Cosmological strong coupling surprises

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Kavli Institute for the Physics and Mathematics of the Universe

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The theory basics

(For me)

1. What is the number of degrees of freedom?

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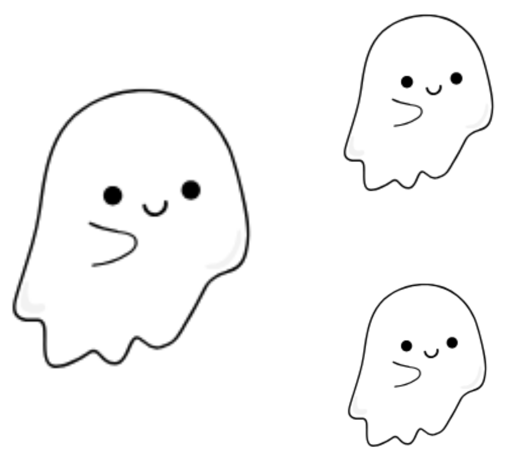
2. How do they behave?

➔ Instabilities?

➔ Ghosts?



Often!

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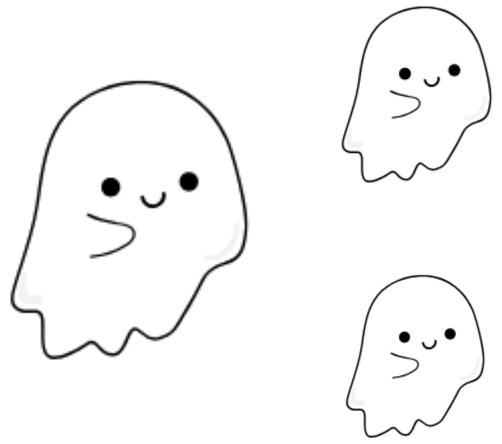

Often!

3. Limits and non-linear terms?

The strong coupling

- **When does the perturbation theory break down?**

$$(\partial^2 + m^2)\chi \sim \frac{g^2\chi^3}{L^2}$$

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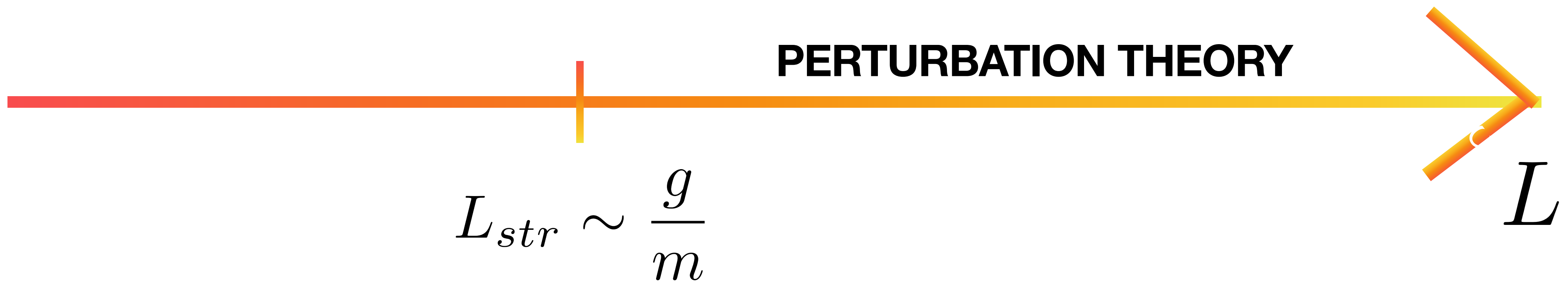

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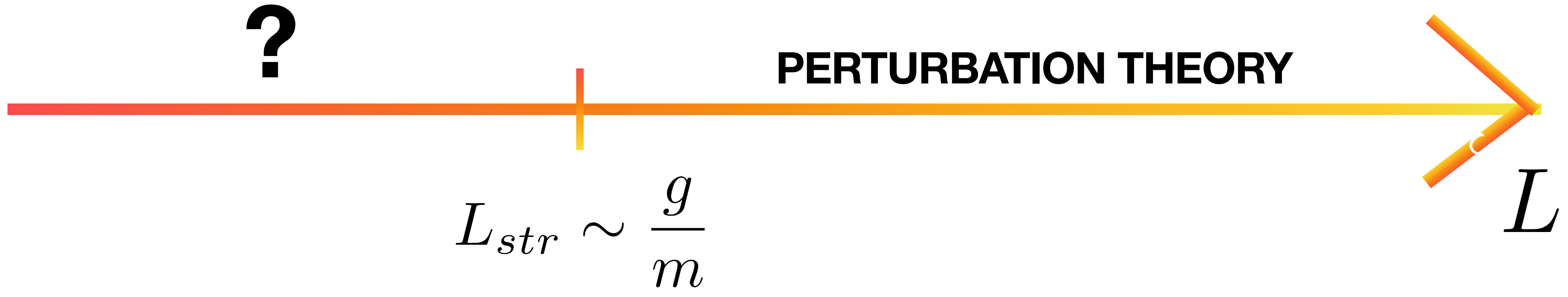

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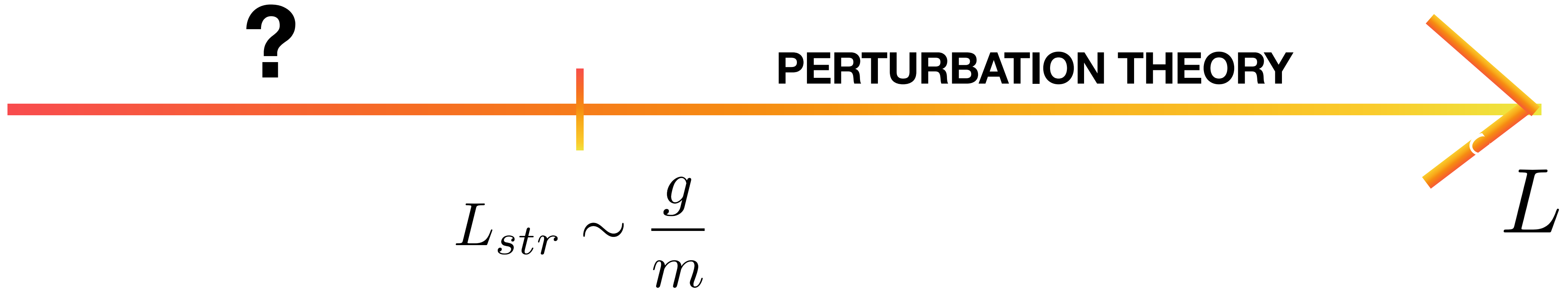

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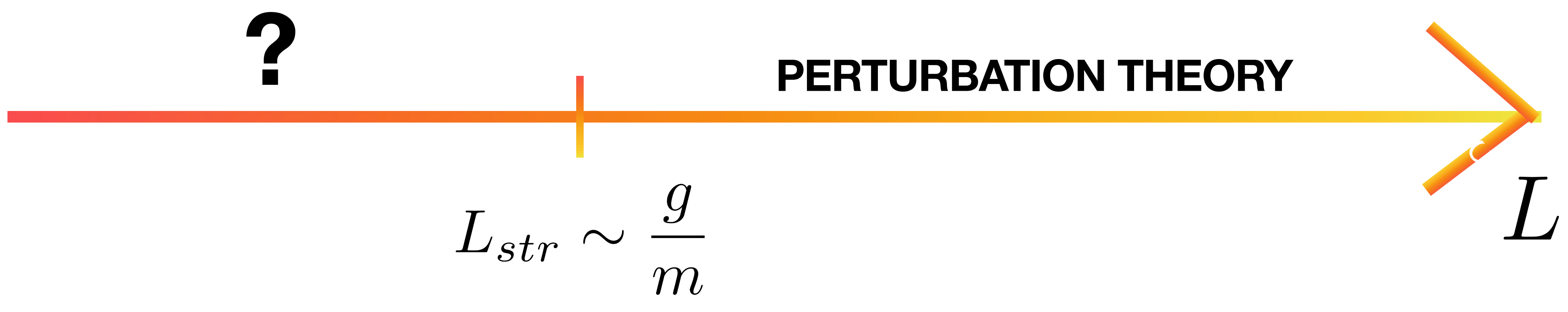

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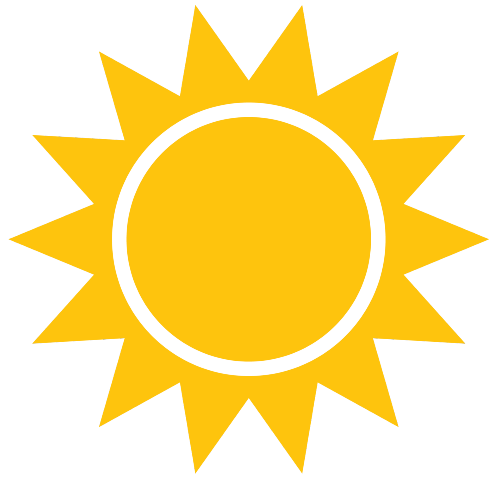

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$$L_{str} \sim \frac{g}{m} \qquad L$$

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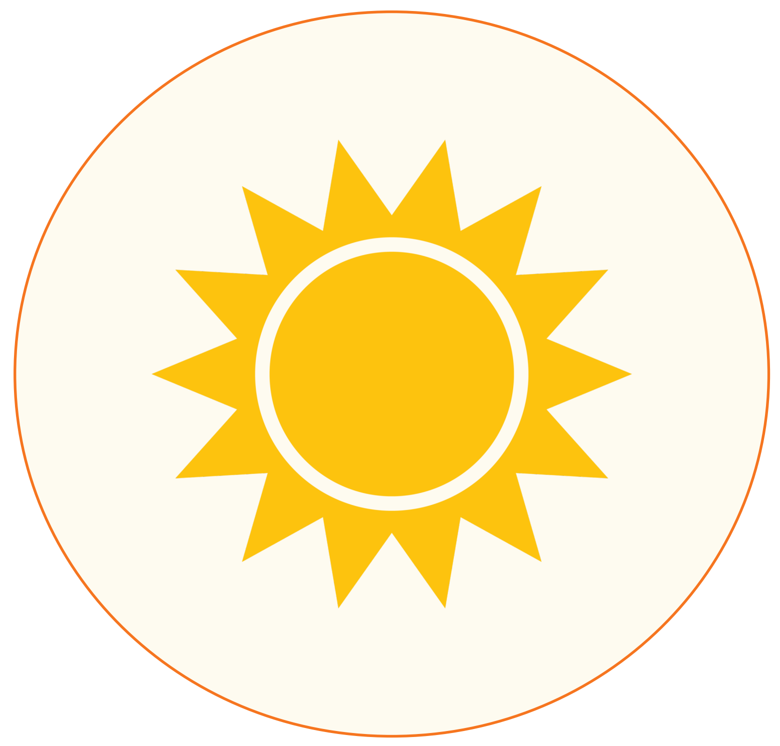

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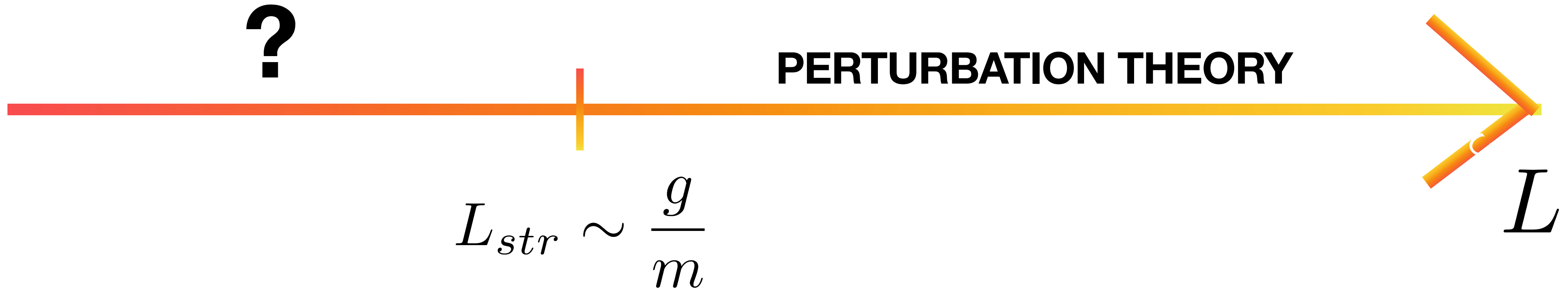

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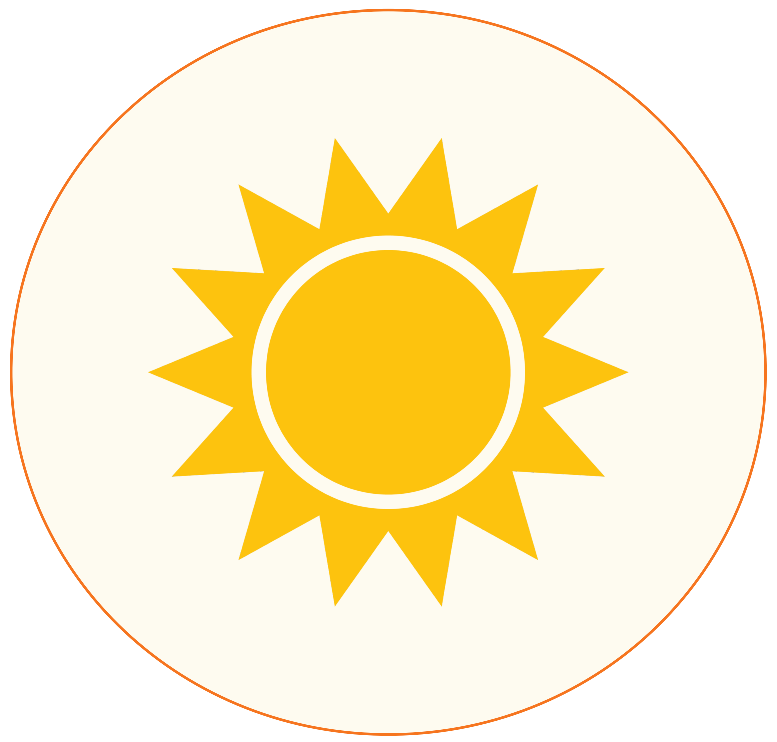

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Look into
A. Hell, JHEP 03, (2022), 167

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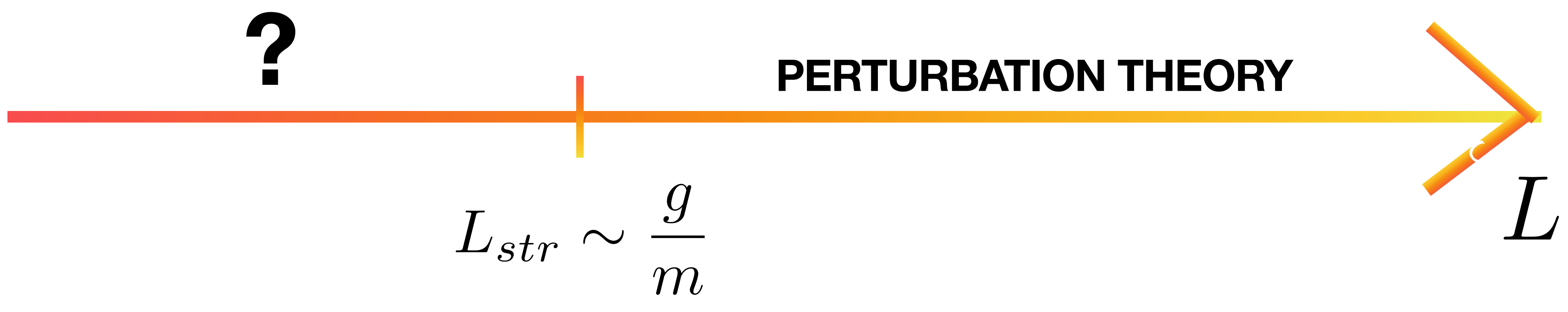

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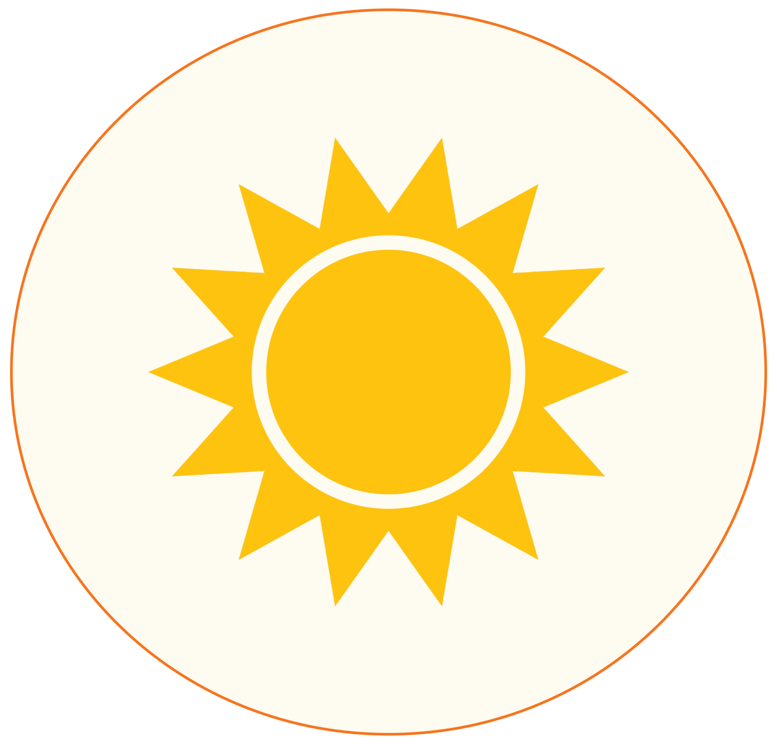

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A. Hell, JHEP 03, (2022), 167

- Self-interacting Proca and Kalb-Ramond fields, R2 gravity

Look into
A. Hell, D. Lüst & G. Zoupanos, JHEP 02 (2024) 039

and
A. Hell, JCAP 01, (2022), 056

In this talk

What is the behavior of
Proca theory with non-minimal coupling to gravity
in cosmological backgrounds?

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}(R-2\Lambda) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu - \frac{1}{2}\left(\beta_1 R A_\mu A^\mu + \beta_2 R^{\mu\nu}A_\mu A_\nu\right)\right]$$

Based on

A. D. Felice and A. Hell, On the cosmological degrees of freedom of Proca field with non-minimal coupling to gravity, 2503.07454

Proca and the non-minimal coupling

- **Vector inflation**

  [2008 Golovnev, Mukhanov and Vanchurin]

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- **Strong coupling of tensor modes in flat spacetime**

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- [And many other works, especially in the applications!]

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1. Background for the metric and the vector field

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3. Find the constraints for the non-propagating fields, solve them and substitute back to the action

Cosmological DOF

FLRW Universe

$$ds^2 = -dt^2 + a^2(t)dx^i dx^j$$

$$A_\mu = \left(A_0(t), 0, 0, 0 \right)$$

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Bianchi T1

$$ds^2 = -dt^2 + a^2(t)dx^2 + b^2(t)\delta_{ij}dx^i dx^j$$

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Procedure:

1. **Background for the metric and the vector field**
2. **Perturb around these solutions**
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Homogeneous and isotropic universe

Perturbations

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + \delta g_{\mu\nu} \qquad A_\mu = A_\mu^{(0)} + \delta A_\mu$$

$$\delta g_{00} = -2\phi \qquad \delta A_0 = A$$

$$\delta g_{0i} = a(t)\left(S_i + B_{,i}\right) \qquad \delta A_i = a(t)A_i^T + \chi_{,i} \qquad S_{i,i} = 0, \quad F_{i,i} = 0, \quad A_{i,i}^T = 0$$

$$\delta g_{ij} = a^2(t)\left(2\psi\delta_{ij} + 2E_{,ij} + F_{i,j} + F_{j,i} + h_{ij}^T\right) \qquad h_{ij,j}^T = 0 \qquad h_{ii}^T = 0$$

Tensor perturbations

$$\mathscr{L} = w_1(t)\dot{h}^T_{ij}\dot{h}^T_{ij} + w_2(t)h^T_{ij}\Delta h^T_{ij} + w_3(t)h^T_{ij}h^T_{ij}$$

Beom →

$$S = \sum_{\sigma=1,2}\int dt d^3k \left[\frac{\left(\left(\beta_1+\beta_2\right)A_0^2 + M_P^2\right)a^3}{8}\dot{h}^\sigma_k\dot{h}^\sigma_{-k} - \frac{a\,k^2\left(A_0^2\beta_1 + M_P^2\right)}{8}h^\sigma_k h^\sigma_{-k},\right]$$

No ghost: $\left(\beta_1+\beta_2\right)A_0^2 + M_P^2 > 0$

Speed of propagation: $c_S^2 = \dfrac{A_0^2\beta_1 + M_P^2}{A_0^2\beta_1 + A_0^2\beta_2 + M_P^2}$

Vector perturbations

Gauge: $F_i = 0$

$$\mathscr{L} = v_1(t)\dot{A}_i^T\dot{A}_i^T + v_2(t)A_i^T\Delta A_i^T + v_3(t)A_i^T A_i^T + v_4(t)S_i\Delta S_i + v_5(t)S_iS_i + v_6(t)A_i^T\Delta S_i$$

$$A_i^T = \sum_{\sigma=1,2}\int \frac{d^3k}{(2\pi)^{\frac{3}{2}}}\varepsilon_i^\sigma v_k^\sigma(t)e^{i\vec{k}\vec{x}} \longrightarrow S = \sum_{\sigma=1,2}\int dtd^3k\left(\frac{a^3}{2}\dot{v}_k^\sigma\dot{v}_{-k}^\sigma + u(t,k)v_k^\sigma v_{-k}^\sigma\right)$$

No ghost: *Always!*

Speed of propagation: $c_S^2 = \dfrac{A_0^2\beta_2^2 + 2A_0^2\beta_1 + 2A_0^2\beta_2 + 2M_P^2}{2A_0^2\beta_1 + 2A_0^2\beta_2 + 2M_P^2}$

Scalar perturbations

$$S = \int dt d^3k \left[e_1(t,k)\dot{\psi}_{2k}\dot{\psi}_{2,-k} + e_2(t)\dot{\psi}_{1k}\dot{\psi}_{1,-k} + e_3(t)\left(\dot{\psi}_{1k}\psi_{2,-k} + \dot{\psi}_{1,-k}\psi_{2,k}\right) + e_4(t)\psi_{2k}\psi_{2,-k} + e_5(t)\psi_{1k}\psi_{1,-k} + e_6(t)\left(\psi_{1k}\psi_{2,-k} + \psi_{1,-k}\psi_{2,k}\right)\right]$$

No ghost conditions:

$$e_1(t) = \frac{3\dot{A}_0^2 A_0^2 a^5 \left(\beta_1 + \frac{\beta_2}{2}\right)^2 \left(\left(\beta_1 + \beta_2\right) A_0^2 + M_P^2\right)}{\left(\left(\beta_1 + \beta_2\right) aHA_0^2 + \dot{A}_0 A_0 \left(\beta_1 + \frac{\beta_2}{2}\right) a + aH M_P^2\right)^2}$$

$$e_2(t) = \frac{a}{2}$$

$$\left(\beta_1 + \beta_2\right) A_0^2 + M_P^2 > 0 \quad \rightarrow \quad \left(\beta_1 + \beta_2\right) > 0$$

Speed of propagation:

$$c_s^2 = 0$$

$$c_s^2 = \frac{3M_P^2 + A_0^2\left(2\beta_2^2 + 3\beta_1 + 4\beta_2\right)}{3\left(M_P^2 + A_0^2(\beta_1 + \beta_2)\right)}$$

Parameter space

$$\mathcal{S}_1 \equiv \left\{\beta_2 \leq -\frac{1}{2}, \lambda < -\frac{1}{4\beta_1 + \beta_2}, \beta_1 < -\frac{\beta_2}{4}, \frac{\beta_2}{2} < \beta_1, -\frac{1}{2\beta_2 + 2\beta_1} < \lambda\right\}$$

$$\mathcal{S}_2 \equiv \left\{-\frac{1}{2} < \beta_2, \lambda < -\frac{1}{4\beta_1 + \beta_2}, \beta_1 < 0, \beta_2 < 0, \frac{\beta_2}{2} < \beta_1, \frac{-4\beta_2^2 - 12\beta_1 - 5\beta_2}{2\left(2\beta_2^2 + 3\beta_1 + 4\beta_2\right)\left(4\beta_1 + \beta_2\right)} < \lambda\right\}$$

$$\mathcal{S}_3 \equiv \left\{0 \leq \beta_1, -\frac{1}{2} < \beta_2, \lambda < -\frac{1}{4\beta_1 + \beta_2}, \beta_1 < -\frac{\beta_2}{4}, \beta_2 < 0, \frac{-4\beta_2^2 - 12\beta_1 - 5\beta_2}{2\left(2\beta_2^2 + 3\beta_1 + 4\beta_2\right)\left(4\beta_1 + \beta_2\right)} < \lambda\right\}$$

Bianchi T1

Background

$$ds^2 = -dt^2 + a^2(t)dx^2 + b^2(t)\delta_{ij}dx^i dx^j$$

$$A_\mu = \left(A_0(t), A_1(t), 0\ , 0 \right)$$

Perturbations

$$\delta A_0 = -A_0 A \qquad \delta A_1 = A_1 \pi_{,x} \qquad \delta A_i = -A_i^T + \chi_{,i} \qquad A_i^T = \varepsilon_{ij} v_{A,j}\,.$$

$$\delta g_{00} = 2\phi \qquad \delta g_{0x} = \omega_{,x} \qquad \delta g_{oi} = v_i + B_{,i} \qquad v_i = \varepsilon_{ij} v_{,j}$$

$$\delta g_{xx} = a^2\psi \qquad \delta g_{xi} = \lambda_{i,x} + \mu_{,xi} \qquad \lambda_i = \varepsilon_{ij}\lambda_{,j} \qquad \delta g_{ij} = b^2(2\tau\delta_{ij} + 2E_{,ij} + h_{i,j} + h_{j,i}), \qquad h_i = \varepsilon_{ij} h_{,j}$$

Bianchi Type I zoo

Even modes

X direction

2 stable modes

2 forward propagating modes $(c_s - A)^2$

2 modes with modified dispersion relation $\omega_k \sim \frac{k^2}{a(t)^2}$

Isotropic subspace

2 stable modes

2 unstable modes $c_s^2 < 0$

2 modes with $c_s^2 = 0$

Odd modes

X direction

2 totally fine modes!

Isotropic subspace

2 totally fine modes AGAIN!

For no-ghost conditions that are satisfied.

So should we forget about this theory?

Discussion

1. FLRW - speed of propagation is 0

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2. Bianchi T1 - speed of propagation is 0
in the subspace

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➔ Reduce the isotropy?

➔ Unstable modes
in the isotropic subspace

➔ More of them appear
or maybe all dissapear!

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2. Bianchi T1 - speed of propagation is 0
in the subspace

→ Strong couplingS

$$NG \sim A_0^2 \left(\beta_1 + \frac{\beta_2}{2}\right)^2$$

→ Reduce the isotropy?

→ Unstable modes
in the isotropic subspace

→ More of them appear
or maybe all dissapear!

Exciting but crazy complicated . . .

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Thank you!

Backup: Proca in flat space-time

$$\mathcal{L}_P = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu$$

$$(A_0, A_i) \qquad A_i = A_i^T + \chi_{,i} \qquad A_{i,i}^T = 0$$

$$\mathcal{L}_P = \frac{1}{2}\Big[A_0\left(-\Delta + m^2\right)A_0 + 2A_0\Delta\dot{\chi} - \left(\dot{\chi}\Delta\dot{\chi} - m^2\chi\Delta\chi\right) + \left(\dot{A}_i^T\dot{A}_i^T - A_{i,j}^T A_{i,j}^T - m^2 A_i^T A_i^T\right)\Big]$$

$$(-\Delta + m^2)A^0 = -\Delta\dot{\chi}$$

$$\rightarrow \quad A_0 = \frac{-\Delta}{-\Delta + m^2}\dot{\chi}$$

$$\mathcal{L}_P = -\frac{1}{2}\left[A_i^T(-\Box + m^2)A_i^T + \chi(-\Box + m^2)\frac{m^2(-\Delta)}{-\Delta + m^2}\chi\right]$$

→ **3 DOF**

Backup: Homogeneous and isotropic universe

Background

$$ds^2 = -dt^2 + a^2(t)dx^i dx^j \qquad A_\mu = \left(A_0(t), 0, 0, 0\right)$$

Equations of motion

$$6\left(\left(\beta_1 + \beta_2\right) A_0^2 + M_P^2\right) H^2 + 12\dot{A}_0 \left(\beta_1 + \frac{\beta_2}{2}\right) A_0 H + \left(A_0^2 m^2 - 2\Lambda M_P^2\right) = 0$$

$$3\left(\left(\beta_2 + \beta_1\right) A_0^2 + M_P^2\right) H^2 + 4A_0\dot{A}_0 \left(\beta_2 + \beta_1\right) H + 2\left(\left(\beta_2 + \beta_1\right) A_0^2 + M_P^2\right) \dot{H} + A_0 \left(\beta_2 + 2\beta_1\right) \ddot{A}_0 + \frac{m^2}{2} A_0^2 + \dot{A}_0^2 \beta_2 - \Lambda M_P^2 + 2\dot{A}_0^2 \beta_1 = 0$$

$$[3\left(\beta_2 + 2\beta_1\right) \dot{H} + 3\left(4\beta_1 + \beta_2\right) H^2 + m^2]A_0 = 0.$$

Backup: Homogeneous and isotropic universe

Equations of motion

$$\dot{H} = -\frac{12\beta_1 H^2 + 3H^2\beta_2 + m^2}{6\beta_1 + 3\beta_2}$$

$$H = \frac{m \tanh\left(-\frac{\sqrt{-12\beta_1 - 3\beta_2}\, m(t - t_0)}{6\beta_1 + 3\beta_2}\right)}{\sqrt{-12\beta_1 - 3\beta_2}}$$

$$\dot{A}_0 = \frac{-3\left(\left(\beta_1 + \beta_2\right) H^2 - m^2\right) A_0^2 + M_P^2\left(-3H^2 + \Lambda\right)}{3\left(2\beta_1 + \beta_2\right) H A_0}$$

→ **Can also solve for the temporal component**

$4\beta_1 + \beta_2 < 0$ & $2\beta_1 + \beta_2 < 0$