

# Time Ordered Kinematic Flow

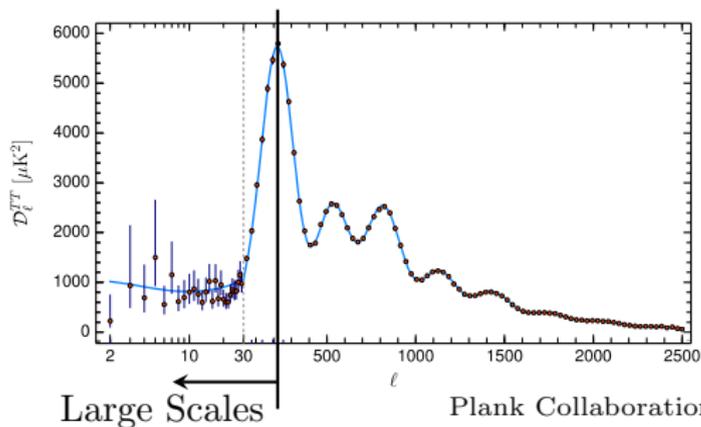
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Based on work with Daniel Baumann, Claudia Fevola, Austin Joyce, Martina Juhnke-Kubitzke, Hayden Lee, Guilherme Pimentel and Tom Westerdijk

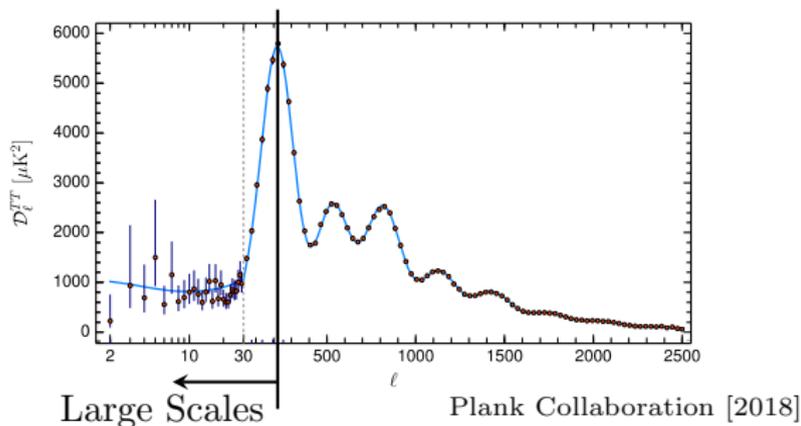
# Why do we care about Inflation?

- ▶ We know that physics depends on energy scales
- ▶ The universe began much hotter and denser than it is today
- ▶ Perturbations in the universe today are primordial



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- ▶ Quantum gravity requires we understand all spacetimes
- ▶ There may be interesting structures to uncover

# Outline

Cosmological Correlators

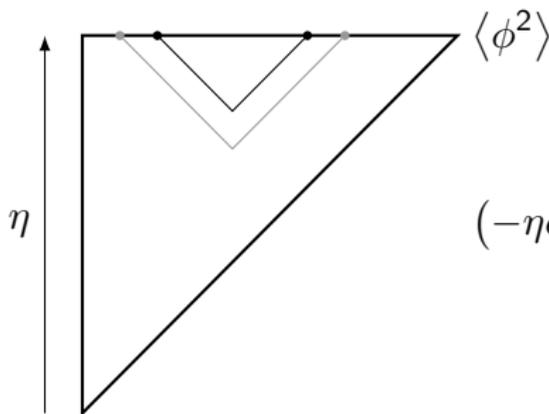
Differential Equations

Geometric Structure

Outlook

# The Reheating Surface

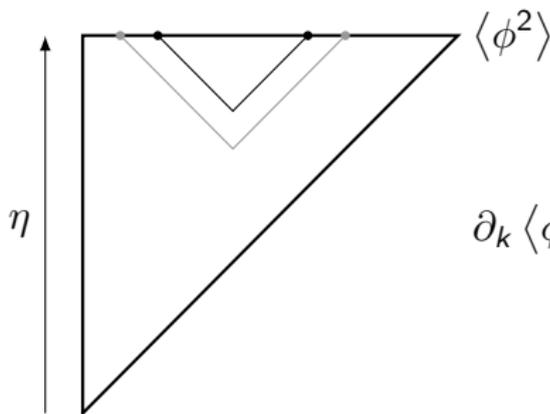
- ▶ We only measure correlation functions at the end of inflation
- ▶ The properties in the bulk are encoded through kinematics
- ▶ This encoding is the goal of the **cosmological bootstrap**
- ▶ As an illustrative example consider the two-point correlator



$$(-\eta\partial_\eta + 3 + k^i\partial_{k_i}) \langle \phi^2 \rangle = 0$$

# The Reheating Surface

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$$\partial_k \langle \phi^2 \rangle = \frac{(2\Delta - 3)}{k} \langle \phi^2 \rangle$$
$$\Rightarrow \langle \phi^2 \rangle \propto k^{2\Delta - 3}$$

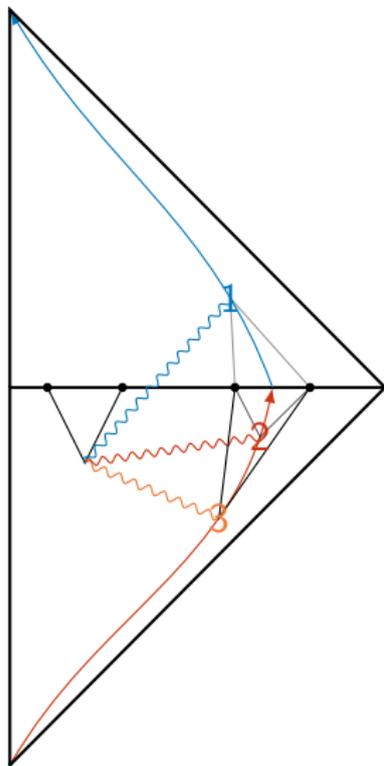
# Higher Point Correlators—Bulk Calculation

- ▶ We evolve the bulk from the vacuum in the past to the reheating surface and then back
- ▶ At the second vertex the physics differs:
  1. Annihilation to vacuum
  2. Particle decay
  3. Vacuum production
- ▶ We will treat each of these cases separately:

$$\langle \phi^4 \rangle_1 = \int d\eta_1 F_1 \int_{-\infty}^0 d\eta_2 \eta_2^{\alpha_2} H_\nu^{(1)}(Y\eta_2) e^{-i(k_3+k_4)\eta_2}$$

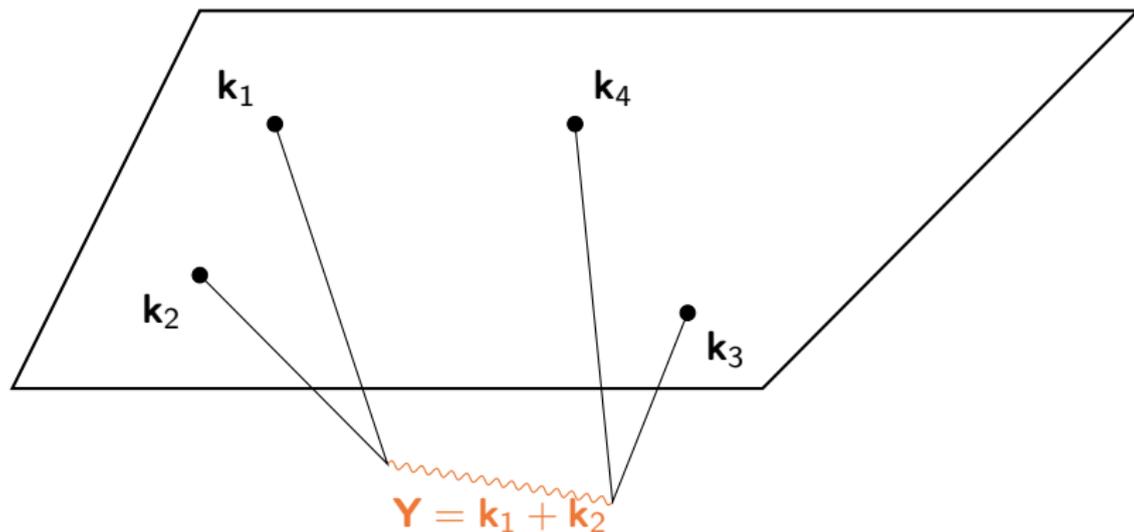
$$\langle \phi^4 \rangle_2 = \int d\eta_1 F_2 \int_{\eta_1}^0 d\eta_2 \eta_2^{\alpha_2} H_\nu^{(1)}(Y\eta_2) e^{i(k_3+k_4)\eta_2}$$

$$\langle \phi^4 \rangle_3 = \int d\eta_1 F_3 \int_{-\infty}^{\eta_1} d\eta_2 \eta_2^{\alpha_2} H_\nu^{(2)}(Y\eta_2) e^{i(k_3+k_4)\eta_2}$$



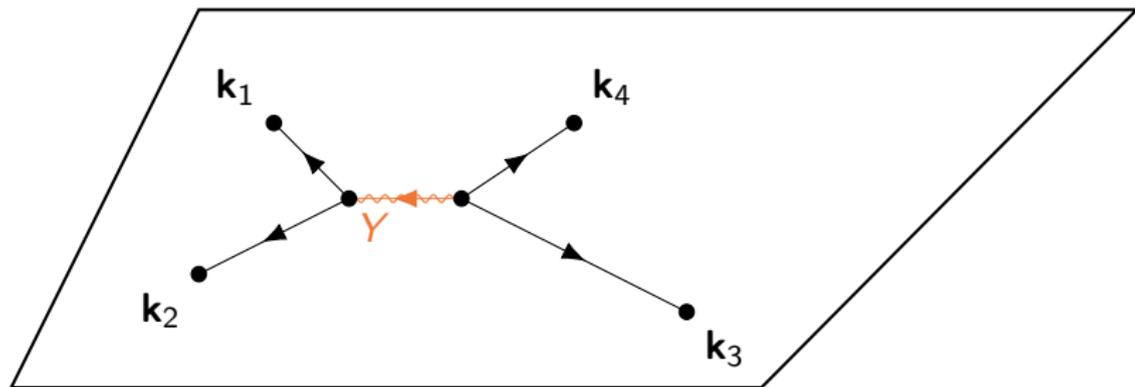
## Moving to the Boundary

- ▶ Consider how this interaction looks on the reheating surface



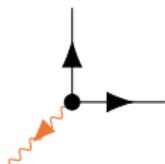
## Moving to the Boundary

- ▶ We still need to specify the time ordering

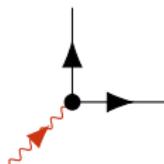


# Tubes

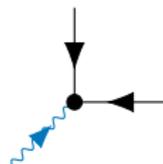
- ▶ The energy dependence comes from the signed sum of energies at a vertex



$$k_3 + k_4 + Y$$



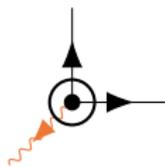
$$k_3 + k_4 - Y$$



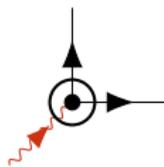
$$-k_3 - k_4 - Y$$

# Tubes

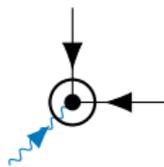
- ▶ We will represent this through a “tube” that defines a subset of vertices



$$k_3 + k_4 + Y$$



$$k_3 + k_4 - Y$$



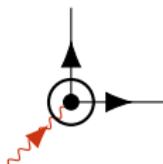
$$-k_3 - k_4 - Y$$

# Tubes

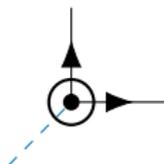
- ▶ We won't be sensitive to the overall sign of this term so we drop it



$$k_3 + k_4 + Y$$



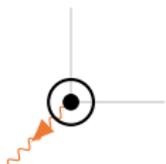
$$k_3 + k_4 - Y$$



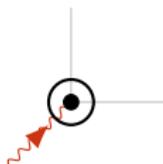
$$k_3 + k_4 + Y$$

# Tubes

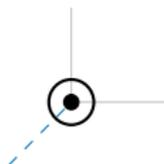
- ▶ As we define the external energy entering each vertex there is no need to explicitly draw those edges



$$k_3 + k_4 + Y$$



$$k_3 + k_4 - Y$$



$$k_3 + k_4 + Y$$

# Toy Model

- ▶ We choose to simplify the problem by looking at a toy model in the hope that we can gain insight into the general problem
- ▶ We assume a power law FRW background

$$ds^2 = \left( \frac{\eta}{\eta_0} \right)^{-2(1+\epsilon)} (-d\eta^2 + dx_i dx^i)$$

- ▶ And consider an interacting conformally coupled scalar

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{12} R \phi^2 - \frac{\lambda}{3!} \phi^3 \right]$$

# Toy Model

- ▶ We choose to simplify the problem by looking at a toy model in the hope that we can gain insight into the general problem
- ▶ We assume a power law FRW background

$$ds^2 = \left( \frac{\eta}{\eta_0} \right)^{-2(1+\epsilon)} (-d\eta^2 + dx_i dx^i)$$

- ▶ We can perform a Weyl transformation to simplify the action

$$S = \int d^4x \left[ -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{\lambda}{3!} (-\eta)^{-1-\epsilon} \phi^3 \right]$$

# Differentiation

- ▶ Consider differentiation of  $\langle \phi^4 \rangle_1$  with respect to  $X_2 = k_3 + k_4$

$$\langle \phi^4 \rangle_1 = \bullet \text{---} \bullet = \int \frac{d\eta_1}{\eta_1^{1+\epsilon}} e^{i(X_1+Y)\eta_1} \int_{-\infty}^0 \frac{d\eta_2}{\eta_2^{1+\epsilon}} e^{-i(X_2+Y)\eta_2}$$

# Differentiation

- ▶ We can rewrite the momentum derivative as a time derivative

$$\partial_{X_2} \bullet \text{---} \bullet = \frac{-1}{X_2 + Y} \int \frac{d\eta_1}{\eta_1^{1+\epsilon}} e^{i(X_1+Y)\eta_1} \int_{-\infty}^0 \frac{d\eta_2}{\eta_2^\epsilon} \partial_{\eta_2} e^{-i(X_2+Y)\eta_2}$$

# Differentiation

- ▶ Finally we integrate by parts

$$\partial_{X_2} \bullet \text{---} \bullet = \frac{\epsilon}{X_2 + Y} \int \frac{d\eta_1}{\eta_1^{1+\epsilon}} e^{i(X_1+Y)\eta_1} \int_{-\infty}^0 \frac{d\eta_2}{\eta_2^{1+\epsilon}} e^{-i(X_2+Y)\eta_2}$$

# Differentiation

- ▶ We can write this expression graphically, using tubes

$$\partial_{x_2} \bullet \text{---} \bullet = \frac{\epsilon}{\bullet \text{---} \odot} \bullet \text{---} \bullet$$

# Differentiation

- ▶ Next consider vacuum production at the second vertex

$$\langle \phi^4 \rangle_3 = \bullet \leftarrow \bullet = \int \frac{d\eta_1}{\eta_1^{1+\epsilon}} e^{i(X_1 - Y)\eta_1} \int_{-\infty}^{\eta_1} \frac{d\eta_2}{\eta_2^{1+\epsilon}} e^{i(X_2 + Y)\eta_2}$$

# Differentiation

- ▶ The extra boundary term is just a contact interaction

$$\partial_{x_2} \bullet \leftarrow \bullet = \frac{\epsilon}{\bullet \leftarrow \odot} \bullet \leftarrow \bullet - \frac{1}{\bullet \leftarrow \odot} \bullet \bullet$$

# Differentiation

- ▶ The final term has the intermediate particle decaying

$$\langle \phi^4 \rangle_2 = \bullet \rightarrow \bullet = \int \frac{d\eta_1}{\eta_1^{1+\epsilon}} e^{i(X_1+Y)\eta_1} \int_{\eta_1}^0 \frac{d\eta_2}{\eta_2^{1+\epsilon}} e^{i(X_2-Y)\eta_2}$$

# Differentiation

- ▶ The different letter comes from its tube

$$\partial_{x_2} \bullet \rightarrow \bullet = \frac{\epsilon}{\bullet \rightarrow \odot} \bullet \rightarrow \bullet + \frac{1}{\bullet \rightarrow \odot} \bullet \bullet$$

# Differentiation

- ▶ We have been forced to introduce a single new function

$$\langle \phi^4 \rangle_4 = \bullet\bullet = \int \frac{d\eta}{\eta^{1+2\epsilon}} e^{i(X_1+X_2)\eta}$$

# Differentiation

- ▶ Its derivative can be read off straightforwardly

$$\partial_{X_2} \bullet\bullet = \frac{2\epsilon}{\textcircled{\bullet\bullet}} \bullet\bullet$$

- ▶ The  $X_1 = k_1 + k_2$  derivatives can be deduced by symmetry

# Four-point Function

- ▶ We represent each of our initial functions by a dot



# Four-point Function

- ▶ The first function just returns itself when we differentiate



# Four-point Function

- ▶ The others both produce the new, collapsed function



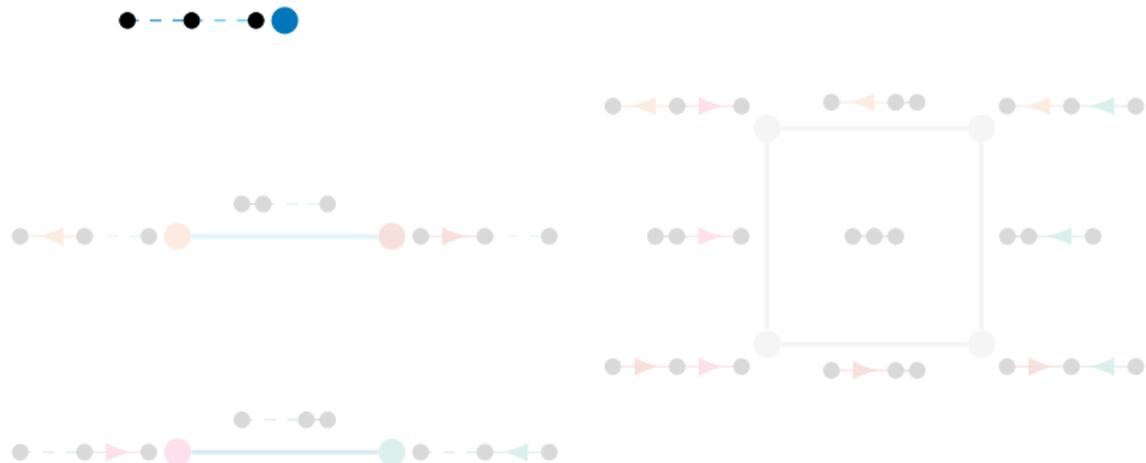
# Four-point Function

- ▶ We represent this by joining the two points with a line

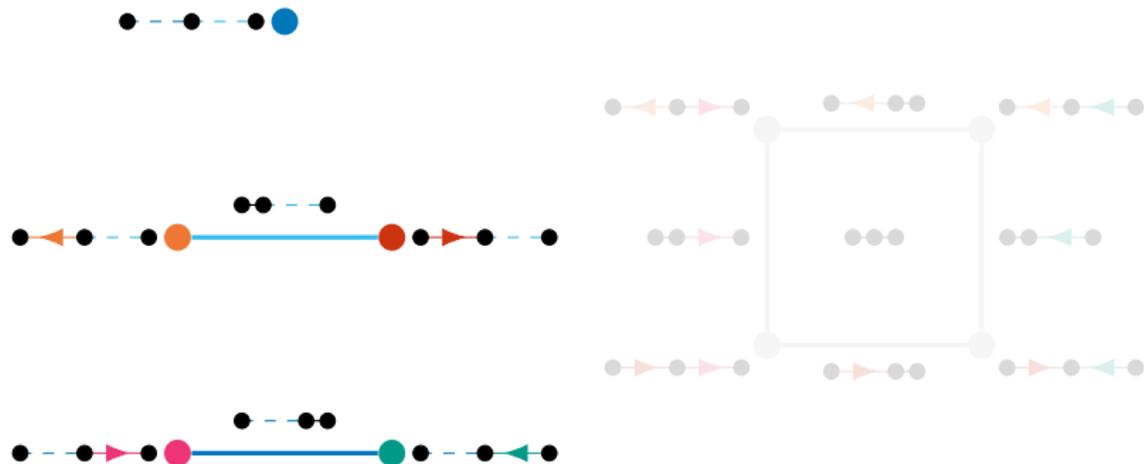


- ▶ Derivatives are local in this space
- ▶ The total correlator is the sum of the vertices
- ▶ At higher points we get more complicated relationships

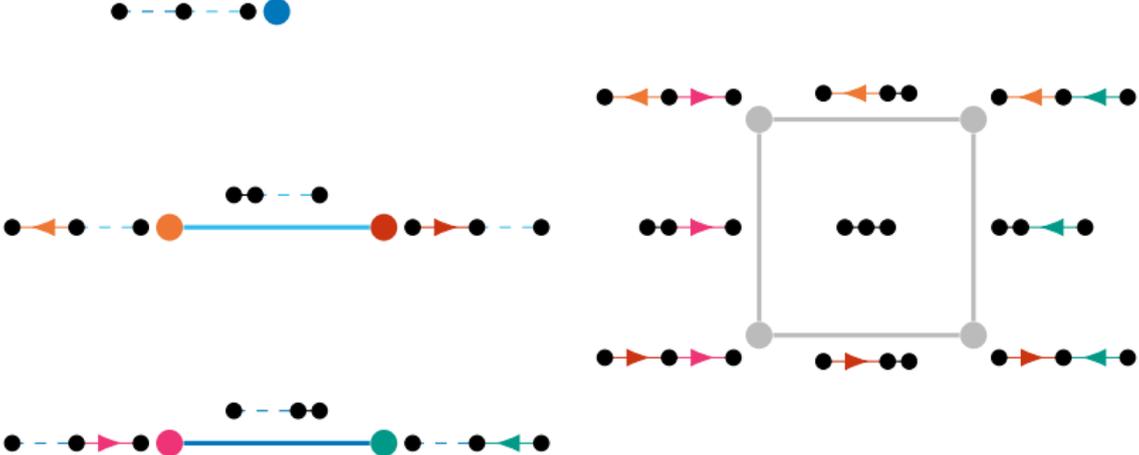
## More Complex Graphs: 5-point function



# More Complex Graphis: 5-point function



# More Complex Graphis: 5-point function



# Summary

- ▶ By considering the bulk basis we simply deduce the kinematic dependence of our correlators
- ▶ We also reveal hidden structures and subspaces that are not obvious from the time integrals themselves
- ▶ There is still much to explore at both a fundamental and technical level:
  - ▶ Massive Fields
  - ▶ Loops
  - ▶ Connecting together diagrams
- ▶ Thank you for listening

# Total Derivatives

- ▶ All kinematic derivatives can be summed up in a single equation using the total derivative

$$d \begin{array}{|c|} \hline \diagdown \\ \hline \end{array} = \left( \epsilon \operatorname{dlog} \begin{array}{|c|} \hline \diagdown \\ \hline \end{array} + \epsilon \operatorname{dlog} \begin{array}{|c|} \hline \diagdown \\ \hline \end{array} \right) \begin{array}{|c|} \hline \diagdown \\ \hline \end{array}$$

$$+ \left( -\operatorname{dlog} \begin{array}{|c|} \hline \diagdown \\ \hline \end{array} + \operatorname{dlog} \begin{array}{|c|} \hline \diagdown \\ \hline \end{array} \right) \begin{array}{|c|} \hline \\ \hline \end{array}$$

$$d \begin{array}{|c|} \hline \text{---} \diagdown \text{---} \\ \hline \end{array} = \left( \epsilon \operatorname{dlog} \begin{array}{|c|} \hline \text{---} \diagdown \text{---} \\ \hline \end{array} + \epsilon \operatorname{dlog} \begin{array}{|c|} \hline \text{---} \diagdown \text{---} \\ \hline \end{array} \right) \begin{array}{|c|} \hline \text{---} \diagdown \text{---} \\ \hline \end{array}$$

$$d \begin{array}{|c|} \hline \\ \hline \end{array} = \left( 2\epsilon \operatorname{dlog} \begin{array}{|c|} \hline \\ \hline \end{array} \right) \begin{array}{|c|} \hline \\ \hline \end{array}$$

# Massive Integrals

- ▶ If we exchange a massive particle the propagators are no longer exponential
- ▶ We can still attempt to integrate by parts in the same way which produces Hankel Functions of other orders
- ▶ Can we still generate a set of flow rules in this case?
- ▶ Will some other geometric structures arise?

# Loop Integrals

- ▶ We can perform this analysis in arbitrary dimension with the goal of performing our loop integrals in dim-reg
- ▶ The different structures in this system have very different complexity after performing a loop integral
- ▶ Can the full loop integral be understood through a new kinematic flow?

# Towards a Deeper Understanding

- ▶ These diagrams are geometrically very simple but does this reveal anything deeper about the physics?
- ▶ The differential equations couple together different diagrams can this be understood through some novel positive geometry?

