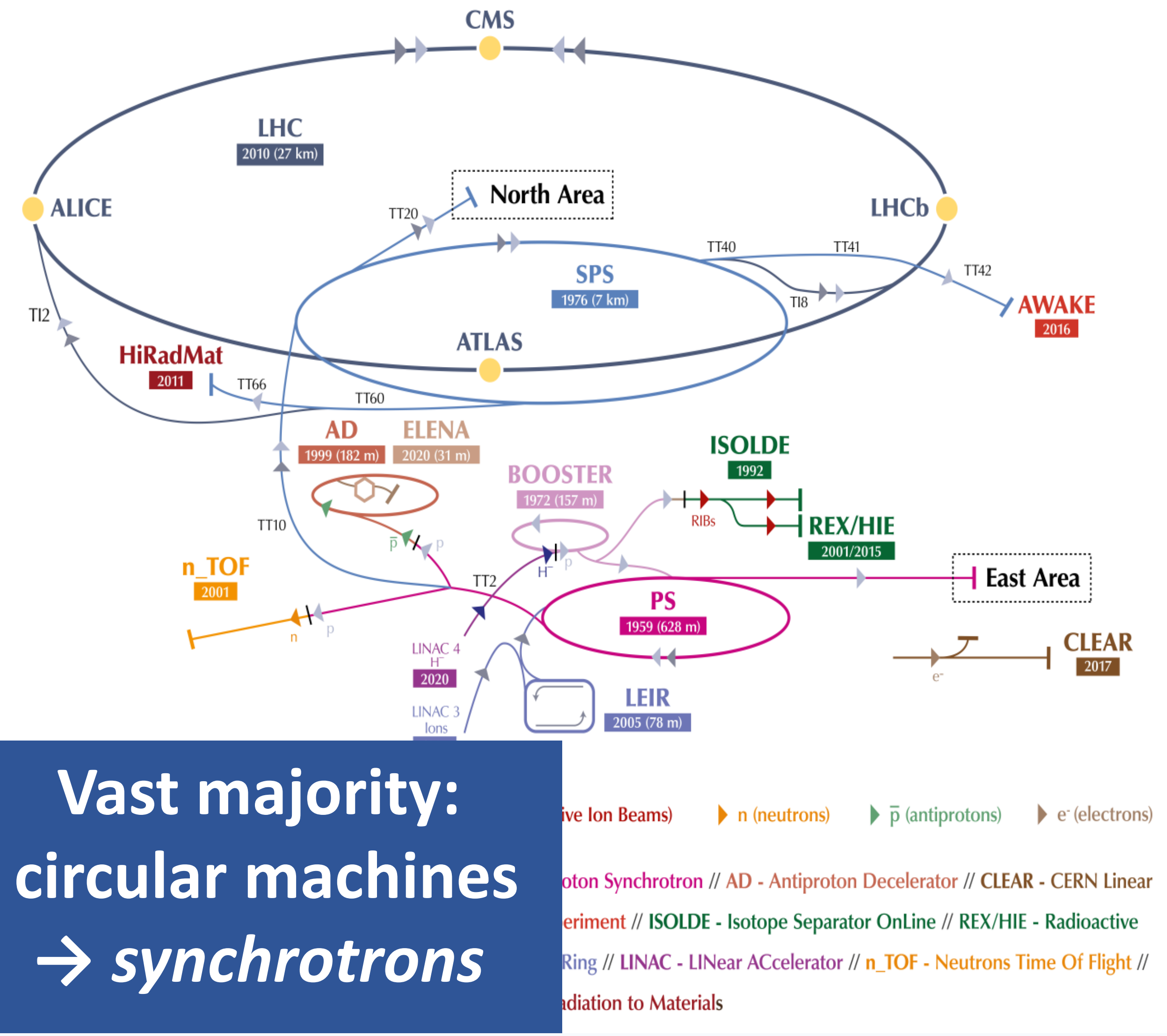


European Synchrotron Radiation Facility (ESRF)

- The **first and brightest** 3rd generation synchrotron radiation source in Europe
- **50 experimental beamlines** using "hard" X-rays produced by interfering magnetic elements (magnetic amplifiers and oscillators) and dipole magnets
- **3500 users/year** from 14 member states perform X-ray spectroscopy experiments for materials science, chemistry, biology, geology, medicine, archaeometry, etc.
- The complex includes a **linear electron accelerator**, a 300-meter booster **synchrotron** and an 844-meter **storage ring**.
- The storage ring shows **record availability of 98%** with an average time between outages of more than 2 days.



CERN Accelerator Complex



Vast majority: circular machines → *synchrotrons*

CERN Proton chain

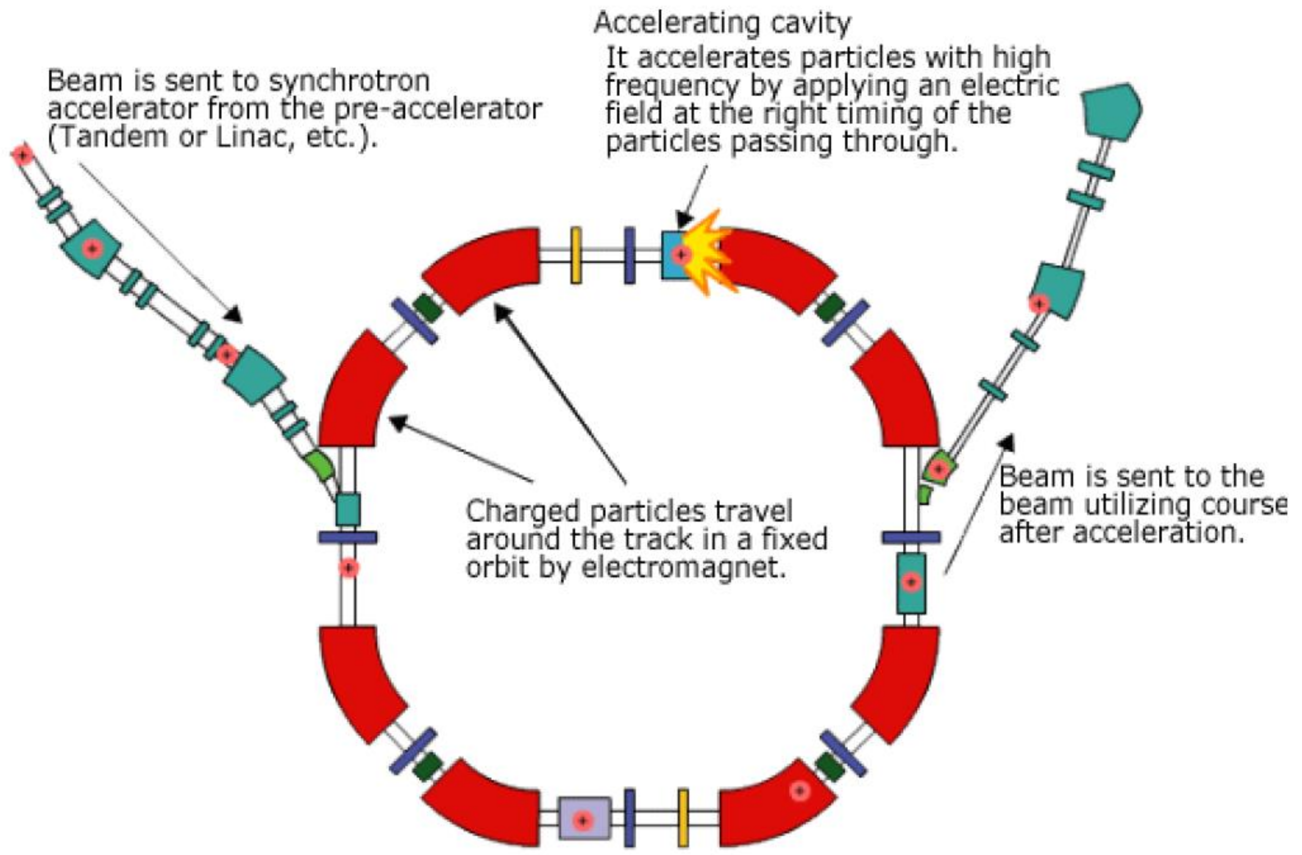
1. **LINAC-4** 160MeV (H-)
2. **Proton Synchrotron Booster** 2GeV
3. **Proton Synchtrotron** 26GeV
4. **Super Proton Synchrotron** 450 GeV
5. **Large Hadron Collider** 7Tev

CERN Ion chain

1. **LINAC-3**
2. **Low Energy Ion Ring**
3. **Proton Synchrotron**
4. **Super Proton Synchrotron**
5. **Large Hadron Collider**

Other facilities & experiments: n_TOF, ISOLDE, East Area, North Area, HiRadMat, AWAKE, CLEAR (electrons), AD & ELENA (Antiprotons)

Main principles of a Synchrotron



The beam needs to be controlled to allow for:

- Long storage times
- Beam quality preservation

Focusing allows better control:

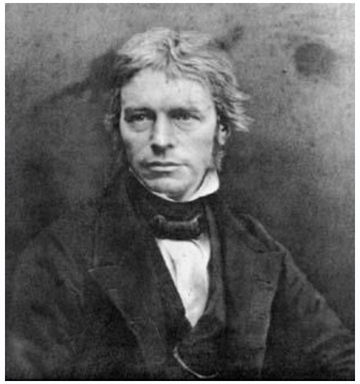
- ***Phase focusing*** **| RF cavities**
- ***Weak focusing*** **| Dipoles**
- ***Strong focusing*** **| Quadrupoles**

Maxwell's equations for electromagnetism

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

Gauss law for electricity

electric field diverges from electric charges



$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}$$

Faraday's law of induction

changing magnetic fields produce electric fields

$$\nabla \cdot \mathbf{B} = 0$$

Gauss law for magnetism

no isolated magnetic poles

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial}{\partial t}\mathbf{E}$$

Ampere-Maxwell law

changing electric fields and currents produce circulating magnetic fields

Lorentz force – *acceleration*

- Force acting on **charged particles** moving under the influence of **electromagnetic fields**

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

or

$$\frac{d\mathbf{p}}{dt} = \frac{d}{dt}(m_0\gamma\mathbf{v}) = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

- The **kinetic energy** (*T*) and the rate of the energy change can be evaluated as:

$$\Delta T = \int \mathbf{F}ds = q\int \mathbf{E}ds + q\int \cancel{(\mathbf{v} \times \mathbf{B})\mathbf{v}dt}$$

$$\frac{dT}{dt} = \mathbf{v}\cdot\mathbf{F} = q\mathbf{v}\cdot(\mathbf{E} + \cancel{\mathbf{v} \times \mathbf{B}}) = q\mathbf{v}\cdot\mathbf{E}$$

- *Magnetic fields do not contribute to the energy change!*

→ ***Acceleration*** can be achieved only by the **electric field**

Lorentz force – *steering*

- Force acting on **charged particles** moving under the influence of **electromagnetic fields**

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

or

$$\frac{d\mathbf{p}}{dt} = \frac{d}{dt}(m_0\gamma\mathbf{v}) = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

- For a particle moving on the longitudinal direction, the **horizontal component** of the Lorentz force yields:

$$\frac{dp_x}{dt} = \mathbf{F_x} = q(E_x - v_z B_y)$$

- Both electric and magnetic fields can contribute to the steering, ***HOWEVER***
- For high energies (approaching the relativistic limit): $\upsilon_z \approx c$ & $\upsilon_z B_y >> E_x$
    - *1 T corresponds to 300 MV/m*

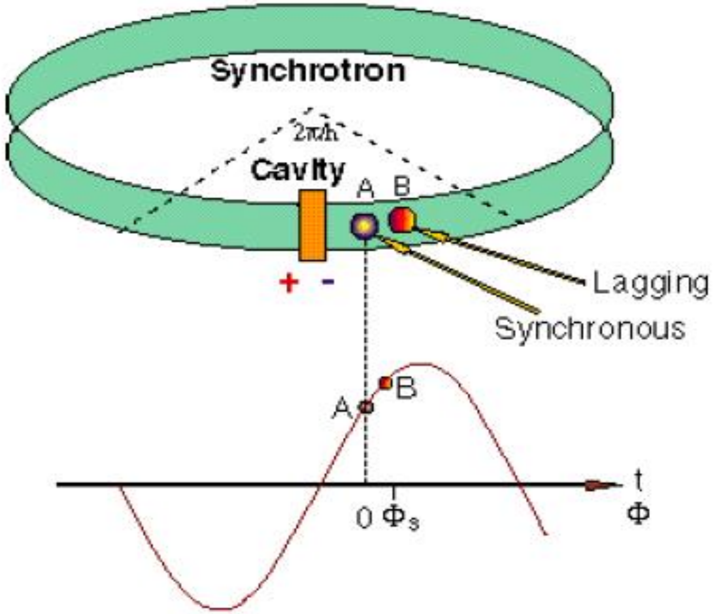
→ **Magnetic fields** much more efficient for ***steering***

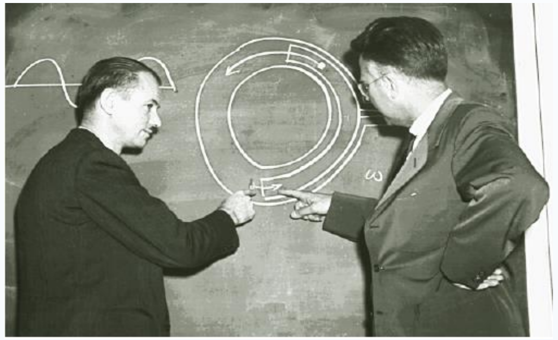
Phase focusing

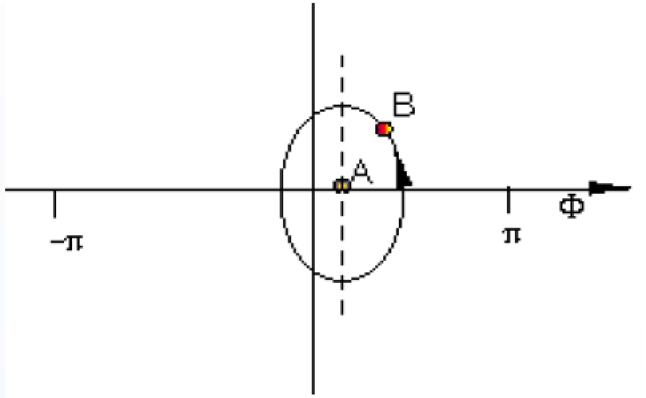
Description applies for low energies !!
More on Monday

- Developed independently by **McMillan** and **Veksler** (1945)
- The **RF cavity** is set such as the particle at the centre of the **bunch** (**synchronous** particle) receives the needed energy
- Voltage in the cavity: $V = V_0 \sin(2\pi\omega_{RF} t) = V_0 \sin(\varphi(t))$
- For no acceleration, synchronous particle phase: $\varphi_s = 0$
- For acceleration, synchronous particle phase: $0<\varphi_s<\pi$ in order to achieve: $\Delta E = V_0 \sin(\varphi(t))$
- Particles arriving **late:** $\varphi > \varphi_s$, → Energy increase ***larger*** than the synchronous particle
- Particles arriving **early:** $\varphi < \varphi_s$, → Energy increase ***smaller*** than the synchronous particle

→ Particles are grouped – ***bunches!***



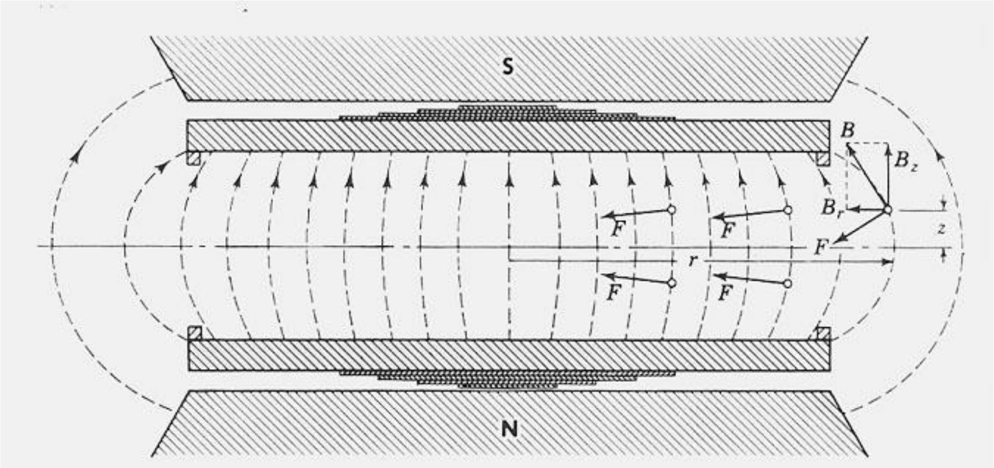




Weak focusing

- Particles entering transversely into a homogenous magnetic field follow circular orbits
- Magnet errors can cause the particles to drift until they get lost

→ A recovering or "***focusing***" force is needed!



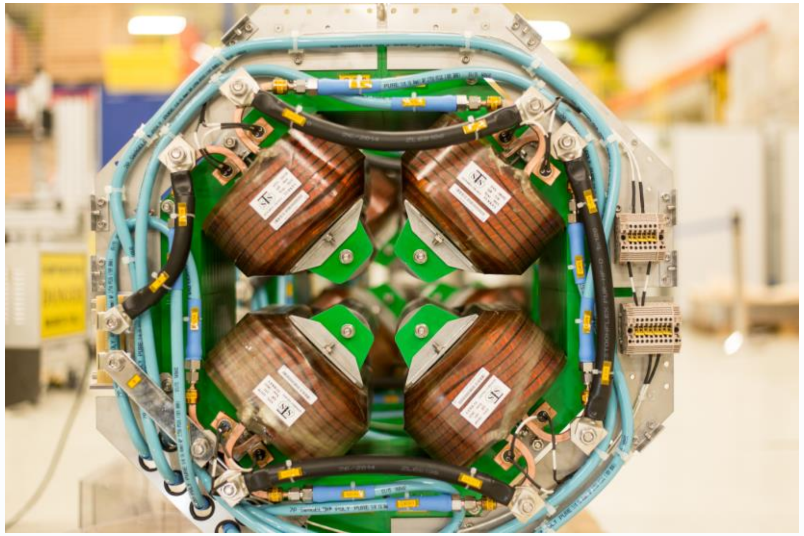
- Such a **focusing** is introduced at the edges of the magnet

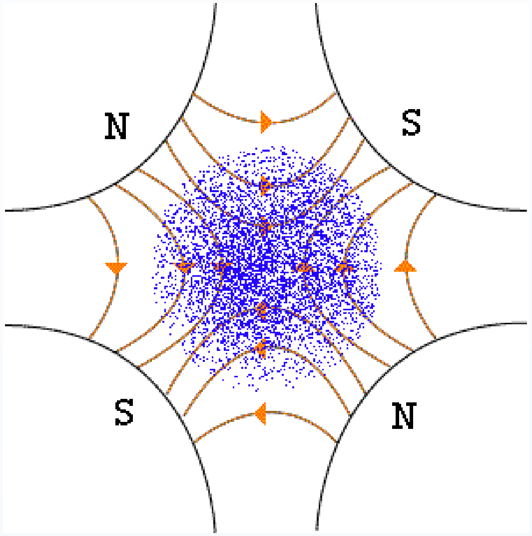
  – due to the disruption of the magnetic field

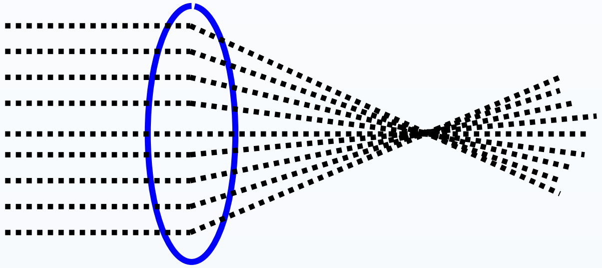
- The transverse components of the magnetic field: $\left(B_x + B_y\right) = B_0\left(-n\frac{y}{r}, 1 - n\frac{x}{r}\right)$, with $n = -\frac{r}{B_0}\frac{\partial B_y}{\partial x}$
- Particles perform linear harmonic oscillations (**betatron**) with frequencies: $\omega_x = \frac{v}{R}\sqrt{1-n}, \omega_y = \frac{v}{R}\sqrt{n}$
- For stable oscillations, **Steenbeck's** condition: 0<n<1

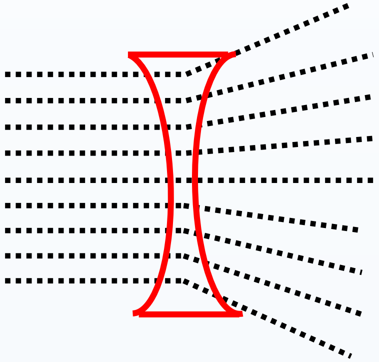
Strong focusing

- Principle developed independently by **Christofilos** (1950) and **Courant, Livingston and Snyder** (1953)
- **No fields** can have a focusing effect in both transverse planes of motion.
- Focusing elements (quadrupoles): act as **focusing in one plane** but **defocusing in the other**
- ***A sequence of such focusing and defocusing fields can give an overall strong focusing***
- The force is proportional to the distance from the axis of the beam
- A succession of ***focusing and defocusing elements*** allow the particles to follow **stable trajectories**, performing small betatronic oscillations around the circular periodic orbit





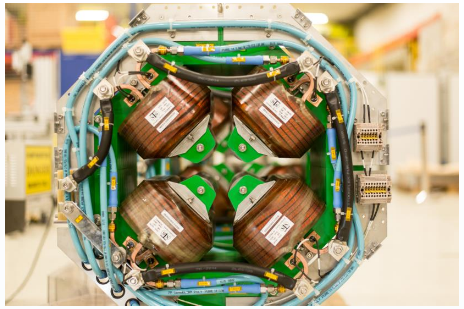


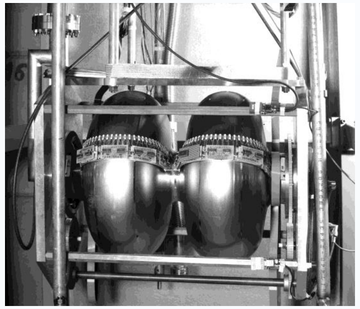


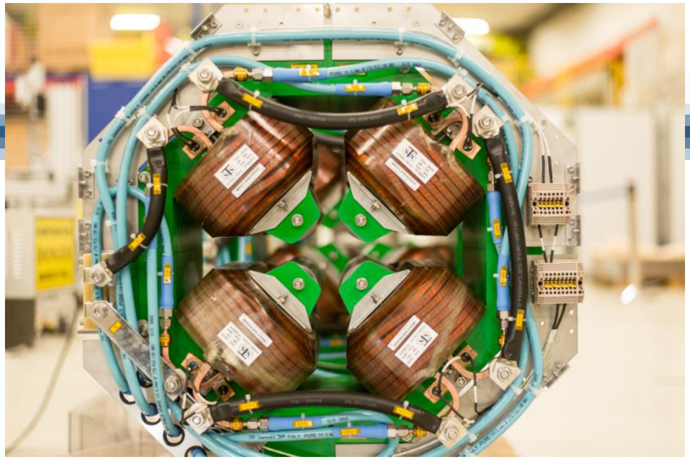
Building Blocks of a Synchrotron

Main components:

Dipole Magnets:	Bending
Quadrupole Magnets:	(De-)Focusing
Higher order magnets:	Corrections
RF cavities:	Acceleration

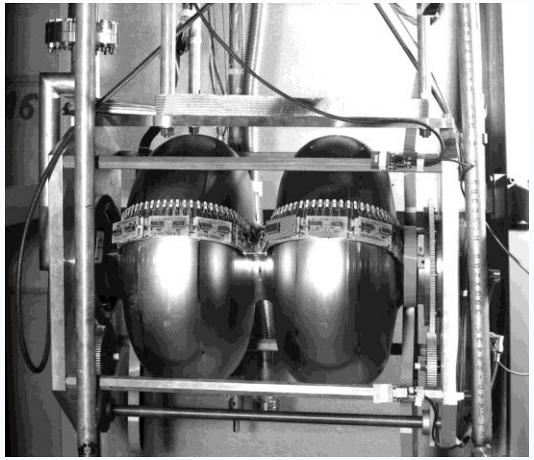






How do particles move under the influence of these elements?

→ Transverse & Longitudinal Beam Dynamics



Why accelerators?

Industry
- Material studies and processing
- ***Food sterilization***
- Ion implantation

Security
- ***Airports & boarders***
- Nuclear security
- Imaging

Energy
- Destroying radioactive waste
- ***Energy production***
- Nuclear fusion
- Thorium fuel amplifier

World wide about ~30,000 particle accelerators are in operation with a large variety of applications.

Health
- Diagnostic and imaging
- X-rays
- ***Cancer therapy***
- Radioisotope production

Less than a fraction of a percent is used for ***Research!***
- **Particle Physics**
- Storage rings & Colliders
- Material science
- Light sources
- R&D

The **large majority** is used in **industry** and **medicine:**

- Industrial applications: ~20,000*
- Medical applications: ~10,000*

*Sources:
A. W. Chao, World Scientific Revies of Accelerator Science and Technology
A. Faus-Golfe, The brave new world of accelerator application
APAE report, Applications of particle accelerators in Europe
S. Sheehy, Applications of accelerators, CAS 2014