Introduction to Longitudinal Beam Dynamics

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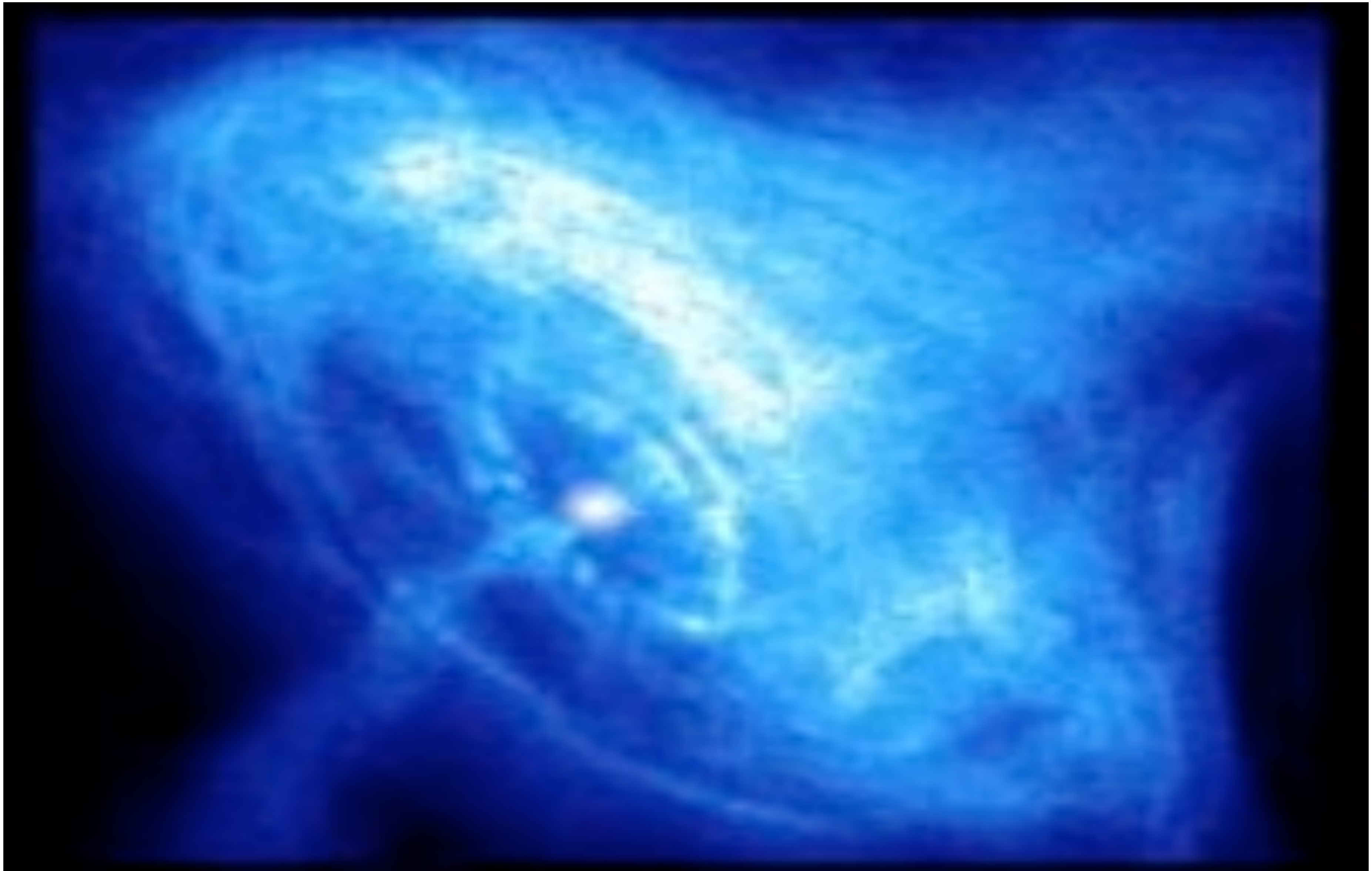

crab nebula, burst of charged particles $E=10^{20}$ eV

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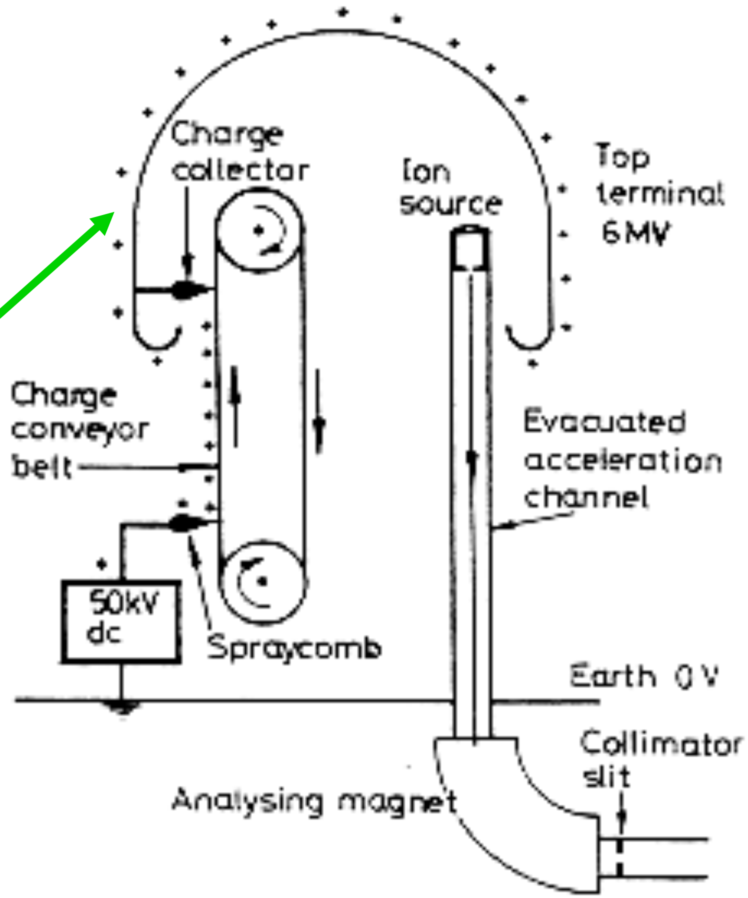
1.) Electrostatic Machines:
(Tandem –) van de Graaff Accelerator (1930 ...)

creating high voltages by mechanical transport of charges



* Terminal Potential: U ≈ 12 ...28 MV
  using high pressure gas to suppress discharge (SF_6)

Problems: * Particle energy limited by high voltage discharges
* high voltage can only be applied once per particle ...
... or twice ?

Energy Gain

... we have to start again from the basics

Lorentz force $\vec{F} = q * (\vec{E} + \vec{v} \times \vec{B})$

in long. direction the B-field creates no force

v* || *B

$$\vec{F} = \frac{d\vec{p}}{dt} = e\vec{E}$$

acc. force is given by the electr. Field

In relativistic dynamics, energy and momentum satisfy the relation:

$$E^2 = E_0^2 + p^2c^2 \qquad (E = E_0 + W)$$

Hence:

$$dE = \int F ds = v dp$$

and the kinetic energy gained from the field along the z path is:

$$dW = dE = eE_z ds \quad \Rightarrow \quad W = e\int E_z ds = eV$$

The „Tandem principle": Apply the accelerating voltage twice ...
... by working with negative ions (e.g. H^-) and stripping the electrons in the centre of the structure

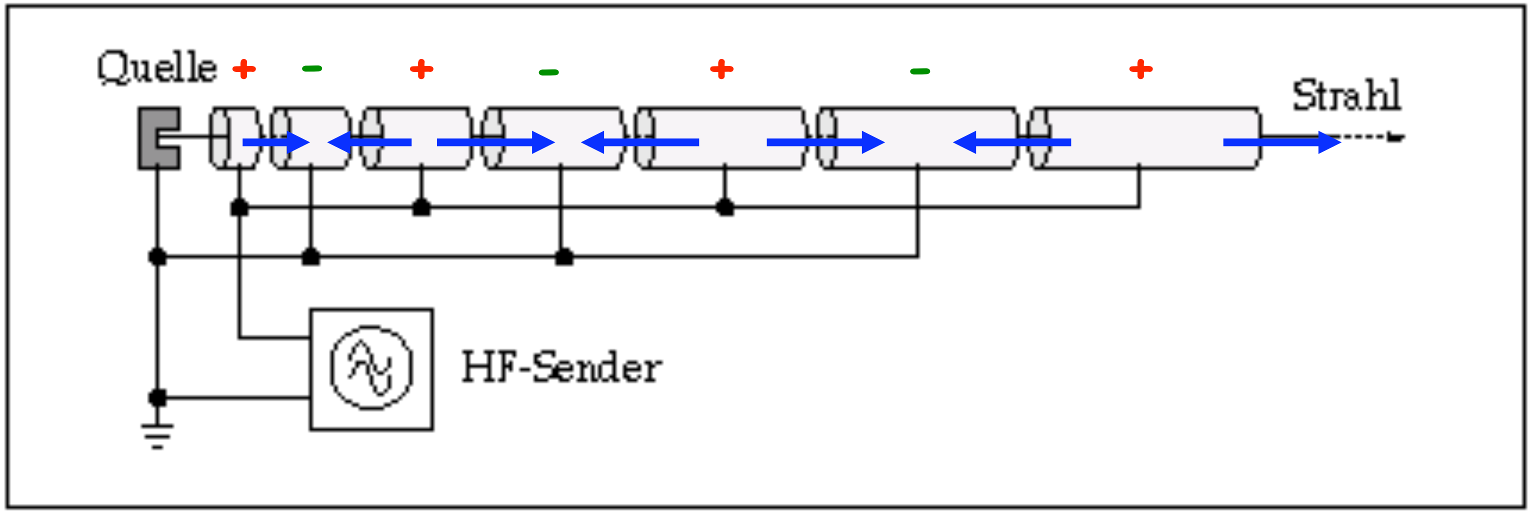
$$dW = dE = eE_z ds \quad \Rightarrow \quad W = e\int E_z ds = eV$$

Electro Static Accelerator: *12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg*

3.) The first RF-Accelerator: „Linac“

1928, Wideroe: how can the acceleration voltage be applied several times to the particle beam

schematic Layout:



Energy gained after n acceleration gaps

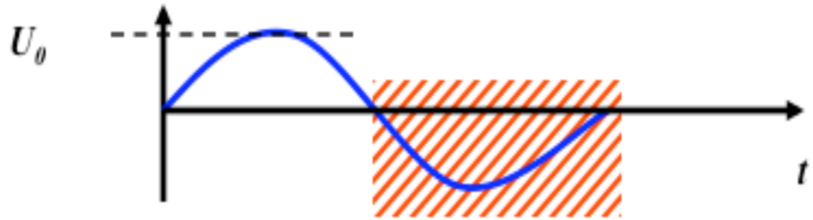
$$E_n = n * q * U_0 * \sin\psi_s$$

n number of gaps between the drift tubes
q charge of the particle
U_0 Peak voltage of the RF System
Ψ_S* *synchronous phase of the particle

* **the problem of synchronisation ... between the particles and the rf voltage**
* **„voltage has to be flipped“ to get the right sign in the second gap**
→ shield the particle in drift tubes during the negative half wave of the RF voltage

Wideroe-Structure: the drift tubes

shielding of the particles during the negative half wave of the RF



Time span of the negative half wave: $\tau_{RF}/2$

Length of the Drift Tube: $l_i = v_i * \frac{\tau_{rf}}{2}$

Kinetic Energy of the Particles $E_i = \frac{1}{2} m v^2$

$$\rightarrow v_i = \sqrt{2E_i/m}$$

$$l_i = \frac{1}{\nu_{rf}} * \sqrt{\frac{i * q * U_{0*\sin\psi_s}}{2m}}$$

valid for non relativistic particles ...

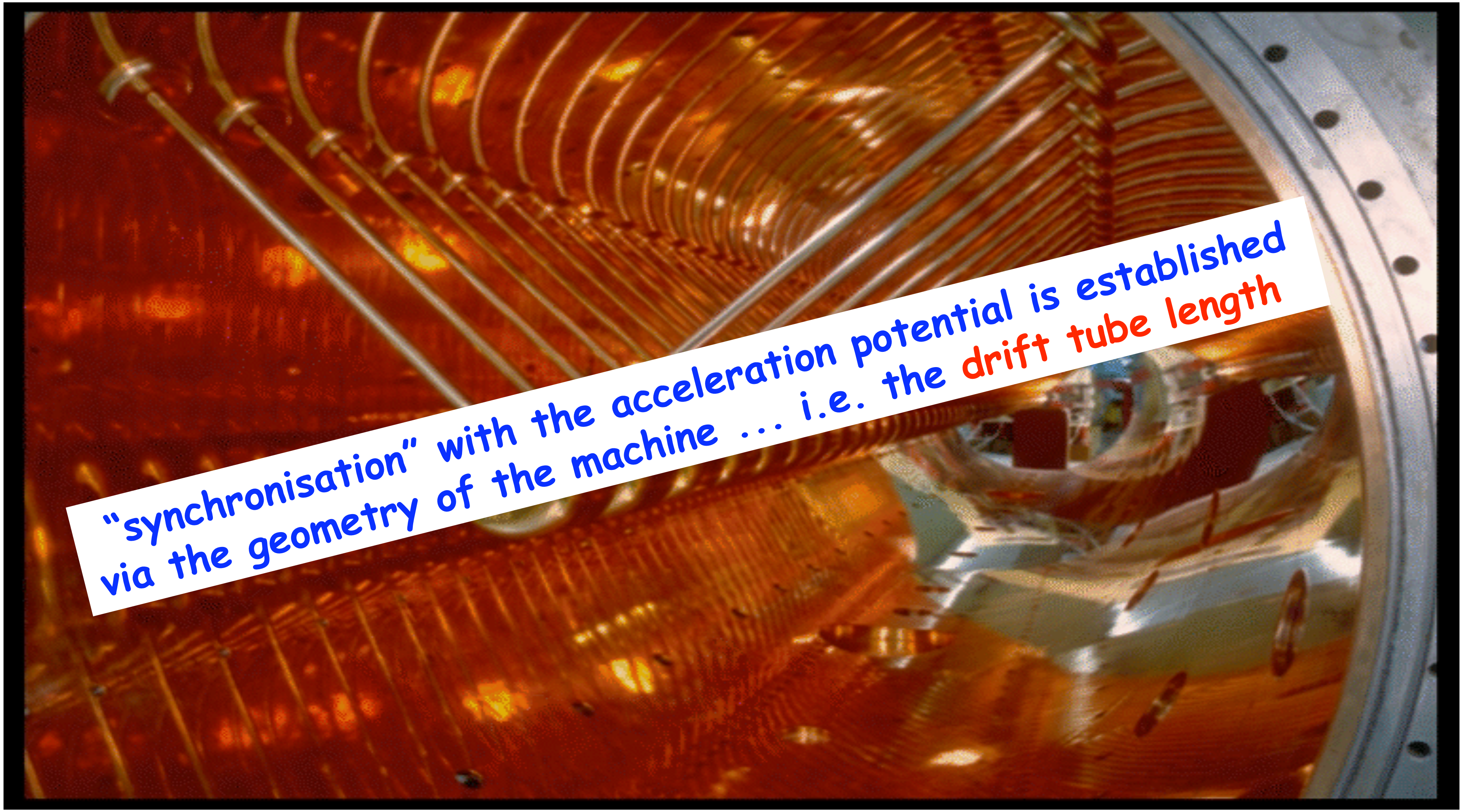
Alvarez-Structure: 1946, surround the whole structure by a rf vessel

Energy: ≈ 20 MeV per Nucleon β ≈ 0.04 … 0.6, Particles: Protons/Ions

GSI: **Unilac, typical Energie ≈ 20 MeV per Nukleon, β ≈ 0.04 … 0.6, Protons/Ions, ν = 110 MHz**

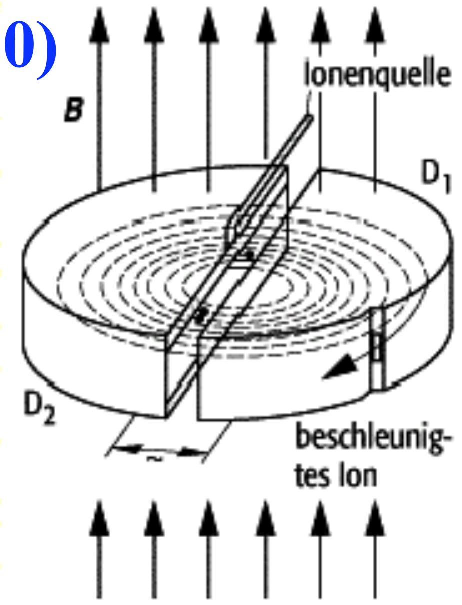
Energy Gain per „Gap“:

$$W = q\,U_0 \sin \omega_{RF}\, t$$



Application: until today THE standard proton / ion pre-accelerator
CERN Linac 4 is being built at the moment

4.) The Cyclotron: (Livingston / Lawrence ~1930)



Idea: B = const, RF = const
Synchronisation particle / RF via orbit

Lorentzforce

$$\vec{F} = q * (\vec{v} \times \vec{B}) = q * v * B$$

circular orbit

$$q * v * B = \frac{m * v^2}{R} \quad \rightarrow \quad B * R = p / q$$

increasing radius for increasing momentum
➔ **Spiral Trajectory**

revolution frequency

$$\omega_z = \frac{v}{R} = \frac{q}{m} * B_z$$

the cyclotron (rf-) frequency is independent of the momentum

rf-frequency = h* revolution frequency, h = "harmonic number"

Cyclotron:

exact equation for revolution frequency:

$$\omega_z = \frac{v}{R} = \frac{q}{\gamma * m} * B_z$$

Cyclotron SPIRAL at GANIL

1.) if v << c $\Rightarrow \gamma \cong 1$

2.) γ increases with the energy

$\Rightarrow$ no exact synchron…

"synchronisation" with the acceleration potential is established
via the spiraling orbit length

Sy…tron

B = constant

$\gamma\, \omega_{RF}$ = constant

ω_{RF} decreases with time

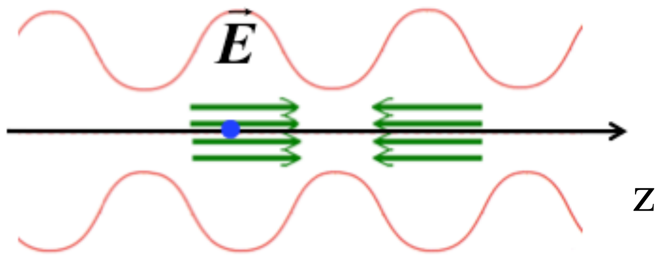
$$\omega_s(t) = \omega_{rf}(t) = \frac{q}{\gamma(t) * m_0} * B$$

keep the synchronisation condition by varying the rf frequency

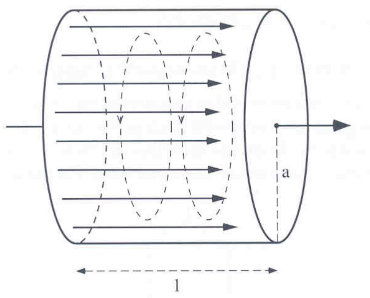
RF Cavities, Acceleration and Energy Gain

$$dW = dE = eE_z ds \quad \Rightarrow \quad W = e\int E_z ds = eV$$

RF acceleration: $V \neq const$



In this case the electric field is oscillating. So it is for the potential. The energy gain will depend on the RF phase experienced by the particle.



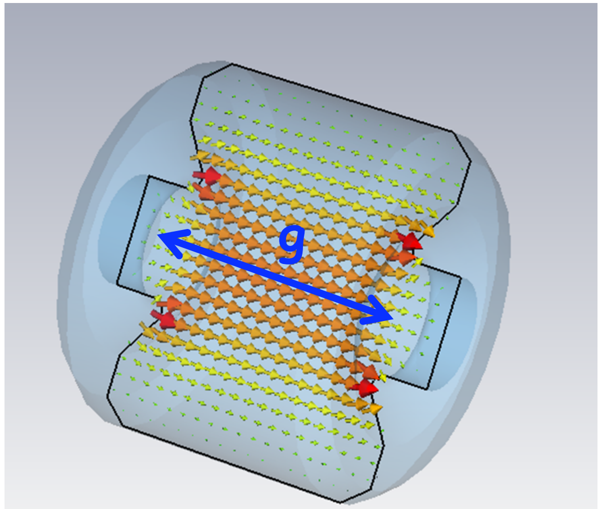
$$\int \hat{E}_z dz = \hat{V}$$

$$E_z = \hat{E}_z \cos \omega_{RF} t \quad = \quad \hat{E}_z \cos \Phi(t)$$

$$W = e\hat{V} \cos \Phi$$

Neglecting the transit time in the gap.

Energy Gain in RF structures: Transit Time Factor



Oscillating field at frequency **ω** (amplitude is assumed to be constant all along the gap)

$$E_z = E_0 \cos\omega t = \frac{V}{g}\cos\omega t$$

Consider a particle passing through the middle of the gap at time t=0 : $z = vt$

The total energy gain is: $$\Delta W = \frac{eV}{g}\int_{-g/2}^{g/2}\cos\omega\frac{z}{v}\,dz$$

$$\Delta W = eV\frac{\sin\theta/2}{\theta/2} = eVT$$

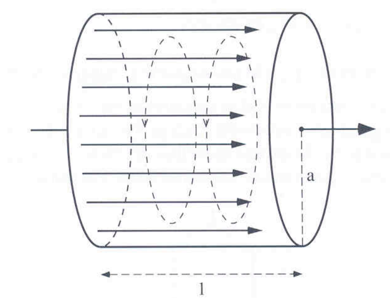
$$T = \frac{\sin\theta/2}{\theta/2}$$ transit time factor (0 < T < 1)

$$\theta = \frac{\omega g}{v}$$ *transit angle*

ideal case: $T = \frac{\sin\theta/2}{\theta/2} \rightarrow 1 \quad \Leftrightarrow \quad \theta/2 \rightarrow 0$

el. static accelertors $\omega \rightarrow 0$

minimise acc. gap $g \rightarrow 0$



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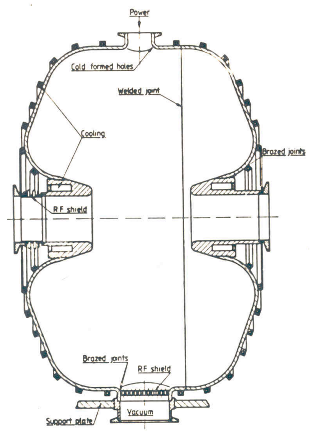
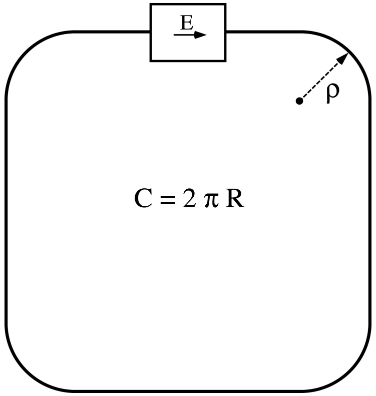


The Synchrotron (Mac Millan, Veksler, 1945)



The synchrotron: Ring Accelerator of const. R where the increase in momentum (i.e. B-field) is automatically synchronised with the correct synchronous phase of the particle in the rf [illegible]ies

"synchronisation" as basic principle of the machine

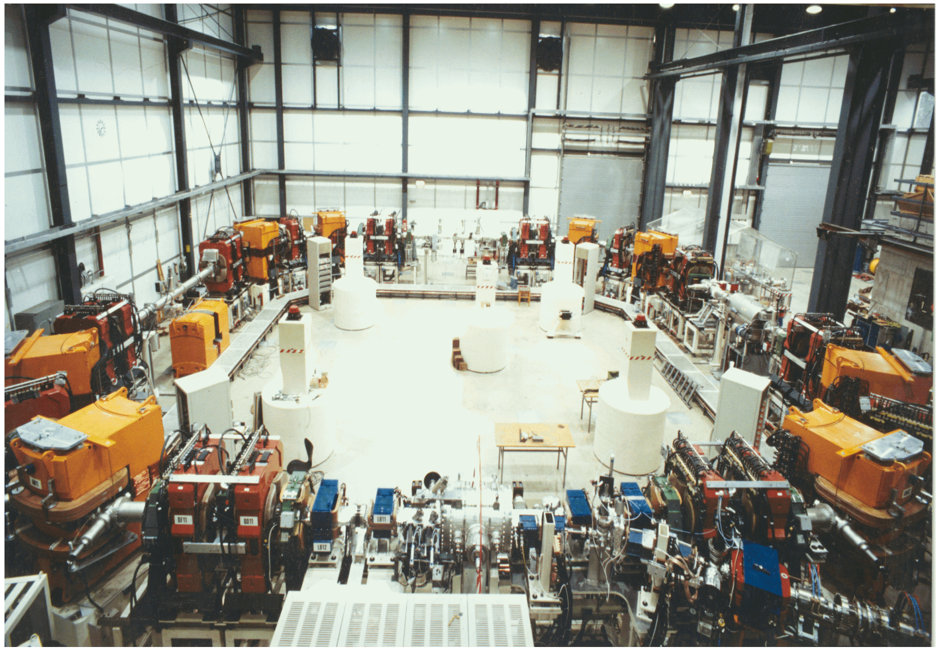
$e\hat{V}$ [illegible] → [illegible]ergy gain per turn

$\Phi = \Phi_s = cte$ → Synchronous particle

$\omega_{RF} = h\omega_r$ → RF synchronism

$\rho = cte \quad R = cte$ → Constant orbit

$B\rho = P/e \Rightarrow B$ → Variable magnetic field

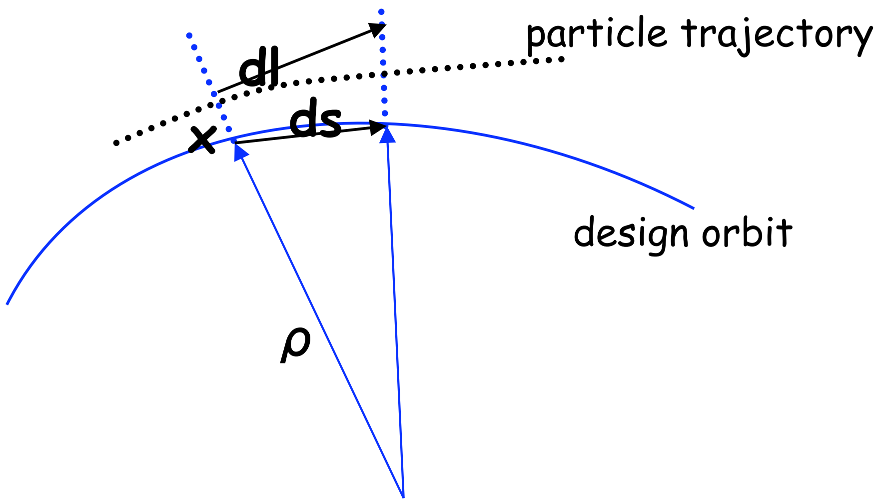


Momentum Compaction Factor: α_p

particle with a displacement x to the design orbit
→ path length dl ...

$$\frac{dl}{ds} = \frac{\rho + x}{\rho}$$

$$\rightarrow \ dl = \left(1 + \frac{x}{\rho(s)}\right) ds$$



circumference of an off-energy closed orbit

$$l_{\Delta E} = \oint dl = \oint \left(1 + \frac{x_{\Delta E}}{\rho(s)}\right) ds$$

remember: $x_{\Delta E}(s) = D(s)\frac{\Delta p}{p}$

$$\delta l_{\Delta E} = \frac{\Delta p}{p} \oint \left(\frac{D(s)}{\rho(s)}\right) ds$$

* The lengthening of the orbit for off-momentum particles is given by the dispersion function and the bending radius.

Definition:

$$\frac{\delta l_\varepsilon}{L} = \alpha_p \frac{\Delta p}{p}$$

$$\rightarrow \alpha_p = \frac{1}{L}\oint\left(\frac{D(s)}{\rho(s)}\right)ds$$

For first estimates assume: $\frac{1}{\rho} = const.$

$$\int_{dipoles} D(s)\,ds \approx l_{\Sigma(dipoles)} \cdot \langle D \rangle_{dipole}$$

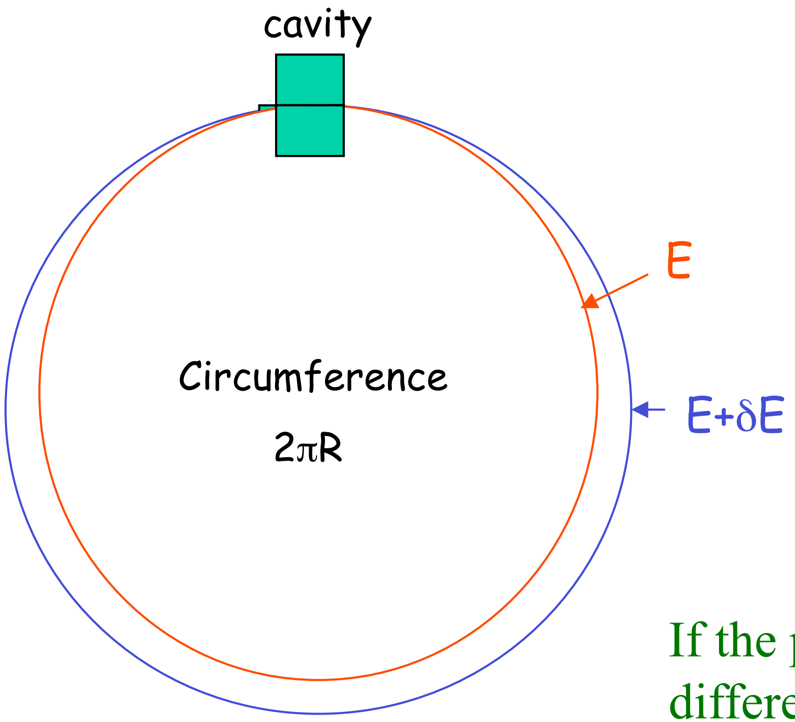
$$\alpha_p = \frac{1}{L}\, l_{\Sigma(dipoles)} \cdot \langle D \rangle \frac{1}{\rho} = \frac{1}{L}\, 2\pi\rho \cdot \langle D \rangle \frac{1}{\rho} \quad \rightarrow \quad \alpha_p \approx \frac{2\pi}{L}\langle D \rangle \approx \frac{\langle D \rangle}{R}$$

Assume: $v \approx c$

$$\rightarrow \quad \frac{\delta T}{T} = \frac{\delta l_\varepsilon}{L} = \alpha_p \frac{\Delta p}{p}$$

α_p combines via the dispersion function the momentum spread with the longitudinal motion of the particle.

Dispersion Effects in a Synchrotron



If a particle is slightly shifted in momentum it will have a different orbit:

$$\alpha = \frac{p}{R}\frac{dR}{dp}$$

This is the "momentum compaction" generated by the bending field.

If the particle is shifted in momentum it will have also a different velocity. As a result of both effects **the revolution frequency changes:**

$$\eta = \frac{p}{f_r}\frac{df_r}{dp}$$

p=particle momentum

R=synchrotron physical radius

f_r=revolution frequency

Dispersion Effects in a Synchrotron

$$\eta = \frac{p}{f_r}\frac{df_r}{dp}$$

$$f_r = \frac{\beta c}{2\pi R} \Rightarrow \frac{df_r}{f_r} = \frac{d\beta}{\beta} - \frac{dR}{R}$$

$$\rightarrow \frac{dR}{R} = \alpha \frac{dp}{p}$$

$$p = mv = \beta\gamma\frac{E_0}{c} \Rightarrow \frac{dp}{p} = \frac{d\beta}{\beta} + \frac{d\left(1-\beta^2\right)^{-\frac{1}{2}}}{\left(1-\beta^2\right)^{-\frac{1}{2}}} = \left(1-\beta^2\right)^{-1}\frac{d\beta}{\beta}$$

$$\rightarrow \frac{d\beta}{\beta} = \frac{1}{\gamma^2}\frac{dp}{p}$$

$$\frac{df_r}{f_r} = \left(\frac{1}{\gamma^2} - \alpha\right)\frac{dp}{p} \longrightarrow \eta = \frac{1}{\gamma^2} - \alpha$$

The change of revolution frequency depends on the particle energy γ and changes sign during acceleration.

Particles get faster in the beginning – and arrive earlier at the cavity: classic regime

Particles travel at v =c and get more massive – and arrive later at the cavity: relativistic regime

boundary between the two regimes: no frequency dependence on dp/p, η =0 "transition energy"

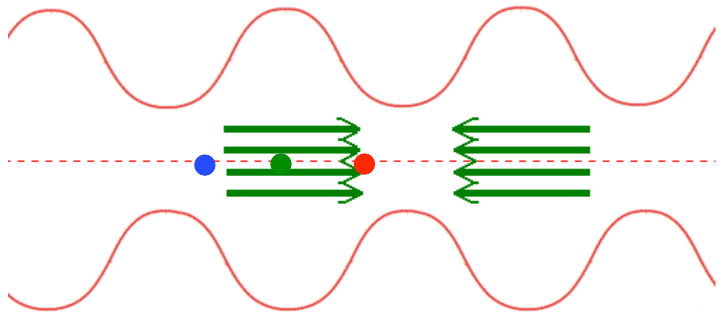
$$\gamma_{tr} = \frac{1}{\sqrt{\alpha}}$$

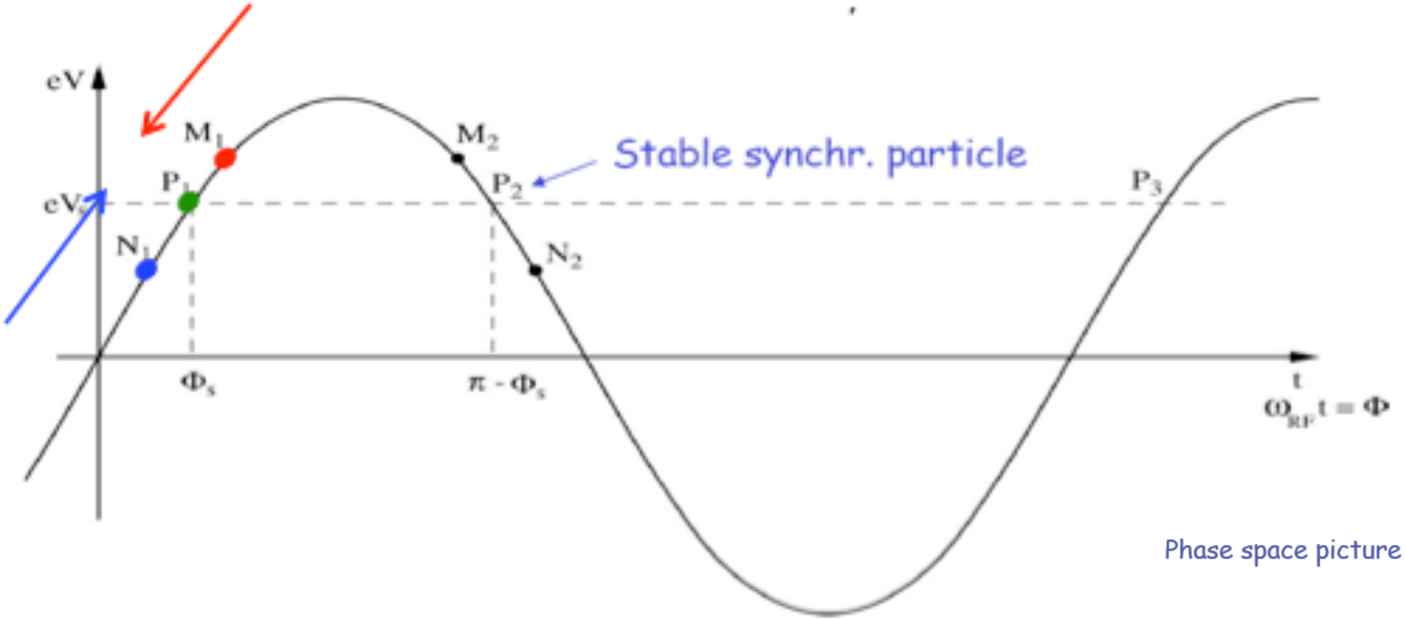
14.) The Acceleration for Δp/p≠0
"Phase Focusing" below transition

ideal particle ●

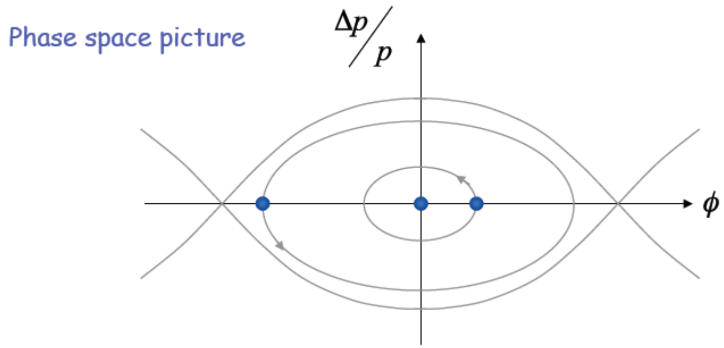
particle with Δp/p > 0 ● **faster**

particle with Δp/p < 0 ● **slower**



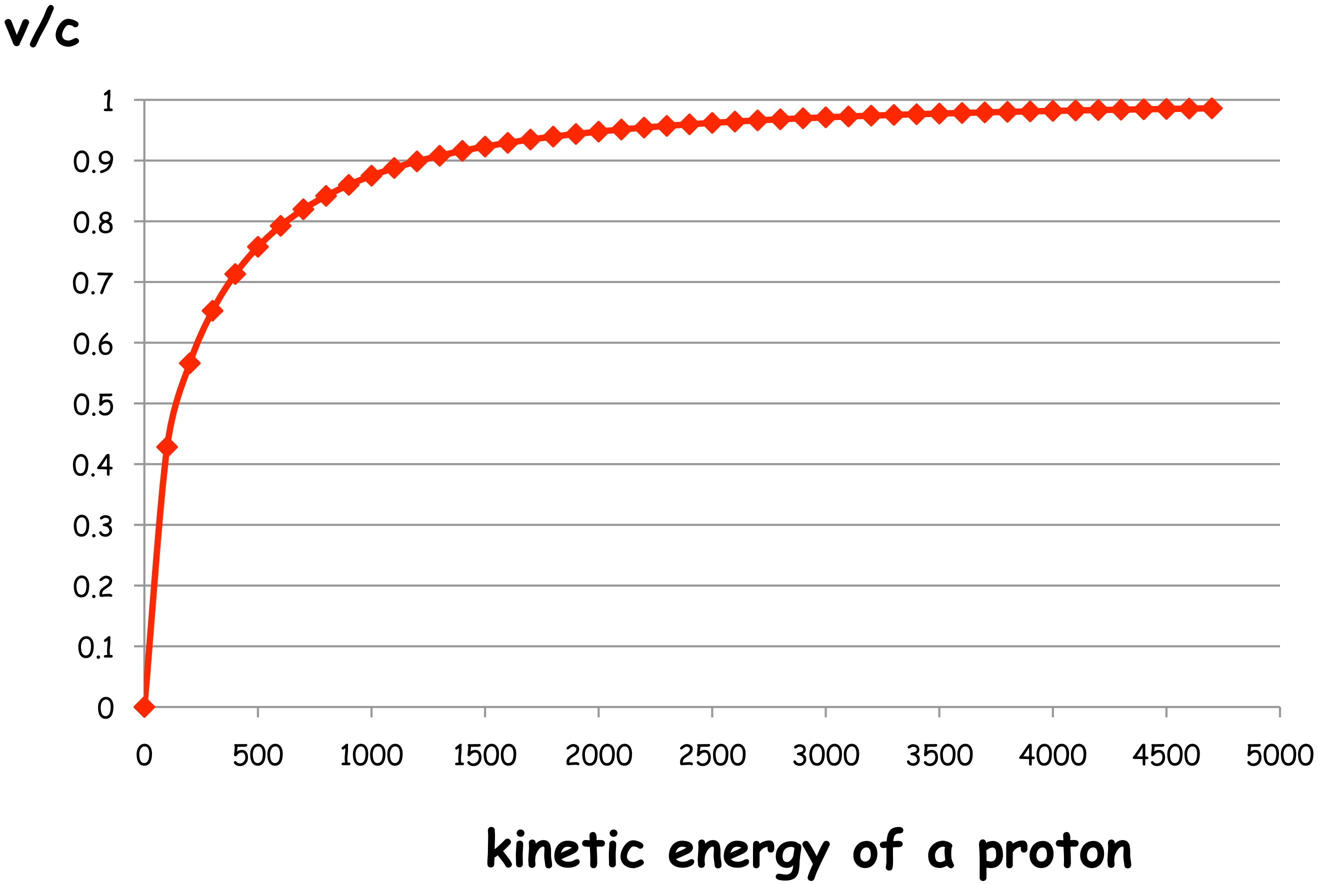


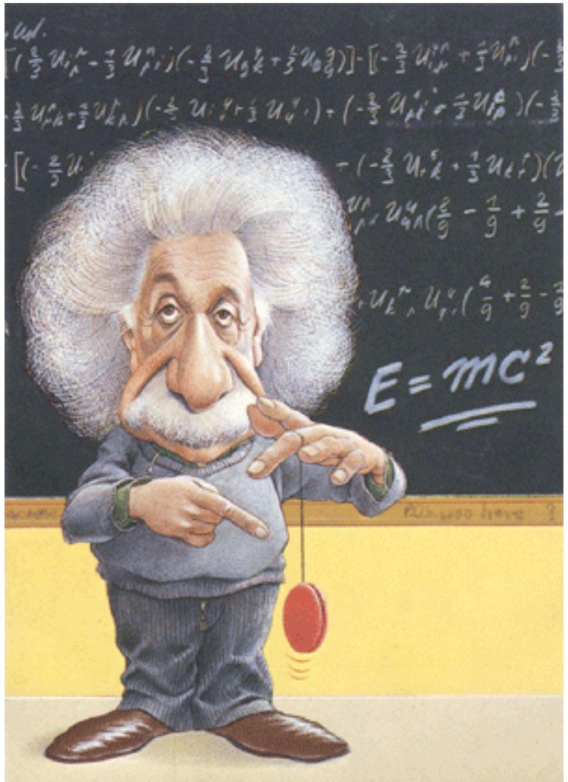
Focussing effect in the longitudinal direction
keeping the particles close together
... forming a "bunch"



... so sorry, here we need help from Albert:

$$\gamma = \frac{E_{total}}{mc^2} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \longrightarrow \frac{v}{c} = \sqrt{1 - \frac{mc^2}{E^2}}$$





... some when the particles do not get faster anymore

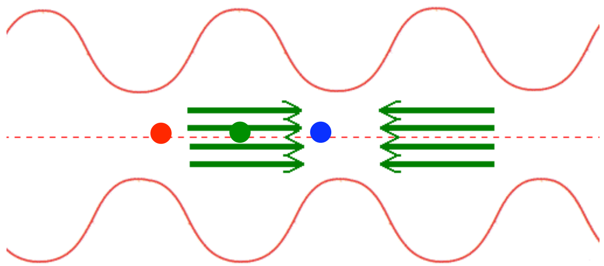
.... but heavier !

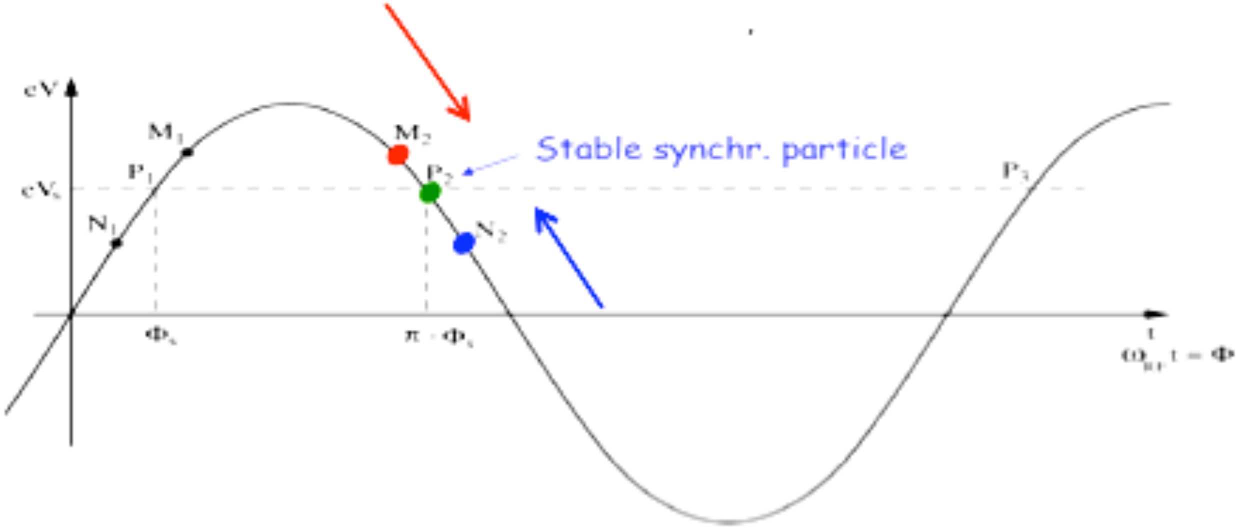
15.) The Acceleration for $\Delta p/p \neq 0$
"Phase Focusing" above transition

ideal particle ●

particle with $\Delta p/p > 0$ ● **heavier**

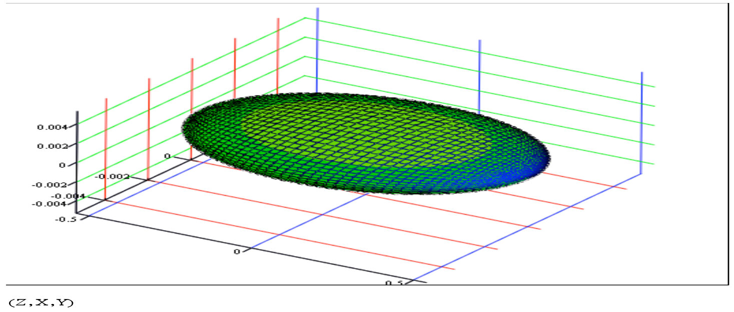
particle with $\Delta p/p < 0$ ● **lighter**



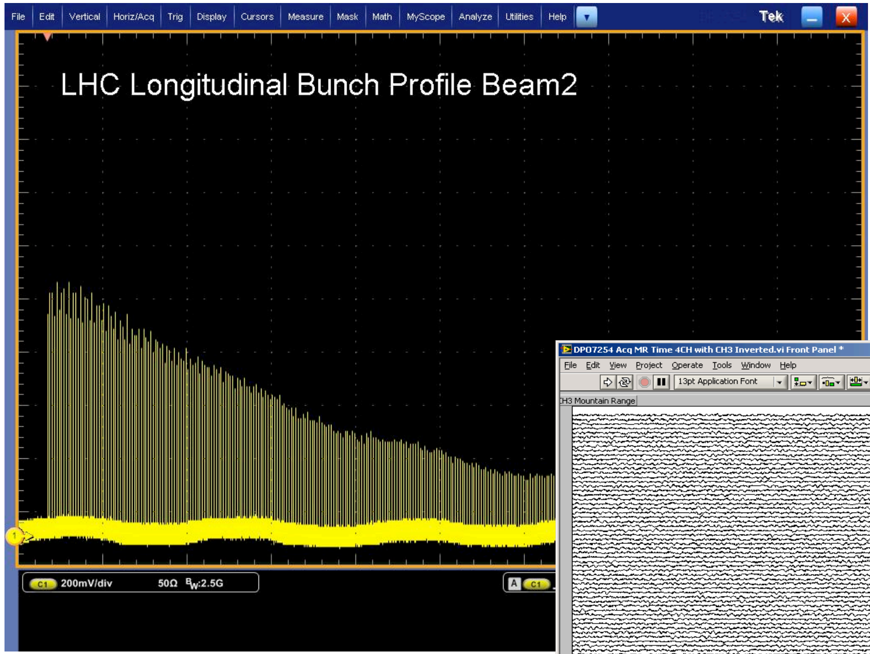


oscillation frequency: $f_s = f_{rev} \sqrt{-\frac{h\alpha_s}{2\pi} * \frac{qU_0 \cos\phi_s}{E_s}}$ **≈ some Hz**

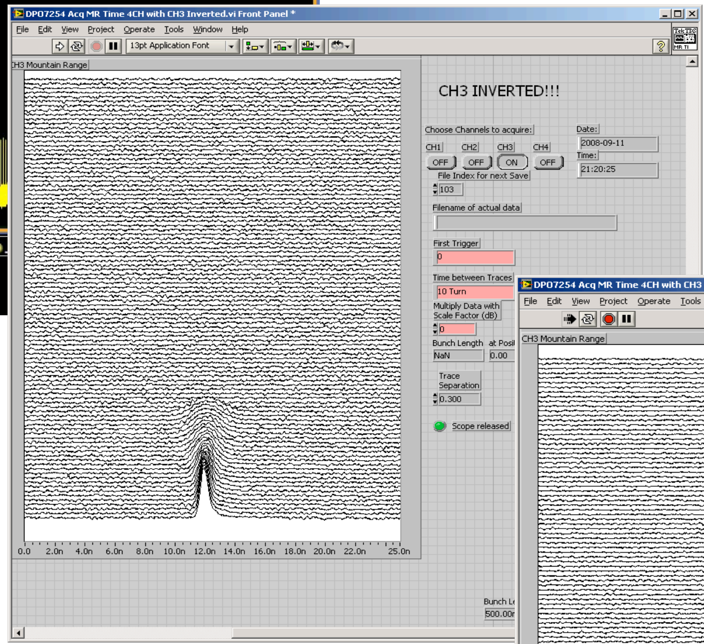
LHC Commissioning: RF

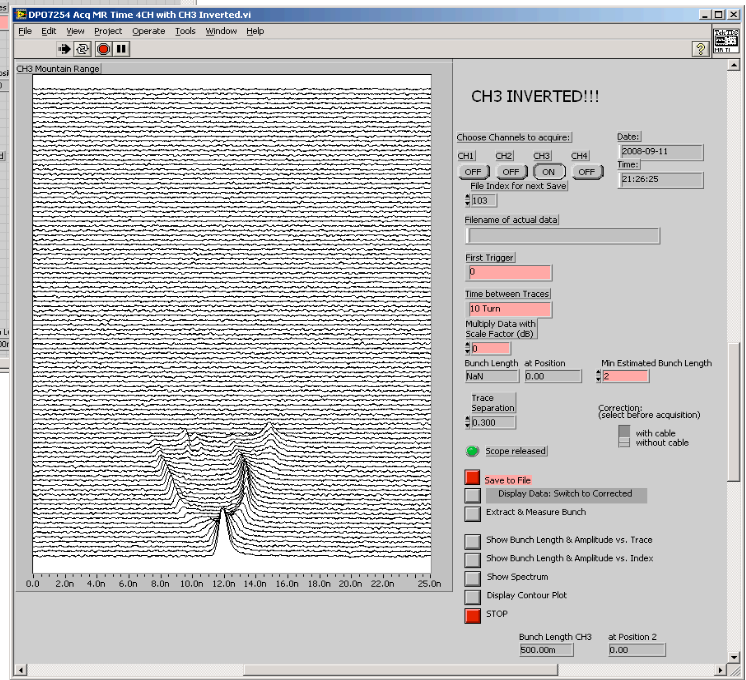


a proton bunch: focused longitudinal by the RF field



RF off





RF on,
wrong phase condition

... and how do we accelerate now ???

with the dipole magnets !

Energy ramping is simply obtained by varying the B field:

$$p = eB\rho \quad \Rightarrow \quad \frac{dp}{dt} = e\rho\,\dot{B} \quad \Rightarrow (\Delta p)_{turn} = e\rho\,\dot{B}T_r = \frac{2\pi\ e\rho\,R\dot{B}}{v}$$

Energy Gain per turn:

$$E^2 = E_0^2 + p^2c^2 \quad \Rightarrow \quad \Delta E = v\Delta p$$

$$\Delta E_{turn} = \Delta W_{turn} = 2\pi e\rho R\dot{B} = e\hat{V}\sin\phi_s$$

* The energy gain depends on the rate of change of the dipole field

* The number of stable synchronous particles is equal to the harmonic number h. They are equally spaced along the circumference.

* Each synchronous particle satifies the relation p = eBρ. They have the nominal energy and follow the nominal trajectory.

The Synchrotron: Frequency Change

During the energy ramping, the RF frequency increases to follow the increase of the revolution frequency : → $\omega_r = \frac{\omega_{RF}}{h} = \omega(B, R_s)$

hence : $$\frac{f_{RF}(t)}{h} = \frac{v(t)}{2\pi R_s} = \frac{1}{2\pi}\frac{e}{m} < B(t) > \quad \Rightarrow \quad \frac{f_{RF}(t)}{h} = \frac{1}{2\pi}\frac{ec^2}{E_s(t)}\frac{r}{R_s}B(t)$$

Since $$E^2 = (m_0 c^2)^2 + p^2 c^2$$

The RF frequency must follow the variation of the **B** field with the law :

$$\frac{f_{RF}(t)}{h} = \frac{c}{2\pi R_s}\left\{\frac{B(t)^2}{(m_0 c^2 / ecr)^2 + B(t)^2}\right\}^{1/2}$$

and as soon as $B > \frac{m_0 c^2}{ecr}$ $$\frac{f_{RF}(t)}{h} \approx \frac{c}{2\pi R_s} = const$$

which is true for LHC at high energy and for electrons from the start

Longitudinal Dynamics: synchrotron motion

We have to follow two coupled variables:

* the energy gained by the particle
* and the RF phase experienced by the same particle.

Since there is a well defined synchronous particle which has always the same phase ϕ_s, and the nominal energy E_s, it is sufficient and elegant to follow other particles with respect to that particle.

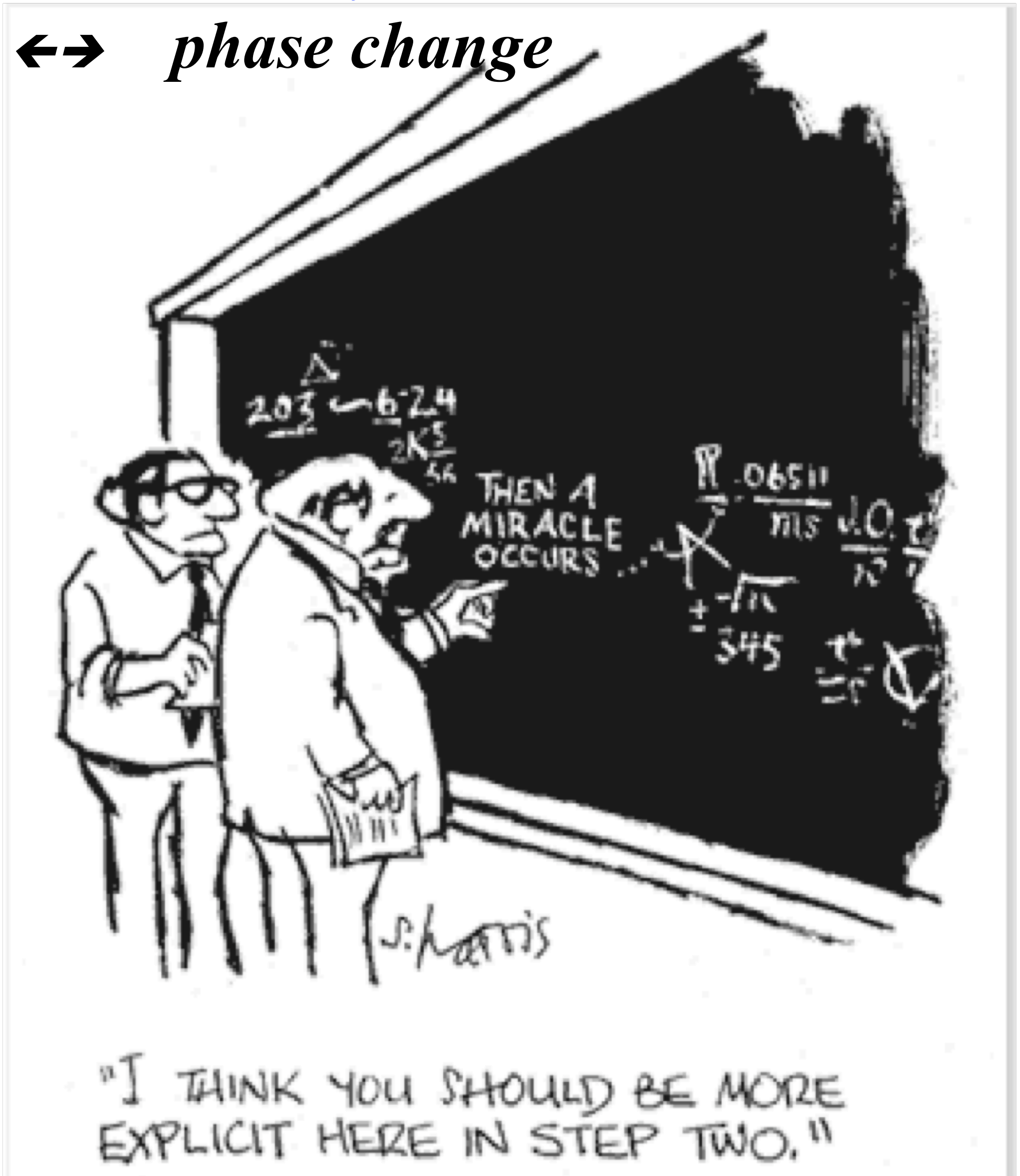
We will introduce the following relative variables:

revolution frequency	:	$\Delta f_r = f_r - f_{rs}$
particle RF phase	:	$\Delta\phi = \phi - \phi_s$
particle momentum	:	$\Delta p = p - p_s$
particle energy	:	$\Delta E = E - E_s$
azimuth angle	:	$\Delta\theta = \theta - \theta_s$

The Equation of Motion:

Energy-Phase Relations in a Synchrotron

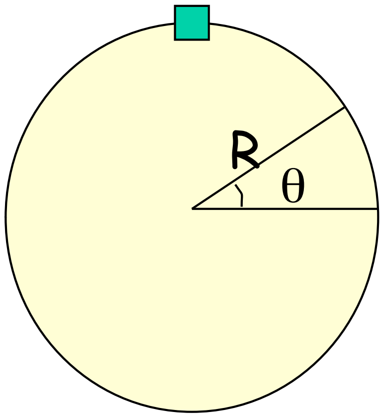
energy offset ←→ phase change



"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

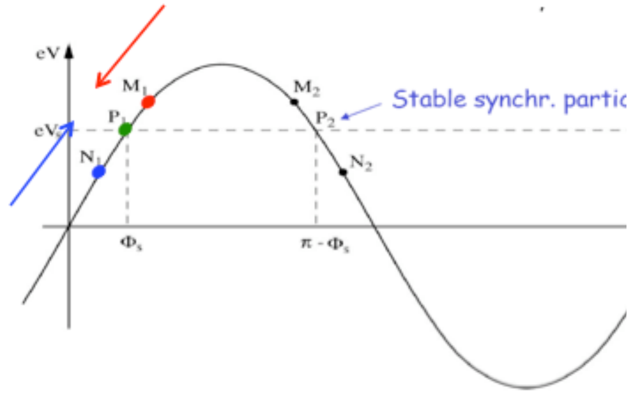
First Energy-Phase Equation:

energy offset ←→ phase change



$$f_{RF} = hf_r \quad \Rightarrow \quad \Delta\phi = -h\Delta\theta \quad \text{with} \quad \theta = \int \omega_r dt$$

For a given particle with respect to the reference one:



$$d\omega_r = \frac{d}{dt}(\Delta\theta) = -\frac{1}{h}\frac{d}{dt}(\Delta\phi) = -\frac{1}{h}\frac{d\phi}{dt}$$

Since: $$\eta = \frac{p_s}{\omega_{rs}}\left(\frac{d\omega_r}{dp}\right)_s \quad \rightarrow \quad dp = \frac{p}{\eta} * \left(\frac{d\omega_r}{dp}\right)_s$$

and from relativity we know: $\Delta E = v_s \Delta p = \omega_{rs} R_s \Delta p$

one gets: $$\frac{\Delta E}{\omega_{rs}} = -\frac{p_s R_s}{h\eta\omega_{rs}}\frac{d(\Delta\phi)}{dt} = -\frac{p_s R_s}{h\eta\omega_{rs}}\dot{\phi}$$

The energy deviation from the synchronous particle depends on the rate of change of the phase.

Second Energy-Phase Equation

energy offset <-> RF voltage

energy gain per turn: $$\Delta E_{turn} = e\hat{V}\sin\phi$$

$$\Delta p_{turn} = \frac{e\hat{V}}{\omega R}\sin\phi$$

$$\Delta E = v\,\Delta p$$
$$v = \omega R$$

momentum rate of change: $$\dot{p} = \frac{\Delta p_{turn}}{T} = \frac{\Delta p_{turn}}{2\pi}\omega = \frac{e\hat{V}}{2\pi R}\sin\phi$$

$$2\pi R\dot{p} = e\hat{V}\sin\phi$$

difference to the synchr. particle:

$$2\pi\,\Delta\left(R\dot{p}\right) = e\hat{V}(\sin\phi - \sin\phi_s)$$

Second Energy-Phase Equation

momentum offset <-> geometry

$$\Delta\left(R\dot{p}\right) = R\dot{p} - R_s\dot{p}_s = \left(R_s + \Delta R\right) * \left(\dot{p}_s + \Delta\dot{p}\right) - R_s\dot{p}_s$$

$$= R_s\dot{p}_s + R_s\Delta\dot{p} + \Delta R\dot{p}_s + \underbrace{\Delta R\Delta\dot{p}}_{\approx 0} - R_s\dot{p}_s$$

$$= R_s\Delta\dot{p} + \Delta R\dot{p}_s = R_s\Delta\dot{p} + \dot{p}_s * \left(\frac{dR}{dp}\right)_s \Delta p = R_s\Delta\dot{p} + \frac{dp_s}{dt}\left(\frac{dR}{dp}\right)_s \Delta p$$

$$= R_s\Delta\dot{p} + \dot{R}_s\Delta p = \frac{d}{dt}(R_s\Delta p) = \frac{d}{dt}\left(\frac{\Delta E}{\omega_s}\right)$$

... put into the green equation ... to get

$$2\pi\,\Delta\left(R\dot{p}\right) = e\hat{V}(\sin\phi - \sin\phi_s)$$

$$2\pi\frac{d}{dt}\left(\frac{\Delta E}{\omega_s}\right) = e\hat{V}(\sin\phi - \sin\phi_s)$$

the rate of energy change is determined by the distance in phase in the sinusoidal rf voltage function

Equations of Longitudinal Motion

$$\frac{\Delta E}{\omega_{rs}} = -\frac{p_s R_s}{h\eta\omega_{rs}} \frac{d(\Delta\phi)}{dt} = -\frac{p_s R_s}{h\eta\omega_{rs}} \dot{\phi}$$

$$2\pi \frac{d}{dt}\left(\frac{\Delta E}{\omega_{rs}}\right) = e\hat{V}(\sin\phi - \sin\phi_s)$$

deriving and combining

$$\frac{d}{dt}\left[\frac{R_s p_s}{h\eta\omega_{rs}} \frac{d\phi}{dt}\right] + \frac{e\hat{V}}{2\pi}(\sin\phi - \sin\phi_s) = 0$$

This rather formidable looking differential equation simplifies a lot if we consider ...

R_s, p_s, ω_s, η as constant (or slowly varying with time).

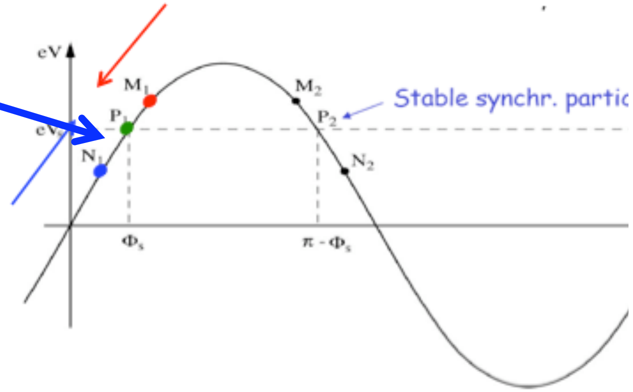
Small Amplitude Oscillations

Let's assume constant parameters R_s, p_s, ω_s and η:

$$\ddot{\phi} + \frac{\Omega_s^2}{\cos\phi_s}\left(\sin\phi - \sin\phi_s\right) = 0 \qquad \textit{with} \qquad \Omega_s^2 = \frac{h\eta\omega_{rs} e\hat{V}\cos\phi_s}{2\pi R_s p_s}$$

Consider now small phase deviations from the reference particle:

$$\sin\phi - \sin\phi_s = \sin(\phi_s + \Delta\phi) - \sin\phi_s \cong \cos\phi_s \Delta\phi$$



and the corresponding linearized motion reduces to a harmonic oscillation:

$$\ddot{\phi} + \Omega_s^2 \Delta\phi = 0$$

stable for $\Omega_s^2 > 0$ and $\mathbf{\Omega_s}$ real

Small Amplitude Oscillations: phase stability

We get a harmonic oscillation of the particle phase with the oscillation frequency

$$\ddot{\phi} + \Omega_s^2 \Delta\phi = 0$$

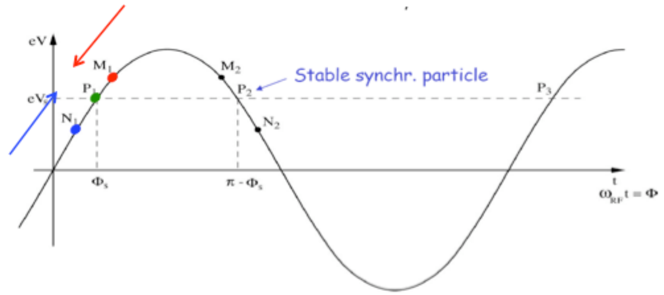
$$\Omega_s = \sqrt{\frac{h\eta\omega_{rs} e\hat{V}\cos\phi_s}{2\pi R_s p_s}}$$

remember $\eta = \frac{1}{\gamma^2} - \alpha$

Stability condition: Ω_s real $\quad \Omega_s^2 > 0$

$\gamma < \gamma_{tr} \qquad \eta > 0 \qquad 0 < \phi_s < \pi/2$

$\gamma > \gamma_{tr} \qquad \eta < 0 \qquad \pi/2 < \phi_s < \pi$



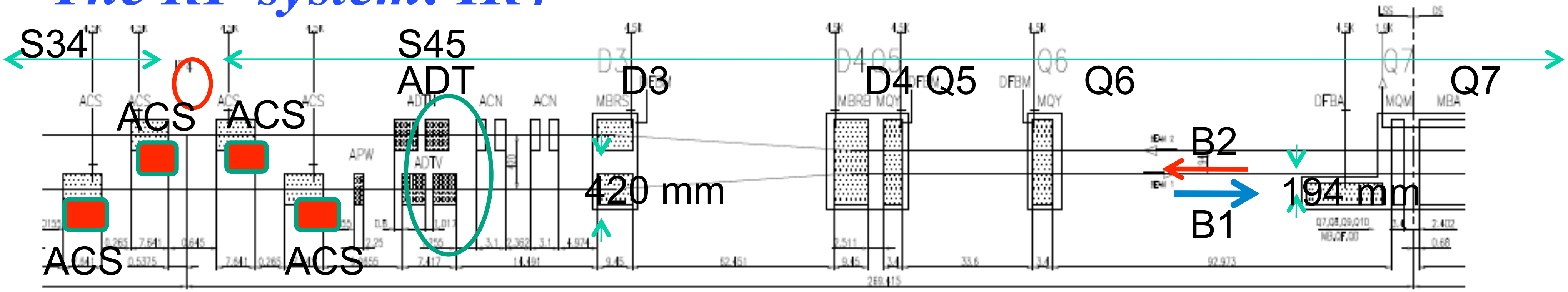
And we will find this situation
“h”-times in the machine

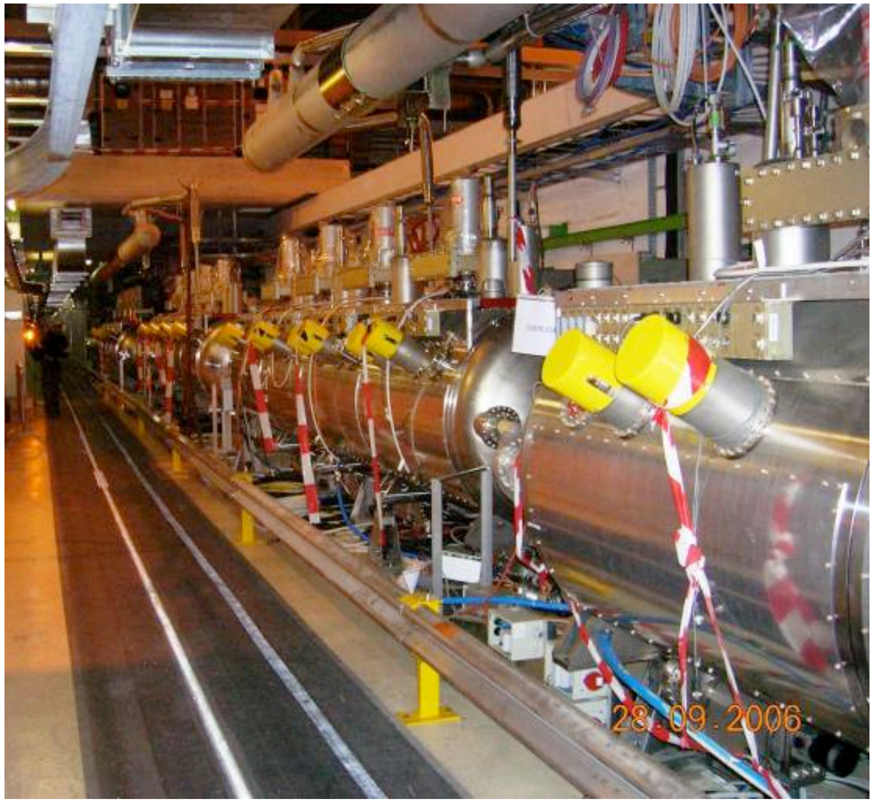
LHC:
35640 Possible Bunch Positions (“buckets”)
2808 Bunches

oscillation frequency depends on
- ***the square root***
- ***of an electrical potential***

-> weak force <-> small frequncy

The RF system: IR4



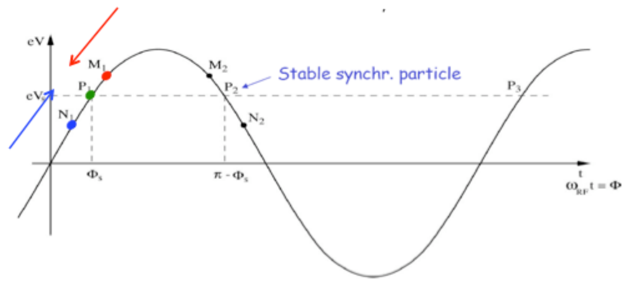


4xFour-cavity cryo module 400 MHz, 16 MV/beam
Nb on Cu cavities @4.5 K (=LEP2)
Beam pipe diam.=300mm

Bunch length (4σ)	*ns*	*1.06*
Energy spread (2σ)	*10^{-3}*	*0.22*
Synchr. rad. loss/turn	*keV*	*7*
Synchr. rad. power	*kW*	*3.6*
RF frequency	*MHz*	*400*
Harmonic number		*35640*
RF voltage/beam	*MV*	*16*
Energy gain/turn	*keV*	*485*
Synchrotron frequency	*Hz*	*23.0*

(small) ... Synchrotron Oscillations in Energy and Phase

$$\ddot{\phi} + \Omega_s^2 \Delta\phi = 0$$



Ansatz: $$\Delta\phi = \Delta\phi_{max} * \cos(\Omega_s t)$$

take the first derivative and put it into the first energy-phase relation $$\Delta E = -\frac{p_s R_s}{h\eta}\frac{d(\Delta\phi)}{dt}$$

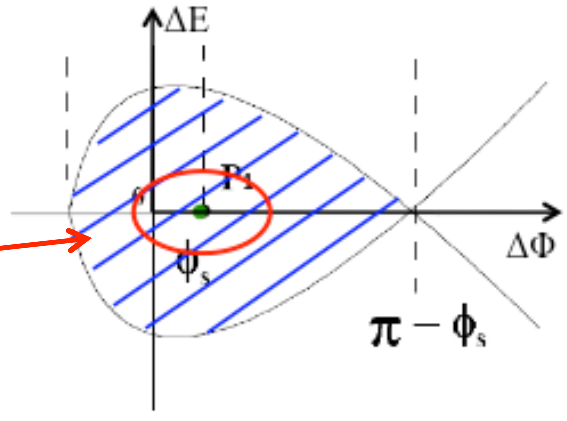
$$\frac{d(\Delta\phi)}{dt} = -\Delta\phi_{max} * \sin(\Omega_s t) * \Omega_s$$

to get the energy oscillations $$\Delta E = \underbrace{\frac{-pR_s\Delta\phi_{max}}{h\eta}}_{\Delta E_{max}} \sin(\Omega_s t)$$

$$\Delta E = \Delta E_{max} * \sin(\Omega_s t)$$

which defines an ellipse in phase space ΔΦ, ΔE:

$$\left(\frac{\Delta\Phi}{\Delta\Phi_{max}}\right)^2 + \left(\frac{\Delta E}{\Delta E_{max}}\right)^2 = 1$$



Large Amplitude Oscillations

Equation of motion: $\ddot{\phi}+\frac{\Omega_s^2}{\cos\phi_s}(\sin\phi-\sin\phi_s)=0$

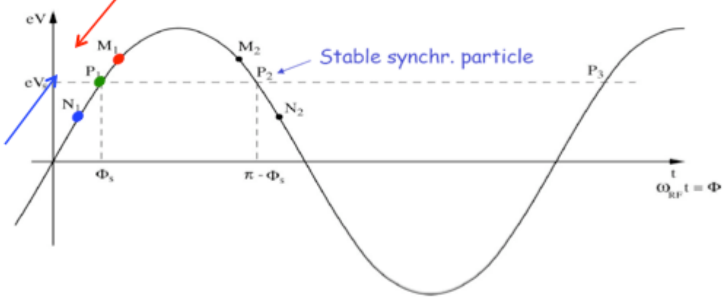
There are two positions (in fact three) where a particle does not get any phase focusing force, $\ddot{\Phi}=0$: at $\Phi = \Phi_s$ (i.e. the ideal position)

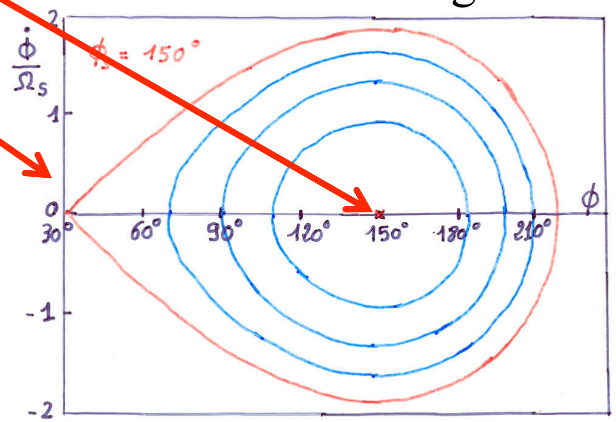
and at $\Phi = \pi-\Phi_s$

When ϕ reaches $\pi-\phi_s$ the force goes to zero and beyond it becomes non restoring. Hence $\pi-\phi_s$ is an extreme amplitude for a stable motion which in the phase space $(\dot{\phi}/\Omega_s, \Delta\phi)$ is shown as closed trajectories.

The phase curve, that belongs to $\Phi = \Phi_s$ separates the stable from the unstable regime

-> "Separatrix"

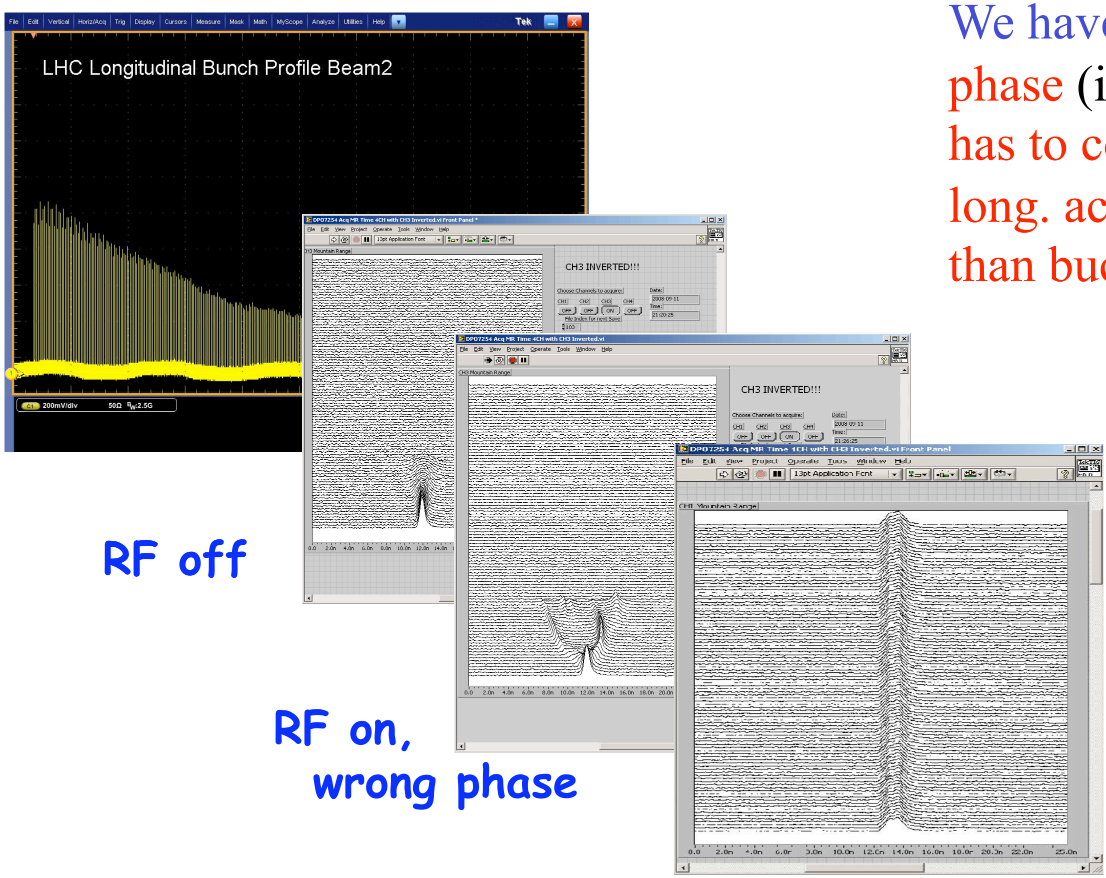




Equation of the separatrix:

$$\frac{\dot{\phi}^2}{2}-\frac{\Omega_s^2}{\cos\phi_s}(\cos\phi+\phi\sin\phi_s)=-\frac{\Omega_s^2}{\cos\phi_s}(\cos(\pi-\phi_s)+(\pi-\phi_s)\sin\phi_s)$$

LHC Commissioning: RF



RF off

RF on,
wrong phase

RF on, phase adjusted,
beam captured

We have to match these conditions:

phase (i.e. timing between rf and injected bunch) has to correspond to ϕ_s

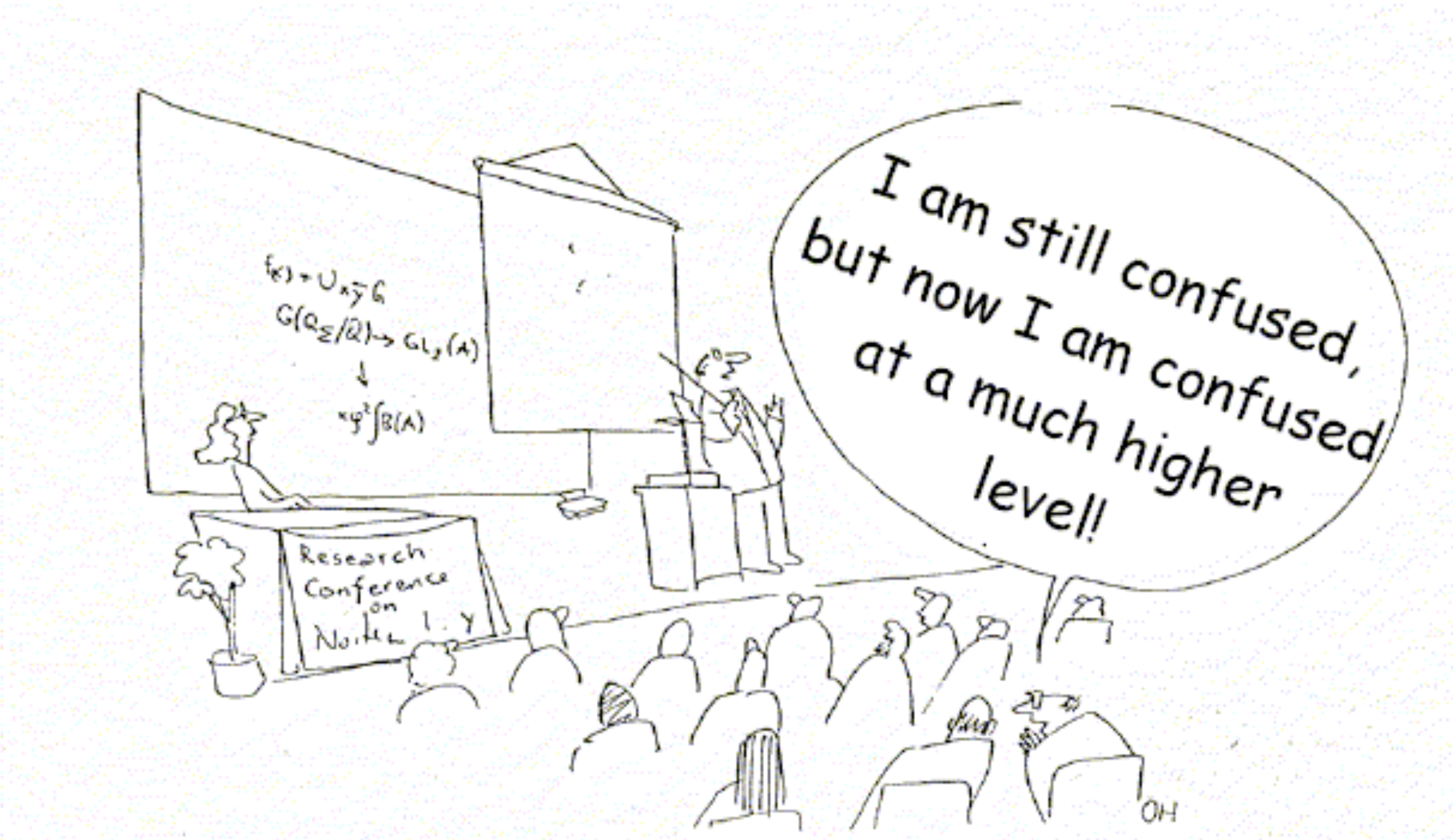
long. acceptance of injected beam has to be smaller than bucket area of the synchrotron.

max stable energy: set $\phi = \phi_s$ and calculate ΔE

$$(\Delta E_{\max})_{sep} = \sqrt{\frac{p_s v_s e V_0}{2\pi h \eta_s}} * \sqrt{\left|4\cos\phi_s - (2\pi - 4\phi_s)\sin\phi_s\right|}$$

LHC injection:
acceptance: 1.4eVs
long emittance: 1.0 eVs

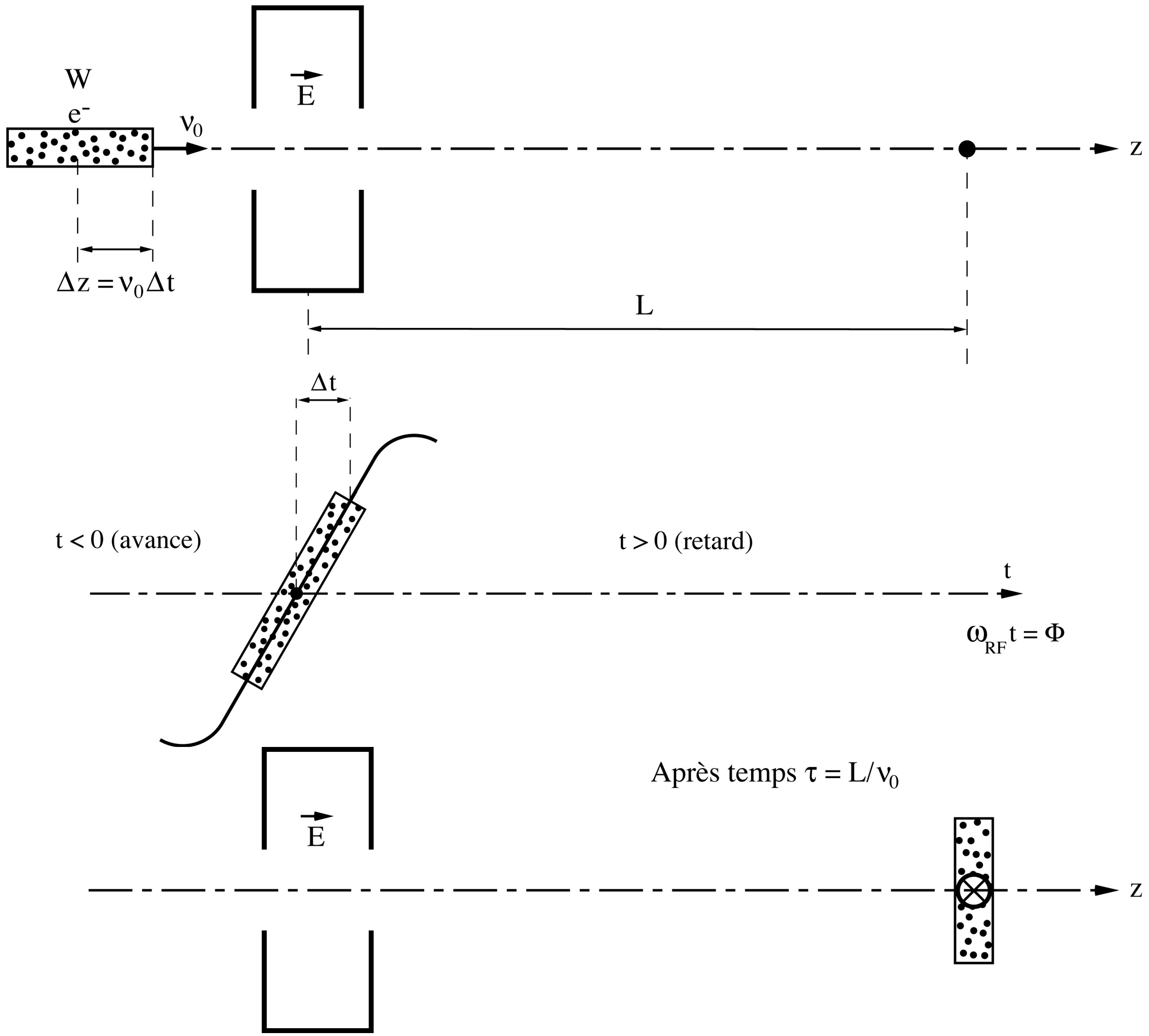
Than'x


I am still confused,
but now I am confused
at a much higher
level!
Research
Conference
on
OH

APPENDIX:

Improved Capture With Pre-buncher

A long bunch coming from the gun enters an RF cavity; the reference particle is the one which has no velocity change. The others get accelerated or decelerated. After a distance L bunch gets shorter while energies are spread: bunching effect. This short bunch can now be captured in the following rf structures.



Improved Capture With Pre-buncher

The bunching effect is a space modulation that results from a velocity modulation and is similar to the phase stability phenomenon. Let's look at particles in the vicinity of the reference one and use a classical approach.

Energy gain as a function of cavity crossing time:

$$\Delta W = \Delta\left(\frac{1}{2} m_0 v^2\right) = m_0 v_0 \Delta v = eV_0 \sin\phi \approx eV_0 \phi$$

$$\Delta v = \frac{eV_0 \phi}{m_0 v_0}$$

Perfect linear bunching will occur after a time delay τ, corresponding to a distance L, when the path difference is compensated between a particle and the reference one:

$$\Delta v.\tau = \Delta z = v_0 t = v_0 \frac{\phi}{\omega_{RF}}$$

(assuming the reference particle enters the cavity at time t=0)

Since L = vτ one gets:

$$L = \frac{2 v_0 W}{eV_0 \omega_{RF}}$$