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The Strong CP Problem in Multi-Higgs Theories

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The Strong CP problem

The QCD Lagrangian has a term which is P and T violating (CP violating)

$$\mathscr{L}_{\text{QCD}} \supset \theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu}$$

<u>θ measures how much is P (CP) violated in the strong interactions</u>

Including weak interactions, θ is shifted by the chiral transformation needed to diagonalize the quark mass matrices

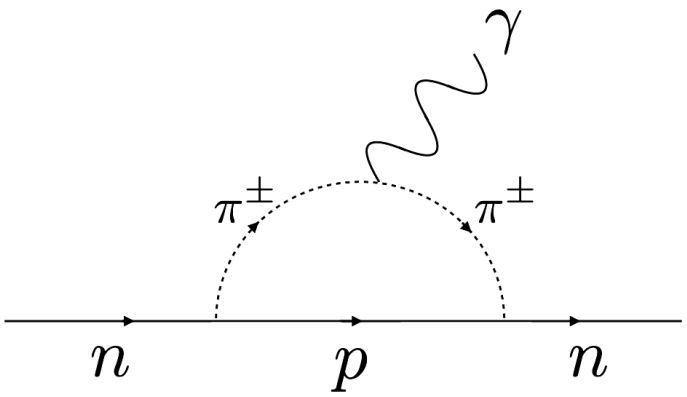
$$\mathscr{L}_{\text{QCD}} \supset \bar{\theta} \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu} = \left(\theta + \arg\det(Y_u Y_d) \right) \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu}$$

Note: There is no reason for these two terms to cancel.

The Strong CP problem

$$\mathscr{L}_{\text{QCD}} \supset \bar{\theta}\,\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\,\tilde{G}^{a\,\mu\nu}$$

How do we measure $\bar{\theta}$?



Naive Dimensional Analysis

PhysRevLett.63.2333

$$d_n = \frac{e\,\bar{\theta}\,m_q}{m_N^2}$$

$$|d_n| \leq 3 \times 10^{-26}\ \text{e} \cdot \text{cm} \implies \boxed{\bar{\theta} \lesssim 10^{-10}}$$

PhysRevD.92.092003

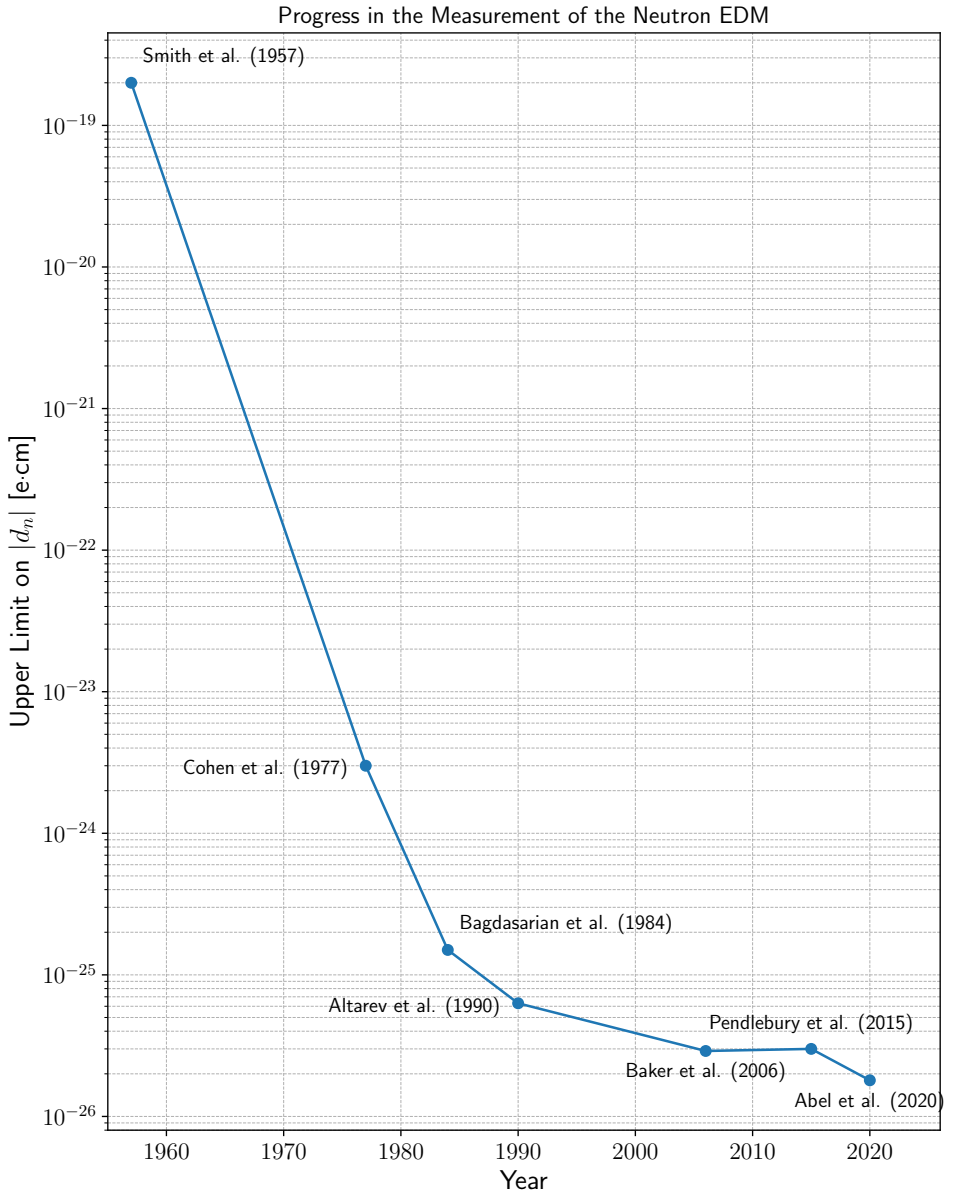
The Strong CP problem

The Strong CP problem is the lack of theoretical understanding of the smallness of $\bar{\theta}$.

$$\mathscr{L}_{\mathrm{QCD}} \supset \bar{\theta}\,\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\,\tilde{G}^{a\,\mu\nu}$$

A few Considerations

- $\bar{\theta}$ is the smallest dimensionless parameter of the SM.
- Every CP phase of the theory generates $\bar{\theta}$.
- CP phase in the weak interaction is large.
- Difficult to advocate an anthropic principle.



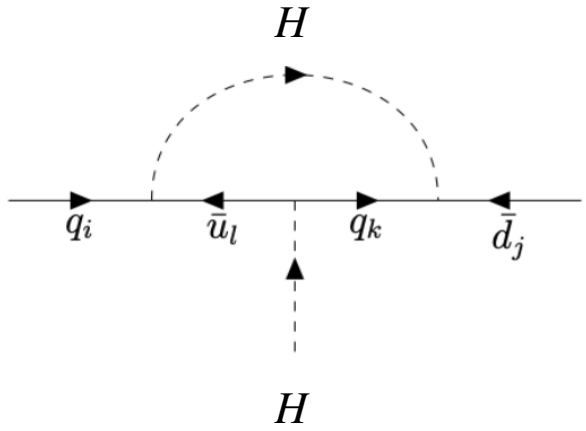
Radiative Corrections in the SM

Λ — $\bar{\theta} = \theta + \arg\det(Y_u Y_d) = 0$

SM

The CKM phase can generate $\bar{\theta}$ radiatively

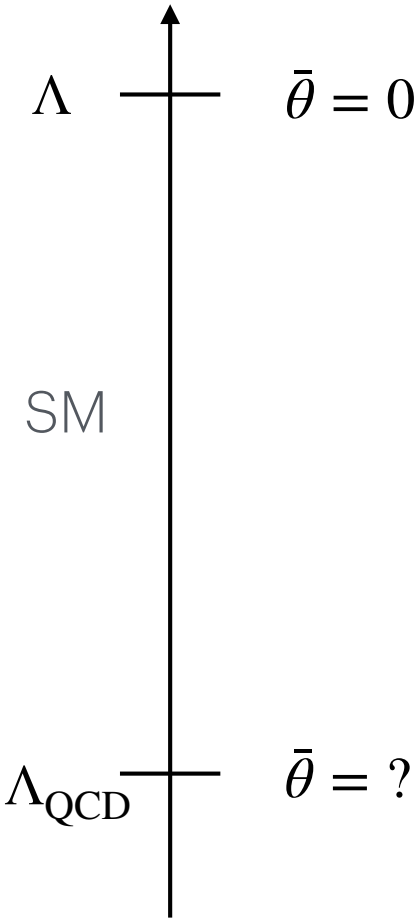
$\Lambda_{\rm QCD}$ — $\bar{\theta} = ?$



$$Y_d + \delta Y_d = Y_d + Y_u Y_u^\dagger Y_d = (\mathbf{1} + Y_u Y_u^\dagger)\, Y_d$$

1 Loop : No corrections to $\bar{\theta}$

Radiative Corrections in the SM

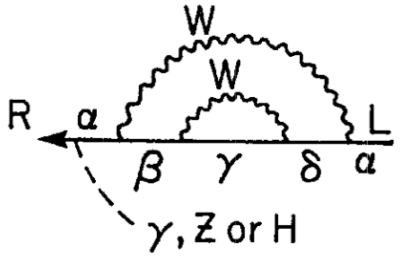


Ellis and Gaillard '79
Nucl.Phys.B150.1979

1 Loop : No corrections to $\bar{\theta}$

2 Loops : No corrections to $\bar{\theta}$

3 Loops : Threshold corrections to $\bar{\theta}$



$$\Delta\bar{\theta} \sim 10^{-16}$$

Experimental limit $\bar{\theta} \lesssim 10^{-10}$

No divergent contributions below 7 loops.

No Strong CP problem if $\bar{\theta}(\Lambda) = 0$ and the SM is the correct EFT below Λ !

Solutions

A Massless quark

If $m_q = 0$, the SM has an additional global chiral $U(1)$ symmetry.

$$u \to e^{i\alpha\frac{\gamma_5}{2}} u \implies \bar{\theta} \to \bar{\theta} + \alpha$$

$\bar{\theta}$ becomes unphysical

Lattice results seem to disfavor this possibility.

The QCD Axion

A global U(1) symmetry, anomalous with QCD and spontaneously broken.

$$\mathscr{L} \supset \left(\frac{a}{f_a} + \bar{\theta}\right) G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu}$$

The axion dynamically drives the CP violating term in the QCD Lagrangian to zero.

Solutions

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The axion dynamically drives the CP violating term in the QCD Lagrangian to zero.

Symmetry

P or CP are symmetries of nature.

$$\bar{\theta} = 0$$

They must be broken at low energies!

CP Solutions

Phys. Rev. D 111, 115012 *Hall, Manzari, Noether*

2506.13853 *Bonnefoy, Hall, Manzari, Noether*

CP Solutions

- The Standard Model gauge structure is compatible with CP

$$\theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu} \xrightarrow{\text{CP}} -\theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu}$$

Hence

$$\theta = -\theta = 0$$

Note: In almost all these frameworks $\bar{\theta}$ can also be π.

For a model that solves this issue see ***Bonnefoy, Hall, Manzari, Scherb,*** **PhysRevLett.131.221802**

CP Solutions

- The Standard Model gauge structure is compatible with CP

$$Y_{ij}\,\bar{\psi}_{L,i}\,\phi\,\psi_{R,j} + Y^{*}_{ij}\,\bar{\psi}_{R,j}\,\phi^{\dagger}\,\psi_{L,i} \xrightarrow{\text{CP}} Y_{ij}\,\bar{\psi}_{R,j}\,\phi^{\dagger}\,\psi_{L,i} + Y^{*}_{ij}\,\bar{\psi}_{L,i}\,\phi\,\psi_{R,j}$$

Hence

$$Y_u, Y_d \in \mathscr{R} \implies \begin{cases} \arg\det(Y_u Y_d) = 0 \\ \delta_{\text{CKM}} = 0 \end{cases}$$

$$\bar{\theta} = \left(\theta + \arg\det(Y_u Y_d)\right) = 0$$

Nelson-Barr Models

Add vector-like quarks q, and scalars η, which spontaneously break CP with complex vevs.

$$\mathscr{L} = \mu \bar{q} q + \lambda \, \eta \, \bar{q} \, d + Y_d \, \bar{Q} \, d \, H$$

$$M_d = \begin{pmatrix} \mu & B \\ 0 & m_d \end{pmatrix}$$

- η must not couple to $\bar{Q}d$
- μ < B for sizable CKM phase
- Dangerous higher dimensional operators and radiative corrections

In this talk we want to find a simpler mechanism.

PhysRevLett.53.329

Phys.Lett.B.136.5–6

What is the minimal extension of the SM that allows to implement a CP solution to the strong CP problem?

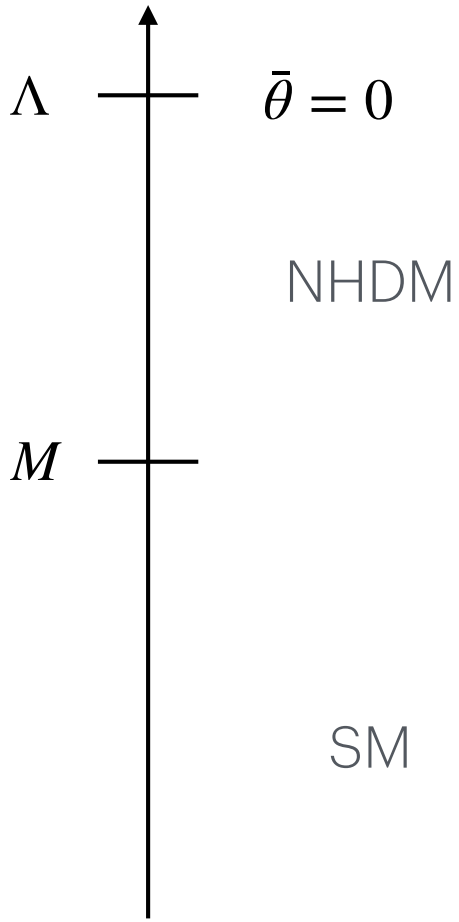
- We want to impose CP in the UV
- We need to break CP to get the CKM phase
- We need to ensure that after breaking $\arg\det(Y_u Y_d) = 0$

N Higgs Solution

L. J. Hall, C. A. Manzari, B. Noether

Phys. Rev. D 111, 115012

EFT picture



Two simple ingredients:

- a scalar sector which softly breaks CP;
- a flavor symmetry, G_{HF}.

N Higgs Solution

Scalar Potential

$$V = \frac{M_\alpha^2}{2}\Phi_i^\dagger\Phi_i + \mu_{ij}^2 e^{i\beta_{ij}}\Phi_\alpha^\dagger\Phi_\beta + \frac{\lambda_{ijkl}}{4}\Phi_i^\dagger\Phi_j\Phi_k^\dagger\Phi_l + h.c.$$

For now all Higgs but one are heavy. Let rotate to the mass basis, moving all phases in the rotation matrix U, such that

$$\Phi_i = U_{ij} H_j$$

And assume $\langle H_1 \rangle = v, \quad m_{H_1} \sim 125\ \text{GeV}, \quad m_{H_{2\ldots n}} \sim M \gg v$

N Higgs Solution

Yukawa Interactions

$$\mathscr{L}_Y = x^{\alpha}_{ij}\, q_i\, \bar{u}_j\, \Phi_\alpha + \tilde{x}^{\alpha}_{ij}\, q_i\, \bar{d}_j\, \Phi^{\dagger}_{\alpha}$$

The quark Yukawa matrices are now complex (CKM phase is possible!)

$$Y_u = x^{\alpha}\, U_{\alpha 1}\,, \qquad Y_d = \tilde{x}^{\alpha}\, U^{*}_{\alpha 1}$$

It remains to find a symmetry that ensures

$$\arg\det(Y_u Y_d) = 0 \qquad \delta_{\mathrm{CKM}} \neq 0$$

N Higgs Solution

Example

N = 2

	Φ_1	Φ_2	q_1	q_2	q_3	$\bar{u}_1$	$\bar{u}_2$	$\bar{u}_3$	$\bar{d}_1$	$\bar{d}_2$	$\bar{d}_3$
$\mathbb{Z}_3$	0	1	0	1	2	1	2	1	1	0	1

$$V(\Phi_1, \Phi_2) = M_{11}\Phi_1^\dagger\Phi_1 + M_{22}\Phi_2^\dagger\Phi_2 + \mu_{12}\left(e^{i\theta}\Phi_2^\dagger\Phi_1 + h.c.\right) + \dots$$

$$\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix} = \begin{pmatrix}\cos\beta & \sin\beta\\ -\sin\beta\, e^{i\theta} & \cos\beta\, e^{i\theta}\end{pmatrix}\begin{pmatrix}H_1\\ H_2\end{pmatrix}$$

$$q\begin{pmatrix}x_{11}\Phi_1 & x_{12}\Phi_1 & x_{13}\Phi_2\\ x_{21}\Phi_2 & x_{22}\Phi_2 & 0\\ 0 & 0 & x_{33}\Phi_1\end{pmatrix}\bar{u} \quad , \quad q\begin{pmatrix}\tilde{x}_{11}\Phi_1^\dagger & 0 & 0\\ 0 & \tilde{x}_{22}\Phi_2^\dagger & \tilde{x}_{23}\Phi_2^\dagger\\ \tilde{x}_{31}\Phi_2^\dagger & \tilde{x}_{32}\Phi_1^\dagger & \tilde{x}_{33}\Phi_1^\dagger\end{pmatrix}\bar{d}$$

N Higgs Solution

Example

N = 2

	Φ_1	Φ_2	q_1	q_2	q_3	$\bar{u}_1$	$\bar{u}_2$	$\bar{u}_3$	$\bar{d}_1$	$\bar{d}_2$	$\bar{d}_3$
$\mathbb{Z}_3$	0	2	0	1	2	0	0	1	0	1	1

$$\mathscr{L}_Y = (q_1 \quad q_2 \quad q_3)\begin{pmatrix} x_{11}U_{11} & x_{12}U_{11} & x_{13}U_{21} \\ x_{21}U_{21} & x_{22}U_{21} & 0 \\ 0 & 0 & x_{33}U_{11} \end{pmatrix}\begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \bar{u}_3 \end{pmatrix} H_1 + (q_1 \quad q_2 \quad q_3)\begin{pmatrix} \tilde{x}_{11}U_{11}^* & 0 & 0 \\ 0 & \tilde{x}_{22}U_{21}^* & \tilde{x}_{23}U_{21}^* \\ \tilde{x}_{31}U_{21}^* & \tilde{x}_{32}U_{11}^* & \tilde{x}_{33}U_{11}^* \end{pmatrix}\begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \end{pmatrix} H_1^\dagger$$

$$\det(Y_u) \propto U_{21}U_{21}U_{11}$$

$$\det(Y_d) \propto U_{21}^* U_{21}^* U_{11}^*$$

$$U = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta e^{i\beta} & \cos\beta e^{i\beta} \end{pmatrix}$$

N Higgs Solution

Example

N = 2

	Φ_1	Φ_2	q_1	q_2	q_3	$\bar{u}_1$	$\bar{u}_2$	$\bar{u}_3$	$\bar{d}_1$	$\bar{d}_2$	$\bar{d}_3$
$\mathbb{Z}_3$	0	2	0	1	2	0	0	1	0	1	1

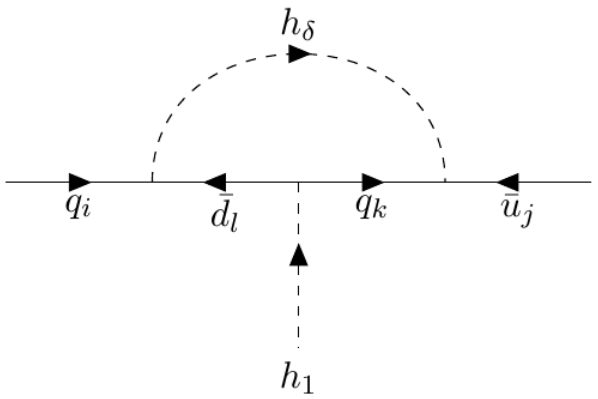
$$
\begin{aligned}
&y_u \approx |x_{11}U_{11}| \ , \quad y_c \approx |x_{22}U_{21}| \ , \quad y_t \approx |x_{33}U_{11}| \ , \\
&y_d \approx |\tilde{x}_{11}U^*_{11}| \ , \quad y_s \approx |\tilde{x}_{22}U^*_{21}| \ , \quad y_b \approx |\tilde{x}_{33}U^*_{11}| \ , \\
&|V_{us}| \approx \frac{|x_{12}U_{11}|}{y_c} \ , \quad |V_{cb}| \approx \frac{|\tilde{x}_{23}U^*_{21}|}{y_b} \ , \quad |V_{ub}| \approx \frac{|x_{13}U_{21}|}{y_t} \ , \\
&|J| \simeq \left| \frac{x_{12}}{x_{22}} \frac{x_{13}}{x_{33}} \frac{\tilde{x}_{23}}{\tilde{x}_{33}} \sin\beta \, \sin 3\theta \right| \ ,
\end{aligned}
$$

We find a very large number of phenomenologically viable theories for different N and G_{HF}.

Radiative Corrections

- The experimental constraint on $\bar{\theta}$ is very stringent, we need to check higher order corrections between Λ and M

$$Y_u = x^\alpha U_{\alpha 1} \quad +$$



NOTE: Similarly for Y_d

Radiative Corrections

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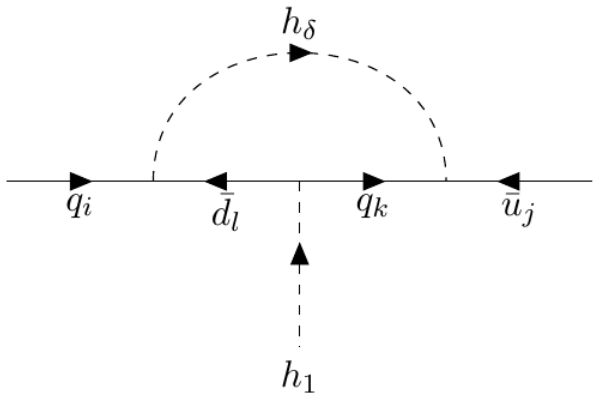
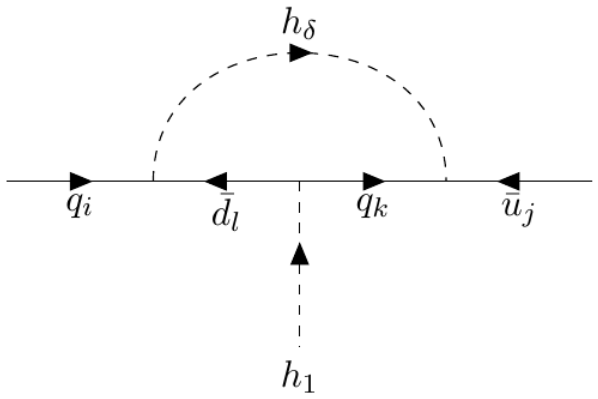


$$\delta y^u = \frac{1}{16\pi^2} \tilde{x}^\alpha \tilde{x}^{\dagger\beta} x^\gamma \, U_{\beta 1} \sum_\delta U^*_{\alpha\delta} U_{\gamma\delta} \left(\frac{1}{\epsilon} + 1 + \log\left(\frac{\mu^2}{M^2_\delta} \right) \right)$$

NOTE: Similarly for Y_d

Radiative Corrections

$$Y_u = x^\alpha U_{\alpha 1} \quad +$$



Using the unitarity of the U matrix, the result reads

$$\delta\bar{\theta} \propto \sum_{i=3}^{N} a_i \log\left(\frac{M_2^2}{M_i^2}\right)$$

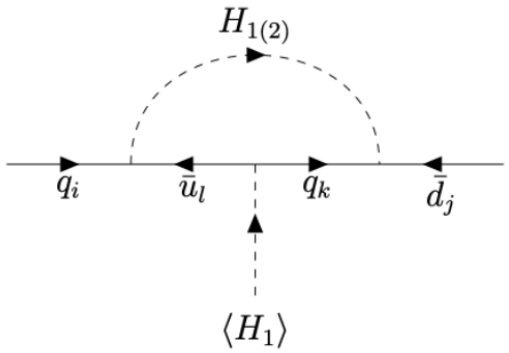
Threshold correction as expected.

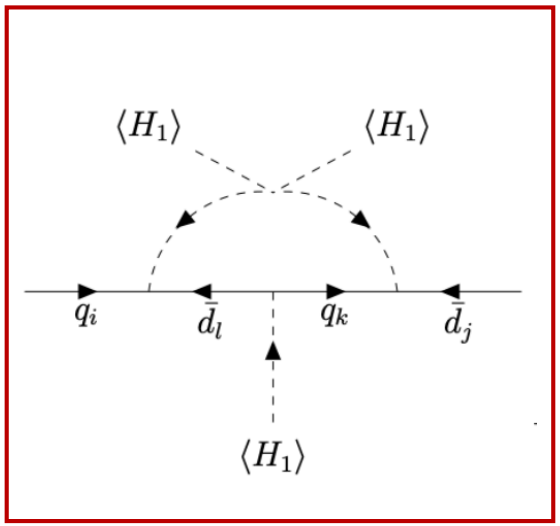
NOTE: Similarly for Y_d

2HDM Theories

Theories with N=2 don't have 1-loop corrections to $\bar{\theta}$

Can the second Higgs be light? Let's include SU(2) breaking effects λv^2





?

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$$V(\Phi_1,\Phi_2) = \mu_{11}\Phi_1^\dagger\Phi_1 + \mu_{22}\Phi_2^\dagger\Phi_2 + \mu_{12}\left(e^{i\alpha}\Phi_2^\dagger\Phi_1 + \text{h.c.}\right)$$
$$+ \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + 2\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + 2\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

$$U = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta e^{i\beta} & \cos\beta e^{i\beta} \end{pmatrix} \qquad \Phi_i = U_{ij} H_j$$

- The scalar potential becomes CP preserving: no CP-even - CP-odd mixing !

2HDM Theories

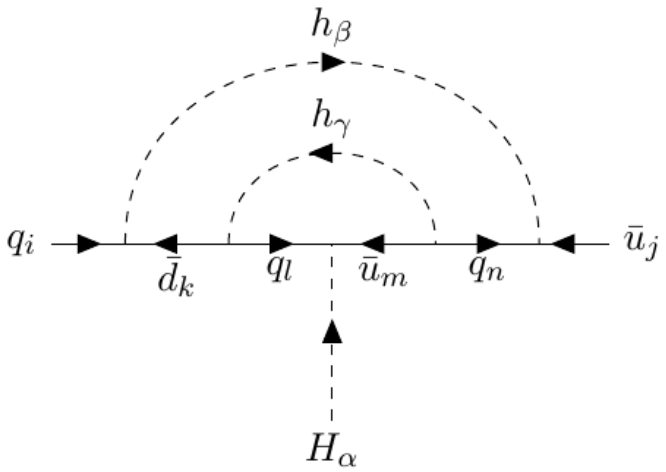
$$H_1 = \begin{pmatrix} G^+ \\ v + H^0 + iG^0 \end{pmatrix} \qquad H_2 = \begin{pmatrix} H^+ \\ R^0 + iI^0 \end{pmatrix}$$

- The scalar potential becomes CP preserving: no CP-even - CP-odd mixing
- The phases are only in the couplings of G^+ (CKM) and H^+

At 1-loop the previous result holds exactly: no strong CP problem

2HDM Theories

2-loop corrections to $\bar{\theta}$



We don't compute these diagrams

- Can prove that mass-independent corrections vanish
- Mass dependent pieces are too small in many theories

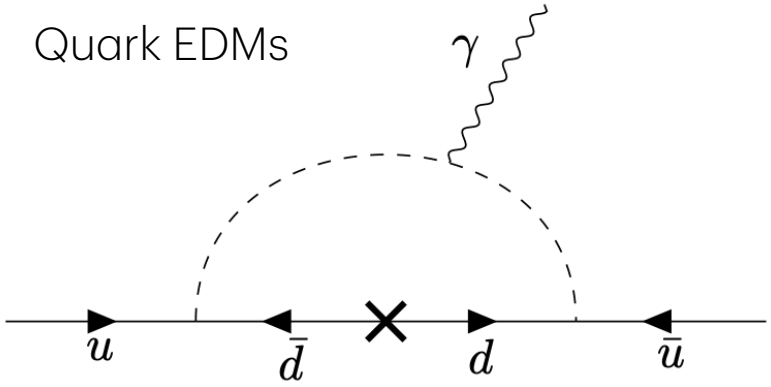
Therefore, there is a class of theories without the strong CP problem, regardless of the mass scale of the additional Higgs bosons.

2HDM - Phenomenology

Neutron EDM

Two Loops Corrections if not 0 can be detectable. In the example shown before $\bar{\theta} \simeq 10^{-12}$

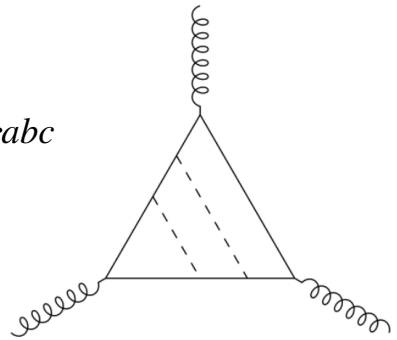
Quark EDMs



They are 0 at second order in perturbative diagonalization of the quark mass matrix. The study of appropriate flavor invariants seems to suggest the result applies to all orders.

Weinberg operator

$$\frac{C_W}{\Lambda^2} G^{a\mu\nu} \tilde{G}^b_{\nu\rho} G^{c\rho}_{\mu} f^{abc}$$



No CP violation below 3 loops. Negligible

2HDM Theories

$$V(\Phi_1,\Phi_2) = \mu_{11}\Phi_1^\dagger\Phi_1 + \mu_{22}\Phi_2^\dagger\Phi_2 + \mu_{12}\left(e^{i\alpha}\Phi_2^\dagger\Phi_1 + \text{h.c.}\right)$$
$$+ \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + 2\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + 2\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

$$\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix} = \begin{pmatrix}\cos\beta & \sin\beta\\ -\sin\beta\, e^{i\theta} & \cos\beta\, e^{i\theta}\end{pmatrix}\begin{pmatrix}H_1\\ H_2\end{pmatrix}$$

$$H_1 = \begin{pmatrix}G^+\\ v + H^0 + iG^0\end{pmatrix} \qquad H_2 = \begin{pmatrix}H^+\\ R^0 + iI^0\end{pmatrix}$$

After diagonalizing the quark mass matrix, H_1 has flavor diagonal couplings, while H_2 has also flavor off-diagonal ones.

2HDM Theories

The two CP-even scalars, H^0 and R^0, mix with angle α.

Let's call the mass eigenstates h and H.

$$\mathscr{L} \supset -\frac{1}{\sqrt{2}}(u,d)\left[\left(c_\alpha\,\hat{y}^{u,d} + s_\alpha\,\hat{z}^{u,d}\right)h + \left(-s_\alpha\,\hat{y}^{u,d} + c_\alpha\,\hat{z}^{u,d}\right)H + i\,\hat{z}^{u,d}\,I^0\right](\bar{u},\bar{d}) \;+ \text{h.c.}$$

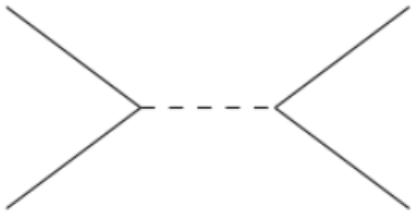
Flavor off-diagonal couplings.

$$s_\alpha \;\simeq\; \cos\beta\sin\beta\left(\cos^2\beta\,(\lambda_1-\lambda_3-\lambda_4) + \sin^2\beta\,(-\lambda_2+\lambda_3+\lambda_4)\right)\frac{v^2}{M^2}$$

2HDM Theories

$\Delta F = 2$ Processes (K, B, D mixing)

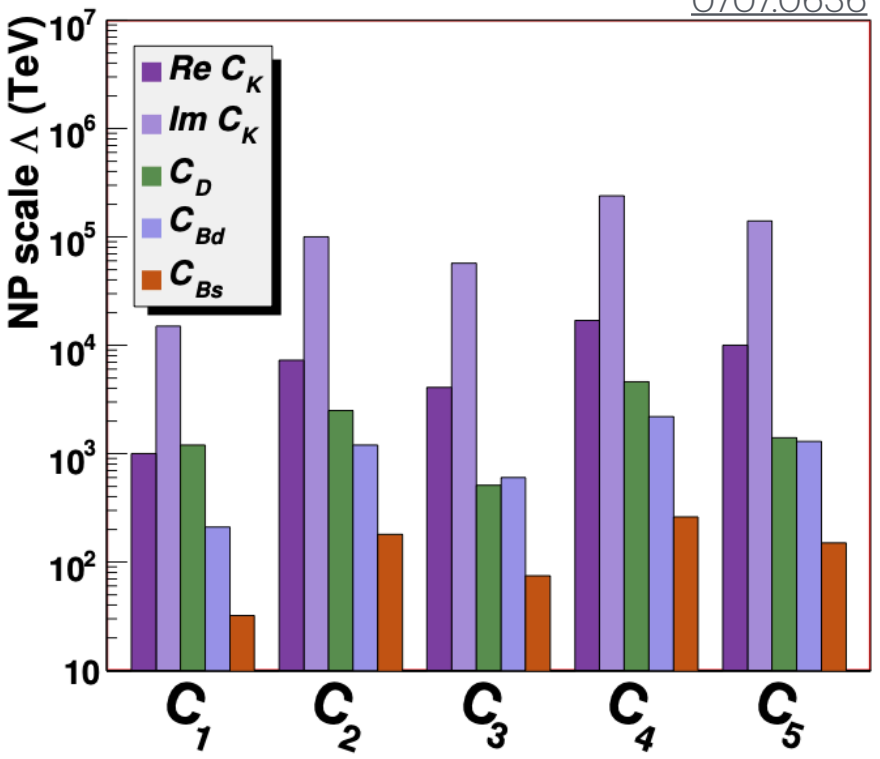
$$\mathscr{L} \supset -\frac{1}{\sqrt{2}}(u,d)\left[\left(c_\alpha\,\hat{y}^{u,d} + s_\alpha\,\hat{z}^{u,d}\right)h + \left(-s_\alpha\,\hat{y}^{u,d} + c_\alpha\,\hat{z}^{u,d}\right)H + i\,\hat{z}^{u,d}\,I^0\right](\bar{u},\bar{d}) + \text{h.c.}$$



No CP violation at tree level.

$$Q_2 = s^\alpha \bar{d}^\alpha\; s^\beta \bar{d}^\beta \qquad Q_4 = s^\alpha \bar{d}^\alpha\; \bar{s}^{\beta\dagger} d^{\beta\dagger}$$

0707.0636

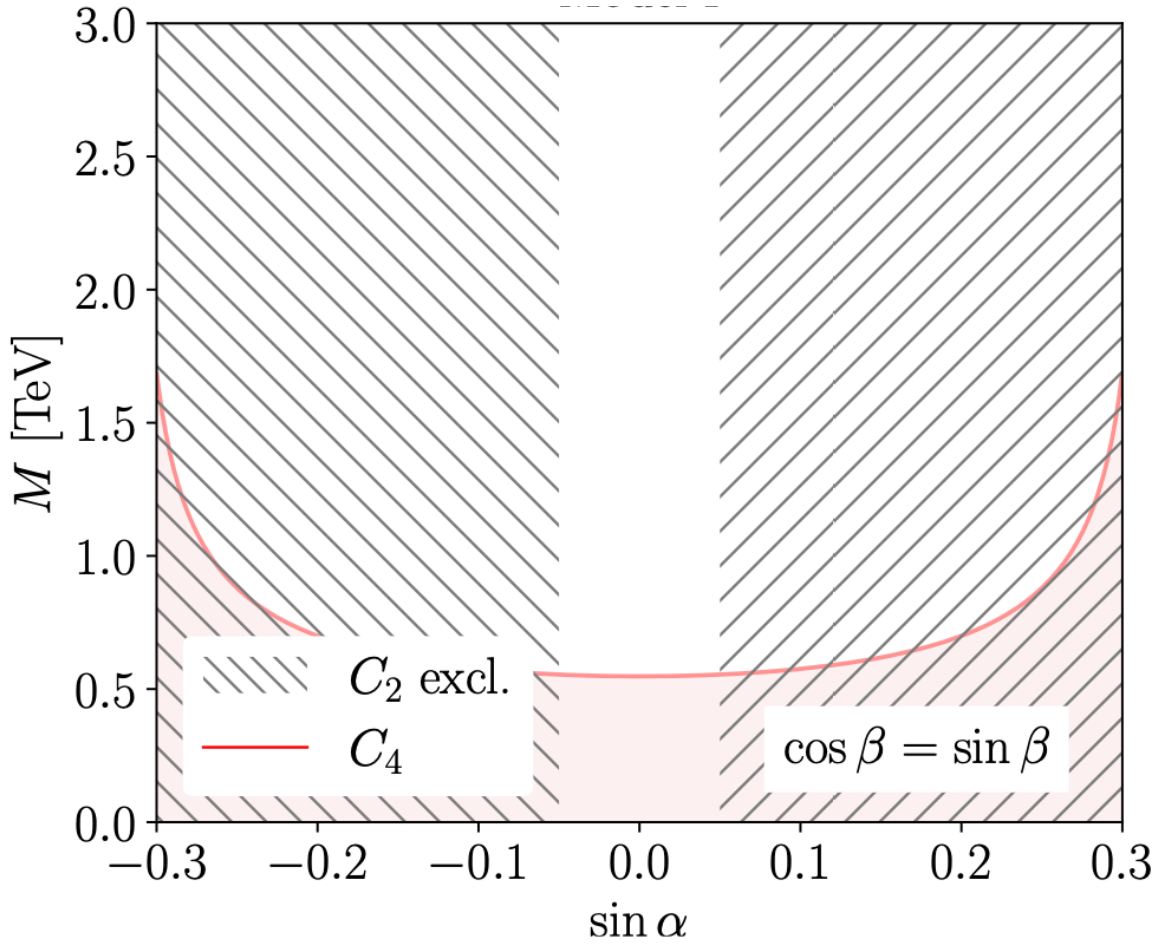


2HDM Theories

$\Delta F = 2$ Processes (K, B, D mixing)

$$C_2 = -\frac{(\hat{z}_{ij}^{u,d} s_\alpha)^2}{4}\left(\frac{1}{m_h^2} - \frac{1}{m_H^2}\right)$$

$$C_4 = -\frac{\hat{z}_{ij}^{u,d}\hat{z}_{ji}^{u,d}}{2}\left(\frac{s_\alpha^2}{m_h^2} + \frac{2 - s_\alpha^2}{m_H^2}\right)$$



2HDM Theories

Modified Yukawa Couplings of the 125 GeV Higgs

$$\mathcal{L} \supset -\frac{m_i^f}{v}\,\bar{f}_i\left(\kappa_i^f + i\,\tilde{\kappa}_i^f\,\gamma_5\right) f_i\,h$$

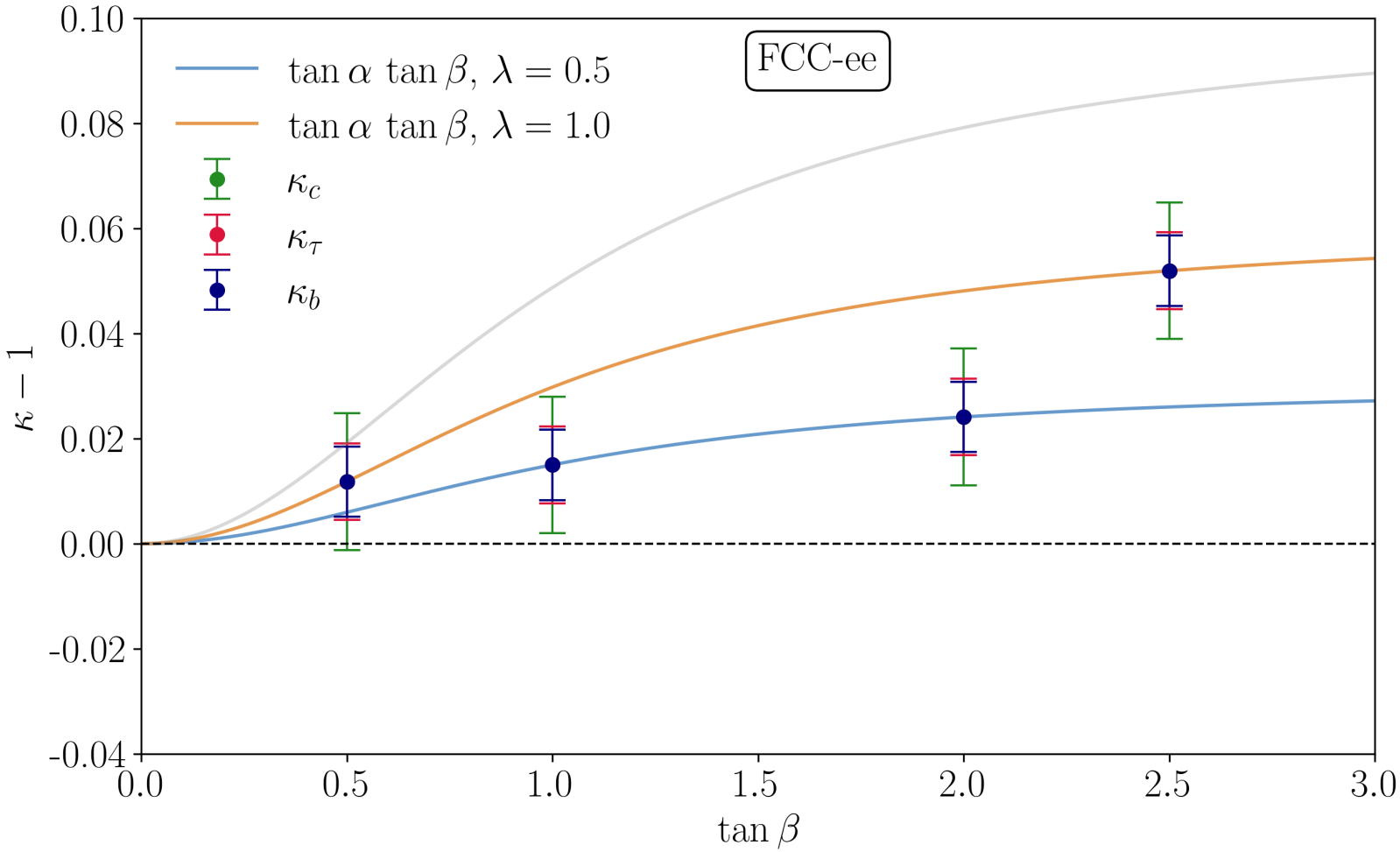
$$|\kappa_i^{u,d,e} - 1| \;\simeq\; \left|\cos\alpha - 1 + \sin\alpha\left(\tan\beta,\ \cot\beta,\ 2\tan\beta + \cot\beta,\ -2\cot\beta - \tan\beta\right)\right|$$

	LHC	HL-LHC	LHeC	HE-LHC		ILC			CLIC			CEPC	FCC-ee	
	ATLAS [38]			S2	S2'	250	500	1000	380	1500	3000		240	365
	%	%	%	%	%	%	%	%	%	%	%	%	%	%
κ_c	$0.03^{+3.02}_{-0.03}$	–	4.1	–	–	2.5	1.3	0.9	4.3	1.8	1.4	2.2	1.8	1.3
κ_t	$0.93^{+0.13}_{-0.06}$	3.3	–	2.8	1.7	–	6.9	1.6	–	–	2.7	–	–	–
κ_b	$0.89^{+0.14}_{-0.11}$	3.6	2.1	3.2	2.3	1.8	0.58	0.48	1.9	0.46	0.37	1.2	1.3	0.67
κ_μ	$1.06^{+0.27}_{-0.30}$	4.6	–	2.5	1.7	15	9.4	6.2	320*	13	5.8	8.9	10	8.9
κ_τ	$0.92^{+0.13}_{-0.07}$	1.9	3.3	1.5	1.1	1.9	0.70	0.57	3.0	1.3	0.88	1.3	1.4	0.73

JHEP01(2020)139

2HDM Theories

Modified Yukawa Couplings of the 125 GeV Higgs



CP Solutions

Follow-up Ideas

Relation to the EW Hierarchy Problem

- Embedding in composite Higgs models.
- Supersymmetric theories.

Relation to Flavor

- Implications for the lepton sector (PMNS, Leptogenesis).
- Guide for Theory of Flavor.

Conclusions

- The Strong CP Problem is the lack of theoretical understanding of the smallness of $\bar{\theta}$.
- It is very important to study the landscape of solutions to best direct the experimental and theoretical effort.
- I discussed a framework with N Higgs bosons, a CP-symmetry and a flavor symmetry.
- 2HDM theories are striking for:
  - minimality;
  - quality of the solution to the strong CP problem;
  - testability (collider, flavor).
- These solutions open up avenues for correlating the strong CP problem with other puzzles in particle physics, such as the flavor problem and the hierarchy problem.

Thank you!

Backup Slides

Introduction

What is the Strong CP Problem ?

The QCD Lagrangian with N flavors has a large global symmetry in the limit of vanishing quark masses.

$$U(N)_L \times U(N)_R$$

Accounting for quark masses

$$m_u,\, m_d \ll \Lambda_{\mathrm{QCD}} \implies U(2)_L \times U(2)_R \quad \text{expected}$$

EXPERIMENTALLY

$$U(2)_V = SU(2)_I \times U(1)_B \quad \text{observed in hadron multiplets}$$

$$U(2)_A \quad \text{broken by quark condensate} \quad \langle \bar{q} q \rangle$$

We expect four Nambu-Goldstone bosons

HOWEVER EXPERIMENTALLY WE OBSERVE ONLY THREE OF THEM

$$m_{\pi^+} \sim m_{\pi^-} \sim m_{\pi^0} \sim 0 \qquad m_\eta \gg m_{\pi^+} \ !$$

$U(1)_A$ PROBLEM

Phys.Rev.D11,3583(1975)3

SOLUTION: $U(1)_A$ is anomalous with QCD

$$\partial_\mu J_5^\mu = \frac{g_s^2 N}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu} \quad \text{with} \quad J_5^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi$$

$$q \to e^{i\alpha\frac{\gamma_5}{2}} q \implies \delta S = \alpha \frac{g_s^2 N}{32\pi^2} \int d^4x G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu}$$

CP Solutions

2HDM Theories

The Flavor Puzzle

The flavor puzzle is transformed in these models. Can we use the strong CP problem as a guide to study theories of flavor?

$$\mathscr{L}_Y = (q_1 \quad q_2 \quad q_3) \begin{pmatrix} 0 & 0 & x_{13}U_{31} \\ 0 & x_{22}U_{11} & 0 \\ x_{31}U_{31} & x_{32}U_{21} & x_{33}U_{11} \end{pmatrix} \begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \bar{u}_3 \end{pmatrix} h_1 + (q_1 \quad q_2 \quad q_3) \begin{pmatrix} 0 & \tilde{x}_{12}U^*_{31} & 0 \\ \tilde{x}_{21}U^*_{31} & \tilde{x}_{22}U^*_{11} & \tilde{x}_{23}U^*_{21} \\ 0 & 0 & \tilde{x}_{33}U^*_{11} \end{pmatrix} \begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \end{pmatrix} h_1^\dagger$$

U_{ij} are constrained from $\bar{\theta}$ at 1-loop $\qquad Y_u = x^\alpha \, U_{\alpha 1} \,, \qquad Y_d = \tilde{x}^\alpha \, U^*_{\alpha 1}$

$$x = \begin{pmatrix} 0 & 0 & 0.4 \\ 0 & 5 \times 10^{-3} & 0 \\ 10^{-1} & 10^{-2} & 1 \end{pmatrix} \qquad \tilde{x} = \begin{pmatrix} 0 & 6 \times 10^{-3} & 0 \\ 5 \times 10^{-3} & 3 \times 10^{-4} & 0.8 \\ 0 & 0 & 0.02 \end{pmatrix}$$

Nelson Barr Mechanism

Add vector-like quarks q, and scalars η, which spontaneously break CP with a complex vev.

$$\mathscr{L} = \mu \bar{q} q + \lambda \eta \, \bar{q} \, d + Y_d \bar{Q} \, d \, H$$

Not so simple...

$$M_d = \begin{pmatrix} \mu & B \\ 0 & m_d \end{pmatrix}$$

- Generally more than 1 η to have CKM phase
- η must not couple to $\bar{Q}d$
- μ < B for sizable CKM phase
- Dangerous higher dimensional operators and radiative corrections

More ingredients than the CP solution described in this talk.

Other Research Directions

- The Strong CP problem.
  - Symmetry solutions
  - Supernovae and Neutron Star Mergers to test the QCD axion

    We got private fundings (~2M) to do R&D with Berkeley Space Lab on satellite network we proposed: GALAXIS. I am leading the theoretical studies.
  - Exotic phases of matter in Neutron Stars and Implications for Axion Emissivity
    - Pion Condensate in Neutron Star Mergers
    - Super Color Conductivity in Massive Neutron Star Cores
- The Flavor Puzzle.
  - Flavor in Grand Unified Theories: Georgi-Jarlskog and Yukawa Structures
  - Extra Dimensions (and GUTs in extra dimensions)
- The Nature of Dark Matter.
  - Model building (e.g. KK gravitons)
  - Constraints from astrophysical observations

Naive Dimensional Analysis

PhysRevLett.63.2333

- Introduce dimensionless reduced couplings

$$g\,O(\phi_1,\ldots,\phi_N) \quad \text{with} \quad [O] = [mass]^D \longrightarrow g\,(4\pi)^{2-N} M^{D-4}$$

$$M = 2\pi F_\pi$$

M is the chiral symmetry breaking scale

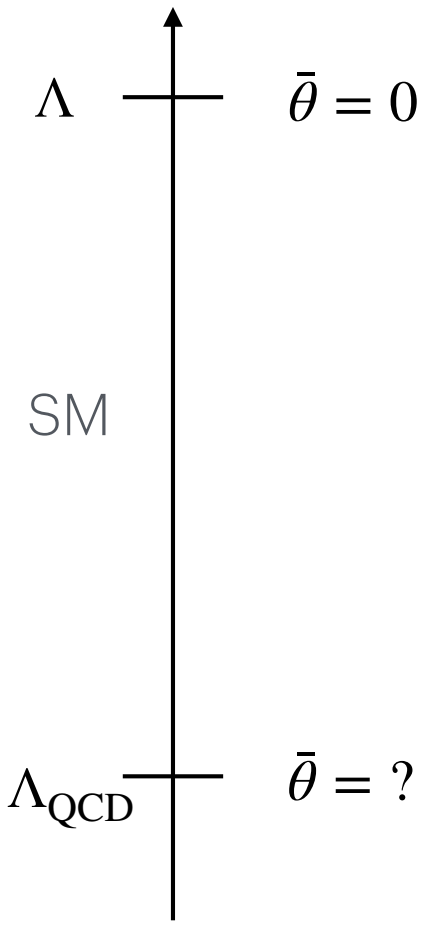
The reduced coupling in the effective hadronic theory at energies below M is given to within an order of magnitude by the product of the reduced couplings of the operators, appearing in the effective Lagrangian at energies above M, that produce this term.

$$e\,\bar{q}\,\gamma_\mu\,A^\mu\,q \quad + \quad \theta\,m_q \bar{q}\gamma_5\,q \qquad\qquad d_n\,\bar{n}\,\gamma_5\,\sigma_{\mu\nu}\,n\,F^{\mu\nu}$$

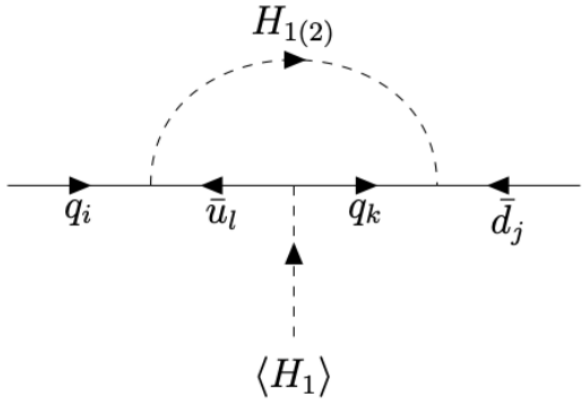
$$\frac{e}{4\pi} \qquad\qquad \frac{\bar{\theta}\,m_q}{M} \qquad\qquad d_n\,\frac{M}{4\pi}$$

$$d_n = \frac{e\,\bar{\theta}\,m_q}{m_N^2}$$

Radiative Corrections in the SM



1 Loop : No corrections to $\bar{\theta}$

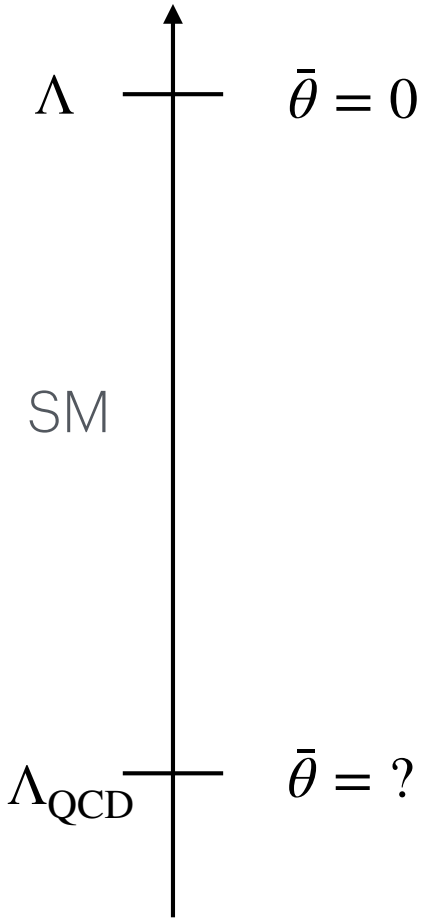


$$Y_d + \delta Y_d = Y_d + Y_u Y_u^\dagger Y_d = (\mathbf{1} + Y_u Y_u^\dagger)\, Y_d$$

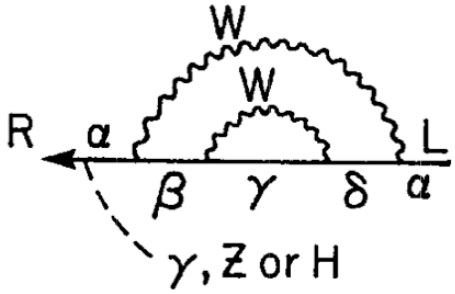
2 Loops : No corrections to $\bar{\theta}$.

Ellis and Gaillard '79
Nucl.Phys.B150.1979

Radiative Corrections in the SM



3 Loops : Threshold corrections to $\bar{\theta}$



Ellis and Gaillard '79
Nucl.Phys.B150.1979

$$\Delta\bar{\theta} = \left(\frac{\alpha}{\pi}\right)^2 s_1^2 s_2 s_3 \sin\delta \frac{m_s^2 m_c^2}{m_W^4} \sim 10^{-16}$$

No divergent contributions below 7 loops.

No Strong CP problem if $\bar{\theta}(\Lambda) = 0$ and the SM is the correct EFT below Λ !

Solutions to the Strong CP Problem

- A massless quark

The SM has an additional global chiral U(1) symmetry $m_u = 0$

$$u \to e^{i\alpha\frac{\gamma_5}{2}} u \implies \bar{\theta} \to \bar{\theta} + \alpha$$

$\bar{\theta}$ Is now unphysical!

Lattice results seem to disfavor this possibility.

Solutions to the Strong CP Problem

- A massless quark
- The axion

We introduce a global chiral U(1) symmetry, anomalous with QCD, and spontaneously broken

$$\mathscr{L}_{\text{QCD}} \supset \left(\bar{\theta} \frac{g_s^2}{32\pi^2} + \frac{a}{f_a} \right) G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu}$$

$$V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{a}{2f_a} + \frac{\bar{\theta}}{2} \right)}$$

The axion dynamically drives the CP violating term in the QCD Lagrangian to zero.

Solutions to the Strong CP Problem

- A massless quark
- The axion
- P or CP

P or CP are exact symmetries of the Lagrangian in the UV $\rightarrow \bar{\theta} = 0$

They have to broken at some lower scale to recover the SM

N Higgs Solution

Example II

	ϕ_1	ϕ_2	ϕ_3	q_1	q_2	q_3	u_1	u_2	u_3	d_1	d_2	d_3
$U(1)_F$	0	2	-1	0	1	-1	-2	1	-1	2	1	-1

N = 3

$$\mathscr{L}_Y = (q_1 \quad q_2 \quad q_3)\begin{pmatrix} 0 & 0 & x_{13}U_{31} \\ 0 & x_{22}U_{11} & 0 \\ x_{31}U_{31} & x_{32}U_{21} & x_{33}U_{11} \end{pmatrix}\begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \bar{u}_3 \end{pmatrix} h_1 + (q_1 \quad q_2 \quad q_3)\begin{pmatrix} 0 & \tilde{x}_{12}U^*_{31} & 0 \\ \tilde{x}_{21}U^*_{31} & \tilde{x}_{22}U^*_{11} & \tilde{x}_{23}U^*_{21} \\ 0 & 0 & \tilde{x}_{33}U^*_{11} \end{pmatrix}\begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \end{pmatrix} h_1^\dagger$$

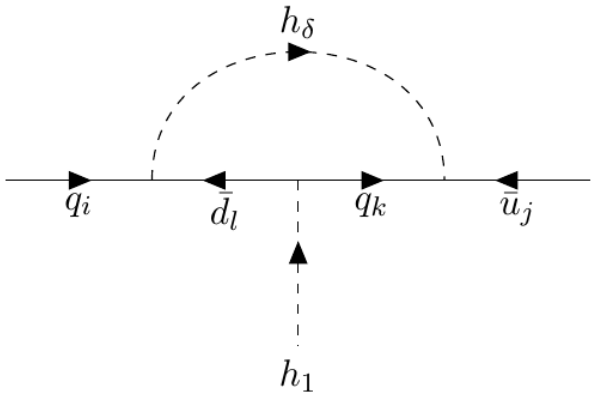
$$\det(Y_u) \propto U_{31}U_{11}U_{31}$$

$$\det(Y_d) \propto U^*_{31}U^*_{31}U^*_{11}$$

$$U = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha e^{i\beta} & \cos\alpha e^{i\beta} \end{pmatrix}$$

Radiative Corrections

$$Y_u = x^\alpha U_{\alpha 1} \quad +$$



Using the unitarity of the U matrix, the result reads

$$\delta Y_u = \frac{1}{16\pi^2} \tilde{x}^\alpha \tilde{x}^{\dagger\beta} x^\gamma \, U_{\beta 1} \left[U^*_{\alpha 1} U_{\gamma 1} \log\left(\frac{M_2^2}{m_h^2}\right) + \sum_{\delta=3}^{N} U^*_{\alpha\delta} U_{\gamma\delta} \log\left(\frac{M_2^2}{M_\delta^2}\right) \right]$$

$$+ \frac{1}{16\pi^2} \tilde{x}^\alpha \tilde{x}^{\dagger\beta} x^\alpha \, U_{\beta 1} \left[\frac{1}{\epsilon} + 1 \right] .$$

$Y_d Y_d^\dagger Y_u$

Only term that can change $\arg\det(Y_u)$

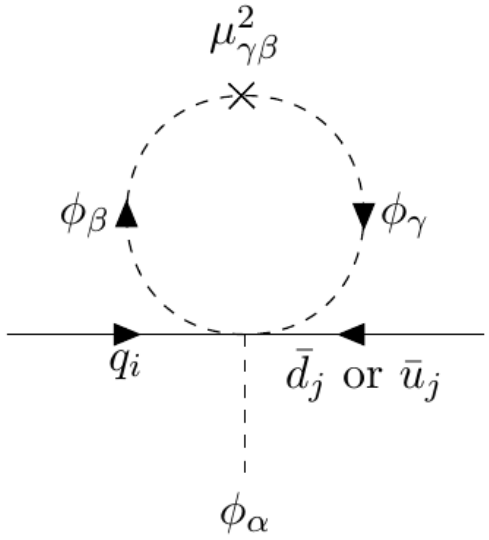
Same structure of Y_u

NOTE: Similarly for Y_d

Higher Dimensional Operators

At dim 6 there are dangerous operators

$$\frac{C_{ijk}}{\Lambda^2} Q\, \bar{u}\, \phi_\alpha\, \phi_\beta\, \phi_\gamma^\dagger$$



$$\delta\bar{\theta} = \frac{1}{8\pi^2} \log\left(\frac{\Lambda}{M}\right) \sum_{\beta\gamma} \frac{\mu^2_{\beta\gamma}}{\Lambda^2} f(U_{\beta\gamma})$$