Brookhaven National Laboratory

U.S. DEPARTMENT OF ENERGY

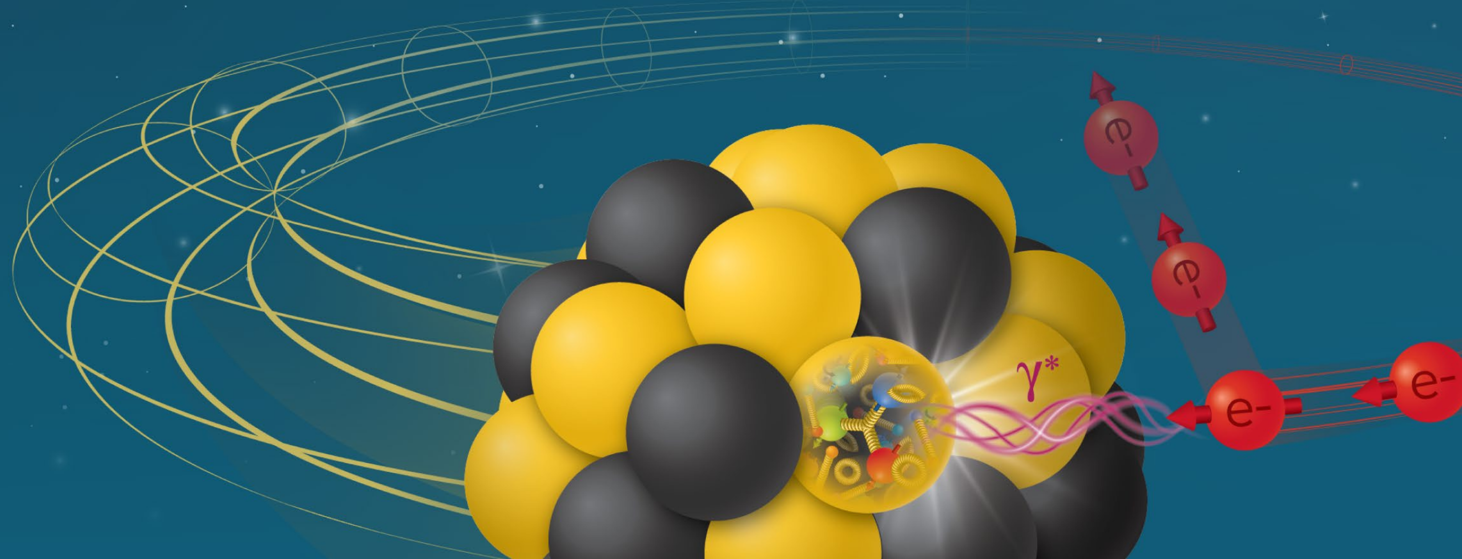
Eddy currents induced by rippling magnets in axially asymmetric vacuum chambers

Boris Podobedov

HiLumi WP2 Meeting

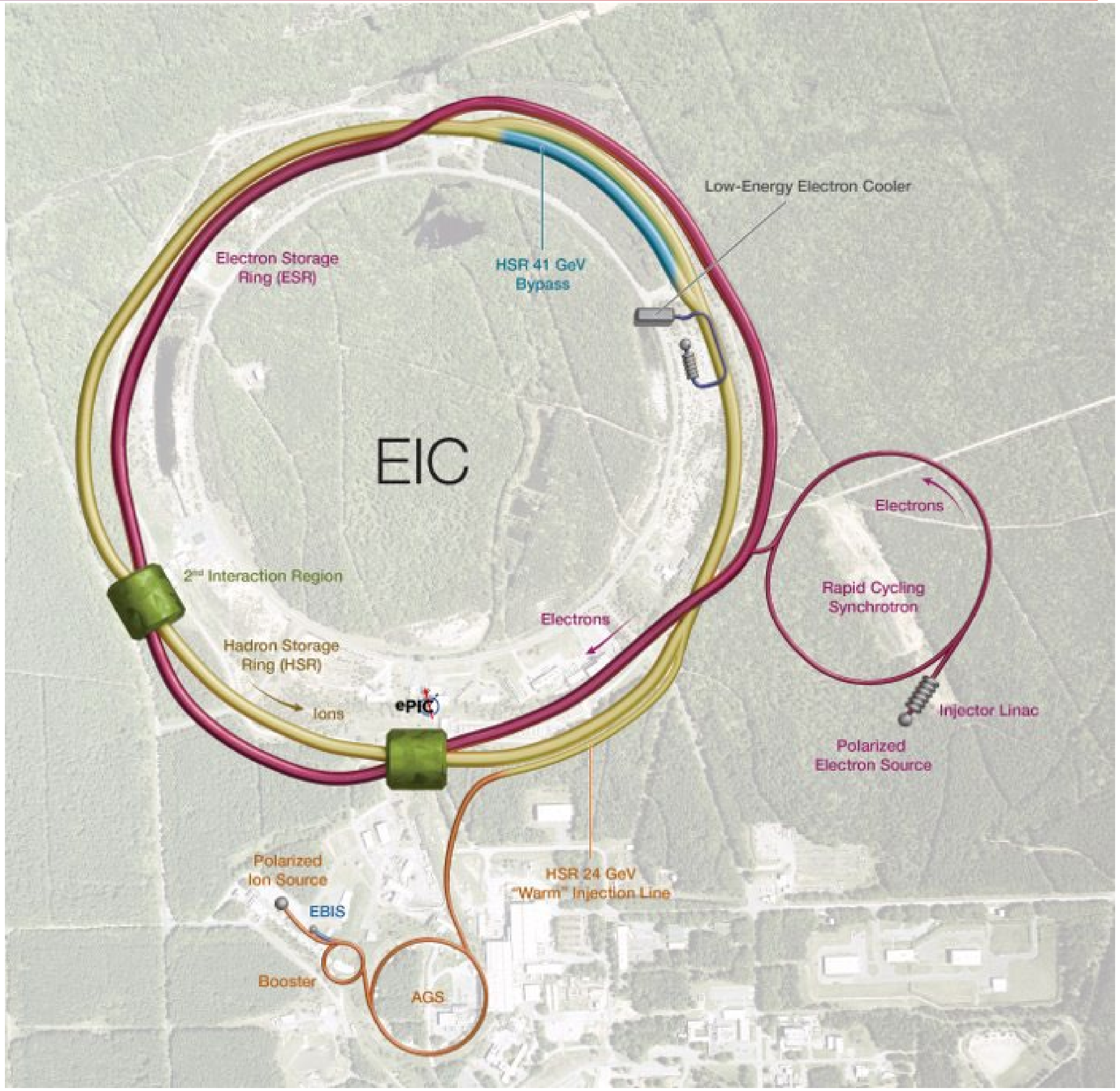
February 13, 2025

Electron-Ion Collider



Talk outline

- Motivation & introduction
- Eddy (aka Foucault) currents, skin effect, and shielding in axially-symmetric vacuum chambers
- New results for (axially asymmetric) EIC ESR chambers
- Implications for the EIC design
    - ESR main magnet power supply ripple specs
    - ~~ESR single dipole power supply powering scheme~~
    - ~~IR orbit feedback (and special fast corrector chambers)~~
    - ~~Modifications of “non-standard” ESR vacuum chambers~~



ESR - Electron Storage Ring (5 GeV – 18 GeV)
HSR - Hadron Storage Ring (40-275 GeV)
IR - Interaction Region(s)

Motivation

- Ideally, beams are matched at IP, incl. transverse offsets and beam sizes
- In reality, there will be some position and size mismatch
- Very stringent dynamic tolerances from beam-beam physics to avoid hadron emittance growth (similar for x and y):

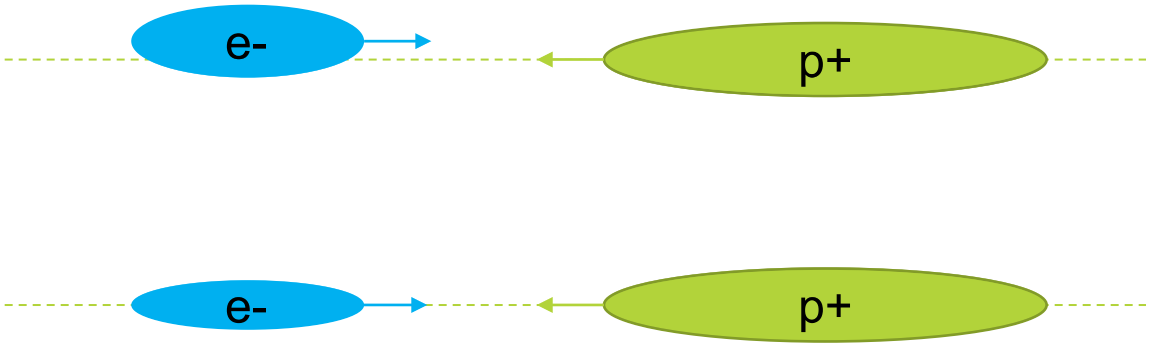
low frequency, f<~700 Hz

$\Delta x_{rms} \leq \sim 1\%\ \sigma_x$, $(\Delta\sigma_x)_{rms} \leq 0.1\%\ \sigma_x$

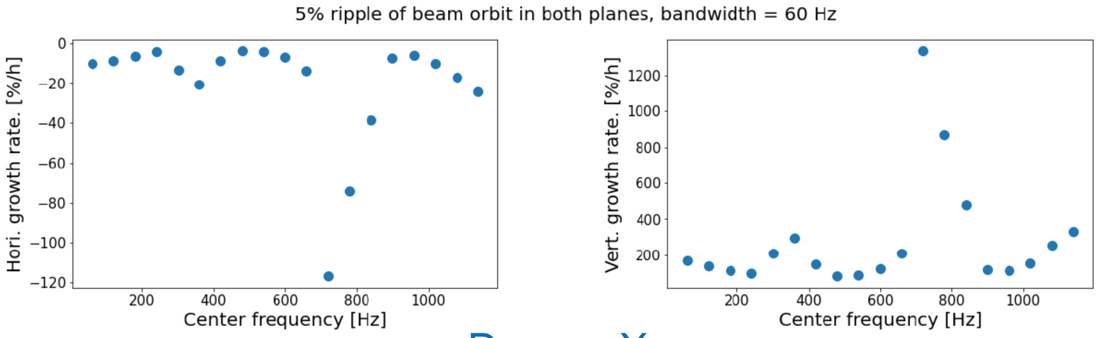
At betatron tune and harmonics, f=[8-40] kHz

$\Delta x_{rms} \leq \sim \sigma_x / 10^4$

- Sizes at IP: $\sigma_x \approx 100\ \mu\text{m}$, $\sigma_y \approx 10\ \mu\text{m}$



Proton H. and V. emit. growth rate with 5% electron orbit ripple

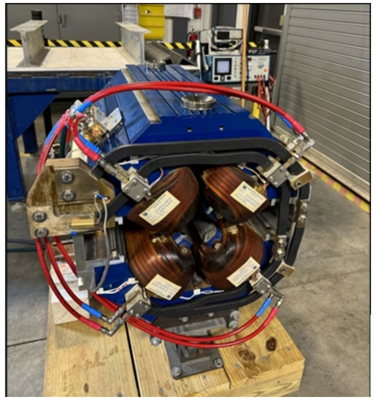


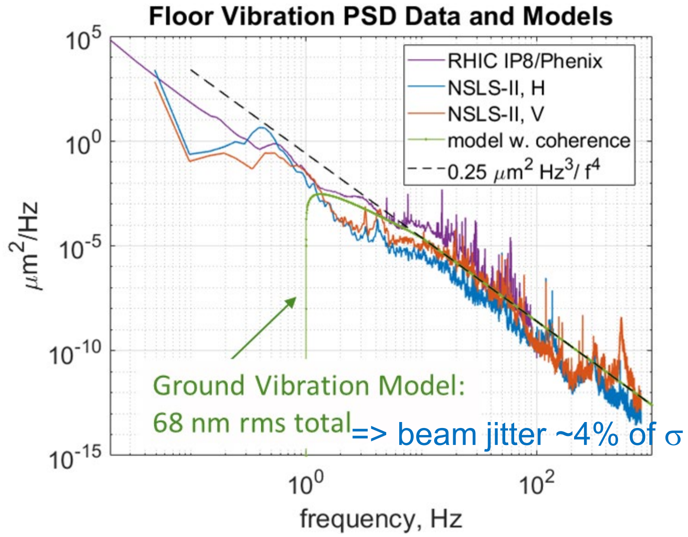
Derong Xu

Motivation cont'd

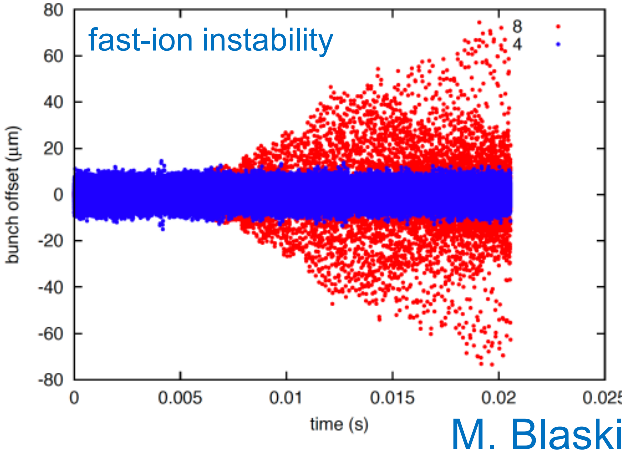
- Hadrons have no damping. They are unforgiving to very small modulations from e-beam via beam-beam kick
- Any system or beam dynamics effect causing beam motions at the IP leads to the hadron emittance growth, e.g.
  - Orbit/size ripple due to magnet PS ripple
  - Orbit ripple due to quad vibrations
  - Coupled bunch collective instabilities
  - Crab RF noise
  - Main RF phase noise (+ ideal crab cavity)
  - …

DC setpoint
+ AC ripple





=> beam jitter ~4% of σ

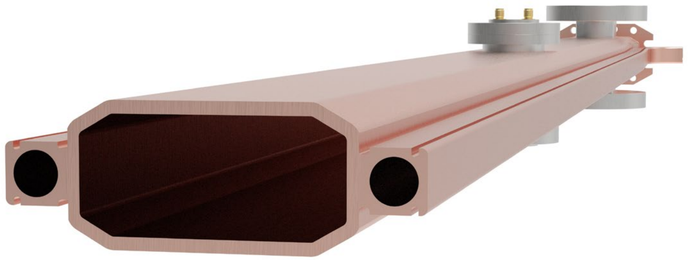


M. Blaskiewicz

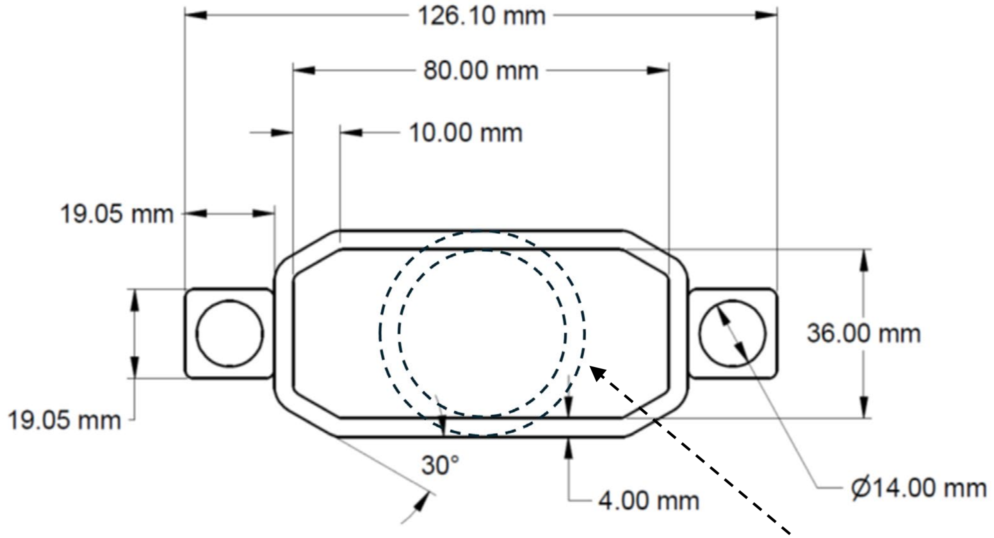
Figure 4.10: Vertical electron bunch offset at a fixed location in the ring for gas densities of

Quantifying and mitigating time-varying magnetic perturbations external to the vacuum chamber requires solid understanding of eddy current shielding

ESR vacuum chamber cross-section



ESR vacuum chamber
in main magnets
(4-mm thick Cu wall)



Axially symmetric approximation
(match height for conservative shielding estimate)

Review of results for axially symmetric chambers

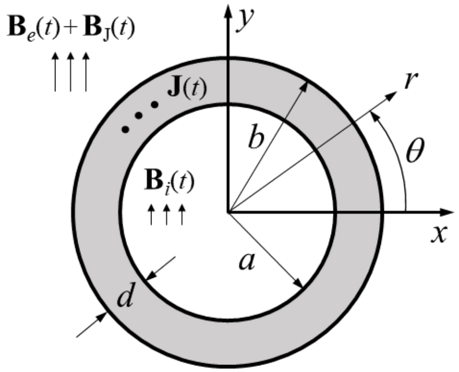
Assumptions

- Axial symmetry (around z-axis)
- Translational symmetry (along z-axis)
- Quasistatic approximation

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}.$$

- Normal skin effect
- Non-magnetic pipe (μ_r=1)



Axial symmetry => all magnetic multipoles separate: external dipole field creates only dipole inside, etc.

Simple case: electrically thin wall

- Given the external AC field B_e calculate the field B_i inside the pipe
- Result: field attenuation and phase lag due to wall eddy currents which create the induced field opposing B_e (Lenz law)
- Express via transfer function (TF), $H(j\omega)$

Skin depth >> wall thickness

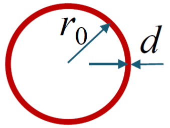
$$B_e(t) = B_0 \sin(\omega t)$$

$$B_i(t) = \frac{B_0}{\sqrt{1+\omega^2\tau^2}} \sin(\omega t - \tan^{-1}(\omega\tau))$$

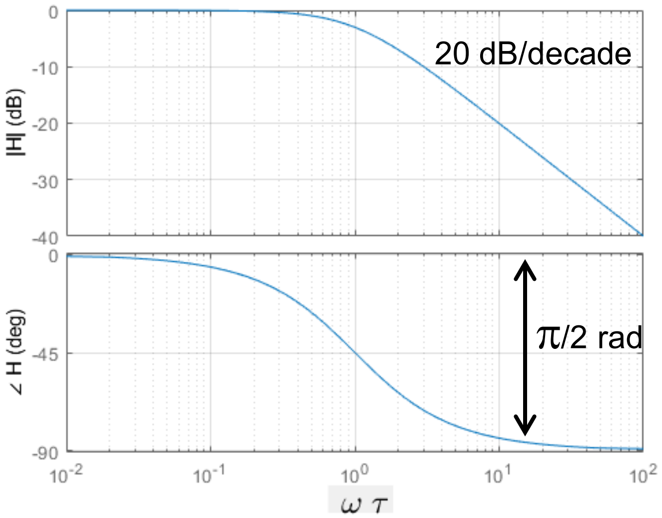
$$H_{dipole}(p) = \frac{\tilde{B}_i(p)}{\tilde{B}_e(p)} = \frac{1}{1+p\tau}, \quad p = j\omega$$

$$\tau = \frac{1}{2}\mu_0 \sigma r_0 d$$

conductivity



Thin-wall chamber solution for the dipole field.
For multipole fields, the time constant is $\tau_m = \tau / m$,
where m = 1,2,3,… stands for dipole, quad, sext, etc.

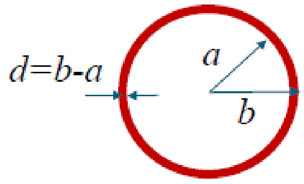


TF with a single pole at
$p_0 = -1/\tau$

Harder case: arbitrary thick wall

We use standard magnetic multipole expansion, $\boxed{A_{m,e} = r^m(a_{m,e}\sin m\theta - b_{m,e}\cos m\theta)}$

The transfer function for any multipole of order m



$$H_m(p) = \frac{\tilde{a}_{m,i}(p)}{\tilde{a}_{m,e}(p)} = \frac{\tilde{b}_{m,i}(p)}{\tilde{b}_{m,e}(p)} = \qquad (5)$$

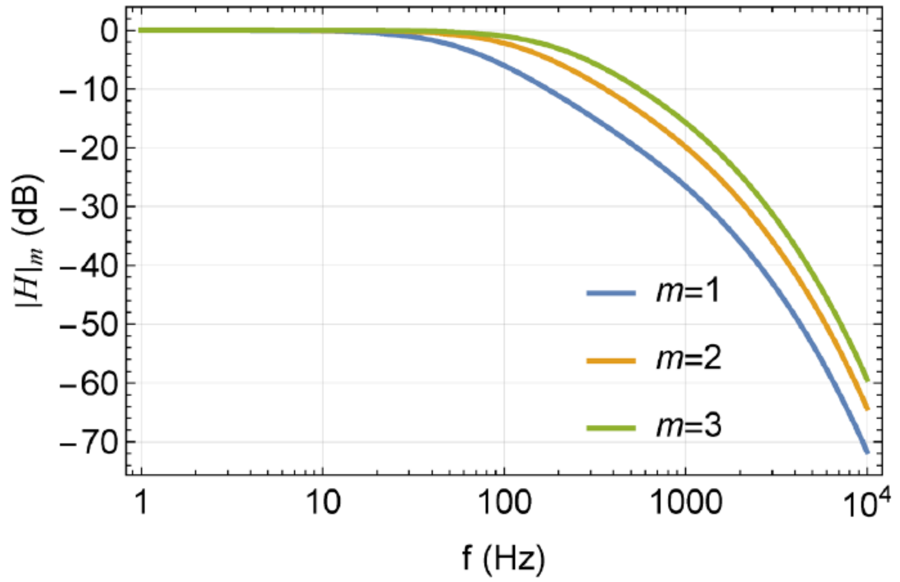
$$\frac{2\,m\left(\frac{b}{a}\right)^m}{a\,b\,q^2\left(K_{m+1}(a\,q)I_{m-1}(b\,q) - I_{m+1}(a\,q)K_{m-1}(b\,q)\right)},$$

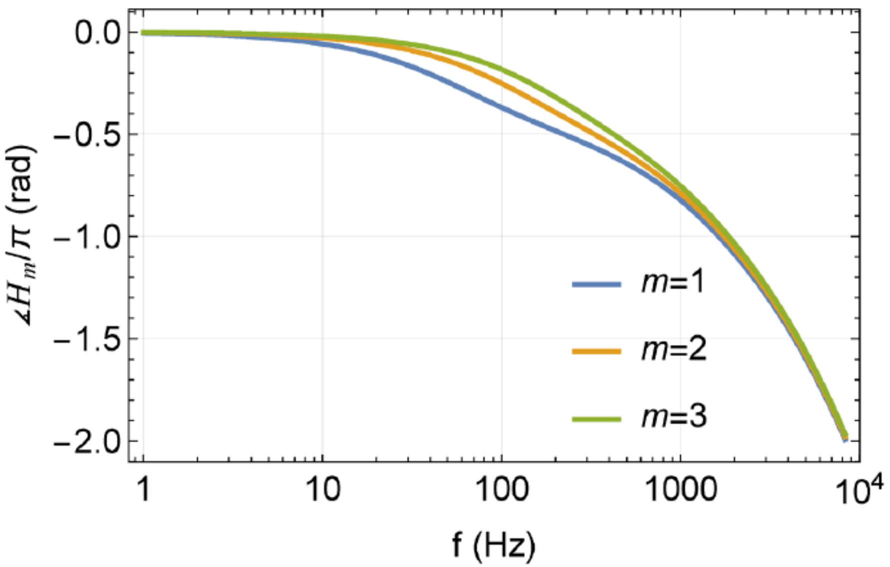
where $q = (\mu_0\sigma p)^{\frac{1}{2}}$, $a < b$ are the inner and outer chamber radii, and $I_m(\ldots)$ and $K_m(\ldots)$ stand for the modified Bessel functions of order m.

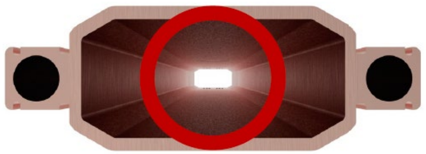
- Maxwell's equations in the quasistatic approximation, for the magnetic vector potential A
- select the m-symmetric solutions of the Poisson equation in the interior and exterior regions
- match the exterior solution to the external field at infinity
- solve the differential equation for A inside the chamber wall with matched boundary conditions
- obtain the transfer function
- solution is exact and self-consistent

- B. Podobedov M. Blaskiewicz, BNL-224904-2023-TECH, 2023
- B. Podobedov M. Blaskiewicz, H. Witte, IPAC'24, MOPC77
- Several papers and books have equivalent formulas, but only for m = 1 (dipole) field!

Field penetration into the ESR chamber







Attenuation and phase lag of the external dipole, quadrupole, and sextupole fields inside the beam pipe

- Drastic difference from the thin-wall solution, especially at high frequency: attenuation >> 20 dB/decade, phase lag >> $\pi/2$

Approximating the TF with a few leading poles

Analysing the zeros of the Bessel functions in the exact TF, we derived a much simpler but accurate TF model:

$$H_m(p) \approx \prod_{n=0}^{N-1} \frac{p_n}{p_n - p}$$

$$p_0 = -m/\tau \qquad \tau = \frac{1}{2}\mu_0 \sigma a d$$

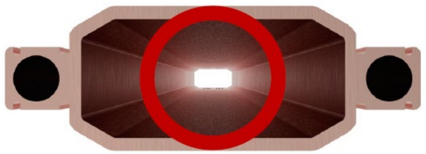
$$p_{n>0} = -n^2 \frac{\pi^2}{\mu_0 \sigma d^2}$$

The dominant pole p_0 agrees with the thin-wall TF model

The $p_{n>0}$ poles are the "skin effect poles". They only depend on the wall thickness d, and occur at frequencies when the skin depth equals $\frac{\sqrt{2}}{\pi}\, d/n$

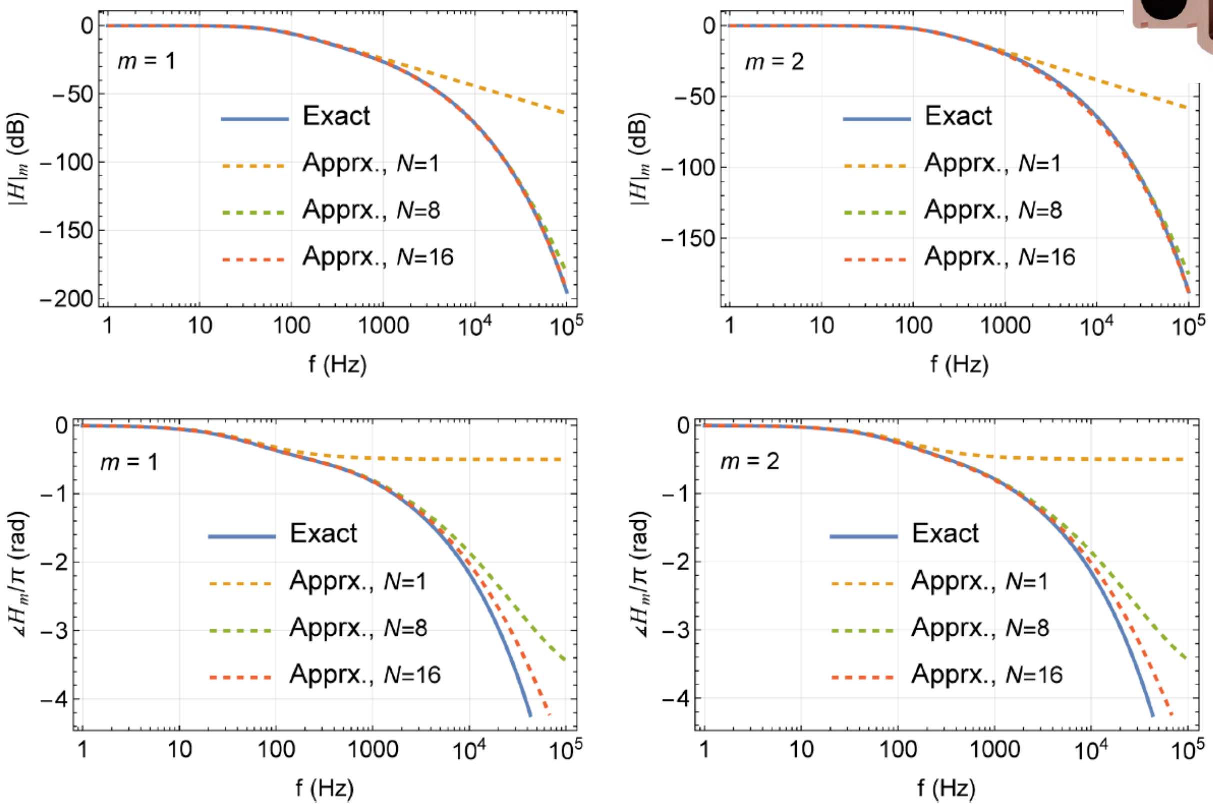
dipole

quadrupole



$|p_0|/2\pi = 60.66$ Hz

$|p_{n>0}|/2\pi = n^2 \times 1347$ Hz



Important observation that will allow us to extend the theory to axially asymmetric beam pipes!

Results for axially asymmetric chambers

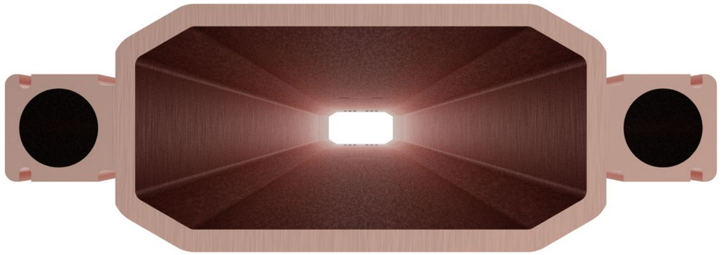
Assumptions

- ~~Axial symmetry (around z-axis)~~
- Translational symmetry (along z-axis)
- Quasistatic approximation

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}.$$

- Normal skin effect
- Non-magnetic pipe (μ_r=1)



No axial symmetry => all magnetic multipoles couple: external dipole field creates quadrupole, sextupole, etc.

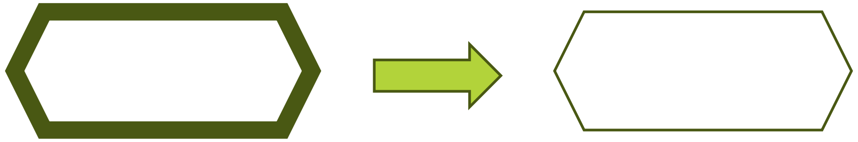
(with mid-plane symmetry dipole creates sextupole, decapole, etc.)

Approach

- All multipoles are coupled – need two indices for the TFs (also different TFs for normal and skew components)
- Unaware of exact solutions => look for an approximate solution for the TF
- Make use of the observations from the axially symmetric case: skin-effect poles depend only on thickness d, they must not be affected much by the overall shape

  => Preserve these poles

  => The problem reduces to that of a thin-wall chamber for which we need to find the dominant pole(s) that capture the overall shape information



$$H_{m,n}(p) = \frac{\tilde{b}_{m,i}(p)}{\tilde{b}_{n,e}(p)}$$

sampled multipole

driving multipole

keep ➡ $H_m(p) \approx \prod_{n=0}^{N-1} \frac{p_n}{p_n - p}$

replace ➡ $p_0 = -m/\tau \qquad \tau = \frac{1}{2}\mu_0 \sigma a d$

replace

keep ➡ $p_{n>0} = -n^2 \frac{\pi^2}{\mu_0 \sigma d^2}$

Approach and main results

- The problem is still hard. No self-consistent solution
- => resort to perturbation theory
- At low frequency, the induced currents are mainly from the external field and self-fields can be ignored
- Then these currents are found from Faraday's law

$$\tilde{J}_m = \sigma r^m p \tilde{b}_{m,e}(p) \cos m\theta$$

- The induced fields are then found from the Biot-Savart law by integrating $\tilde{J}_n$ over the chamber cross-section. These fields are inversely related to the pole values.
- E.g. the dominant pole p_0 of $H_{m,m}(p) = p_0/(p-p_0)$ is

$$p_0^{-1} = -\frac{2^m}{m}\frac{\mu_0 \sigma d}{4\pi}\oint (\cos m\,\theta)^2\, ds$$

Can be done analytically by splitting x-section into line segments
If σ or d vary around the x-section, they go inside the integral

- For $m \neq n$, the low-frequency scaling is $H_{m,n}(p) \sim p$, hence there must be two additional poles, given by similar expressions with powers of $\cos(m\theta)$ and $\cos(n\theta)$

Driving multipole $\tilde{b}_{m,e}$

↓

Induced current
$J_m \sim \cos m\,\theta$

↓

Induced multipoles
$\tilde{b}_{m,i}$, $\tilde{b}_{m\pm1,i}$, $\tilde{b}_{m\pm2,i}$, …

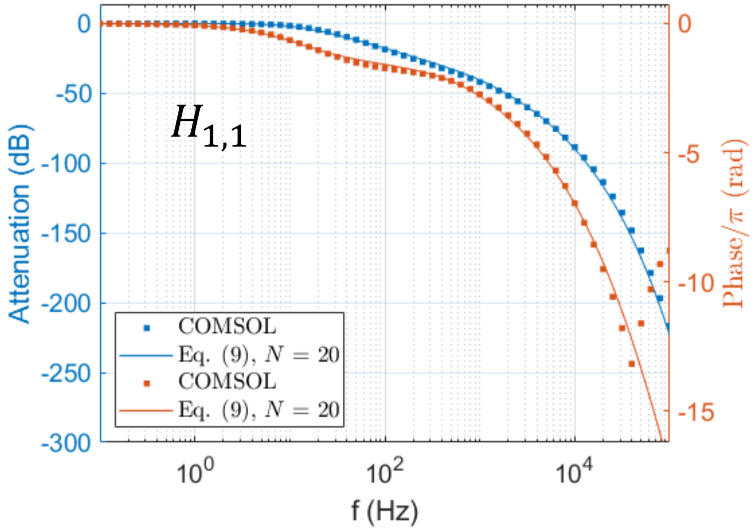
↓

More induced currents
$J_m \sim \cos m\,\theta, \cos(m \pm 1)\theta,$
$\cos(m \pm 2)\,\theta, \ldots$

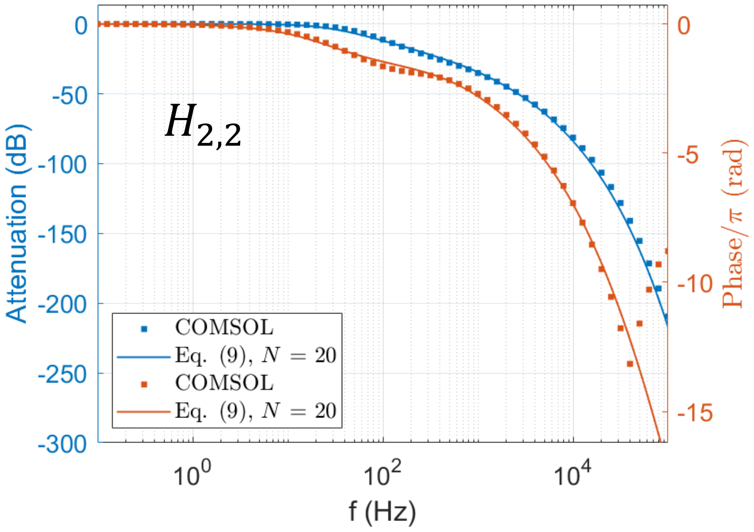
↓

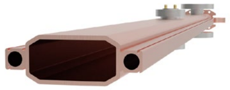
Eddy current shielding by the beam pipe

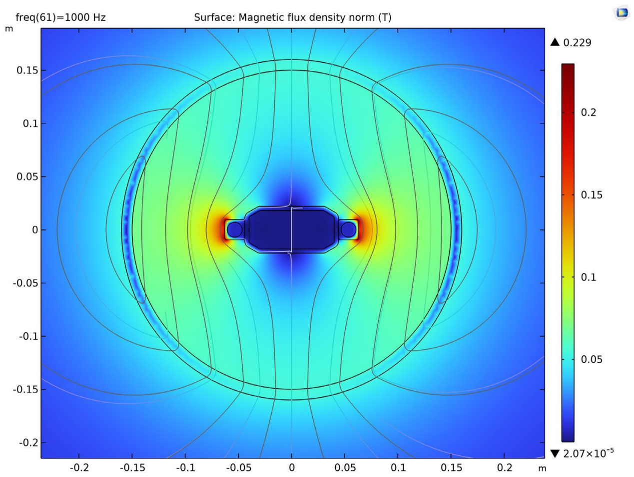
Dipole



Quadrupole

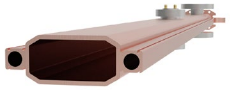




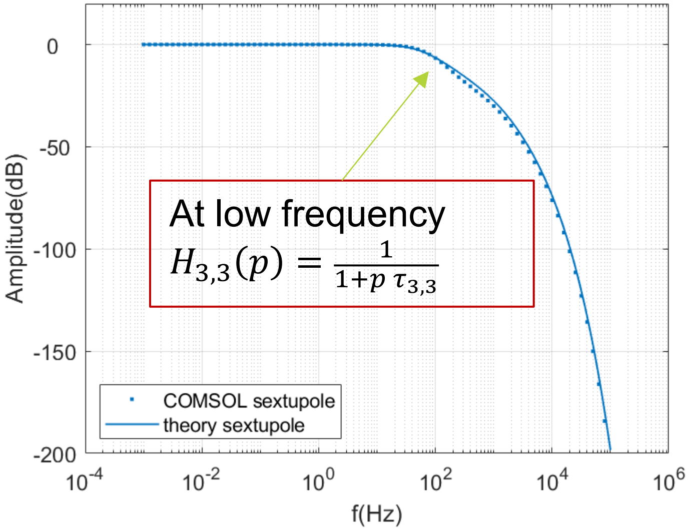


- Plot the TF from the external B-field, to the same multipole symmetry B-field inside pipe
- Good agreement with FEM simulations
- Small higher-order multipoles are also excited, but they are irrelevant for the shielding problem
- (They are important for ramping machines, e.g. EIC RCS (dipole-induced sextupole), HSR (transition-quad-jump-induced octupole), etc.)

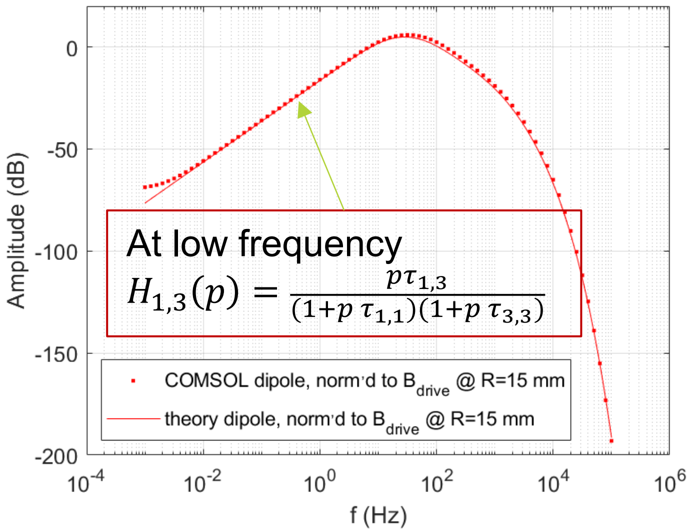
Shielding and excitation of other multipoles



Sextupole-induced sextupole



Sextupole-induced dipole



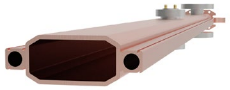
$$p_{m,n} = -1/\tau_{m,n}$$

sampled multipole

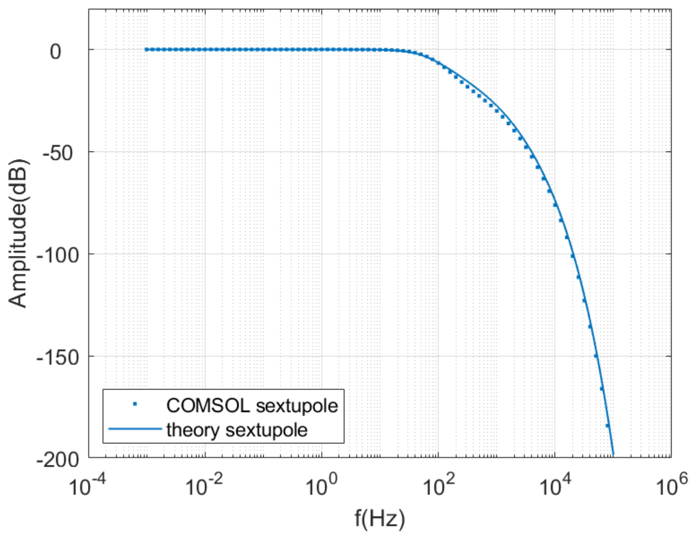
driving multipole

- Perturbation-theory TF expressions are shown in figures (higher order "skin-effect poles" are omitted)
- Good agreement with COMSOL, including the phase (not shown)
- But only after we figured out how to properly separate the induced dipole and sextupole fields in COMSOL output

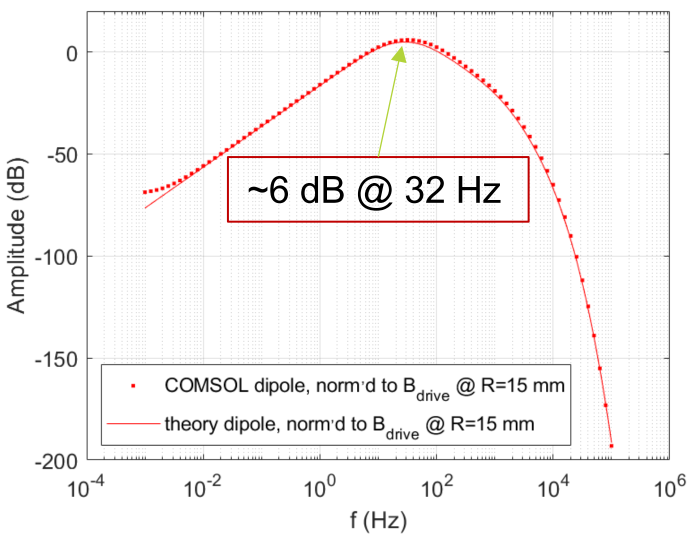
Shielding and excitation of other multipoles cont'd



Sextupole-induced sextupole



Sextupole-induced dipole



- Sextupole is special, being the lowest multipole which excites a lower-order dynamic multipole (dipole), for top-bottom & left-right chamber symmetry
- This effect is new TTBOMK; a paper with Mike and Holger is in preparation
- Practical relevance: the maximum dipole amplitude is ~2 times the sextupole drive field at R=15 mm
- The resulting dipole kick (=> IP orbit ripple) is the dominant driver for the sextuple PS ripple spec

Power supply current ripple specs

$$\frac{\delta V}{V} = p(f)\frac{\delta I}{I}$$

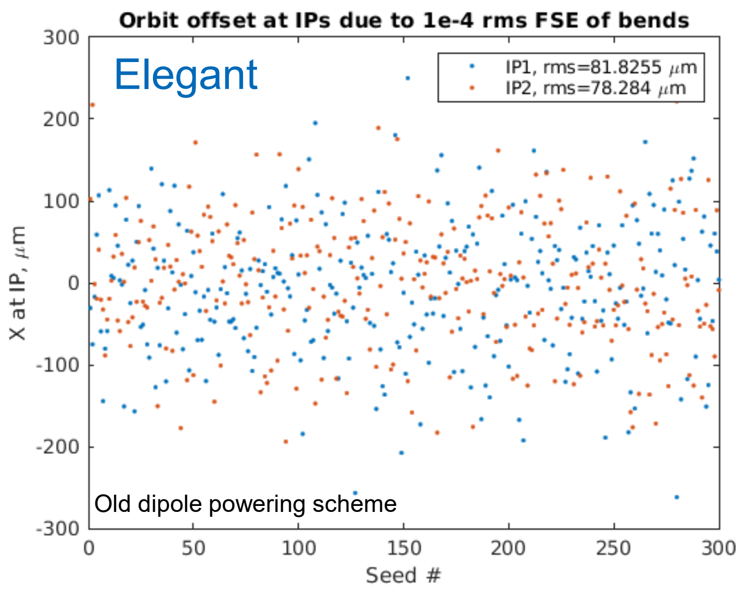
L/R in magnets

$$\frac{\delta I}{I} = q(f)\frac{\delta B}{B}$$

Attenuation due to chamber eddy currents + other effects (to be ignored)

ESR magnet PS ripple specs at low frequency

- Estimating the effect of rippling ESR PS to the orbit or size mismatch at IP is straightforward, except for
- Tedious bookkeeping (many magnet families, different lattices and operating energies, non-trivial normalization, etc.)
- Example: add small random B-field error to the relevant magnets, calculate closed orbits, calculate rms at IP over many error sets, scale to match the 1% of σ beam-beam requirements, normalize to I_{max}, scale for chamber shielding



Max. rms current ripple normalized to the max. PS current over all operating beam energies

		dipole	quadrupole	sextupole
No credit for shielding ⇨	1 Hz - 1 kHz	10 ppm (new single PS scheme, with IR orbit feedback)	5 ppm	20 ppm
Shielding credited ⇨	1 kHz - 4 kHz	100 ppm	50 ppm	200 ppm
	Reason	IP orbit stability	IP size stability	IP orbit stability

WARNING
SUBJECT TO CHANGE

Challenging but doable!

ESR dipole PS ripple specs at high frequency

- Look at resonantly driven oscillations of e-beam around the closed orbit due to ESR dipoles rippling close to the electron betatron frequency
- The worst case is when this is close to the proton betatron frequency
- Do the e-beam oscillations at IP meet the beam-beam requirement, $\Delta x_{rms} \approx \sigma_x/10^4$?
- Yes, for the PS ripple amplitude below the one shown in figure
- The specification is not overly restrictive (esp. for PS switching frequency > 20 kHz)
- This is due to a huge amount of chamber attenuation at these frequencies

BNL-226416-2024-TECH

EIC-ADD-TN-054

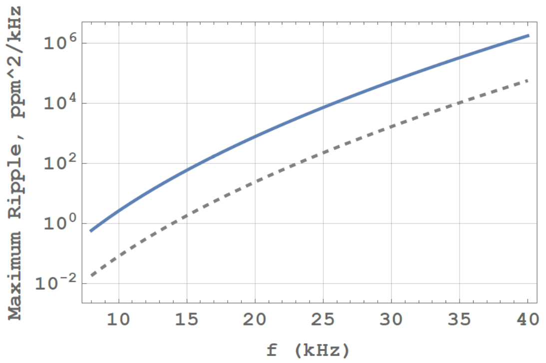


FIG. 4. (solid) Maximum allowable rms current ripple normalized to the maximum operating dipole PS current; (dashed) same for axially-symmetric chamber approximation and the (now obsolete) original powering scheme with $I_{\rm dipole} \propto E$ for most dipoles.

B. Podobedov, M. Blaskiewicz, IPAC'24, MOPC76

Conclusion

- Eddy current shielding in axially symmetric vacuum pipes is well understood analytically for arbitrary pipe wall thickness and magnetic field multipole order.
- The shielding transfer function was approximated using a physically intuitive rational function, where all but the dominant pole depend only on the wall thickness.
- This approach was extended to non-circular chambers while preserving the same expression for the non-dominant poles. A method for determining the dominant pole was provided, showing good agreement with FEM simulations.
- A previously unexplored effect of lower-order multipoles induced by an oscillating higher-order multipole magnet was quantified and found to be consistent with FEM results.
- These findings helped establish stringent power supply ripple specifications for the EIC ESR, particularly at low frequencies where shielding is insufficient.
- Further applications at the EIC and beyond are being actively explored