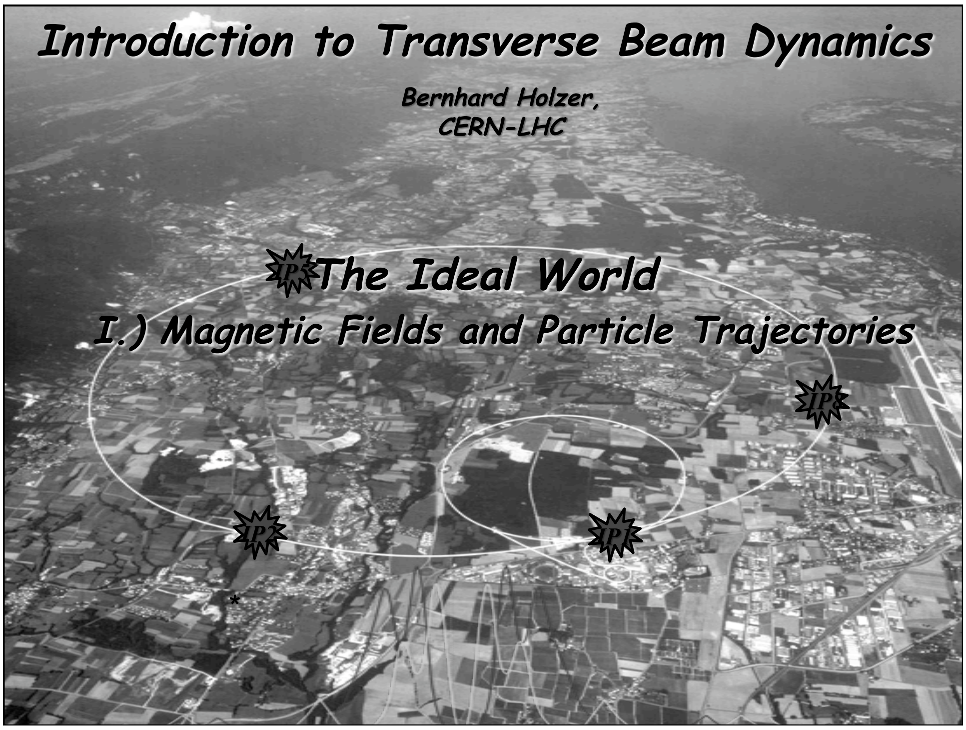


Introduction to Transverse Beam Dynamics

Bernhard Holzer,
CERN-LHC

The Ideal World

I.) Magnetic Fields and Particle Trajectories

1.) Introduction and Basic Ideas

„ ... in the end and after all it should be a kind of circular machine“
→ need transverse deflecting force

Lorentz force $\vec{F} = q * (\cancel{\vec{E}} + \vec{v} \times \vec{B})$

typical velocity in high energy machines: $v \approx c \approx 3*10^{8}\, {}^{m}\!/_{s}$

Example:

$$B = 1T \quad \rightarrow \quad F = q*3*10^{8}\,\frac{m}{s}*1\frac{Vs}{m^{2}}$$

$$F = q*\underbrace{300\,\frac{MV}{m}}_{E}$$

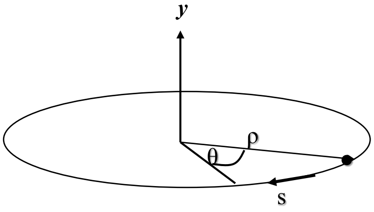
equivalent el. field ...

technical limit for el. field:

$$E \leq 1\,\frac{MV}{m}$$

old greek dictum of wisdom:
if you are clever, you use magnetic fields in an accelerator wherever it is possible.

The ideal circular orbit



circular coordinate system

condition for circular orbit:

Lorentz force $F_L = e\,v\,B$

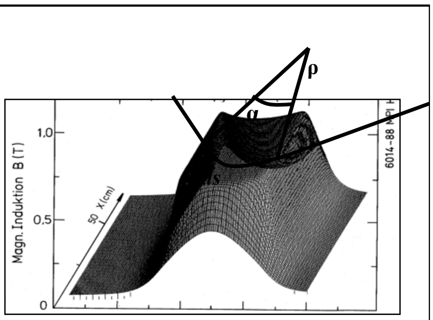
centrifugal force $F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}$

$$\frac{\gamma\, m_0\, v^{\cancel{2}}}{\rho} = e\,\cancel{v}\,B$$

$$\boxed{\frac{p}{e} = B\,\rho}$$

B ρ = "beam rigidity"

The Magnetic Guide Field



field map of a storage ring dipole magnet

Example LHC:

$$\left.\begin{array}{l} B = 8.3\,T \\ p = 7000\,\frac{GeV}{c} \end{array}\right\}$$

$\rho = 2.53\ km \longrightarrow$ ***2πρ = 17.6 km***
≈ 66%

convenient units:

$$B = [T] = \left[\frac{Vs}{m^2}\right] \qquad p = \left[\frac{GeV}{c}\right]$$

rule of thumb: $\frac{1}{\rho} \approx 0.3\,\frac{B[T]}{p[GeV/c]}$

„normalised bending strength“

Example LHC:

7000 GeV Proton storage ring
dipole magnets N = 1232
l = 15 m
q = +1 e

$$\int B\,dl \approx N\,l\,B = 2\pi\ p/e$$

$$B \approx \frac{2\pi\ \ 7000\ \ 10^{9}\,eV}{1232\ \ 15\,m\ \ 3\ 10^{8}\,\frac{m}{s}\ \ e} = 8.3\,Tesla$$

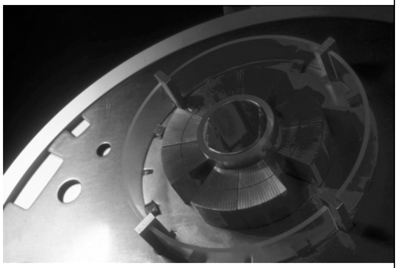
2.) Quadrupole Magnets:

required: *focusing forces to keep trajectories in vicinity of the ideal orbit*
linear increasing Lorentz force
linear increasing magnetic field $B_y = g\,x \qquad B_x = g\,y$

normalised quadrupole field:

gradient of a quadrupole magnet: $$g = \frac{2\mu_0 nI}{\boxed{r^2}}$$

$$\longrightarrow \qquad k = \frac{g}{p/e}$$



LHC main quadrupole magnet

$$g \approx 25 \ldots 220\ \ T/m$$

simple rule: $$k = 0.3\,\frac{g(T/m)}{p(GeV/c)}$$

what about the vertical plane:
... Maxwell

$$\vec{\nabla}\times\vec{B} = \cancel{\vec{j}} + \cancel{\frac{\partial\vec{E}}{\partial t}} = 0 \qquad \Rightarrow \qquad \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y}$$

3.) The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \cancel{\frac{1}{2!}\,m\,x^2} + \cancel{\frac{1}{3!}\,n\,x^3} + \ldots$$

only terms linear in x, y taken into account dipole fields
quadrupole fields

Separate Function Machines:

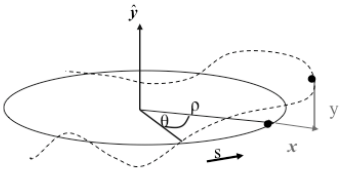
Split the magnets and optimise them according to their job:

bending, focusing etc

Example:
heavy ion storage ring TSR

* ***man sieht nur***
dipole und quads → linear

The Equation of Motion:



general radial acceleration $$a_r = \frac{d^2\rho}{dt^2} - \rho\left(\frac{d\theta}{dt}\right)^2$$

general trajectory: $\rho \rightarrow \rho + x$

$$F = m\frac{d^2}{dt^2}(x+\rho) - \frac{mv^2}{x+\rho} = e\,B_y\,v$$

$$B_y = B_0 + x\frac{\partial B_y}{\partial x}$$

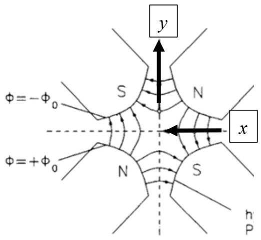
$$x'' + x\,(\frac{1}{\rho^2} - k) = 0$$

* *Equation for the vertical motion:*

$\frac{1}{\rho^2} = 0$ ***no dipoles … in general …***

$k \leftrightarrow -k$ ***quadrupole field changes sign***

$$y'' + k\;y = 0$$



4.) Solution of Trajectory Equations

Define … hor. plane: $K = 1/\rho^2 - k$

… vert. Plane: $K = k$

$$\left.\begin{array}{l} \\ \\ \end{array}\right\} \qquad \boldsymbol{x'' + K\, x} = 0$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: $x(s) = a_1 \cdot \cos(\omega s) + a_2 \cdot \sin(\omega s)$

general solution: linear combination of two independent solutions

$$x'(s) = -a_1\omega \sin(\omega s) + a_2\omega \cos(\omega s)$$

$$x''(s) = -a_1\omega^2 \cos(\omega s) - a_2\omega^2 \sin(\omega s) = -\omega^2 x(s) \qquad \longrightarrow \qquad \omega = \sqrt{K}$$

general solution:

$$x(s) = a_1 \cos(\sqrt{K} s) + a_2 \sin(\sqrt{K} s)$$

---

determine a_1, a_2 by boundary conditions:

$$s = 0 \longrightarrow \begin{cases} x(0) = x_0 \quad , \quad a_1 = x_0 \\ x'(0) = x'_0 \quad , \quad a_2 = \dfrac{x'_0}{\sqrt{K}} \end{cases}$$

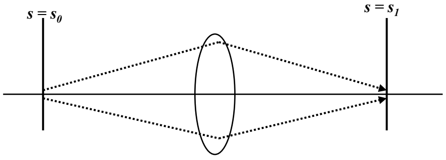
Hor. Focusing Quadrupole K > 0:

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x'_0 \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x'_0 \cdot \cos(\sqrt{|K|}s)$$

For convenience expressed in matrix formalism:

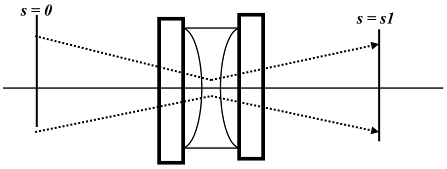
$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$



$$M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|}s) & \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s \\ -\sqrt{|K|} \sin(\sqrt{|K|}s) & \cos(\sqrt{|K|}s) \end{pmatrix}_0$$

hor. defocusing quadrupole:

$$x'' - K\,x = 0$$



Remember from school:

$$f(s) = \cosh(s) \quad , \quad f'(s) = \sinh(s)$$

Ansatz: $x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s)$

$$M_{def\,oc} = \begin{pmatrix} \cosh\sqrt{|K|}l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}l \\ \sqrt{|K|}\sinh\sqrt{|K|}l & \cosh\sqrt{|K|}l \end{pmatrix}$$

drift space:

K = 0

$$M_{drif\,t} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

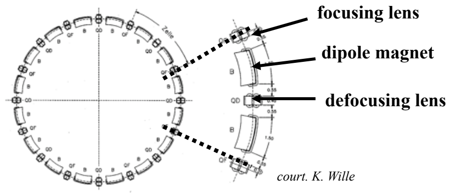
! with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“

Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

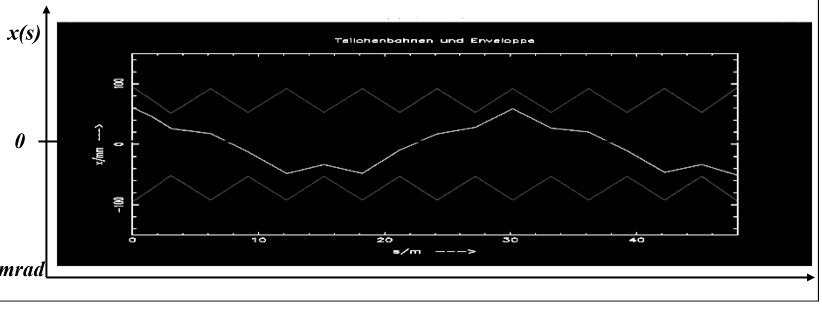
$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D*}.....$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2, s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$



court. K. Wille

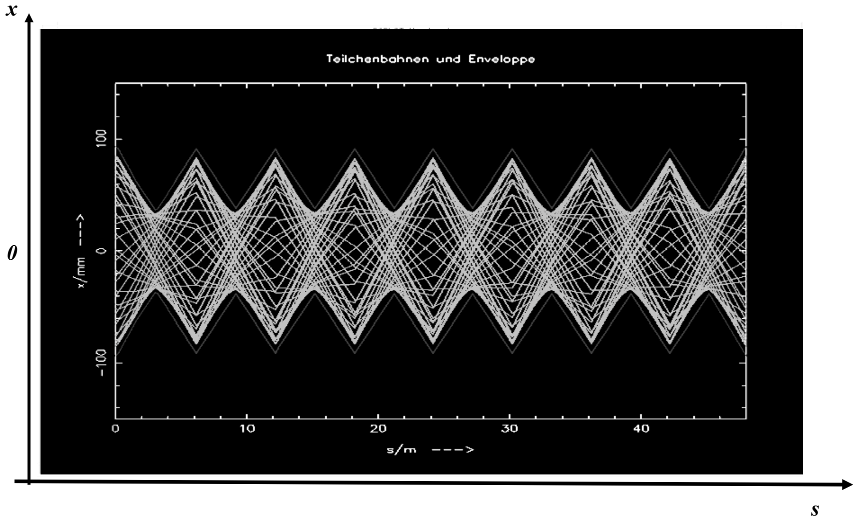
in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „



typical values in a strong foc. machine:
$x \approx$ mm, $x' \le$ mrad

Question: what will happen, if the particle performs a second turn ?

... or a third one or ... 10^{10} turns



Astronomer Hill:

differential equation for motions with periodic focusing properties
„Hill's equation"

Example: particle motion with periodic coefficient

equation of motion: $x''(s) - k(s)x(s) = 0$

restoring force ≠ const,
k(s) = depending on the position s
k(s+L) = k(s), periodic function

we expect a kind of quasi harmonic oscillation: amplitude & phase will depend on the position s in the ring.

5.) The Beta Function

General solution of Hill´s equation:

(i) $$x(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)} \cdot \cos(\psi(s) + \phi)$$

ε, Φ = integration constants determined by initial conditions

β(s) periodic function given by focusing properties of the lattice ↔ quadrupoles

$$\beta(s + L) = \beta(s)$$

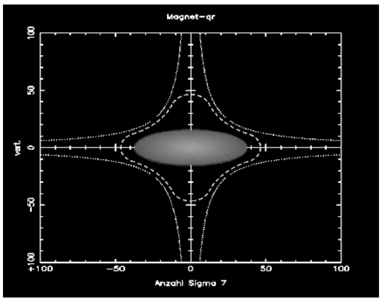
Inserting (i) into the equation of motion …

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)}$$

Ψ(s) = „phase advance“ of the oscillation between point „0“ and „s“ in the lattice.
For one complete revolution: number of oscillations per turn „Tune“

$$\boldsymbol{Q_y} = \frac{1}{2\pi}\oint \frac{\boldsymbol{ds}}{\beta(\boldsymbol{s})}$$

The Beta Function

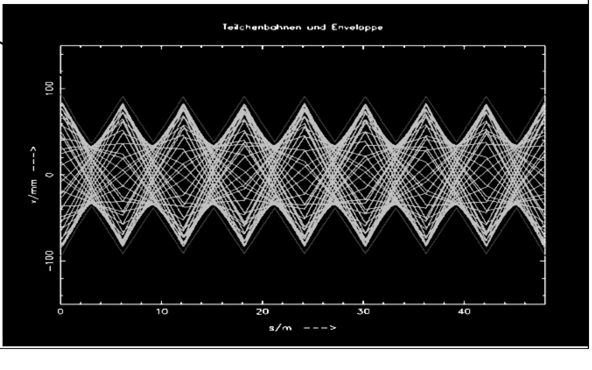


Amplitude of a particle trajectory:

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \varphi)$$

Maximum size of a particle amplitude

$$\boxed{\hat{x}(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}}$$



β determines the beam size
(... the envelope of all particle trajectories at a given position “s” in the storage ring.

It reflects the periodicity of the magnet structure.

6.) Beam Emittance and Phase Space Ellipse

general solution of Hill equation

$$\begin{cases} (1) \quad x(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}\,\cos(\psi(s)+\phi) \\ (2) \quad x'(s) = -\dfrac{\sqrt{\varepsilon}}{\sqrt{\beta(s)}}\,\{\alpha(s)\cos(\psi(s)+\phi)+\sin(\psi(s)+\phi)\} \end{cases}$$

from (1) we get

$$\cos(\psi(s)+\phi) = \frac{x(s)}{\sqrt{\varepsilon}\,\sqrt{\beta(s)}}$$

$$\alpha(s) = \frac{-1}{2}\beta'(s)$$

$$\gamma(s) = \frac{1+\alpha(s)^2}{\beta(s)}$$

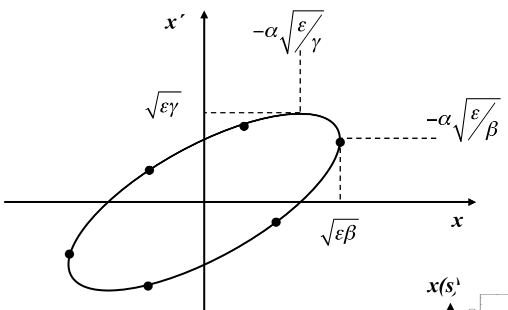
Insert into (2) and solve for ε

$$\varepsilon = \gamma(s)\,x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)\,x'^2(s)$$

**** ε is a constant of the motion … it is independent of „s“***
**** parametric representation of an ellipse in the x x‘ space***
**** shape and orientation of ellipse are given by α, β, γ***

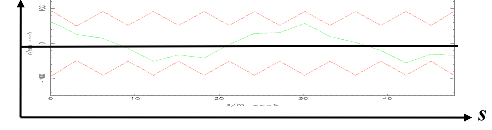
Beam Emittance and Phase Space Ellipse

$$\varepsilon = \gamma(s)\,x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)\,x'^2(s)$$



Liouville: in reasonable storage rings
area in phase space is constant.

A = π*ε=const



ε beam emittance = woozilycity of the particle ensemble, intrinsic beam parameter,
cannot be changed by the foc. properties.
Scientifiquely speaking: area covered in transverse x, x´ phase space … and it is constant !!!

Phase Space Ellipse

$$\varepsilon = \gamma(s)\, x^2(s) + 2\alpha(s) x(s) x'(s) + \beta(s)\, x'^2(s)$$

$$\alpha(s) = \frac{-1}{2}\beta'(s)$$

$$\gamma(s) = \frac{1+\alpha(s)^2}{\beta(s)}$$

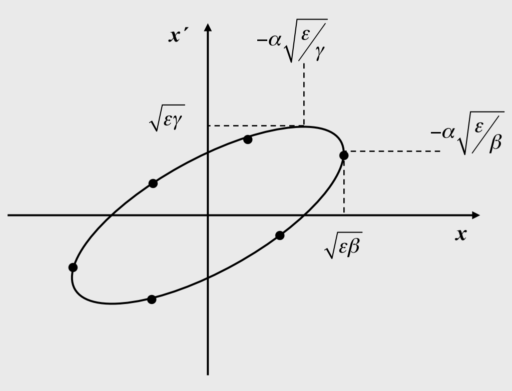
$$\longrightarrow \quad \varepsilon = \frac{x^2}{\beta} + \frac{\alpha^2 x^2}{\beta} + 2\alpha \cdot xx' + \beta \cdot x'^2$$

… solve for x´ $x'_{1,2} = \dfrac{-\alpha \cdot x \pm \sqrt{\varepsilon\beta - x^2}}{\beta}$

… and determine $\hat{x}'$ *via:* $\dfrac{dx'}{dx} = 0$

$$\longrightarrow \quad \hat{x}' = \sqrt{\varepsilon\gamma}$$

$$\longrightarrow \quad \hat{x} = \pm\alpha\sqrt{\varepsilon/\gamma}$$

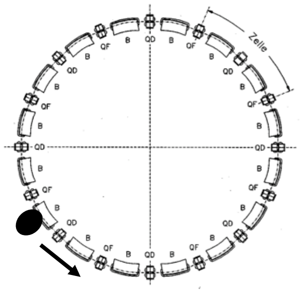


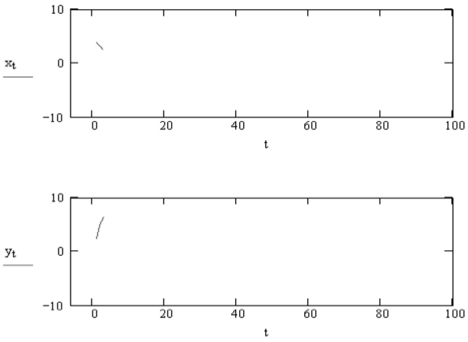
shape and orientation of the phase space ellipse depend on the Twiss parameters β α γ

Particle Tracking in a Storage Ring

Calculate x, x´ for each linear accelerator element according to matrix formalism

plot x, x´as a function of „s"

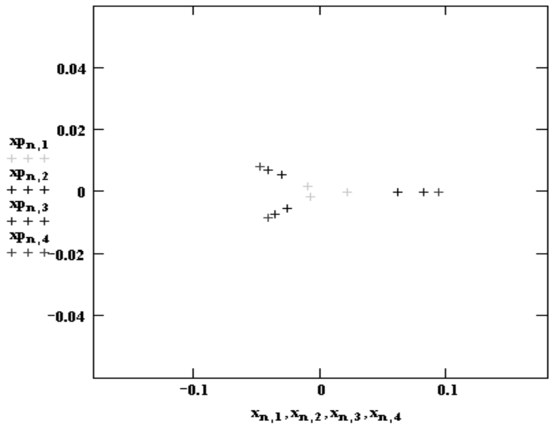




… and now the ellipse: A beam of 4 particles

each having a slightly different emittance:

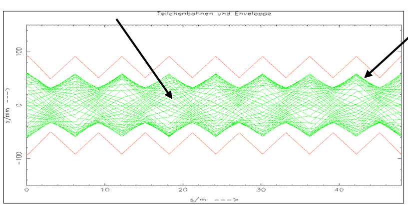
note for each turn x, x´at a given position „s_1“ and plot in the phase space diagram



Emittance of the Particle Ensemble:

$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}\cdot\cos(\Psi(s)+\phi)$

$\boxed{\hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}}$



single particle trajectories, N ≈ 10^{11} per bunch

Gauß Particle Distribution:

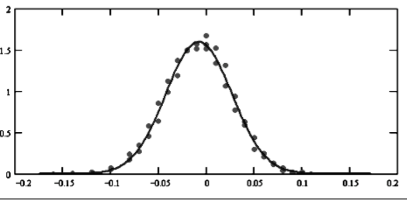
$$\rho(x) = \frac{N\cdot e}{\sqrt{2\pi}\sigma_x}\cdot e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}}$$

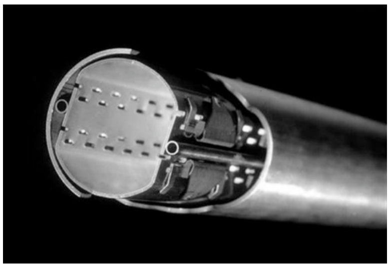
particle at distance 1 σ from centre ↔ 68.3 % of all beam particles

LHC: $\beta = 180\,m$

$\varepsilon = 5*10^{-10}\,m\,rad$

$\sigma = \sqrt{\varepsilon*\beta} = \sqrt{5*10^{-10}m*180\,m} = 0.3\,mm$

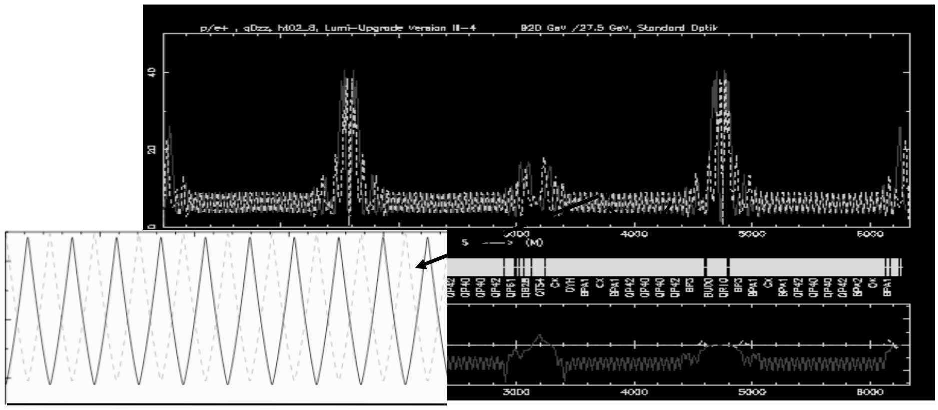




aperture requirements: r_0 = 12 * σ

The FoDo-Lattice

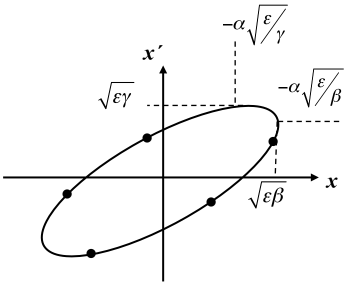
A magnet structure consisting of focusing and defocusing quadrupole lenses in alternating order with nothing in between.
(Nothing = elements that can be neglected on first sight: drift, bending magnets, RF structures ... and especially experiments...)



Starting point for the calculation: in the middle of a focusing quadrupole
Phase advance per cell μ = 45°,
→ calculate the twiss parameters for a periodic solution

7.) Liouville during Acceleration

$$\varepsilon = \gamma(s)\,x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)\,x'^2(s)$$



Beam Emittance corresponds to the area covered in the x, x´ Phase Space Ellipse

Liouville: Area in phase space is constant.

But so sorry ... ε ≠ const !

Classical Mechanics:

phase space = diagram of the two canonical variables
position & momentum
x ***p_x***

$$p_j = \frac{\partial L}{\partial \dot{q}_j} \quad ; \quad L = T - V = kin.\,Energy - pot.\,Energy$$

According to Hamiltonian mechanics:
phase space diagram relates the variables q and p

q = position = x
p = momentum = γmv = mcγβ$_x$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad ; \quad \beta_x = \frac{\dot{x}}{c}$$

Liouvilles Theorem: $\int p\,dq = const$

for convenience (i.e. because we are lazy bones) we use in accelerator theory:

$$x' = \frac{dx}{ds} = \frac{dx}{dt}\frac{dt}{ds} = \frac{\beta_x}{\beta}$$

where β$_x$= v$_x$/ c

$$\int p\,dq = mc\int \gamma\beta_x dx$$

$$\int p\,dq = mc\gamma\beta\underbrace{\int x'dx}_{\varepsilon}$$

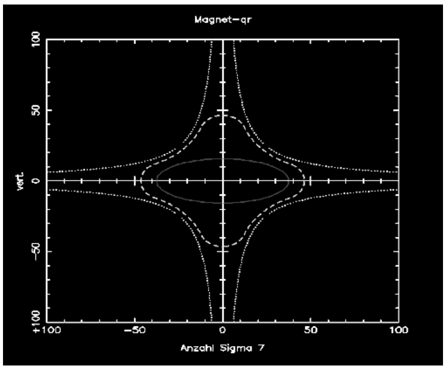
$$\Rightarrow \quad \varepsilon = \int x'dx \propto \frac{1}{\beta\gamma}$$

the beam emittance shrinks during acceleration ε ~ 1 / γ

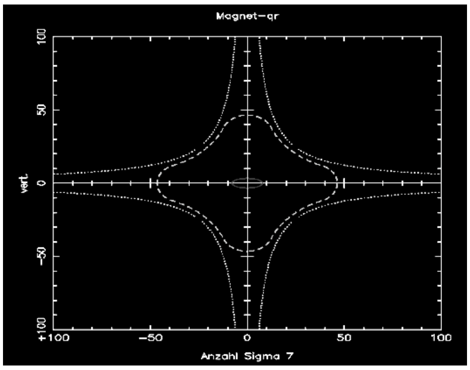
Example: HERA proton ring

injection energy: 40 GeV ***γ = 43***
flat top energy: 920 GeV ***γ = 980***

emittance ε (40GeV) = 1.2 * 10 $^{-7}$
ε (920GeV) = 5.1 * 10 $^{-9}$



7 σ beam envelope at E = 40 GeV



… and at E = 920 GeV

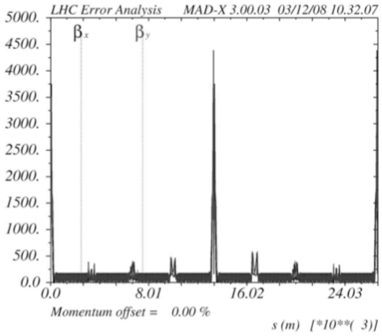
Nota bene:

1.) A proton machine … or an electron linac … needs the highest aperture at injection energy !!!
as soon as we start to accelerate the beam size shrinks as $\gamma^{-1/2}$ in both planes.

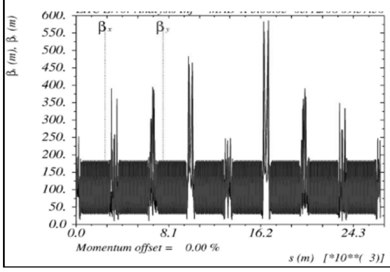
$$\sigma = \sqrt{\varepsilon\beta}$$

2.) At lowest energy the machine will have the major aperture problems,
→ here we have to minimise $\hat{\beta}$

3.) we need different beam optics adopted to the energy:
A Mini Beta concept will only be adequate at flat top.



LHC mini beta optics at 7000 GeV

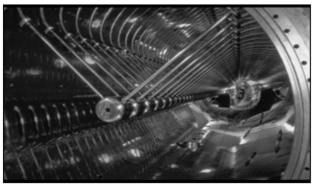


LHC injection optics at 450 GeV

8.) Problem: panta rhei … RF Acceleration

(Heraklit: 540-480 v. Chr.)

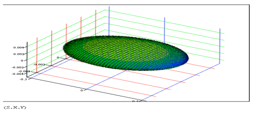
The „ Δp / p ≠ 0" Problem



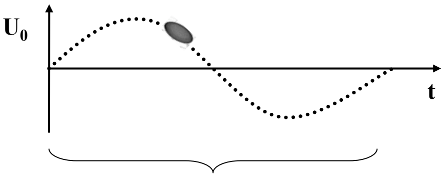
drift tube structure (GSI Unilac)

Energy Gain per „Gap":

$$\boxed{W = q\, U_0 \sin \omega_{RF} t}$$



Bunch length of Electrons ≈ 1cm



$\lambda = 60\, cm$

$$\left.\begin{array}{l} f_{rf} = 500\,MHz \\ c = \lambda\, f \end{array}\right\} \lambda = 60\, cm$$

$$\sin(90^o) = 1 \qquad \frac{\Delta U}{U} = 6.0\ 10^{-3}$$
$$\sin(84^o) = 0.994$$

typical momentum spread of an electron bunch: $\frac{\Delta p}{p} \approx 1.0\ 10^{-3}$

9.) Dispersion:

$$x'' + x(\frac{1}{\rho^2} - k) = \frac{\Delta p}{p} \cdot \frac{1}{\rho}$$

general solution:

$$x(s) = x_h(s) + x_i(s)$$

$$\begin{cases} x''_h(s) + K(s) \cdot x_h(s) = 0 \\ x''_i(s) + K(s) \cdot x_i(s) = \frac{1}{\rho} \cdot \frac{\Delta p}{p} \end{cases}$$

Normalise with respect to Δp/p:

$$D(s) = \frac{x_i(s)}{\Delta p / p}$$

Dispersion function D(s)

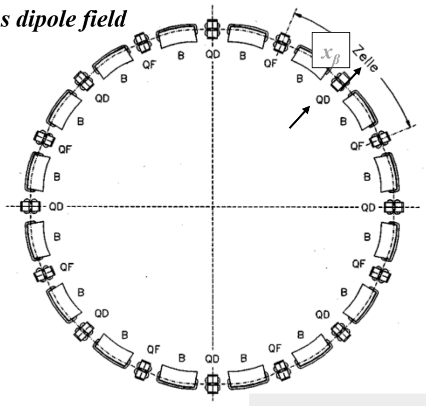
**** is that special orbit, an ideal particle would have for Δp/p = 1***

**** the orbit of any particle is the sum of the well known x_β and the dispersion***

**** as D(s) is just another orbit it will be subject to the focusing properties of the lattice***

Dispersion

Example: homogeneous dipole field



it for Δp/p > 0

$$D(s) \cdot \frac{\Delta p}{p}$$

$$x(s) = x_\beta(s) + D(s) \cdot \frac{\Delta p}{p}$$

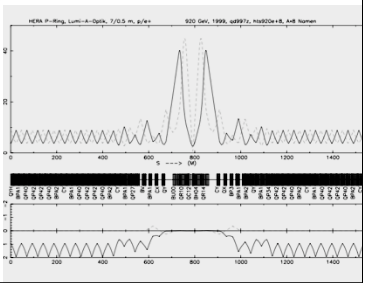
Amplitude of Orbit Oscillation
contribution due to Dispersion
≈ beam size
→Dispersion must vanish at
the collision point

Example

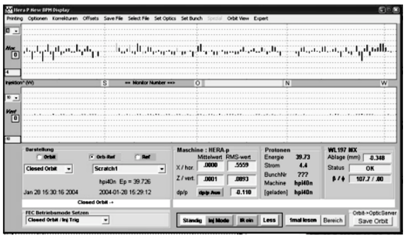
$$x_\beta = 1 \ldots 2\ mm$$

$$D(s) \approx 1 \ldots 2\, m$$

$$\Delta p / p \approx 1 \cdot 10^{-3}$$



Dispersion is visible

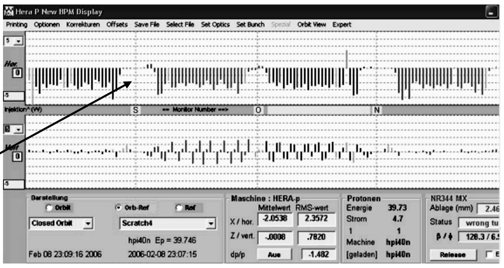


HERA Standard Orbit

HERA Dispersion Orbit

dedicated energy change of the stored beam
→ closed orbit is moved to a
dispersions trajectory

$$x_D = D(s) * \frac{\Delta p}{p}$$



Closed Orbit
Scratch4
hpi40n Ep = 39.746
Feb 08 23:09:16 2006
2006-02-08 23:07:15

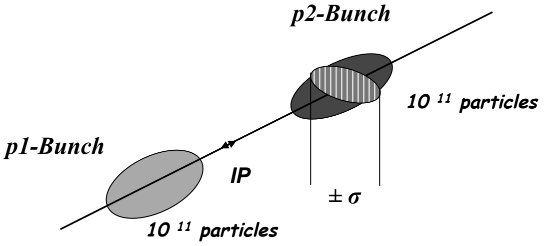
Maschine : HERA-p
Mittelwert RMS-wert
X / hor. -2.0538 2.3572
Z / vert. -.0008 .7820
dp/p Aus -1.482

Protonen
Energie 39.73
Strom 4.7
1 1
Machine hpi40n
(geladen) hpi40n

NR344 MX
Ablage (mm) 2.46
Status wrong tu
β / ϕ 128.3 / 6.

Attention: at the Interaction Points
we require D=D´= 0

11.) Luminosity



Example: Luminosity run at LHC

$\beta^*_{x,y} = 0.55\ m$

$\varepsilon_{x,y} = 5 * 10^{-10}\ rad\ m$

$\sigma_{x,y} = 17\ \mu m$

$f_0 = 11.245\ kHz$

$n_b = 2808$

$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

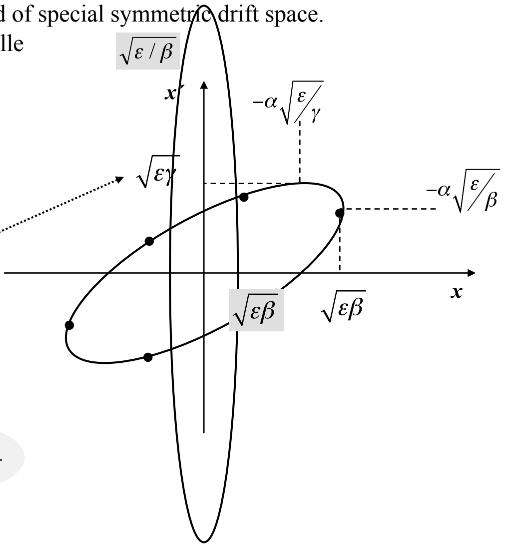
$I_p = 584\ mA$

$$L = 1.0 * 10^{34}\ {}^{1}\!/_{cm^2 s}$$

Mini-β Insertions: Betafunctions

A mini-β insertion is always a kind of special symmetric drift space.

→greetings from Liouville



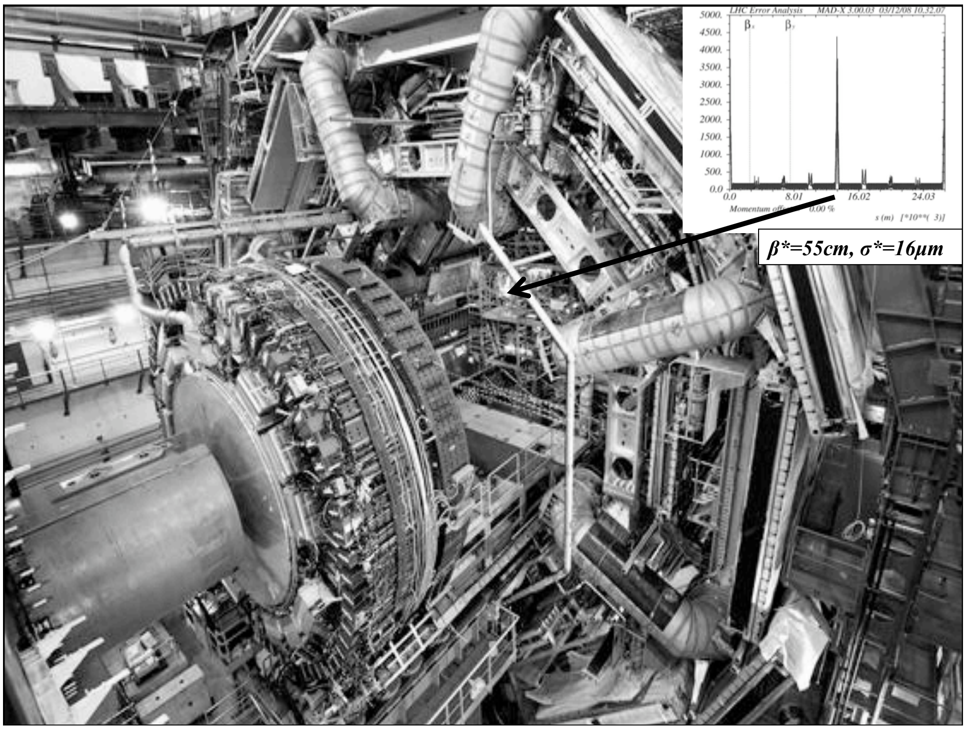
$$\alpha^* = 0$$

$$\gamma^* = \frac{1+\alpha^2}{\beta} = \frac{1}{\beta^*}$$

$$\sigma'^* = \sqrt{\frac{\varepsilon}{\beta^*}}$$

$$\beta^* = \frac{\sigma^*}{\sigma'^*}$$

at a symmetry point β is just the ratio of beam dimension and beam divergence.



β*=55cm, σ*=16μm

APPENDIX: The equation of motion:

Linear approximation:

**** ideal particle → design orbit***

**** any other particle → coordinates x, y small quantities***
x,y << ρ

→ magnetic guide field: only linear terms in x & y of B
have to be taken into account

Taylor Expansion of the B field:

$$B_y(x) = B_{y0} + \frac{dB_y}{dx}x + \frac{1}{2!}\frac{d^2B_y}{dx^2}x^2 + \frac{1}{3!}\frac{eg''}{dx^3} + \dots$$

normalise to momentum
p/e = Bρ

$$\frac{B(x)}{p/e} = \frac{B_0}{B_0\rho} + \frac{g*x}{p/e} + \frac{1}{2!}\frac{eg'}{p/e} + \frac{1}{3!}\frac{eg''}{p/e} + \dots$$

Equation of Motion:

Consider local segment of a particle trajectory
... and remember the old days:
(Goldstein page 27)

radial acceleration:

$$a_r = \frac{d^2\rho}{dt^2} - \rho\left(\frac{d\theta}{dt}\right)^2$$

Ideal orbit: $\rho = const, \quad \frac{d\rho}{dt} = 0$

Force: $F = m\rho\left(\frac{d\theta}{dt}\right)^2 = m\rho\omega^2$

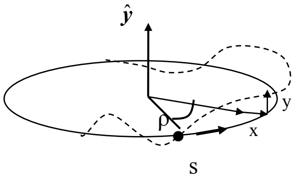
$$F = mv^2 / \rho$$

general trajectory: $\rho \rightarrow \rho + x$

$$F = m\frac{d^2}{dt^2}(x+\rho) - \frac{mv^2}{x+\rho} = e\,B_y\,v$$

$$F = m\frac{d^2}{dt^2}(x+\rho) - \frac{mv^2}{x+\rho} \;=\; e\,B_y\,v$$

①　　②



① $$\frac{d^2}{dt^2}(x+\rho) = \frac{d^2}{dt^2}x$$ *… as ρ = const*

② *remember: x ≈ mm , ρ ≈ m … → develop for small x*

$$\frac{1}{x+\rho} \approx \frac{1}{\rho}(1-\frac{x}{\rho})$$

Taylor Expansion

$$f(x) = f(x_0) + \frac{(x-x_0)}{1!}f'(x_0) + \frac{(x-x_0)^2}{2!}f''(x_0) +$$

$$m\frac{d^2x}{dt^2} - \frac{mv^2}{\rho}(1-\frac{x}{\rho}) = eB_y v$$

---

guide field in linear approx.

$$B_y = B_0 + x\frac{\partial B_y}{\partial x}$$

$$m\frac{d^2x}{dt^2} - \frac{mv^2}{\rho}(1-\frac{x}{\rho}) = ev\left\{B_0 + x\frac{\partial B_y}{\partial x}\right\}$$ | : m

$$\frac{d^2x}{dt^2} - \frac{v^2}{\rho}(1-\frac{x}{\rho}) = \frac{e\,v\,B_0}{m} + \frac{e\,v\,x\,g}{m}$$

independent variable: t → s

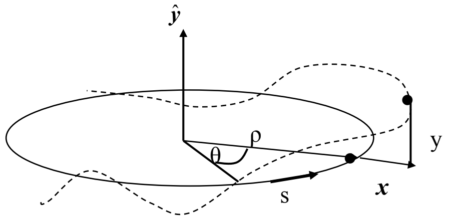
$$\frac{dx}{dt} = \frac{dx}{ds}\frac{ds}{dt}$$

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{ds}\frac{ds}{dt}\right) = \frac{d}{ds}\left(\underbrace{\frac{dx}{ds}}_{x'}\underbrace{\frac{ds}{dt}}_{v}\right)\frac{ds}{dt}$$

$$\frac{d^2x}{dt^2} = x''\,v^2 + \cancel{\frac{dx}{ds}\frac{dv}{ds}}\,v$$

$$x''v^2 - \frac{v^2}{\rho}(1-\frac{x}{\rho}) = \frac{e\,v\,B_0}{m} + \frac{e\,v\,x\,g}{m}$$ | : v^2

$$x'' - \frac{1}{\rho}(1-\frac{x}{\rho}) = \frac{e\ B_0}{mv} + \frac{e\ x\,g}{mv}$$

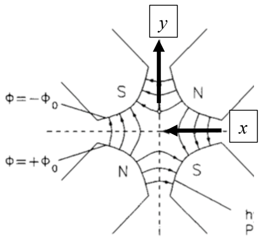


$$x'' - \frac{1}{\rho} + \frac{x}{\rho^2} = \frac{B_0}{p/e} + \frac{x\,g}{p/e}$$

$$x'' - \cancel{\frac{1}{\rho}} + \frac{x}{\rho^2} = -\cancel{\frac{1}{\rho}} + k\,x$$

$$\boxed{x'' + x\,(\frac{1}{\rho^2} - k) = 0}$$

* *Equation for the vertical motion:*



$\frac{1}{\rho^2} = 0$ *no dipoles … in general …*

$k \leftrightarrow -k$ *quadrupole field changes sign*

$$y'' + k\ y = 0$$

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