Beam Extraction and Transport

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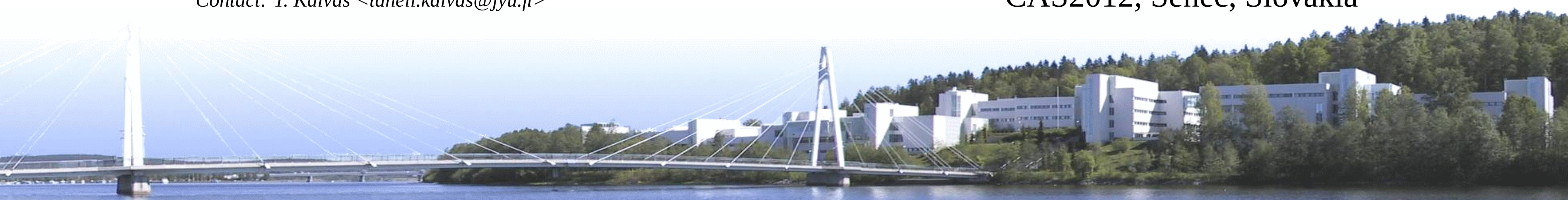
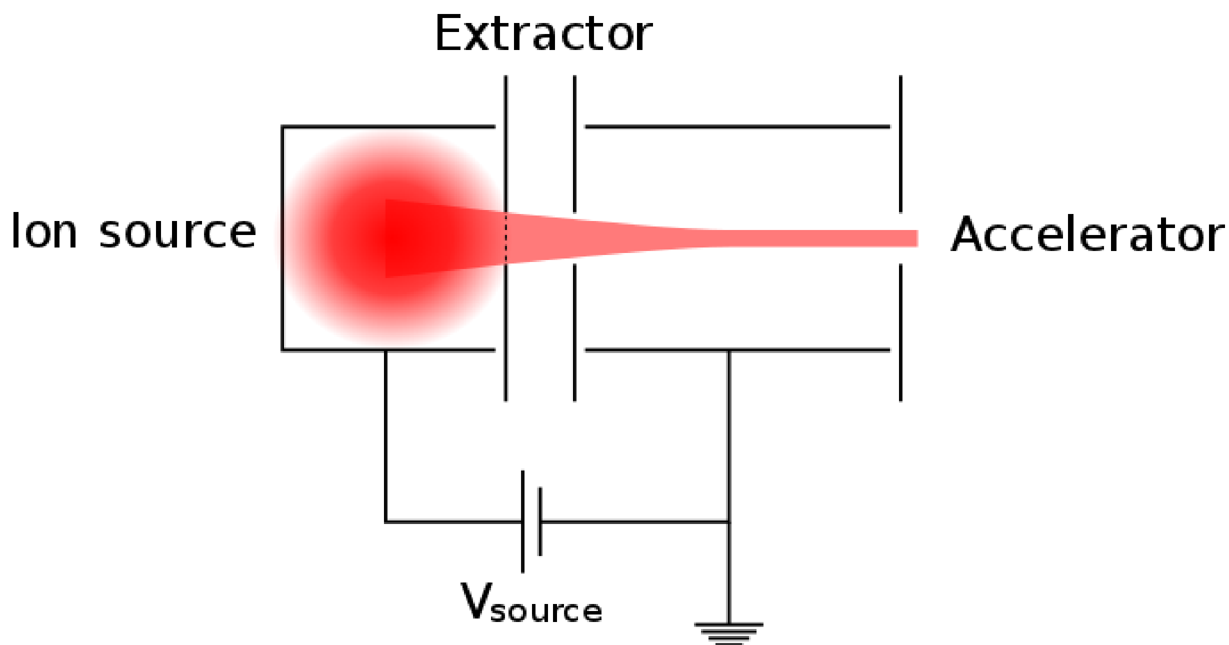


Presentation outline

- Introduction to ion source extraction systems
- Emittance
- Low energy beam transport
  - Matrix codes
  - Trajectory tracing codes
  - Beam line elements
  - Space charge, beam potential and compensation
- Beam extraction from plasma
  - Child-Langmuir law
  - Pierce angle
  - Plasma sheath models for positive and negative ions
- Examples

Basic beam extraction and transport

The extractor takes the plasma flux $J = \frac{1}{4} q n \bar{v}$ and forms a beam with energy $E = q(V_{\text{source}} - V_{\text{gnd}})$ transporting it to the following application.



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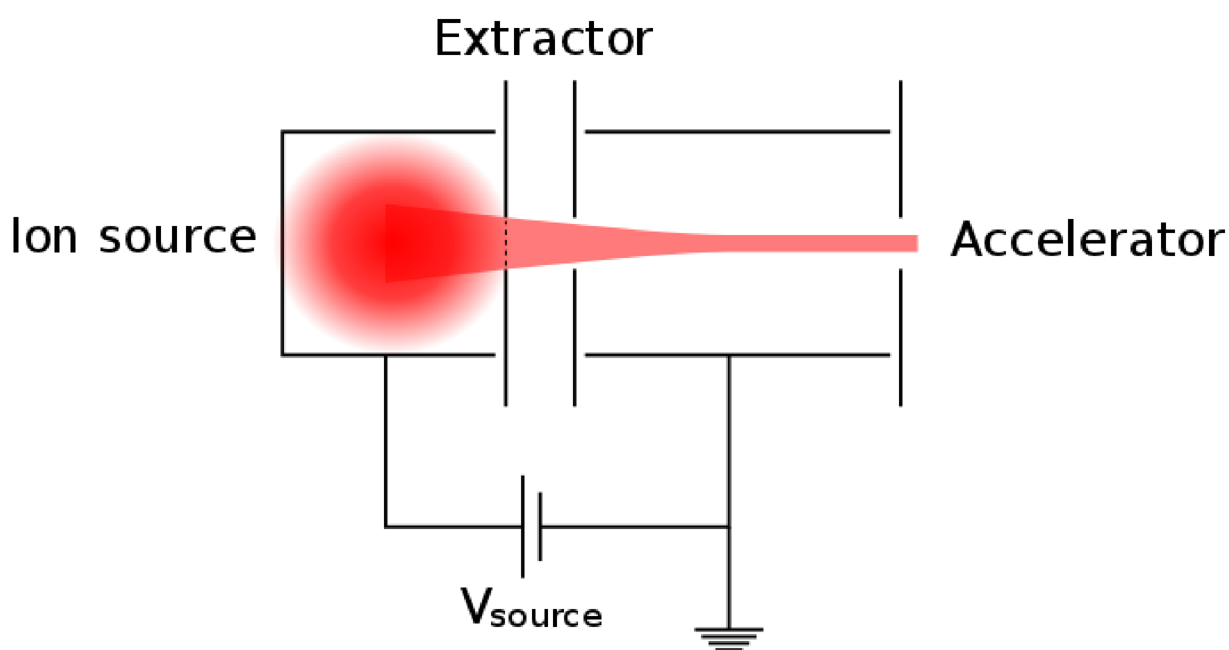

Simple?

Extraction complications

- Plasma-beam transition physics
  - Plasma parameters: density, potential, temperature, etc
  - Beam intensity, quality, uniformity, species
- Application requirements for beam spatial and temporal structure
  - Need for focusing, chopping, etc
- Space charge
- Practical engineering constraints
  - Space for diagnostics, pumping, etc
  - Materials, power supplies, money

Emittance

Emittance

Traditionally the emittance is defined as the 6-dimensional volume limited by a countour of particle density in the (x, p_x, y, p_y, z, p_z) phase space. This volume obeys the Liouville theorem and is constant in conservative fields.

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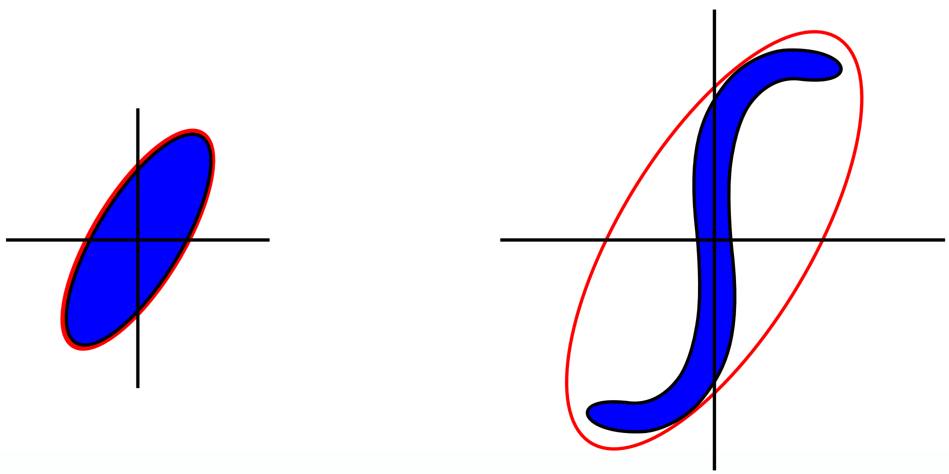
With practical accelerators a more important beam quality measure is the volume of the elliptical envelope of the beam bunch. This is not conserved generally — only in the case where forces are linear.



Transverse emittance

The transverse emittances are 4 and 2-dimensional reductions of the 6-dimensional definition, usually assuming that p_z is constant and replacing p_x with $x' = p_x/p_z$ and p_y with $y' = p_y/p_z$. The 2D emittance ellipse then becomes

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \epsilon_x,$$

where scaling

$$\beta\gamma - \alpha^2 = 1$$

is chosen. The ϵ_x is the product of the half-axes of the ellipse (A/π) and α, β and γ are known as the Twiss parameters defining the ellipse orientation and aspect ratio.

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Because of the connection between the area of the ellipse and ϵ there is confusement on which is used in quoted numbers. Sometimes π is included in the unit of emittance (π mm mrad) to emphasize that the quoted value is not the area, but the product of half-axes as defined here.

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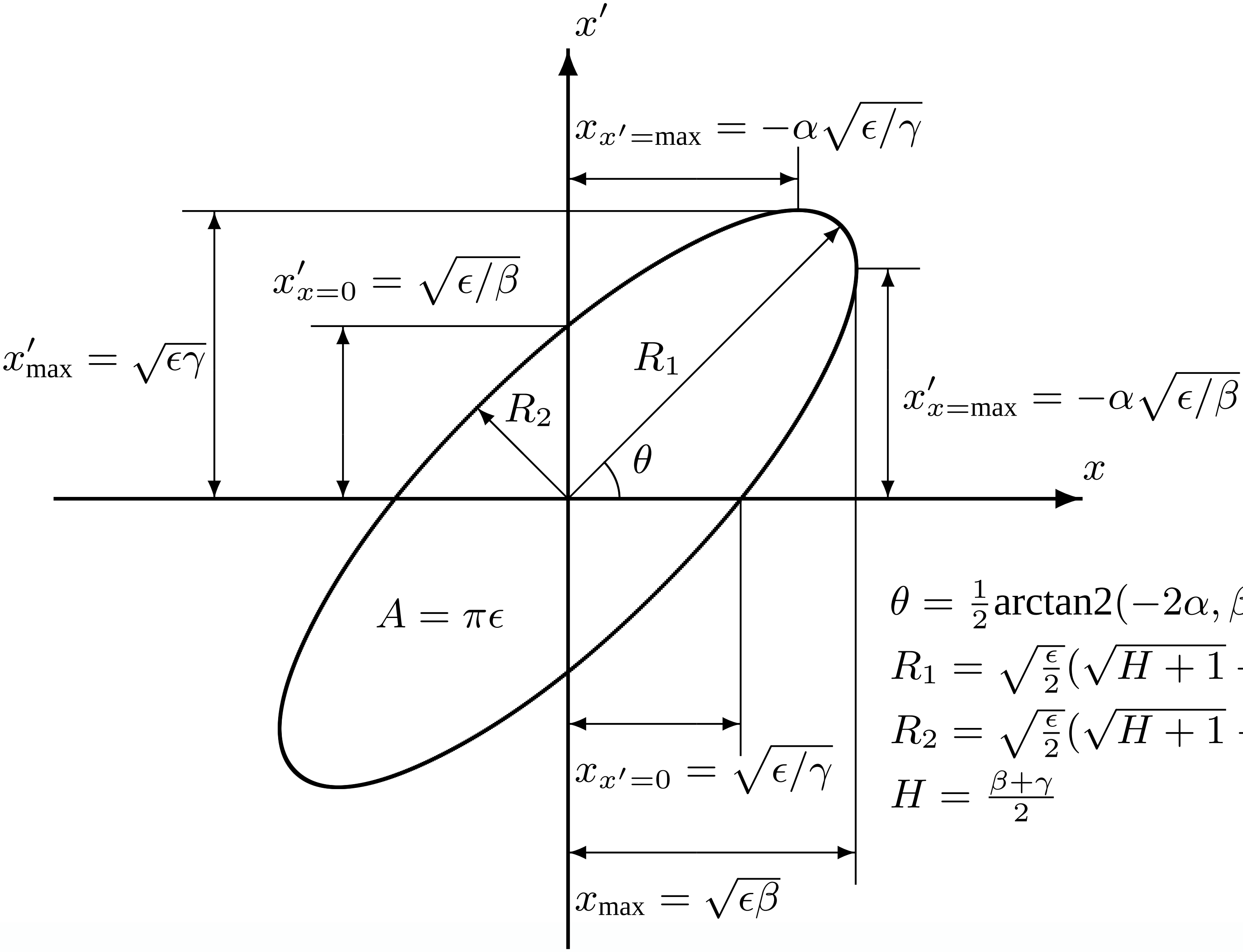
Ellipse geometry

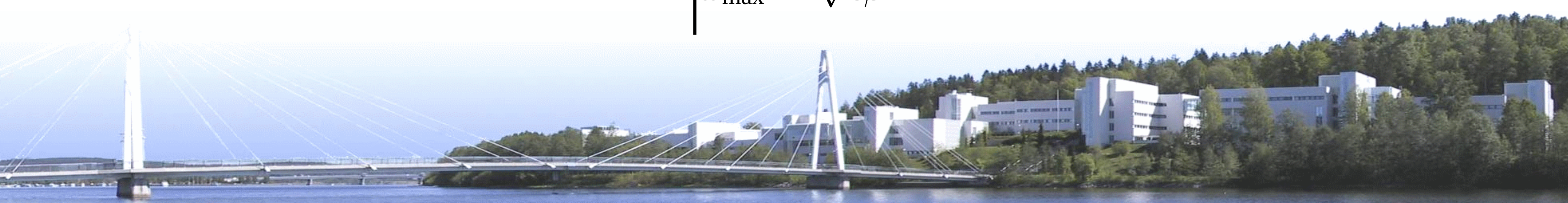


$$\theta = \frac{1}{2}\text{arctan2}(-2\alpha, \beta - \gamma)$$

$$R_1 = \sqrt{\frac{\epsilon}{2}}(\sqrt{H+1} + \sqrt{H-1})$$

$$R_2 = \sqrt{\frac{\epsilon}{2}}(\sqrt{H+1} - \sqrt{H-1})$$

$$H = \frac{\beta+\gamma}{2}$$



Emittance envelope

How to define the "envelope"?

Numerous algorithms exist for defining the ellipse from beam data. Often a minimum area ellipse containing some fraction of the beam is wanted (e.g. $\epsilon_{90\%}$). Unfortunately this is difficult to produce in a robust way.

A well-defined way for producing the ellipse is the rms emittance:

$$\epsilon_{\text{rms}} = \sqrt{< x'^2 >< x^2 > - < xx' >^2},$$

and similarly the Twiss parameters

$$\begin{aligned} \alpha &= -\frac{< xx' >}{\epsilon}, \\ \beta &= \frac{< x^2 >}{\epsilon}, \\ \gamma &= \frac{< x'^2 >}{\epsilon}, \end{aligned}$$

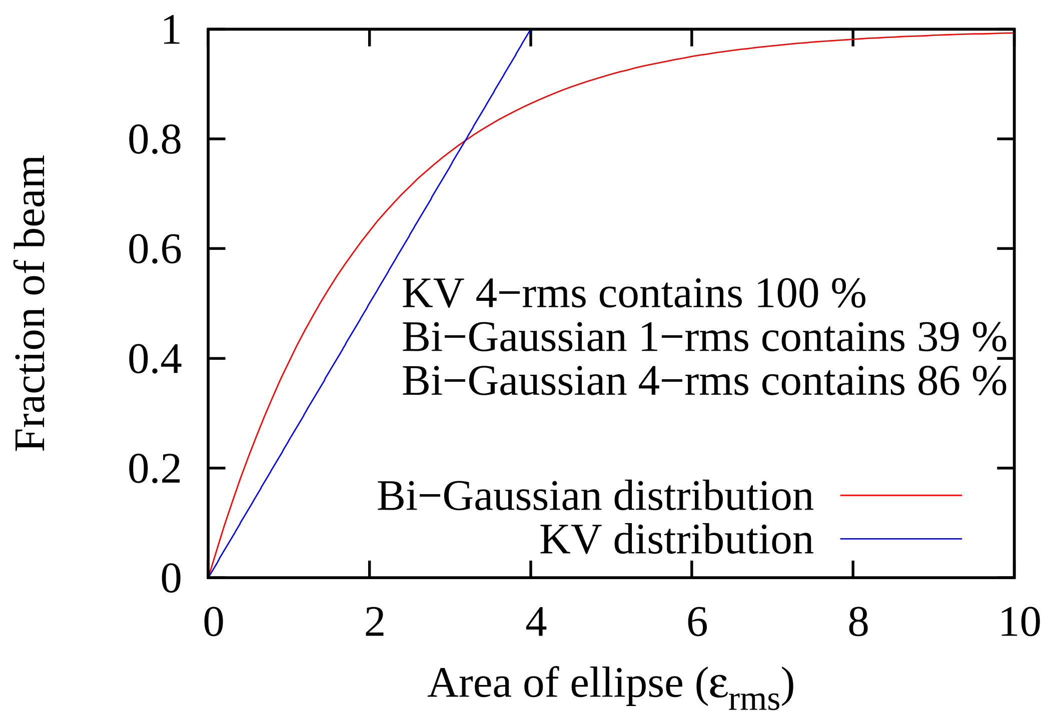
where

$$\begin{aligned} < x^2 > &= \frac{\iint x^2 I(x,x')\mathrm{d}x\mathrm{d}x'}{\iint I(x,x')\mathrm{d}x\mathrm{d}x'}, \\ < x'^2 > &= \frac{\iint x'^2 I(x,x')\mathrm{d}x\mathrm{d}x'}{\iint I(x,x')\mathrm{d}x\mathrm{d}x'}, \\ < xx' > &= \frac{\iint xx' I(x,x')\mathrm{d}x\mathrm{d}x'}{\iint I(x,x')\mathrm{d}x\mathrm{d}x'}. \end{aligned}$$

Assuming $< x >= 0$ and $< x' >= 0$.

Meanining of rms emittance

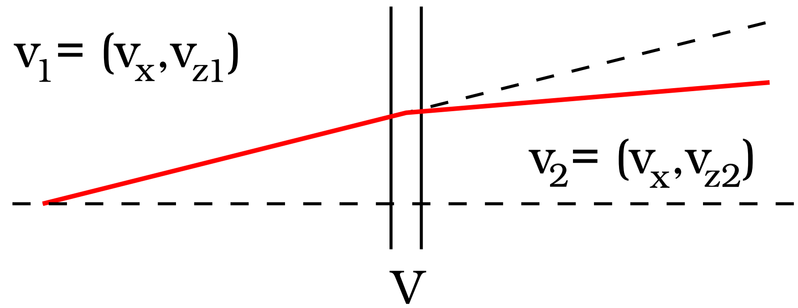
How much beam does the rms ellipse contain?



Depends on the distribution shape. For real simulated or measured distributions there is no direct rule.

Normalization of emittance

The transverse emittance defined in this way is dependent on the beam energy. If p_z increases, $x' = p_x/p_z$ decreases.



The effect is eliminated by normalizing the velocity to c, which gives

$$x'_n = \frac{p_x}{p_{z1}}\frac{v_{z1}}{c} = \frac{v_x}{c} = \frac{p_x}{p_{z2}}\frac{v_{z2}}{c}.$$

Normalized emittance is

$$\epsilon_n = \epsilon\frac{v_z}{c}$$

Emittance from plasma temperature

Assume circular extraction hole and Gaussian transverse ion distribution

$$I(x,x') = \frac{2}{\pi r^2}\sqrt{r^2 - x^2}\ \sqrt{\frac{m}{2\pi kT}} \exp\left(\frac{-m(x'v_z)^2}{2kT}\right).$$

The rms emittance can be integrated using the definition and normalized

$$\epsilon_{\text{rms,n}} = \frac{1}{2}\sqrt{\frac{kT}{m}}\frac{r}{c}.$$

Similarly for a slit-beam extraction

$$\epsilon_{\text{rms,n}} = \frac{1}{2}\sqrt{\frac{kT}{3m}}\frac{w}{c}.$$

Larger aperture $\Rightarrow$ more beam, weaker quality

Emittance from solenoidal B-field

If a circular beam starts from a solenoidal magnetic field (ECR) particles receive a azimuthal thrust of

$$v_\theta = r_0 \frac{qB}{2m},$$

when exiting the magnetic field. Far from solenoid the motion is cylindrically symmetric and

$$r' = \frac{v_r}{v_z} = \frac{v_\theta}{v_z} = \frac{qBr_0}{2mv_z}$$

The emittance of the beam is

$$\epsilon_{\text{rms}} = \frac{1}{4} r_0 r' = \frac{qBr_0^2}{8mv_z}$$

and normalized

$$\epsilon_{\text{rms,n}} = \frac{qBr_0^2}{8mc}$$

Low Energy Beam Transport

Beam line elements

Beam control happens with electromagnetic forces a.k.a. ion-optics.

The classic beam line elements are also in use at low energies:

Electrostatic

- Diode (accel or decel gap)
- Einzel lens
- Dipole
- Quadrupole

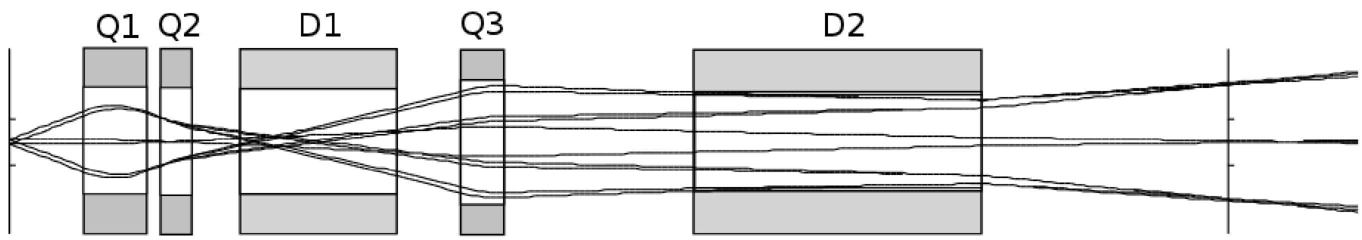
Magnetic

- Solenoid
- Dipole
- Quadrupole
- Multipole

Tools of trade

- Ion-optical software based on Nth-order approximation of trajectories (commonly used at higher energies)
- Electromagnetic field programs: POISSON SUPERFISH, FEMM, RADIA-3D, VECTOR FIELDS (OPERA), COMSOL MULTIPHYSICS, LORENTZ, etc. Some with and some without particle tracking capability.
- Specialized ion source extraction software.
- Many other specialized programs for modelling beam space charge compensation, bunching, cyclotron injection, collisional ion source plasmas, etc. with PIC-MCC type of methods.

Traditional transfer matrix optics



Treats ion-optical elements (and drifts) as black boxes with transfer matrices describing the effect to trajectories. In TRANSPORT $X = (x, x', y, y', l, \delta p/p)$

$$X_i(1) = \sum_j R_{ij} X_j(0) + \sum_{jk} T_{ijk} X_j(0) X_k(0) + \cdots$$

Ideal 1st order quadrupole:

$$R = \begin{pmatrix} \cos kL & \frac{1}{k}\sin kL & 0 & 0 & 0 & 0 \\ -k\sin kL & \cos kL & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh kL & \frac{1}{k}\sinh kL & 0 & 0 \\ 0 & 0 & k\sinh kL & \cosh kL & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Traditional transfer matrix optics

- Matrices based on analytic formulation, numerical integration of fields or fitting experimental/simulation data.
- The whole system can be described with one matrix:

$$R_{\text{system}} = R_N \cdots R_2 \cdot R_1$$

- Can also transport elliptical envelopes in addition to trajectories:

$$\sigma_1 = R\sigma_0 R^T, \text{ where}$$

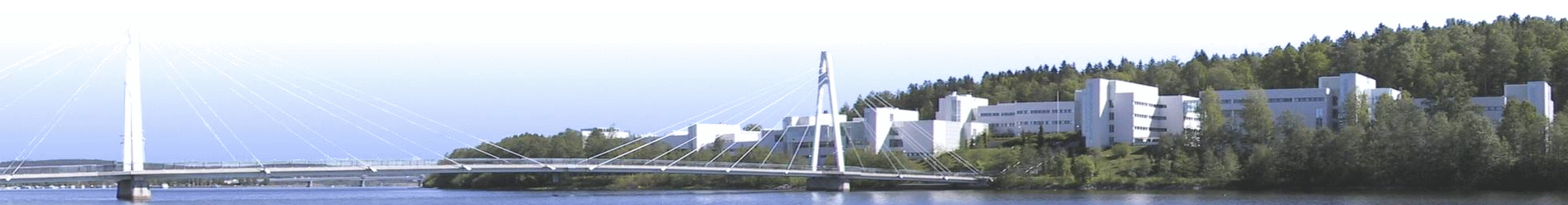
$$\sigma = \epsilon \begin{pmatrix} \beta & -\alpha \\ \alpha & \gamma \end{pmatrix}$$

- Advantage: calculation is fast (automatic optimization, etc)
- May include additional space charge induced divergence growth for beam envelopes and/or rms emittance growth modelling for particle distributions.

Codes of this type

- TRANSPORT — One of the classics, up to 2nd or 3rd order calculation, no space charge
- COSY INFINITY — Up to infinite order, no space charge
- GIOS — Up to 3rd oder, space charge of KV-beam
- DIMAD — Up to 3rd oder, space charge of KV-beam
- TRACE-3D — Mainly linear with space charge of KV-beam
- PATH MANAGER (TRAVEL) — Up to 2nd order, more advanced space charge modelling for particle distributions (mesh or Coulomb)

Some of the codes are more suitable for low energies, choose carefully!



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Differences to high energy transport

Now $v \ll c$ and J is large

- **Space charge** plays a major role
- Beam generated B-field is negligible.
- Several ion species
- Beam line elements often not well separated (no drift spaces in between).
- Complex electrostatic electrode shapes used.
- Nonlinear effects are significant!

Traditional Nth order transfer matrix optics cannot be used (well) close to ion sources. More fundamental methods are needed.

$\Rightarrow$ Particle tracking method

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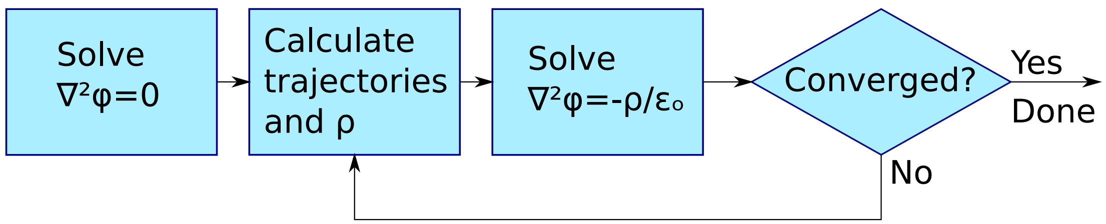
Particle tracking codes

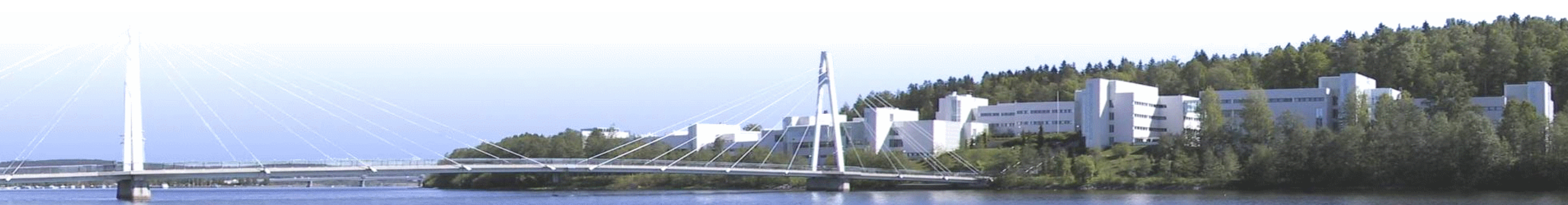
Particle tracking codes for ion source extraction and LEBT systems:

- Calculation of electrostatic fields in electrode geometry including space charge effects.
- Calculation/importing of magnetostatic fields.
- Tracking of particles in the fields.
- Diagnostics and other supportive methods.



Available codes of this type

- IGUN — Plasma modelling for negative and positive ions, 2D only
- PBGUNS — Plasma modelling for negative and positive ions, 2D only
- SIMION — Simple 3D E-field solver and particle tracer, low quality space charge modelling, no plasma
- KOBRA — More advanced 3D E-field solver, positive ion plasma modelling, PIC capability
- LORENTZ — State of the art 3D EM solver and particle tracer with a lot of capabilities, no plasma modelling
- IBSIMU — Plasma modelling for negative and positive ions, 1D–3D E-field solver



Ion Beam Simulator

IBSimu is an ion optical code package made especially for the needs of ion source extraction design. Using Finite Difference Method (FDM) in a regular cartesian mesh the code can model

- Systems of electrostatic and magnetic lenses
- High space charge beams (low energy)
- Positive and negative multispecies 3D plasma extraction

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The code is made as a C++ library and is released freely under GNU Public Licence*.

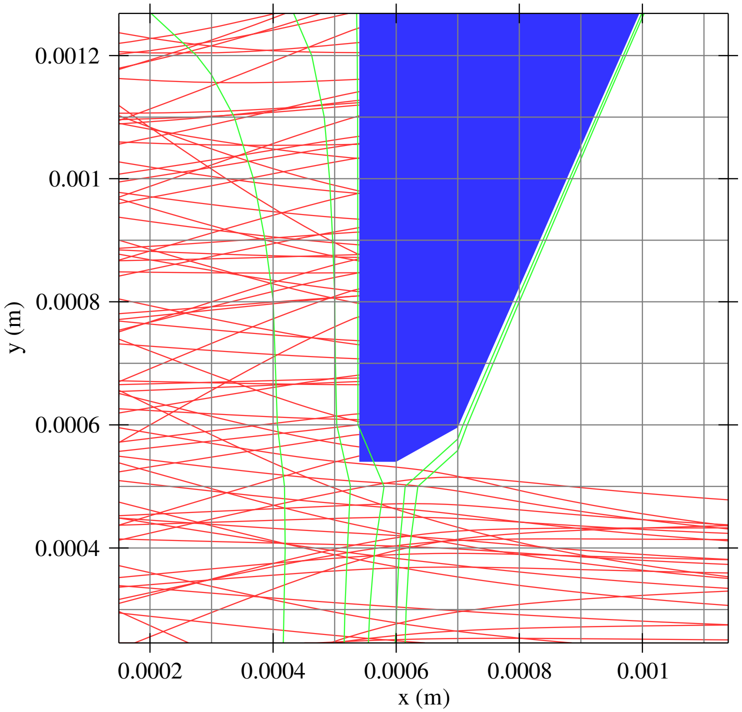
- Highly versatile and customizable.
- Can be used for batch processing and automatic tuning of parameters.

*) *http://ibsimu.sourceforge.net/*

Ion optics with FDM

Calculation is based on evenly sized square cartesian grid(s):

- Solid mesh (node type): vacuum, solid, near solid, neumann boundary condition, ...
- Electric potential
- Electric field
- Magnetic field
- Space charge density
- Trajectory density



Electrostatic field solver

Poisson's equation

$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0}$$

Finite Difference representation for vacuum node i:

$$\frac{\phi_{i-1} - 2\phi_i + \phi_{i+1}}{h^2} = -\frac{\rho_i}{\epsilon_0},$$

Neumann boundary node i:

$$\frac{-3\phi_i + 4\phi_{i+1} - \phi_{i+2}}{2h} = \frac{d\phi}{dx}$$

and Dirichlet (fixed) node i:

$$\phi_i = \phi_{\text{const}}$$

1D example

Solve a 1D system of length $L = 10$ cm, charge $\rho = 1 \cdot 10^{-6}$ C/m^3 and boundary conditions

$$\frac{\partial \phi}{\partial x}(x = 0) = 0 \text{ V/m} \qquad \text{and} \qquad \phi(x = L) = 0 \text{ V}.$$

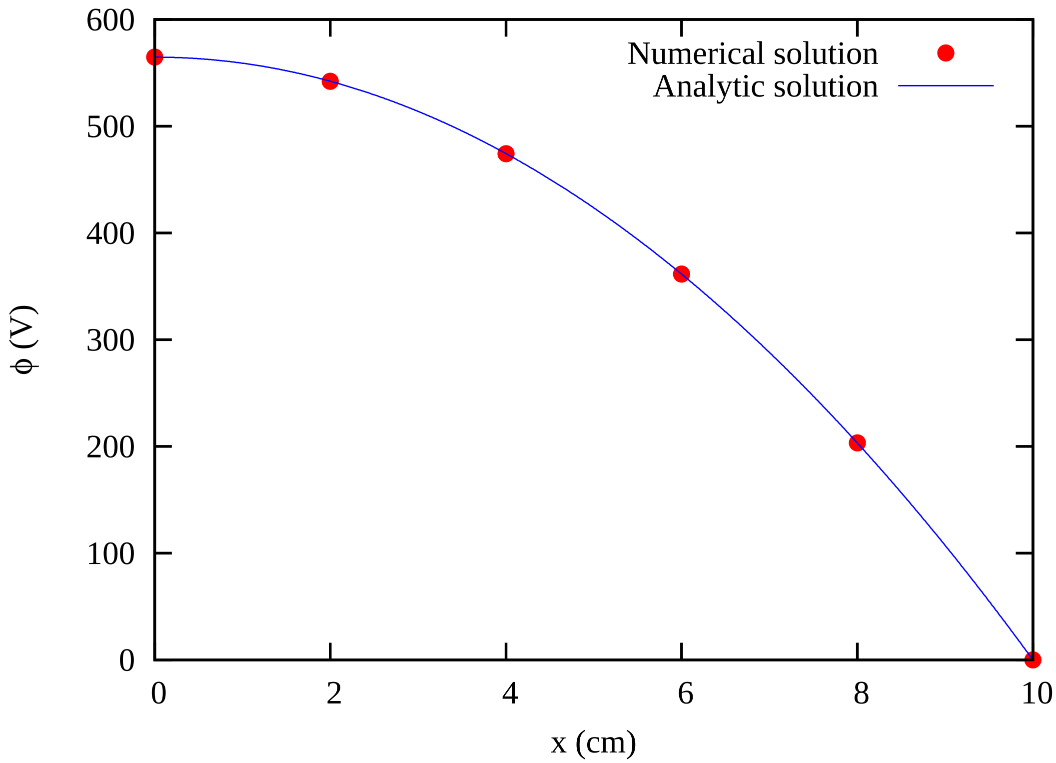
The system is discretized to $N = 6$ nodes. Problem in matrix form:

$$\begin{pmatrix} -3 & 4 & -1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \end{pmatrix} = \begin{pmatrix} 2h\frac{\partial \phi}{\partial x}(0) \\ -h^2\frac{\rho}{\epsilon_0} \\ -h^2\frac{\rho}{\epsilon_0} \\ -h^2\frac{\rho}{\epsilon_0} \\ -h^2\frac{\rho}{\epsilon_0} \\ \phi(L) \end{pmatrix}$$

Solving the matrix equation we get ...

1D example

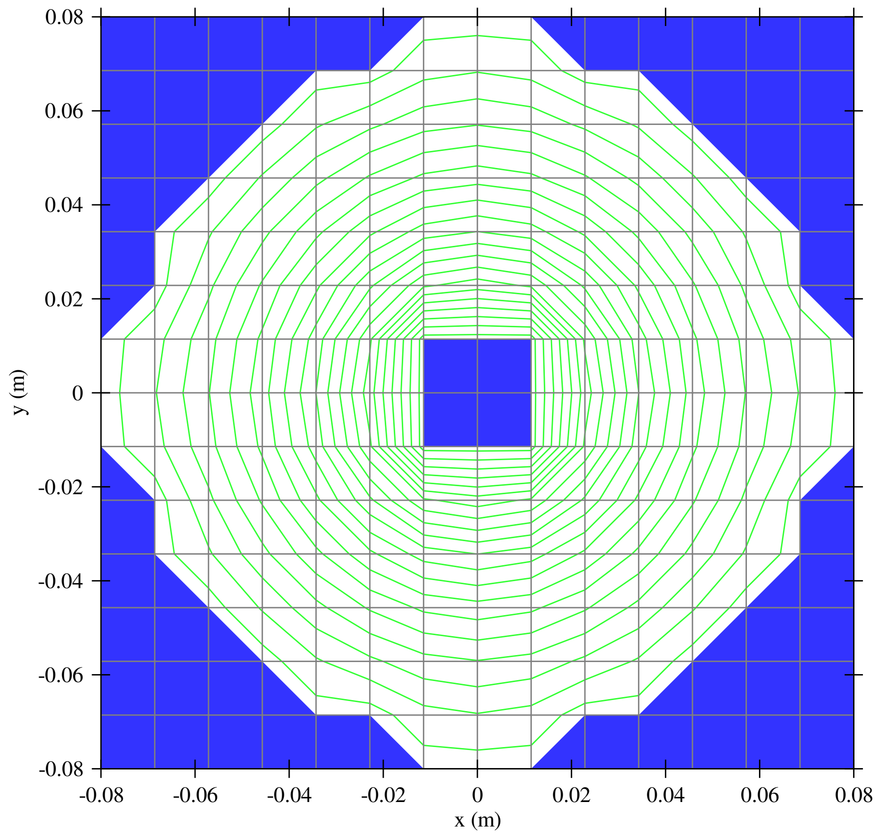
... perfect agreement with analytic result



but only because of flat charge distribution and boundaries defined exactly at node locations.

Jagged boundaries

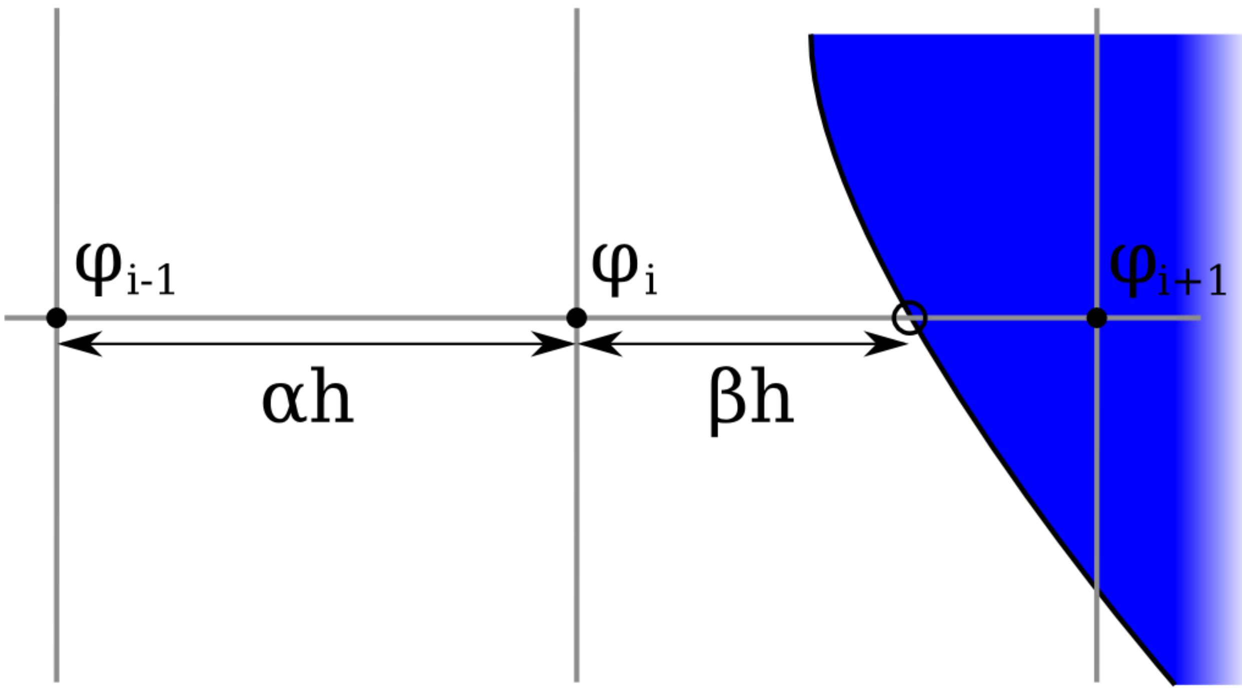
In higher dimensions basic FDM generally suffers from jagged boundaries (nodes don’t coincide with surfaces).



Smooth boundaries

Derivatives in Poisson's equation featured with uneven distances

$$\frac{\beta\phi(x_0-\alpha h)-(\alpha+\beta)\phi(x_0)+\alpha\phi(x_0+\beta h)}{\frac{1}{2}(\alpha+\beta)\alpha\beta h^2}=-\frac{\rho(x_0)}{\epsilon_0}$$



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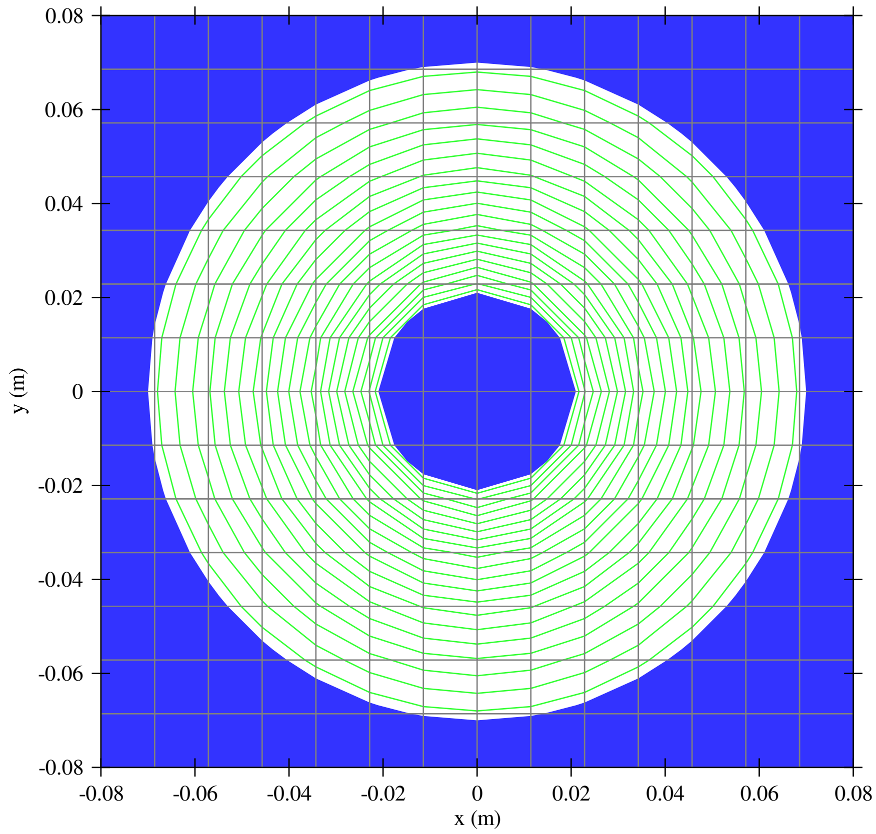
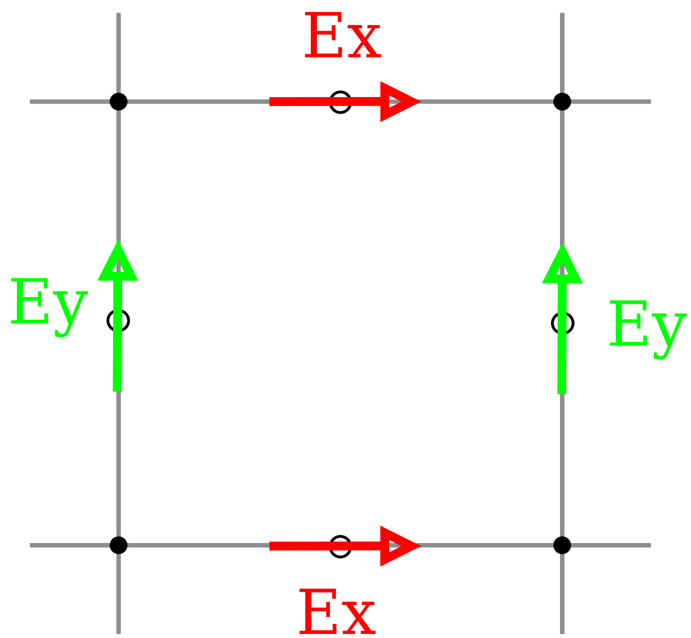
Smooth boundaries

A much better solution with smooth boundaries is achieved.



Electric field calculation

Electric field is calculated between the nodes simply by $E = \frac{V}{h}$.



Electric field nodes between potential nodes.

Trajectory calculation

Population of virtual particles is calculated with following properties:

- Charge: q
- Mass: m
- Current carried: I
- Time, position and velocity coordinates:
  - 2D: (t, x, v_x, y, v_y)
  - Cylindrical symmetry: $(t, x, v_x, r, v_r, \omega)$, $\omega = \frac{d\theta}{dt}$
  - 3D: $(t, x, v_x, y, v_y, z, v_z)$

Trajectory calculation

Calculation of trajectories done by integrating the equations of motion

$$
\begin{aligned}
\frac{dx}{dt} &= v_x \\
\frac{dy}{dt} &= v_y \\
\frac{dz}{dt} &= v_z \\
\frac{dv_x}{dt} &= a_x = \frac{q}{m}(E_x + v_y B_z - v_z B_y) \\
\frac{dv_y}{dt} &= a_y = \frac{q}{m}(E_y + v_z B_x - v_x B_z) \\
\frac{dv_z}{dt} &= a_z = \frac{q}{m}(E_z + v_x B_y - v_y B_x)
\end{aligned}
$$

Trajectory calculation

... and in cylindrical symmetry:

$$
\begin{aligned}
\frac{dx}{dt} &= v_x \\
\frac{dr}{dt} &= v_r \\
\frac{dv_x}{dt} &= a_x = \frac{q}{m}(E_x + v_r B_\theta - v_\theta B_r) \\
\frac{dv_r}{dt} &= a_r + r\omega^2 = \frac{q}{m}(E_y + v_\theta B_x - v_x B_\theta) + r\omega^2 \\
\frac{d\omega}{dt} &= \frac{1}{r}(a_\theta - v_r\omega) = \frac{1}{r}\left(\frac{q}{m}(v_x B_r - v_r B_x) - 2v_r\omega\right),
\end{aligned}
$$

where $v_\theta = r\frac{d\theta}{dt} = r\omega$

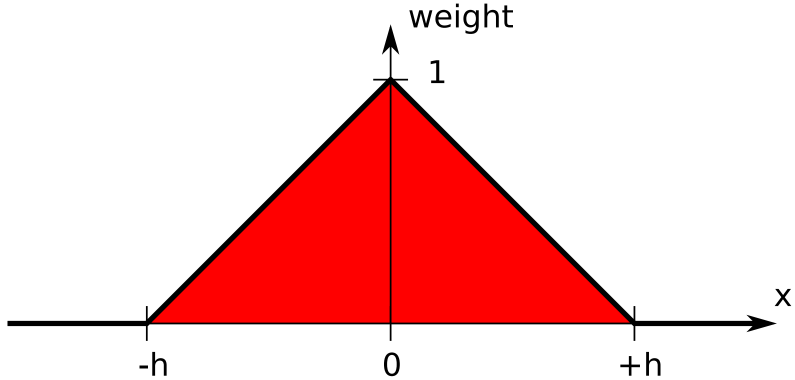
Space charge deposition

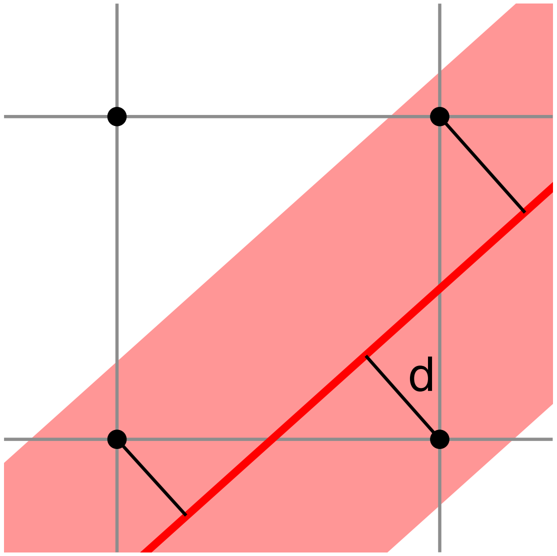
Particle trajectories deposit space charge to the geometry

$$\rho = \frac{I}{Av},$$

where A is the cross section of the particle.

Linear/bilinear weighing used (finite particle size):





Several particles needed per mesh for smooth space charge field.

Emittance growth

The rms emittance can grow and shrink:

- Particle-particle scattering
- EM-field fluctuations
  - Power supply ripples
  - Plasma instabilities
- Nonlinear fields in electrostatic and magnetic optics
- Nonlinear fields from beam/plasma space charge
- Collimation
- Simulation artefact: mesh induced emittance growth

Typically accelerator systems are designed to be as linear as possible.

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Beam space charge effects

Assuming constant space charge of the beam $\rho = J/v$. In cylindrical case one can calculate the E-fields from Gauss law:

$$
\begin{aligned}
E &= \frac{I}{2\pi\epsilon_0 v}\frac{r}{r_{\text{beam}}^2}, r < r_{\text{beam}} \\
E &= \frac{I}{2\pi\epsilon_0 v}\frac{1}{r}, r > r_{\text{beam}}
\end{aligned}
$$

and the potential in the beam tube:

$$
\begin{aligned}
\phi &= \frac{I}{2\pi\epsilon_0 v}\left[\frac{r^2}{2r_{\text{beam}}^2} + \log\left(\frac{r_{\text{beam}}}{r_{\text{tube}}}\right) - \frac{1}{2}\right], r < r_{\text{beam}} \\
\phi &= \frac{I}{2\pi\epsilon_0 v}\log\left(\frac{r}{r_{\text{tube}}}\right), r > r_{\text{beam}}
\end{aligned}
$$

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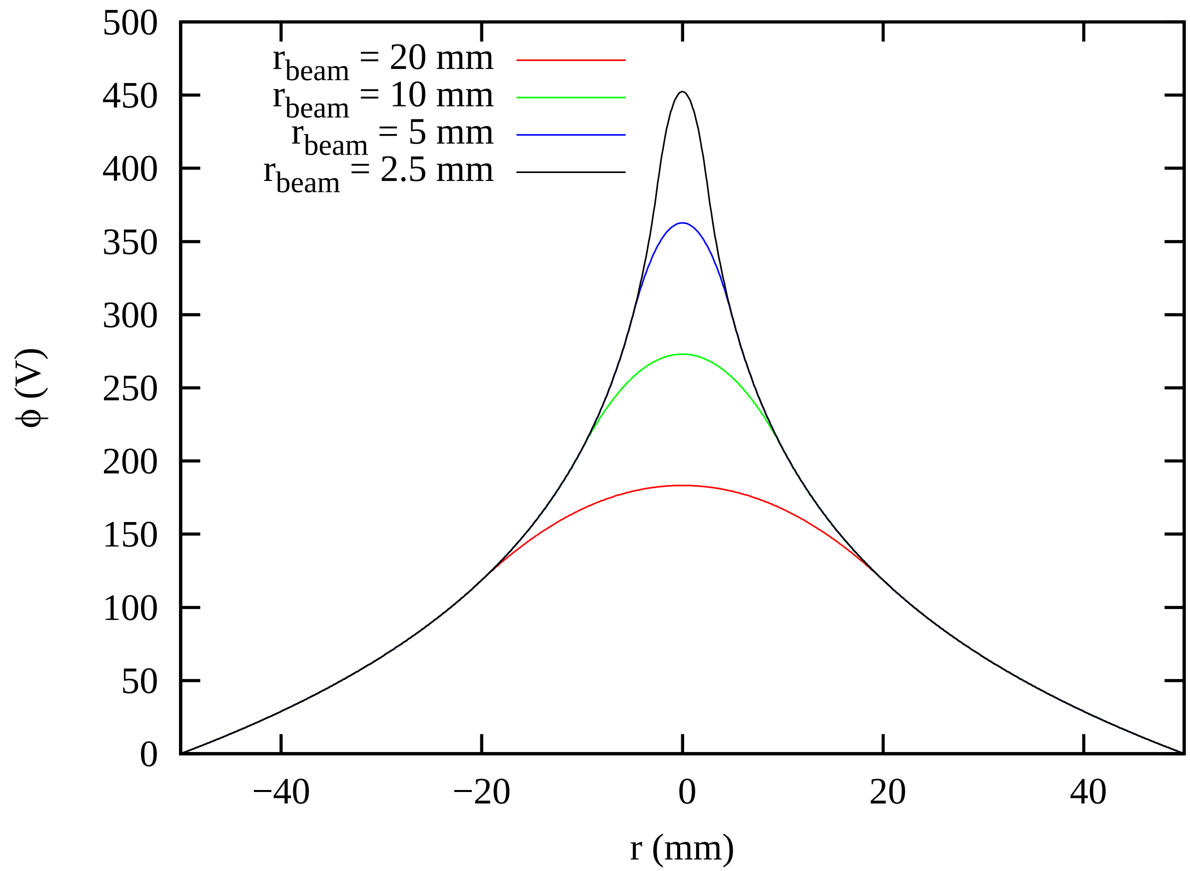
Beam space charge effects

Potential in a 100 mm tube with a 10 mA, 10 keV proton beam



Beam space charge blow-up

Ion at the beam boundary experiences a repulsive force

$$F_r = qE_r = ma_r = \frac{qI}{2\pi\epsilon_0 r v_z}.$$

The particle acceleration is

$$a_r = \frac{d^2r}{dt^2} = \frac{d^2r}{dz^2}\frac{d^2z}{dt^2} = v_z^2\frac{d^2r}{dz^2}.$$

Therefore

$$\frac{d^2r}{dz^2} = \frac{1}{v_z^2}a_r = K\frac{1}{r}, \text{ where}$$
$$K = \frac{qI}{2\pi\epsilon_0 m v_z^3}.$$

The DE can be integrated after change of variable $\lambda = \frac{dr}{dz}$ and gives

$$\frac{dr}{dz} = \sqrt{2K\log(r/r_0)},$$

assuming $\frac{dr}{dz} = 0$ at $z = 0$.

Beam space charge blow-up

The solution is separable and can be again integrated to a final solution

$$z = \frac{r_0}{\sqrt{2K}} F\left(\frac{r}{r_0}\right), \text{ where}$$

$$F\left(\frac{r}{r_0}\right) = \int_{y=1}^{r/r_0} \frac{dy}{\sqrt{\log y}}.$$

(1) Low divergence was assumed to be able to use equation for E_r.

(2) Constant v_z was assumed (beam potential changes neglected).

Example: Parallel zero-emittance beam of $^{181}\mathrm{Ta}^{20+}$ accelerated with 60 kV has initial radius of $r_0 = 15$ mm. The size of a 120 mA beam after a drift of 100 mm can be solved from $F(r/r_0) = 1.189$, which gives $r = 20$ mm.

Linear effect $\Rightarrow$ no rms emittance growth.

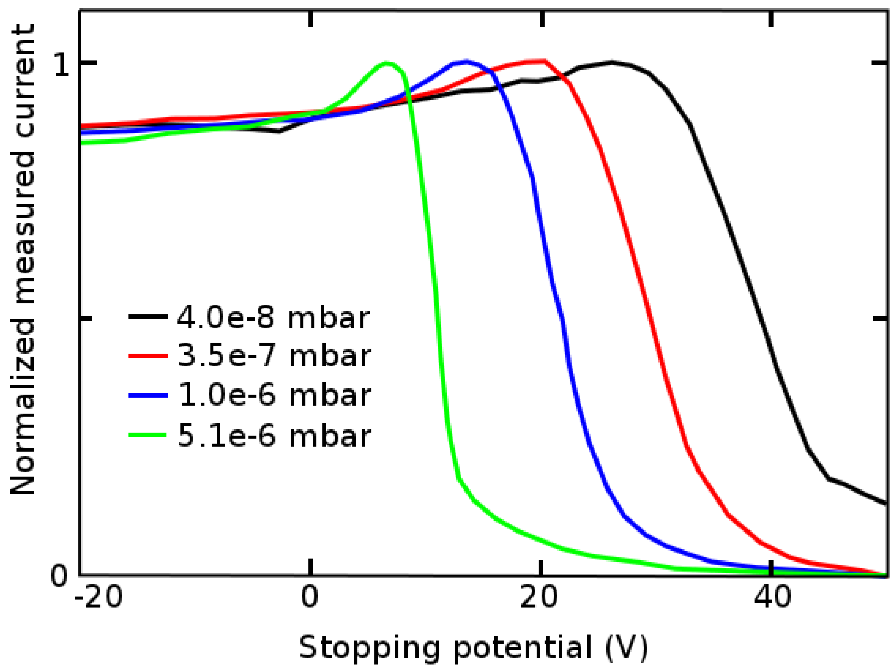
Beam space charge compensation

Transport of high-intensity, low-energy beams can be difficult due to space charge blow-up. Beam compensation helps in low E-field areas.

- Background gas ionization: e^- and X^+ created within the beam.
- Opposite sign to beam trapped in beam potential, while same sign particles accelerated out $\Rightarrow$ decreasing beam potential.
- Secondary electron emission from beam halo hitting beam tube providing compensating particles for positive beams.
- Also methods for active compensation: running electron beam in opposite direction of the main beam.
- Usually increased by feeding background gas into the beamline.

Beam space charge compension

Measurement of ion energy distribution ejected from beam



Reproduced from D. S. Todd, BIW 2008

Gives an indication of the compensation degree.

Beam space charge compension

Compensation by thermal particles trapped in the beam potential is difficult to estimate. Creation rate

$$\frac{d\rho_c}{dt} = Jn\sigma_c$$

$$\tau = \frac{\rho_{\mathrm{beam}}}{\left(\frac{d\rho_c}{dt}\right)} = \frac{1}{vn\sigma_c}$$

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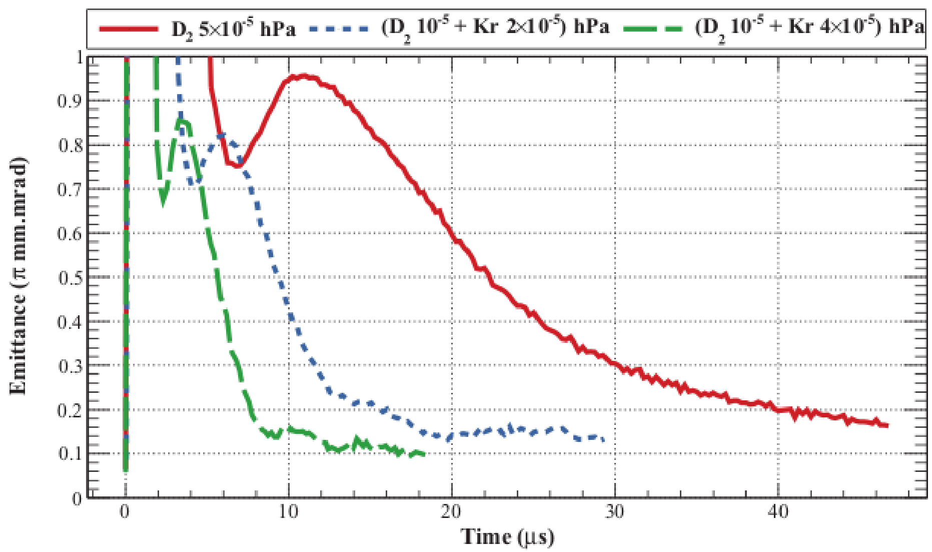
Pulsed beams may or may not be long enough for reaching equilibrium.



From N. Chauvin, ICIS 2011

Beam space charge compension

If creation rate is high, the SCC is finally limited by leakage of compensating particles from the potential well as SCC approaches 100 %.

Electrons are fast $\Rightarrow$ X^+ SCC < 100 %

Ions are slow $\Rightarrow$ X^- overcompensation is possible.

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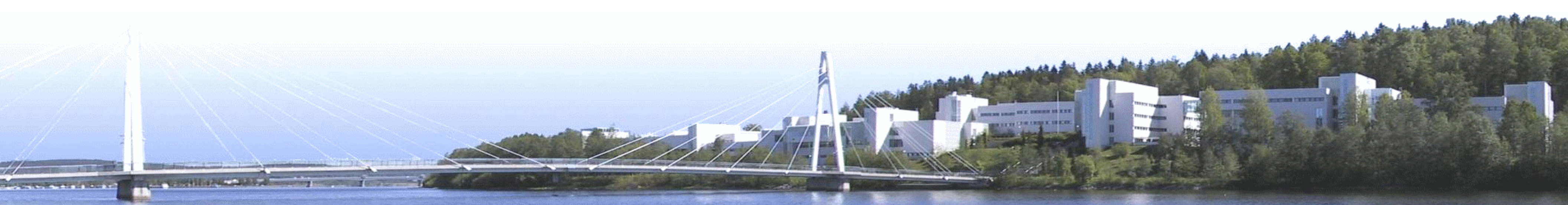
SCC is location dependent because compensating particles move in the potential well. Leakage in the beam ends cause at least local loss of SCC. Leakage may be limited by accelerating einzel lens or by magnetic fields.

Background gas causes beam losses. Typically a 1–2 % sacrifice is sufficient for good SCC.



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Background gas causes beam losses. Typically a 1–2 % sacrifice is sufficient for good SCC. Modelling:

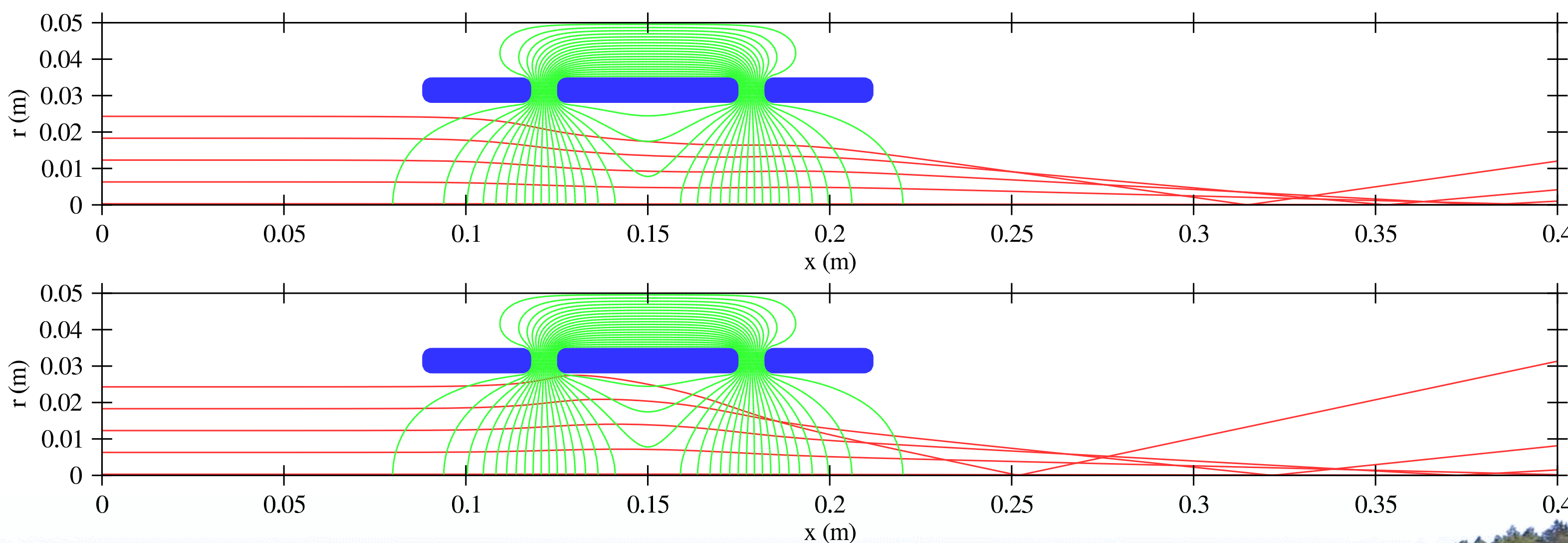
- Simple model for SCC: scaling the effective beam current globally or locally with a SCC-factor.
- PIC simulation (for example WARP or SOLMAXP) with modelling of trapped particle dynamics

Einzel focusing

Einzel is a cylindrically symmetric focusing lens, which is characterized by voltage ratio

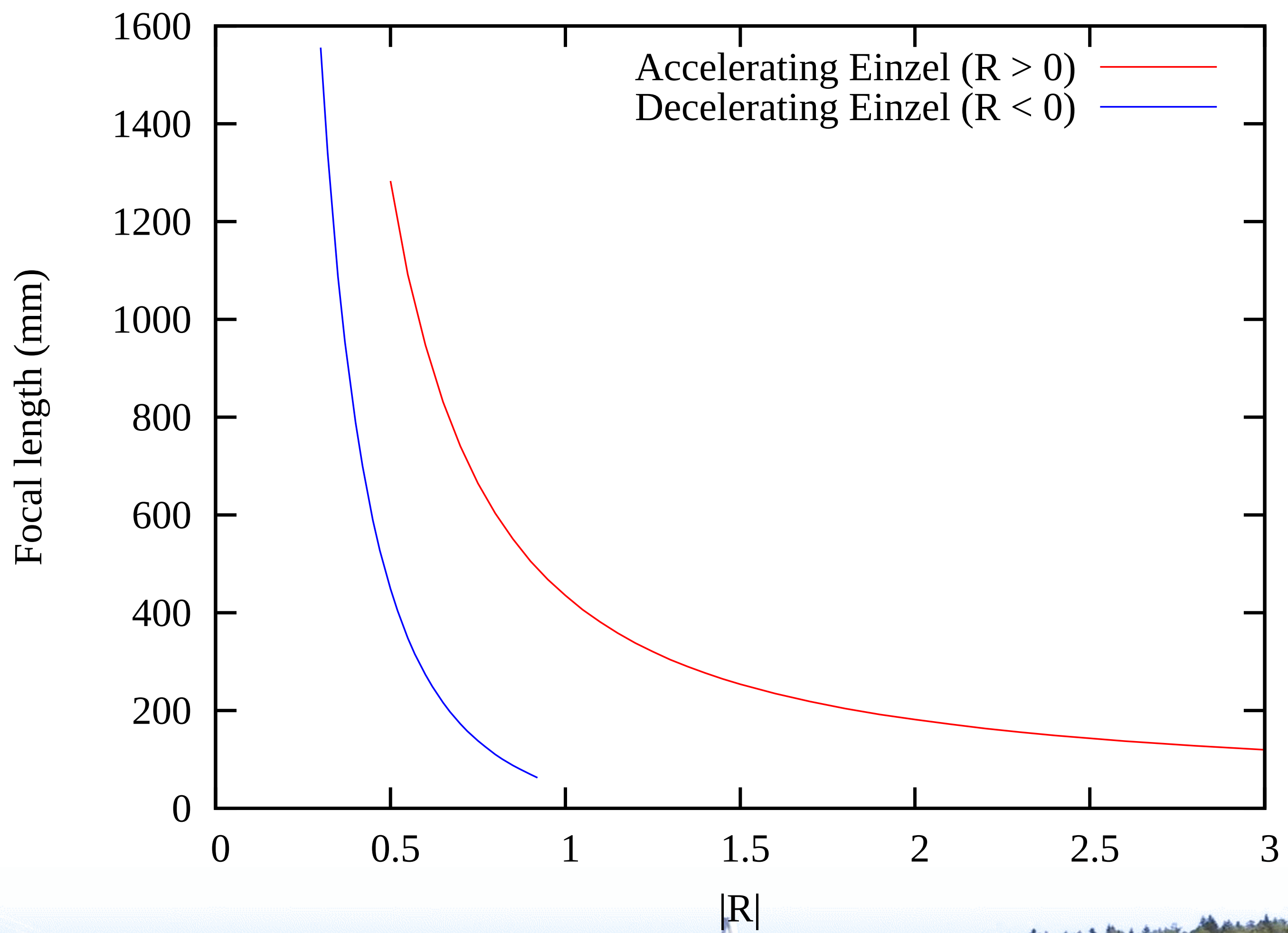
$$R = \frac{V_{\mathrm{einzel}} - V_{\mathrm{tube}}}{V_{\mathrm{tube}} - V_0},$$

where V_{einzel} is the center electrode potential, V_{tube} is the beam tube potential and V_0 is the potential where particle kinetic enrgy is zero. The einzel lens can be accelerating ($R > 0$) or decelerating ($R < 0$).



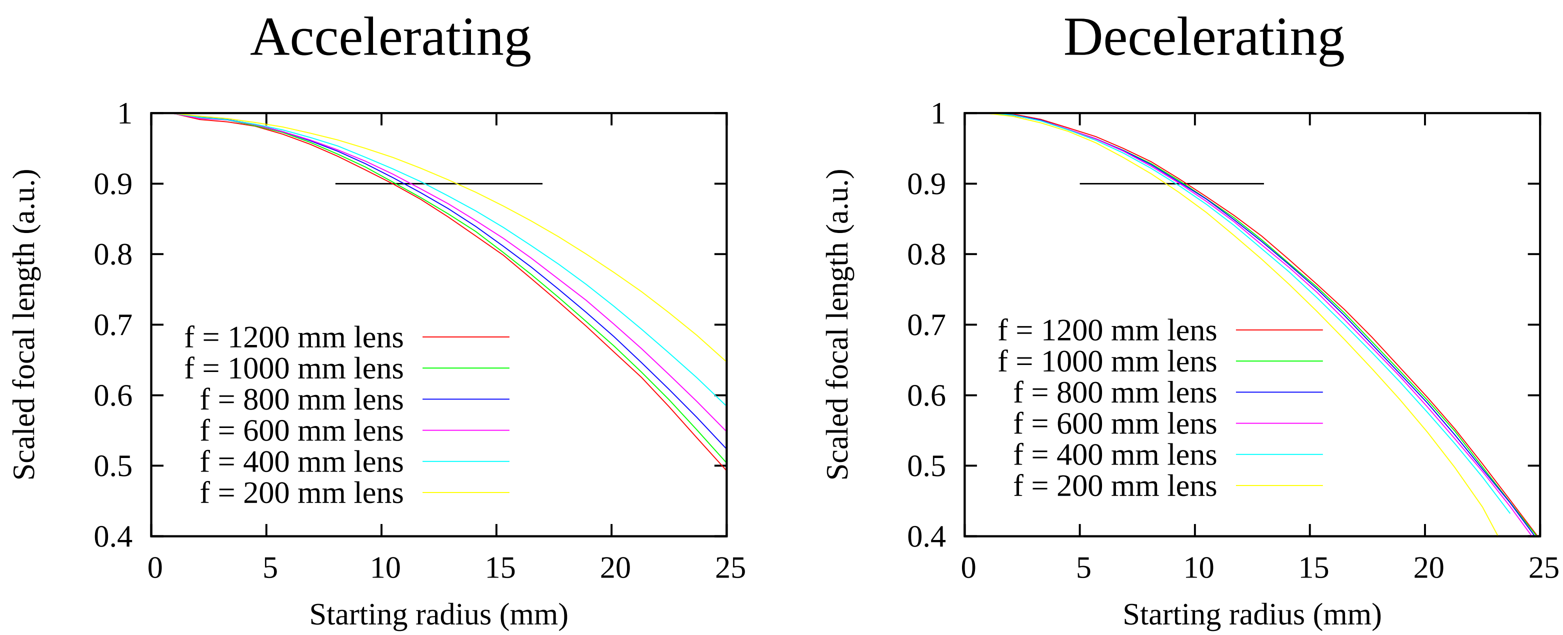
Einzel focusing

Focusing power as a function of R.



Einzel focusing

Focal length changes with particle radius: aberrations



- Beam should fill less than half of the Einzel radius (28 mm in the example case).
- Accelerating should be preferred if not voltage/E-field limited (less aberrations, limits space charge compensation leakage)

Magnetic solenoid lens

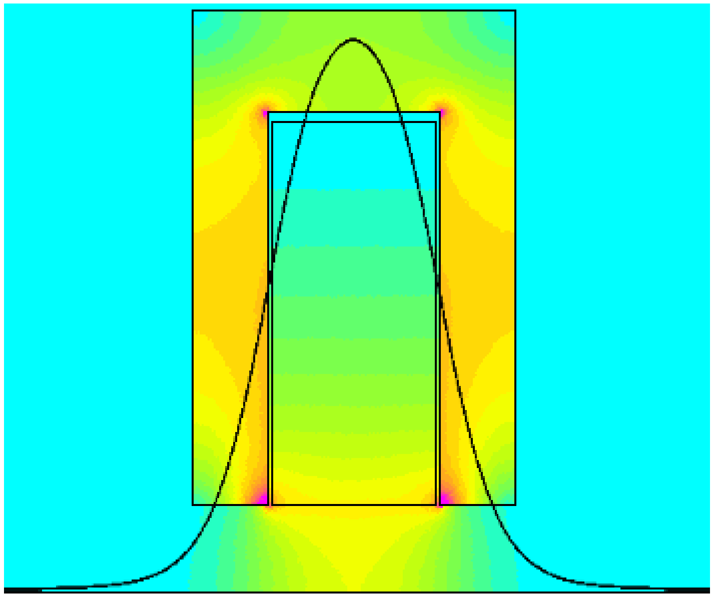
Magnetic equivalent to Einzel lens

Solenoid field using on-axis field:

$$\begin{aligned} B_z(r,z) &\approx B_0(z) \\ B_r(r,z) &\approx -\frac{1}{2}B_0(z)'r \end{aligned}$$

Focal length of solenoid

$$\frac{1}{f} = \frac{q^2}{8Em}\int B_z^2 dz$$



Magnetic solenoid lens

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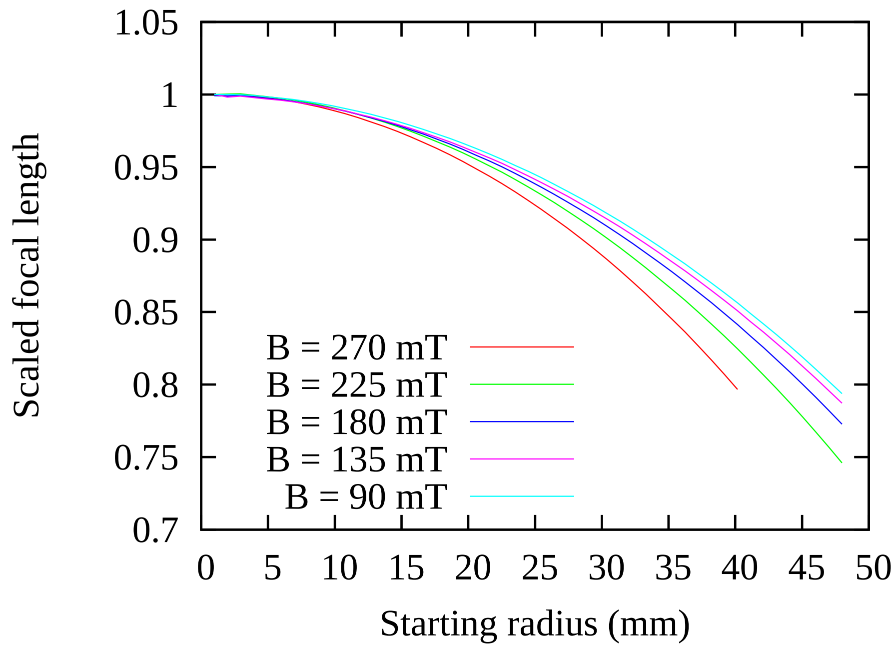
Solenoid spherical aberrations

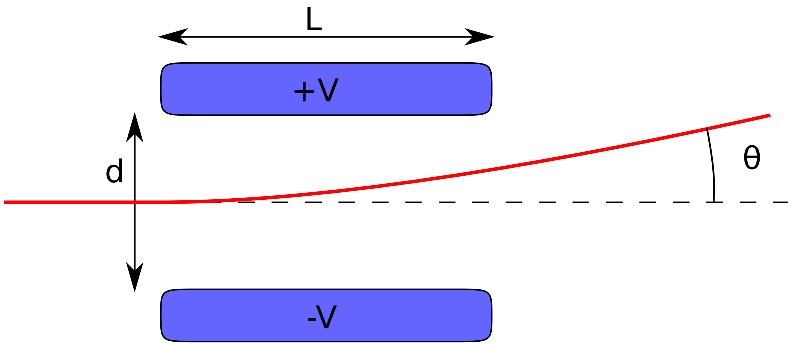


Filling about half of the bore leads to ∼5–10 % focal length variation.

Parallel plates for beam deflection

Simplest possible electrostatic dipole

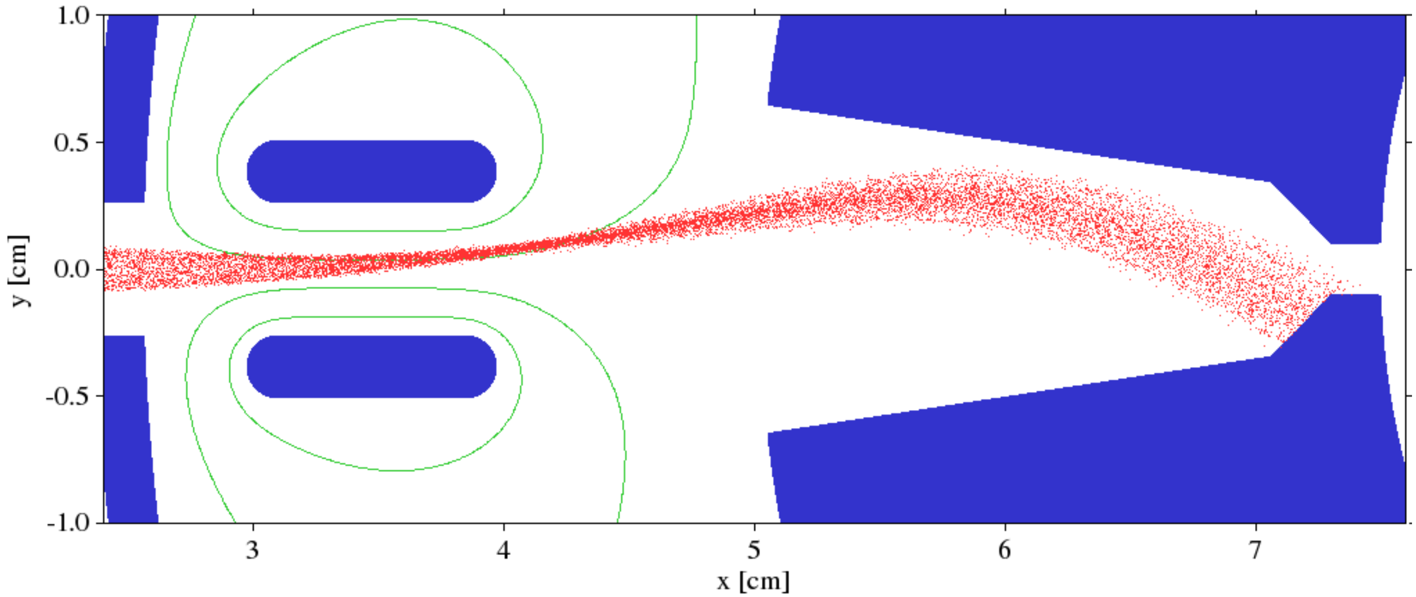
$$
\begin{aligned}
v_z^2 &= 2\frac{q}{m}V_{\mathrm{acc}} \\
v_x &= a_x \Delta t = \frac{q}{m}E_x\frac{L}{v_z} \\
\theta &\approx \frac{v_x}{v_z} = \frac{q}{m}E_x\frac{L}{v_z^2} \\
\theta &= \frac{V_{\mathrm{plate}}L}{V_{\mathrm{acc}}d}
\end{aligned}
$$



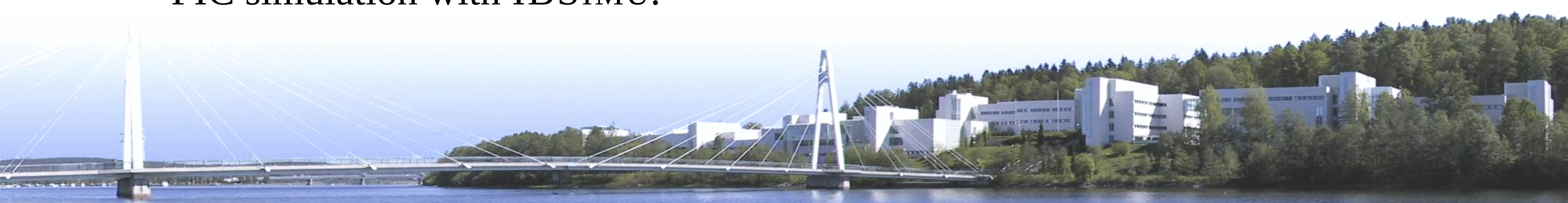
Good example: q and m do not effect trajectories in electrostatic systems.

Parallel plates for beam chopping

Fast beam chopping can be done with parallel plates: LBNL built neutron generator using 15 ns rise-time ± 1500 V switches for generating 5 ns beam pulses.



PIC simulation with IBSimu.



Magnetic beam deflection

Cyclotron radius

$$
\begin{aligned}
r &= \frac{mv_z}{qB} \\
&= \frac{1}{B}\sqrt{\frac{2mV_{\mathrm{acc}}}{q}} \\
r\theta &\approx L \\
\theta &= LB\sqrt{\frac{q}{2mV_{\mathrm{acc}}}}
\end{aligned}
$$

Valid for small angles

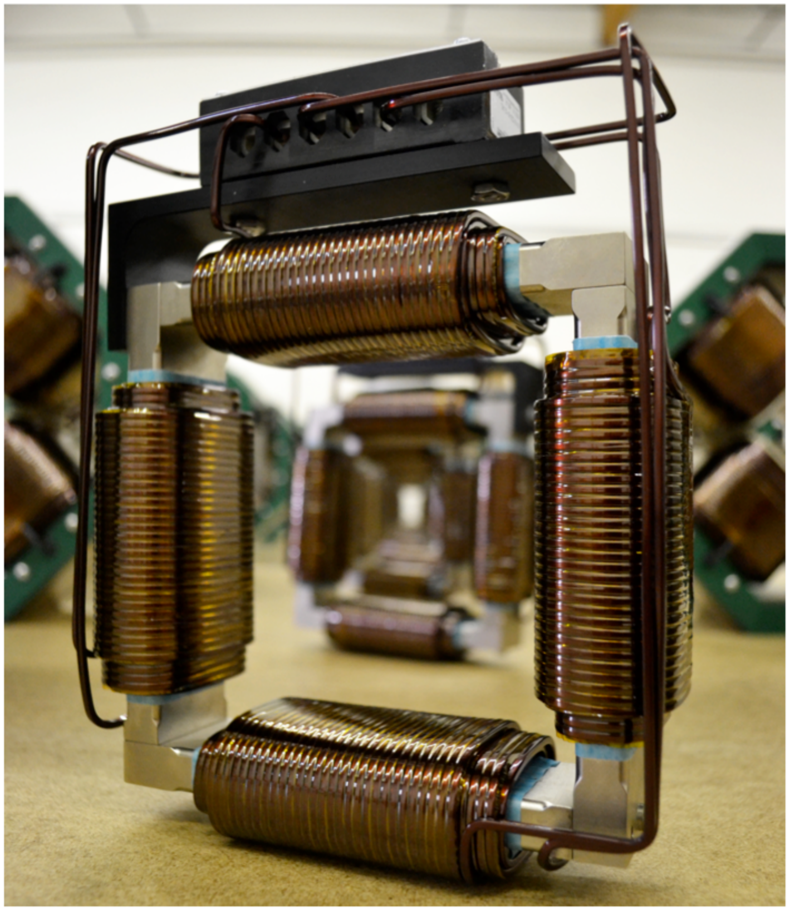
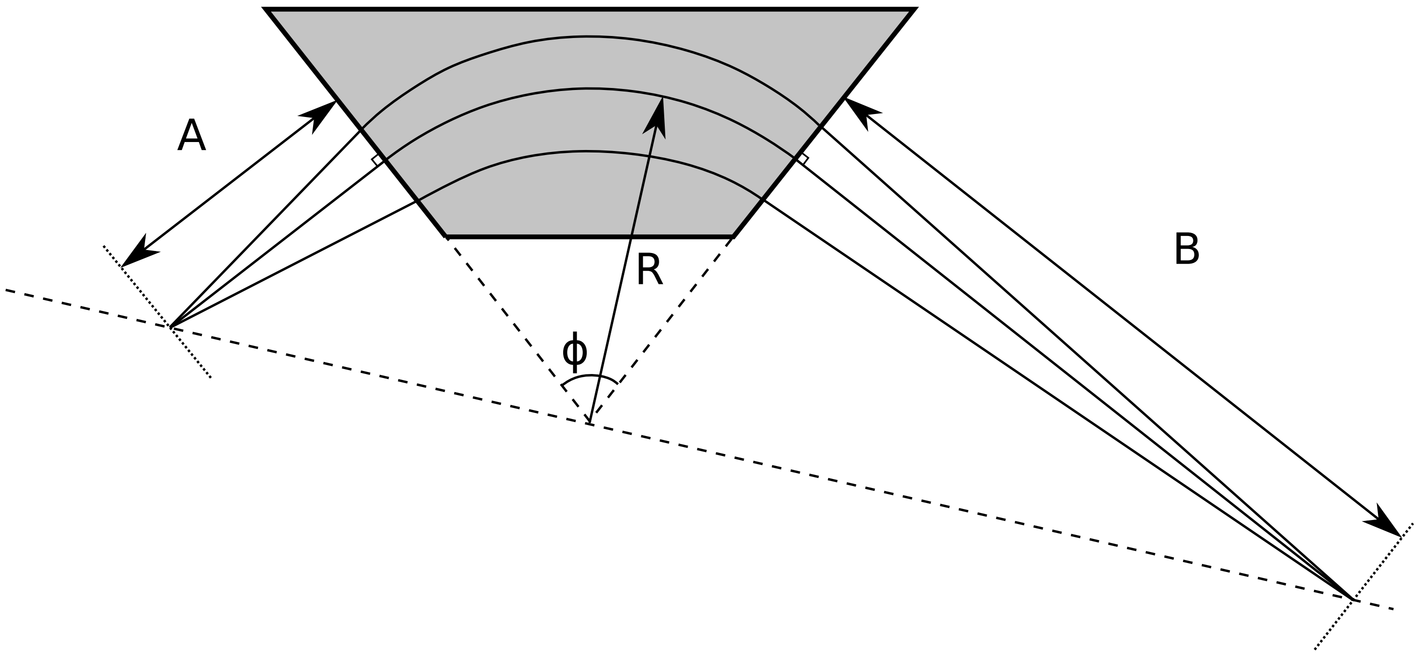


Image from Radia Beam Technologies

Magnetic dipole lenses

Homogenous sector magnet focuses in bending plane (x)



Barber's rule: center of curvature and two focal points are on a straight line

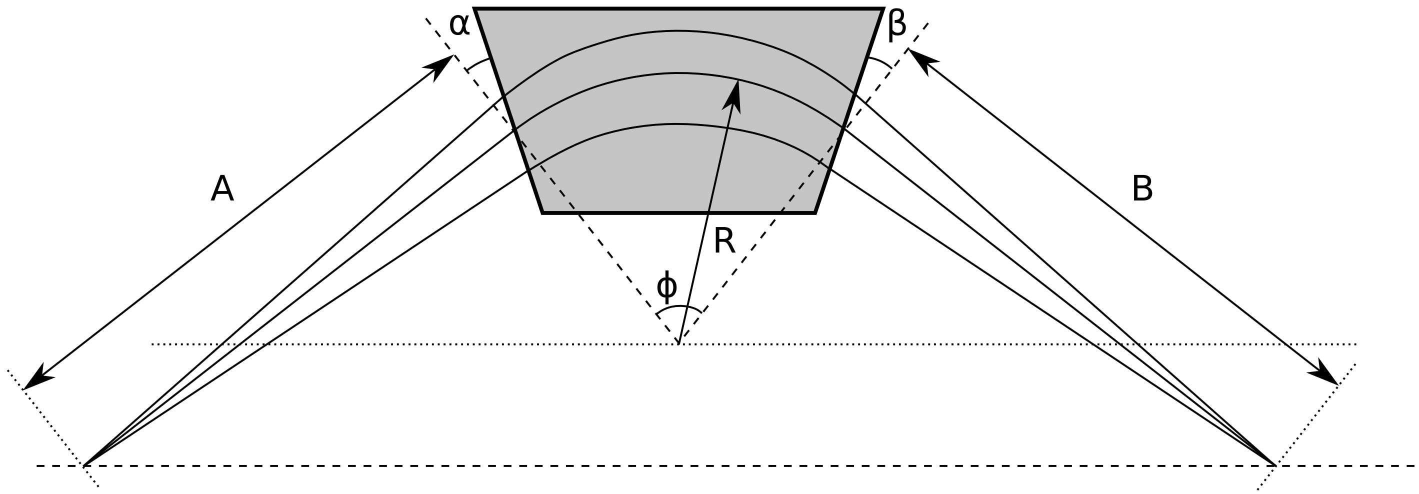
For symmetric setup: $A = B = R/\tan(\frac{\phi}{2})$

For a 90 degree magnet: $A = B = R$

No focusing power in transverse plane (y)

Magnetic dipole lenses

If magnet edge angles deviate from 90°, the focusing power in x-direction can be adjusted.

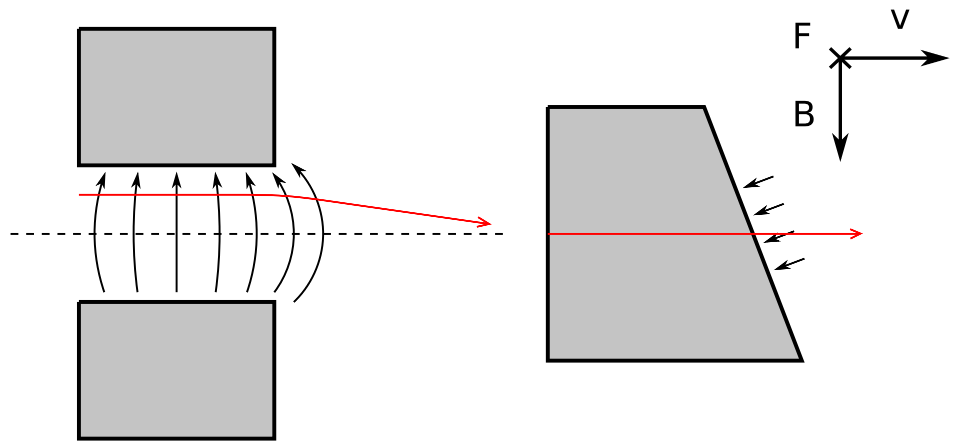


Positive angle (as shown in figure) $\Rightarrow$ less focusing power in x-direction.

Negative angle $\Rightarrow$ more focusing power in x-direction.

Magnetic dipole lenses

The fringing fields provide focusing in y-direction if edge angle (α and β) positive.



Focusing in x-direction can be traded for y-focusing: $f_y = \frac{R}{\tan(\alpha)}$

Important case: symmetric (same focal length in x and y) double focusing dipole:

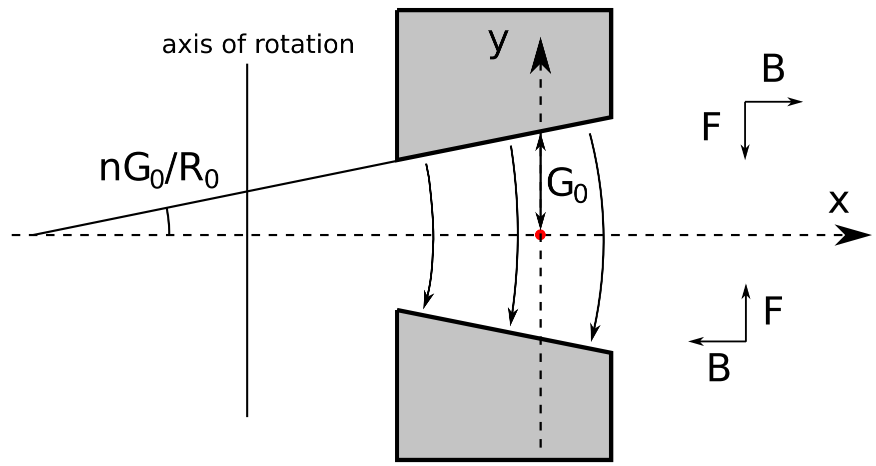
$$2\tan(\alpha) = 2\tan(\beta) = \tan(\frac{\phi}{2})$$

$$A = B = \frac{2R}{\tan(\frac{\phi}{2})}$$

For $\phi = 90^\circ$, $\alpha = \beta = 26.6^\circ$ and $A = B = 2R$.

Magnetic dipole lenses

Radially inhomogenous sector magnet



Magnetic field approximation from $\nabla \times B = 0$:

$$B_y(x,y) = B_0 \left(1 - n\frac{x}{R_0} + \cdots\right)$$
$$B_x(x,y) = B_0 \left(n\frac{y}{R_0} + \cdots\right)$$

Radial focusing if $n < 1$, vertical focusing if $n > 0$, symmetric at $n = \frac{1}{2}$.

Magnetic dipole applications

Important applications for magnetic dipoles

- Species analysis/selection
- Switching magnets

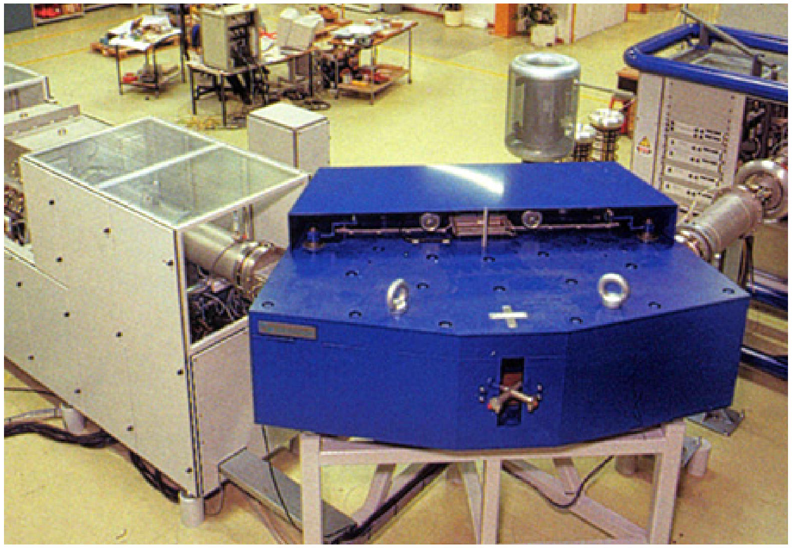


Image from Danfysik

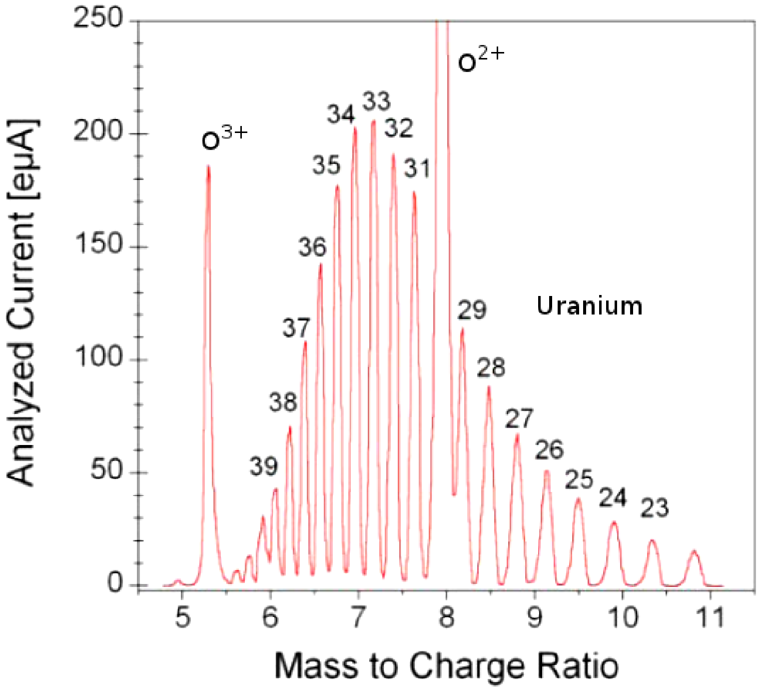
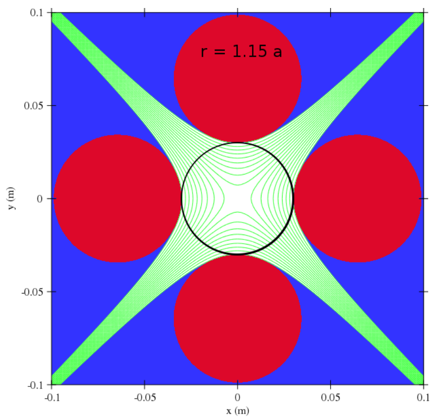
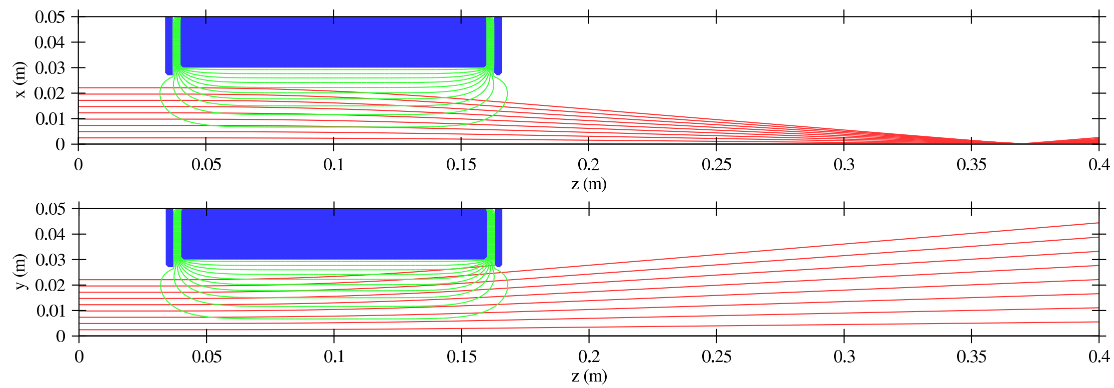


Image from D. Leitner, BIW 2010

Electrostatic quadrupole focusing

Electrostatic quadrupole: ideally hyperbolic electrodes, cylindrical ok



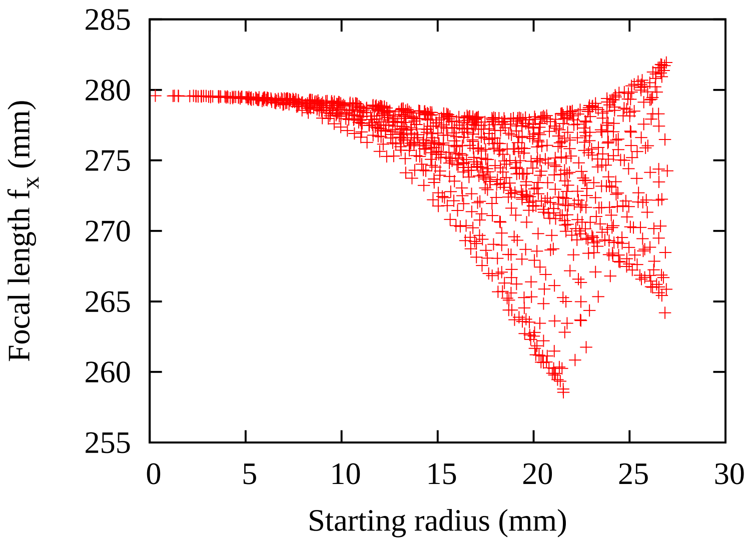


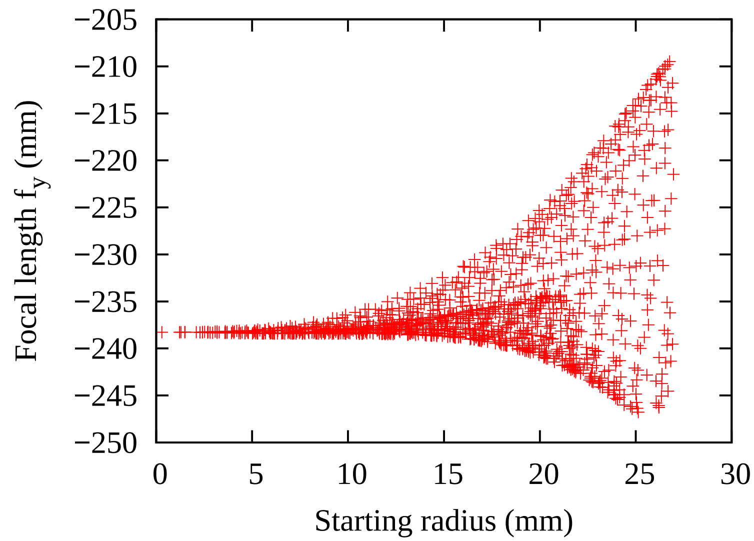
$$
\begin{aligned}
1/f_x &= k\tan(kw) \\
1/f_y &= -k\tanh(kw) \text{ ,where } k^2 = \frac{V_{\text{quad}}}{G_0 V_{\text{acc}}}
\end{aligned}
$$

- Used as doublets or triplets for focusing in both directions.
- Can also provide beam steering if electrodes independently controlled.

Electrostatic quadrupole focusing

Aberrations as a function of trajectory radius

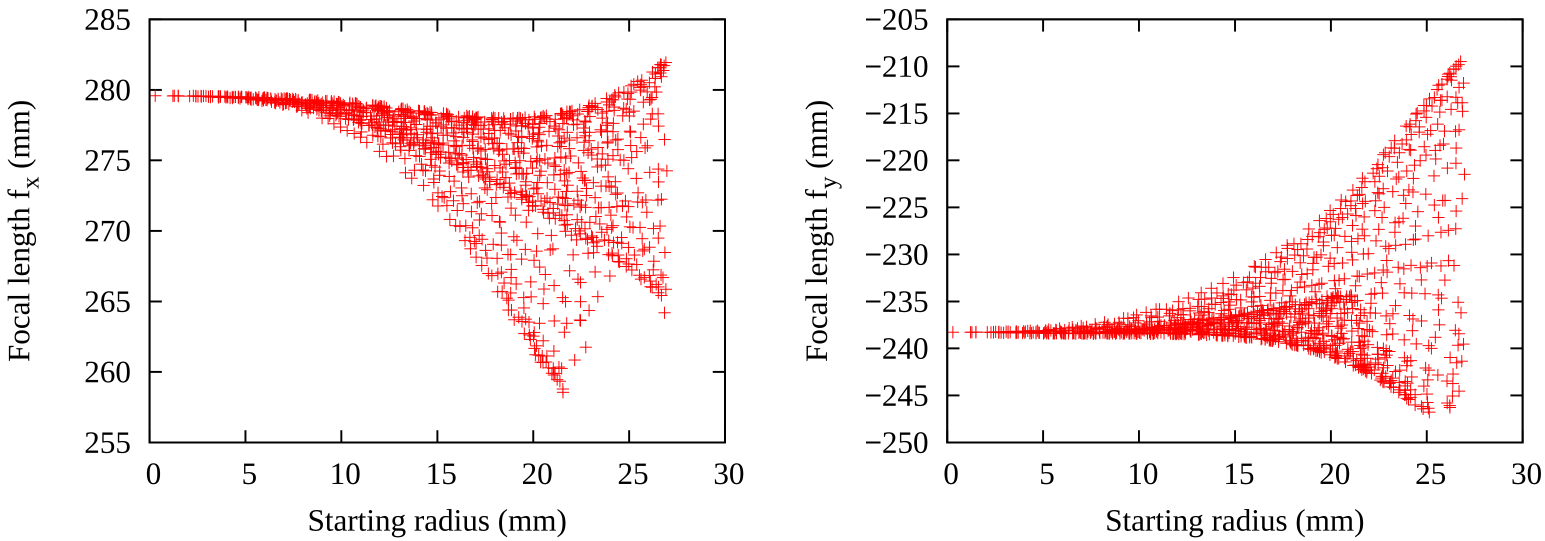




Less than 5 % aberration at $r < r_{\text{max}}$.

Electrostatic quadrupole focusing

Aberrations as a function of trajectory radius



Less than 5 % aberration at $r < r_{\mathrm{max}}$.

Magnetic quad the same with $k_B^2 = \frac{qB}{G_0 m v_z}$.

Electrostatic vs magnetic LEBT

- Electrostatic fields do not separate ion species.
  - Same focusing for all species.
  - Magnetic: separation of important (minor) beam.
- Electrostatic lenses are more compact.
- Power efficiency: Einzel $\sim$1 W, Solenoid $\sim$1000 W, water cooling usually required for magnetic elements.
- Space charge compensation can be conserved in magnetic lenses.
- Magnets are spark-free.

Beam Extraction from Plasma

Plasma-beam interface

Ions are extracted from a plasma ion source

1. Full space charge compensation ($\rho_- = \rho_+$) in the plasma

2. No compensation in extracted beam (single polarity)

Plasma-beam interface

Ions are extracted from a plasma ion source

1. Full space charge compensation ($\rho_- = \rho_+$) in the plasma
2. No compensation in extracted beam (single polarity)

The boundary is often thought as a sharp surface known as the *plasma meniscus* dividing the two areas.

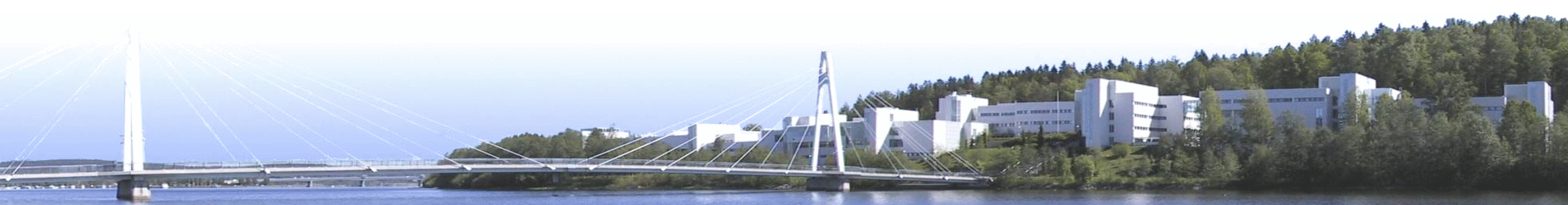
- Works as a thought model.
- In reality compensation drops going from plasma to beam in a transition layer with thickness $\sim \lambda_D \Rightarrow$ plasma sheath.
- E-field in extraction rises smoothly from zero.

Plasma flux

The plasma flux to a surface is

$$J = \frac{1}{4} q n \bar{v} = q n \sqrt{\frac{kT}{2\pi m}}$$

Extraction hole: ion beam samples plasma species with weight $\propto m^{-1/2}$.



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Extraction hole: ion beam samples plasma species with weight $\propto m^{-1/2}$.

Plasma flux sets the maximum current extractable

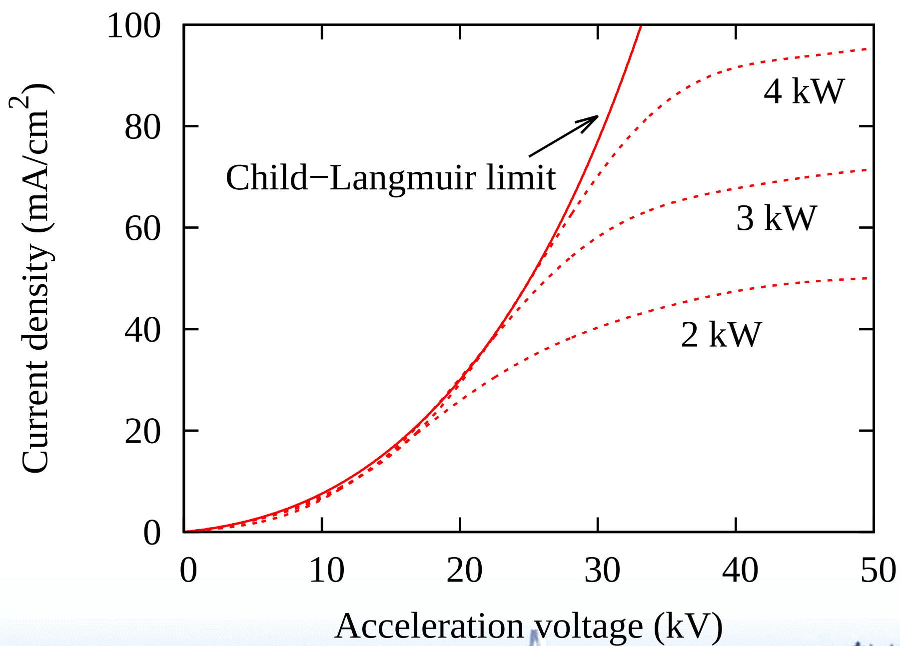
$$I = J A_{\text{meniscus}},$$

where the area of plasma meniscus $A_{\text{meniscus}} \neq A_{\text{aperture}}$ and therefore not quite constant. N-dimensional simulations needed for better estimates.

Child-Langmuir law

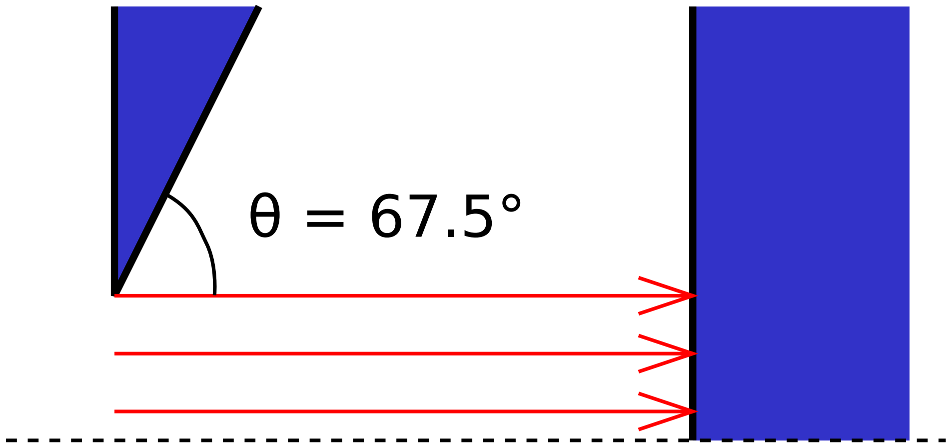
Ion beam propagation may also be limited by space charge. The 1D Child-Langmuir law gives the maximum current density for the special case where the beam is starting with $v_0 = 0$ (not plasma).

$$J = \frac{4}{9}\epsilon_0 \sqrt{\frac{2q}{m}} \frac{V^{3/2}}{d^2}.$$



Plasma electrode shape

For electrons starting from a flat surface with $v_0 = 0$ a perfectly perpendicular beam can be achieved with so-called **Pierce geometry**.



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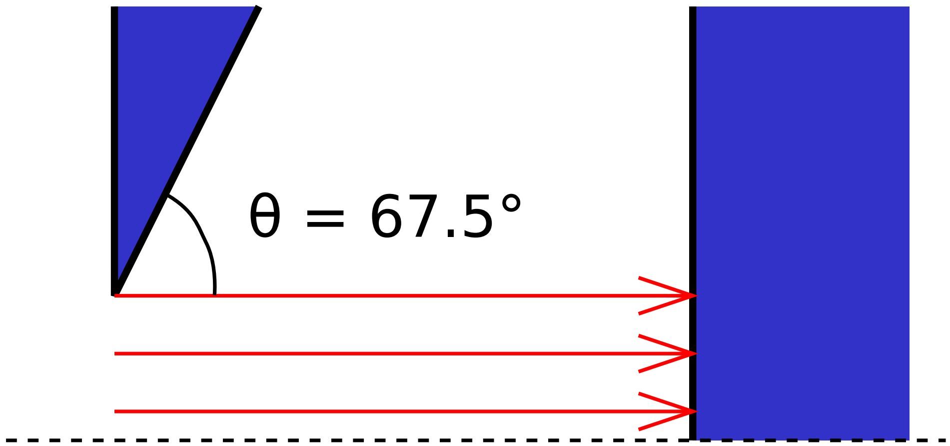


For ion sources, there is no magic geometry because the plasma sheath shape plays a major role in the optics of the plasma-electrode to puller-electrode gap.

Thermal plasma sheath

Classic 1D plasma sheath theory: In an electron-ion plasma a positive plasma potential is formed due to higher mobility of electrons. Situation is described by Poisson equation

$$\frac{d^2U}{dx^2} = -\frac{en_0}{\epsilon_0}\left[\sqrt{1-\frac{2eU}{m_i v_0^2}} - \exp\left(\frac{eU}{kT_e}\right)\right],$$

where the entering the sheath have an initial velocity

$$v_0 > v_{\text{Bohm}} = \sqrt{\frac{kT_e}{m_i}}$$

or energy

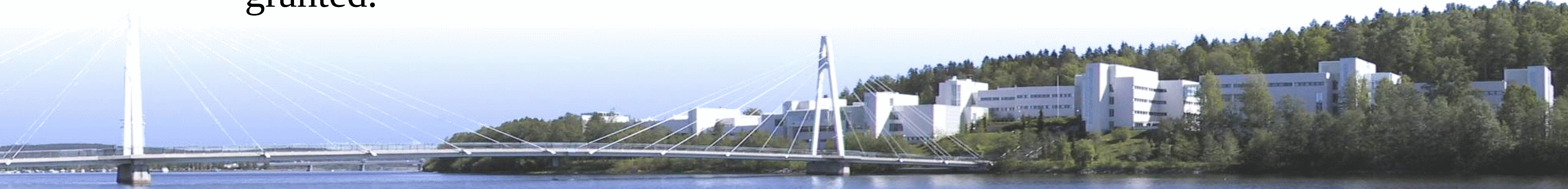
$$E_0 > \frac{1}{2} m_i v_{\text{Bohm}}^2 = \frac{1}{2} kT_e.$$

Model applies quite well for positive ion plasma extraction.

Positive ion plasma extraction model

Groundbreaking work by S. A. Self, *Exact Solution of the Collisionless Plasma-Sheath Equation*, Fluids **6**, 1762 (1963) and J. H. Whealton, *Optics of single-stage accelerated ion beams extracted from a plasma*, Rev. Sci. Instrum. **48**, 829 (1977):

- Model has been used very successfully for describing positive ion extraction systems since.
- Assumptions: no ion collisions, no ion generation, electron density only a function of potential (no magnetic field).
- Take the model with a semiempirical approach and use it as a tool proving to yourself that it works for your case — don't take it for granted.

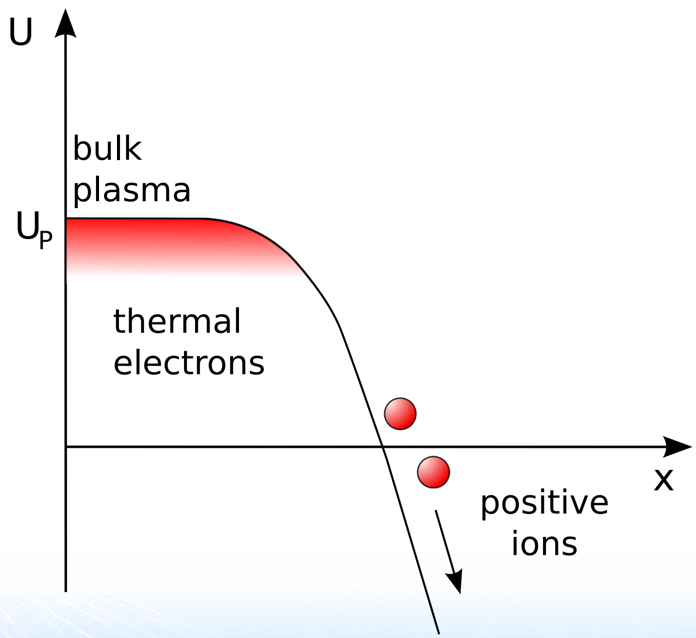


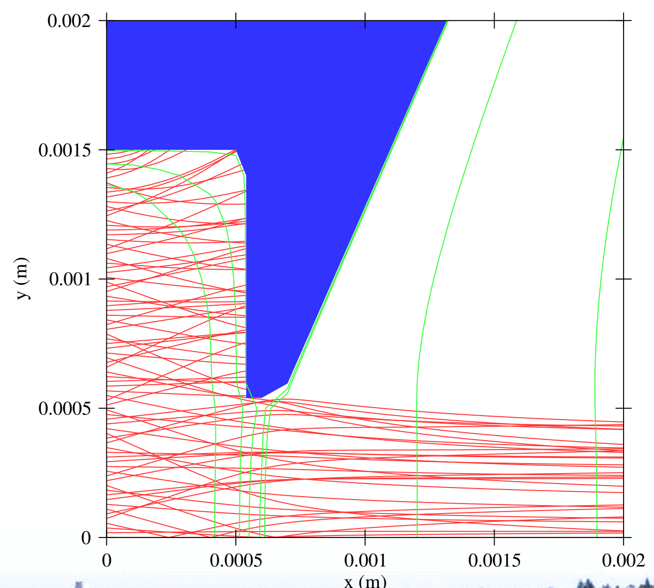
Positive ion plasma extraction model

Modelling of positive ion extraction

- Ray-traced positive ions entering sheath with initial velocity
- Nonlinear space charge term (analytic in Poisson's equation):

$$\rho_\mathrm{e} = \rho_\mathrm{e0} \exp\left(\frac{U - U_P}{kT_e/e}\right)$$





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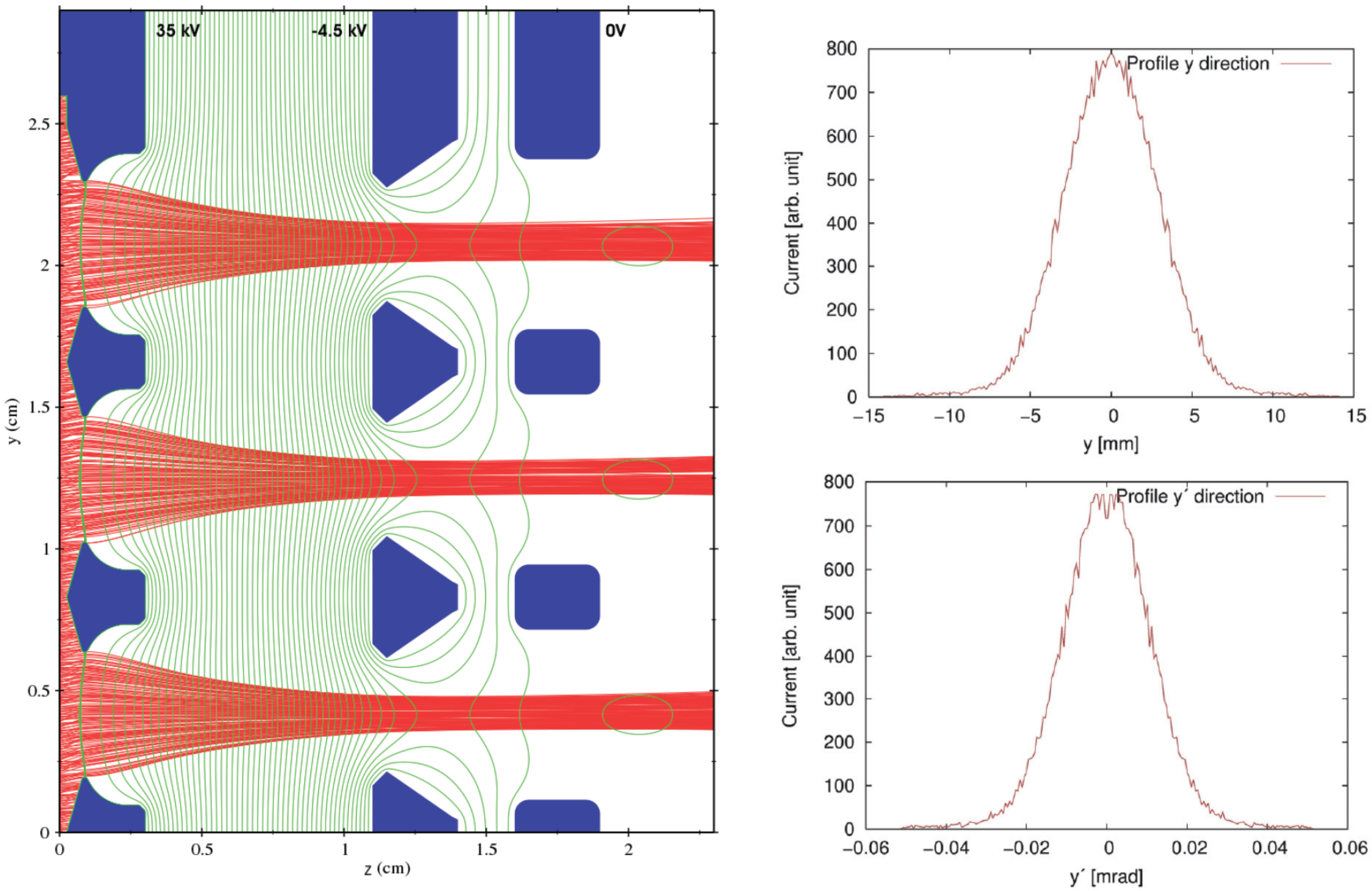
Example: Triode extraction

Three dimensional modelling of slit-beam system for PPPL

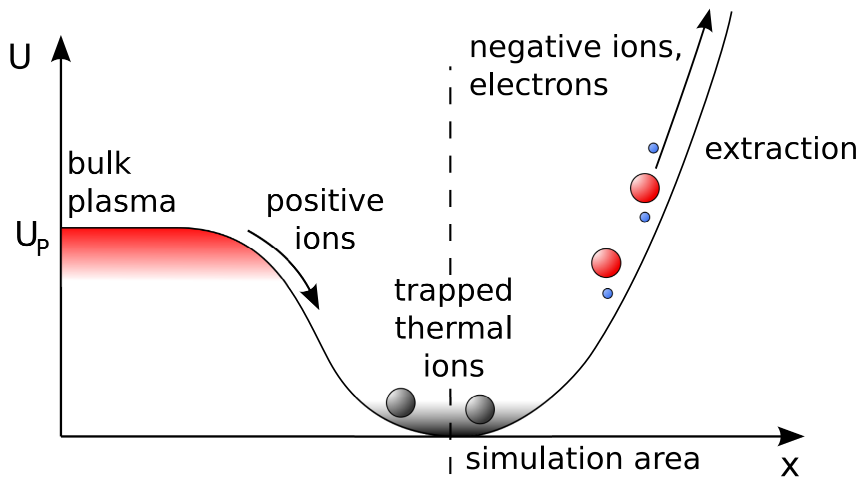


ICIS 2007, J. H. Vainionpaa, et. al., Rev. Sci. Instrum. 79, 02C102 (2008)

Negative ion plasma extraction model

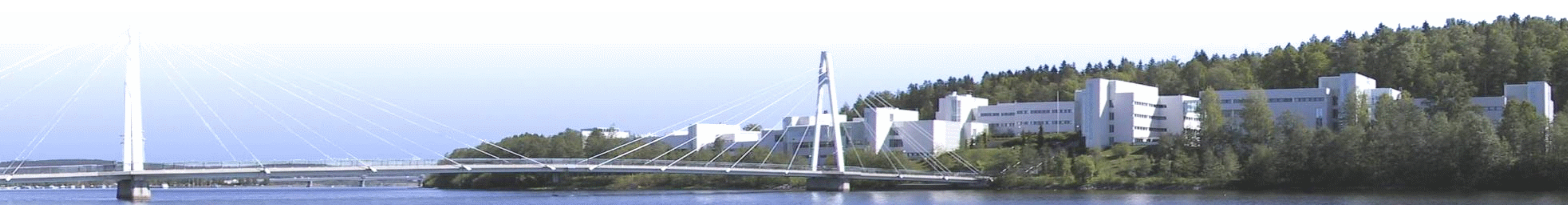
Modelling of negative ion extraction

- Ray-traced negative ions and electrons
- Analytic thermal and fast positive charges
- Magnetic field suppression for electrons inside plasma



$$\rho_{\mathrm{th}} = \rho_{\mathrm{th}0} \exp\left(\frac{-eU}{kT_i}\right)$$

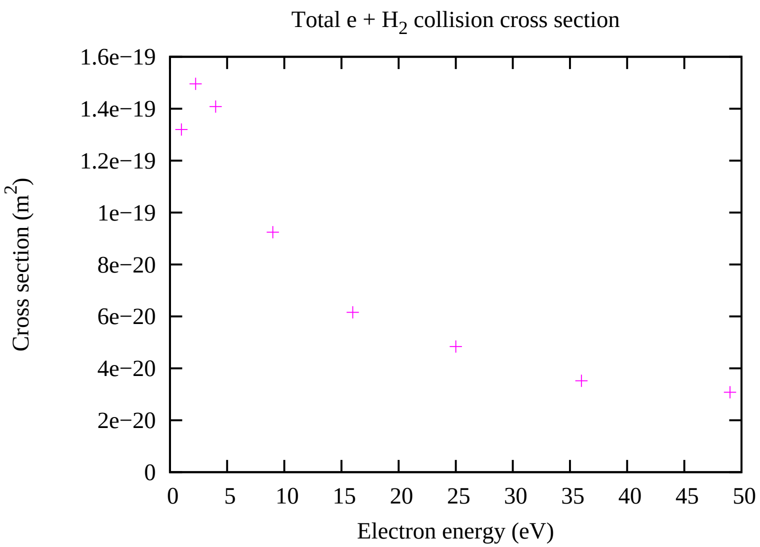
$$\rho_{\mathrm{f}} = \rho_{\mathrm{f}0} \left(1 + \mathrm{erf}\left(\frac{eU}{E_i}\right)\right)$$

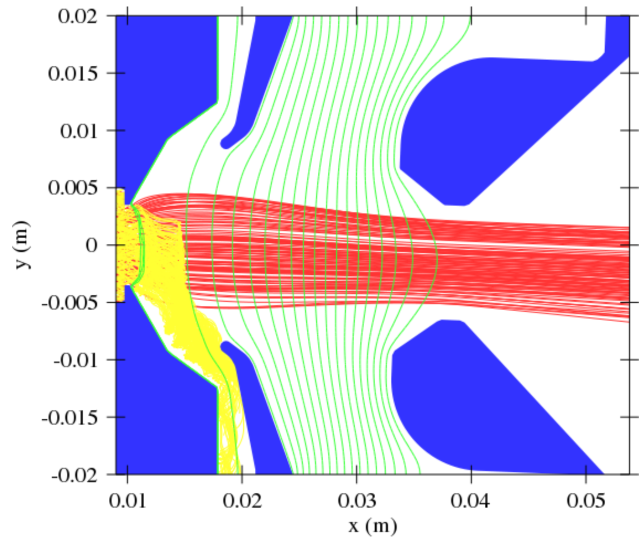


Negative ion plasma extraction model

Magnetic field suppression for electrons inside plasma

- Electrons highly collisional until velocity large enough
- Magnetic field suppression for electrons inside plasma



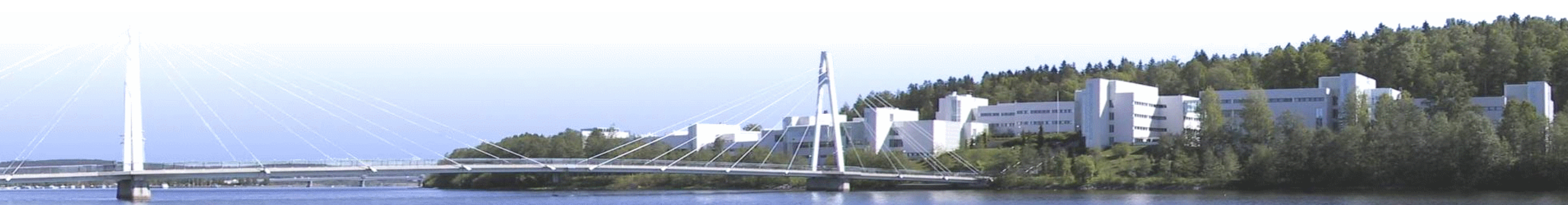


Difficulties in modelling extraction systems

Amount of parameters fed to the model is quite large

- Extracted species: J_i, T_i, v_0
- Positive ion plasma model: T_e, U_P
- Negative ion plasma model: T_i, E_i/T_i, gas stripping loss of ions
- All: space charge compensation degree and localization in LEBT

Methods: educated guessing (literature data), plasma measurements and matching to beam measurements (emittance scans).



Electron dumping

Negative ion source extraction systems need to dispose of the co-extracted electrons ⇒ magnetic elements needed

- Solenoidal focusing field (LANSCE, BNL)
- Source dipole B-field (ISIS Penning)
- Dipole field bending e^- to dump, source tilt for ions
- Dipole-antidipole dump and correction.

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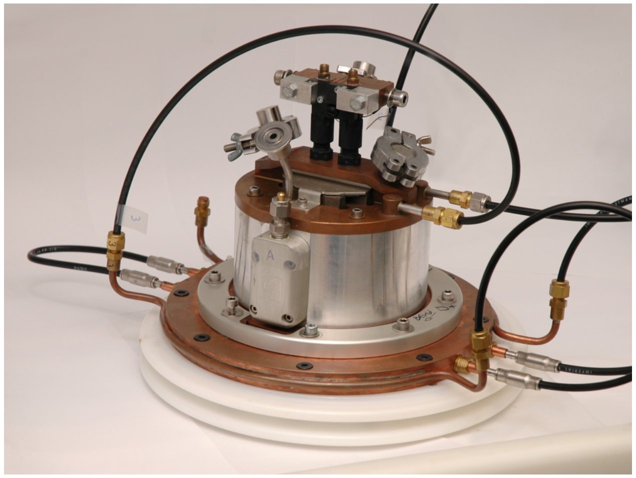
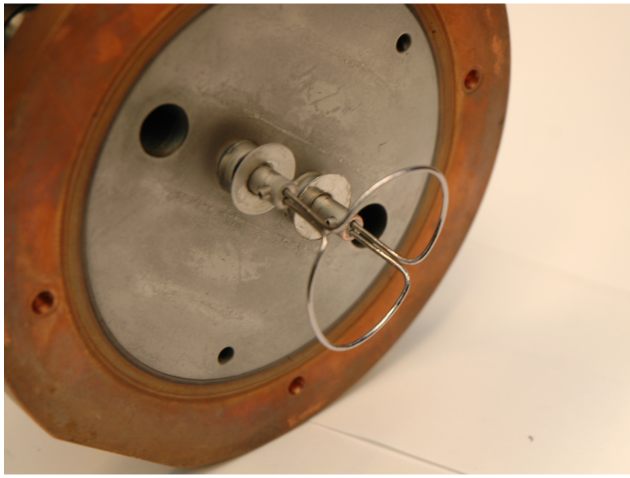
Practical boundary conditions:

- X-ray generation
- Heat load on dump (continuous, peak)
- Current load on power supplies

Design project example

K150 cyclotron at the Texas A&M needed a H^-/D^- source and extraction





Using spare LBNL style H^- multicusp ion source. Requirements:

- DC beam of 1 mA H^- and 0.5 mA D^-.
- Beam energy from 5 keV to 15 keV.

Texas A&M: Extraction requirements

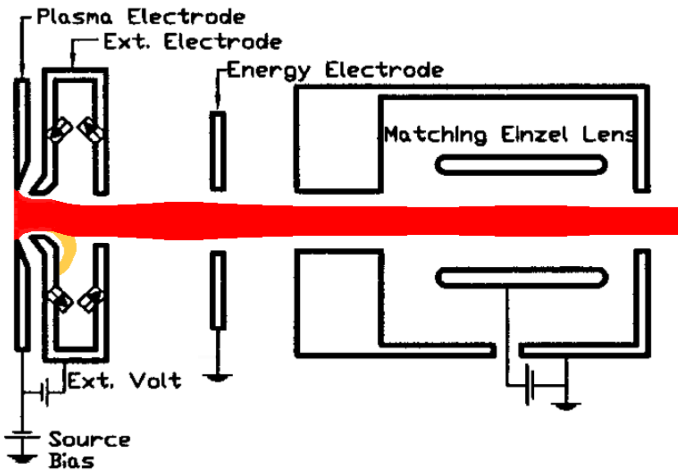
The application at the cyclotron needed a new H^-/D^- extraction for 1 mA:

- Negative ion extraction design is dominated by the necessary removal of co-extracted electrons (Factor of 10–20 more than ions).

Texas A&M: Extraction requirements

The application at the cyclotron needed a new H^-/D^- extraction for 1 mA:

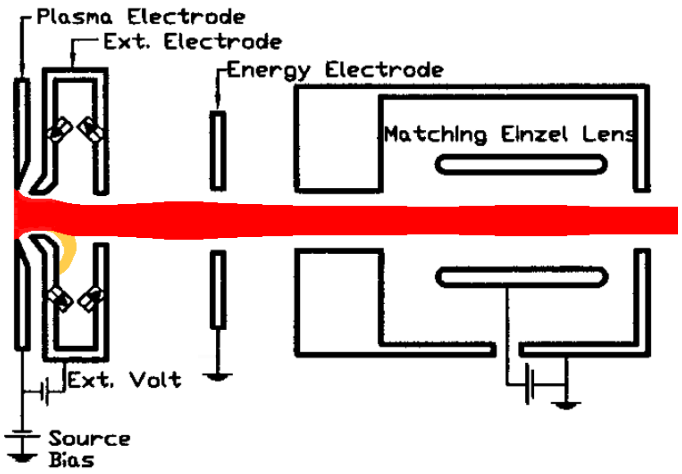
- Negative ion extraction design is dominated by the necessary removal of co-extracted electrons (Factor of 10–20 more than ions).
- Design by T. Kuo for newer TRIUMF sources has fixed energy at puller electrode and two anti-parallel B-fields for removing electrons and returning the H^- back to original angle.



Texas A&M: Extraction requirements

The application at the cyclotron needed a new H^-/D^- extraction for 1 mA:

- Negative ion extraction design is dominated by the necessary removal of co-extracted electrons (Factor of 10–20 more than ions).
- Design by T. Kuo for newer TRIUMF sources has fixed energy at puller electrode and two anti-parallel B-fields for removing electrons and returning the H^- back to original angle.



- With the LBNL source, this is not possible, because of internal filter field extends to extraction. Going with simple dipole field, tilted source design and fixed energy at tilt.

Texas A&M: Extraction design

First the geometry, electrde voltages and plasma parameters were optimized using cylindrically symmetric simulations (fast).

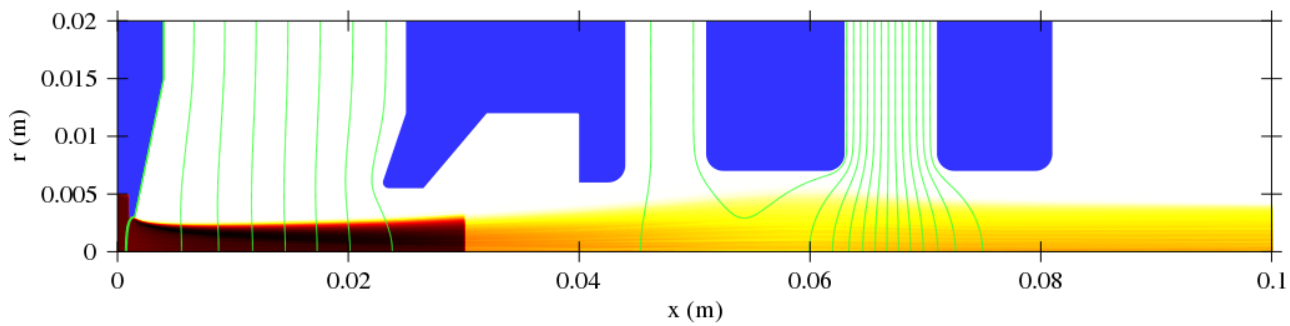
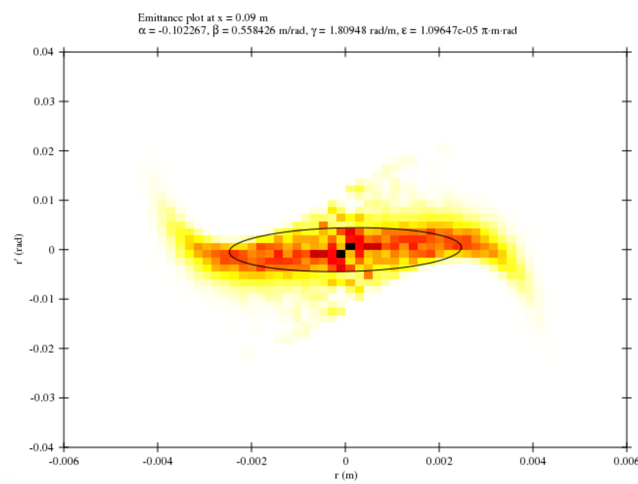


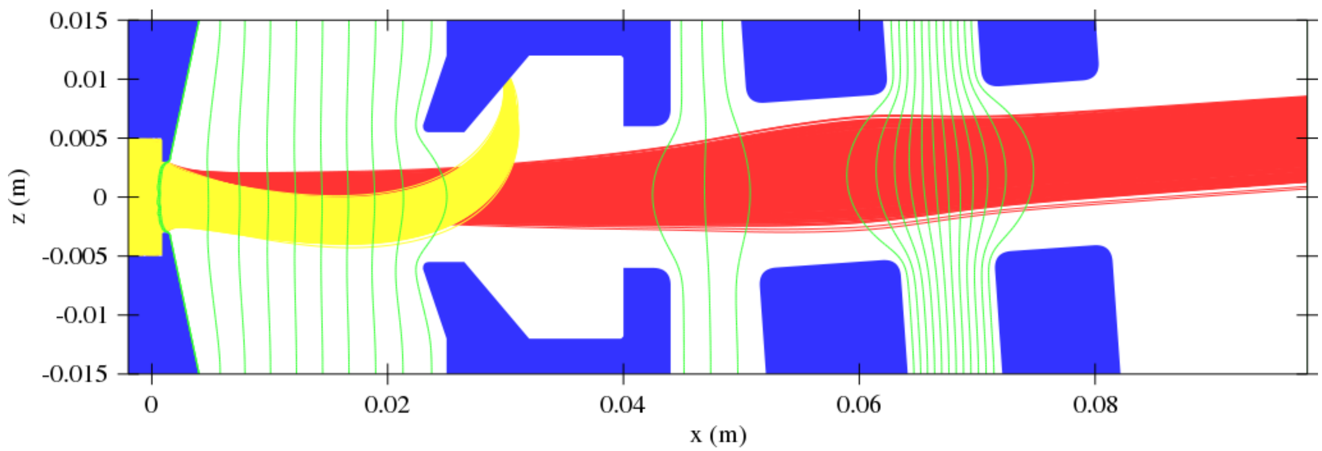
Table of electrode voltages

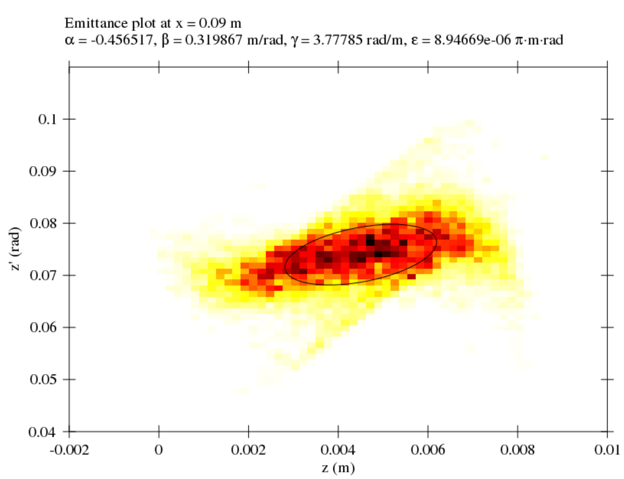
HV	Puller	Einzel
-5	+1	-3.2
-8	-2	-5.8
-12	-6	-8.2
-15	-9	-10.5

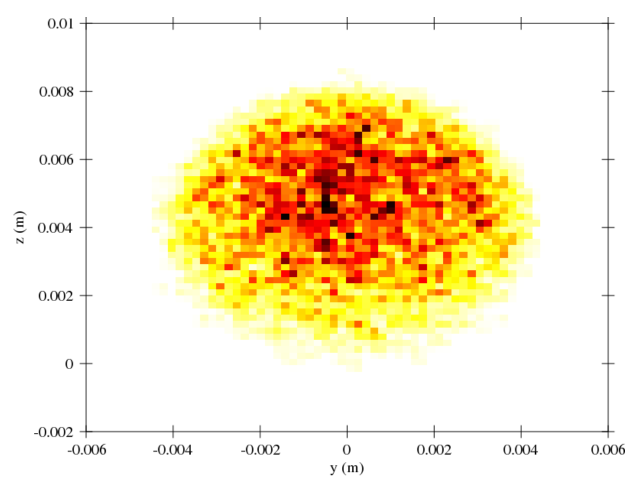


Texas A&M: 3D geometry design

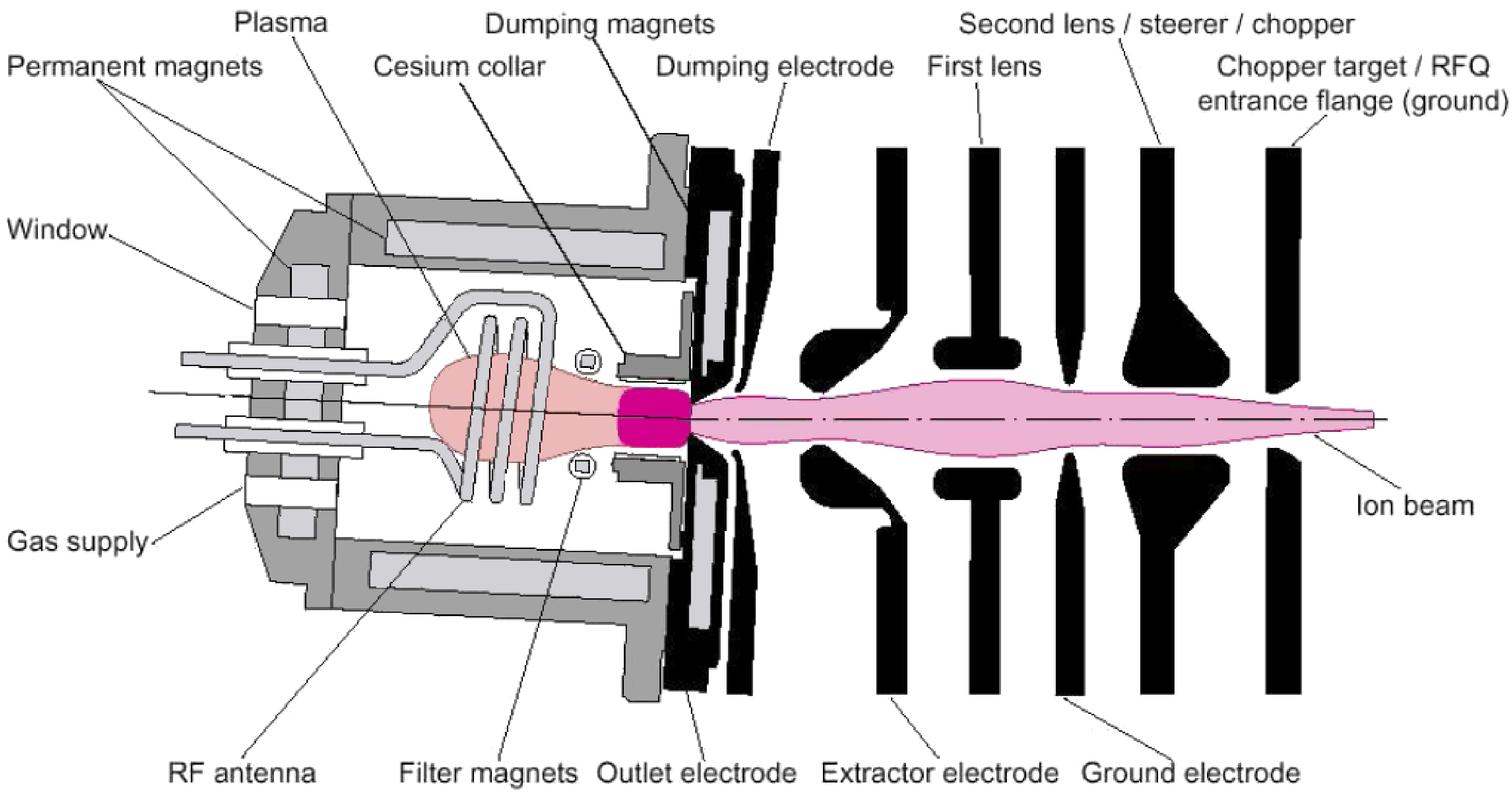
Geometry was optimized for low-aberration emittance and centered beam







Example: SNS ion source baseline extraction



SNS: plasma parameters

Previously, the same plasma parameters were used as in other published simulation work. Fine tuning was now made made to match results to experimental emittance data.

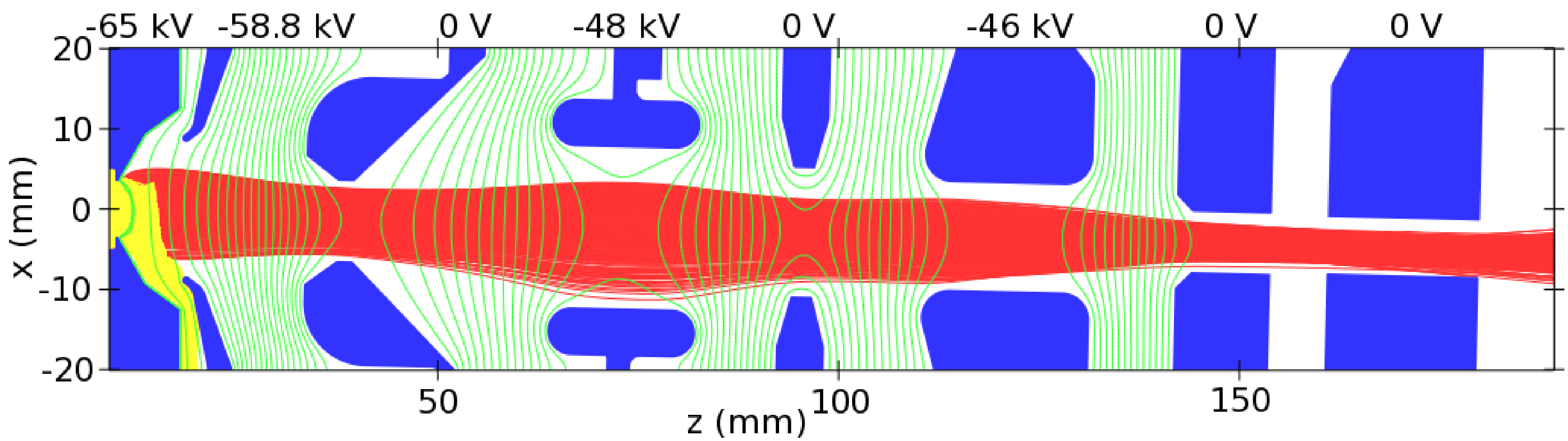
SNS: plasma parameters

Previously, the same plasma parameters were used as in other published simulation work. Fine tuning was now made made to match results to experimental emittance data.

- Transverse temperature of e$^-$ and H$^-$ $T_t = 2.0$ eV
- Plasma potential $U_P = 15$ V
- Emitted electron to ion ratio $I_{e^-}/I_{H^-} = 10$
- Thermal positive ion to negative ion ratio $\rho_{X^+}/\rho_{H^-} = 0.5$
- Initial energy of particles $E_0 = 2.0$ eV

SNS: Extraction simulation

Tilted SNS extraction delivering 64 mA of H^- beam to the RFQ.



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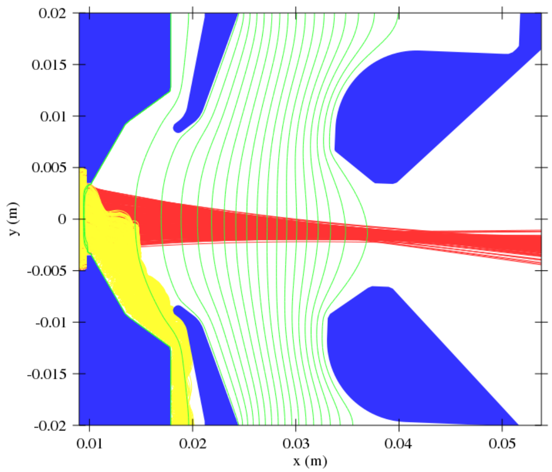
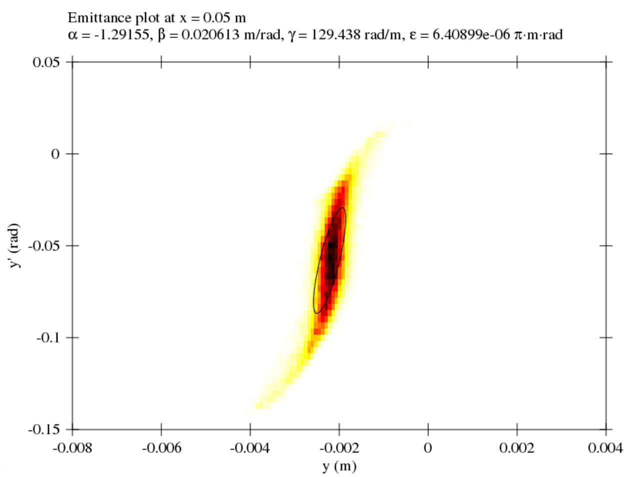
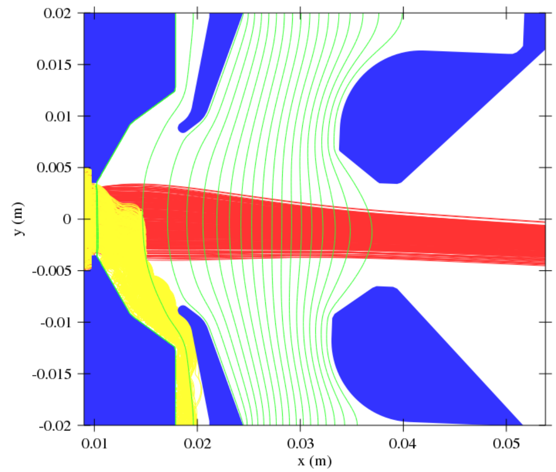
Plasma-beam transition behaviour

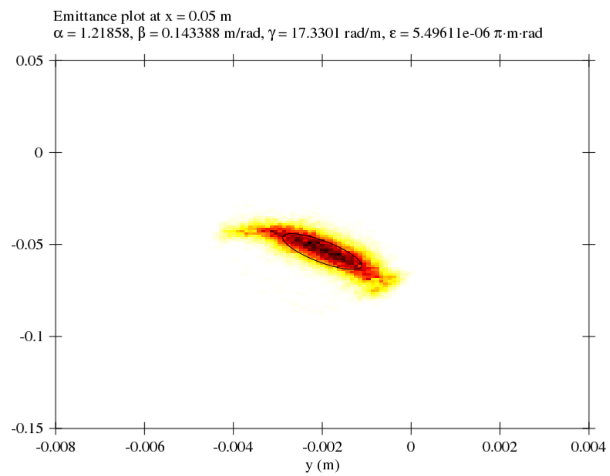
30 mA/cm^2



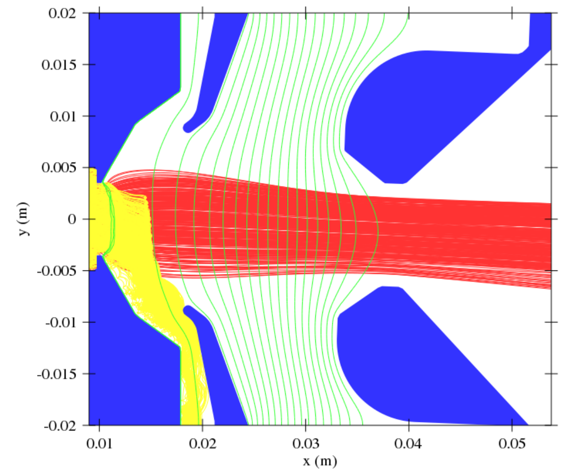


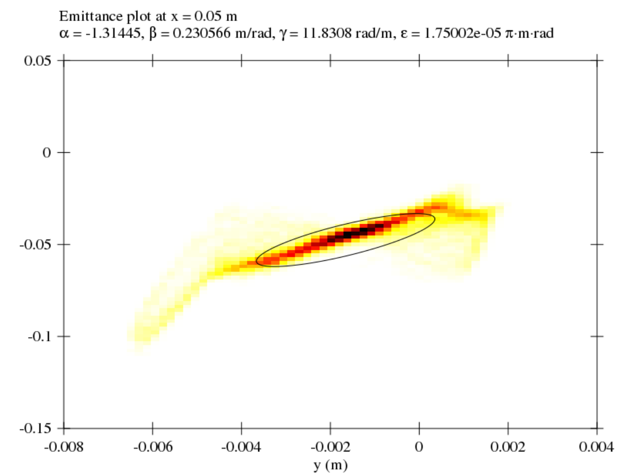
60 mA/cm^2

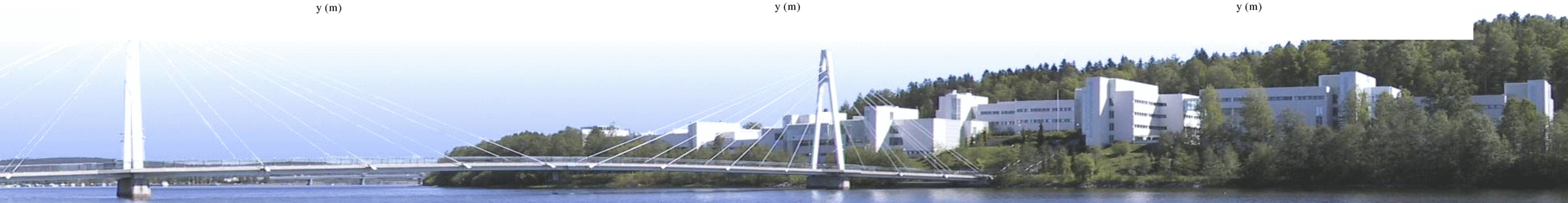




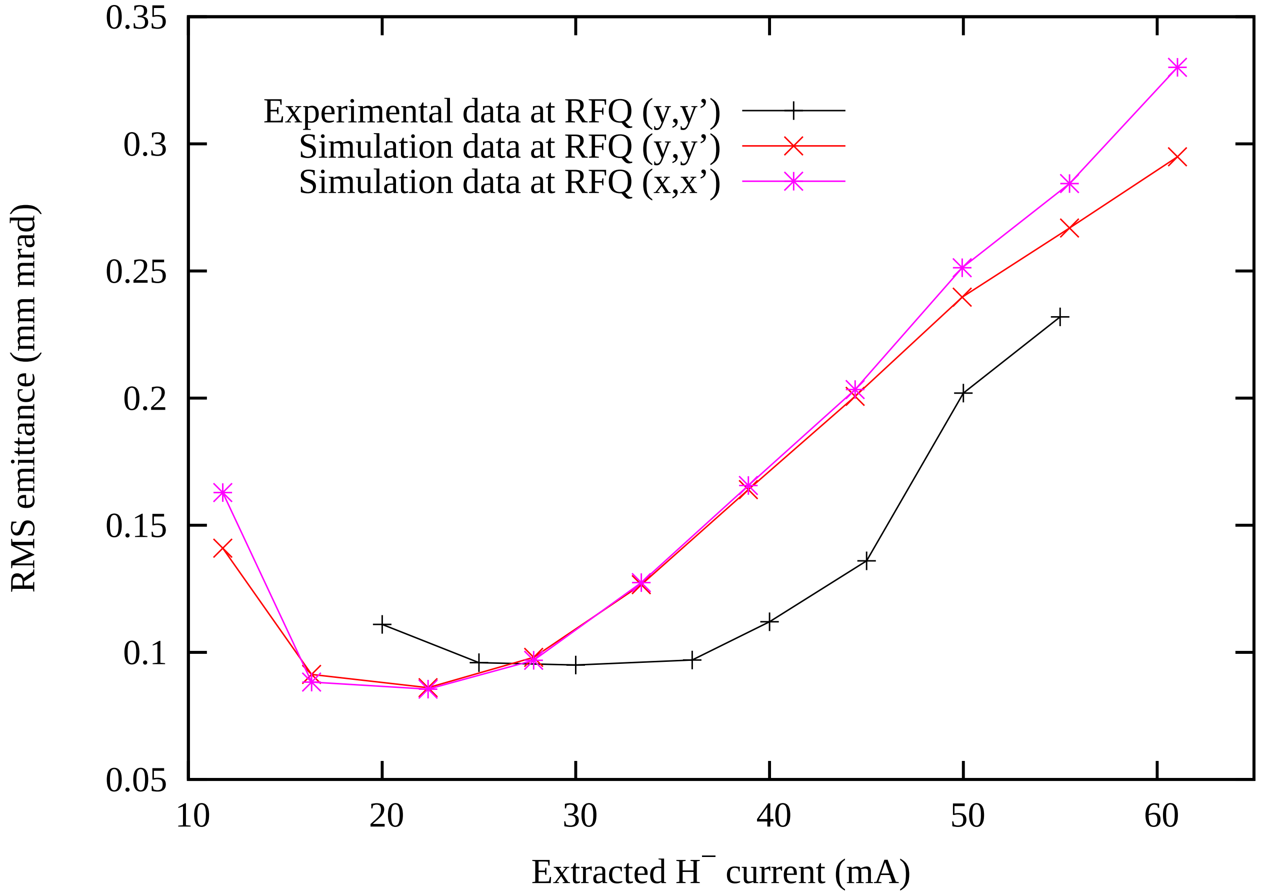
120 mA/cm^2







Emittance comparison



Experimental emittance data: B. X. Han, RSI 81 02B721 (2010)

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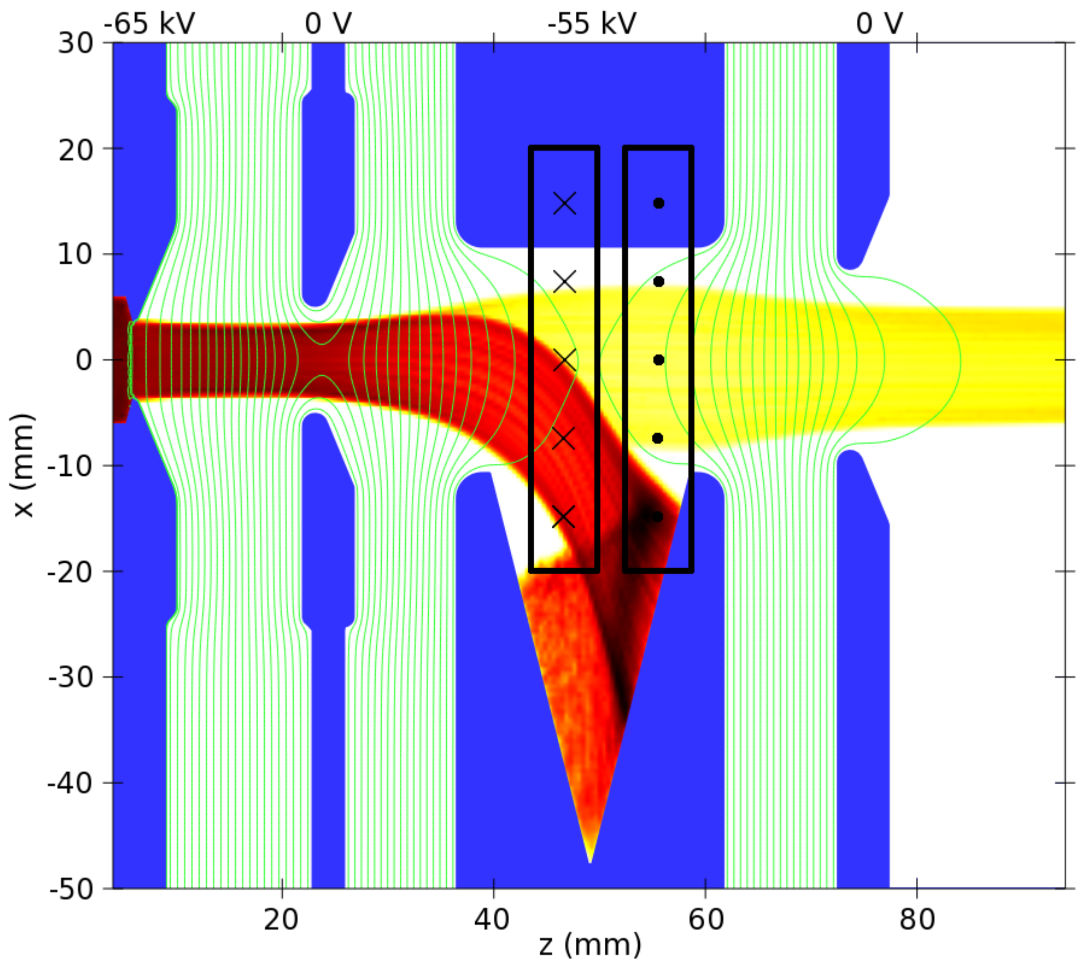
Proposed design



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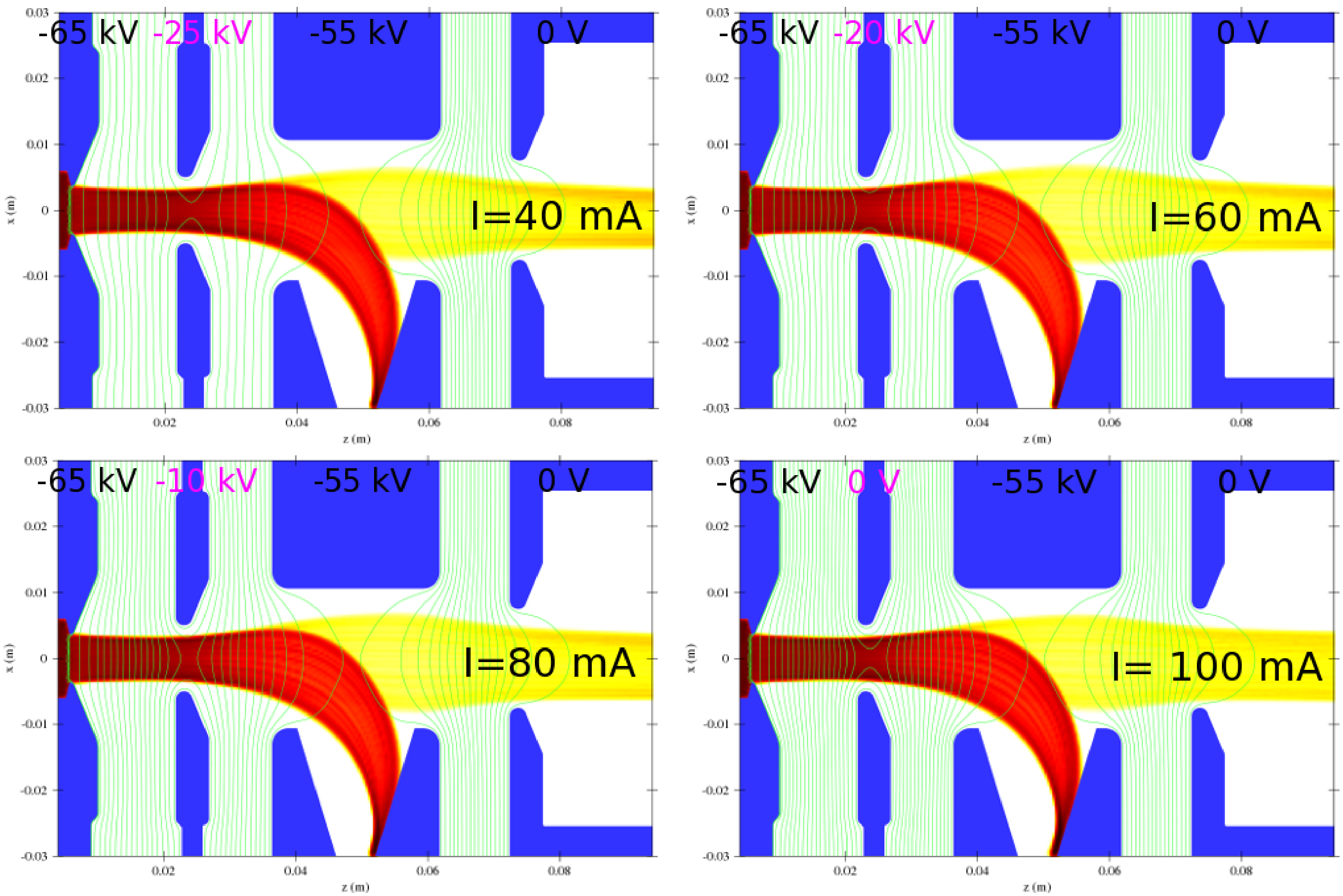
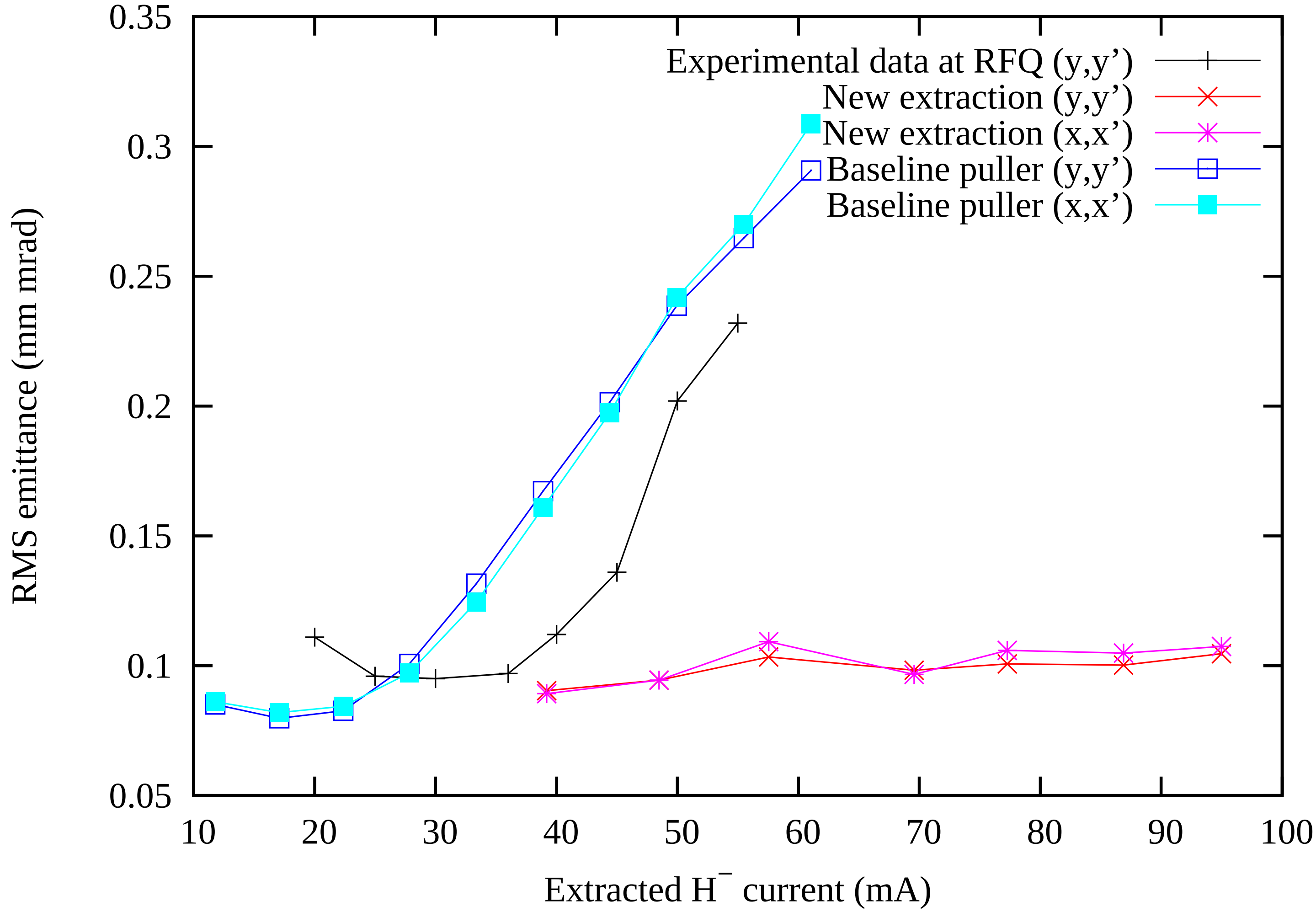
Puller voltage adjust



Emittance comparison



Extraction adjustability

Important example: even if a magnetic LEBT is used for beam transport, a diode extraction is not sufficient because it has no adjustability!

An electrostatic extraction system must be a triode system or a diode + Einzel at minimum to be able to **adjust** to changing plasma conditions.

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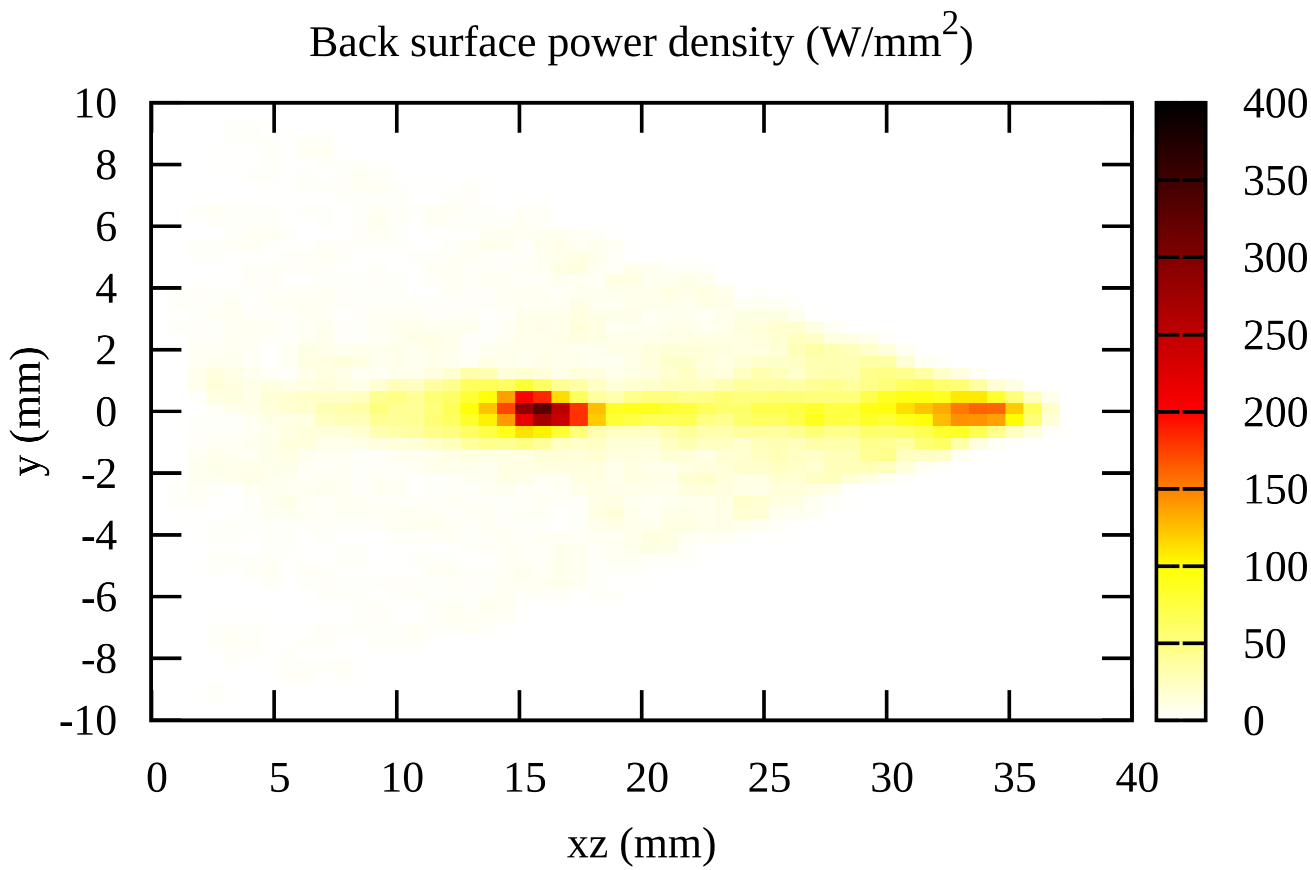
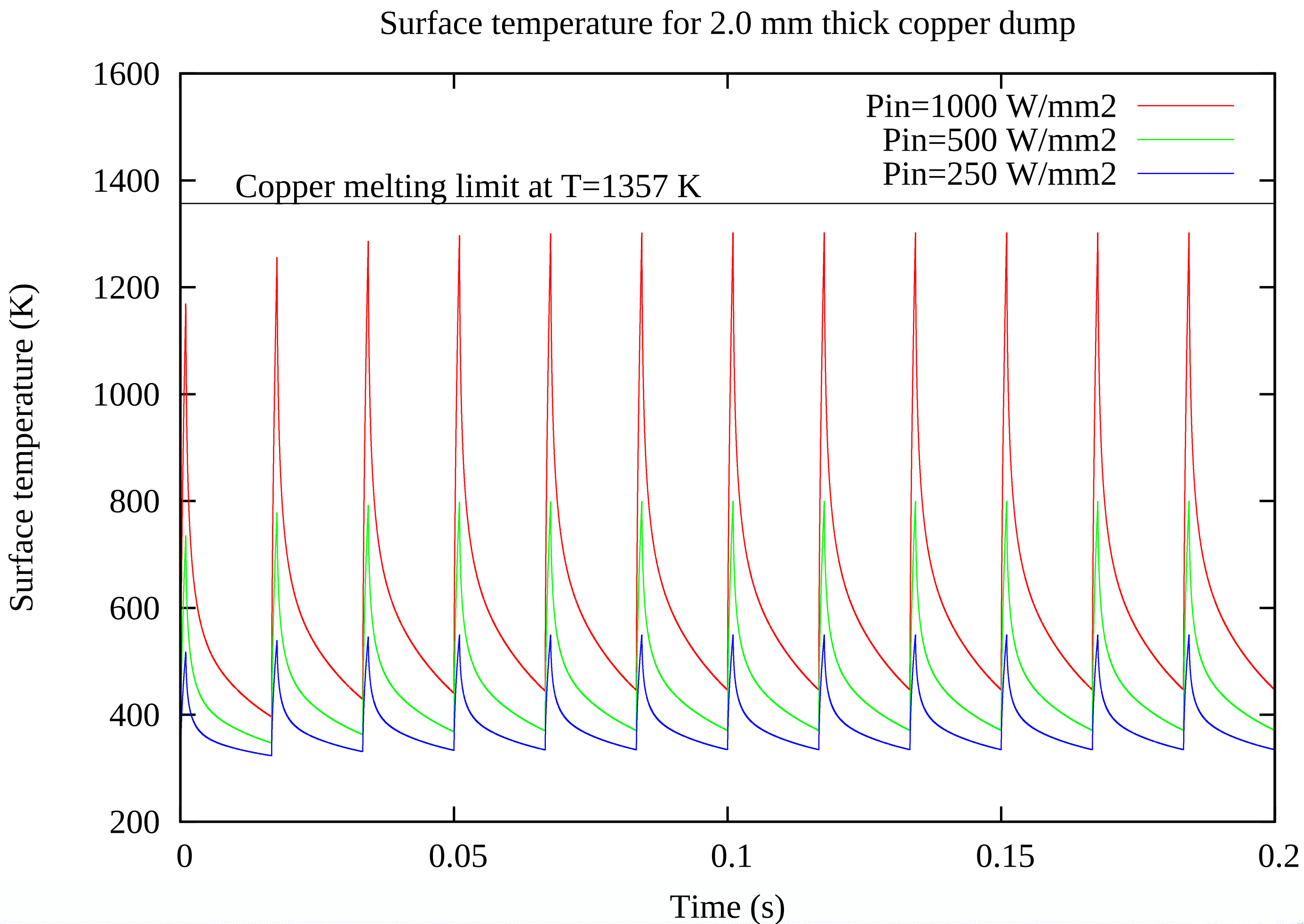
Power density on dump

Assuming 100 mA of H^- and e^- to H^- ratio of 10



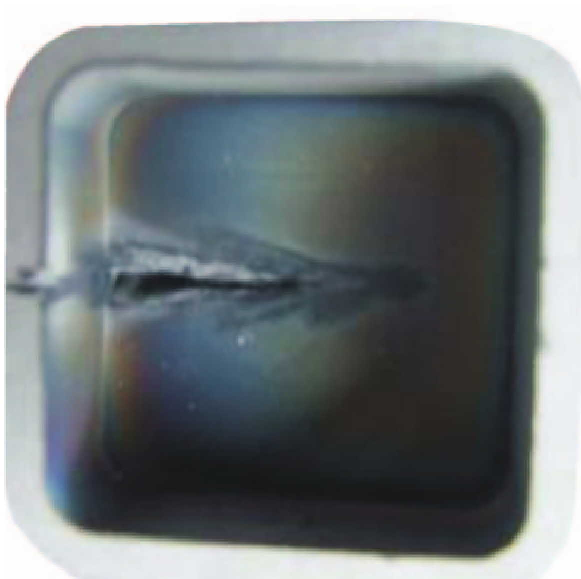
Thermal considerations

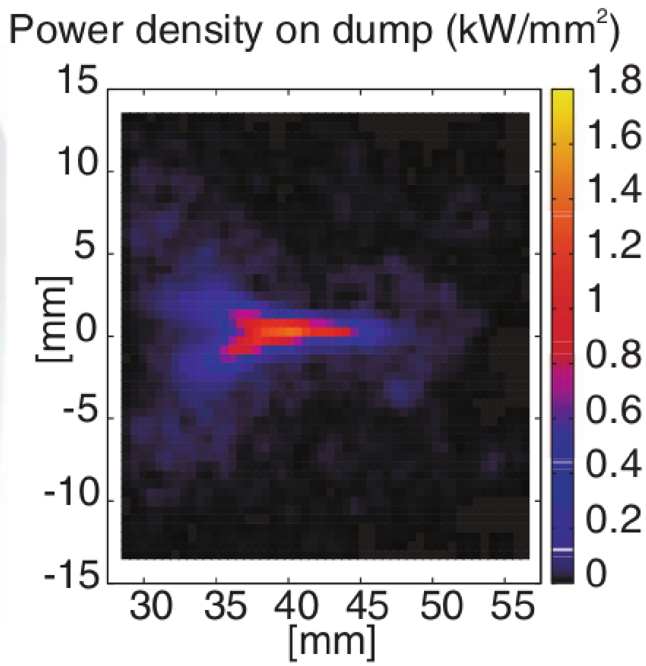
SNS pulse pattern of 60 Hz, 1 ms beam on.



Thermal considerations

If you fail to take it in account

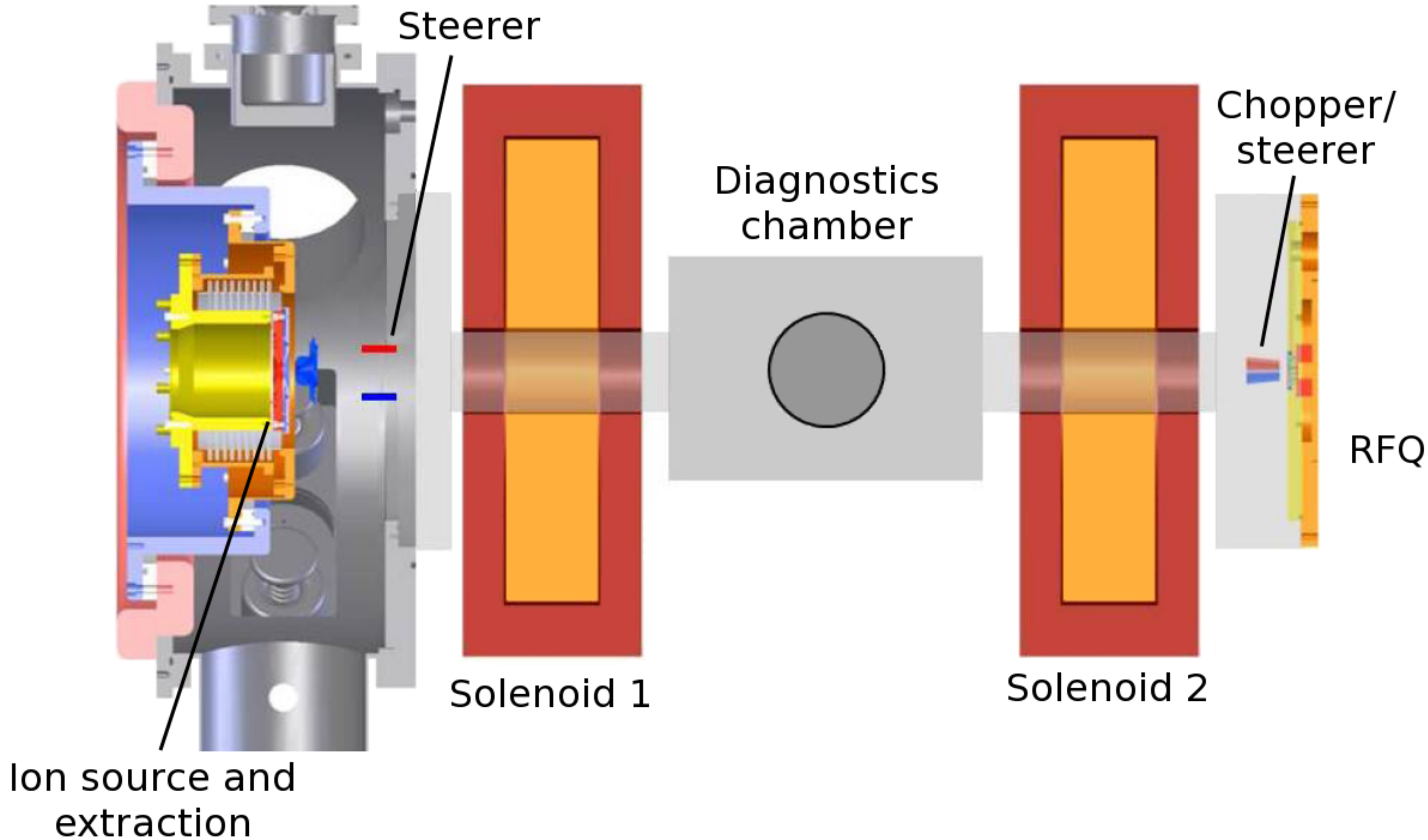




Linac4 graphite electron dump test results, Ø. Midttun, ICIS 2011

Magnetic LEBT

Proposed future magnetic LEBT for SNS



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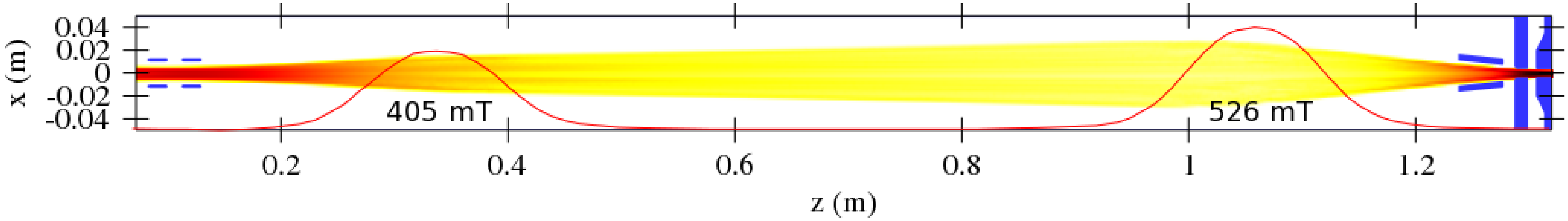
Magnetic LEBT simulations

Simulation of 60 mA beam, 90 % compensation in LEBT assumed



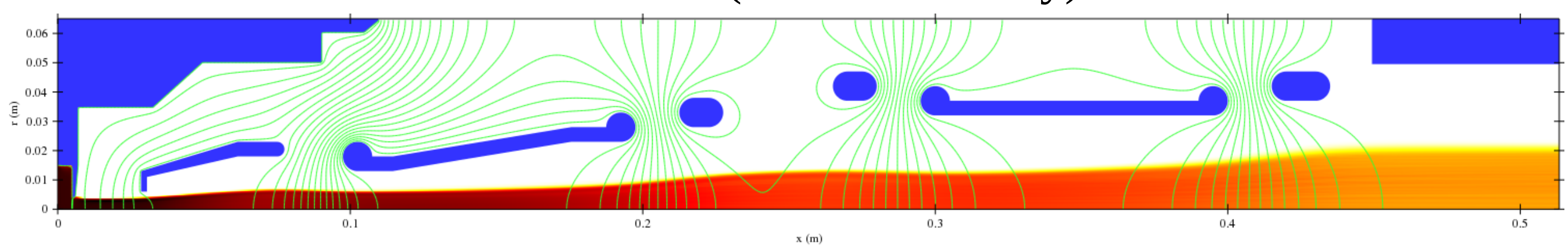
- Magnetic LEBT throughput calculated with beam tracer software.
- Usually long magnetic LEBT systems calculated with matrix codes.
- But: space charge induced emittance growth in this case is important.

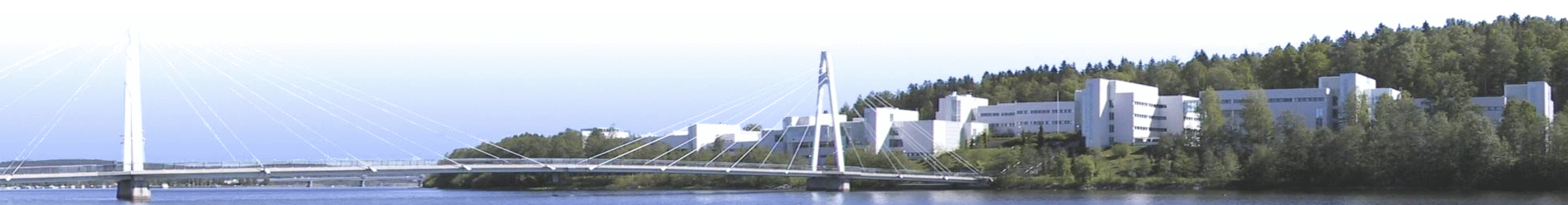
Example: JYFL 14 GHz ECR

At JYFL we are working on improving the injection line from ECR ion sources to the K-130 cyclotron.

A beam tracing code is used to calculate the electrostatic extraction (first 50 cm) affected of course by the ECR magnetic field stray fields. Old extraction modelled to gain confidence on simulations.

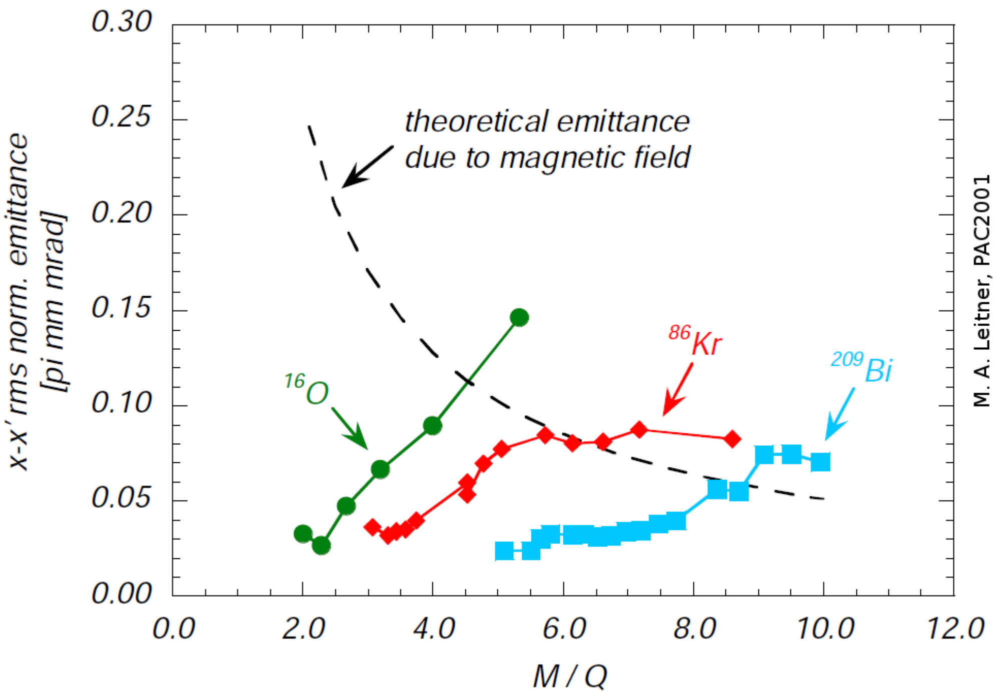
New extraction (installed in May)





Difficulty of ECR simulations

The emittance from the ECR plasma is dominated by the magnetic field:



The experimental data doesn't fit ⇒ species are not extracted from a homogenous plasma. High Q species are concentrated closer to axis.

Difficulty of ECR simulations

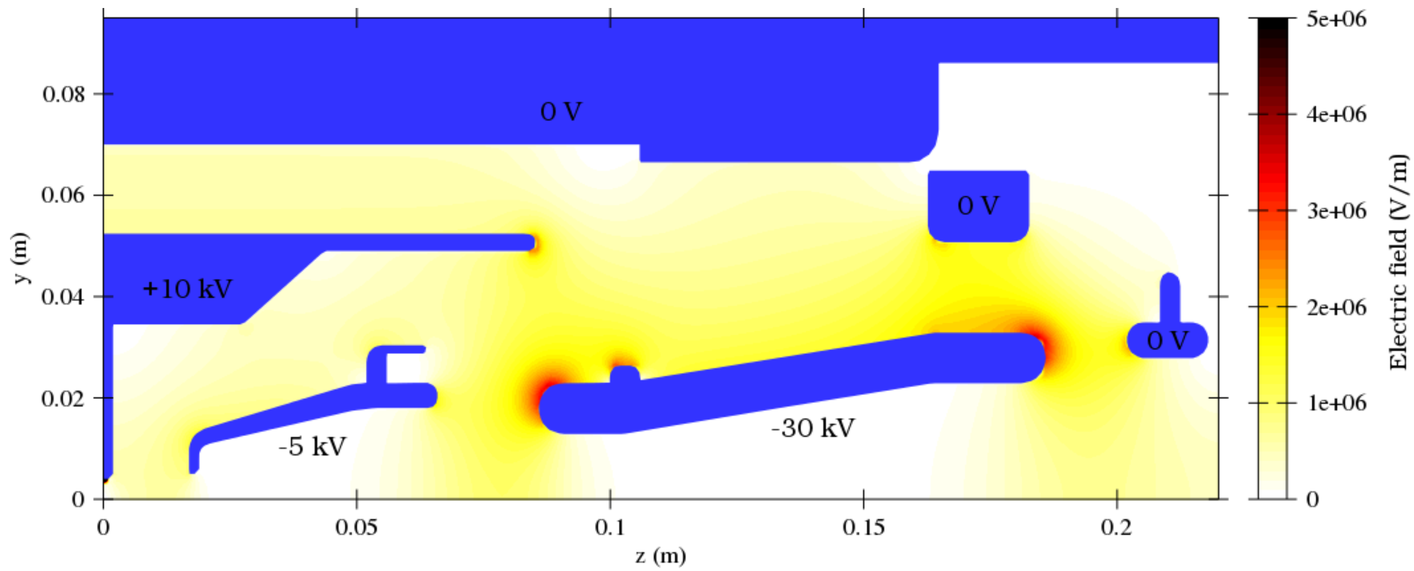
ECR plasma parameters:

- Beam contains (usually) several isotopes m_i with several charge states q_j each
- All of the species have intensity J
- Common T_t, E_0
- Starting distribution? $r < r_{\mathrm{max}}$ or more complicated (triangular) shapes?
- MB compensating electrons?

A lot of unknowns. For multipurpose ECR (like at JYFL) a single optimization is not relevant. Different case for a fixed beam system.

High voltage considerations

Sparking limits need to be considered when designing extraction systems.



Surface field of 5 MV/m was taken as a limit in the new JYFL ECR extraction. Maximum E-field with no sparking is a function of many parameters: surface smoothness, vacuum, density of charged particles in system, etc.

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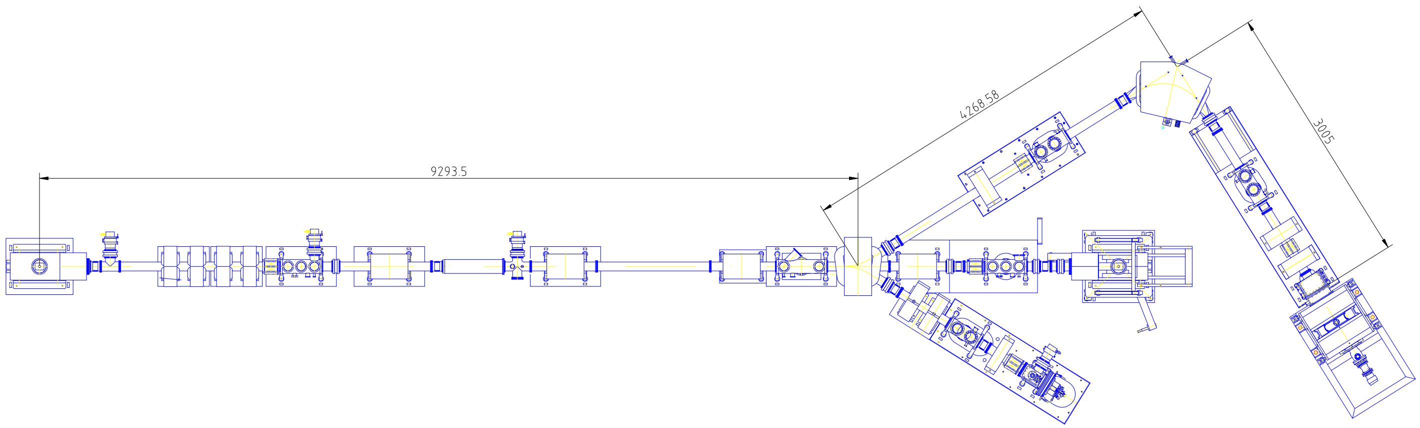
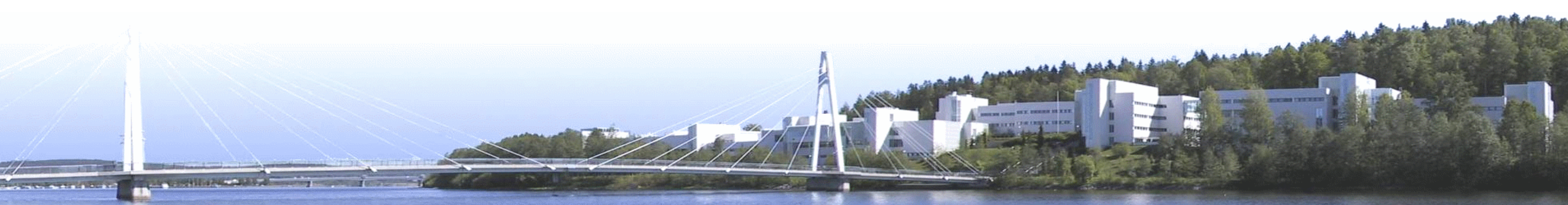
JYFL ECR

Plasma and beam tracing simulations are used to give a starting point for the matrix code used for rest of the beamline.



Final words

- Take your time when analyzing/designing extraction systems, there is a huge number of issues that need to be taken in account.
- Be clear when communicating about emittance.
- Provide enough adjustment knobs for extraction systems, especially for plasma-puller system (gap, voltage, plasma density).
- Use simulations and experiments hand-in-hand. Doing only simulations will lead to garbage-in, garbage-out type of science.
- Choose your matrix code wisely.



Thank you for your attention!