Track parameters in full and fast simulation at CLD

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1 Introduction

Charged particles and especially their displaced tracks are crucial for jet tagging. This motivates us to study tracks at CLD, comparing their parameters in fast and full simulation.

2 Parametrization of a track

When charged particles move in a magnetic field (here B_z), they follow a trajectory of a helix. The measurement of the trajectory is called a track. A helix is fully described by five parameters, however, the conventions differ. We choose the following convention:

1. d_0: transverse impact parameter; describes the smallest distance of the helix to the primary vertex in the xy-plane.
2. ϕ: azimuthal angle of the momentum vector on the helix at point of closest approach.
3. ω: curvature; $\omega = \frac{q \cdot B_z \cdot c}{\sin\theta \cdot |\vec{p}|} \propto \frac{1}{p_t}$; although named like a frequency, ω refers to the curvature in units of 1/mm.
4. z_0: longitudinal impact parameter; defined as the distance from the point of closest approach of the track to the primary vertex along z.
5. $\tan\lambda$: tangent of the dip angle λ of the helix in the r-z-plane; in other words, the angle between $\vec{p}$ and p_t, so that $\tan\lambda = \frac{p_z}{p_t}$.

A graphical illustration of a helix and some parameters can be seen in Figure 1.

As the measurement of a track holds uncertainties, we are also interested in the covariance matrix. The covariance matrix of a five-dimensional vector results in 15 values: 5 diagonal values which correspond to the variance and are by definition greater zero and 10 off-diagonal values as a real covariance matrix is symmetric.

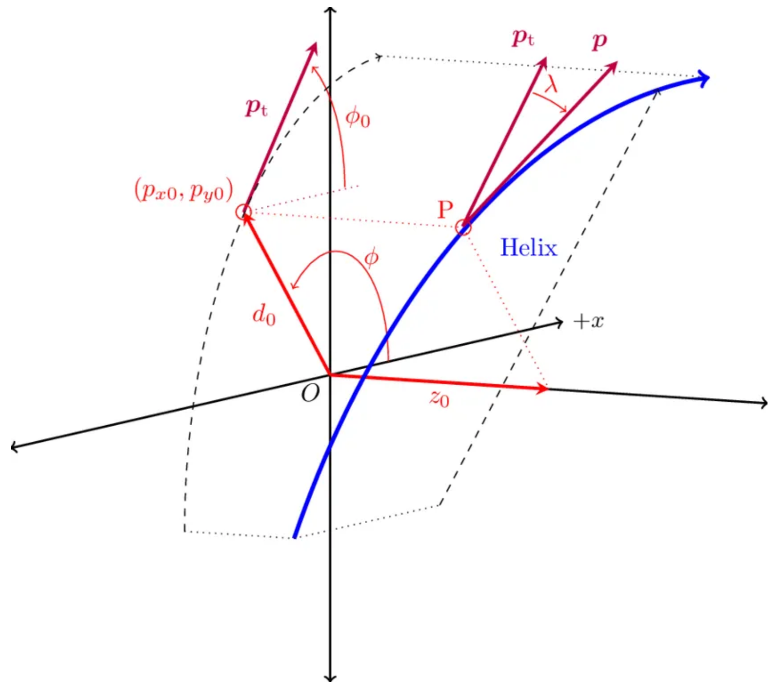

Figure 1: Graphical illustration of helix (blue) and some descriptive parameters (z_0, d_0, λ, p_t, ϕ). Note: ϕ in the text is labeled as ϕ_0 in this image.

3 Fast vs. full simulation parametrization

We use all track parameters for tagging including the covariance matrix C_{ij} except for the curvature c and $\tan\lambda$. We also use other variables that are strongly related to them such as the signed impact parameters in 2D and 3D and their significances and the jet-track distance in 3D and its significance [1]. In fast and full simulation, we can retrieve the basic track parameters straight forward. But we need to shift the coordinate system, so we describe the helix with respect to the primary vertex. Only then, impact parameters become meaningful.

Full simulation EDM4HEP use the same convention for the track parametrization as we do, see the definition of the `edm4hep::TrackState`. Be careful that ω although named like a frequency refers to the curvature in units of 1/mm.

Fast simulation DELPHES also uses the same definition of the 5 track parameters. It uses the half-curvature at some point which needs to be accounted for when switching frameworks. The covariance matrix also follows the convention.

4 Comparison of the track parameters in fast vs. full simulation

We compare the track parameters for $H \to u\bar{u}$, the five helix parameters in Figure 2 and the 15 covariance parameters in Figure 4.

4.1 The five helix parameters

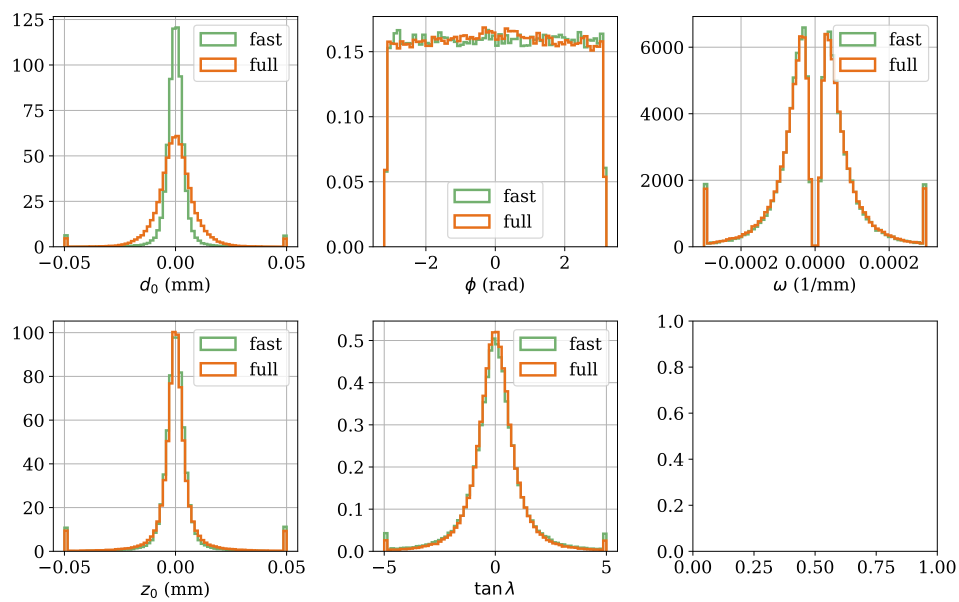

Figure 2: Five helix parameters (d_0, ϕ, ω, z_0, $\tan\lambda$ for leading tracks in fast (green) and full (orange) simulation for 25000 jets.

We retrieve the parameters d_0, ϕ, ω and z_0 straight forward from `TrackState` and calculate $\tan\lambda = \frac{1}{\tan\theta}$.

All parameters show accordance but the traverse impact parameter d_0 that is more spread out in full simulation. We expect the spread to be around 2-5 μm [2] which is the case for both impact parameters d_0 and z_0, only d_0 in full simulation has a larger spread of $\sigma = 5.88 \pm 0.07$ μm, see Figure 3.

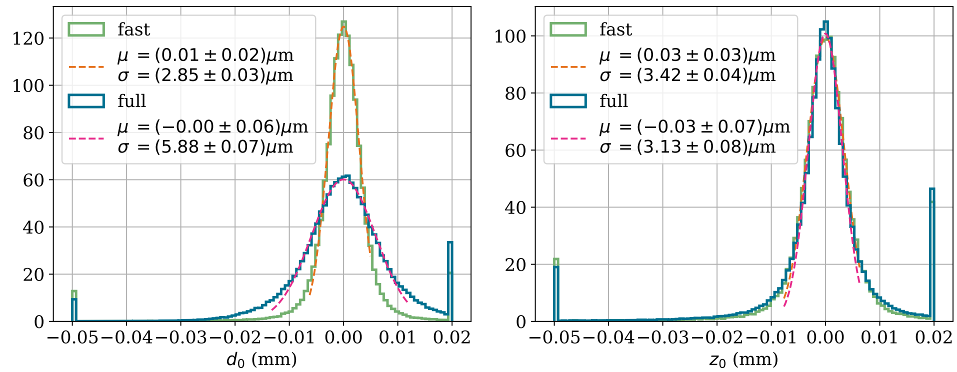

Figure 3: Impact parameters d_0 (left) and z_0 (right) in fast (full) simulation in green (blue) with Gaussian fits in orange (pink).

To investigate where the deviation in the transverse IP d_0 comes from, we plot the five parameters against each other in 2D histograms, see Figure 5 for fast and Figure 6 for full simulation. We see a strong derivation in the correlation of d_0 to ϕ when comparing fast and full simulation. While d_0 has the same resolution over ϕ in fast simulation, the distribution smears out around $\pm\frac{\pi}{2}$ in full simulation. We do not know the reason for this but guess it might come from detector geometry.

4.2 The covariance matrix of the helix parameter

The covariance matrix is also straight forward retrieved with `track.covMatrix`, so the convention used in the setup is crucial. In Figure 4 show the diagonal elements in the first column, the off diagonal elements in the second and third. We see many deviations in the off-diagonal elements. While cov(ω, d_0) and cov(ϕ, ω) (first two elements of the third column) differ in their values as fast simulation is a lot more spread out, the most worrying part is the inconsistent sign convention. 6 out of the 10 off-diagonal elements show inconsistency in the range when comparing full and fast simulation. In these 6 cases (cov(ϕ, $\tan\lambda$), cov(ϕ, z_0), cov(d_0, d_0), cov(ω, z_0), cov(d_0, $\tan\lambda$), cov(ω, $\tan\lambda$) that are the last three elements in the second and third column), fast simulation has only positive (or negative) values while the distribution of the full simulation values smears around zero. Most surprising is cov(d_0, z_0) that is only positive in fast simulation while d_0 and z_0 show distributions centered around zero (so also negative).

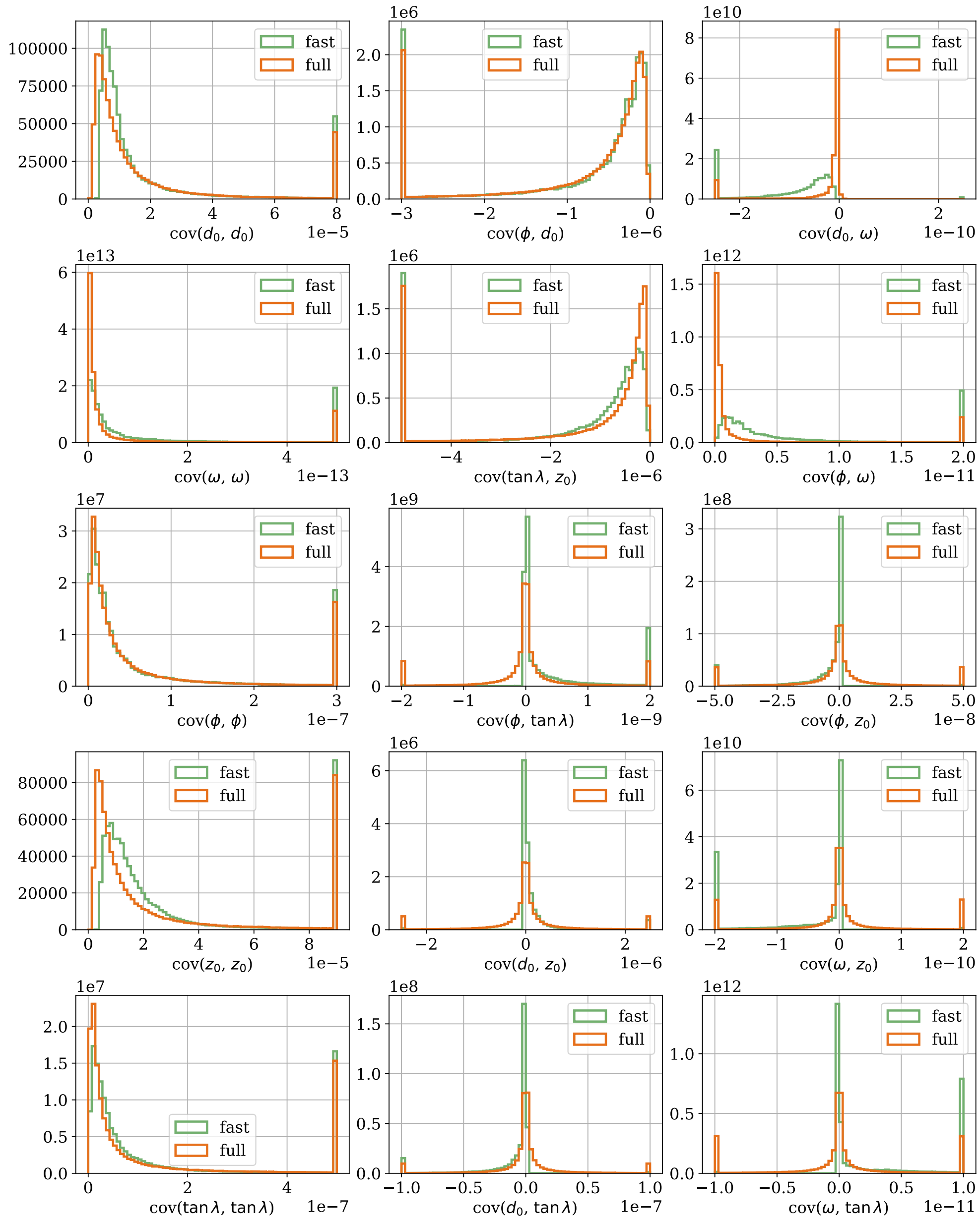

Figure 4: 15 covariance parameters for leading tracks in fast (green) and full (orange) simulation for 25000 jets.

5 Open questions

1. Why does d_0 has a larger spread in full simulation than expected (6 μm instead of 3 μm)?

2. Why does the covariance matrix differ comparing fast vs. full simulation although the definitions of the track parameters are the same? Why does the distribution of the off-diagonal elements of the covariance matrix center around zero in full simulation while it is only positive/negative in some cases in fast simulation?

6 Notes

Some notes and comments while doing this study:

1. Checking the units: Why has $\omega = \frac{q \cdot B_z \cdot c}{p_t}$ the unit of $\frac{1}{\mathrm{mm}}$? When calculating ω the speed of light c is defined as $c = 2.99792458 \cdot 10^8 \cdot 10^{-9} \cdot 10^{-3}$. The 10^{-3} accounts for converting from $\frac{1}{\mathrm{m}}$ to $\frac{1}{\mathrm{mm}}$ while the 10^{-9} account for the momentum be defined in GeV. The momentum is NOT defined in GeV/c, so that we turn out with $\frac{1}{\mathrm{m}}$ not $\frac{1}{\mathrm{s}}$. So we calculated the units of ω as $[\frac{q \cdot B_z \cdot c}{p_t}] = \frac{\mathrm{As} \cdot \frac{\mathrm{kg}}{\mathrm{As}^2} \frac{\mathrm{m}}{\mathrm{s}}}{10^{-9} \cdot \frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^2}} \cdot 10^{-9} \cdot 10^{-3} = \frac{1}{\mathrm{mm}}$

2. You can find the code for these plots on GitHub.

3. You can find a GitHub issue discussing the d_0 discrepancy here.

Appendix

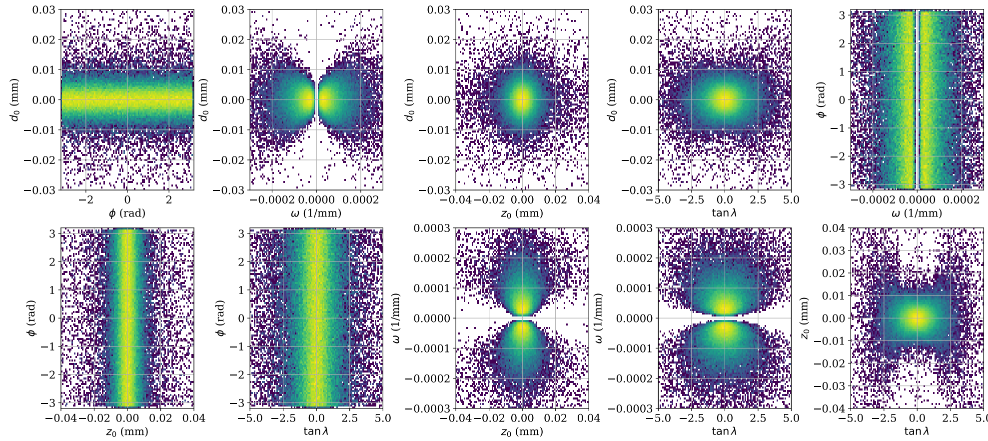

Figure 5: Logarithmic 2D plots of fast simulation track parameters of 250 000 $H \to u\bar{u}$ jets

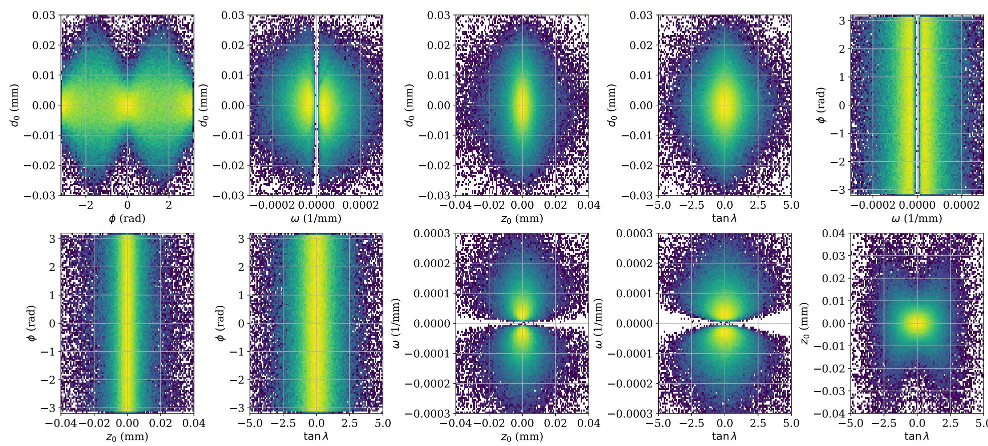

Figure 6: Logarithmic 2D plots of full simulation track parameters of 250 000 $H \to u\bar{u}$ jets

References

[1] Sara Aumiller, Dolores Garcia, and Michele Selvaggi. *Jet-Flavor Tagging Performance at FCC-ee*, November 2024.

[2] Franco Bedeschi, Loukas Gouskos, and Michele Selvaggi. Jet flavour tagging for future colliders with fast simulation. *The European Physical Journal C*, 82(7), July 2022.