Strong Coupling Constant Determination from Dijet Events in Thrust

Gherardo Vita

QCD Seminar - CERN, 10 Feb 2025

Based on:

"On Determining α_s from Dijets in e^+e^- Thrust"

M. A. Benitez, A. H. Hoang, V. Mateu, I. W. Stewart, **GV** [2412.15164]

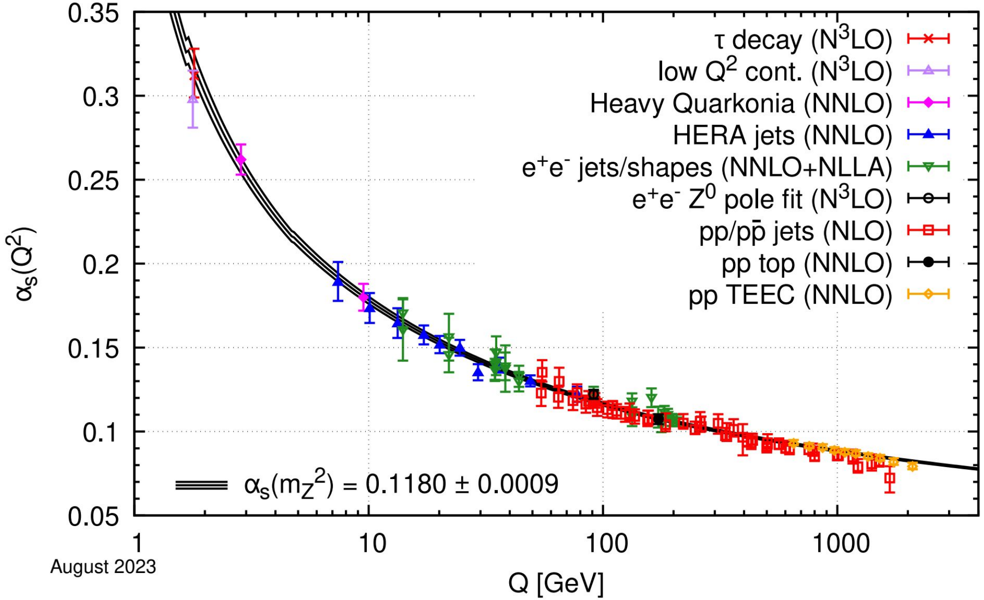
Strong coupling

- Strength of QCD interactions is a free parameter of the Standard Model. Pretty obvious why we care about it.

- In QED the coupling is small at **low energy** so we have a plethora of extremely precise measurements, but for **QCD** confinement complicates this.

- But, as we all know, α_s is a "running" coupling, so for energy scales above the GeV the coupling is **small** → it is *perturbative*, and so are its evolution equations

- Use **collider physics experiments** to measure it



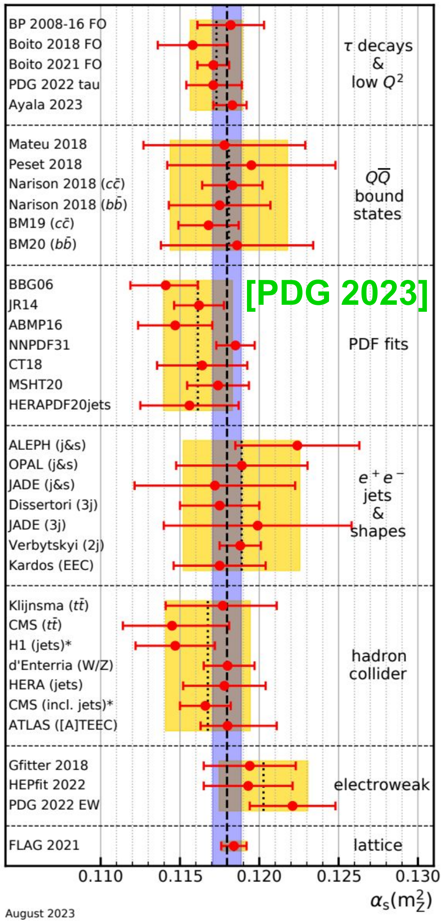
[PDG 2023]

Review of Particle Physics - PDG World Average

- Reference point for the value is conventionally chosen to be 91.2 GeV, the mass of the Z boson.
- Many different ways proposed to measure it. Results that meet certain quality standards are regularly reported in the Particle Data Group **(PDG)**.
- Precision is at the **percent** level (to be compared to the 10^{-8} of the QED coupling)!
- Lattice is by far the most precise at the moment.
- Determination of the world average value: [https://pdg.lbl.gov/2024/reviews/rpp2024-rev-qcd.pdf]

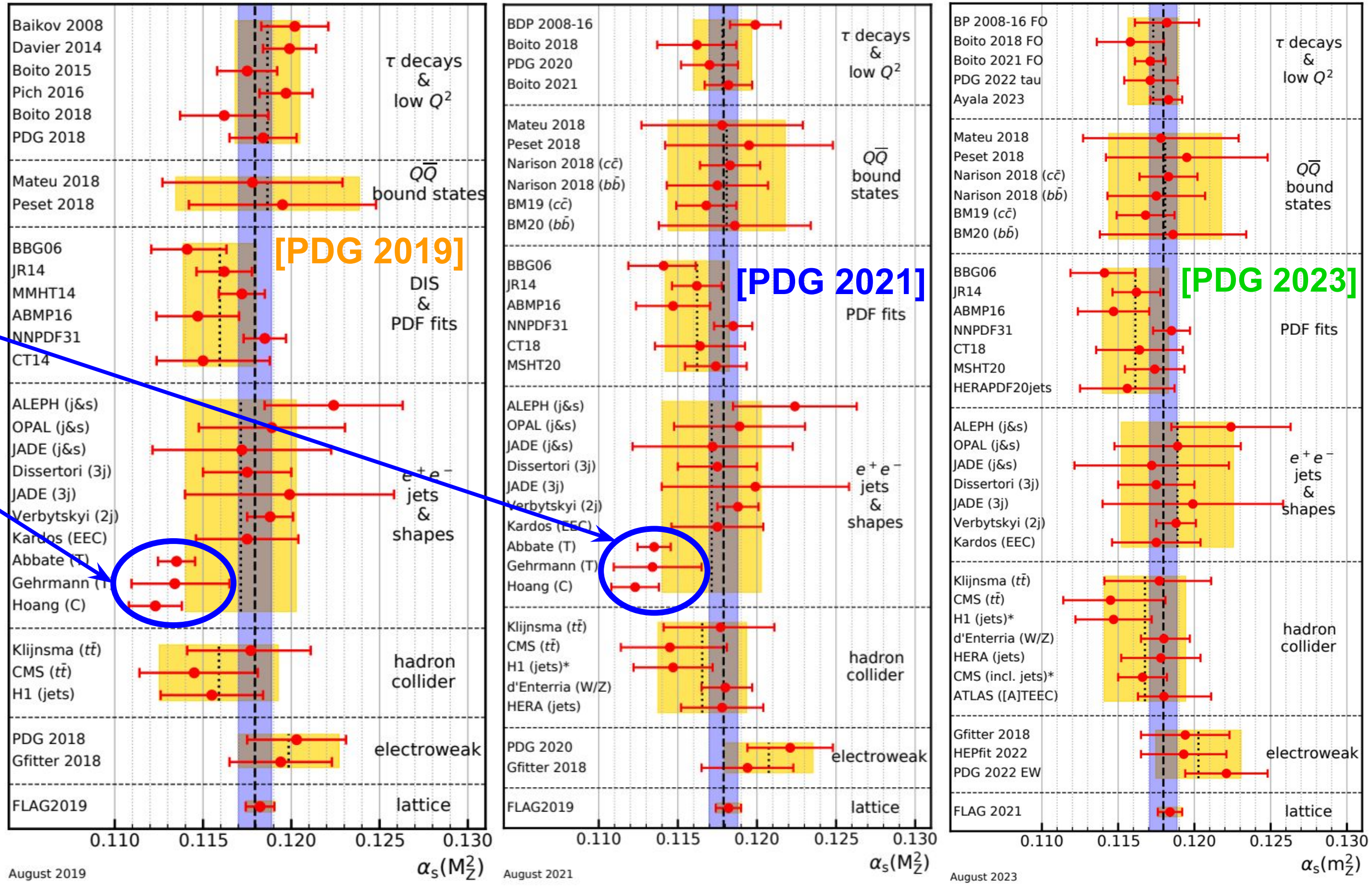
$$\alpha_s(m_Z^2) = 0.1180 \pm 0.0009 \qquad \text{(PDG 2023 average)}$$

$$\alpha_s(m_Z^2) = 0.1175 \pm 0.0010 \qquad \text{(PDG 2023 without lattice)}$$



Review of Particle Physics - PDG World Average

The last edition of the PDG removed three extractions from event shapes

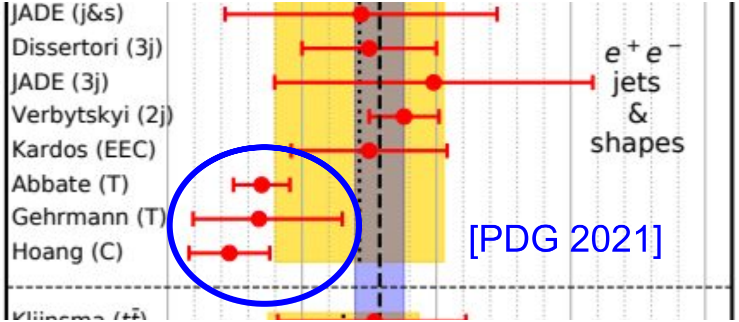


Review of Particle Physics - PDG World Average

"Thrust at N3LL with Power Corrections and a Precision Global Fit for αs(mZ)"
[Abbate, Fickinger, Hoang, Mateu, Stewart **2010**]

"Power corrections in the dispersive model for a determination of the strong coupling constant from the thrust distribution"
[Gehrmann, Luisoni, Monni **2012**]

"A Precise Determination of αs from the C-parameter Distribution"
[Hoang, Kolodrubetz, Mateu, Stewart **2015**]



- These are analyses from 2010-2015 based on **thrust** and **C-parameter** employing high order resummation, *analytic* models for hadronization effects. They all obtained a low central value.

- The motivation for removal are questions raised over the years about their estimate of
  - Non perturbative uncertainties
  - Resummation uncertainties
  - Hadron mass effects

The key finding in Ref. [688] is that non-perturbative corrections computed in the three-jet region significantly deviate from those computed in the two-jet limit and hence the aforementioned fits based on power corrections in the two-jet limit result in smaller values of $\alpha_s(m_Z^2)$. Another important observation is that the inclusion of resummation effects introduces a relatively substantial ambiguity outside the two-jet limit. Additionally, other factors such as the choice of mass-scheme used to extend the definition of event shapes to massive hadrons can have significant effects.

These findings are inconsistent with the very small experimental, hadronization, and theoretical uncertainties of only 2, 5, and 9 per-mille, respectively, as reported in Refs. [683, 685]. For these reasons, we exclude the results of Refs. [683–685] from the average. Determinations based on corrections for non-perturbative hadronization effects using QCD-inspired Monte Carlo generators have also faced criticism due to the differing nature of parton-level simulations compared to fixed-order calculations. However, these determinations typically exhibit a more conservative theoretical uncertainty.

Quantum Chromodynamics - PDG 2023 update [2312.14015]

A fresh look at Thrust Fits

- This has motivate a reanalysis of these fits. Which we have started in this work [2412.15164]

““On Determining αs(mZ) from Dijets in e+e- Thrust”
[Benitez, Hoang, Mateu, Stewart, **GV 2024**]

- This new analysis is based only on **thrust,** updating the work in [1006.3080]

“Thrust at N3LL with Power Corrections and a Precision Global Fit for αs(mZ)”
[Abbate, Fickinger, Hoang, Mateu, Stewart **2010**]

- I’ll refer to [1006.3080] as the ***2010 analysis*** and to [2412.15164] as the ***2024 analysis***.
- Main goal of the 2024 analysis is to
  - Update perturbative ingredients that have become available in the meantime
  - Address criticisms that have come since 2010 on certain aspects of the 2010 analysis
- This also implies that several ingredients that entered the 2010 analysis, that have not raised concerns were not discussed in this update, eg. QED, bottom mass, Omega2, etc. Also decided not to include N4LL resummation in this analysis to change the minimum of ingredients w.r.t. 2010

Strong coupling extraction from Event Shapes

- So now let's take a step back...

- Event shapes in electron-positron collisions (like thrust, EEC, C-parameter) are natural candidates to extract α_s mainly because:

  - They can be measured very precisely compared to other collider measurements

  - They are very sensitive to the coupling constant since they start at $\mathcal{O}(\alpha_s)$

  - We have plenty of data from LEP (and even older colliders, like PETRA, with lower CMEs)

  - We can make precise theoretical predictions for them

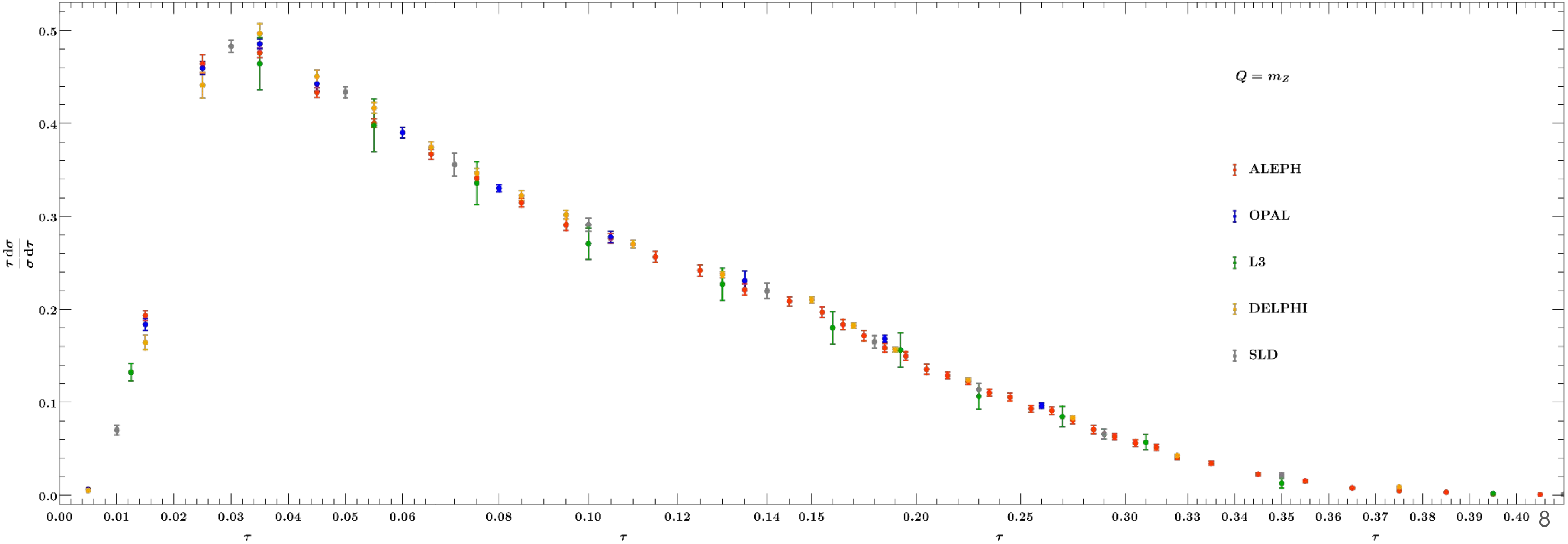
The Thrust Distribution

E. Farhi , Phys. Rev. Lett. 39 (1977) 1587

- We'll focus on thrust

$$T \equiv \max_{\vec{n}} \frac{\sum_j |\vec{p}_j \cdot \vec{n}|}{\sum_j |\vec{p}_j|}$$

$$\tau = 1 - T$$



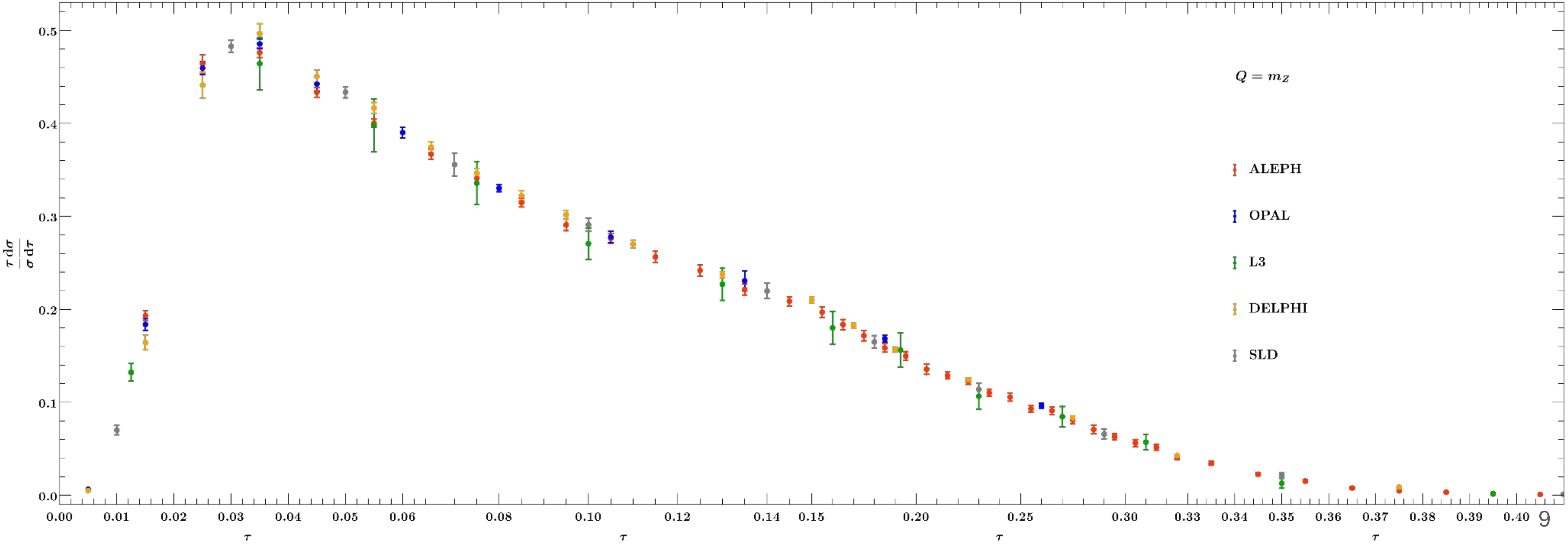
The Thrust Distribution

E. Farhi , Phys. Rev. Lett. 39 (1977) 1587

Event shape observable, constructed s.t. for $\tau \ll 1$ events take **dijet** shape => radiation constrained to be soft or collinear to thrust axis

$$T \equiv \max_{\vec{n}} \frac{\sum_j |\vec{p}_j \cdot \vec{n}|}{\sum_j |\vec{p}_j|}$$

$$\tau = 1 - T$$



The Thrust Distribution

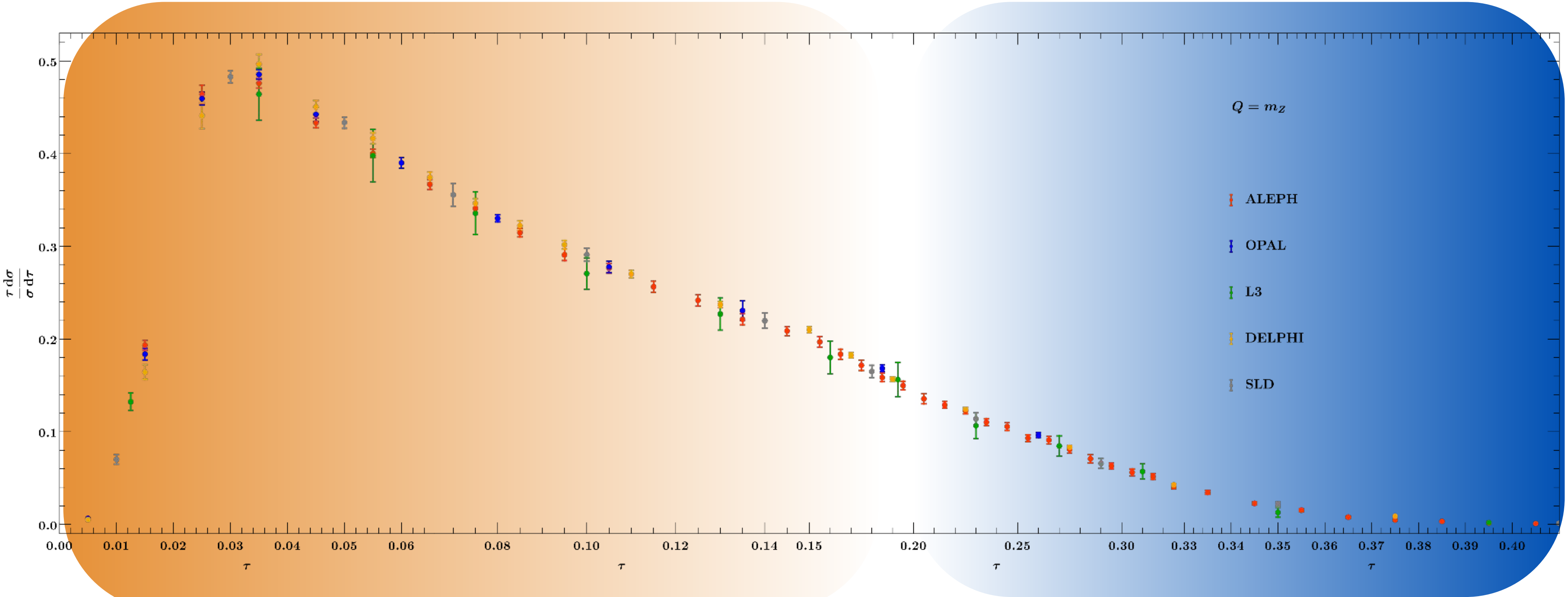
E. Farhi , Phys. Rev. Lett. 39 (1977) 1587

Dijet region

$$\tau Q \ll Q$$

Three/multi-jet region

$$\tau Q \sim Q$$



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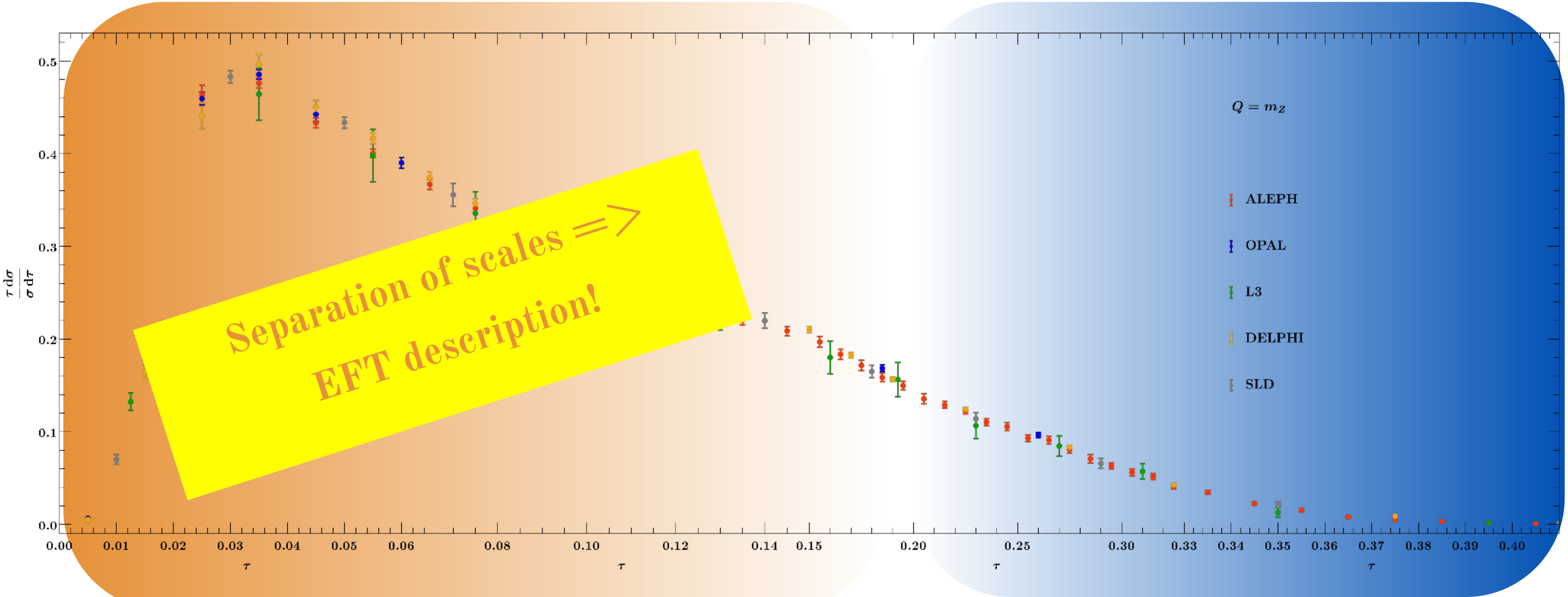


Dijet Factorization in SCET

- We can use Soft and Collinear Effective Theory (**SCET**) to derive **factorization theorem** to describe the leading asymptotic of the thrust distribution for $\tau Q \ll Q$
- Schematically, one starts from the **operatorial definition in QCD** of a distribution in electron-positron annihilation

$$\frac{d\sigma}{de} = \frac{1}{2Q^2} \sum_X \int d^4x \, e^{iq\cdot x} \sum_{i=V,A} L^i_{\mu\nu} \langle 0| j_i^{\mu\dagger}(x) |X\rangle \langle X| j_i^\nu(0) |0\rangle \, \delta(e - e(X))$$

$$e(X) = \frac{1}{Q} \sum_{i\in X} f_e(\eta_i) \left|\mathbf{p}_i^{\mathrm{T}}\right|$$

- Different event shapes will have different measurements

$$f_\tau(\eta) = e^{-|\eta|}, \qquad f_B(\eta) = 1, \qquad f_C(\eta) = \frac{3}{\cosh\eta}$$

Dijet Factorization in SCET

- Order by order in the power expansion, soft and collinear modes decouple allowing a factorization in terms of **Hard**, **Jet**, and **Soft** functions

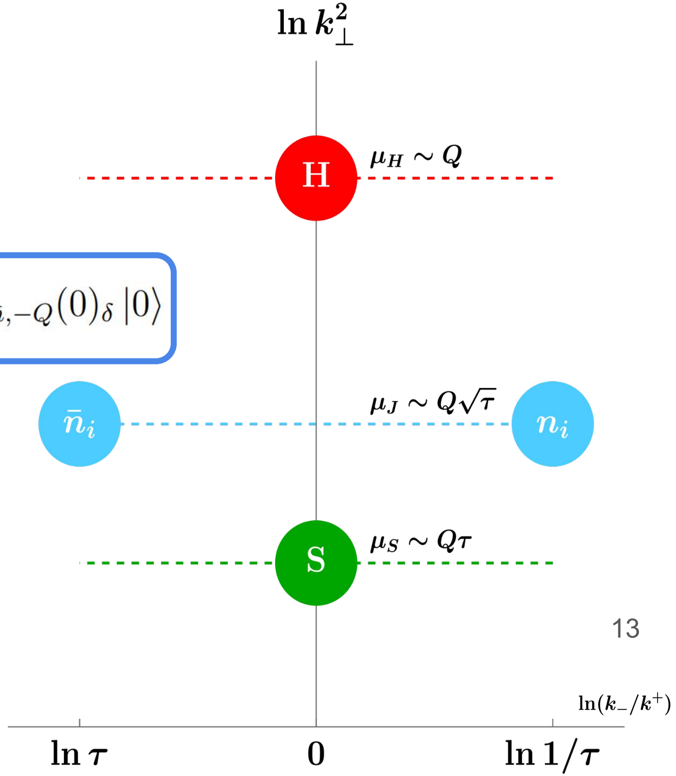
$$\frac{d\sigma}{de} = \frac{1}{6Q^2} \sum_{n} |C_{n\bar{n}}(Q,-Q;\mu)|^2 \int d^4x \int de_n\, de_{\bar{n}}\, de_s\, \delta(e - e_n - e_{\bar{n}} - e_s)$$
$$\times \frac{1}{N_C^2} \mathrm{Tr}\, \langle 0| \chi_{n,Q}(x)_\beta \delta(e_n - \hat{e}_n) \bar{\chi}_{n,Q}(0)_\gamma |0\rangle \, \mathrm{Tr}\, \langle 0| \bar{\chi}_{\bar{n},-Q}(x)_\alpha \delta(e_{\bar{n}} - \hat{e}_{\bar{n}}) \chi_{\bar{n},-Q}(0)_\delta |0\rangle$$
$$\times \mathrm{Tr}\, \langle 0| \overline{Y}_{\bar{n}}^\dagger(x) Y_n^\dagger(x) \delta(e_s - \hat{e}_s) Y_n(0) \overline{Y}_{\bar{n}}(0) |0\rangle \sum_{i=V,A} L^i(\bar{\Gamma}_i^\mu)_{\alpha\beta} (\Gamma_{i\mu})_{\gamma\delta}\,,$$

Gauge invariant collinear quark field

$$\chi_n = (W_n^\dagger \xi_n)$$

Soft Wilson Line

$$Y_n(x) = \overline{\mathrm{P}}\, \exp\Big(-ig \int_0^\infty ds\, n \cdot A_s(ns+x) \Big)$$



Beyond the Dijet limit

- Discussion so far valid for dijet limit. But what about corrections?
- For precision strong coupling extraction crucial to include "non-singular" corrections, i.e. higher orders in the $\tau Q \ll Q$ expansion.
- These are captured up to order αs^3 by fixed-order predictions, e.g.

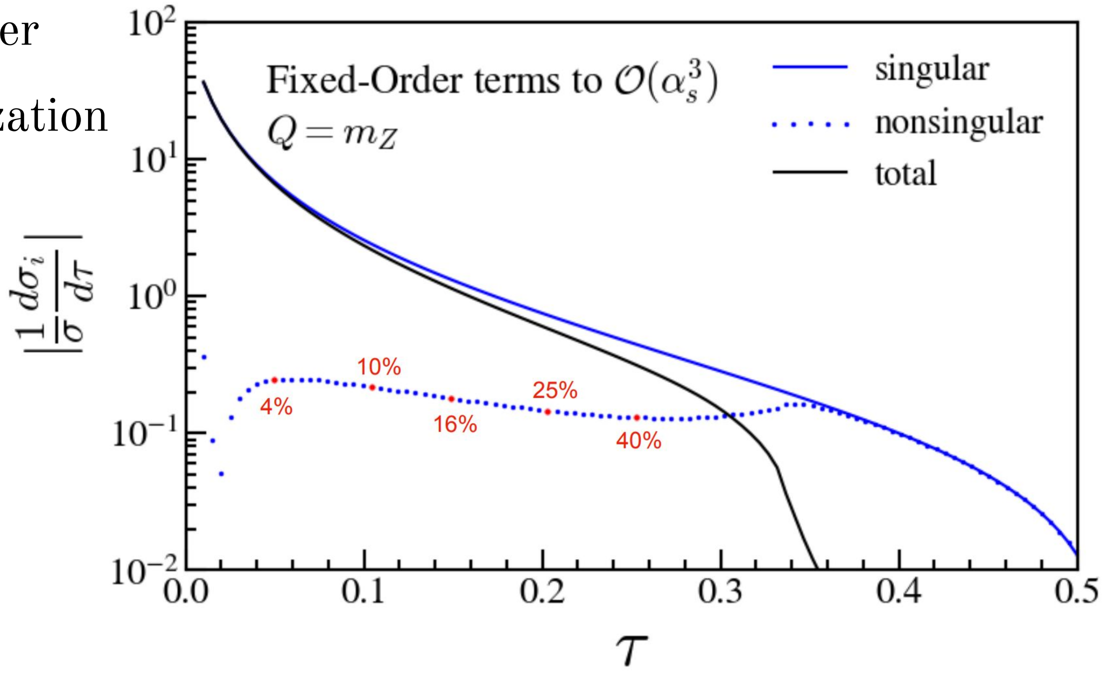
EERAD3: Event shapes and jet rates in electron-positron annihilation at order α_s^3

A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, G. Heinrich

Jet production in the CoLoRFulNNLO method: event shapes in electron-positron collisions

Beyond the Dijet limit

- Non-singular is difference between full fixed order prediction and fixed order expansion of leading power factorization
- Corrections are important, but non-singular terms subdominant until ~ 0.3
- This gives a practical way to quantitatively distinguish Dijet vs trijet/multijet region

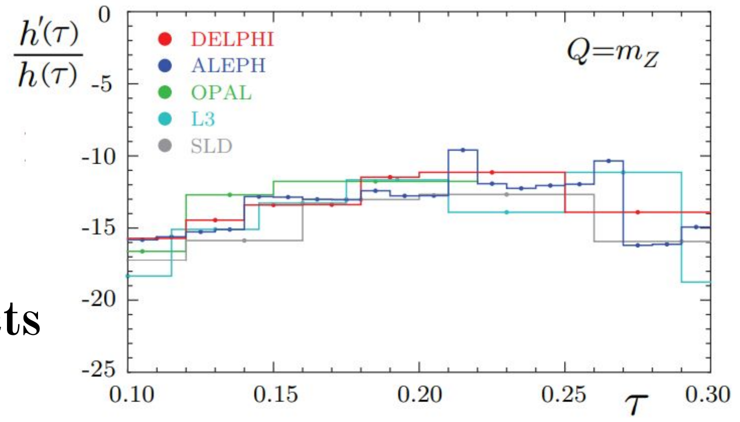


Need of NP effects

- So ideally one uses dijet factorization for resummation, supplements it with fixed order correction, and fits the strong coupling constant to very high precision.
- Not so fast....
- It is well known that event shapes have important **non perturbative corrections**
- In first approximation behave as a **shift** $\tau \to \tau - 2\Lambda/Q$ with $\Lambda \sim \Lambda_{\rm QCD}$
- One can estimate impact of the shift on fit

$$\frac{\delta\alpha_s}{\alpha_s} \simeq \frac{2\Lambda}{Q}\frac{h'(\tau)}{h(\tau)} \Rightarrow \delta\alpha_s/\alpha_s \simeq -(9\pm 3)\%$$

for $Q = m_Z \quad \Lambda = 0.3\,\text{GeV}$

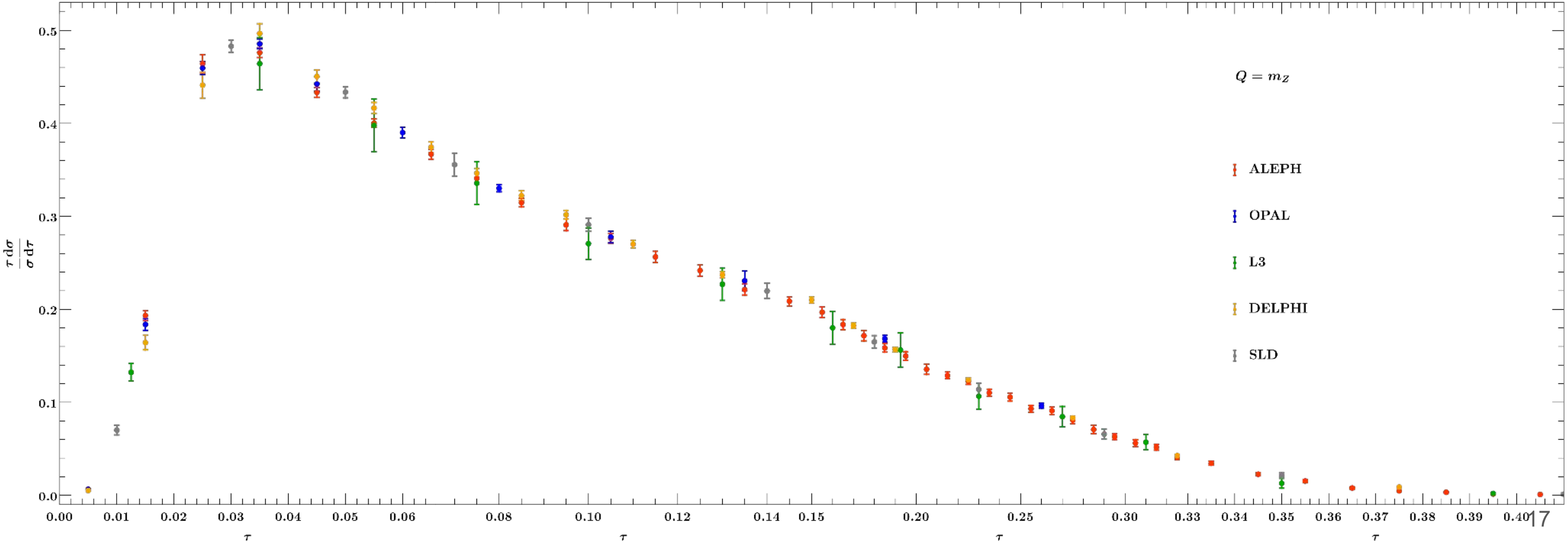


- => Mandatory to have good control over **NP effects** for percent level extractions

The Thrust Distribution - Scales

In QCD, thrust is actually a function of **three** scales: $\Lambda_{\rm QCD}\ ,\ \ \tau Q\ ,\ \ Q$

For $\Lambda_{\rm QCD} \ll Q$ their hierarchy defines 3 regions: **peak**, **tail**, and **far-tail**



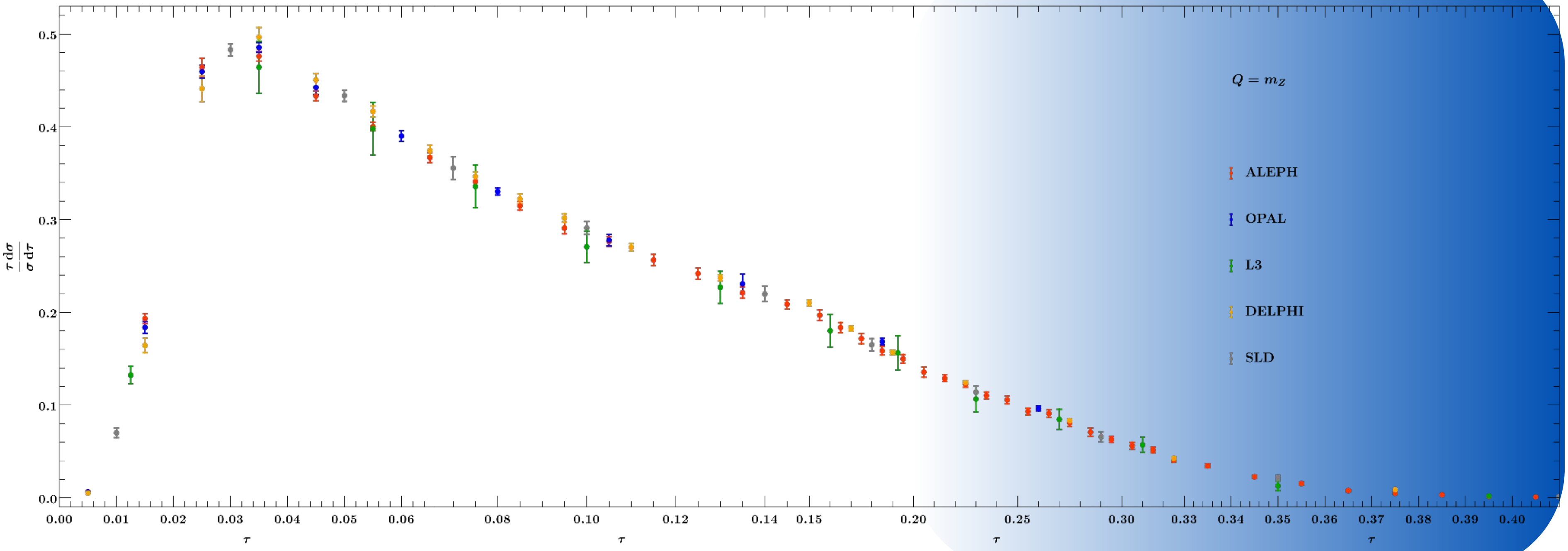
The Thrust Distribution - Far Tail region

No clear distinction between hard scale and observable $\tau Q \sim Q$

Distribution is well described by perturbation theory. Multi jet events dominate. Non-perturbative corrections are small, $\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{Q}\right)$

Far-tail / 3(multi)jet region

$$\Lambda_{\rm QCD} \ll \tau Q \sim Q$$



The Thrust Distribution - Tail region

All scales are separated.

Events have clear dijet shape.

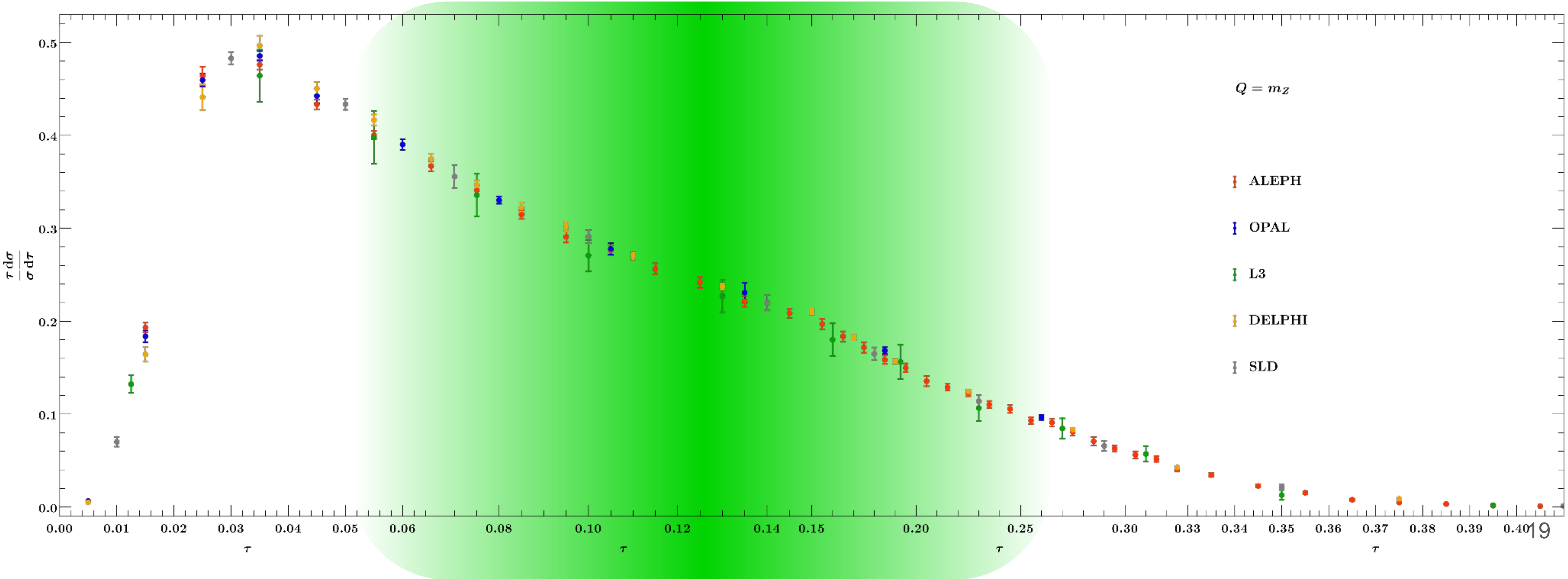
Tail region

$$\Lambda_{\mathrm{QCD}} \ll \tau Q \ll Q$$

Soft and collinear radiation dominates,

but $\Lambda_{\mathrm{QCD}} \ll \tau Q$ guarantees clear

separation between perturbative and NP modes

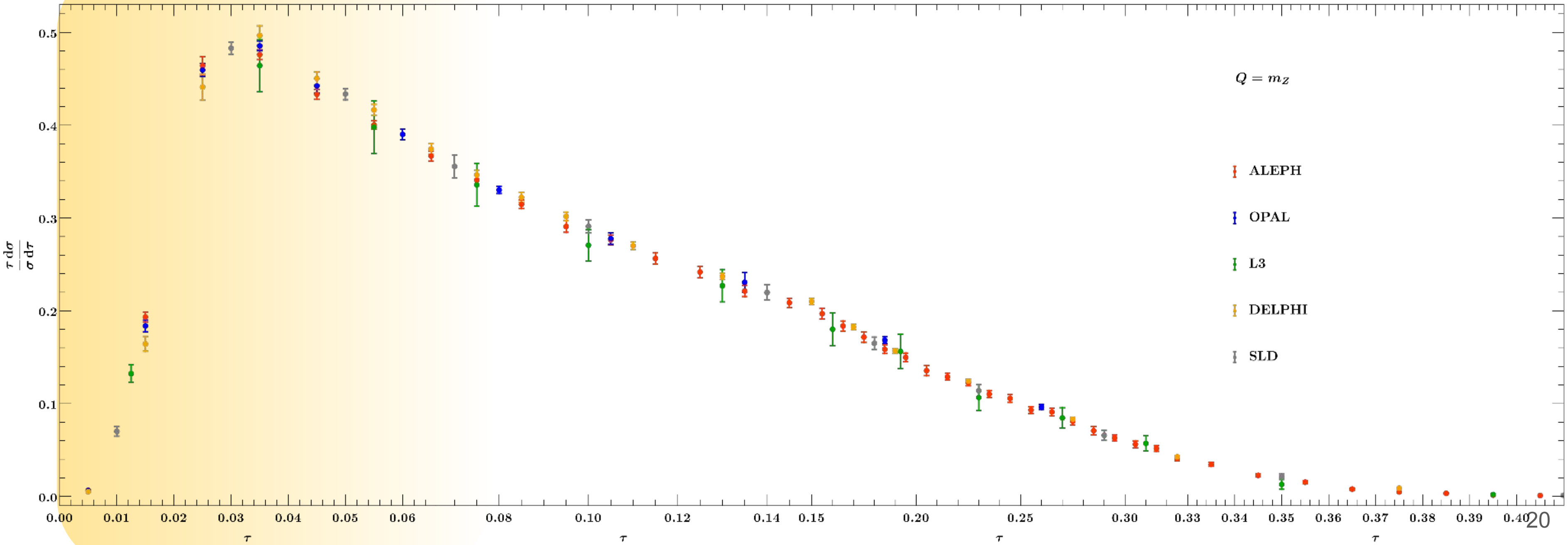


The Thrust Distribution - Peak region

Peak region

$$\Lambda_{\rm QCD} \sim \tau Q \ll Q$$

Here the distribution is heavily affected by **non-perturbative dynamics.** No clear distinction between hadronization scale and observable $\Lambda_{\rm QCD} \sim \tau Q$



The Thrust Distribution - the 3 regions

Peak region

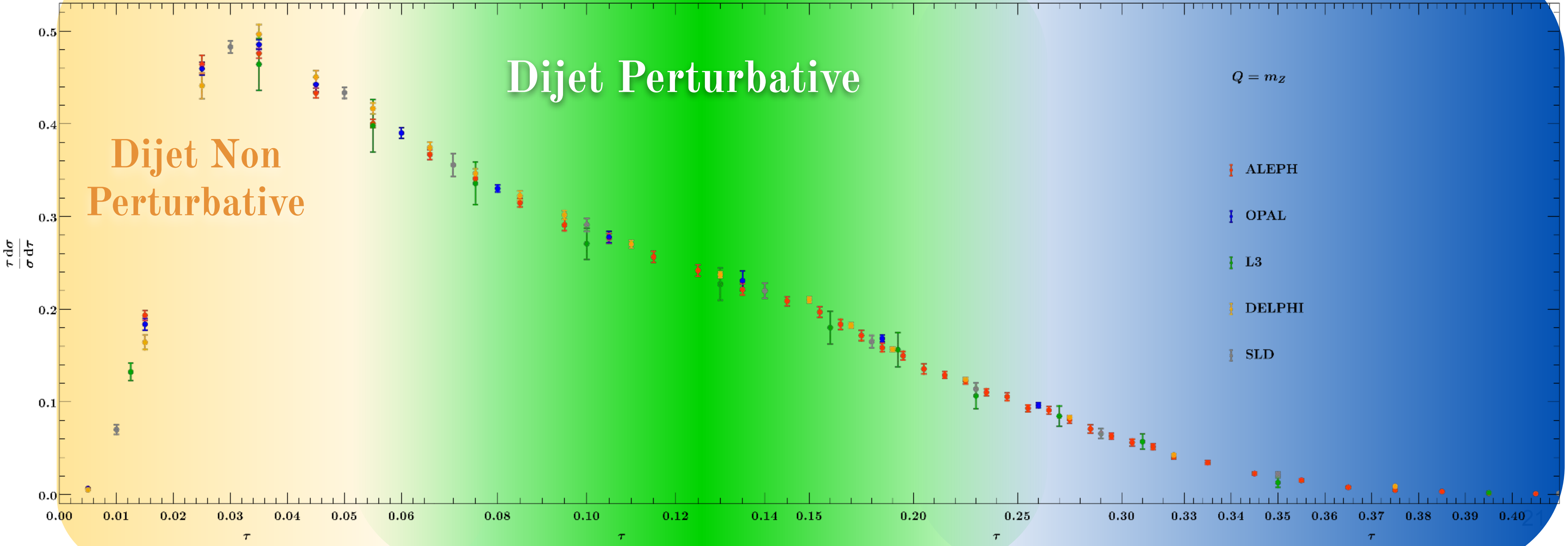
$\Lambda_{\text{QCD}} \sim \tau Q \ll Q$

Tail region

$\Lambda_{\text{QCD}} \ll \tau Q \ll Q$

Far-tail / 3(multi)jet region

$\Lambda_{\text{QCD}} \ll \tau Q \sim Q$

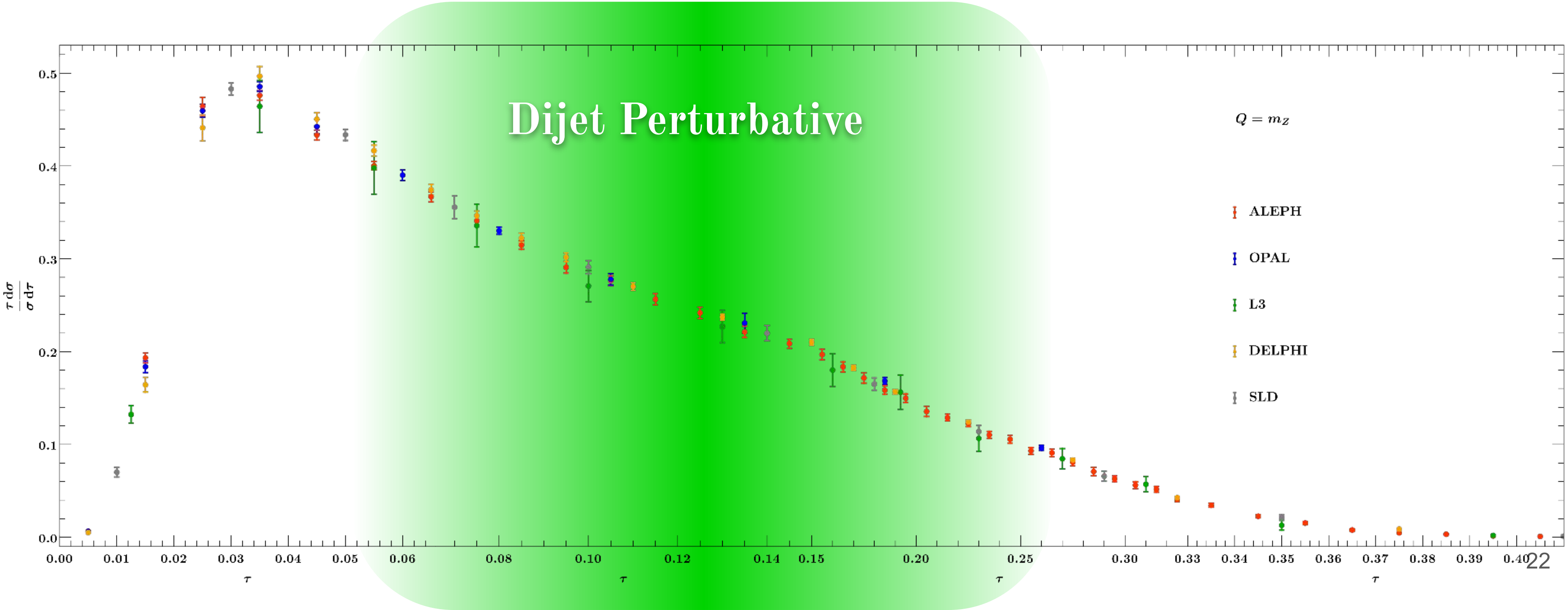


The Thrust Distribution - the fit region

Choose tail region for fit

Tail region

$$\Lambda_{\rm QCD} \ll \tau Q \ll Q$$



Precision Predictions in tail region

Theory prediction:

Tail region $\Lambda_{\rm QCD} \ll \tau Q \ll Q$

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}_{\rm s}}{d\tau} + \frac{d\hat{\sigma}_{\rm ns}}{d\tau} \right)\left(\tau - \frac{k}{Q}\right) F_\tau[k - 2\bar{\Delta}(R, \mu_s)]$$

Singular distribution

Describes the main contribution to the distribution in this region.

Use SCET dijet factorization in perturbative regime

Non-singular terms

Accounts for the corrections to the $\tau Q \ll Q$ expansion.

Use fixed order codes.

Shape function

Accounts for the corrections to the $\Lambda_{\rm QCD} \ll \tau Q$ expansion.

Fully accounted by the non-perturbative part of the dijet factorization theorem

Theory Ingredients: Singular cross section

$$\frac{d\hat{\sigma}_s^{QCD}}{d\tau}(\tau) = Q\sum_I \sigma_0^I\, H_Q^I(Q,\mu_H)\, U_H(Q,\mu_H,\mu) \int ds\, ds'$$
$$\times J_\tau(s',\mu_J)\, U_J^\tau(s-s',\mu,\mu_J) \int dk'\, U_S^\tau(k',\mu,\mu_S)$$
$$\times e^{-2\frac{\delta(R,\mu_S)}{Q}\frac{\partial}{\partial\tau}}\, S_\tau^{part}\left(Q\tau - \frac{s}{Q} - k', \mu_S\right).$$

Accuracy	$H,\ J,\ S$	$\Gamma_{cusp}(\alpha_s)$	$\gamma(\alpha_s)$	$\beta(\alpha_s)$
LL	Tree level	1-loop	–	1-loop
NLL	Tree level	2-loop	1-loop	2-loop
NLL′	1-loop	2-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	3-loop
NNLL′	2-loop	3-loop	2-loop	3-loop
N^3LL	2-loop	4-loop	3-loop	4-loop
N^3LL'	3-loop	4-loop	3-loop	4-loop

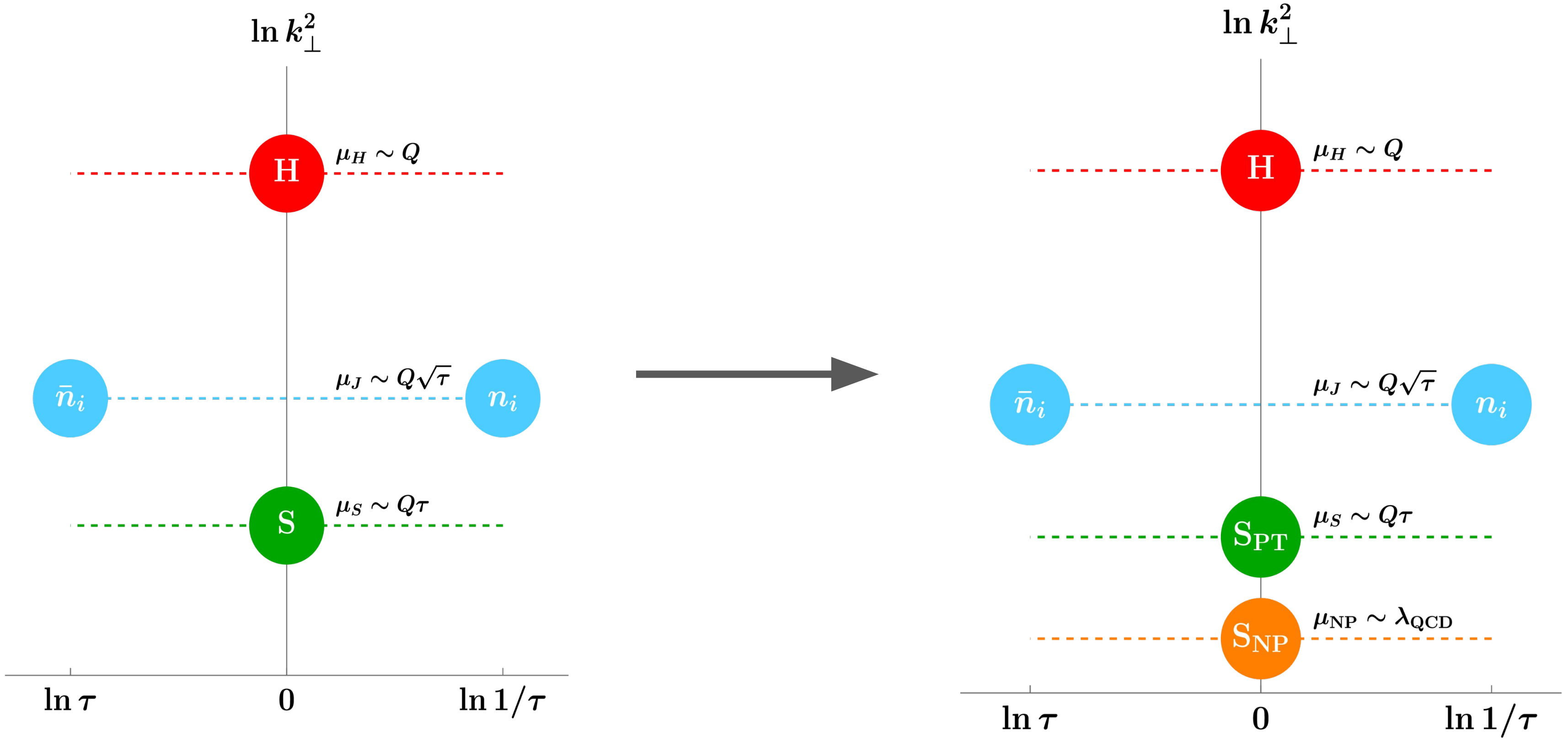
- **Hard** function to 3 loops
- **Jet** function to 3 loops
- **Soft** functions to 3 loops
- Evolution factors U to 3 loops

=> N3LL’ resummation

- H, J, S referred as “boundary” because they give the boundary conditions of the RGE differential equations.
- U are evolution factors encoding the solution of the RGE

Non Perturbative Corrections

- Leading Non Perturbative (NP) effects enter only the soft function.



NP Corrections - Soft Function OPE

- Definition of the soft function is valid also non perturbatively

$$S_\tau(k,\mu) = \frac{1}{N_c}\langle 0|\mathrm{tr}\ \overline{Y}^T_{\bar n} Y_n \delta(k - i\hat\partial) Y_n^\dagger \overline{Y}^*_{\bar n}|0\rangle_\mu$$

- In the region we are interested in we have

$$k \sim Q\tau \gg \mathcal{O}(\Lambda_{\mathrm{QCD}})$$

- We can perform an **Operator Product Expansion** for S

[Lee, Sterman '06] [2010 analysis]

$$S_\tau(k,\mu) = \hat S_\tau(k,\mu) - \frac{\mathrm{d}\hat S_\tau(k,\mu)}{\mathrm{d}k} 2\bar\Omega_1 + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{k^3}\right)$$

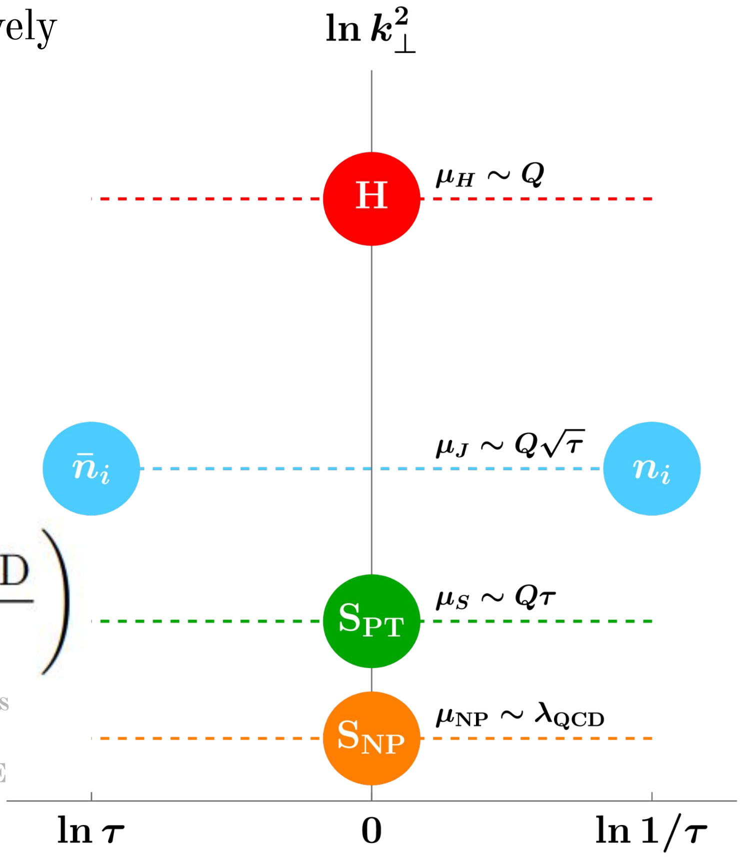
Full Soft Function

Perturbative Soft Function (zero-th order of the OPE)

Matching coefficient (in this case trivially related to derivative of PT SF)

NP Operator Matrix Element (first order of the OPE)

Higher orders in the soft function OPE expansion



NP Corrections - O1 Matrix Element

$$S_\tau(k,\mu) = \hat{S}_\tau(k,\mu) - \frac{\mathrm{d}\hat{S}_\tau(k,\mu)}{\mathrm{d}k} 2\bar{\Omega}_1 + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{k^3}\right)$$

Full Soft Function

Perturbative Soft Function (zero-th order of the OPE)

Matching coefficient (in this case trivially related to derivative of PT SF)

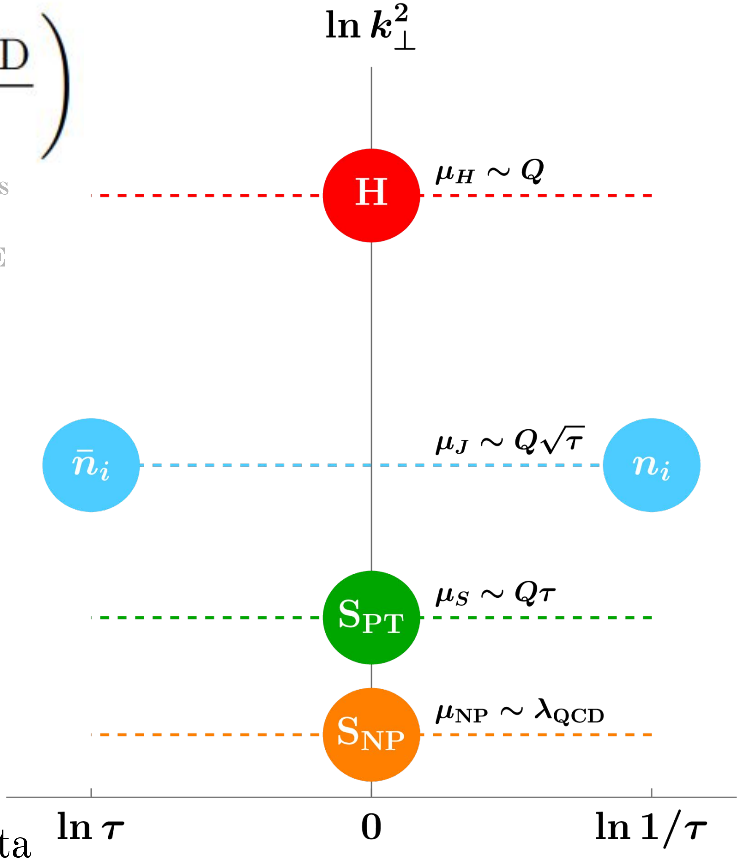
NP Operator Matrix Element (first order of the OPE)

Higher orders in the soft function OPE expansion

$$\bar{\Omega}_1 = \frac{1}{N_c}\langle 0|\mathrm{tr}\ \overline{Y}^T_{\bar{n}}(0) Y_n(0) \hat{\mathcal{E}}_T(0) Y^\dagger_n(0) \overline{Y}^*_{\bar{n}}(0)|0\rangle\,.$$

$$\hat{\mathcal{E}}_T(\eta)|X\rangle = \sum_{i\in X} p_i^\perp \delta(\eta - \eta_i)|X\rangle$$

- This gives field theoretical handle on non perturbative corrections in this region.
- NP Matrix Element clearly defined. Will be fitted from data



NP Corrections - the Shape Function

- Having understood this, we can look at the problem more systematically
- Separate perturbative and NP contributions by defining a **shape function**

$$S_\tau(k,\mu) = \int_0^k \mathrm{d}k' \, \hat{S}_\tau(k-k',\mu) \, F_\tau(k')$$

Full Soft Function — $S_\tau(k,\mu)$

Standard perturbative Thrust Soft Function — $\hat{S}_\tau(k-k',\mu)$

Shape function/ Soft function modification — $F_\tau(k')$

- **OPE** is then just the **multiple expansion** of the shape function

$$F_\tau(k) = \delta(k) + \sum_{i=1}^{\infty} 2^i \bar{\Omega}_i \, \delta^{(i)}(k)$$

$$\int_0^\infty \mathrm{d}k \, F_\tau(k) = 1$$

- NP matrix elements are related to **moments** of the shape function

$$\bar{\Omega}_i = \int_0^\infty \mathrm{d}k \left(\frac{k}{2}\right)^i F_\tau(k)$$

Precision Predictions in tail region

Theory prediction:

Tail region $\Lambda_{\rm QCD} \ll \tau Q \ll Q$

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}_{\rm s}}{d\tau} + \frac{d\hat{\sigma}_{\rm ns}}{d\tau} \right) \left(\tau - \frac{k}{Q} \right) F_\tau[k - 2\bar{\Delta}(R, \mu_s)]$$

Singular distribution

Resummed to N3LL'

Non-singular terms

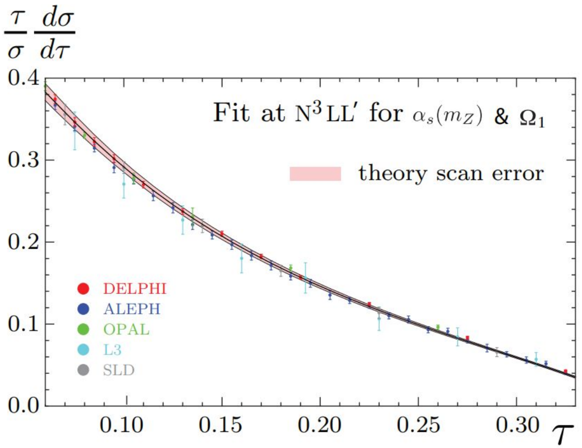
All fixed order corrections up to $O(\alpha s^3)$

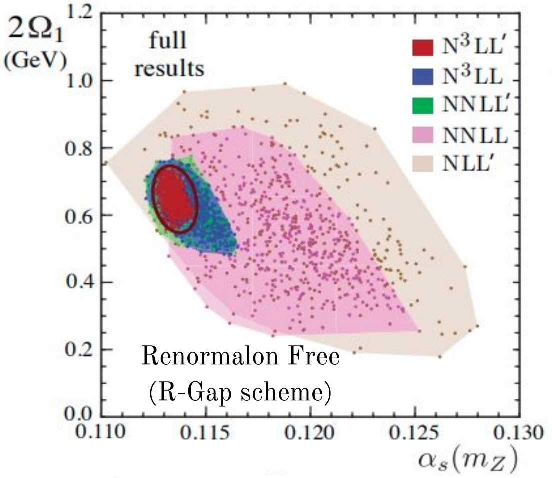
Shape function

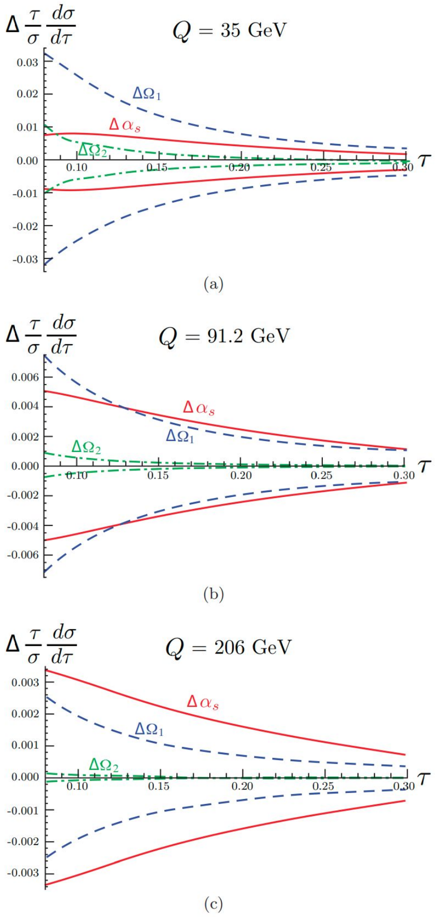
Inclusion of first moment. To be fitted together with αs

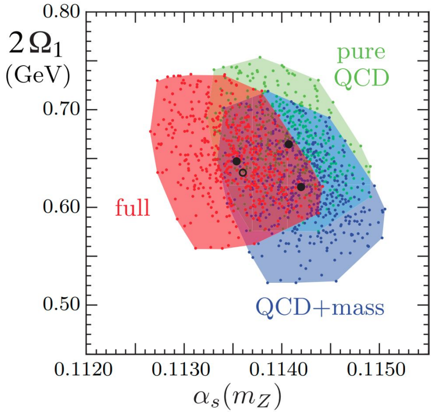
2010 analysis

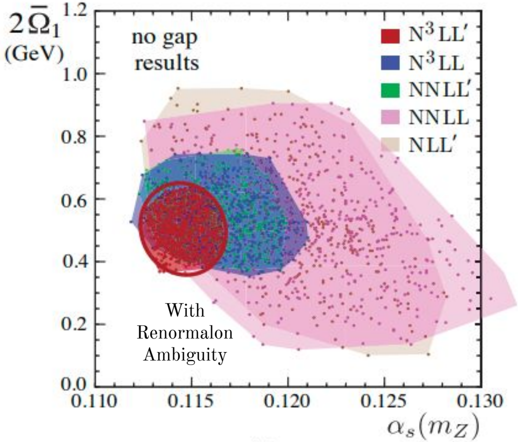
[Abbate, Fickinger, Hoang, Mateu, Stewart 1006.3080]





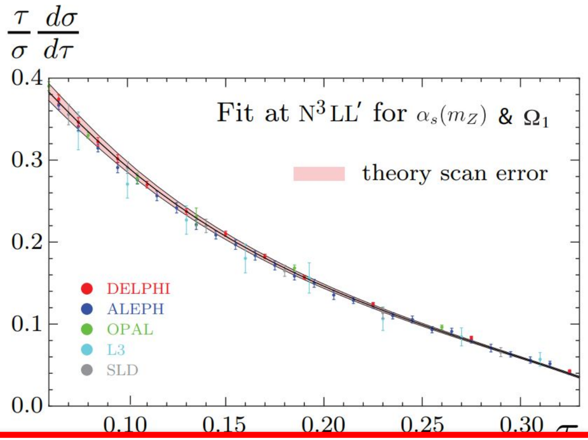


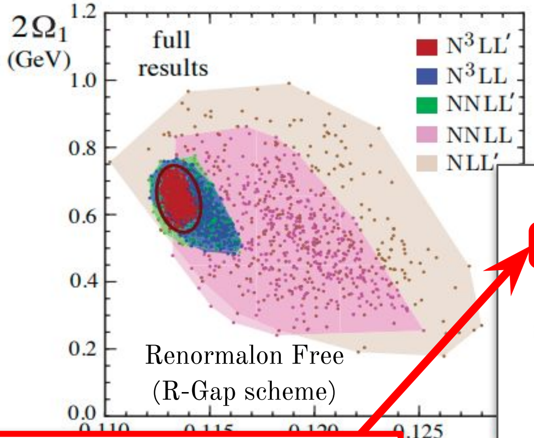


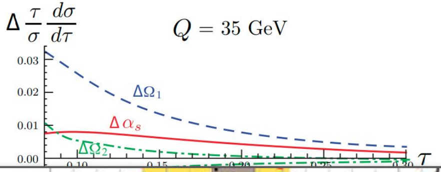


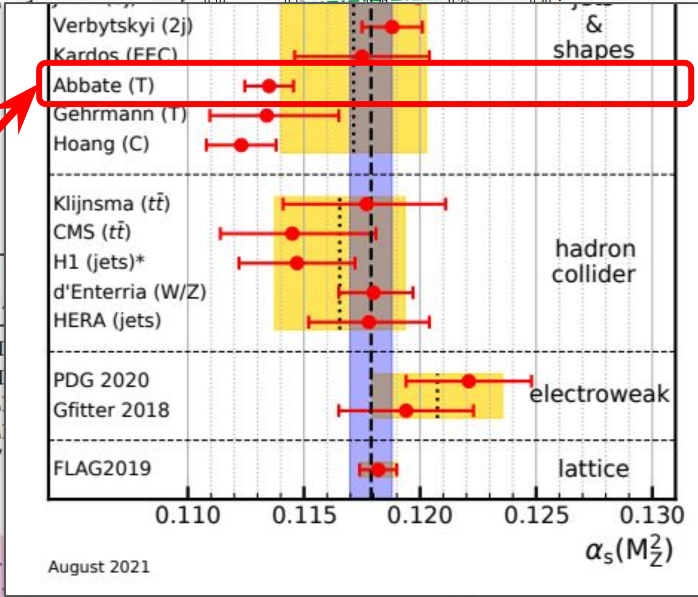
2010 analysis

[Abbate, Fickinger, Hoang, Mateu, Stewart 1006.3080]



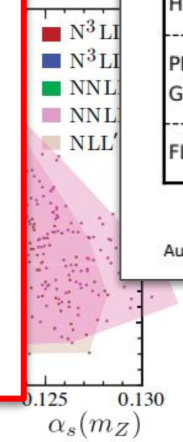


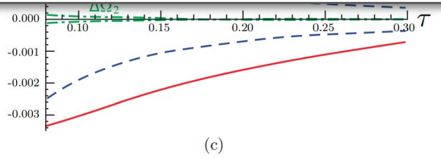




$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\rm exp} \pm (0.0005)_{\rm hadr} \pm (0.0009)_{\rm pert},$$

$$\Omega_1(R_\Delta,\mu_\Delta) = 0.323 \pm (0.009)_{\rm exp} \pm (0.013)_{\Omega_2} \pm (0.020)_{\alpha_s(m_Z)} \pm (0.045)_{\rm pert}\ {\rm GeV},$$



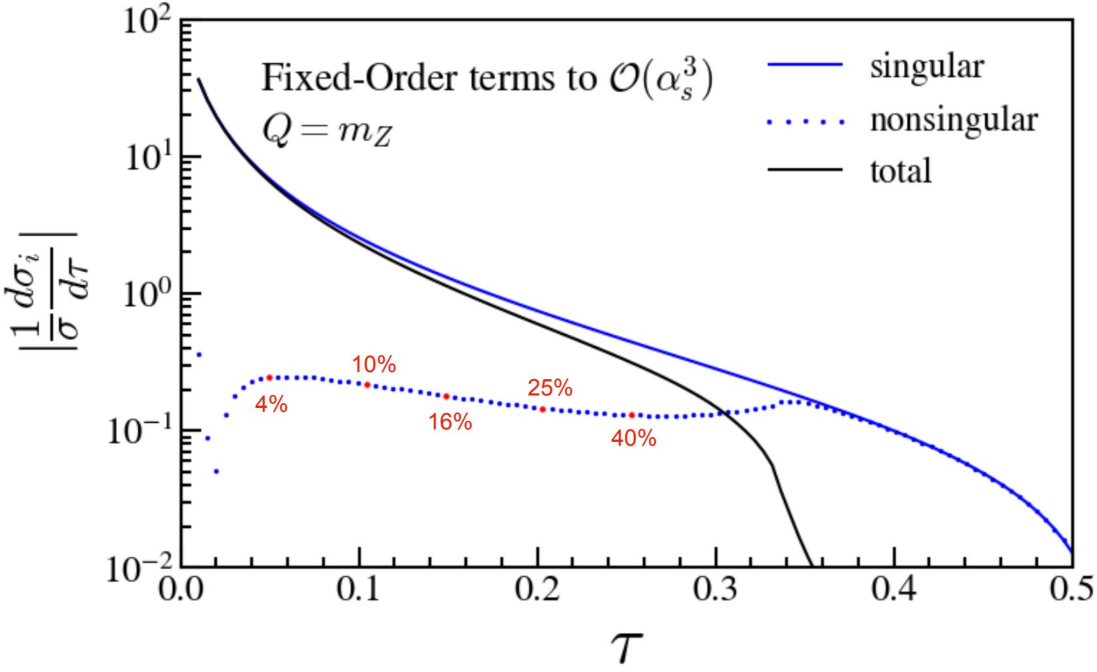


Questions on 2010 analysis

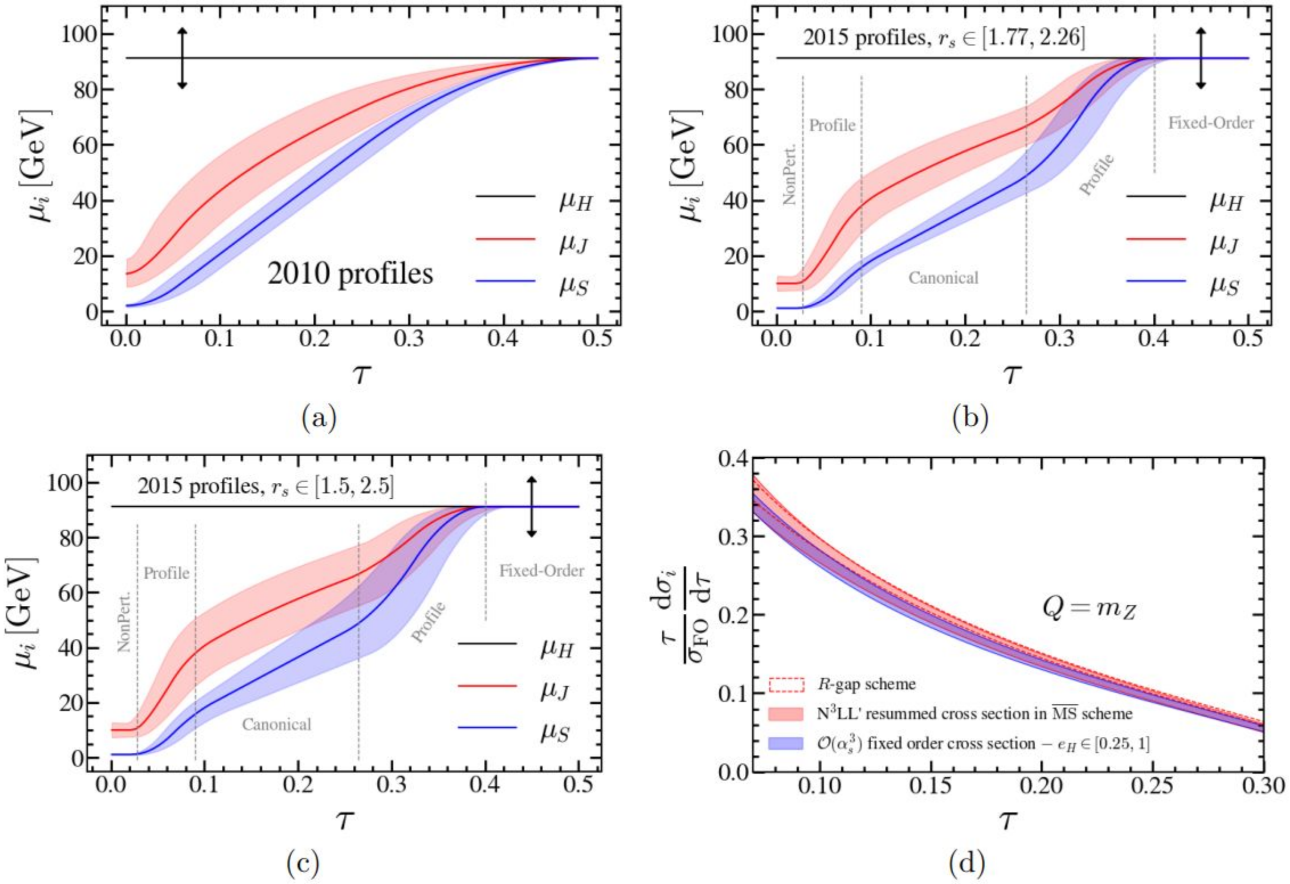
- Fit range up to 0.33. Is dijet formulation still valid?
- 3 jet power corrections
- Stability over modifications of fit range
- Estimate of Perturbative Uncertainties
- Hadron mass effects

2024 analysis

- Reduced fit range to 0.15.
  - Still plenty of data.
  - Lower energy data less important.
  - Clearly dijet dominated

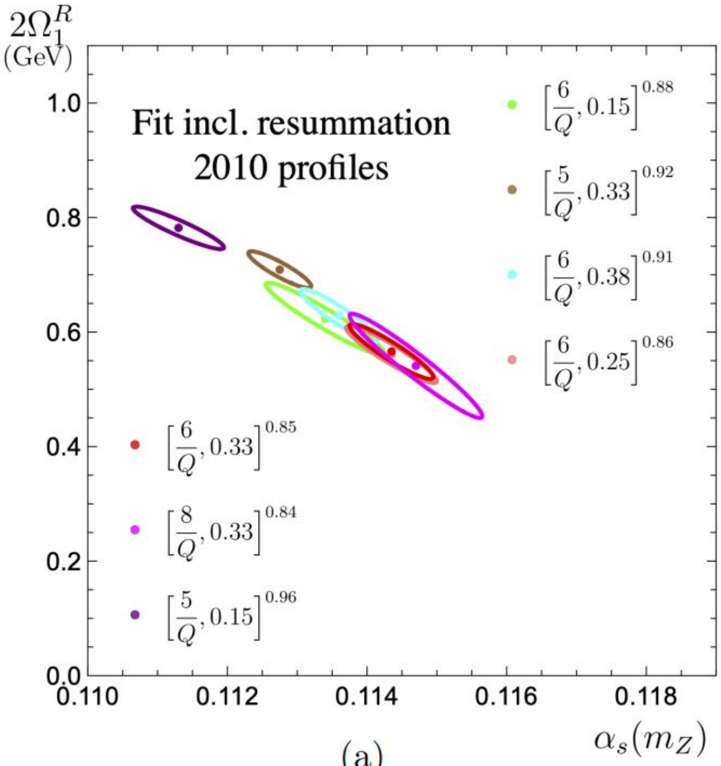


2024 analysis - Employed 2015 profiles

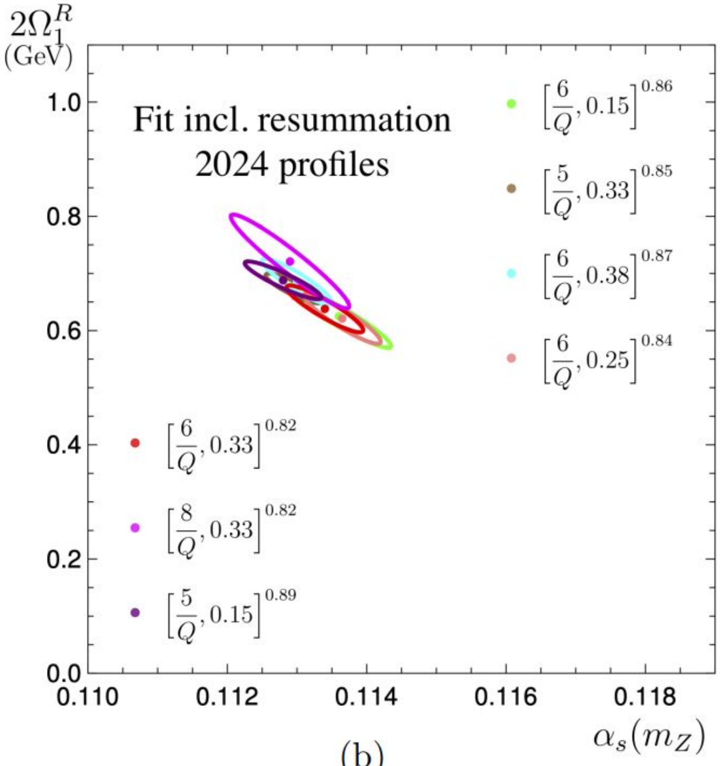


2024 analysis - Stability

- Stability over fit range improved with new profiles.



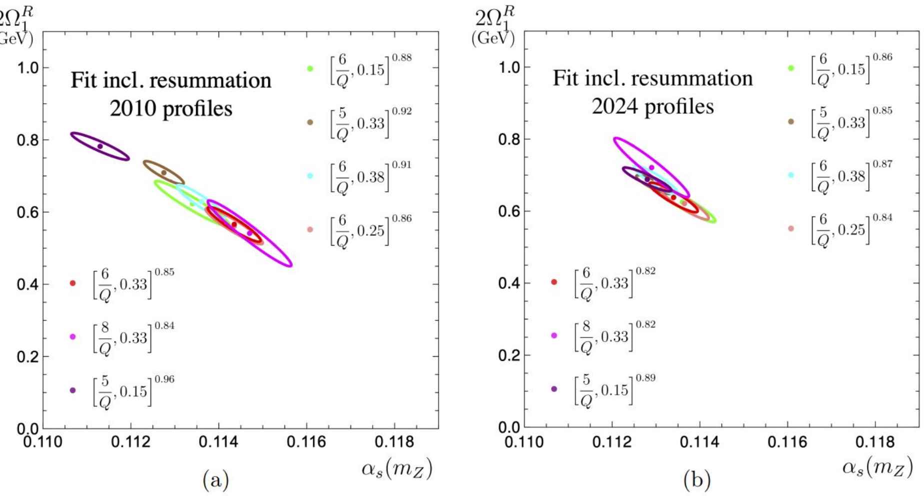
(a)

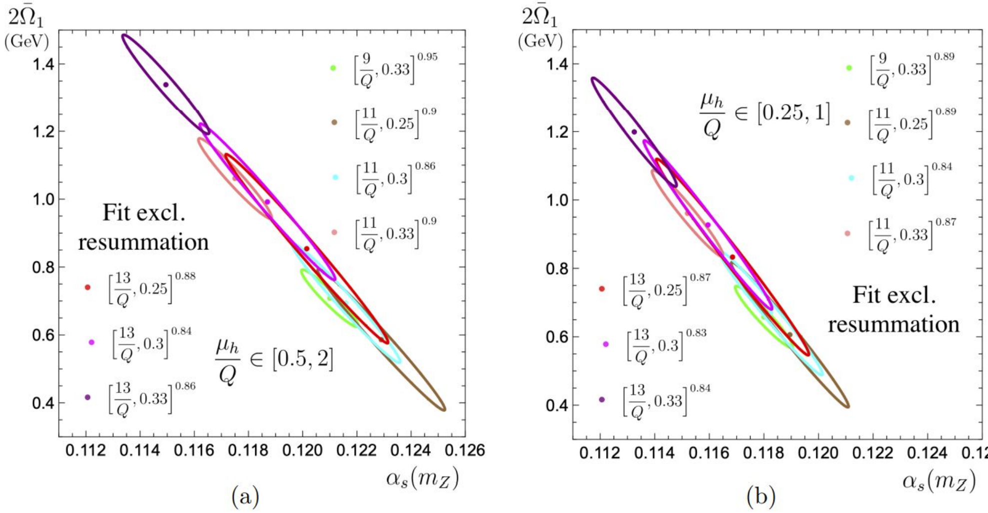


(b)

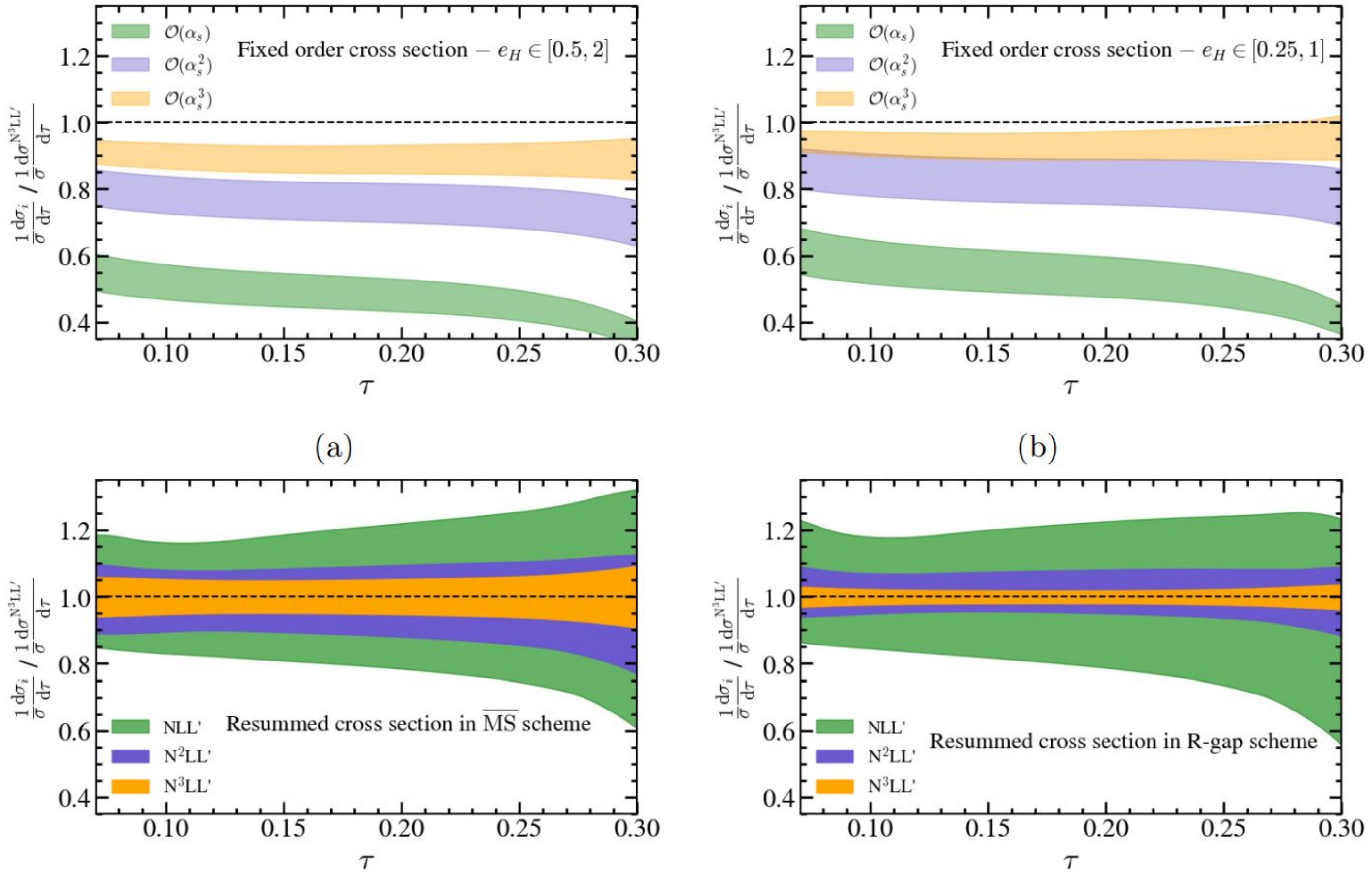
2024 analysis - Stability

- Stability over fit range much better with resummation then with fixed order only

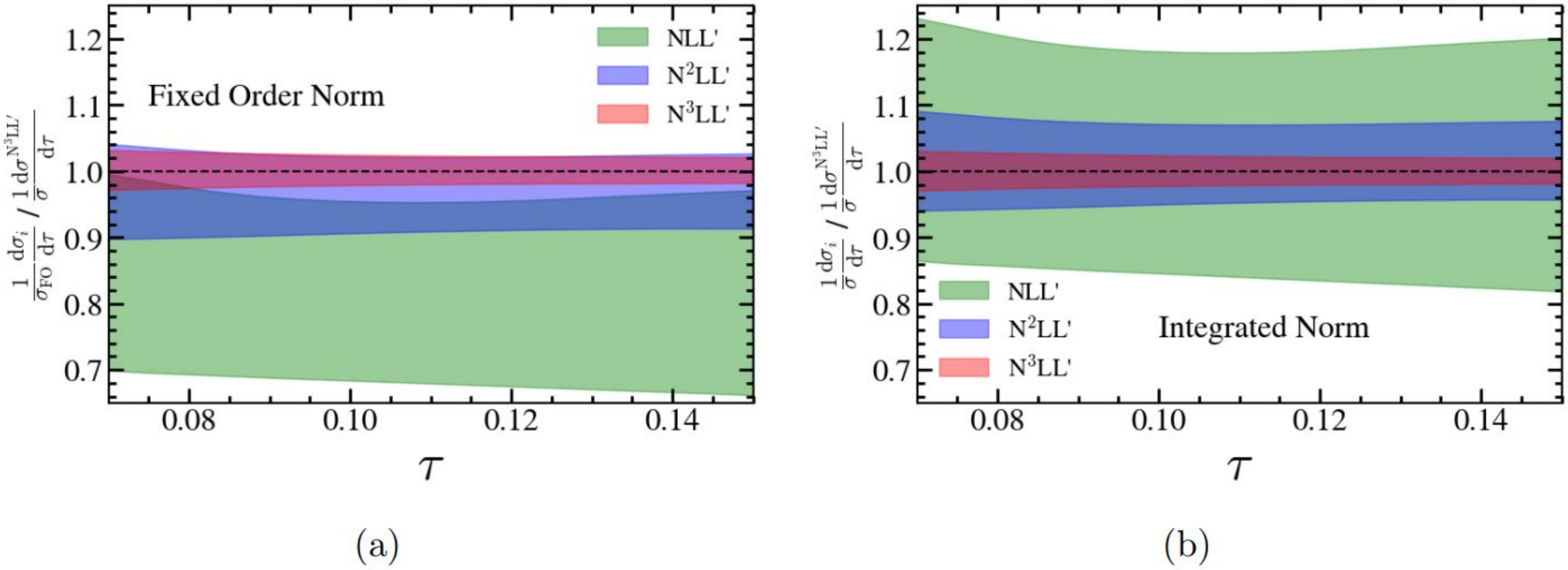




2024 analysis - Perturbative Uncertainty



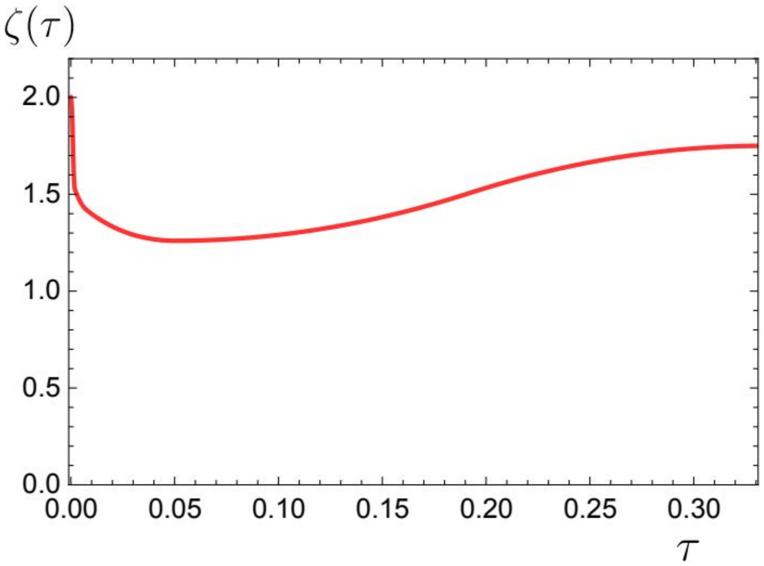
2024 analysis - Perturbative Uncertainty



2024 analysis - 3 jet power corrections

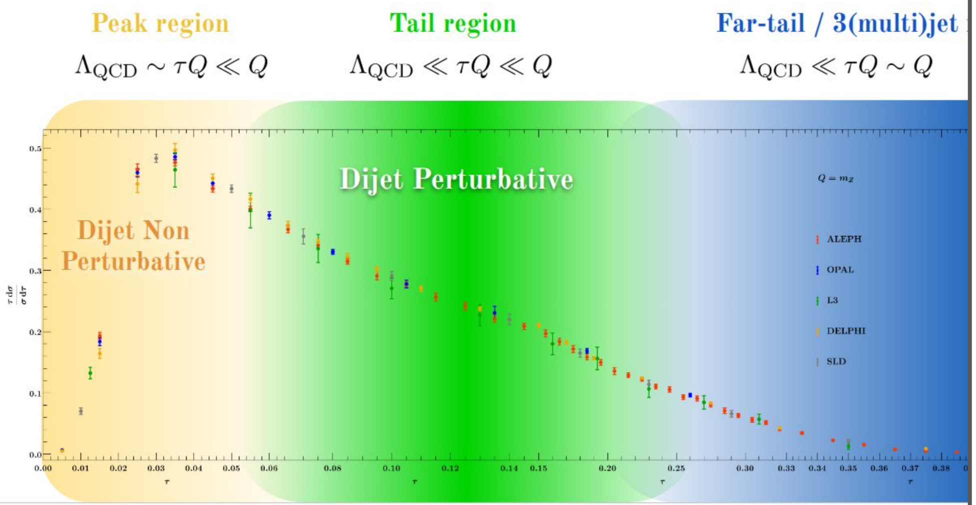
- What if non perturbative corrections have non trivial dependence on thrust value?

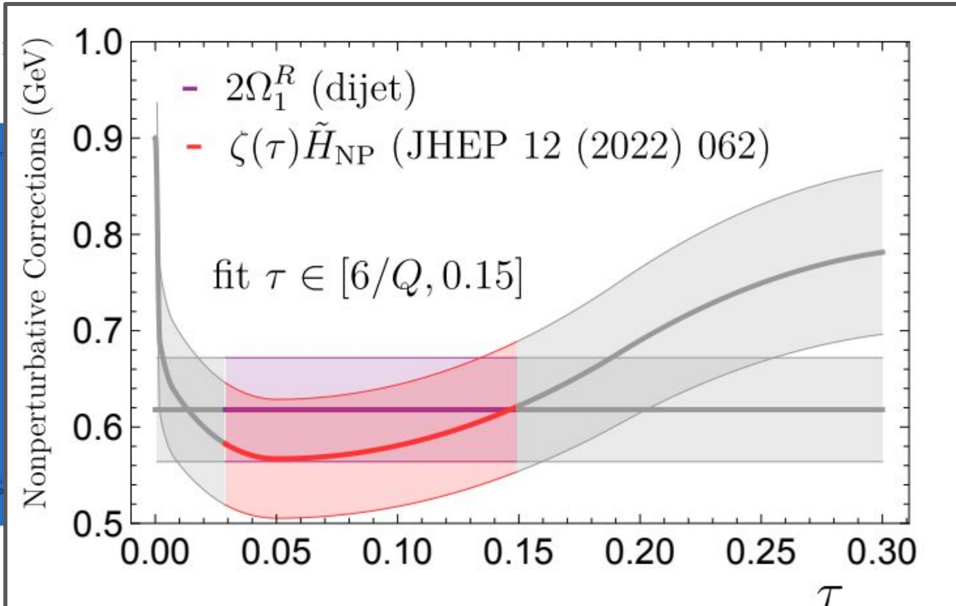
$$\bar{\Omega}_1 = \frac{1}{N_c}\langle 0|\mathrm{tr}\, \overline{Y}^T_{\bar{n}}(0) Y_n(0) \hat{\mathcal{E}}_T(0) Y^\dagger_n(0) \overline{Y}^*_{\bar{n}}(0)|0\rangle\,.$$



2024 analysis - 3 jet power corrections

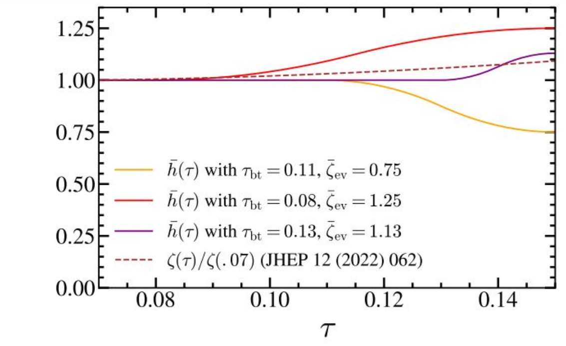
- Normalization determined by fit, not fixed. Unless dramatic shape effects, very little impact on fit.
- Value at 0 is unphysical in QCD due to $\Lambda_{\rm QCD}$. At some point $\Lambda_{\rm QCD} \sim \tau Q$ so any assumption that NP scale is smallest breaks down.



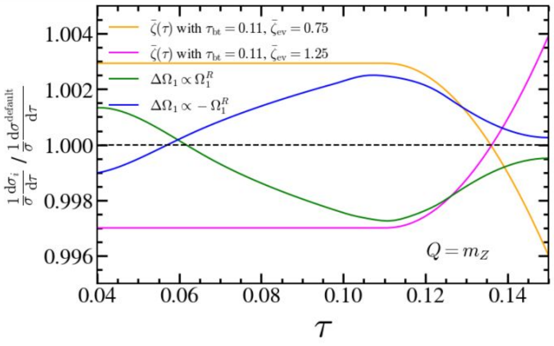


2024 analysis - 3 jet power corrections estimate

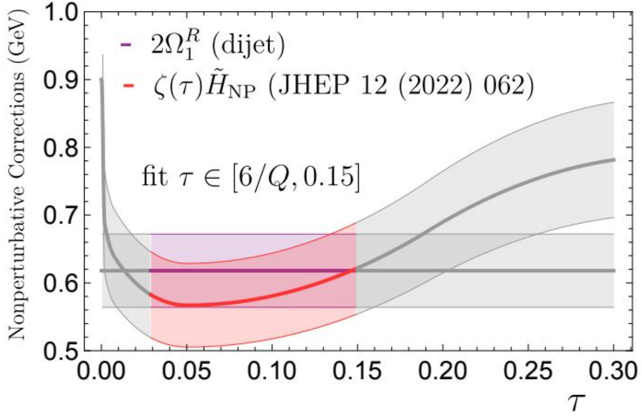
- We included even larger variations. Found negligible impact on extraction, as expected

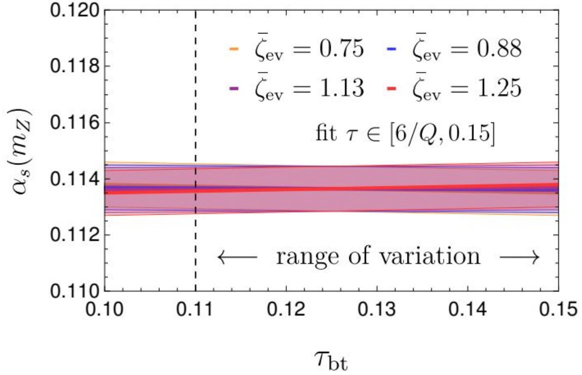


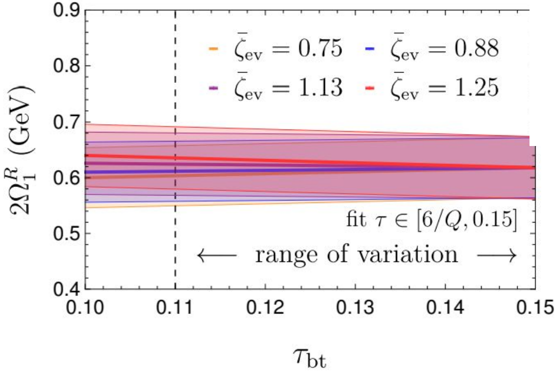
(a)



(b)

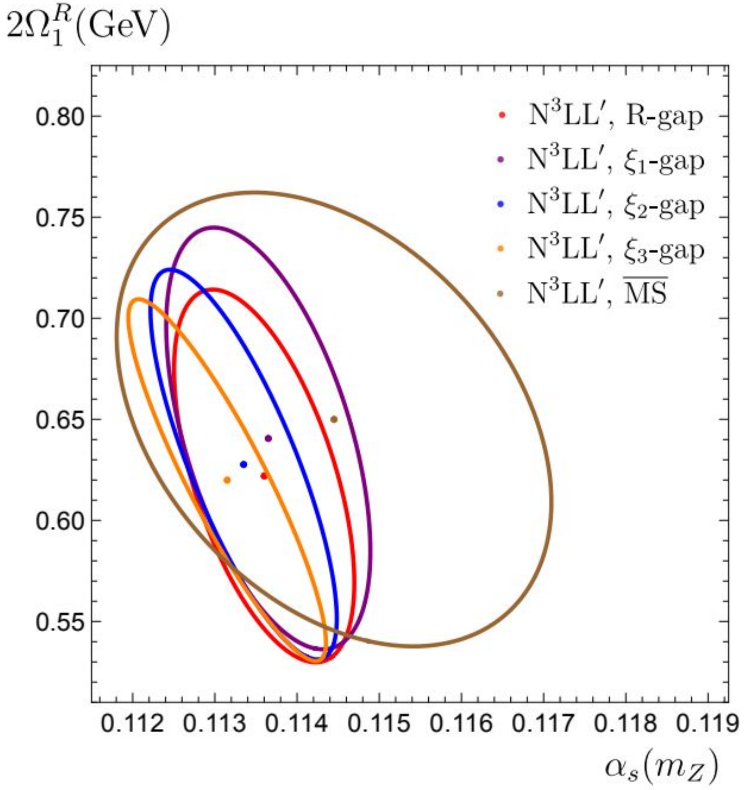


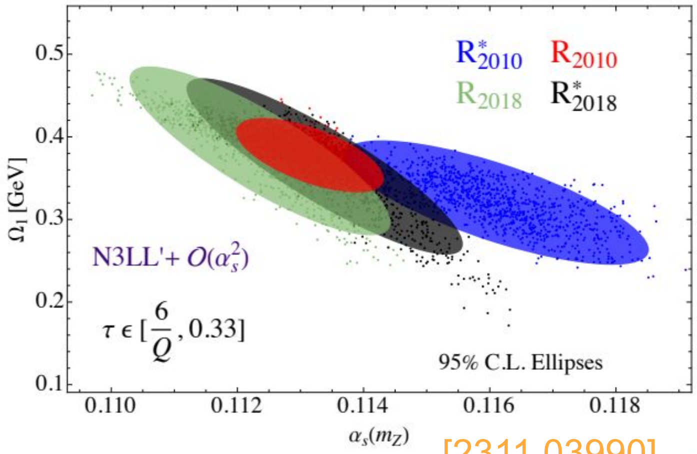




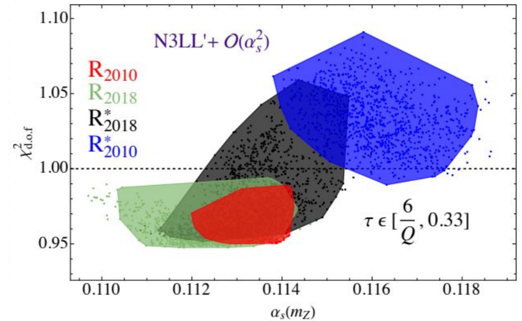
2024 analysis - Gap Scheme dependence

- We find very small gap scheme dependence at this order





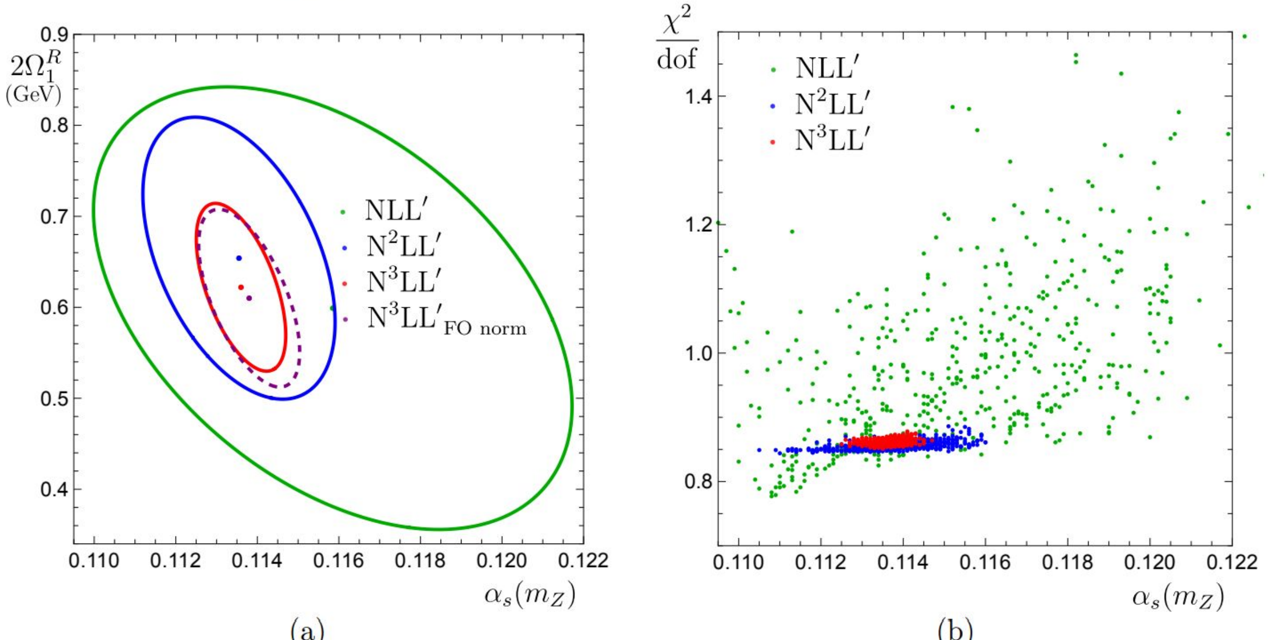
[2311.03990]



2024 - Results

$$\alpha_s(m_Z) = 0.1136 \pm 0.0012_{\text{tot}}\,,$$
$$\Omega_1^R = 0.311 \pm 0.049_{\text{tot}}\ \text{GeV}\,,$$
$$\chi^2/\text{dof} = 0.86\,.$$

QCD only
No QED nor
bottom mass



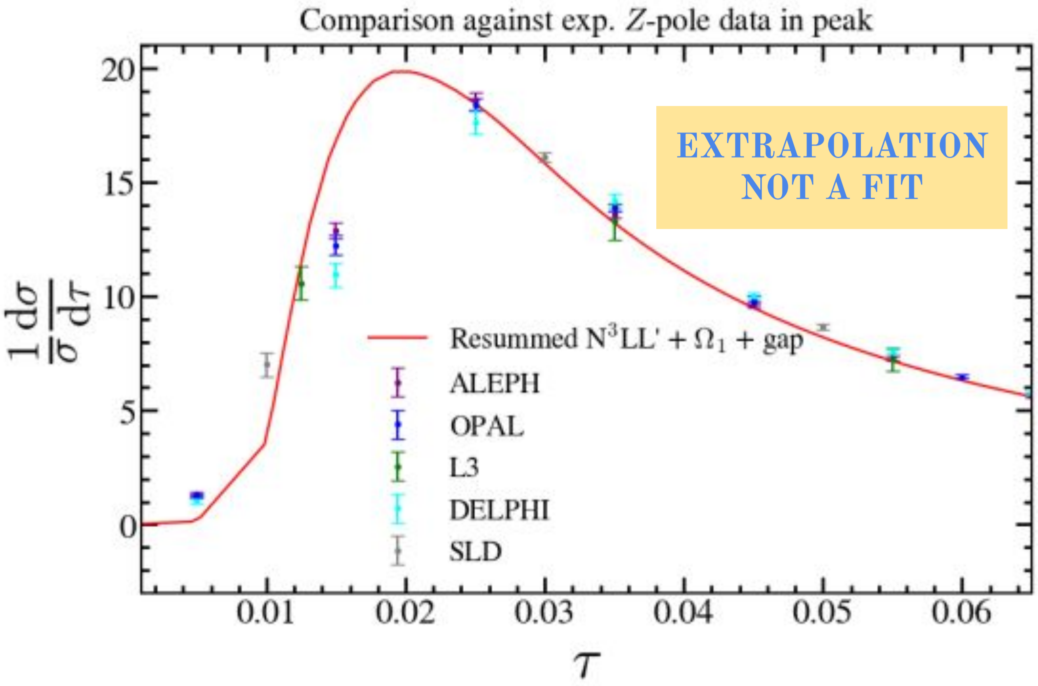
2024 - Uncertainties Break Down

	$\delta\alpha_s(m_Z)$	$\delta\Omega_1^R$	Included in [50]
Experiment	0.0003	0.010	✓
Ω_1/α_s	0.0007	0.026	✓
Total Experiment + $\boldsymbol{\Omega_1/\alpha_s}$	**0.0008**	**0.028**	✓
Ω_2 hadronization	0.0002	0.013	✓
3jet hadronization	0.0002	0.010	
Subleading power dijet	0.0002	0.004	
Total subleading hadronization	**0.0003**	**0.017**	
Perturbative	**0.0008**	**0.037**	✓
Total	**0.0012**	**0.049**	

2024 - Agreement outside of fit range

Peak region

$$\Lambda_{\rm QCD} \sim \tau Q \ll Q$$

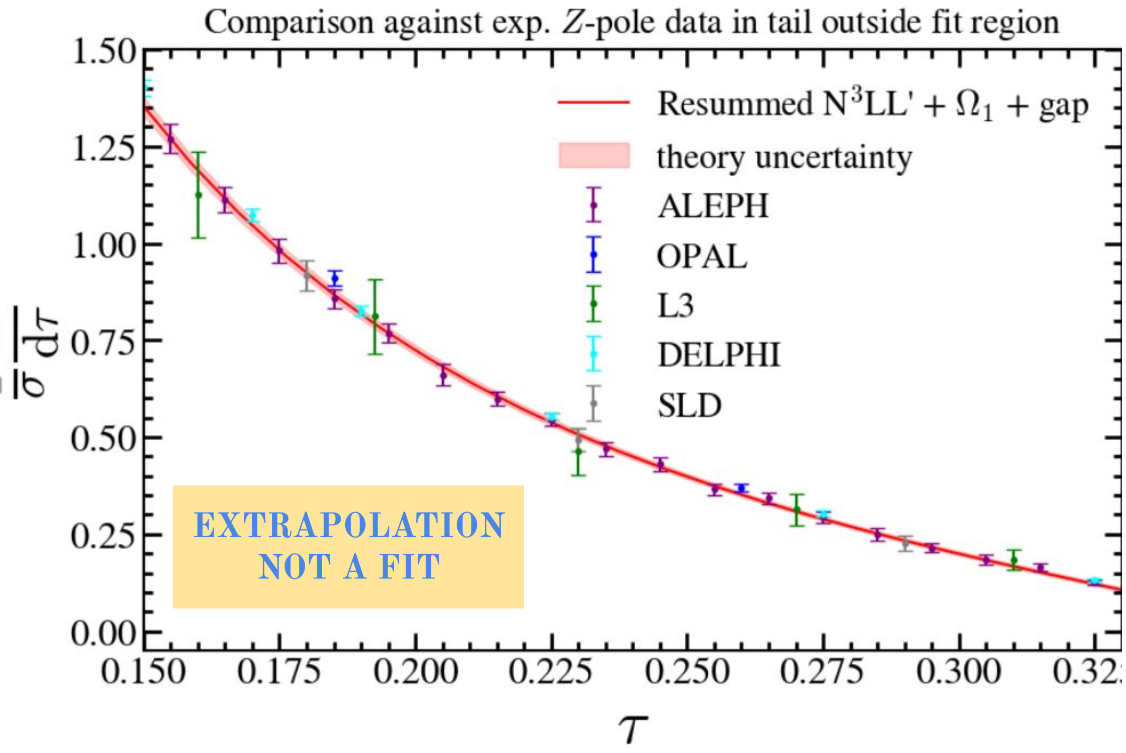


- Here the prediction is missing all the higher order moments of the shape function.

- Description not terrible, but very dependent on form of shape function

2024 - Agreement outside of fit range

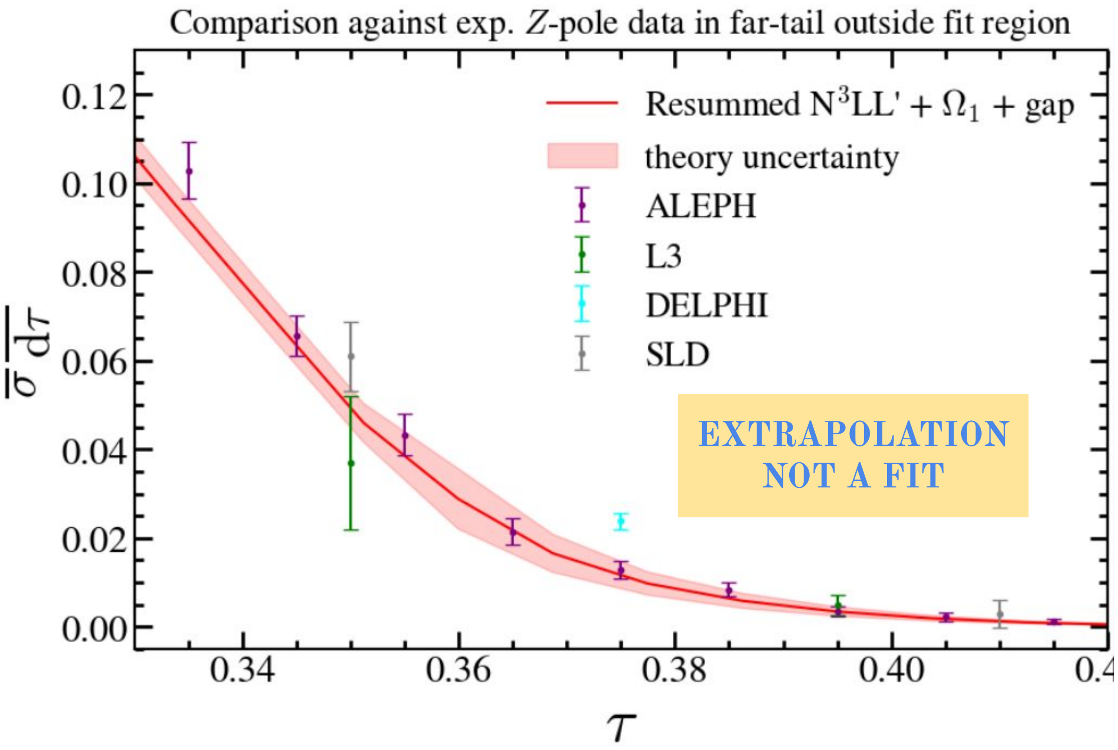
Tail region above 0.15 (2010 range) $\Lambda_{\rm QCD} \ll \tau Q \ll Q$



- Spectacular agreement.
- Here the prediction is only missing additional power corrections beyond dijet limit
- Effects of these seems small

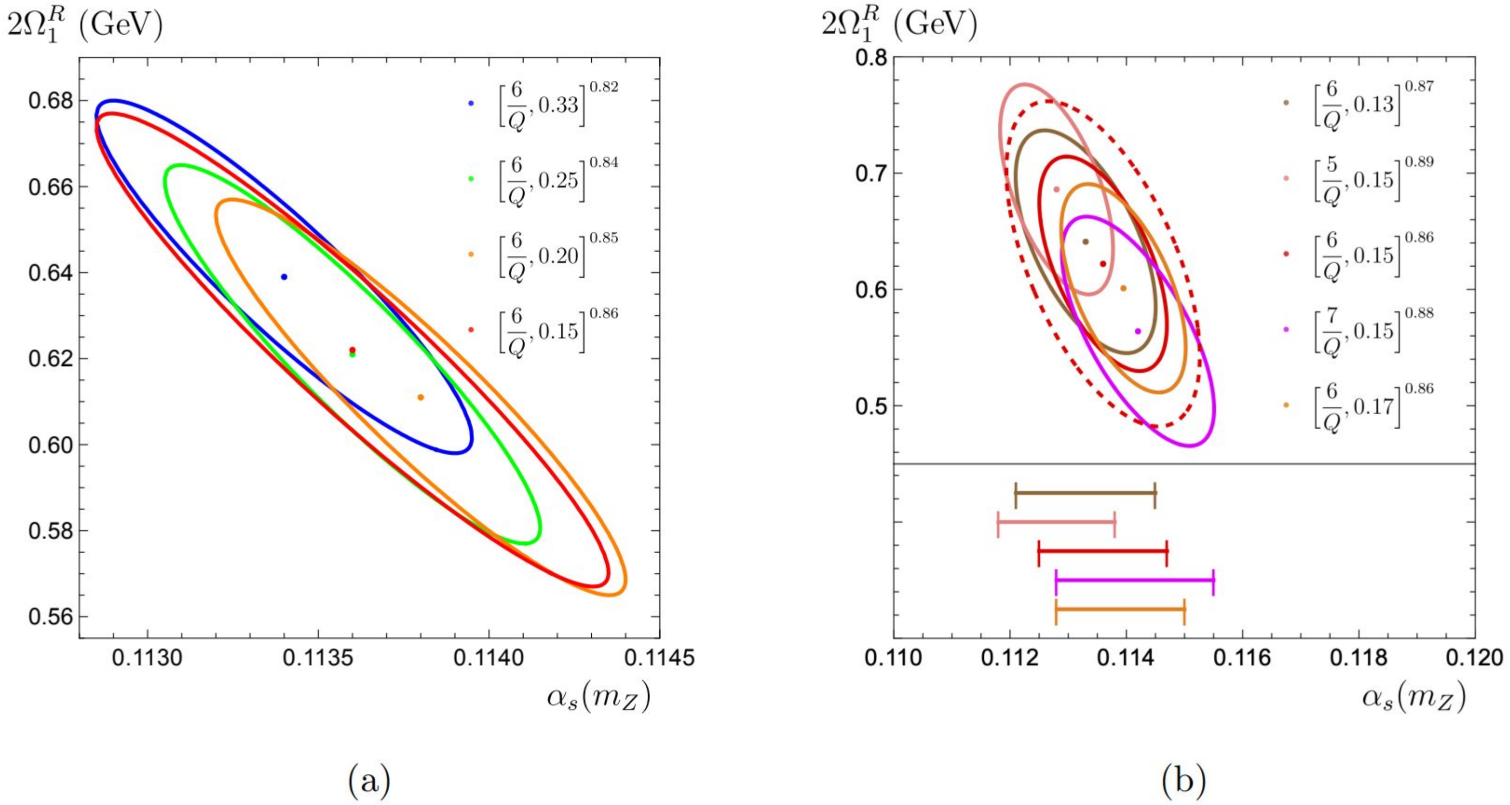
2024 - Agreement outside of fit range

Far-tail / 3(multi)jet region $\Lambda_{\mathrm{QCD}} \ll \tau Q \sim Q$



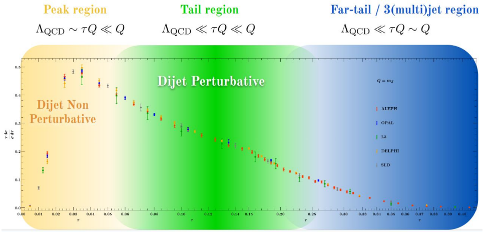
- Spectacular agreement.
- Effects of non pertubative effects beyond dijet limit again seem very small
- Missing shoulder resummation also seems within uncertainties

2024 - Fits over different ranges



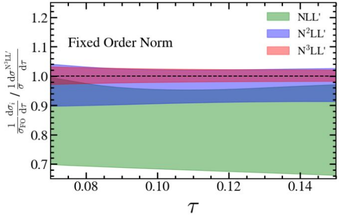
Conclusions

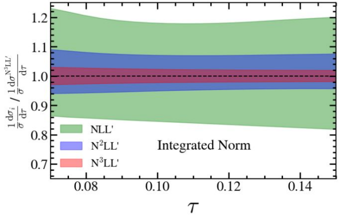
- ➢ Discussed reanalysis of thrust data
- ➢ Described EFT treatment of regions and perturbative ingredients
- ➢ Revisited estimate of Perturbative and NP Uncertainties
- ➢ Presented fit results, breakdown of uncertainties, and extrapolation outside fit range

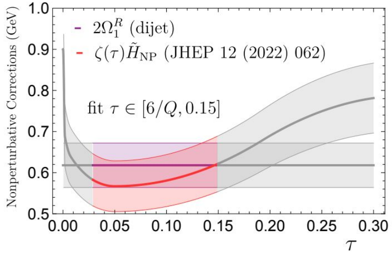


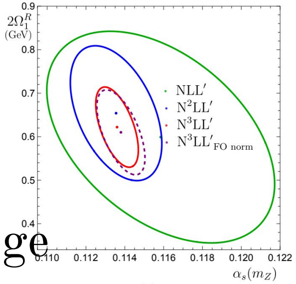
$$S_\tau(k,\mu) = \hat{S}_\tau(k,\mu) - \frac{\mathrm{d}\hat{S}_\tau(k,\mu)}{\mathrm{d}k} 2\bar{\Omega}_1 + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{k^3}\right)$$

Full Soft Function — Perturbative Soft Function (zero-th order of the OPE) — Matching coefficient (in this case trivially related to derivative of PT SF) — NP Operator Matrix Element (first order of the OPE) — Higher orders in the soft function OPE expansion

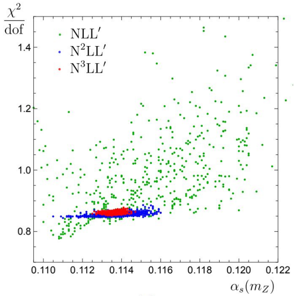




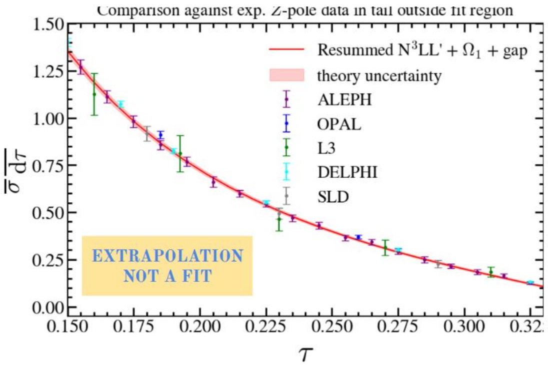




(a)

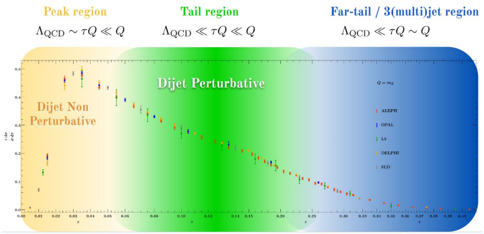


(b)



Outlook

- ➢ Extend analysis to N4LL
- ➢ Look for updates on experimental reanalysis of LEP data
- ➢ Extend shape function formalism to peak fits
- ➢ Many interesting opportunities in improving our understanding of NP effects if FCC can provide data at various energies



$$S_\tau(k,\mu) = \hat{S}_\tau(k,\mu) - \frac{d\hat{S}_\tau(k,\mu)}{dk} 2\bar{\Omega}_1 + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{k^3}\right)$$

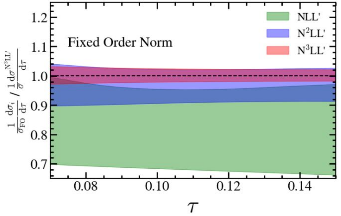
Full Soft Function

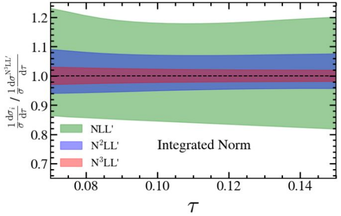
Perturbative Soft Function (zero-th order of the OPE)

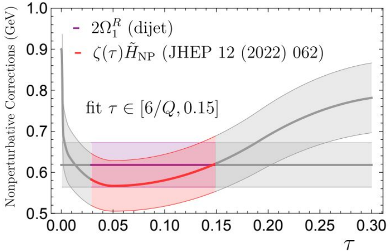
Matching coefficient (in this case trivially related to derivative of PT SF)

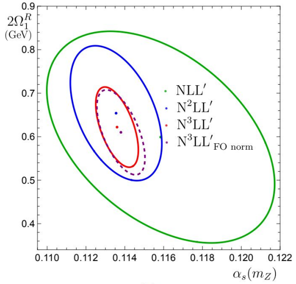
NP Operator Matrix Element (first order of the OPE)

Higher orders in the soft function OPE expansion

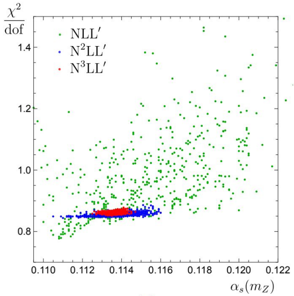








(a)



(b)