# modeling

Peter Spädtke
GSI Darmstadt

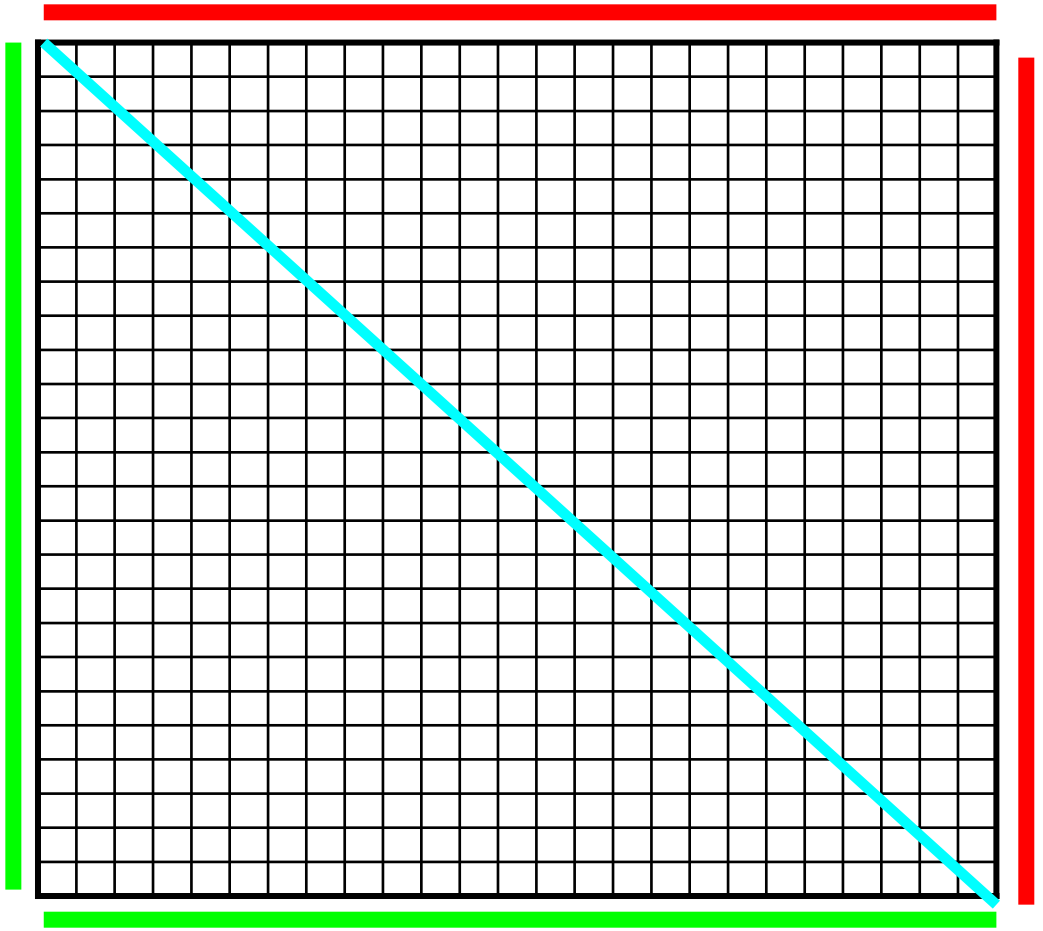
# outline

- some remarks about hardware and software
- model for a drifting ion beam
- model for ion sources
  - fixed emitter
  - volume ion sources
  - laser ion sources
  - Penning ion sources
  - electron cyclotron resonance ion sources
  - $H^-$ ion sources
- conclusion

benchmark

1. create a 2D mesh with 101 * 101 nodes.
2. Assign -1 respectively 1 to the boundaries
3. solve the difference equation for each node(except on the boundary):
   $\Phi(i,j) = (\Phi(i+1,j) + \Phi(i-1,j) + \Phi(i,j+1) + \Phi(i,j-1))/4$
4. Execute 100 iterations

After 100 iterations the achieved solution can be compared with the analytical solution, which is 0.5 on the diagonal.



computer (ancient time)

ancient time = before 1990

laptop (today)

| Hardware | language | CPU time (s) R*4 | R*8 |
|---|---|---|---|
| McIntosh | Pascal | 60000 | --- |
| | Basic | 54000 | --- |
| C64 | Assembler | 18000 | --- |
| Atari ST | Assembler | 1800 | --- |
| IBM PC | Fortran | 7055 | --- |
| VAX8700 | Fortran | 10.5 | 15.8 |
| IBM3090 | Fortran | 2.61 | --- |
| Cray XMP | Fortran | 2.2 | --- |
| laptop2012 | Fortran | 0.031 | 0.031 |

**Memory restrictions for a 3D grid:**

| | |
|---|---|
| computer (ancient time)<br>ancient time = before 1990 | 30 * 30 * 30 nodes |
| laptop (today) | 500 * 500 * 500 nodes |

**Remark to Dan Faircloth's talk, titled** *Technological aspects: High Voltage*

**of course there are many programs available for field calculations-even in 3D. If it comes to extraction simulation, the KOBRA3-INP code should be mentioned – instead of some he did mention, which are not possible to use for that application.**

# Space Charge Compensation of a Charged Particle Beam

assume a positive ion beam (cw)

building up of a positive space charge potential will occur.

repulsive action of all charges on each other will create a diverging beam.

the primary ion beam will have collisions with surrounding boundaries (beam tube, …) and with residual gas atoms.

already presented by N.Chauvin on Thursday

ions and electrons will be created.

the space charge potential of the positive ion beam is attractive for electrons, ions will be repelled from the positive potential.

the electrons will be trapped within this space charge potential, and therefore the space charge potential will be reduced.

final state: cancellation of electrostatic forces caused by the space charge.

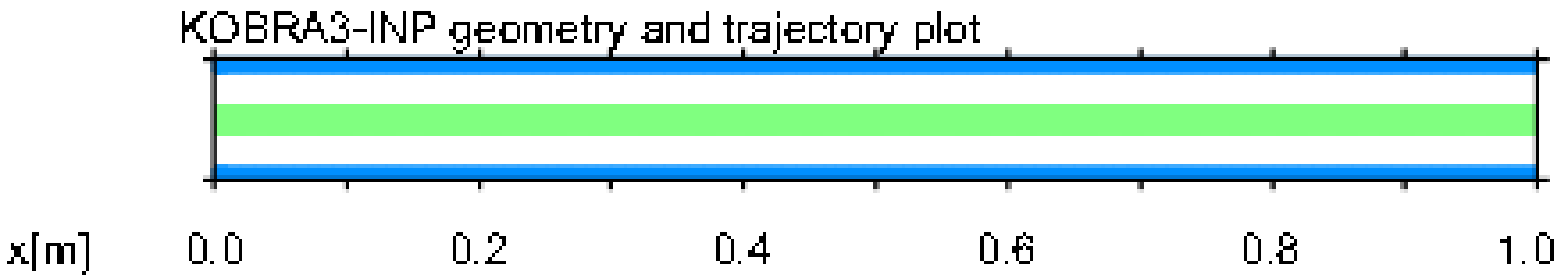
# Space Charge Compensation of a Charged Particle Beam

**… of course only, if there is no leakage for electrons along the drifting ion beam!**

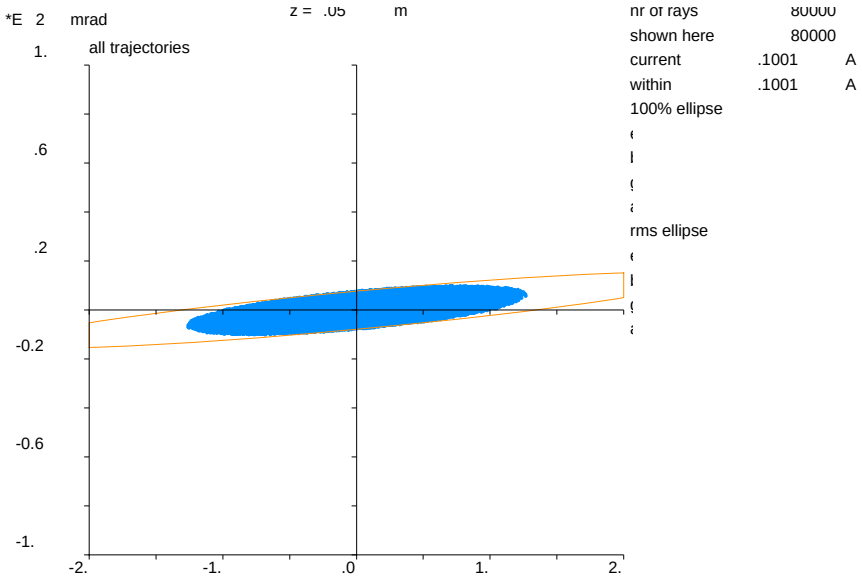
# Space Charge Compensation of a Charged Particle Beam

- How strong is the effect?
  - Compare both solutions: full space charge and no space charge
- If the difference is negligible: problem solved
- If the effect is not negligible
  - Assuming a net current
  - Assuming a plasma model
    - No external field
    - With external electric field
    - With external magnetic field

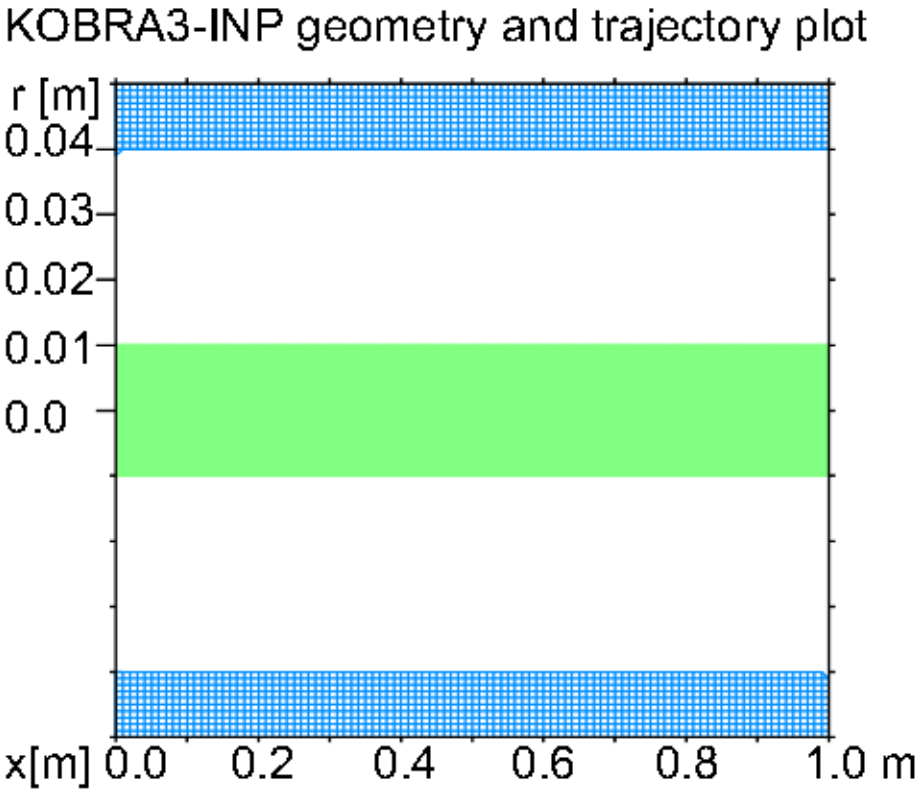
# Laplacian solution (space charge = 0)



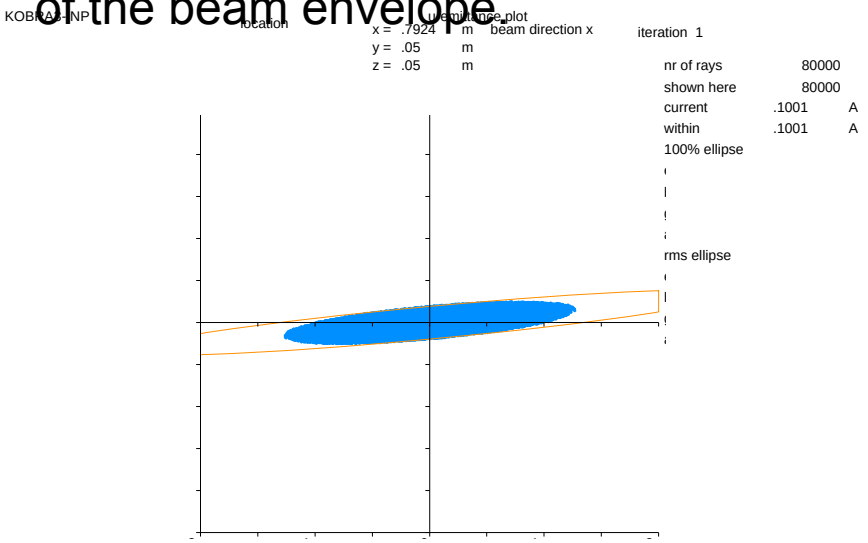

- Assume a drifting ion beam (singly charged Argon, 35keV energy, small emittance and small divergence (→0)).
- no external fields.
- beam radius stays constant along a drift section.



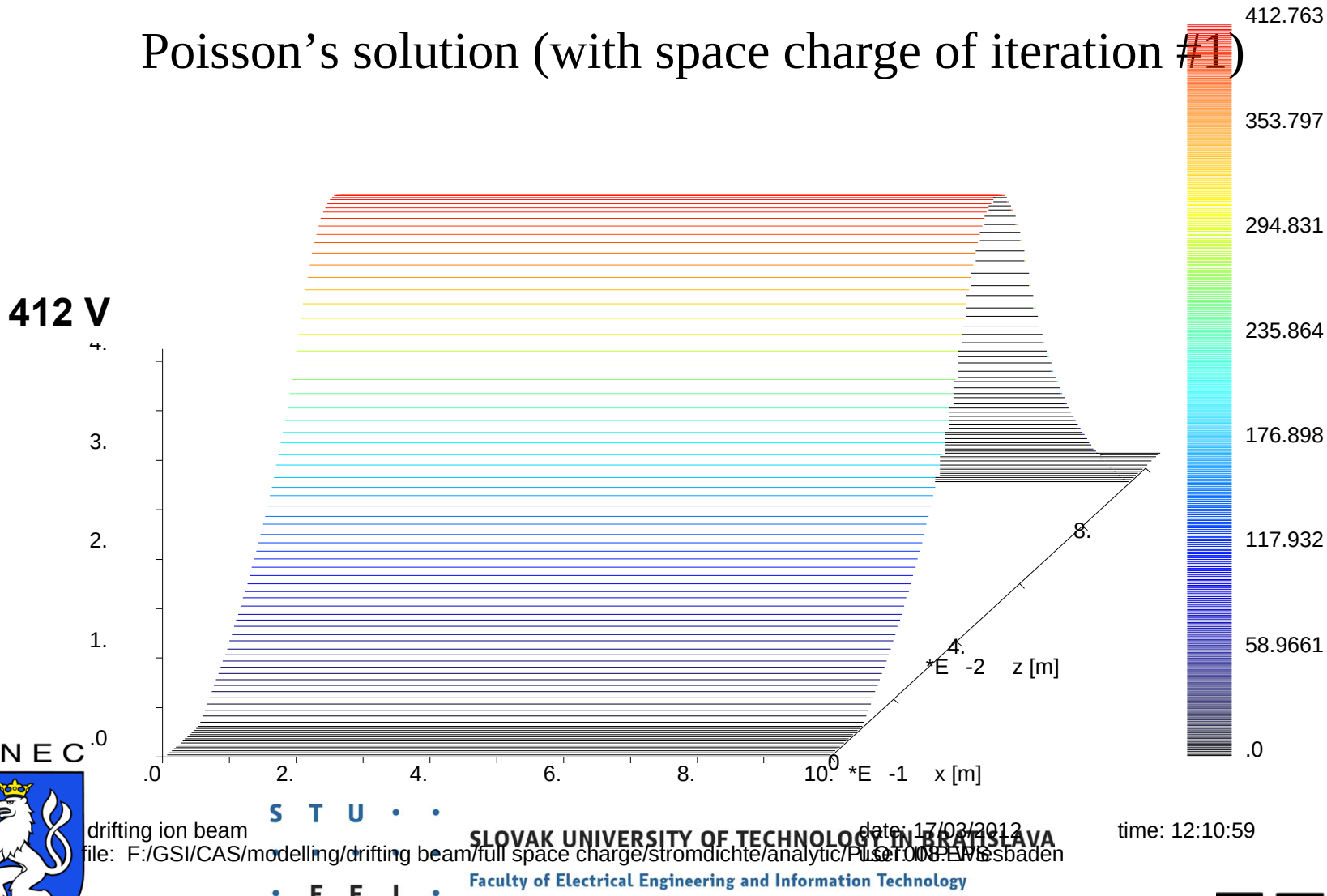
# Laplacian solution (space charge = 0)



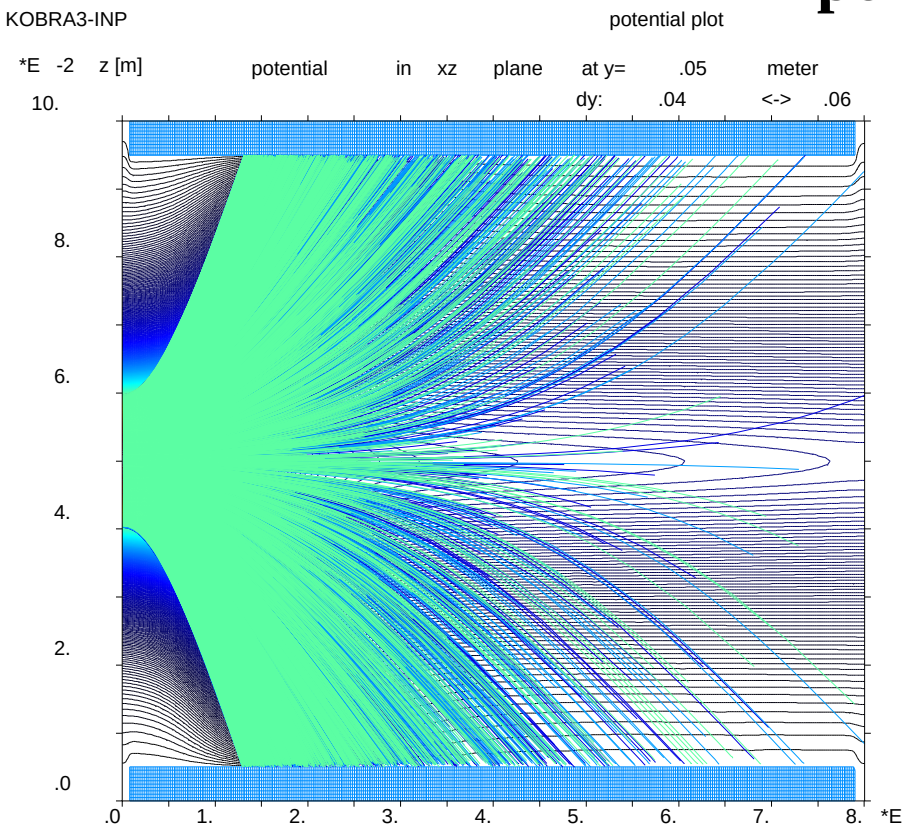

This is the same picture as in the previous slight, just the transverse scale has been increased, to show more details of the beam envelope



# Poisson's solution (with space charge of iteration #1)



# poisson



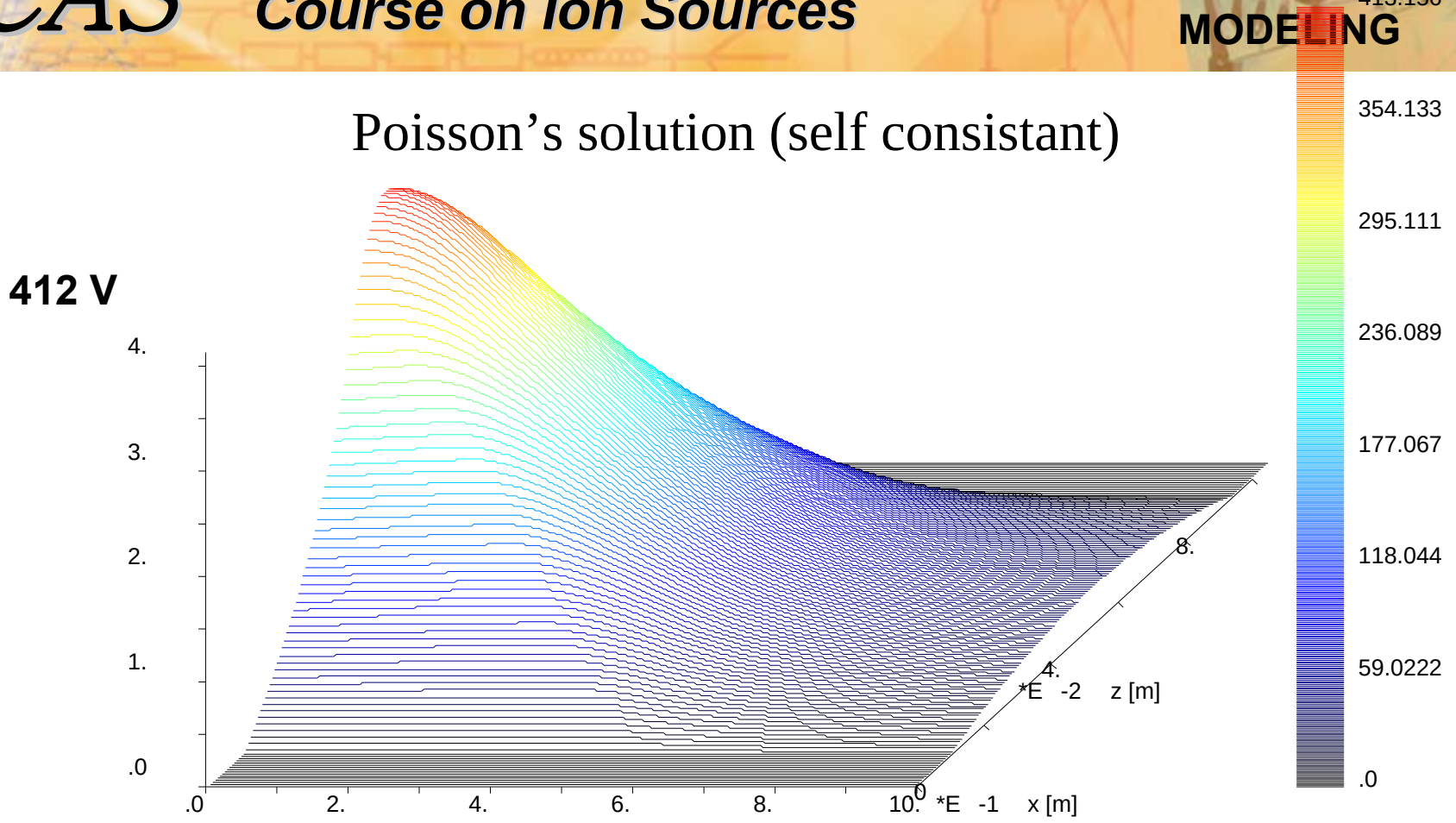

Using the full space charge of the 5mA, 35keV $Ar^+$-beam, it 'explodes'. The potential at the beginning increases to 412V!

However, this is not a self consistent solution! The space charge distribution has changed drastically compared to the Laplace solution.

A series of iterations* have to be executed, until a self consistency will be achieved.

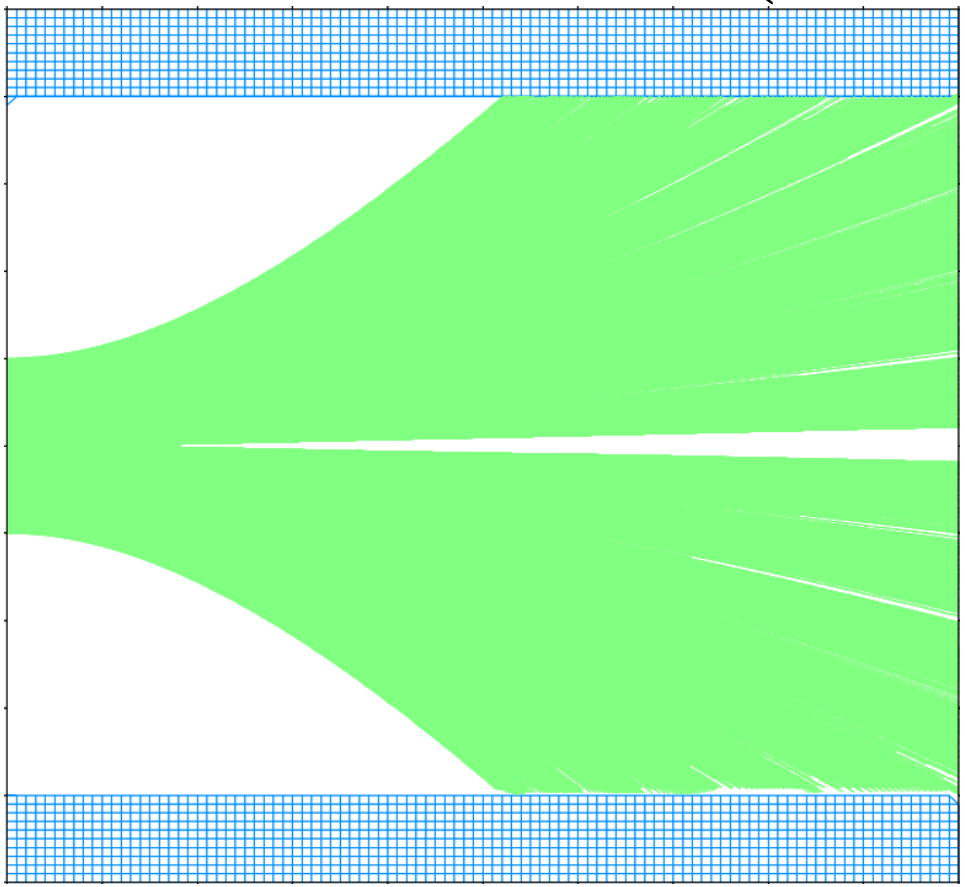
*iteration means: using the actual space charge distribution generated by ray tracing to calculate a new potential distribution.

# Poisson's solution (self consistant)

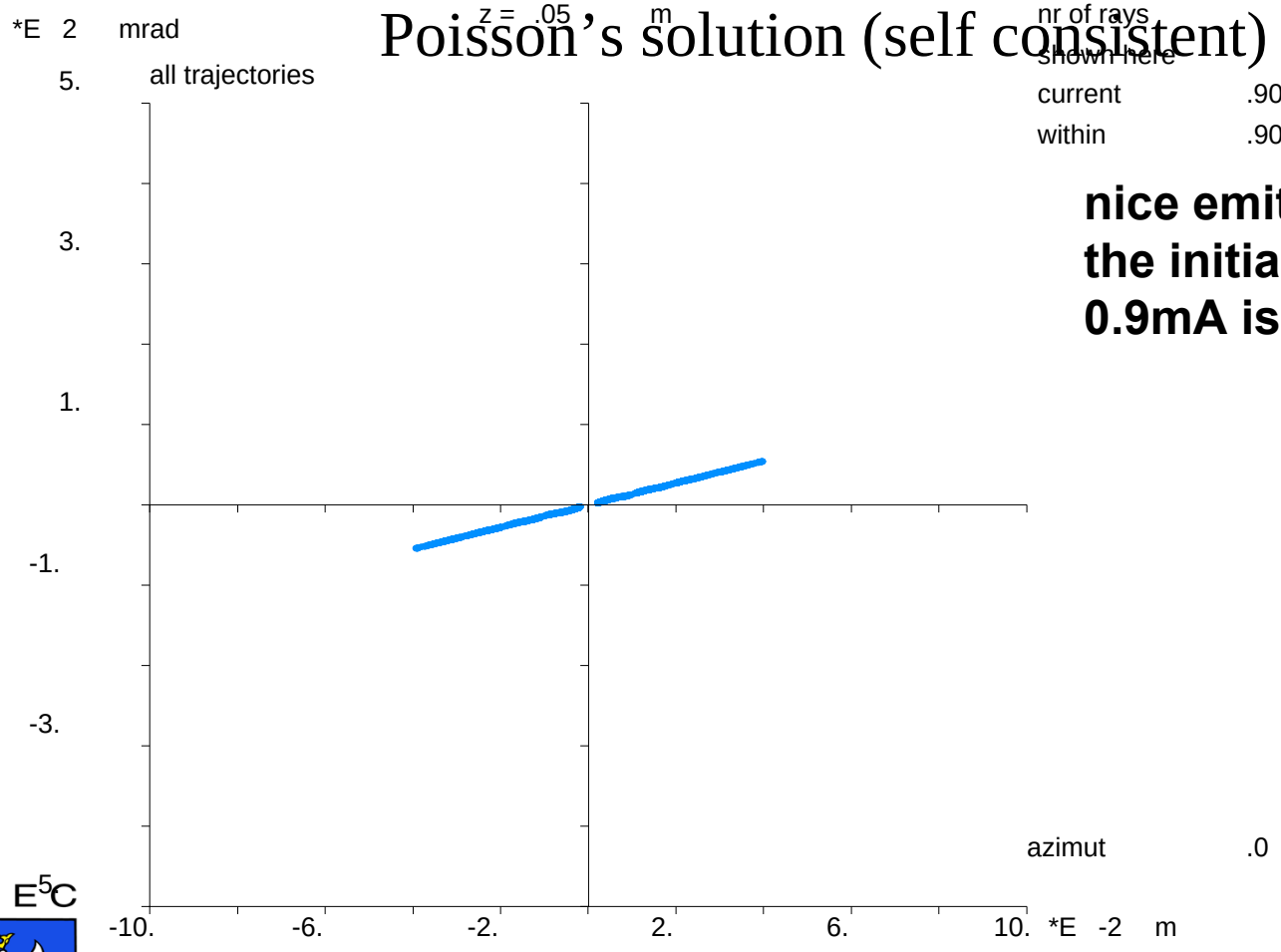



drifting ion beam
file: F:/GSI/CAS/modelling/driftingBeam/FullSpaceCharge/stromdichte/PLOT002.DAT
date: 31/05/2012 time: 21:14:04
user: INP Wiesbaden

# Poisson's solution (self consistent)



# Poisson's solution (self consistent)




**nice emittance, but from the initially 5 mA only 0.9mA is left after 1m.**

# Poisson's solution self consistent, assuming 90% space charge compensation.

**Instead of doing the simulation is done with 5 mA, it is made with 0.5 mA only (90% space charge compensation).**

**The result will be a less divergent beam, the emittance will be as nice as before, and a 5 mA current with 90% space charge compensation can be transported. But is the model correct?**

## Beam plasma model

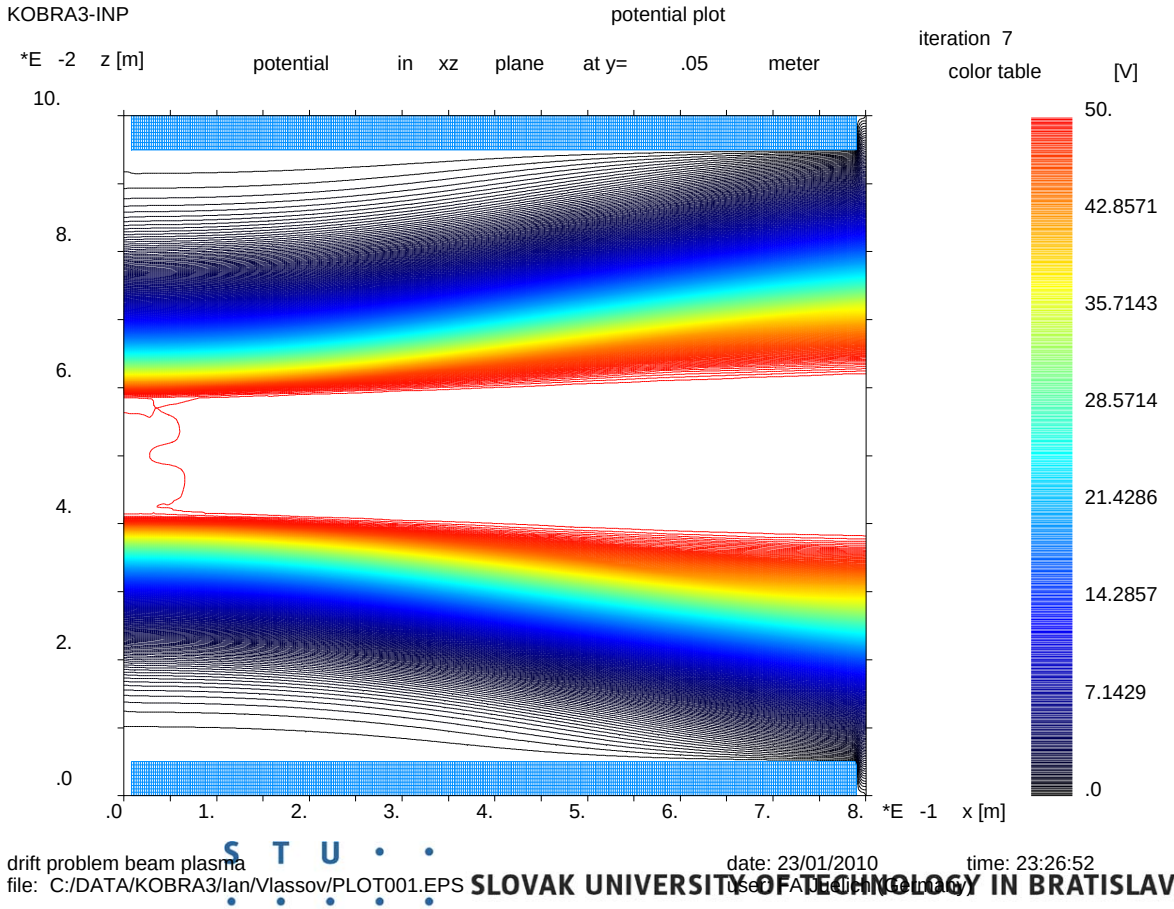
**Self consistant, assuming ~90% space charge compensation, resulting beam potential would be 50V.**

**Let's now assume, that the electron density inside the beam can be described as function from the potential (a justification will follow later → Self's model).**

$$n_e = n_{e0} * \exp(-q(\Phi_b-\Phi)/kT_e)$$

| | |
|---|---|
| $n_e$ | electron density |
| $n_{e0}$ | electron density at the beam center |
| **q** | charge state |
| **Be** | beam potential |
| **Φ** | potential at (x,y,z) |
| **K** | Boltzmann's constant |
| $T_e$ | electron temperature |

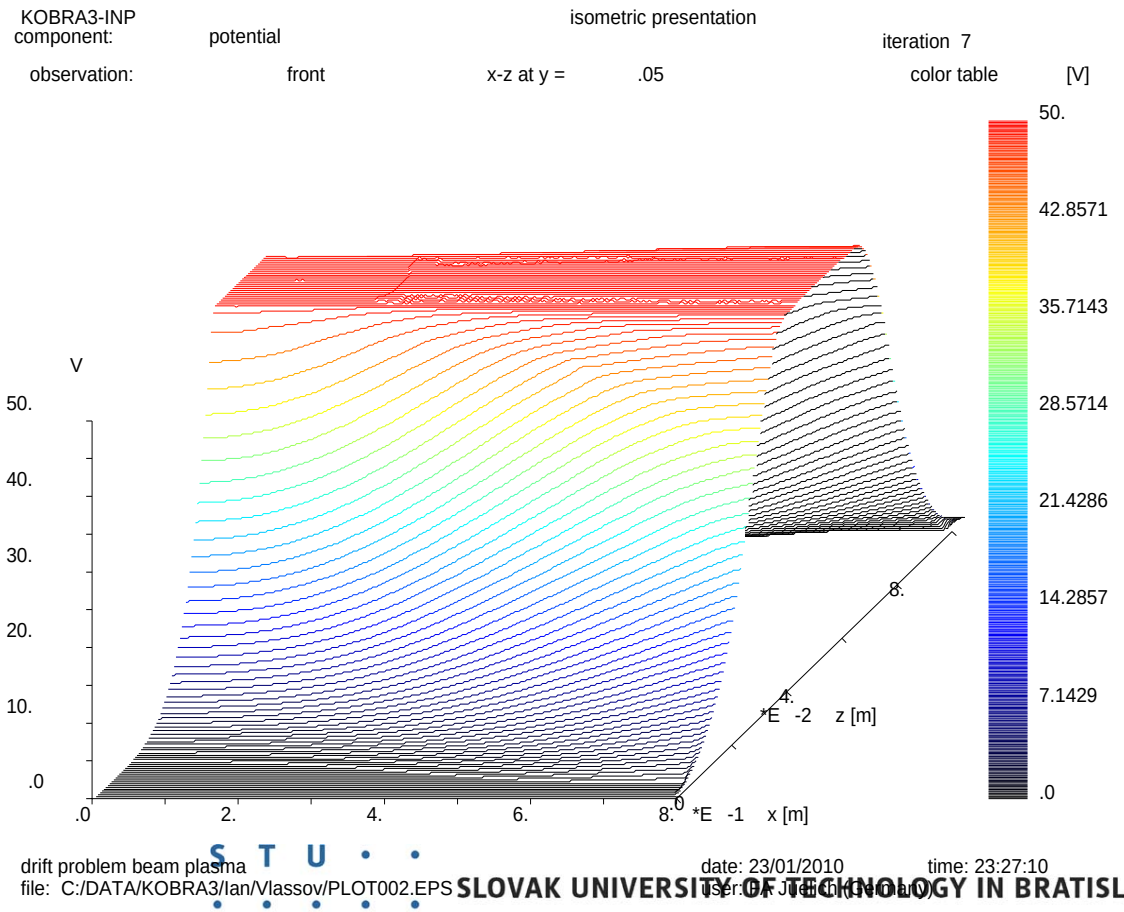
# Beam plasma




**Consequence:**

**No field inside the beam, only a gradient at the beam edge.**

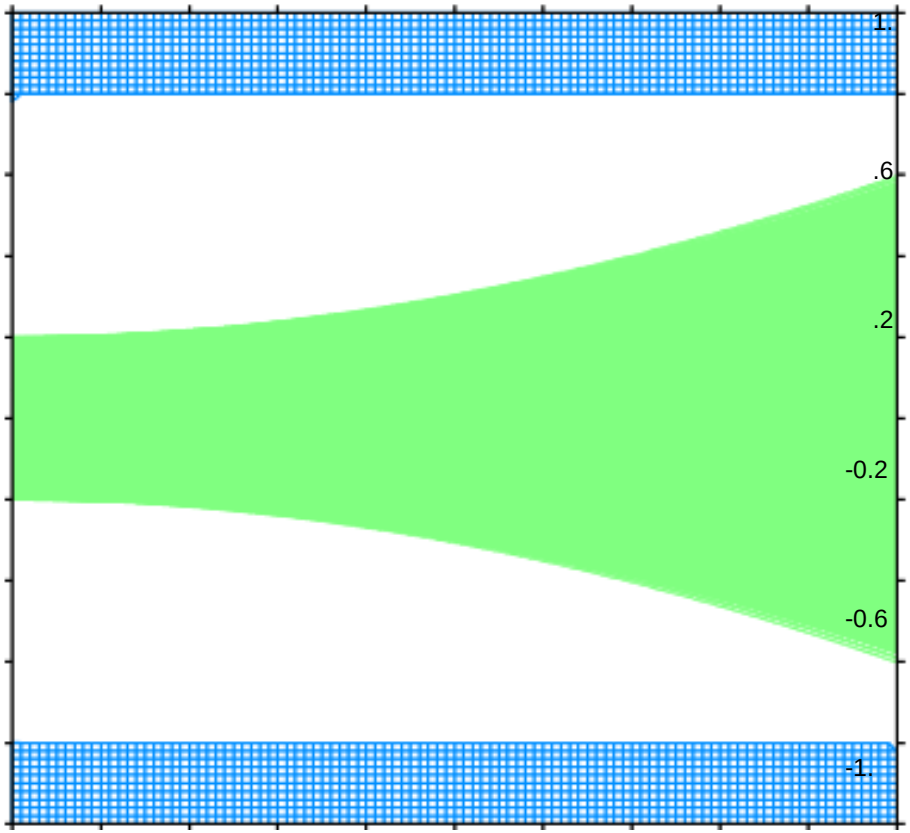
# Beam plasma

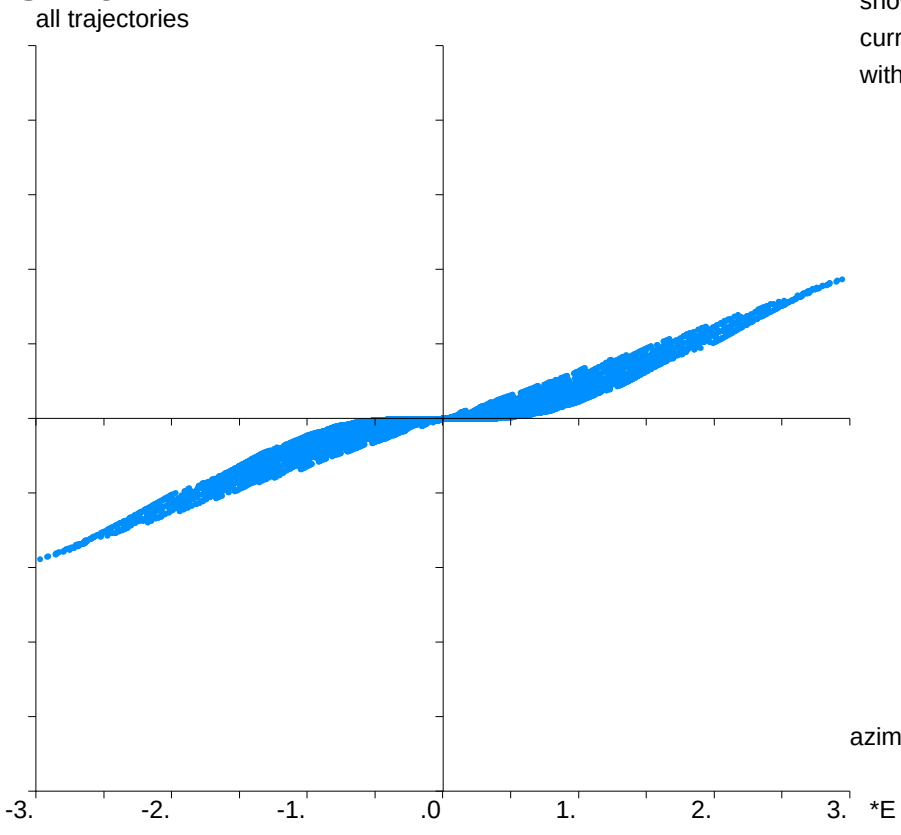



Only another presentation…

# MODELING

## Beam plasma




Some aberrations will occur at the beam edge.

If additional external electric fields exist, these fields will add to the field created by the space charge.

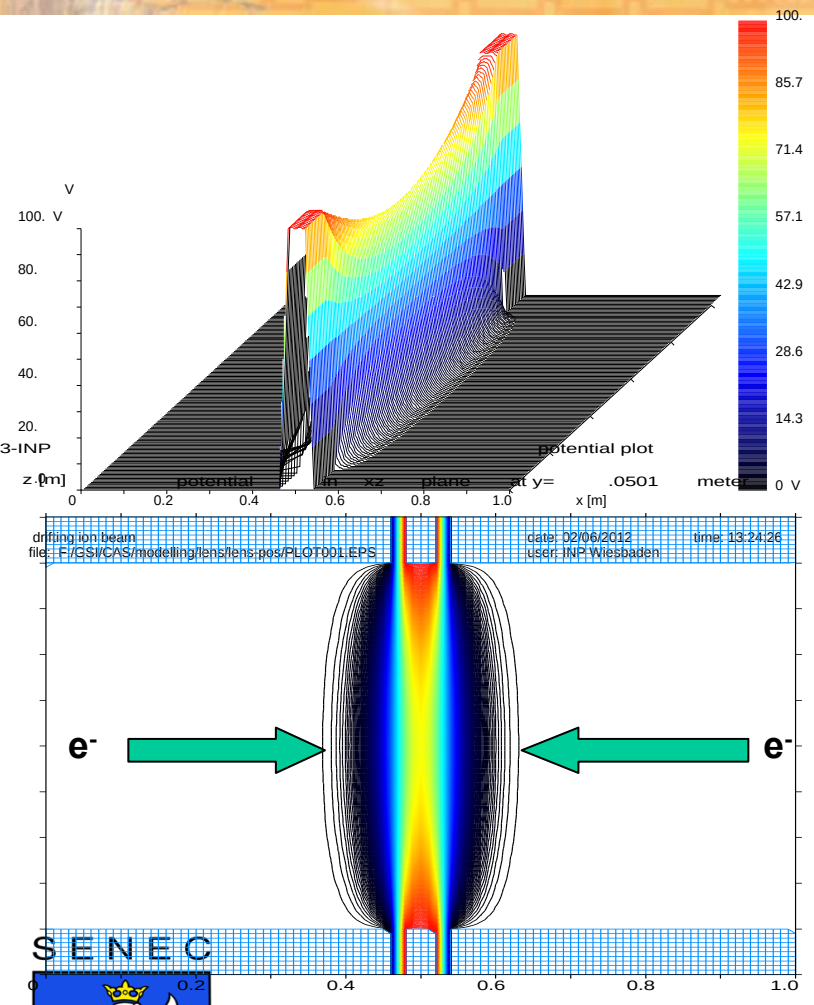
• in case of an additional positive potential, electrons will be extracted from the beam if the potential is high enough, otherwise an equilibrium with partial space charge compensation will be formed. Example: assume a ring electrode in the beam line, positively biased (next slight).

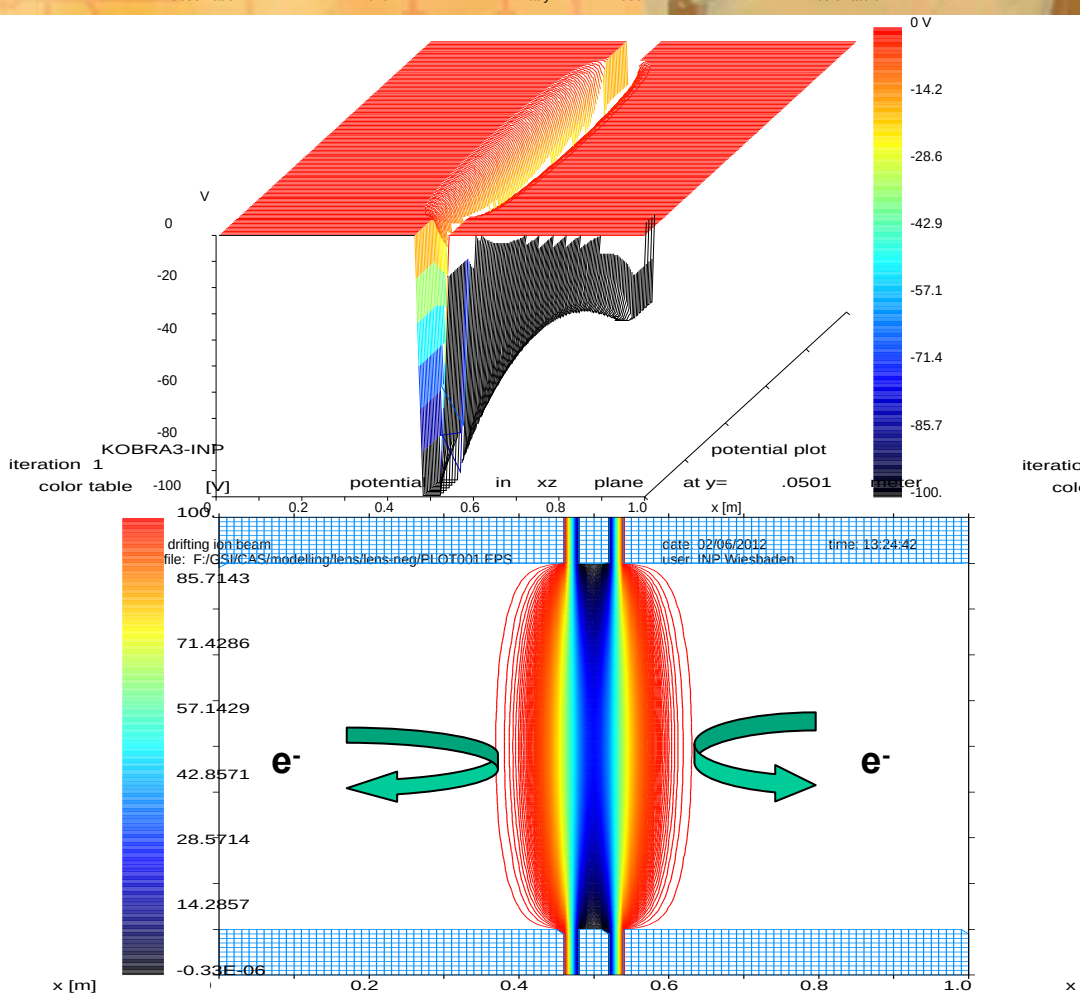
• in case of an additional negative potential, electrons will be repelled from these regions, where a negative potential have been created. Example: assume a ring electrode in the beam line, negatively biased (next slight).

# CAS Course on Ion Sources

**29.05. - 8.06.2012 Senec**

## MODELING




SENEC

S T U

F E I

SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA

Faculty of Electrical Engineering and Information Technology

GSI

**Conclusion: any electric potential beside ground potential within the beam line should be avoided.**

**However, this is not always possible: extraction, post acceleration by a dc voltage, entering a rf accelerator are examples, where it is not possible.**

**In these cases, a negative ring electrode, as short as possible should be used to keep the electrons within the beam. The potential has be chosen such, that on axis a negative potential ensures the required shielding.**

If additional external magnetic fields exist, these fields will influence the moveability of charged particles.

Typically the charged particles can move freely in the direction of the field line, but they cannot move perpendicular to the magnetic field line.

As an example: a charge to mass separation is typically made by a magnetic dipole field. Space charge compensating electrons are not able to travel together with the ion beam through that dipole field. However, inside the dipole field electrons will be created (by collisions) which will oscillate along the magnetic field line through the space charge potential of the ion beam and will compensate it.

To investigate the problem of space charge compensation of a drifting beam, we already have used a very simplified model, namely the model to describe the initial condition of such a beam.

We simply have launched an ion beam described by the occupied area in phase space with several assumptions regarding the distribution of particles within that phase space.

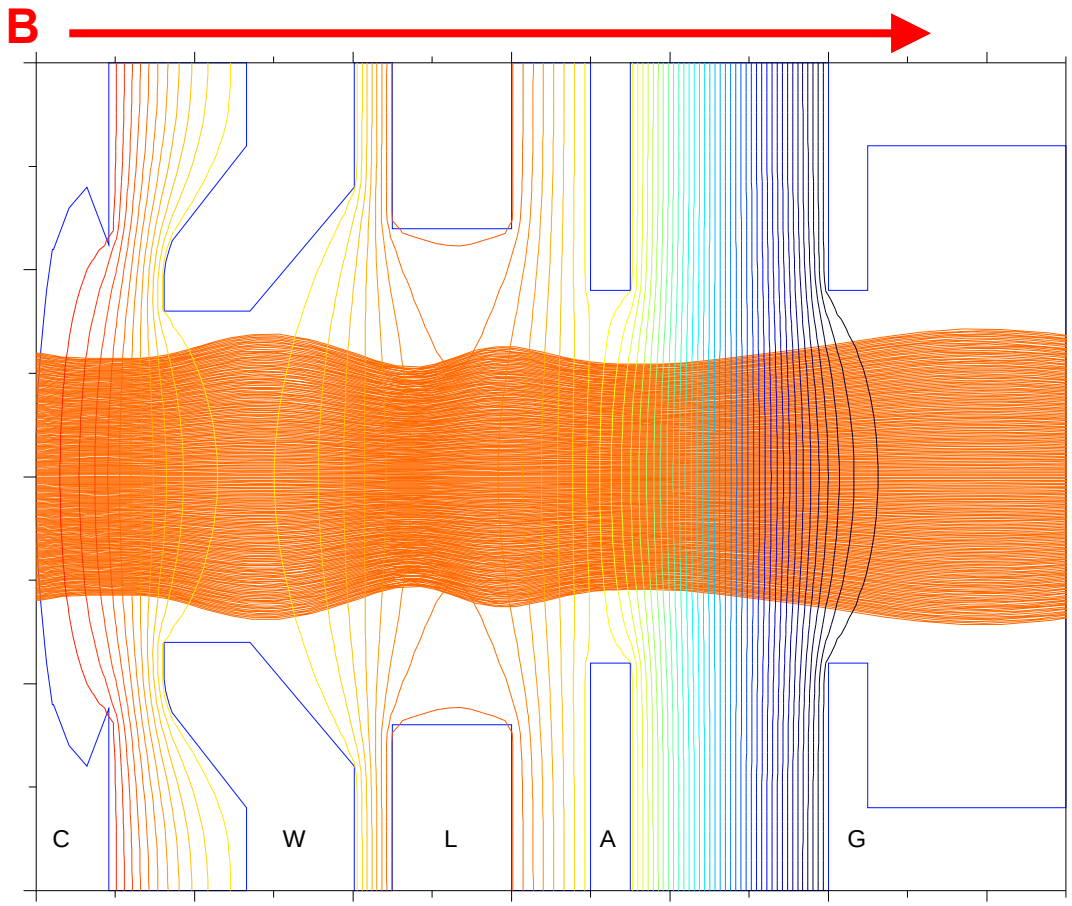
If the source of the ion beam is known, a better model for the initial conditions of the beam can be used.

# Extraction of a Charged Particle Beam from a source

Specialised model required !

- Fixed emitter
  - Electron Gun
  - Surface Ionization
  - Laser Ion Source
- Plasma emitter
  - high current – multi cusp confinement
  - PIG with radial extraction
  - ECR with axial extraction

# Electron gun model




1. Solve Laplace
2. Calculate the current for each trajectory according Child's law.
3. Take a certain percentage of that current, and solve Poisson's equation.
4. Increase the current and iterate as long ΔΦ across a small distance d does not vanish.

$$j = 4/9\ \varepsilon_0 \sqrt{2q/m}\ \Phi^{3/2}/d^2$$

This model can be applied for electron guns (fixed cathode) and for surface ionization sources, Liquid Metal Ion Sources (LMIS). It can give accurate results for space charge limited flow only.

If the flux is emission limited, Child's law cannot be applied; a fixed value of current for each trajectory has to be assumed instead, according the emission capability.

# Plasma ion sources

## *volume ion source model:*

Only ions are simulated, the space charge of electrons is calculated analytically by a one dimensional dependency on the potential. Of course, because $\Phi(x,y,z)$ depends on all three coordinates, this is a 3D model.

$$n_e = n_{e0} * \exp(-q(\Phi_{pl}-\Phi)/kT_e)$$

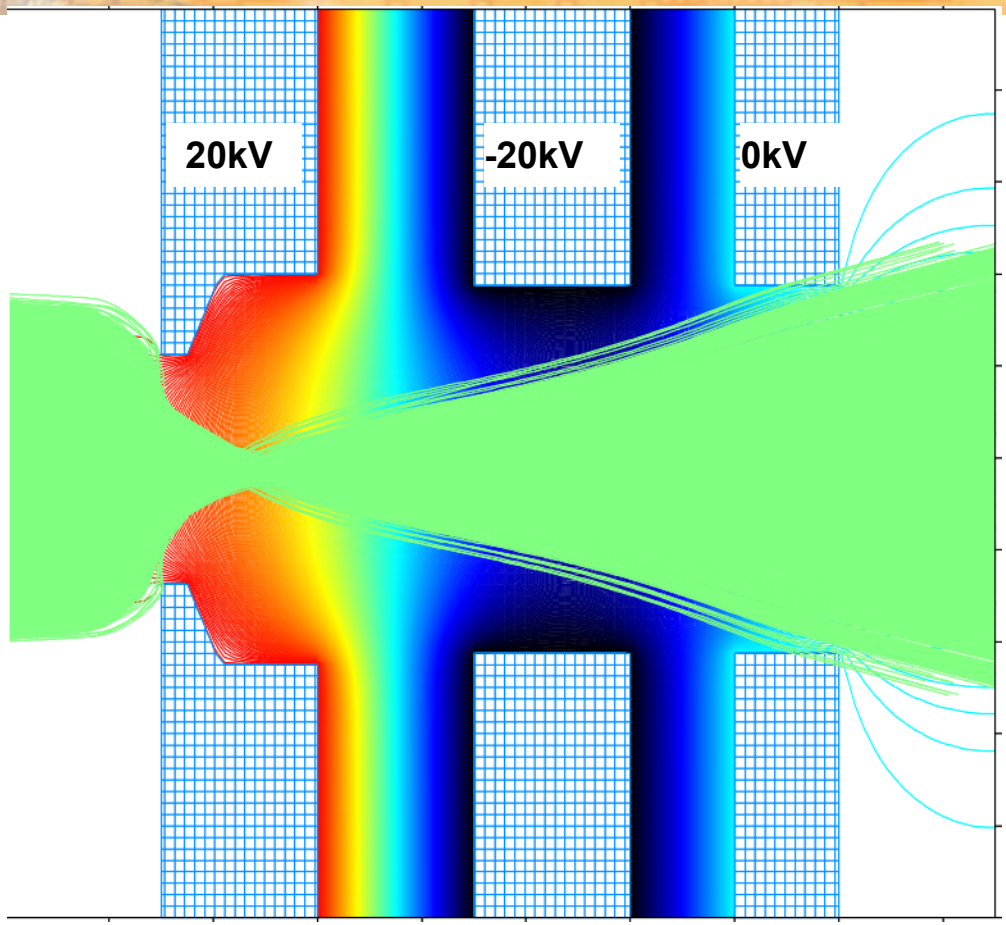
**S.A. Self, Phys. Fluids 6, p.1762, 1963.**

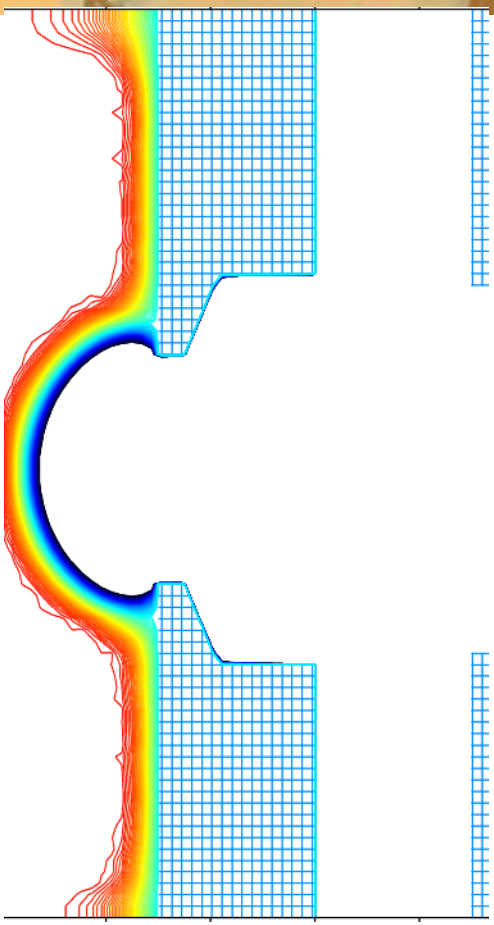
| | |
|---|---|
| $n_e$ | electron density |
| $n_{e0}$ | electron density within the undisturbed plasma |
| $q$ | charge state |
| $\Phi_{pl}$ | plasma potential |
| $\Phi$ | potential at (x,y,z) |
| $K$ | Boltzmann's constant |
| $T_e$ | electron temperature |

# Simulation of Ion Beam Extraction

- Space charge forces are 0 inside the plasma
- Full space charge within the extraction system
- The plasma boundary can be described by Self's law
- after extraction the beam tends to create and to trap electrons (this will create a second plasma boundary between the extracted beam and the drifting beam!)

# MODELING

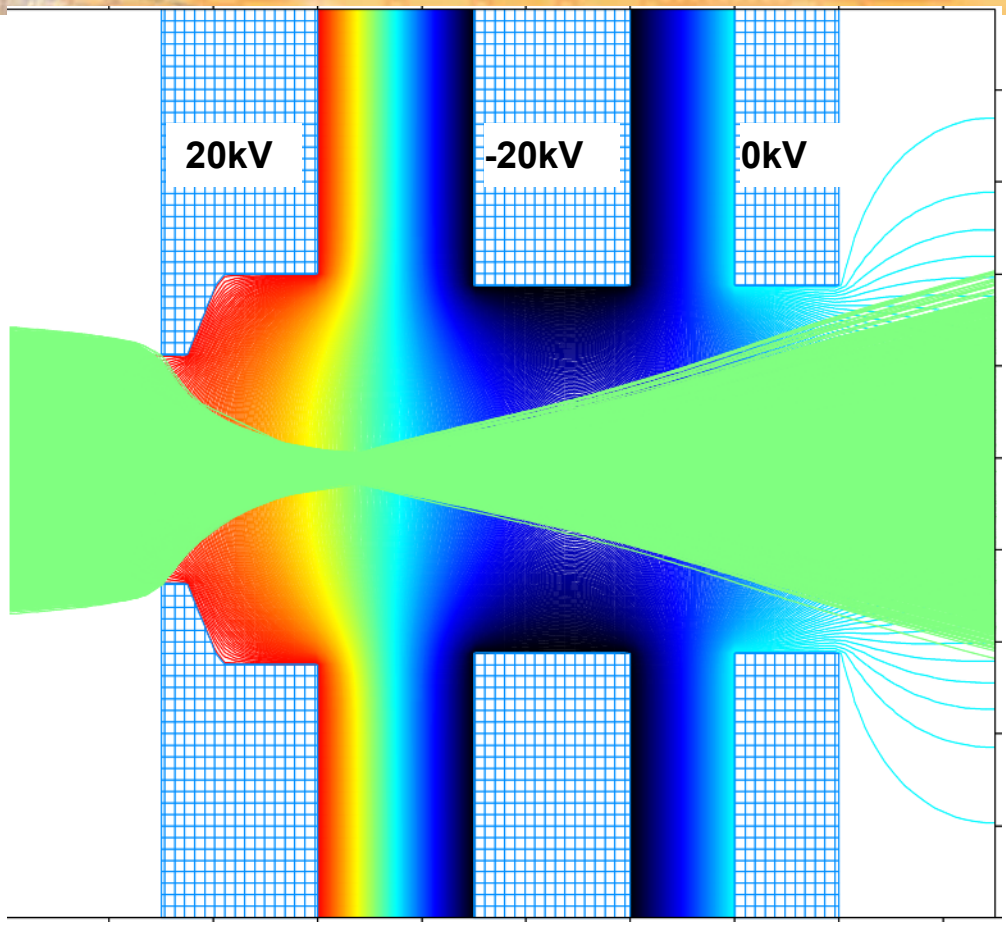


$Ar^+$

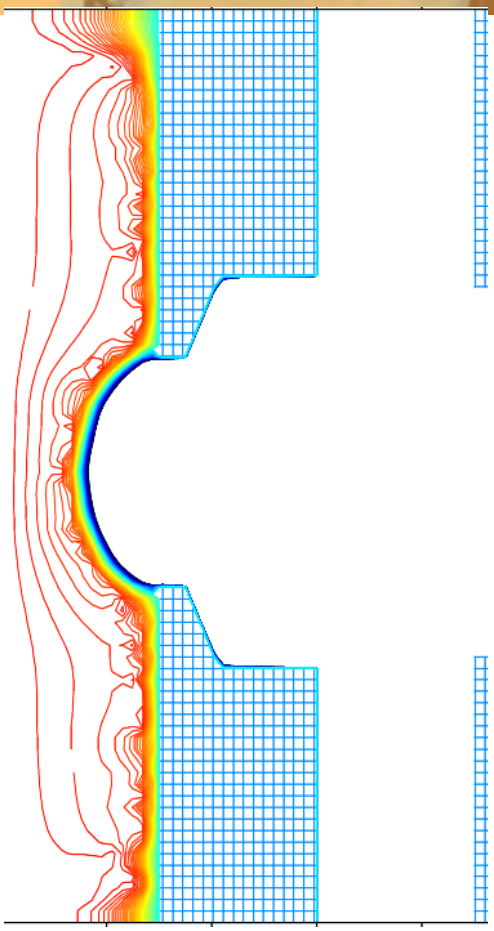
$J=16.8\ A/m^2$

# MODELING

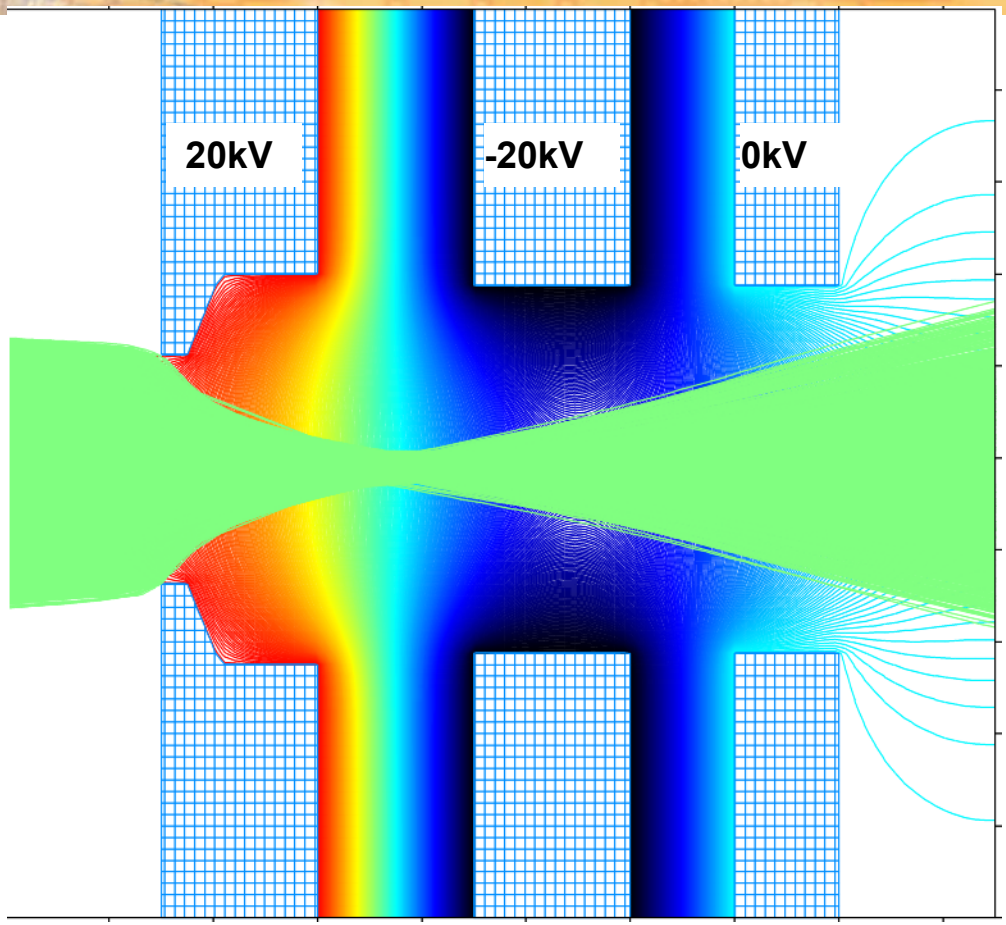


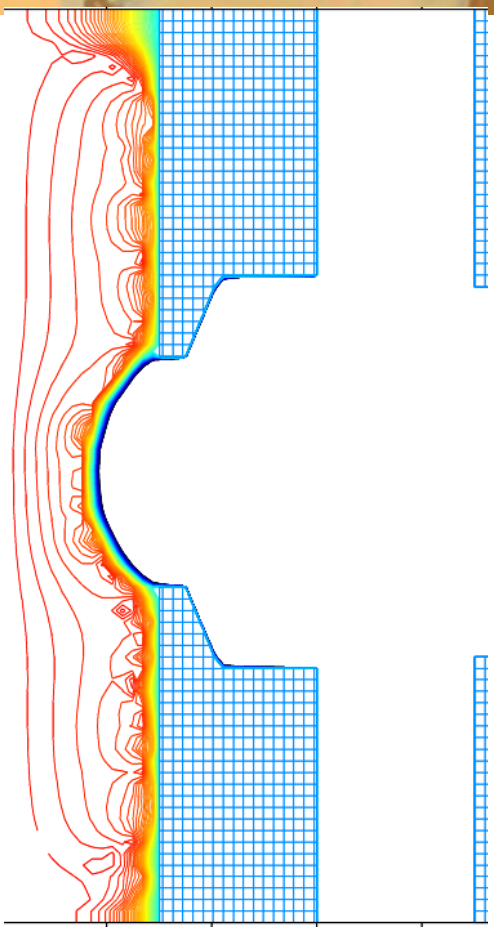

$Ar^+$

$J=75\ A/m^2$



# MODELING

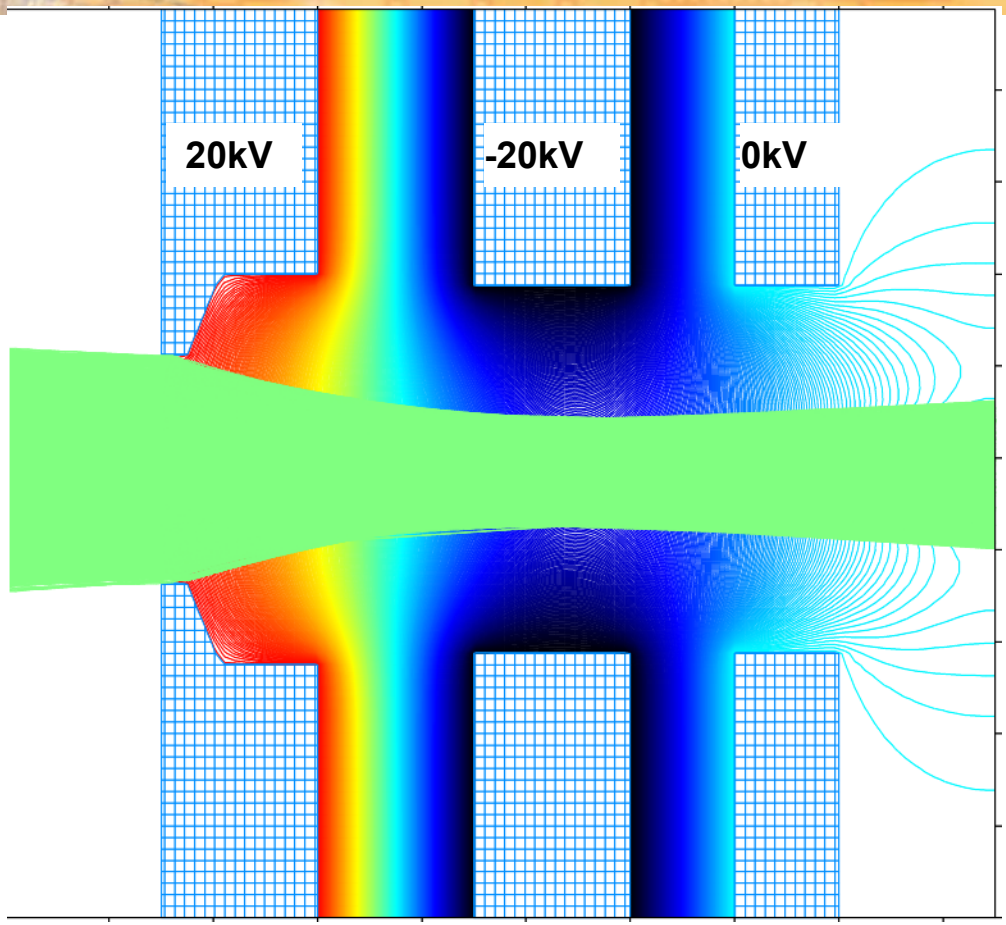


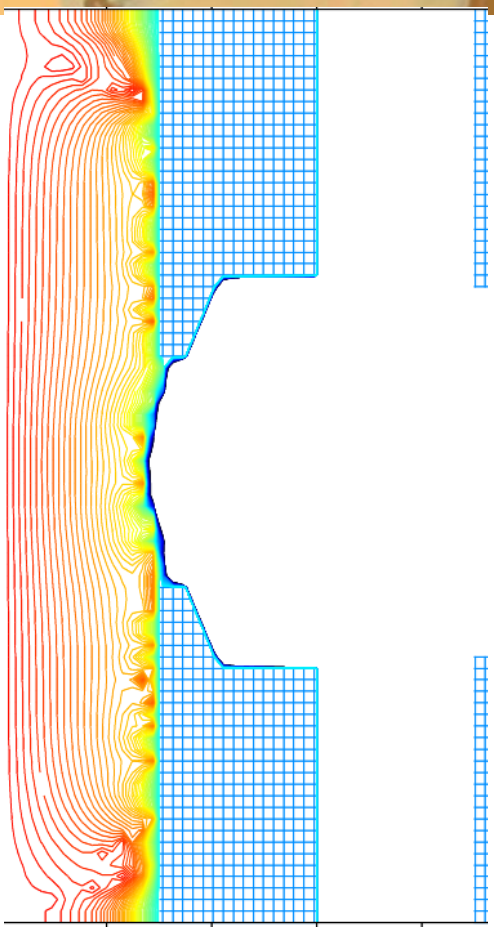



$Ar^+$

$J=100\ A/m^2$

# MODELING

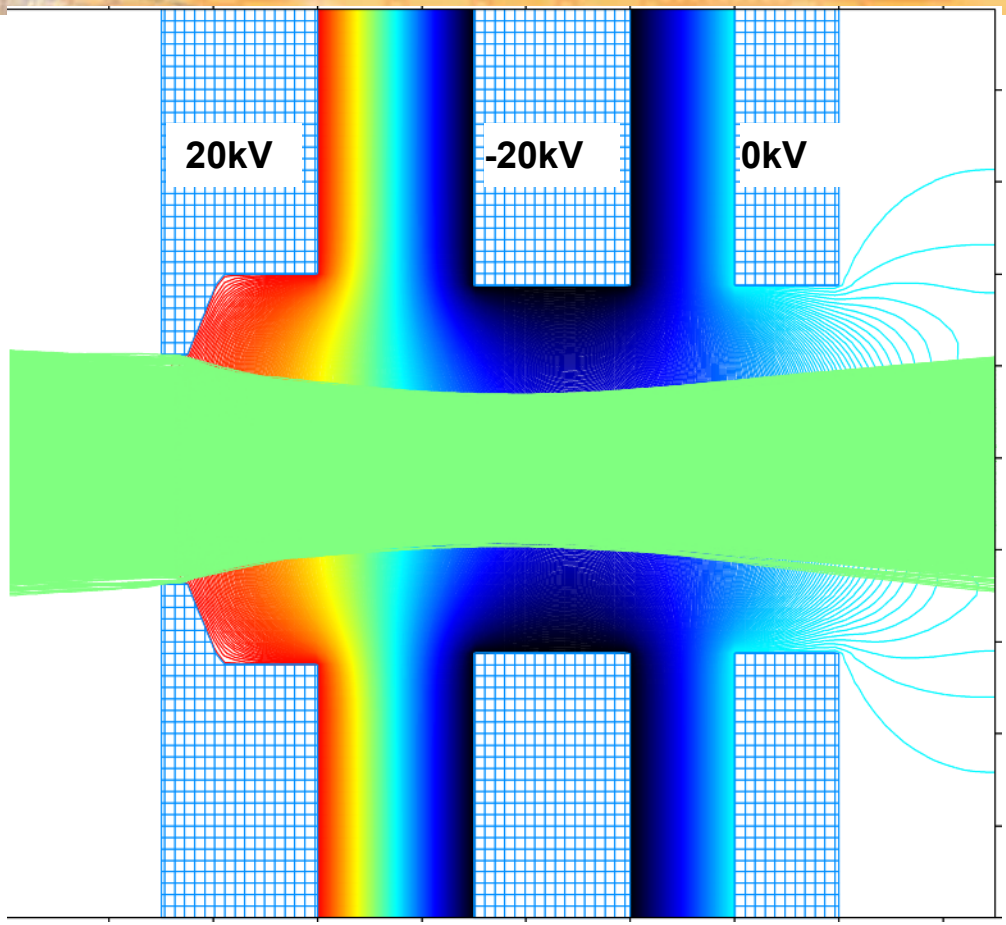


$Ar^+$

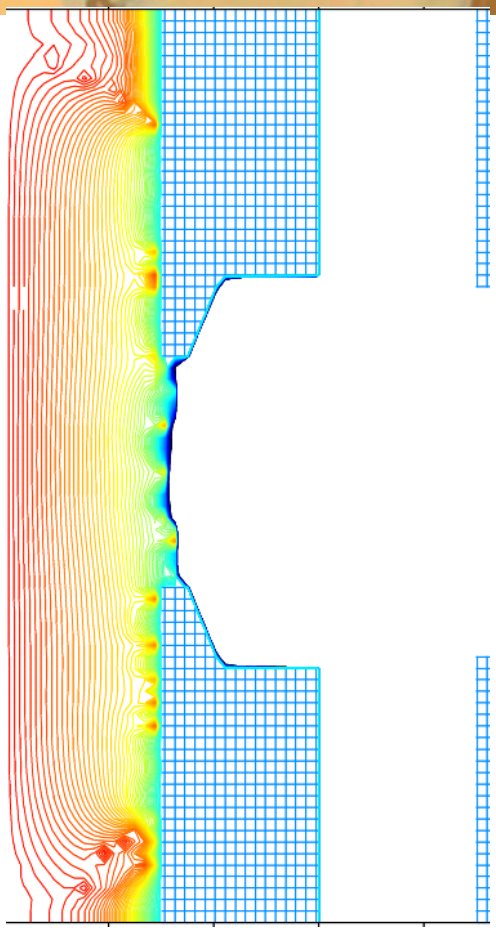
$J=500\ A/m^2$

# MODELING

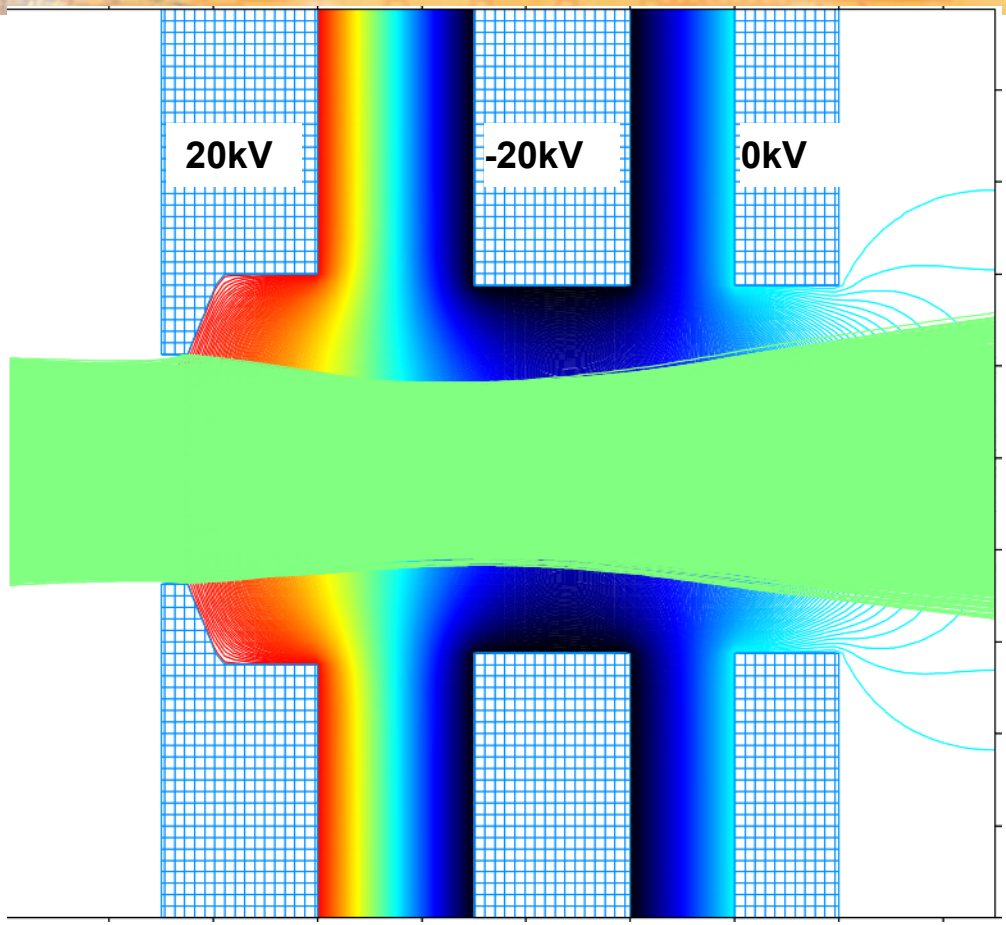


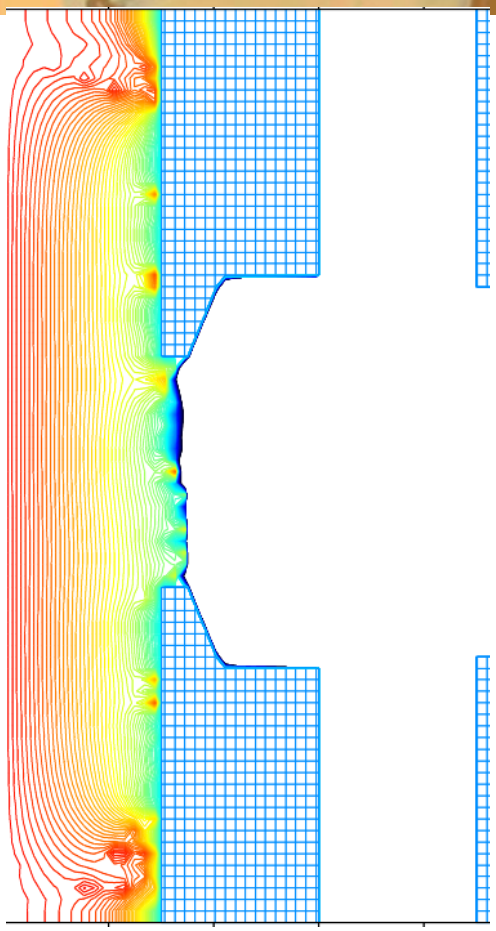

$Ar^+$

$J=750\ A/m^2$



# MODELING



$Ar^+$

$J=1000\ A/m^2$

## Conclusion

**Child's law is valid!**

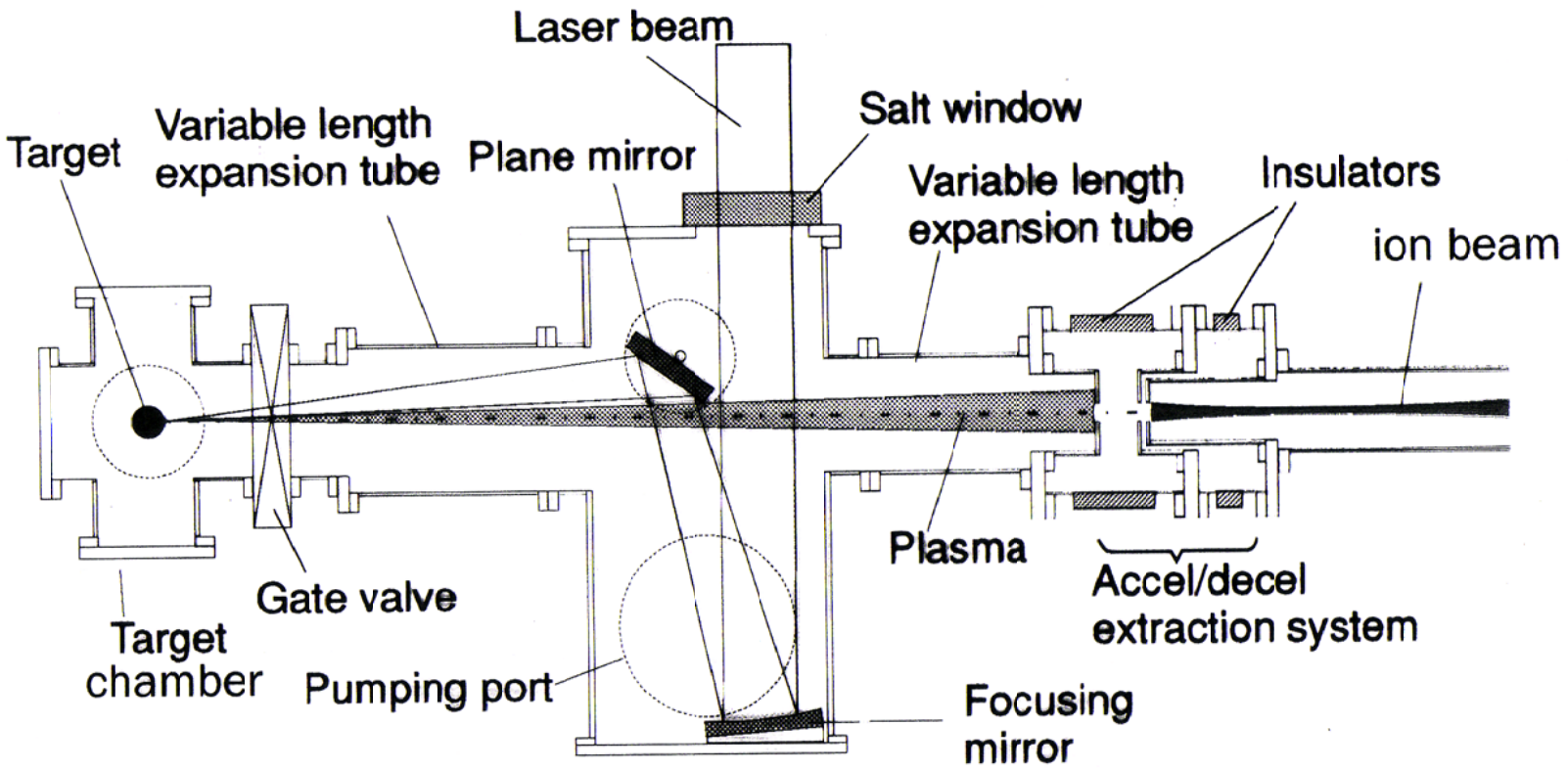
**But be aware: to increase the current it is not sufficient to rise the extraction voltage only, the particle density has to be increased inside the plasma accordingly (do the same thing on both sides of the equation)!**

$$j = 4/9\ \varepsilon_0\ \sqrt{2q/m}\ \Phi^{3/2}/d^2$$

**A scan for the different parameters is recommended:**

- **beam shape as function of current density**
- **beam intensity within a given acceptance as function of current density**
- **emittance as function of electron temperature**

# laser ion sources


Laser beam
Salt window
Target
Variable length expansion tube
Plane mirror
Variable length expansion tube
Insulators
ion beam
Plasma
Accel/decel extraction system
Gate valve
Target chamber
Pumping port
Focusing mirror

## *volume ion source model:*

Only ions are simulated, the space charge of electrons is calculated analytically by a one dimensional dependency on the potential. Of course, because $\Phi(x,y,z)$ depends on all three coordinates, this is a 3D model.

$$n_e = n_{e0} * \exp(-q(\Phi_{pl}-\Phi)/kT_e)$$

**S.A. Self, Phys. Fluids 6, p.1762, 1963.**

| | |
|---|---|
| $n_e$ | electron density |
| $n_{e0}$ | electron density within the undisturbed plasma |
| $q$ | charge state |
| $\Phi_{pl}$ | plasma potential |
| $\Phi$ | potential at (x,y,z) |
| $K$ | Boltzmann's constant |
| $T_e$ | electron temperature |

**Yes, you are right. You have seen that slice before, the same model as for the volume source can be used here, but …**

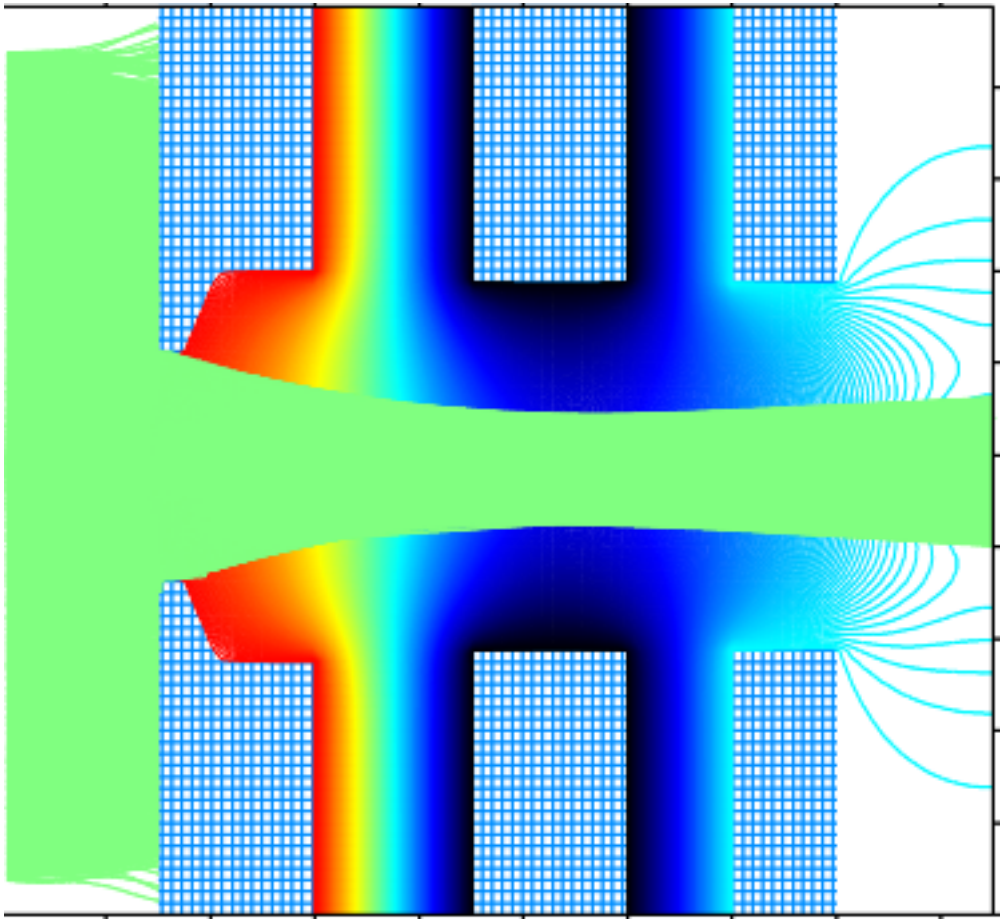
depending on laser power, a drift velocity has to be added, due to the explosive behavior of the plasma. The exact amount of additional velocity depends on laser power density on the target.

the same model is applicable for MEVVA ion sources.

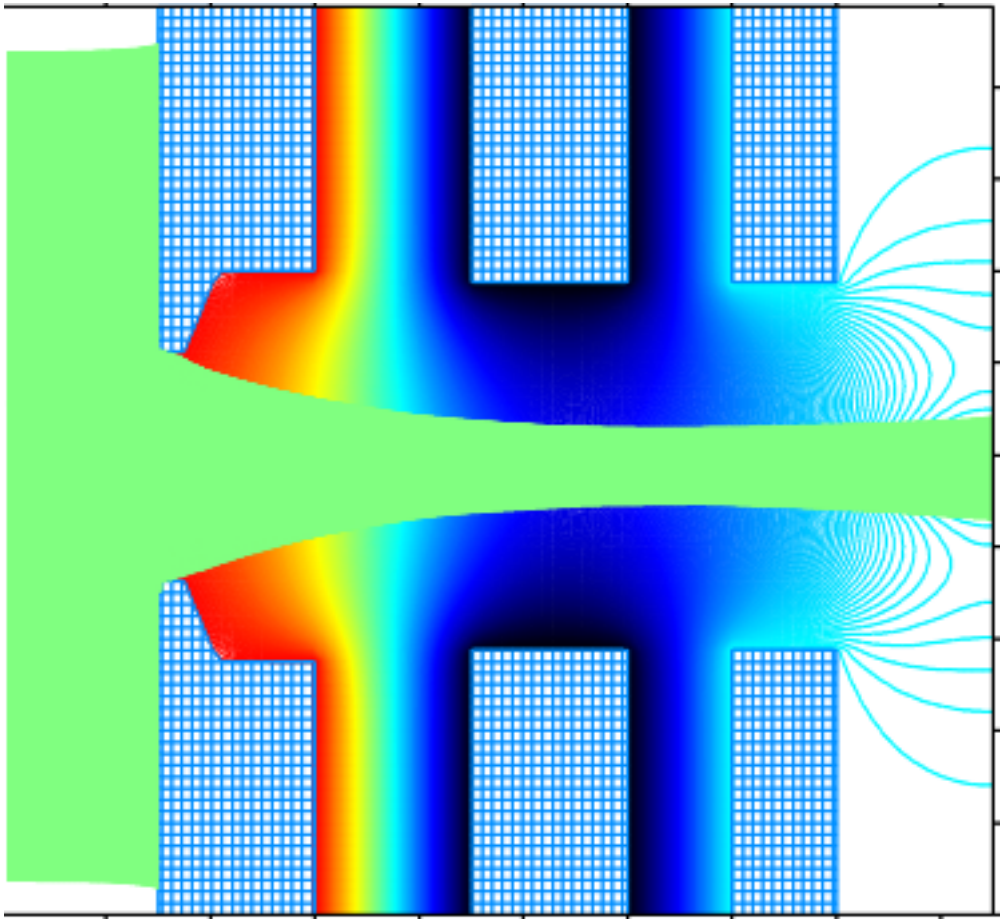
the high momentum spread can be used to increase the pulse length on the cost of decreasing pulse current.

for simplicity and comparison, the same extraction geometry and applied voltages have been used as for the volume source.

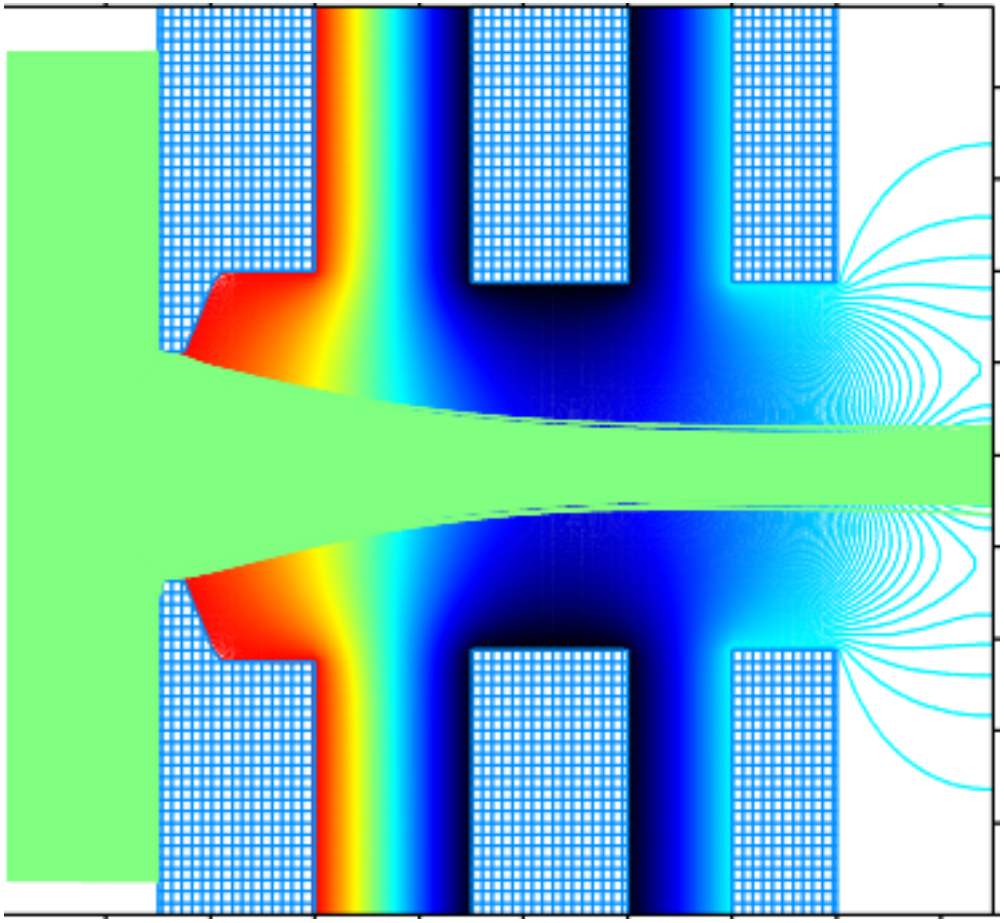
additional drift velocity according to 5eV



additional drift velocity according to 50eV



additional drift velocity according to 500eV



## Conclusion

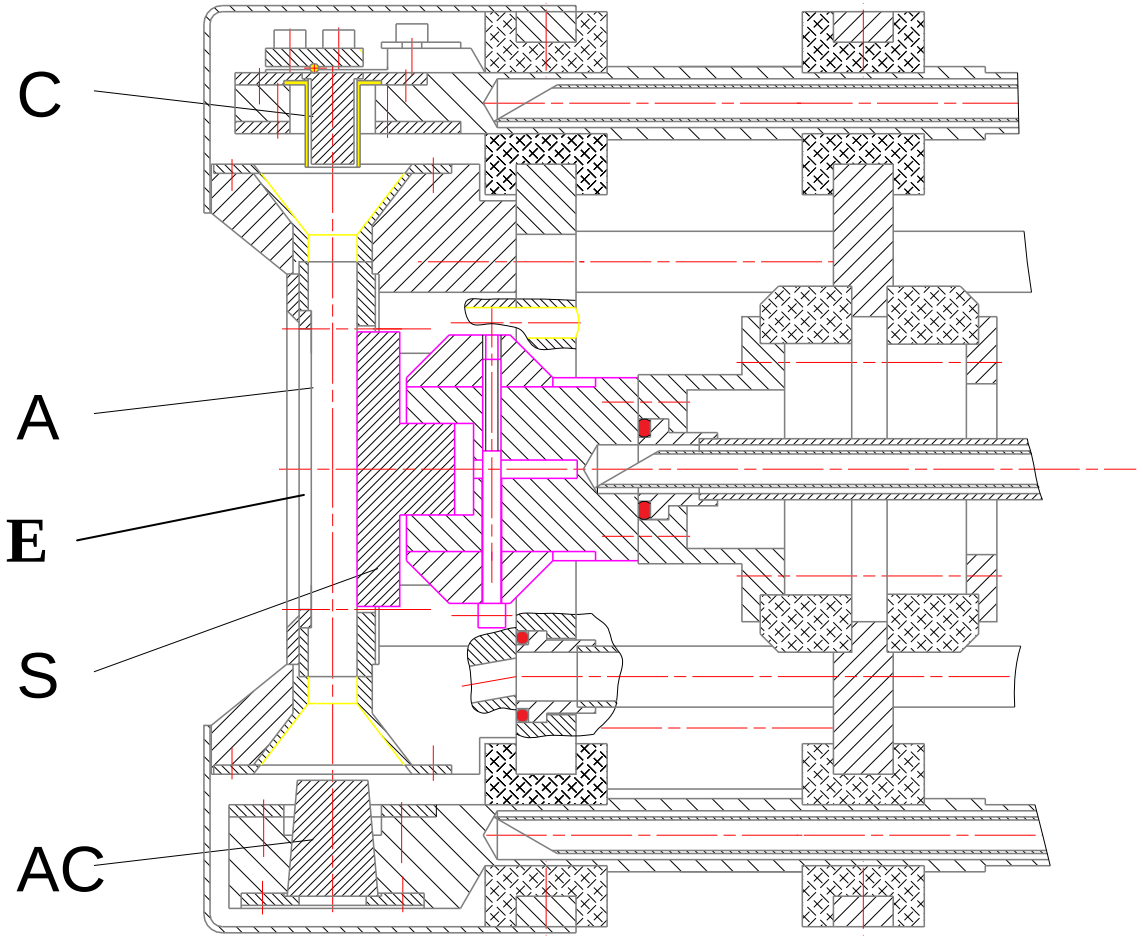
- Space charge effects becomes smaller, because of higher velocity.
- For the same reason, the influence of the Laplace fields decreases.

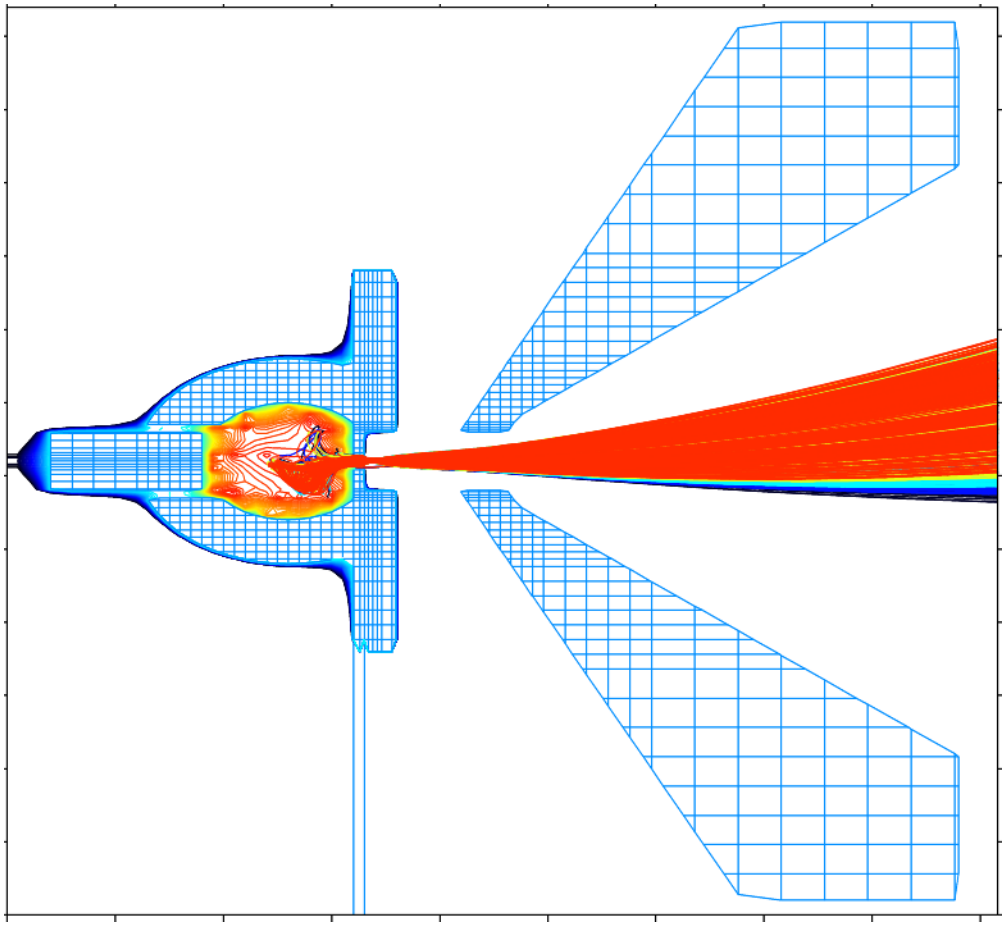
# Penning ion source

# Penning ion source

(radial extraction)

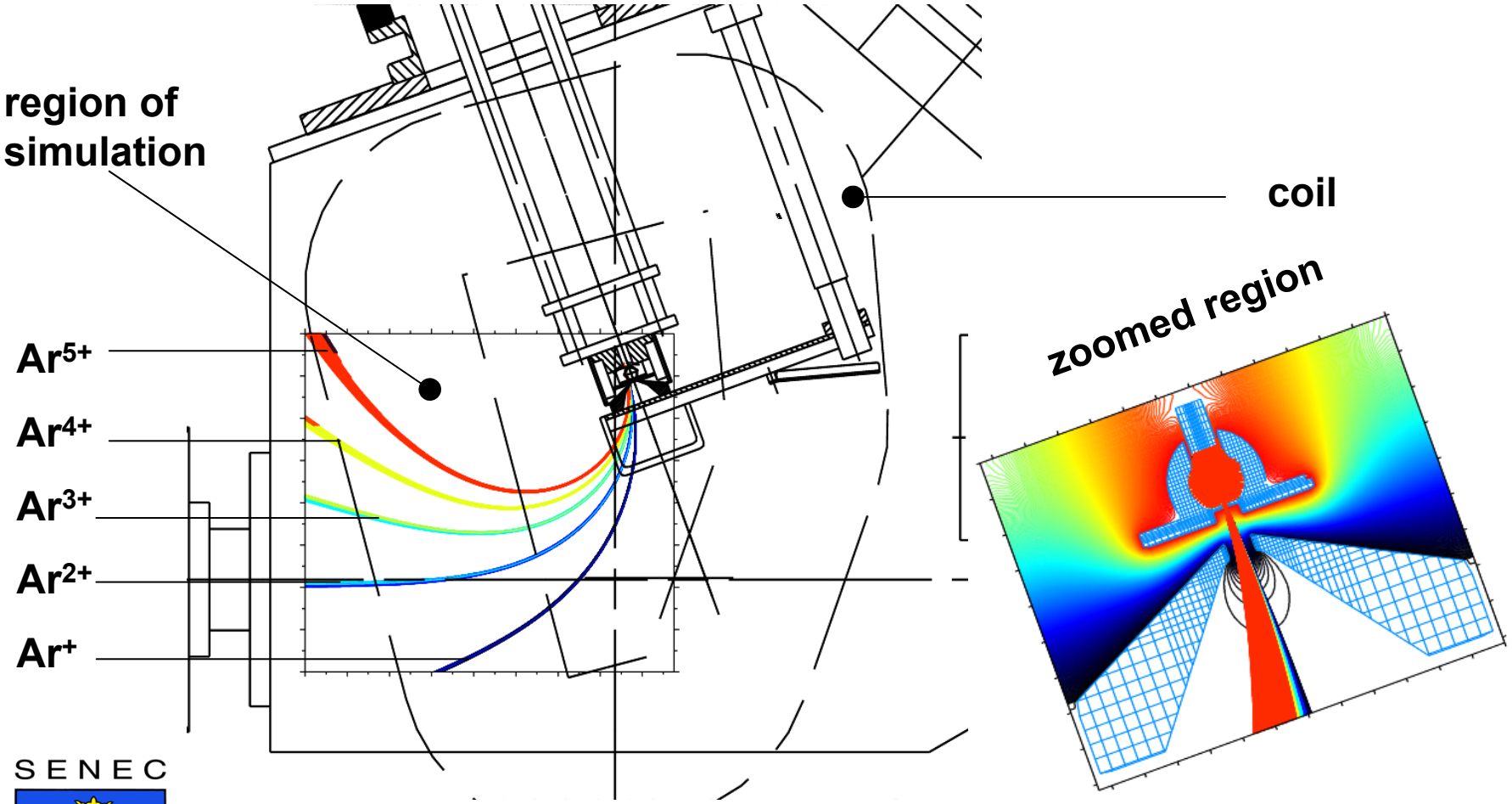
C cathode
A anode
S sputter electrode
AC anticathode
E extraction
B magnetic field



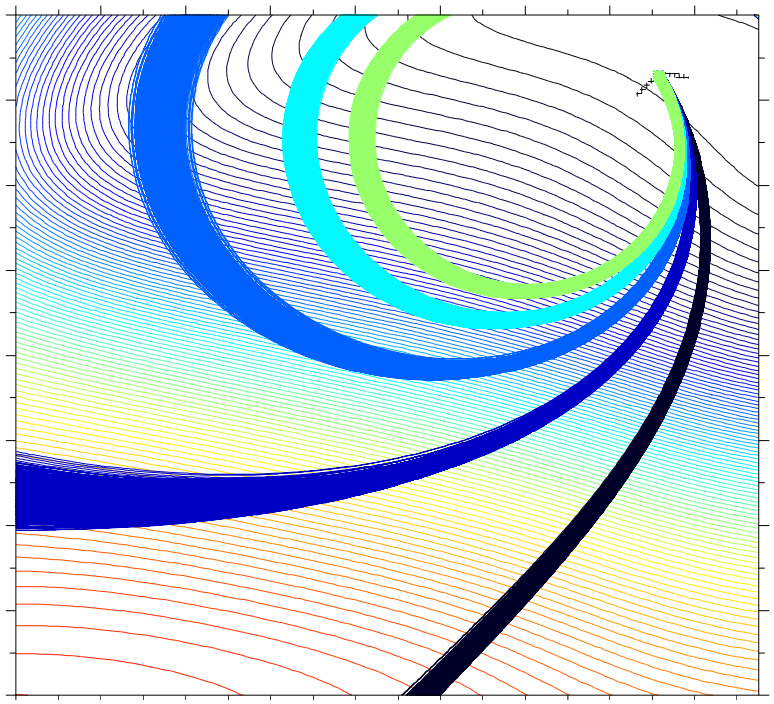


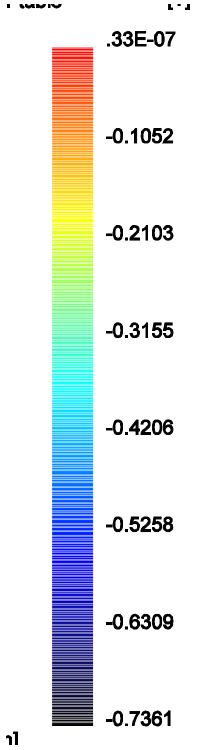
Pig in gaseous mode

medium current

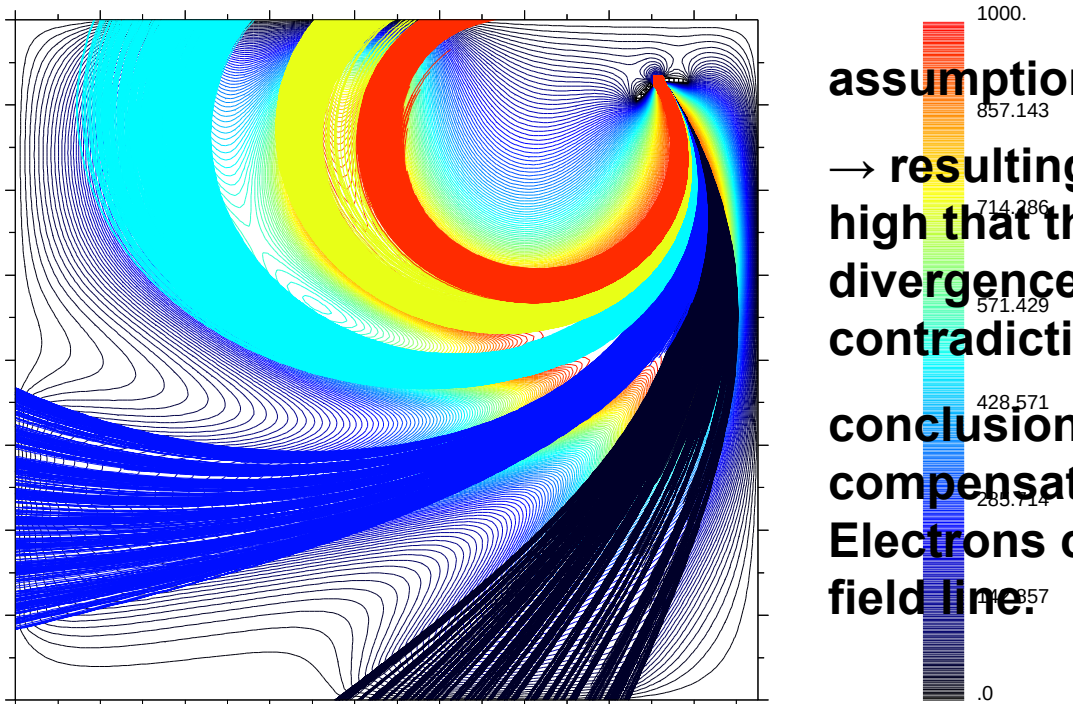

region of simulation
coil
zoomed region
Ar5+
Ar4+
Ar3+
Ar2+
Ar+

magnetic flux density






**Using the final coordinates from the extraction simulation as starting conditions within the magnetic field distribution show the separation of the different charge states.**



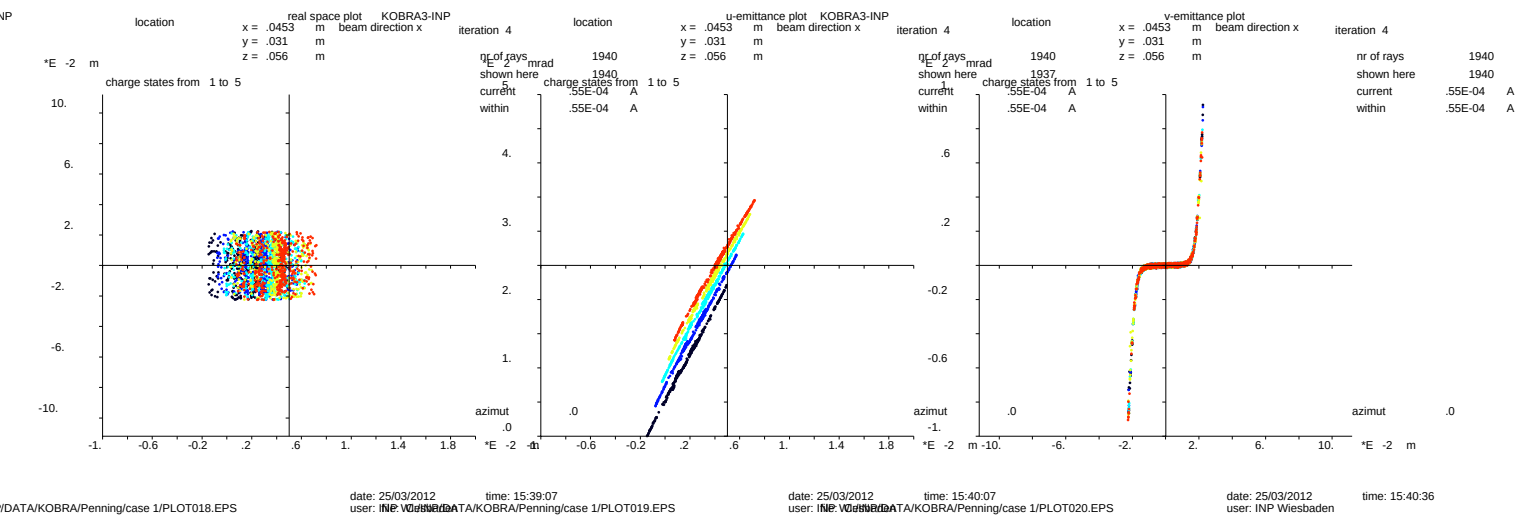

**assumption: no space charge compensation**

**→ resulting electrostatic potential is such as high that the beam diameter and beam divergence becomes very large which is in contradiction with experimental results.**

**conclusion: the beam is space charge compensated even in the dipole magnet. Electrons can oscillate along the magnetic field line.**

Pig transport through the dipole

INP Wiesbaden (GERMANY)
DATE: 08/20/99 TIME: 13:10



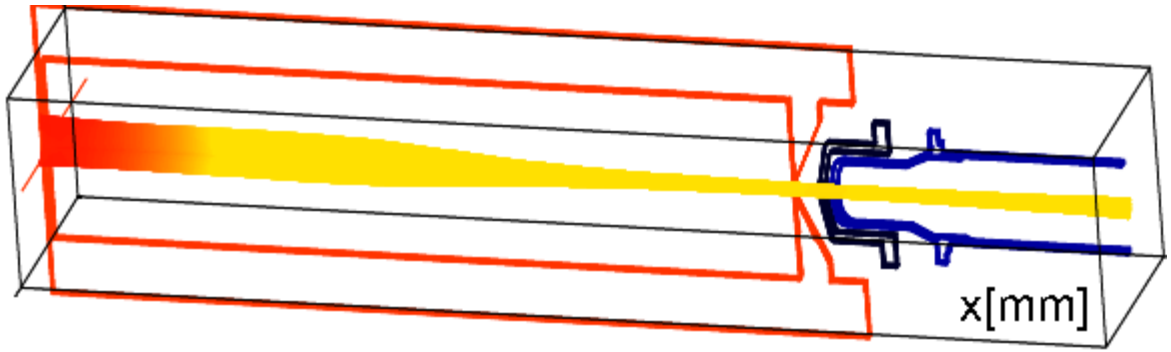

**real space** **horizontal** **vertical**

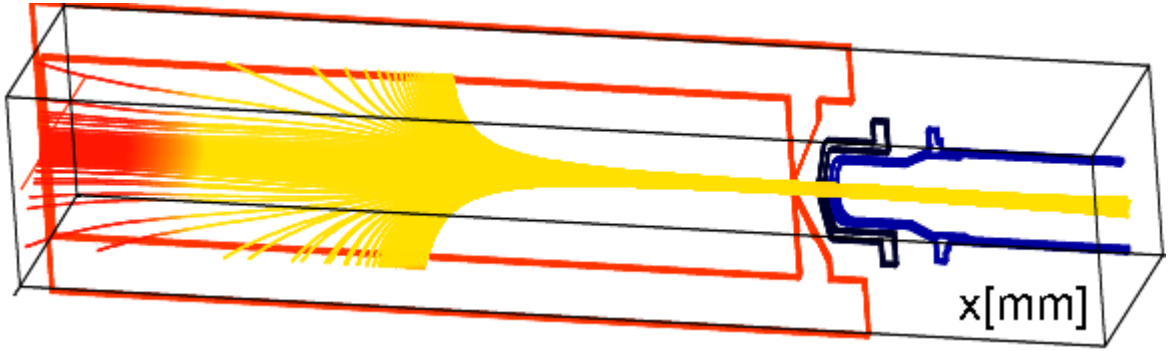
**emittance**

**Typical profile and emittance for a PIG source.**

# Electron cyclotron resonance ion sources

## Geometry and magnetic flux density for the MS-ECRIS



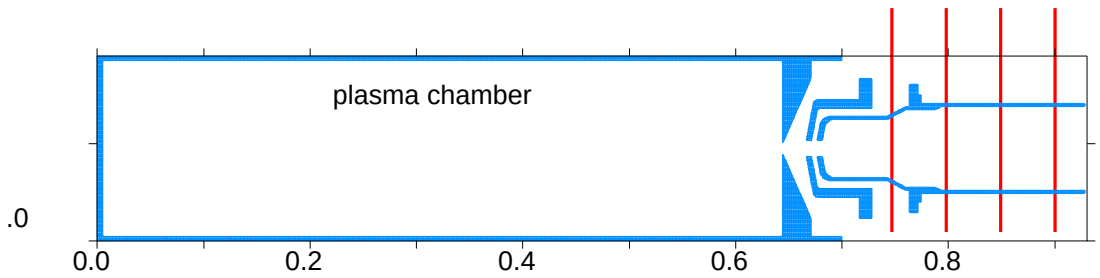

Strong magnetic fields for the plasma confinement (magnetic flux density has been calculated with Tosca®):

- electrons and ions are magnetized (Larmor radii are small).
- magnetic field lines going through the extraction aperture indicate the possible paths of ions to be extracted.
- but the field lines depend on specific coil settings.

• Using the coordinates and the local magnetic flux density of the magnetic filed lines going through the extraction electrode aperture provide starting coordinates for the ions.

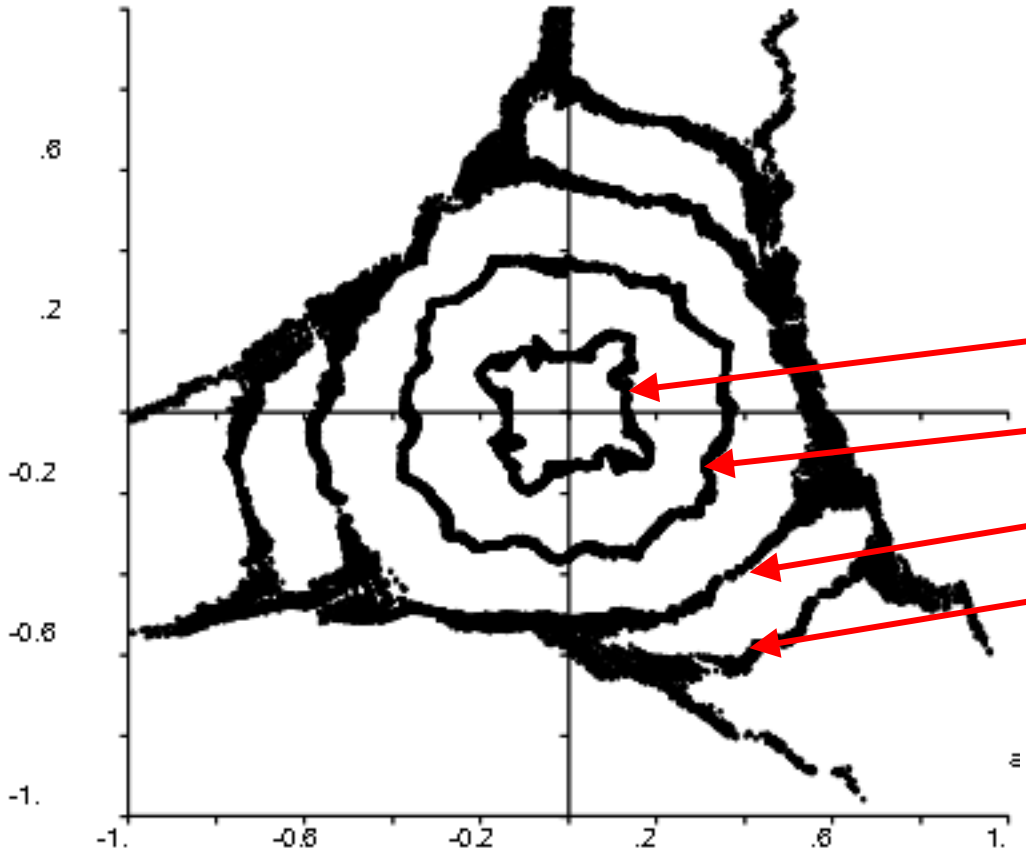
• The ‘structure’ of the extracted beam is because magnetic field lines have been traced for discrete coordinates only ($r_i$,$x_0$,θ): $r_i$=1,2,3,4mm, $x_0$=0.64m, θ = 0,360°. This structure is of course artificial, but interpretation is much easier.

4 different planes where profiles, emittances, and momentum space are shown on the next slights.




MS ECRIS 28GHz
file: C:/INP/DATA/KOBRA/MS ECRIS/case h2/PLOT098.EPS
date: 12/04/2012
user: INP Wiesbaden
time: 14:32:58

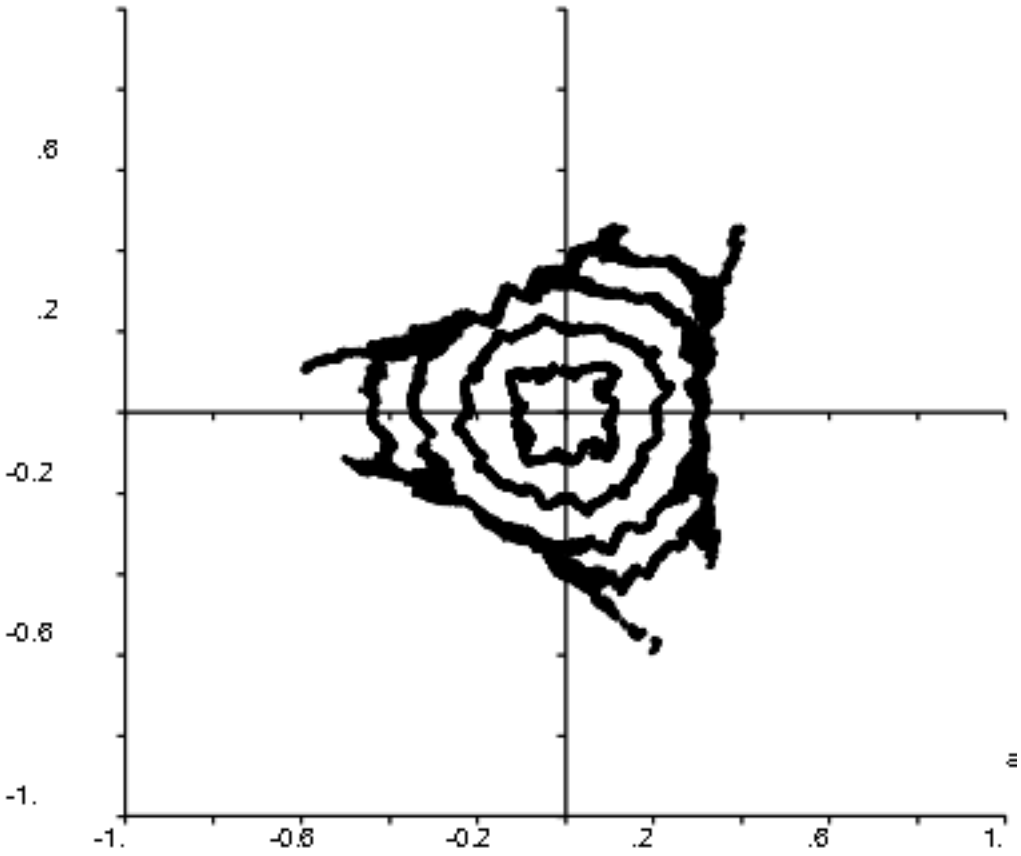
Profile at X = 0.75m




starting conditions from field lines with different radius at the extraction aperture

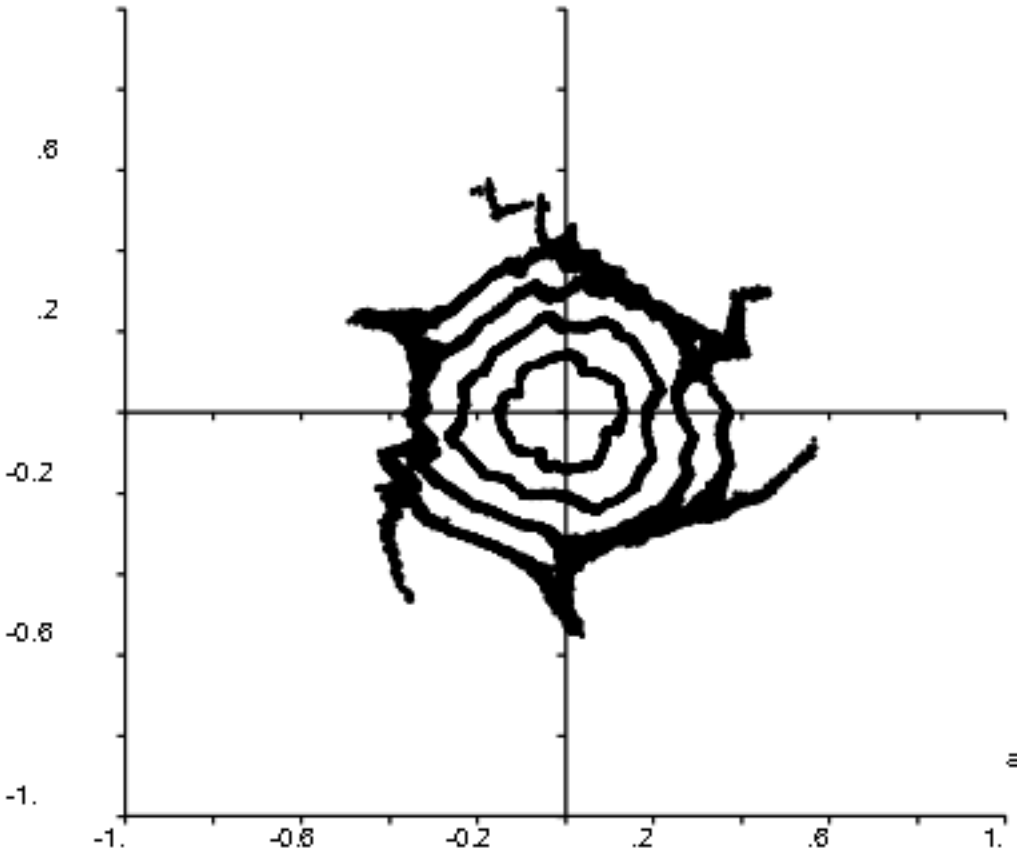
- 1 mm
- 2 mm
- 3 mm
- 4 mm

Scale +/- 1cm for both axis



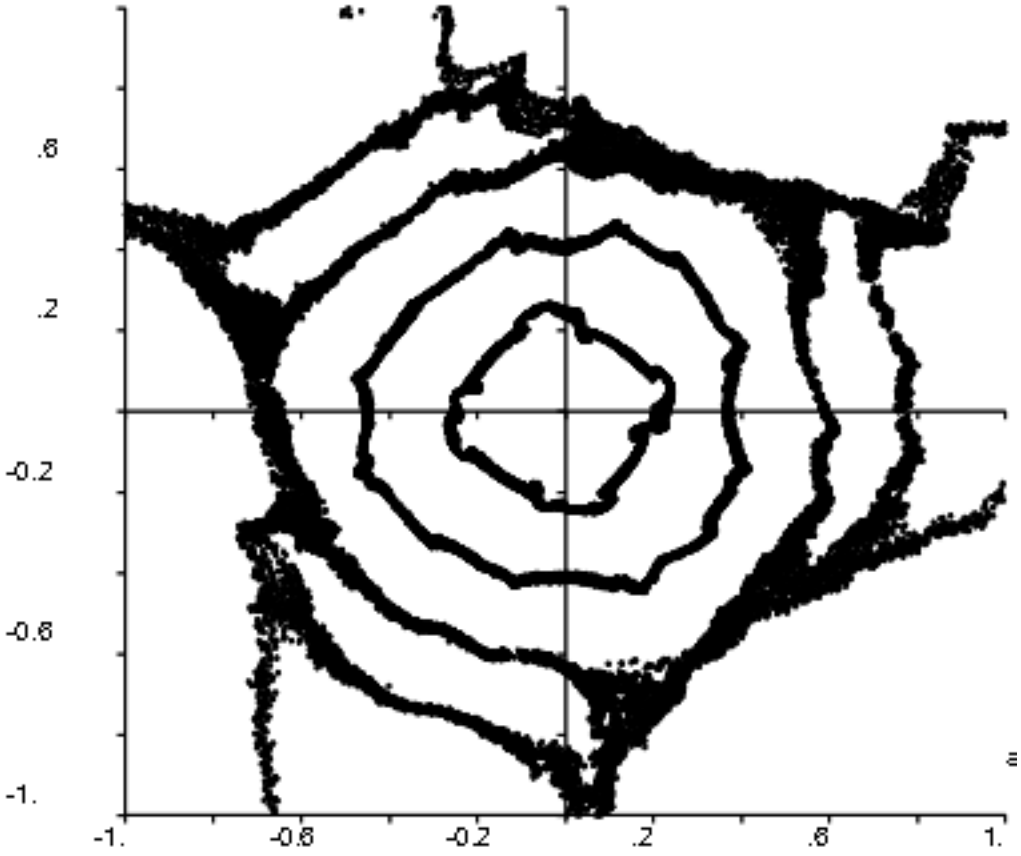

Profile at X = 0.80m

Scale +/- 1cm for both axis



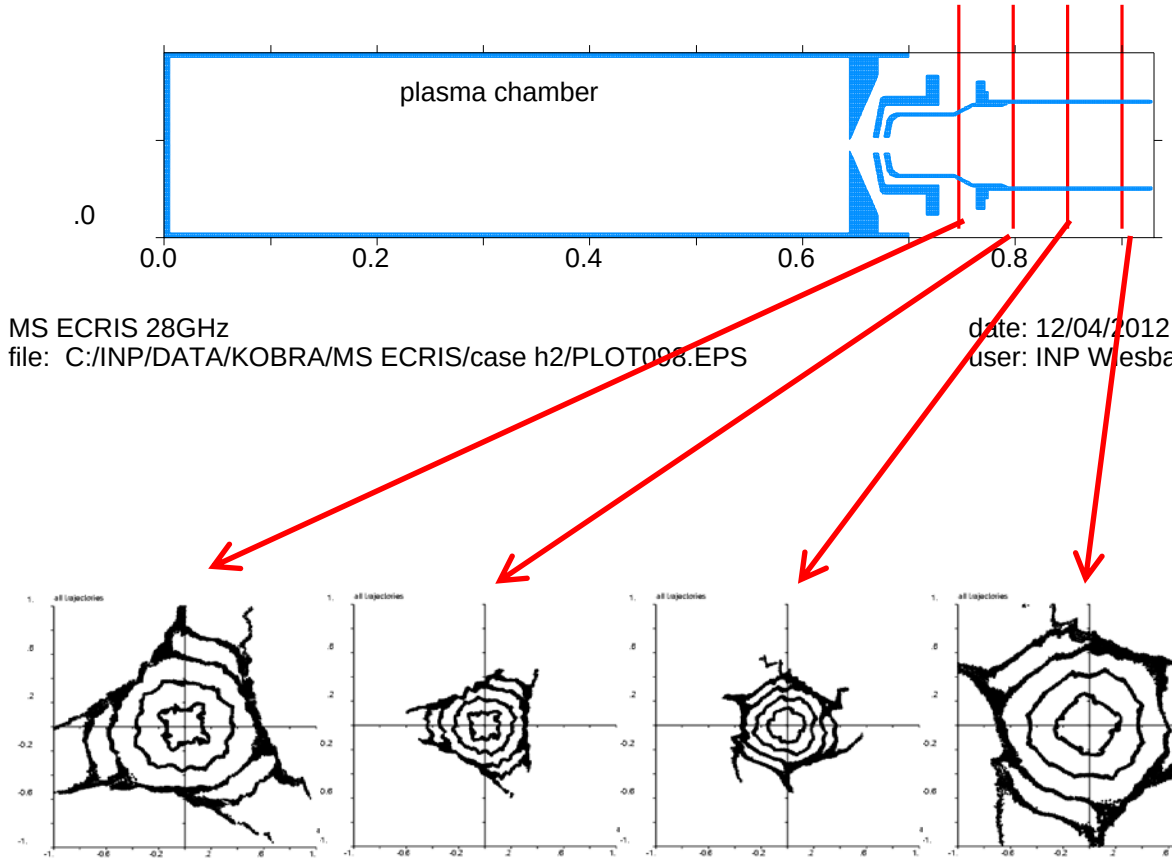

Profile at X = 0.85m

Scale +/- 1cm for both axis




Profile at X = 0.90m

Scale +/- 1cm for both axis




Development of ion beam profile along the path within the stray field

## A very short remark to 'emittances'

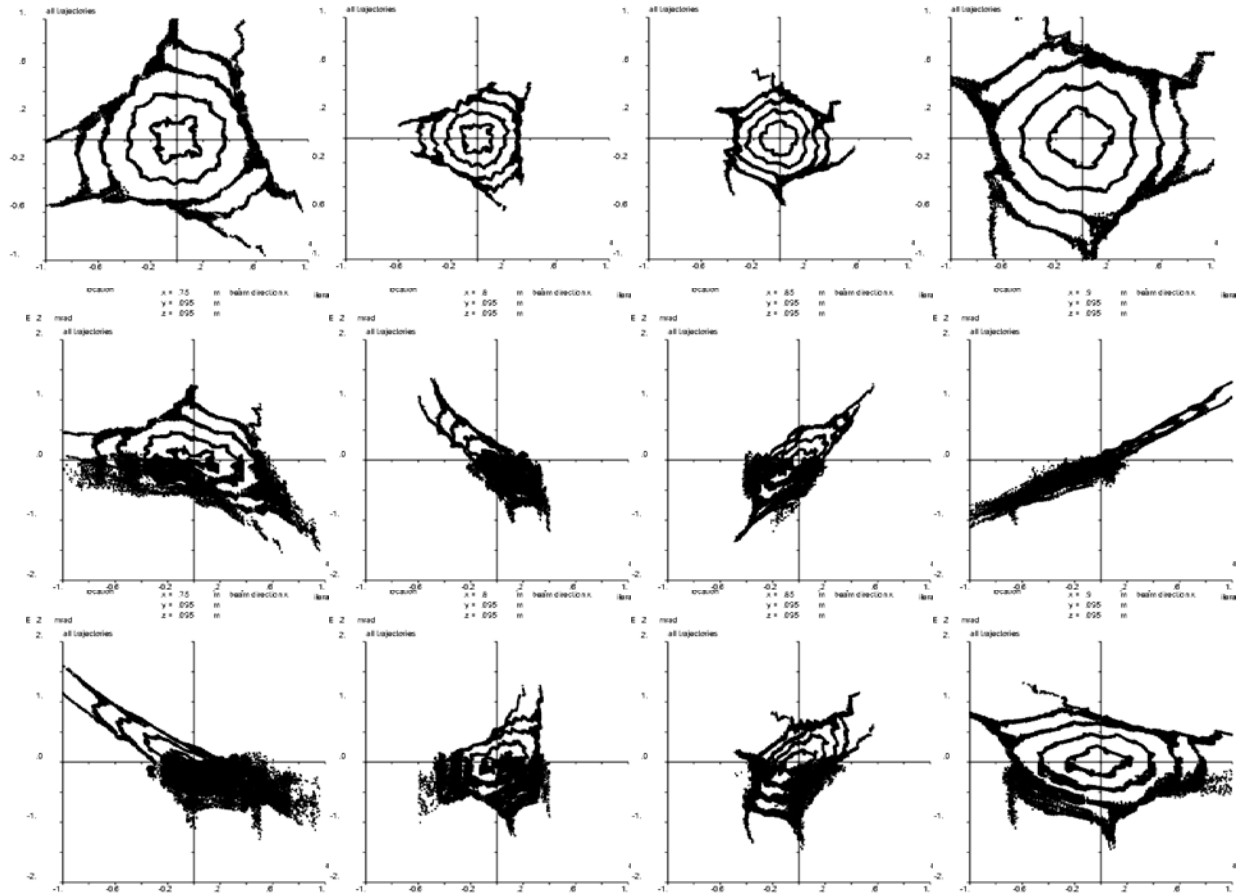
Everybody knows (at least you should know) that a particle is absolutely exact described, if all coordinates of this particle for a given time t are given: $(x, y, z, p_x, p_y, p_z)_t$ where x,y,z describe the spatial coordinates and $p_x$, $p_y$, $p_z$ are the momenta in each direction. Most human beings having a problem to draw this six-dimensional so called phase space (me too). Therefore, a projection of this 6D space into a 2D plotting plane is made.

Nobody has any problem to read the projection into the real space $(y, z)_{x,t}$ if y, z are the transverse coordinates, and x the beam direction, and t the actual time. Typically this is called than a beam profile.

As there are different possibilities of projection, one can do such a projection in any direction. Not to get confused the other projections have different names, like emittance or momentum space (or similar).

Liouville has shown that the six-dimensional volume occupied by a number of particles stays constant under certain conditions (only conservative forces, no collisions, may be more…). If the acting forces are independent in each plane one can even do calculus independently in these sub-spaces. We are saying in that case that the planes are de-coupled. Which is in general not the case, when magnetic fields are present or space charge forces are acting.

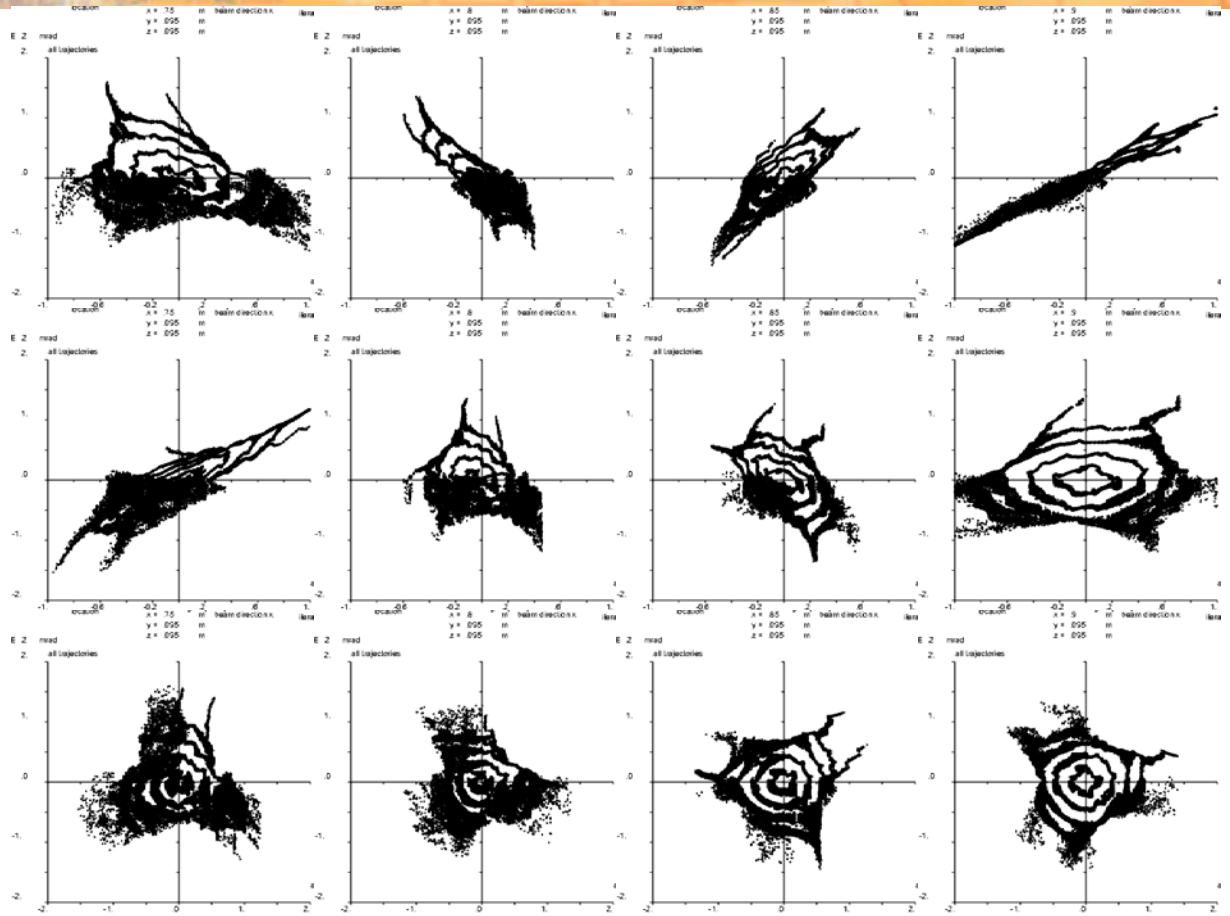
Where is the problem?



real space y-z

emittance y-y’

mixed phase space y-z’



emittance z-z’

mixed phase space z-y’

momentum space y’-z’

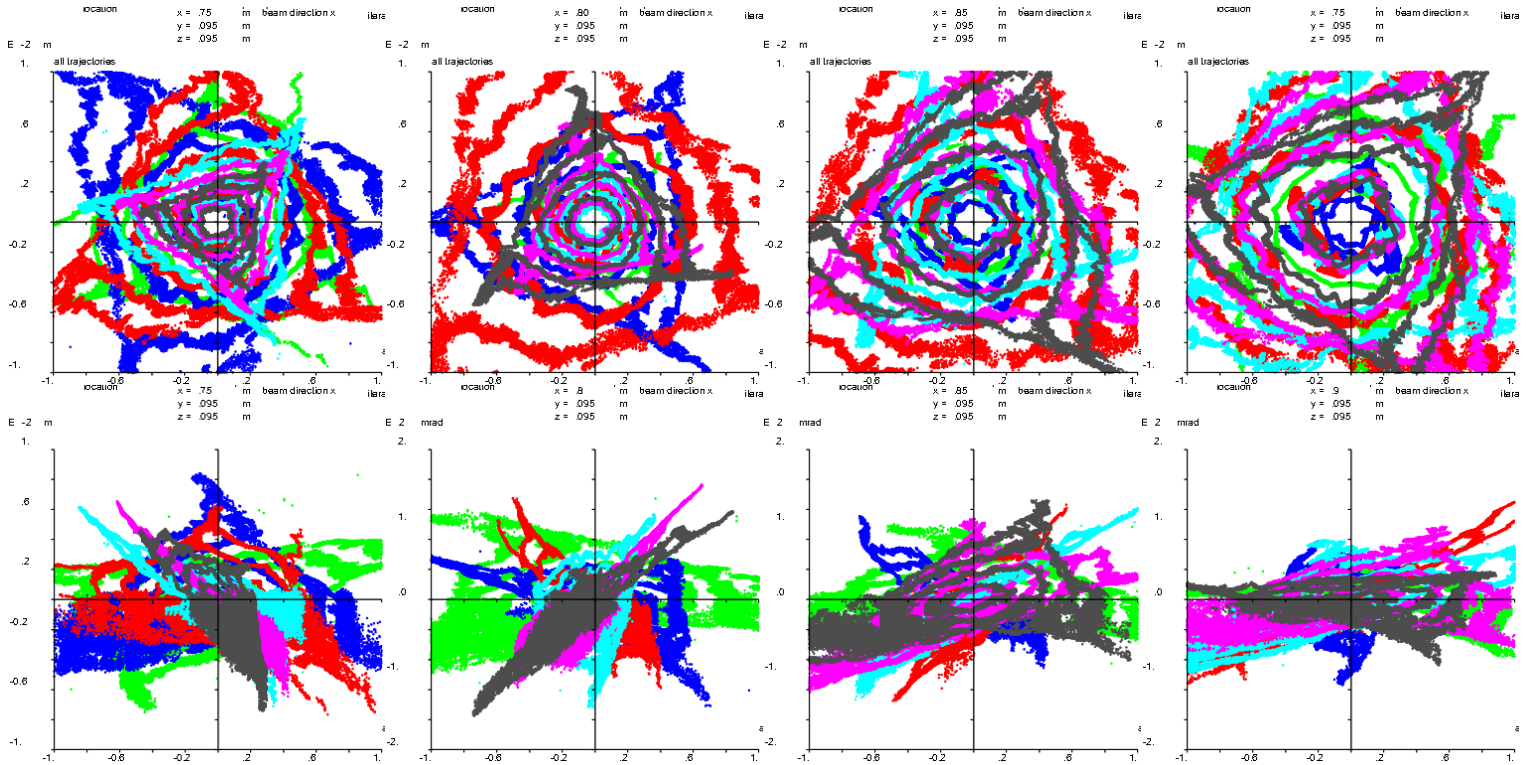
**These figures have shown the Ar3+ charge state only…**

**Let's add some other charge states:**

# CAS Course on Ion Sources

**29.05. - 8.06.2012 Senec**

**MODELING**




**profile y-z**

scale +/- 1cm

**emittance y-y'**

scale +/- 1cm

+/- 200 mrad

**X = 0.75 0.8 0.85 0.9 m**

$Ar^{6+}$ $Ar^{5+}$ $Ar^{4+}$ $Ar^{3+}$ $Ar^{2+}$ $Ar^{+}$

SENEC

STU FEI

SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA

Faculty of Electrical Engineering and Information Technology

GSI

## Conclusion

The stray field of the solenoid from the source already focus the extracted beam.

Different charge states are treated differently.

Ions extracted on one radius are generated at different longitudinal positions, depending on the azimuth, having therefore different $\int \mathbf{B} ds \neq 0$ !!

The current of each trajectory will depend on the plasma density at the individual starting coordinate. See lectures given by Luigi Celona: *New Plasma Heating Methods* and Santo Gammino: *High Intensity, Highly Charged Ions*.

Child's law should be valid, but only locally at each magnetic field line. At some starting coordinates it might be emission limited, depending where the magnetic field line has its origin.

# Model for $H^-$

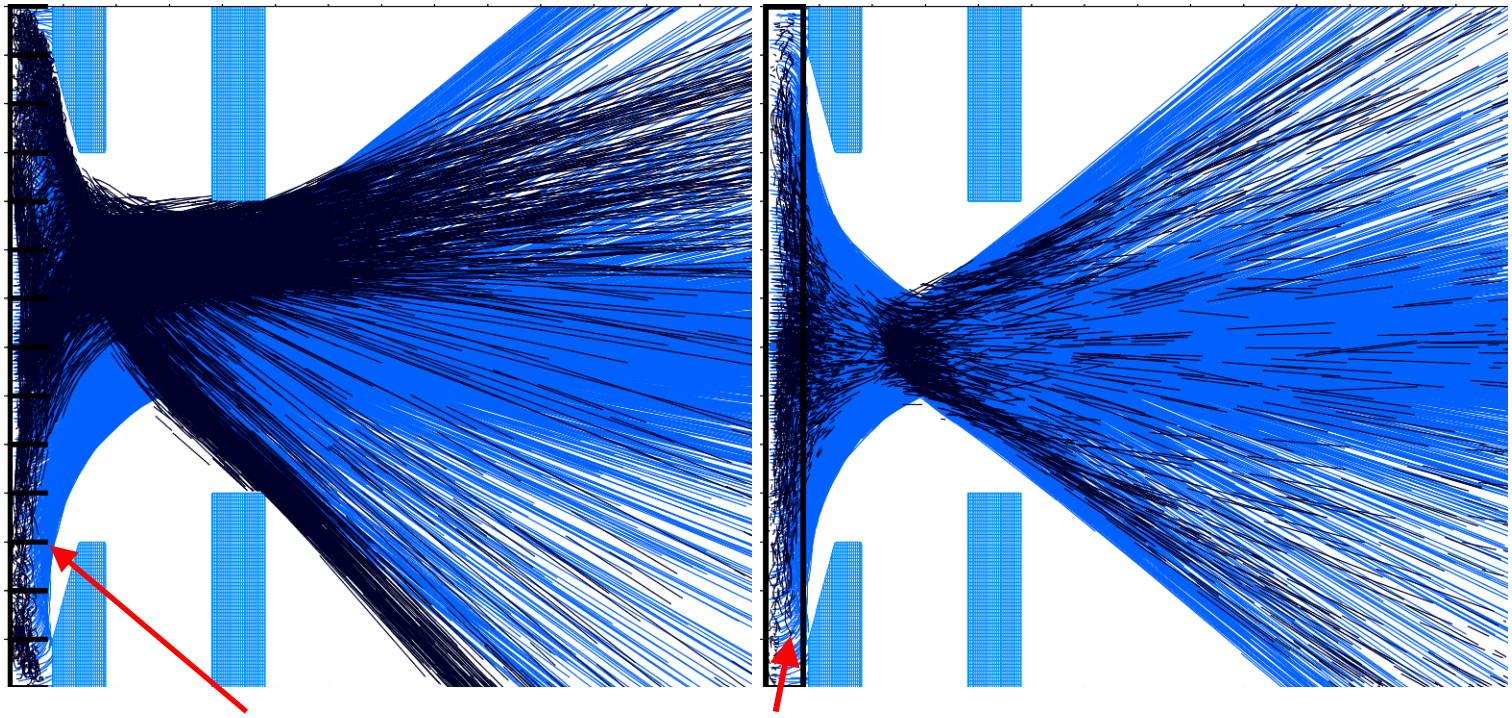
***volume ion source model:***

Only negative ions and electrons are simulated, the space charge of positive ions is calculated analytically by a one dimensional dependency on the potential. Of course, because Φ(x,y,z) depends on all three coordinates, this is a 3D model.

$$N_{i+} = n_{i+0} * \exp(-q(\Phi_{pl}-\Phi)/kT_{i+})$$

| | |
|---|---|
| $N_{i+}$ | positive ion density |
| $N_{i+0}$ | positive ion density within the undisturbed plasma |
| **q** | charge state |
| $\Phi_{pl}$ | plasma potential |
| **Φ** | potential at (x,y,z) |
| **K** | Boltzmann's constant |
| $T_{i+}$ | positive ion temperature |

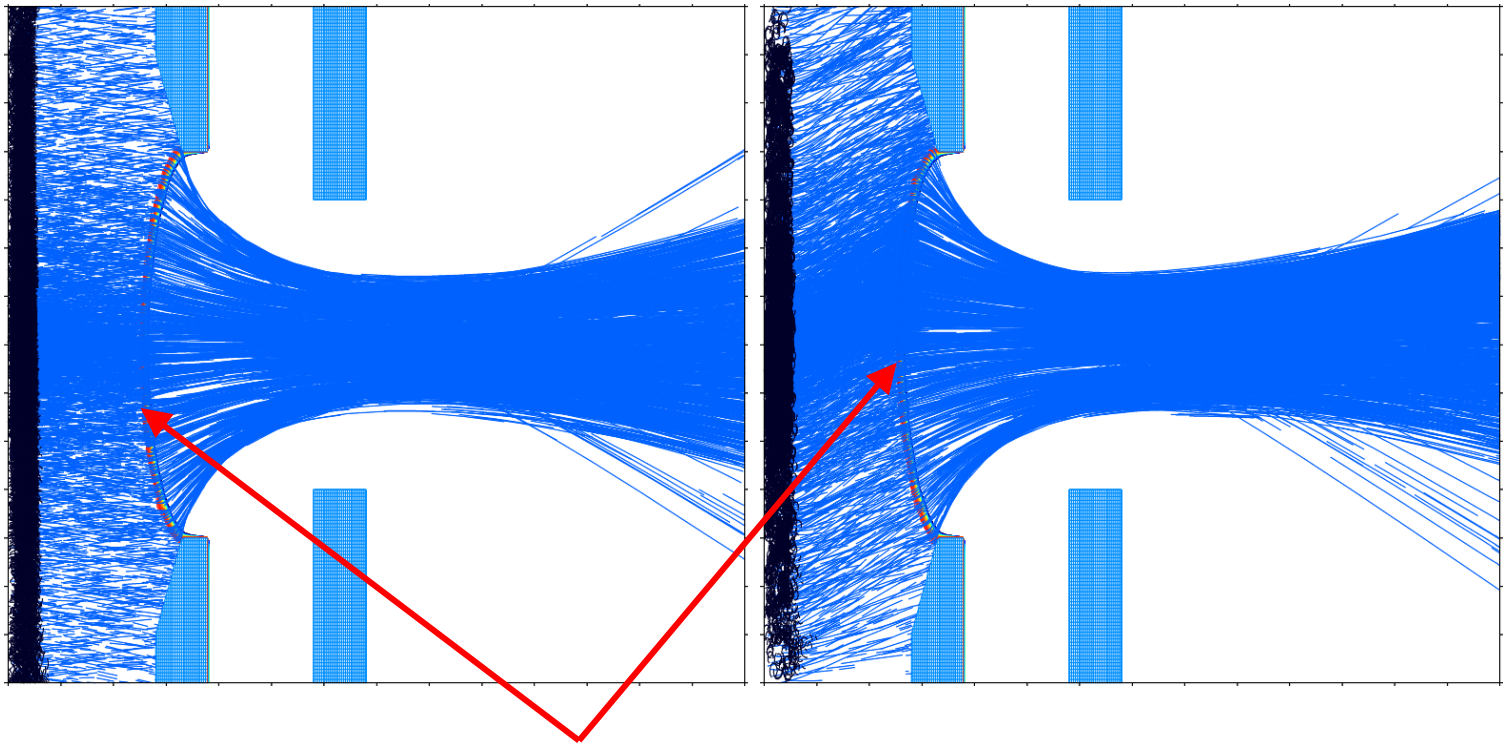
Self's model with reversed potentials, electrons are replaced by positive ions.



Conductor creating a dipole filter field

all electrons are extracted, the magnetic filter is not working as expected.

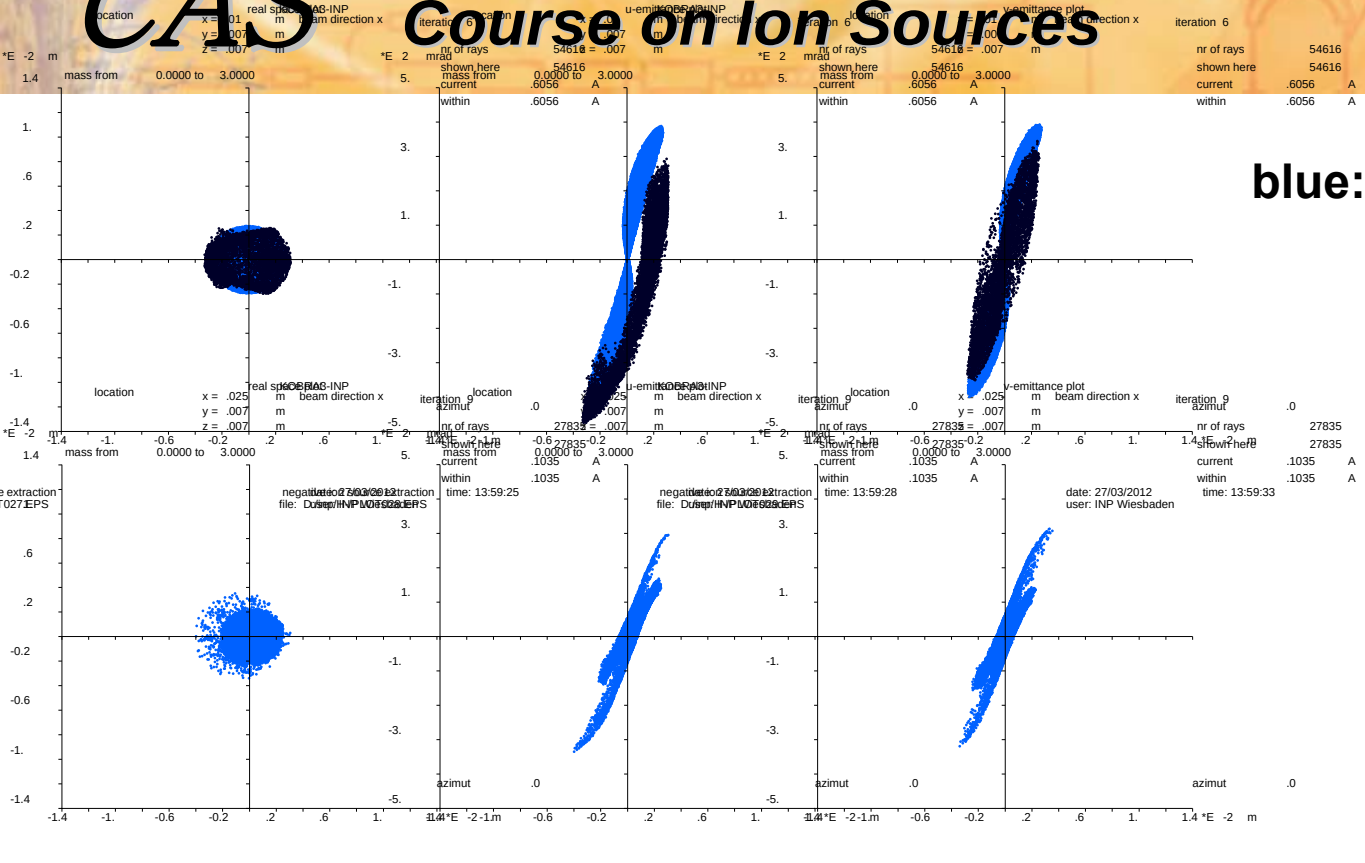
let us try to move the filter back 4mm away from the extraction.




all electrons are now trapped within the plasma.

see also the plasma boundary.

# MODELING



**blue: $H^-$ black: electrons**

**The only difference between both simulations is, that the magnetic filter is shifted into the plasma volume by 4mm.**

**This shows how sensitive the simulation can be, also indicating that the initial conditions of particle tracking and the boundary conditions for solving Poisson's equation are critical.**

## Conclusion (this time final!)

I hope that I have shown, that the used model for each ion source has to be specialized.

To perform simulations only is a bad idea, but doing experiments without modeled simulation is also not a good idea.

Only if simulations show a good agreement with experiments, an optimization of the system can be performed with a high probability of success.