In these lectures I will describe a general set of methods for studying the properties of integrals over rational and algebraic functions

⇒ we will focus on their analytic structure — namely, where they develop singularities and discontinuities

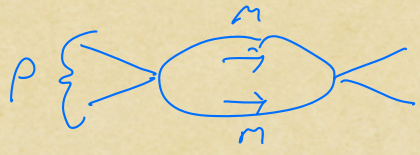
⇒ so while we will primarily study Feynman integrals that contribute to flat-space scattering amplitudes (and in some cases cosmological correlators), the hope is that these methods may also prove useful in the study of other physical systems

In the context of scattering amplitudes, though, let us motivate why we might be interested in studying analytic properties

(1) first, analytic properties encode many of the implications of physical principles such as causality, locality, and unitarity

⇒ for example, we will study the singularities of the bubble integral in great detail in these lectures, and will encounter the threshold and pseudothreshold singularities

on the physical Riemann sheet →



$p^2 = 4m^2$

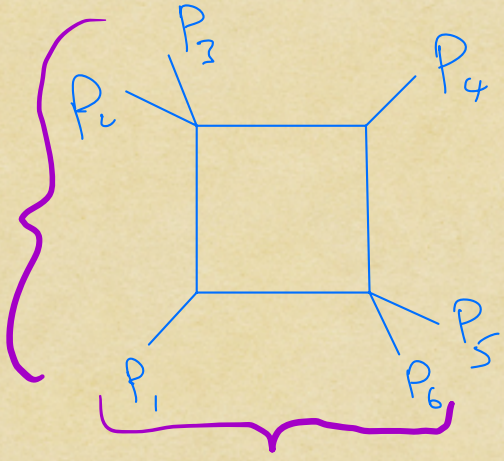
only on the second sheet →

$p^2 = 0$

⇒ causality dictates that only the threshold singularity should appear in physically-realizable kinematics

⇒ we will see how this property follows from the placement of the physical integration contour of the bubble integral

⇒ more nontrivially, causality tells us that certain discontinuities are incompatible with each other (I will define what I mean by discontinuities here in more detail in the second lecture)



$s = (p_1 + p_2 + p_3)^2$

$t = (p_1 + p_5 + p_6)^2$

$$\text{Disc}_{t=0}\left(\text{Disc}_{s=0}(I)\right) = \text{Disc}_{s=0}\left(\text{Disc}_{t=0}(I)\right) = 0$$

(2) A second motivation comes from the hope to bootstrap amplitudes directly from their physical and mathematical properties

⇒ Namely, given how hard it is to compute scattering amplitudes and Feynman integrals, the dream would be to just predict a set of properties these quantities must have, and directly construct the full space of functions that have these properties

⇒ if the list of properties is restrictive enough, this will lead us to a unique function that must be the amplitude or integral we are after