

Status of Fluctuation Analysis at CERN SPS Energies



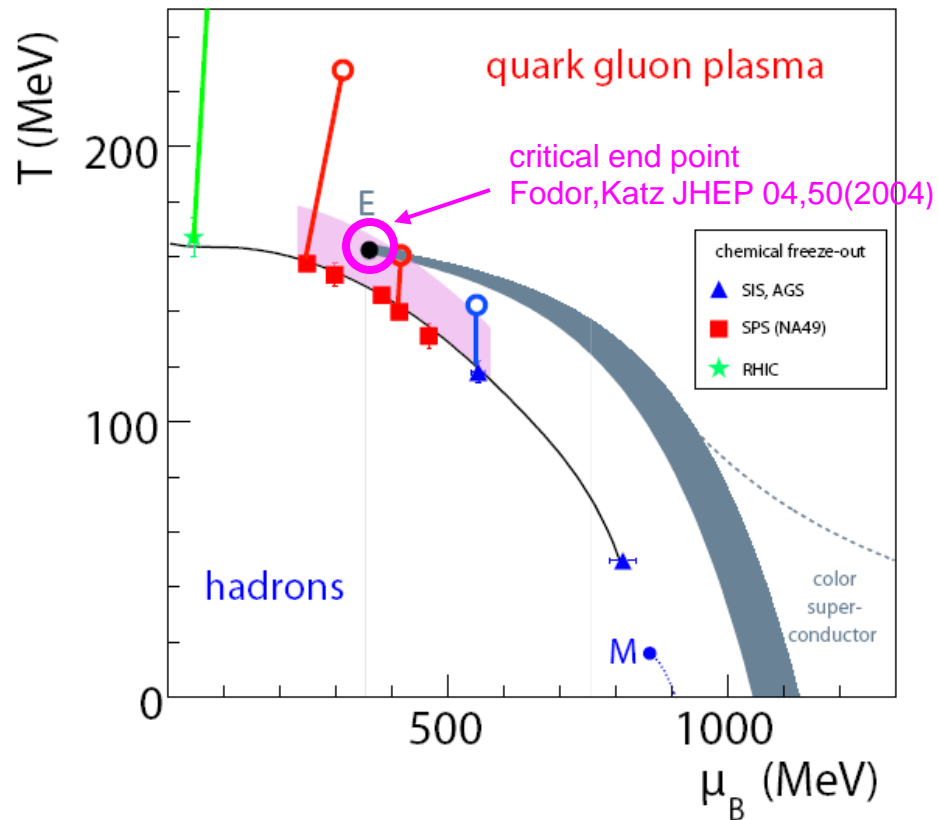
P. Seyboth
Max-Planck-Institut für Physik, München
and
Jan Kochanowski University, Kielce



- confirmation of onset of deconfinement
- results on fluctuations
 - particle ratios , NA49/STAR disagreements
 - search for critical point of strongly interacting matter
- future programs



Exploration of phase diagram of strongly interacting matter



only central collisions
are considered here

- QCD considerations suggest a 1st order phase boundary ending in a critical point
- hadro-chemical freeze-out points are obtained from statistical model fits to measured particle yields
- T and μ_B approach phase boundary and estimated critical point at SPS
- evidence of onset of deconfinement from rapid changes of hadron production properties
- search for indications of the critical point as a maximum in fluctuations

evidence for the onset of deconfinement (1)

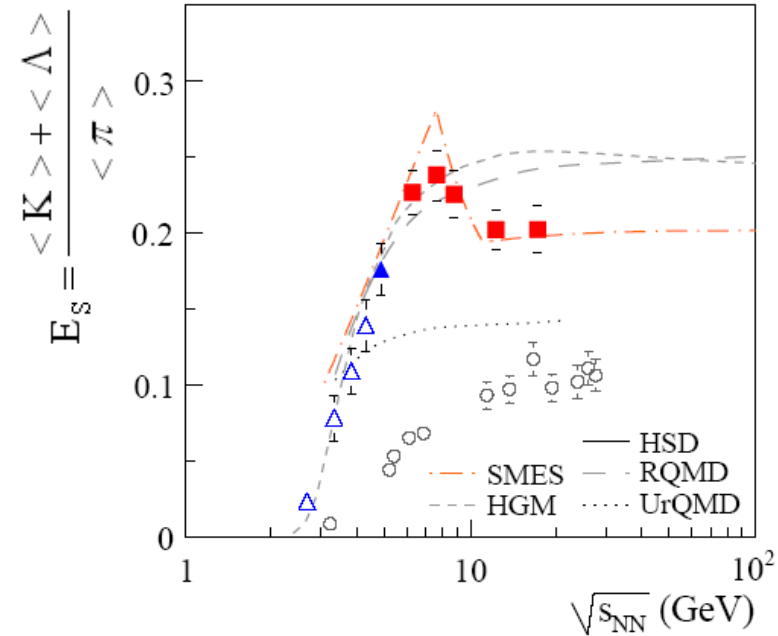
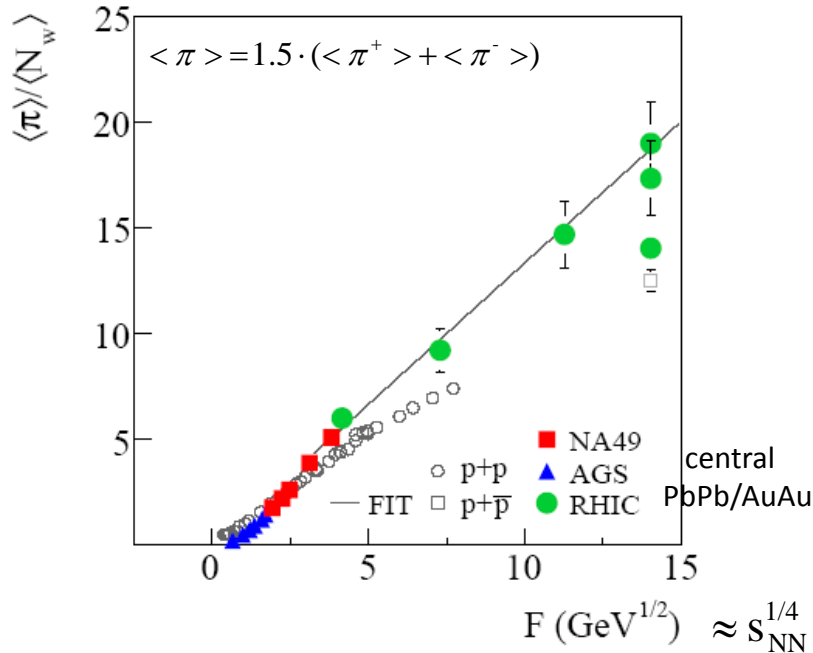
the kink

pion yield per participant

the horn

ratio of strange particle to pion yield

NA49,C.Alt et al.,PRC77,024903(2008)



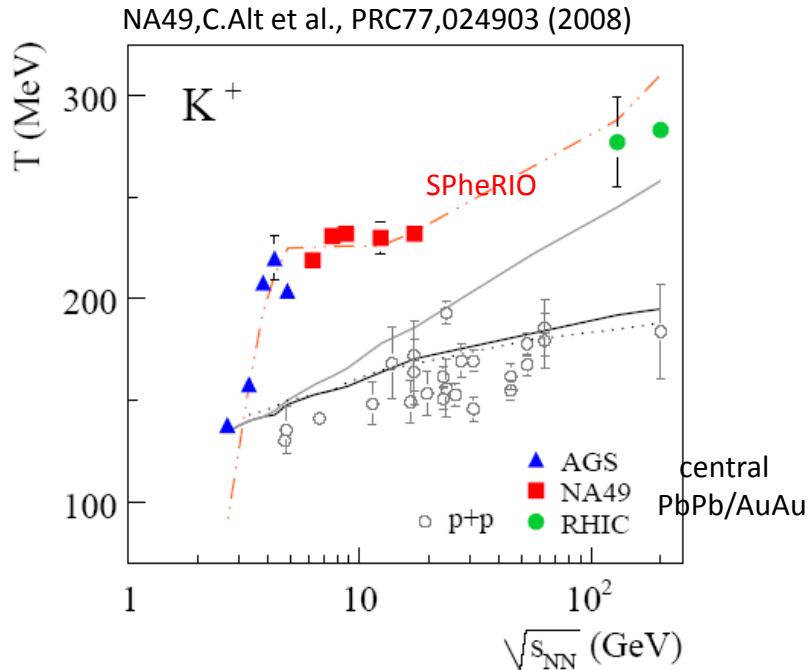
- π yield related to entropy production
- steeper increase in A+A suggests 3-fold increase of initial d.o.f

- E_s related to strangeness/entropy ratio
- plateau consistent with prediction for deconfinement

evidence for the onset of deconfinement (2)

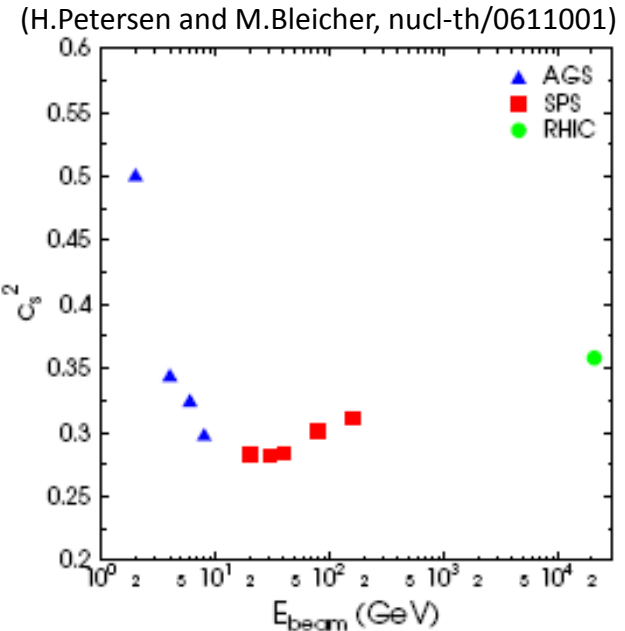
the step

shape of transverse mass spectra



the dale

estimate of sound velocity



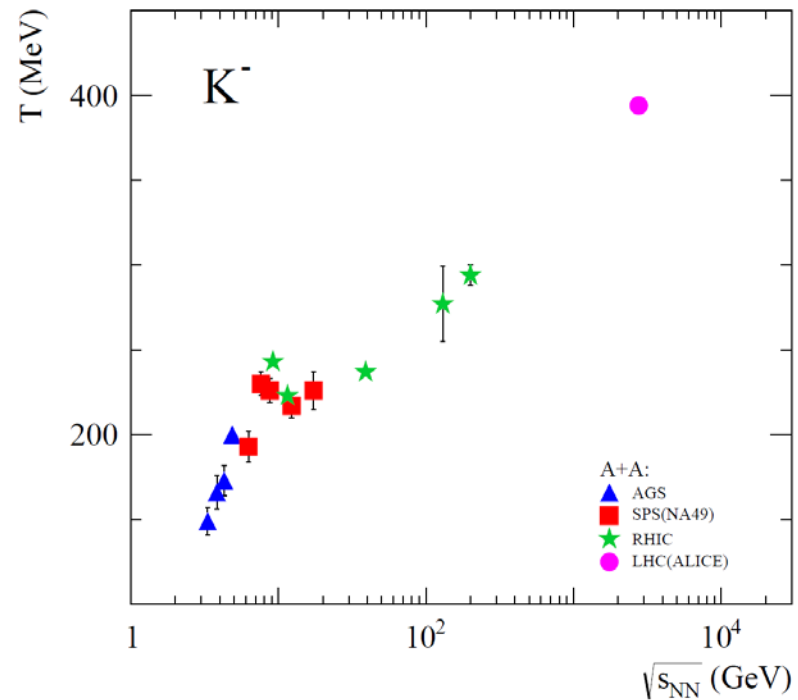
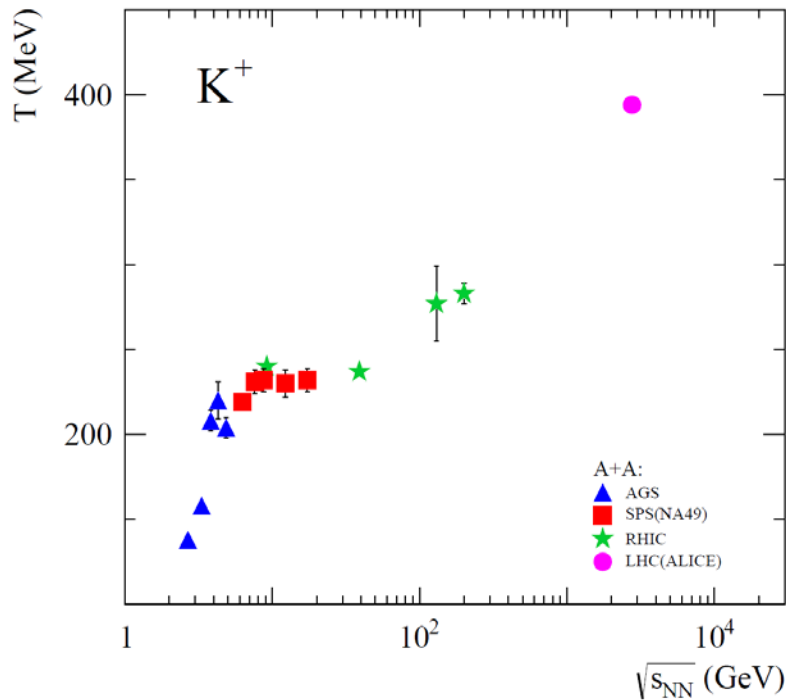
softening of transverse (step) and longitudinal (minimum of c_s) features of EoS due to mixed phase (soft point of EoS)

rapid changes of hadron production properties at low SPS energy most naturally explained by onset of deconfinement

NA49, C. Alt et al., PRC77,024903(2008); M. Gazdzicki et al., arXiv:1006.1765

confirmation by recent STAR and ALICE results (1)

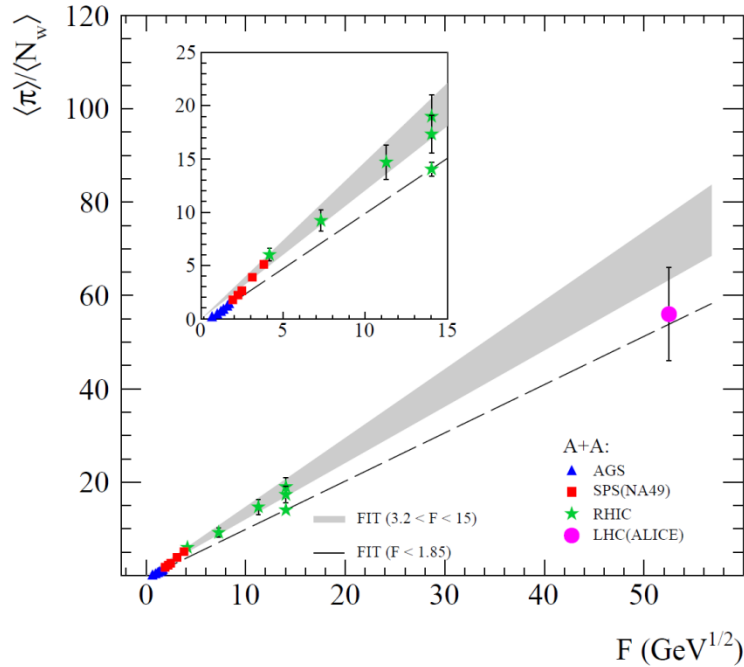
the step



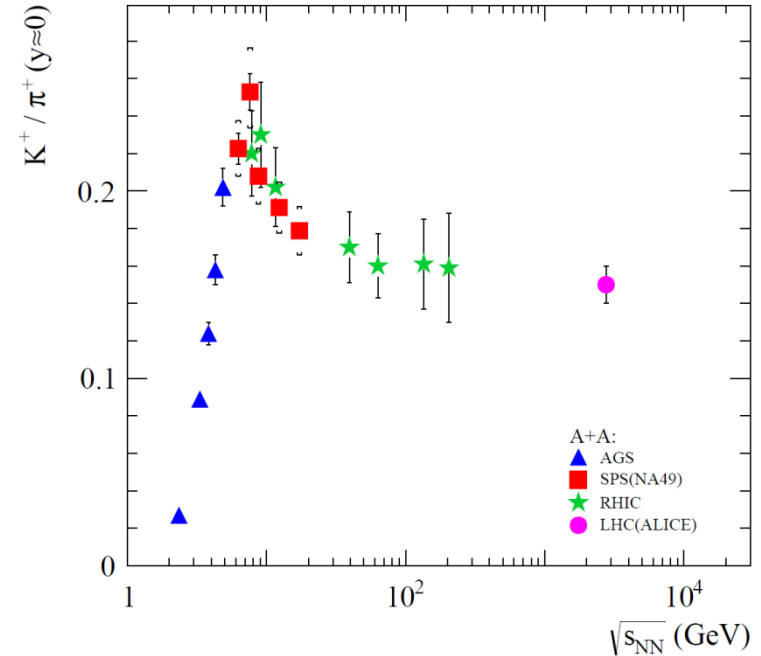
- STAR low energy results confirm the step
- continued rise for ALICE/LHC (increase of radial flow)

confirmation by recent STAR and ALICE results (2)

the kink



the horn



- estimate obtained from ALICE data not inconsistent with extrapolation
- preliminary STAR results confirm horn
- K/π constant above SPS as expected

onset of deconfinement at 30A GeV remains the most natural scenario

fluctuation measures studied by NA49

- σ_{dyn} measure of dynamical particle ratio fluctuations (K/π , p/π , K/p)

$$\sigma_{dyn} = \text{sign}(\sigma_{data}^2 - \sigma_{mix}^2) \sqrt{|\sigma_{data}^2 - \sigma_{mix}^2|} ; \quad \sigma_{dyn}^2 = |v_{dyn}|$$

- e-by-e fit of particle multiplicities required in NA49
- mixed events used as reference
- **$1/N_W$ dependence** V.Koch,T.Schuster PRC81,034910

- Φ_x measure of fluctuations of observable x ($\langle p_T \rangle$, $\langle \phi \rangle$, Q , ...)

$$\Phi_x = \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\langle Z^2 \rangle} ;$$

$$Z = x - \langle x \rangle ,$$

$$Z = \sum_{i=1}^N (x_i - \langle x \rangle)$$

M.Gazdzicki and S.Mrowczynski, Z.Phys.C54,127(1992)

- superposition model: $\Phi_x(A+A) = \Phi_x(N+N)$
- independent particle emission: $\Phi_x = 0$
- **Φ_x strongly intensive fluctuation measure independent of $\langle N_W \rangle$ and its fluctuations**

- scaled variance ω of the multiplicity distribution $P(n)$

$$\omega = \frac{\text{Var}(n)}{\langle n \rangle} = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle}$$

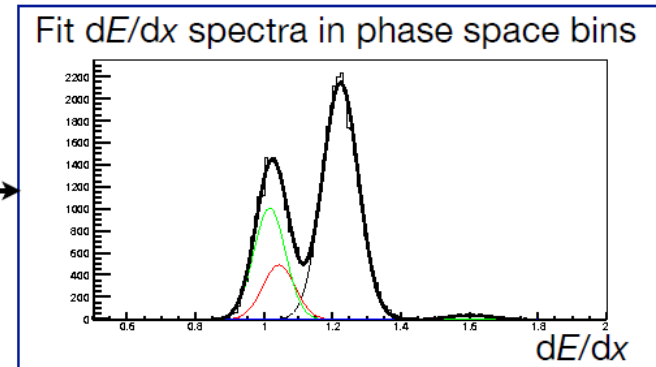
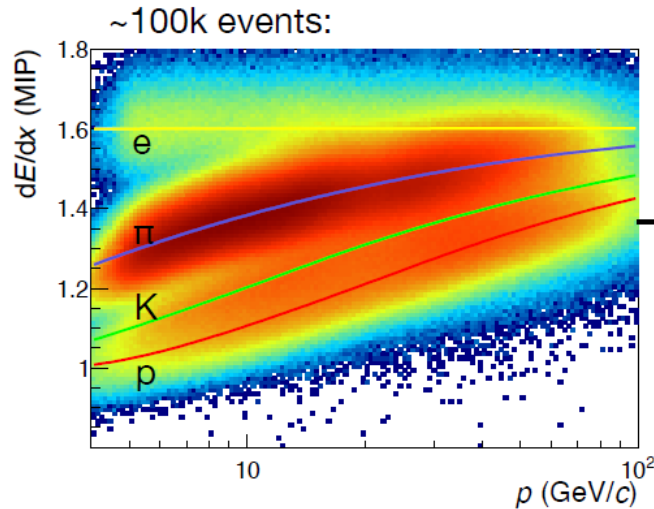
- intensive fluctuation measure
- independent particle emission: $\omega = 1$
- superposition model: $\omega(A+A) = \omega(N+N) + \langle N_W \rangle \omega_{NW}$
- **ω sensitive to fluctuations of N_W**

- intermittency of low mass $\pi^+\pi^-$ pair and proton number in p_T space



event-by-event identified particle ratio fluctuations

NA49 identification procedure

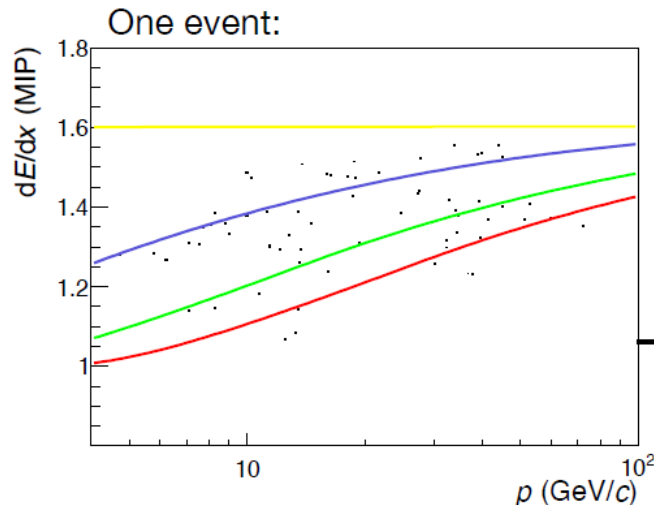


Inclusive probability density function ($p, dE/dx$)

$$F(\vec{p}, (dE/dx) | \Theta) = \sum_{(m)} r^{(m)} \left(\vec{p} | \Theta^{(m)} \right) f_p^{(m)}(dE/dx)$$

($m = \pi, K, p, \dots$)

Extract hadron multiplicities with maximum likelihood from each event

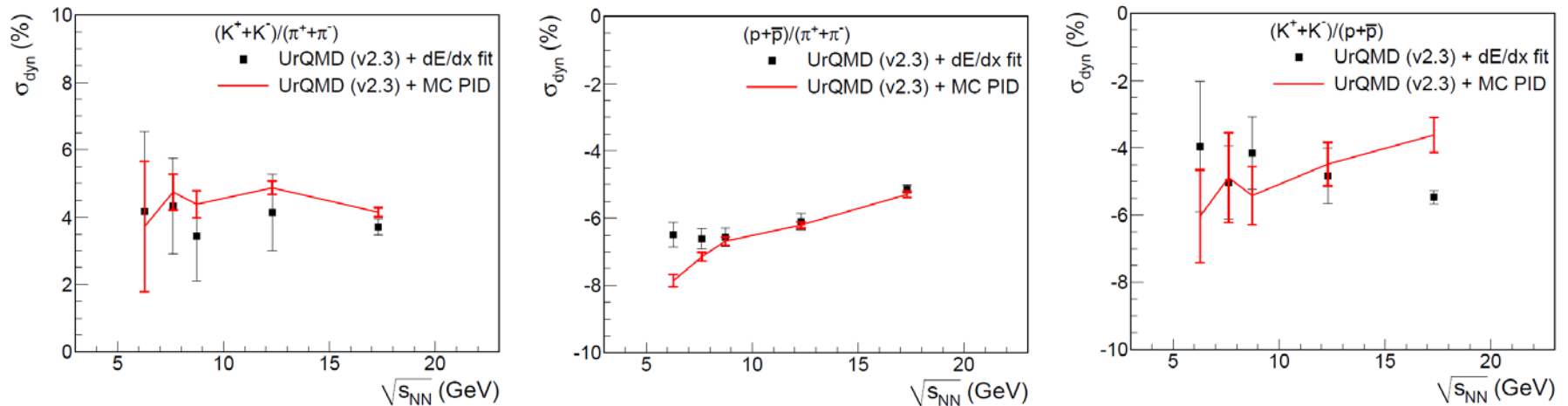


fit procedure introduces correlations

artificial correlations introduced by the fit procedure are quantified by applying the same analysis procedure to mixed events and subtracted

$$\sigma_{dyn} = \text{sign}(\sigma_{data}^2 - \sigma_{mix}^2) \sqrt{|\sigma_{data}^2 - \sigma_{mix}^2|}, \quad \sigma = \frac{\sqrt{\text{Var}(A/B)}}{\langle A/B \rangle}$$

UrQMD simulation demonstrates validity of the method:



differences mostly insignificant, taken into systematic errors

equivalence of σ_{dyn} and v_{dyn}

$$\sigma_{dyn}^2 \approx \left(\frac{\langle A(A-1) \rangle}{\langle A^2 \rangle} + \frac{\langle B(B-1) \rangle}{\langle B^2 \rangle} - 2 \frac{\langle AB \rangle}{\langle A \rangle \langle B \rangle} \right) = v_{dyn}$$

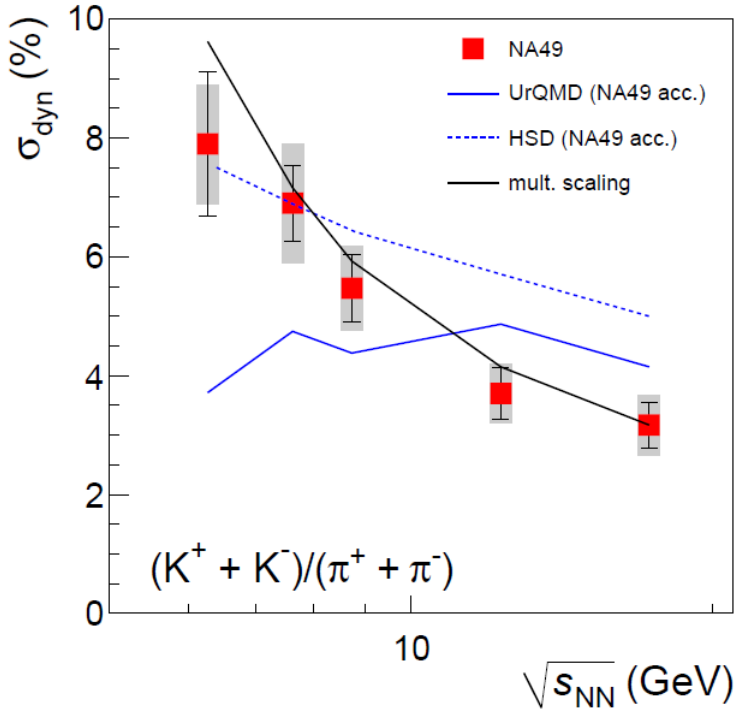
generic multiplicity dependence

Koch, Schuster PRC81,034910(2010)

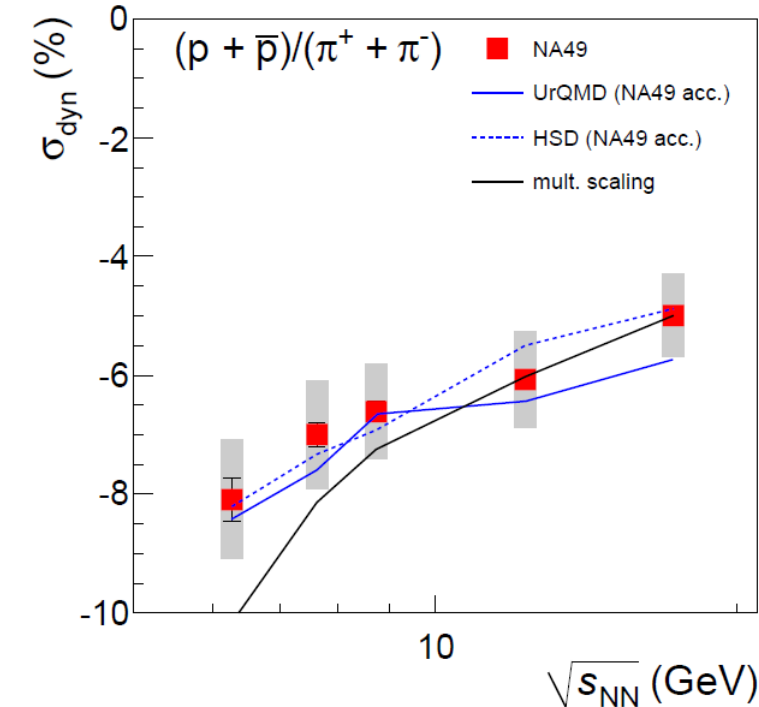
$$= \left(\frac{1}{\langle A \rangle} C_{AA} + \frac{1}{\langle B \rangle} C_{BB} - \frac{2}{\sqrt{\langle A \rangle \langle B \rangle}} C_{AB} \right)$$

experimental results from NA49

NA49 results: PRC79,044910(2009) for 3.5 % most central Pb+Pb collisions



generic multiplicity scaling :
Jeon,Koch PRL83,5435(1999)



$$\sigma_{\text{dyn}} \propto \sqrt{\frac{1}{\langle A \rangle} + \frac{1}{\langle B \rangle}} \quad (\text{"Poisson" scaling})$$

σ_{dyn} positive, rise towards low \sqrt{s}
effect of deconfinement ?

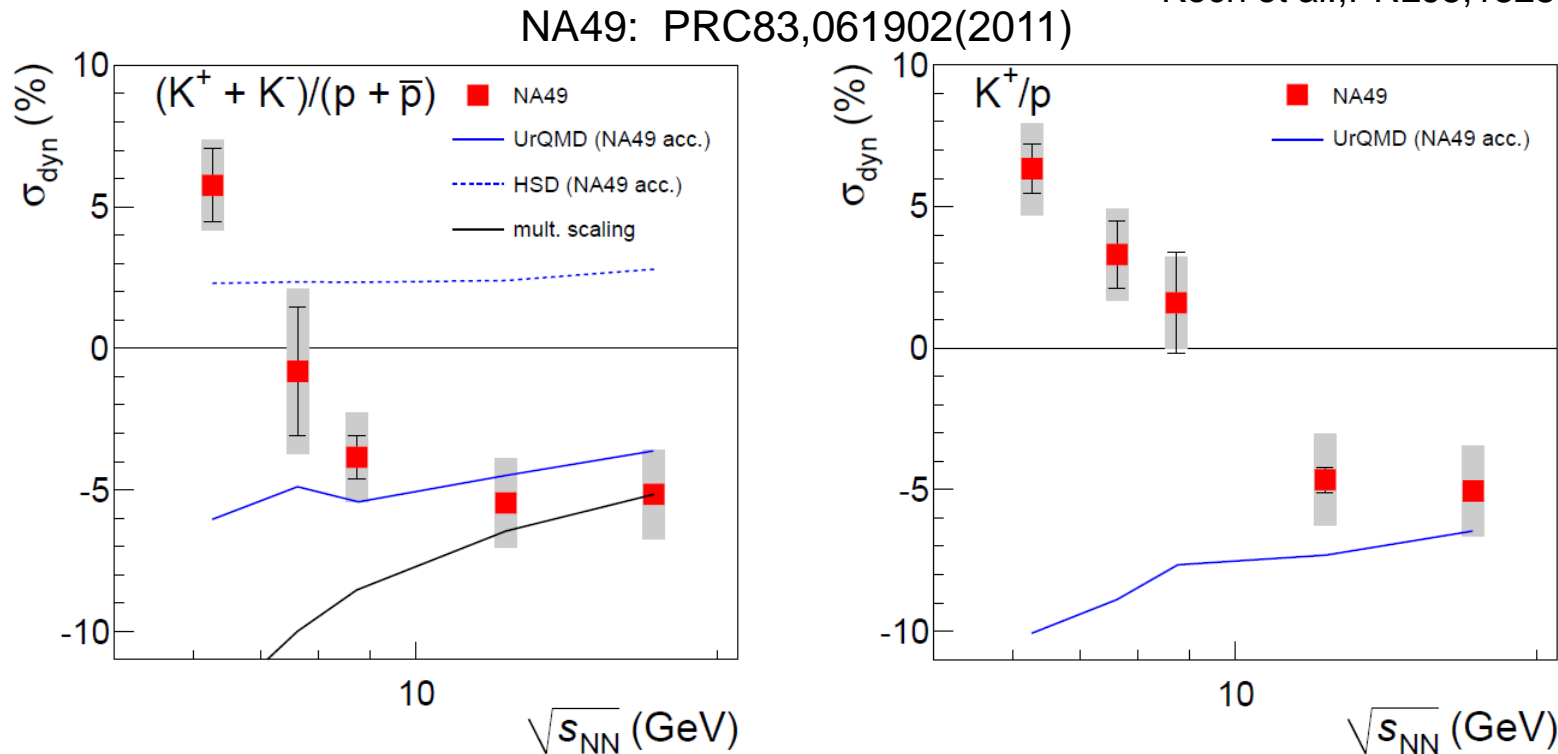
σ_{dyn} negative, rise towards high \sqrt{s}
effect of nucleon resonances

Gorenstein et al., PLB585,237



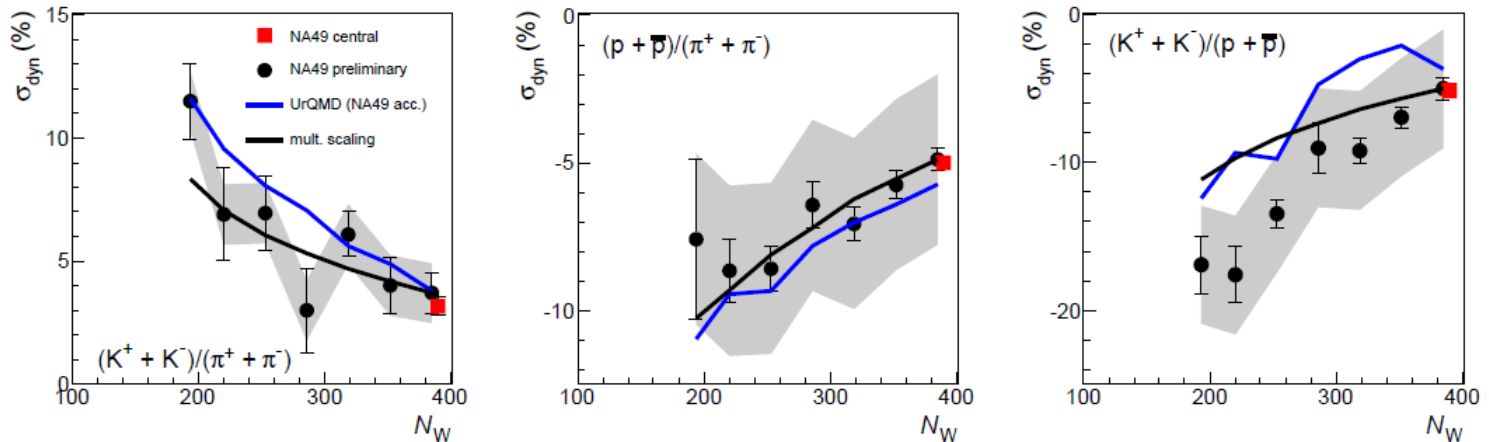
K/p fluctuations probe correlation of baryon number B and strangeness S
 hadron gas: production of S unrelated to B is allowed (strange mesons)
 deconfined: S produced in conjunction with B (quarks)
 Correlation coefficient C_{BS} can be estimated, precise relation to σ_{dyn} ?

Koch et al., PRL95,182301(2005)



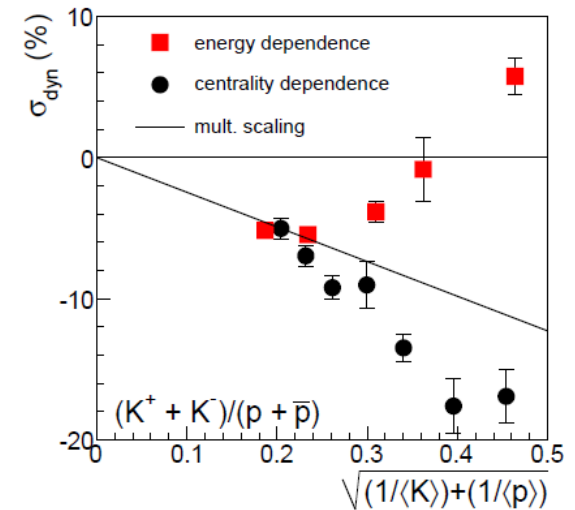
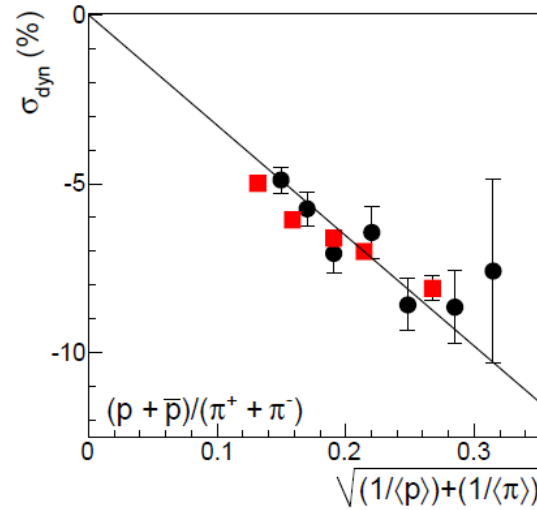
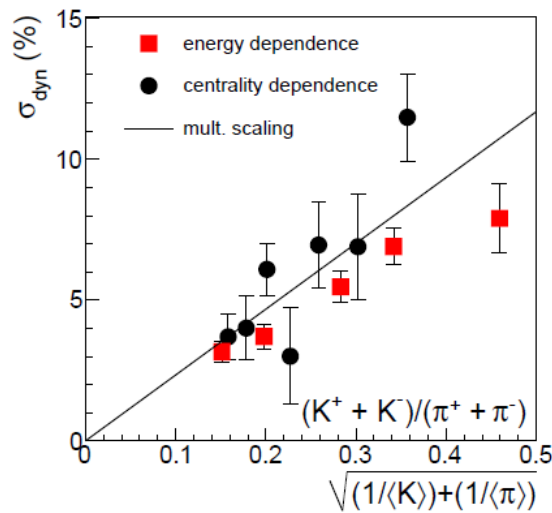
- generic multiplicity scaling ruled out, qualitative change of correlation
- sign change near 30A GeV related to onset of deconfinement ?

centrality dependence in Pb+Pb collisions at 158A GeV



- multiplicity scaling and UrQMD model describe fluctuations for K/π and p/π ratio
- disagreement for K/p ratio

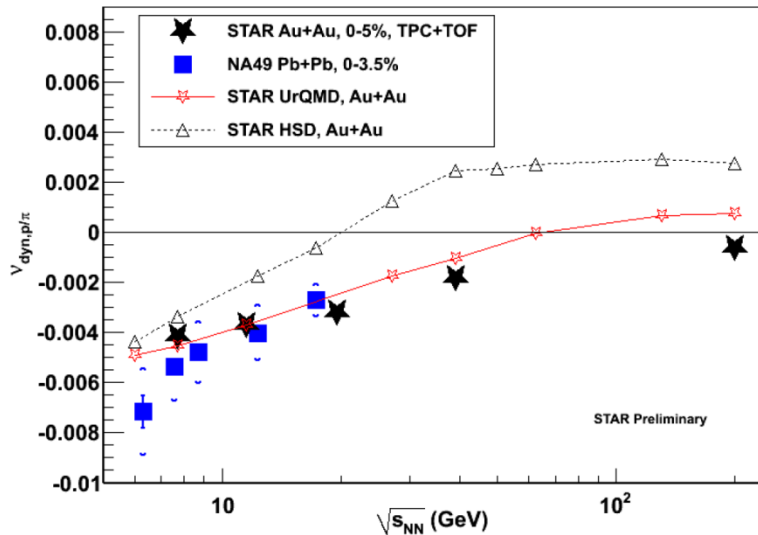
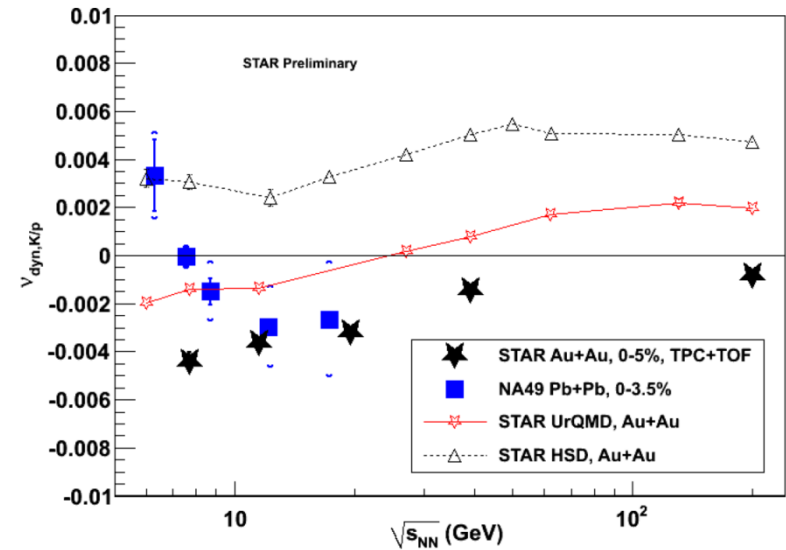
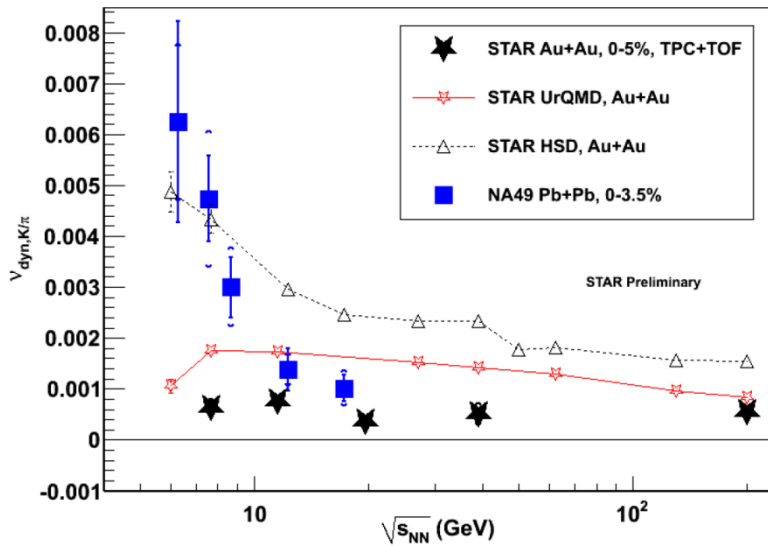
test of simultaneous multiplicity scaling for energy and centrality dependence



generic scaling

- works for K/π and p/π fluctuations
- cannot describe K/π fluctuations

discrepancy NA49 / STAR low energy results

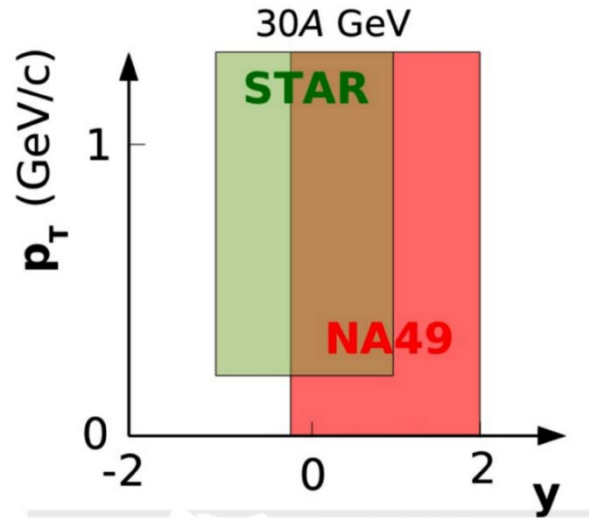


figures by T. Tarnowsky
(STAR, SQM2011)

conversion via: $V_{dyn} = \sigma_{dyn}^2$

- analysis procedures were carefully checked , no problems found
- NA49 and STAR acceptance and centrality selection differ significantly

schematic
sketch



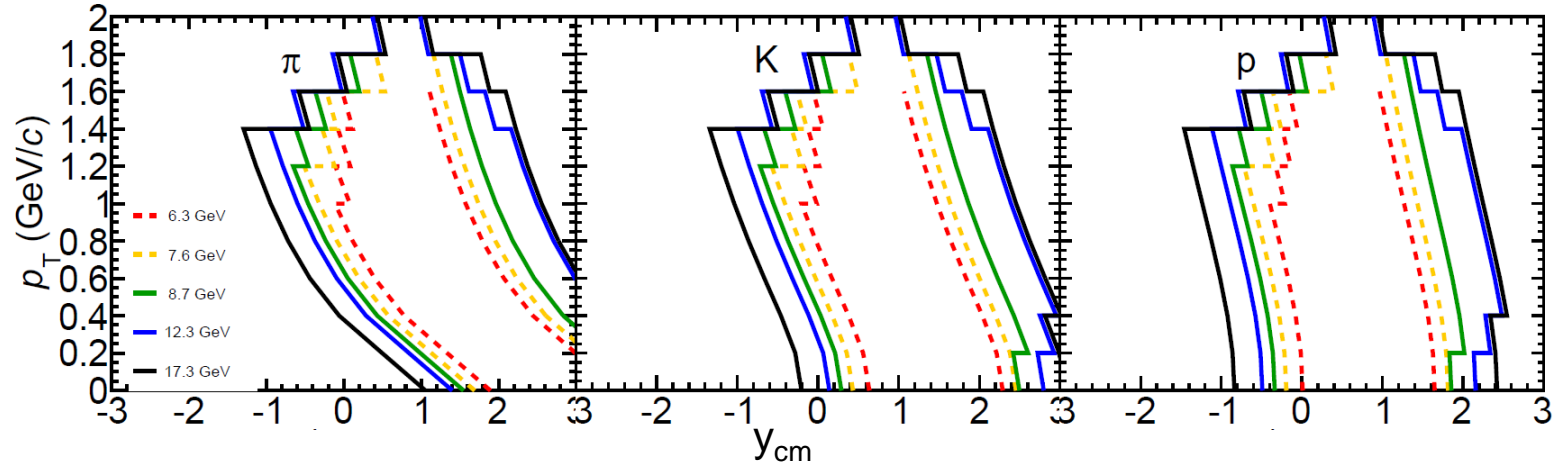
- study of discrepancies is continuing
- NA49 will apply new analysis procedure not requiring e-by-e fits
 - (1) deconvolution of effect of dE/dx resolution from first and second moments of the multiplicity distribution (identity method)
 - (2) calculation of strongly intensive fluctuation measure Ψ directly from the corrected moments

M.Gazdzicki et al. PRC83,054907(2011)

M.Gorenstein PRC84,024902(2011)

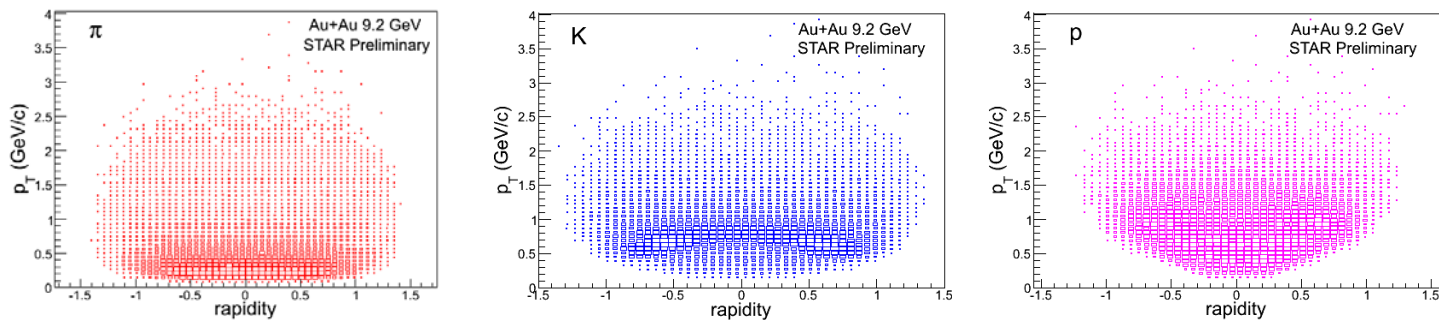
details of acceptance in NA49 and STAR

NA49:



slight energy dependence and up/down azimuthal wedge missing

STAR:



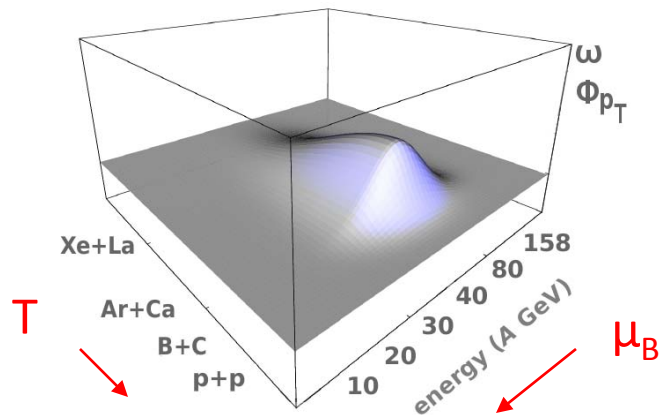
independent of energy and full azimuthal acceptance



critical point search strategy of NA49 and NA61

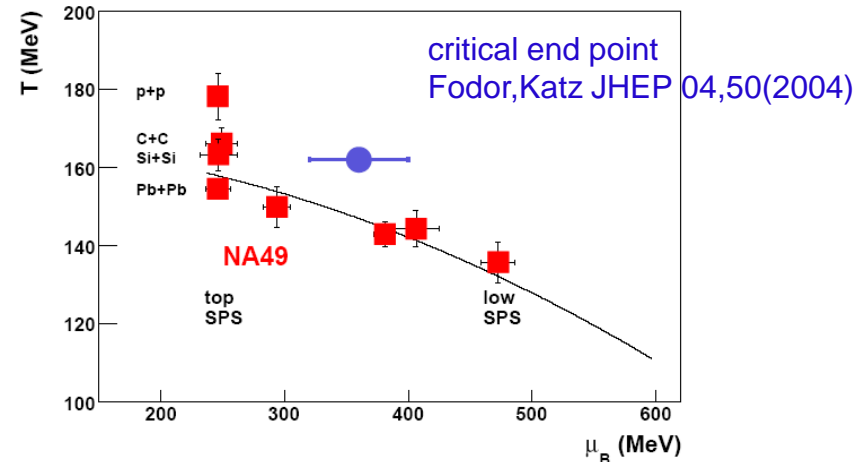
2-dimensional (T, μ_B) scan of phase diagram

expected “hill” of fluctuations



freeze-out points from stat. model

Becattini et al, PRC73, 044905 (2006)



- deconfinement necessary for observing CP effect (above 30A GeV)
- freeze-out occurs close to the critical point
- expected size of fluctuation signals ($\sim \xi^2$) limited by short lifetime and size of collision system (correlation lengths $\sim 3 - 6$ fm for Pb+Pb)

(M.Stephanov, K.Rajagopal, E.Shuryak, PRD60,114028(1999))

- freeze-out close enough to CP ?
- can fluctuation signals survive later fireball evolution ??

estimates of effects due to the critical point

correlation length ξ at the critical point not divergent but limited by finite size and lifetime of the fireball

parameterization: $\xi = \min(c_1 A^{1/3}, c_2 A^{1/9})$
size lifetime

(M.Stephanov, private comm.)

suggesting $\xi(\text{Pb+Pb}) = 3 \rightarrow 6 \text{ fm}$
 $\xi(\text{p+p}) = 1 \rightarrow 2 \text{ fm}$

range of correlation effect estimated from QCD calculations:

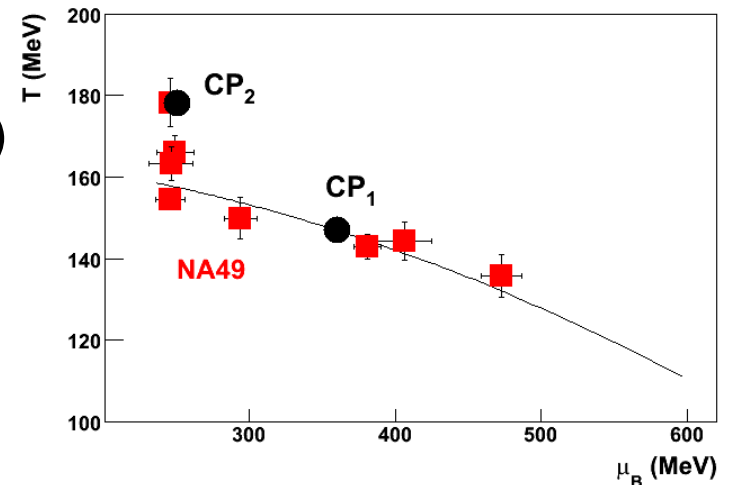
$$\sigma(\mu_B) = 30 \text{ MeV}, \sigma(T) = 10 \text{ MeV}$$

(Hatta,Ikeda,PRD67,014028(2003))

considered examples:

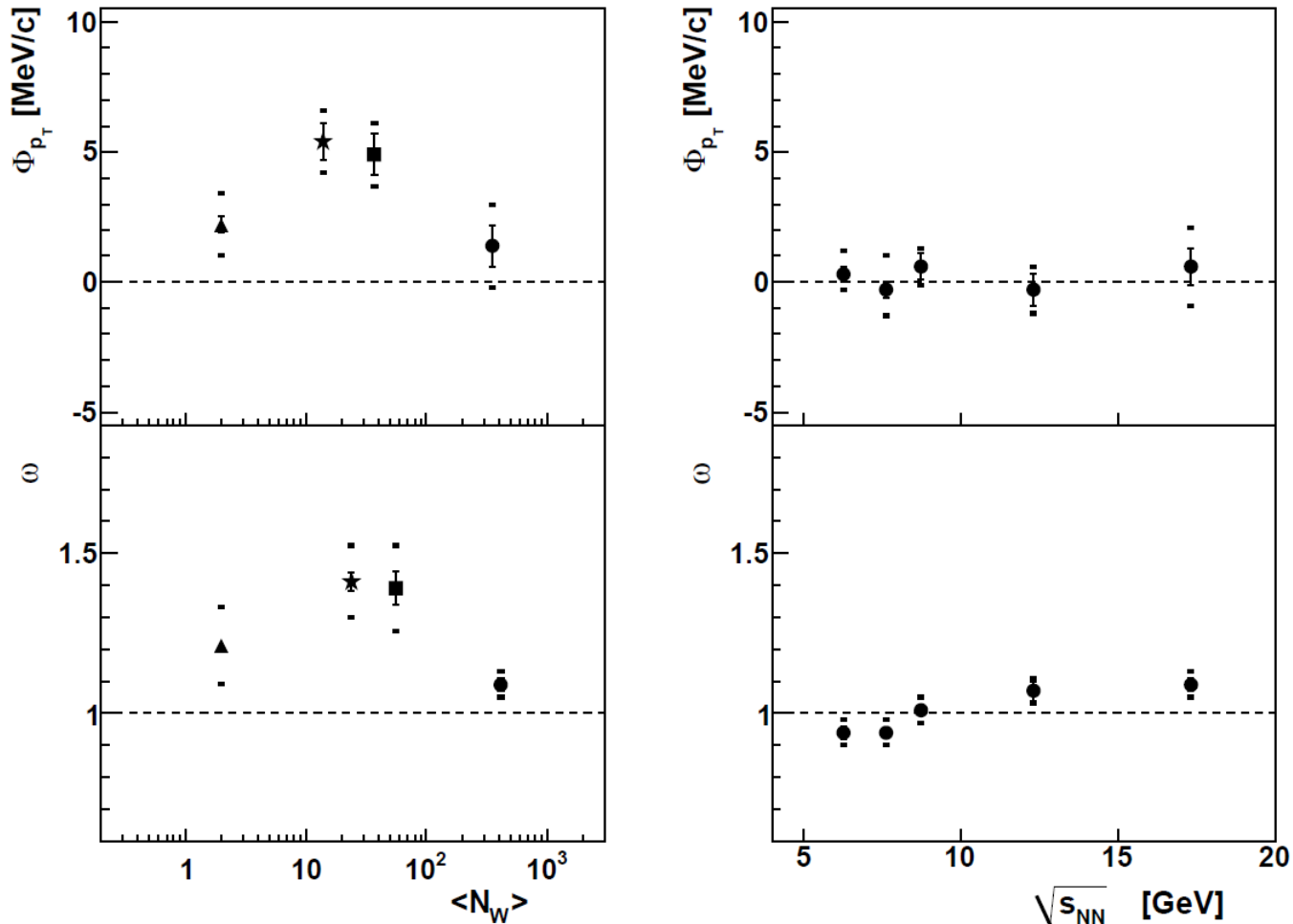
- $\mu_B = 360 \text{ MeV}$ (lattice QCD,Fodor-Katz)
 $T = 147 \text{ MeV}$ (chem. freeze-out line)
- $\mu_B = 250 \text{ MeV}$ (data 158A GeV)
 $T = 178 \text{ MeV}$ (fit of p+p data)

freeze-out points from
Becattini et al, PRC73, 044905 (2006)

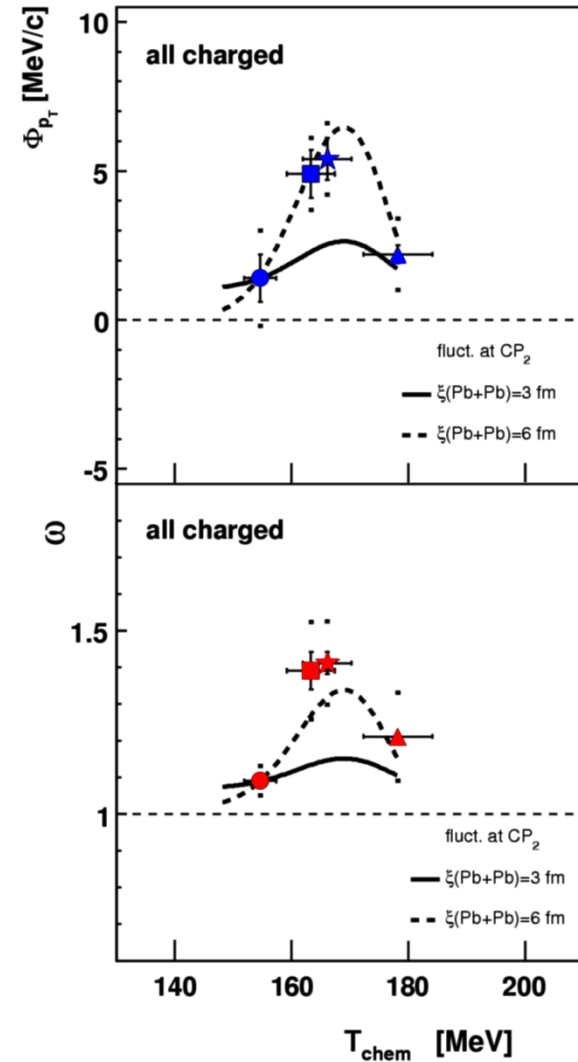


results of critical point search (ω, Φ_{pT}) by NA49

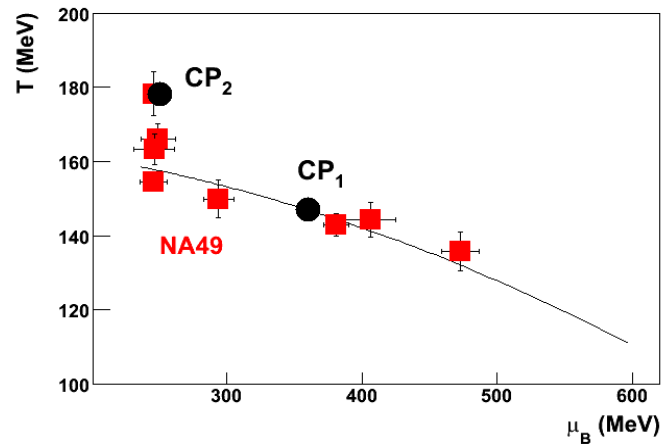
dependence on
system size: pp, CC, SiSi, PbPb energy (central PbPb)



map onto T, μ_B phase diagram using statistical model fits

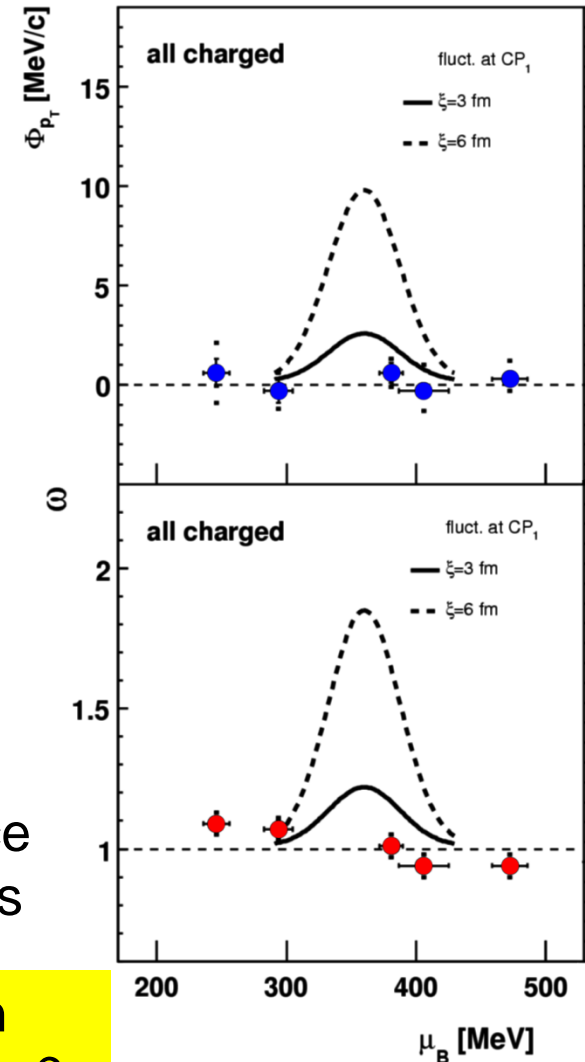


T.Anticic et al., PRC70, 034902 (2004)
 C.Alt et al., PRC75, 064904 (2007)
 C.Alt et al., PRC78, 034914 (2008)
 T.Anticic et al., PRC79, 044904 (2009)
 B.Lungwitz, NA49 thesis (2008)



smooth energy dependence
 in central Pb+Pb collisions

hint of peak at 158A GeV in
 nuclear size (T) dependence ?



$\Phi_{p_T}^{(3)}$: 3rd moment of $\langle p_T \rangle$ fluctuations

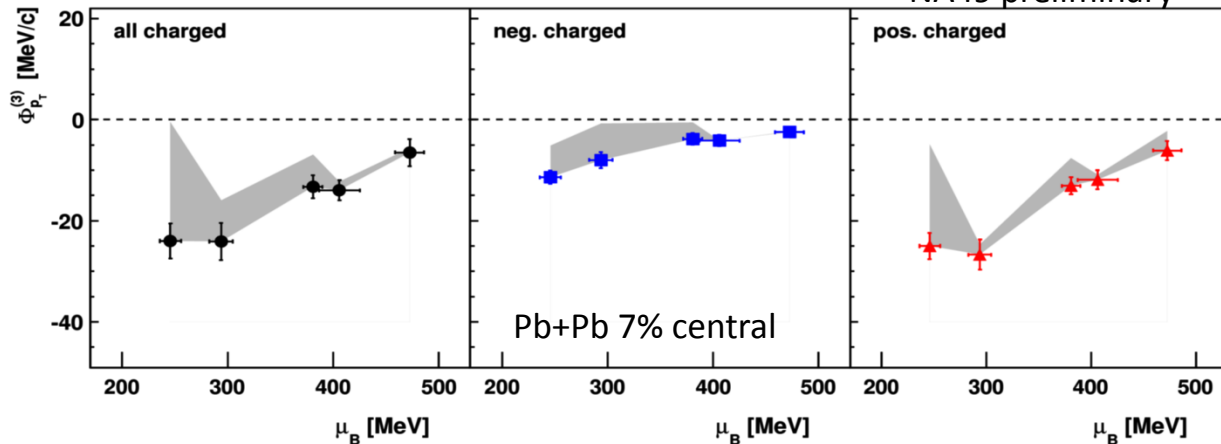
K.Grebieszkow and M.Bogusz, NA49 preliminary

$$\Phi_{p_T}^{(n)} = \left(\frac{\langle Z_{p_T}^2 \rangle}{\langle N \rangle} \right)^{1/n} - \left(Z_{p_T}^{\bar{n}} \right)^{1/n}$$

$\Phi_{p_T}^{(3)}$ has strongly intensive property like Φ_{p_T}

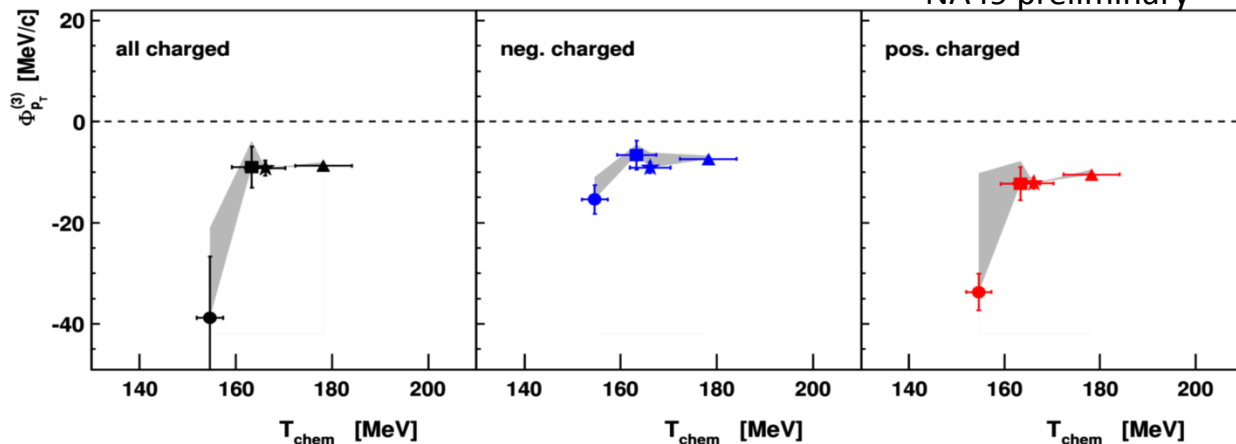
(S.Mrowczynski, Phys.Lett.B465,8(1999))

NA49 preliminary



higher moments are expected to be more sensitive to fluctuations

NA49 preliminary



systematic errors are large

no theoretical predictions yet

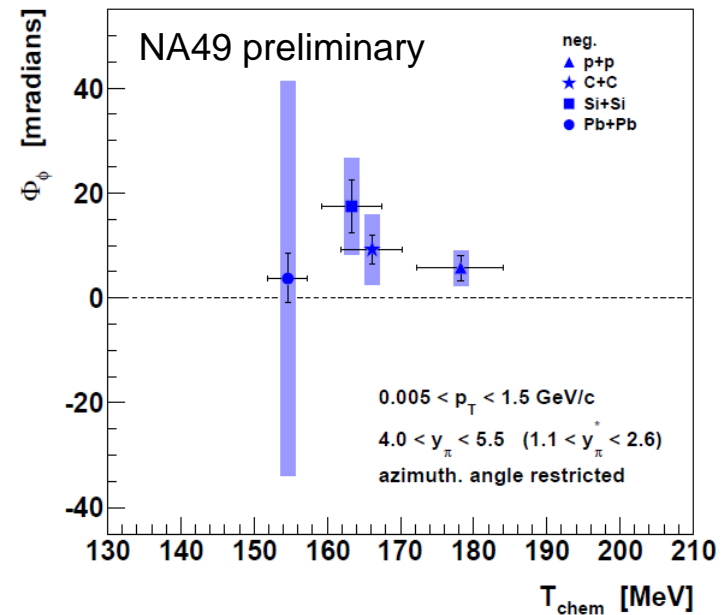
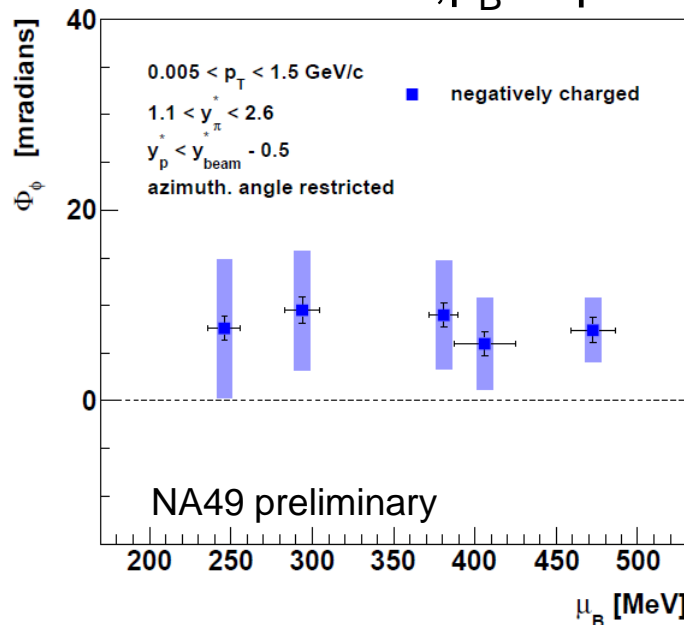


Φ_ϕ : fluctuations of average azimuthal angle

K.Grebieszkow, NA49 preliminary

- plasma instabilities (S.Mrowczynski, Phys.Lett. B314,118(1993))
- flow fluctuations (S.Mrowczynski,E.Shuryak,Act.Phys.Pol.B34,4241(2003))
- onset of deconfinement, critical point

T, μ_B dependence in central collisions:

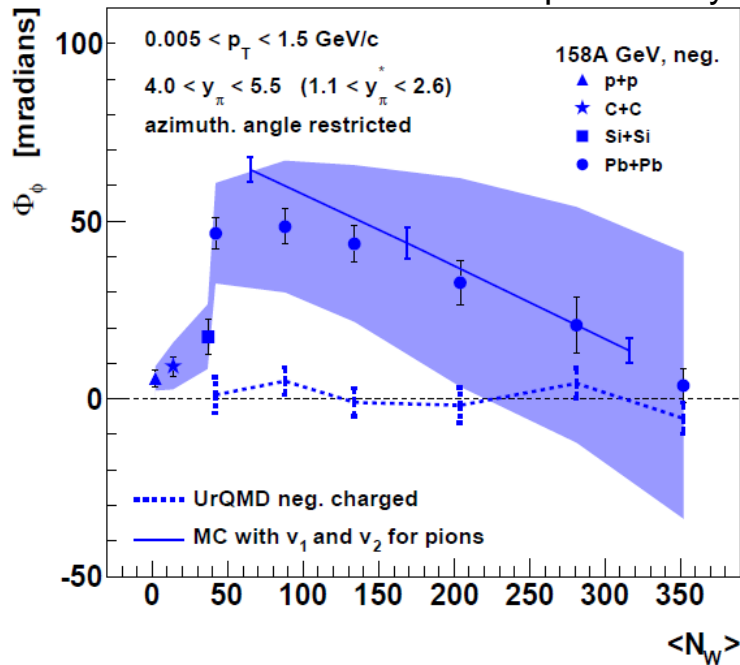


- no significant energy (μ_B) dependence in central Pb+Pb collisions
- perhaps hint of maximum in nuclear size (T) dependence

system size dependence at 158A GeV:

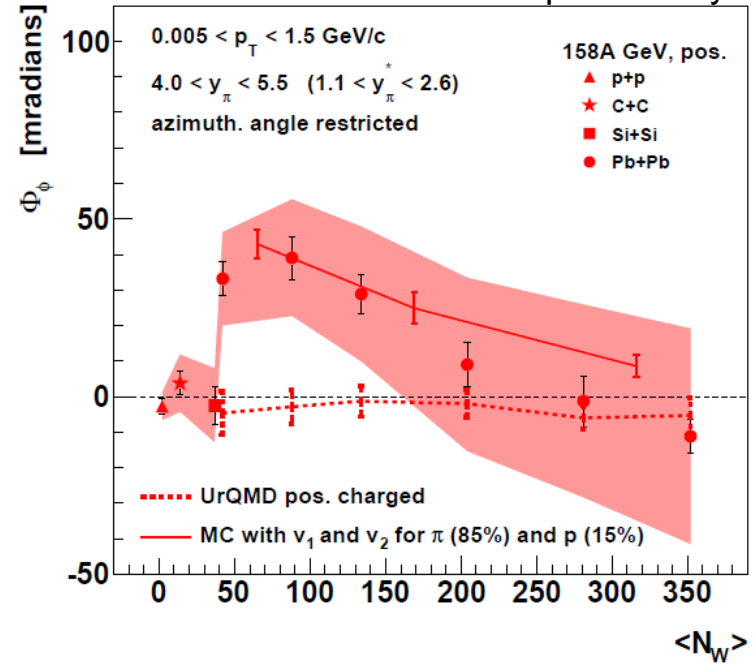
negative charge

NA49 preliminary



positive charge

NA49 preliminary



- simulation of flow describes centrality dependence in Pb+Pb collisions
- UrQMD model shows no effect, flow too small ?

intermittency in particle production as signal of the critical point

N.Antoniou et al., NPA693,799(2001); PRL97,032002(2006)

- at the critical point local density fluctuations with power-law singularity expected both in configuration and momentum space
 - σ field: density of σ particles, related to low-mass $\pi^+\pi^-$ pairs
 - baryonic density: related to net baryon number (\approx protons)
- experimental observation via factorial moments in p_T space:
(subdivided into M bins in $p_{T,x}$ and $p_{T,y}$)
predicted intermittency index at critical point: $\Phi_2 = 2/3, 5/6$
- estimate combinatorial background by mixed events and subtract

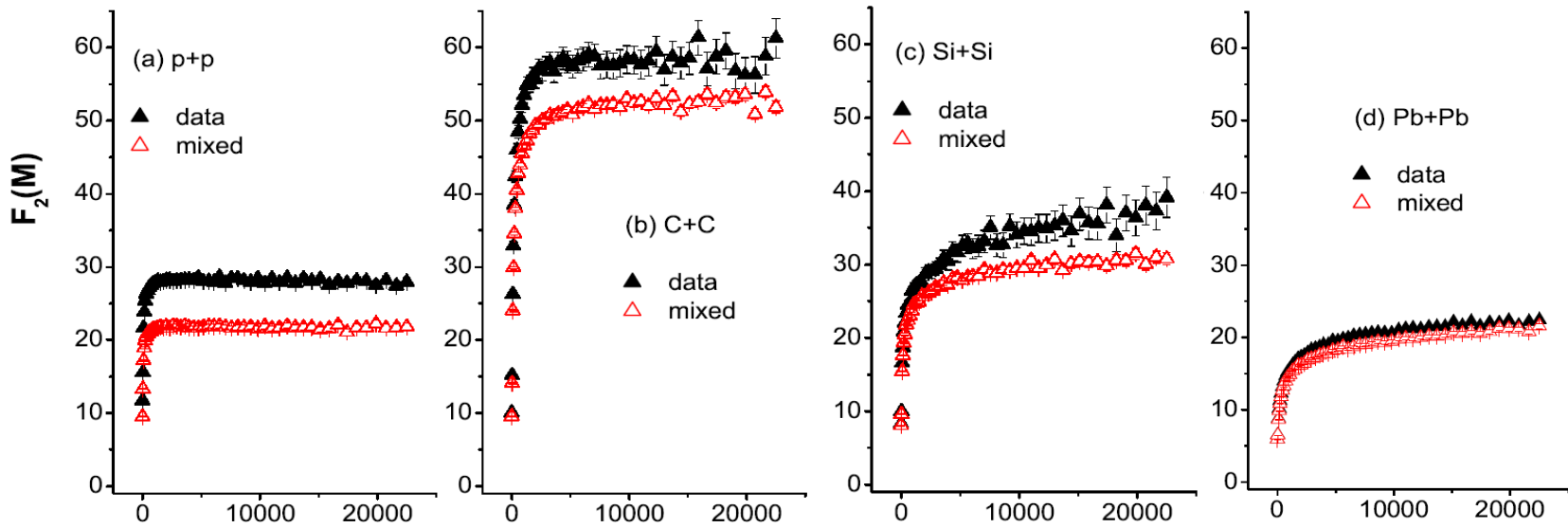
$$\Delta F_2(M) = F_2^{data} - F_2^{mix} \propto M^{2\Phi_2}$$

$\sigma \rightarrow \pi^+\pi^-$ intermittency analysis

- use $\pi^+\pi^-$ pairs near threshold to reduce combinatorial background
- exclude Coulomb correlation region near pair mass threshold

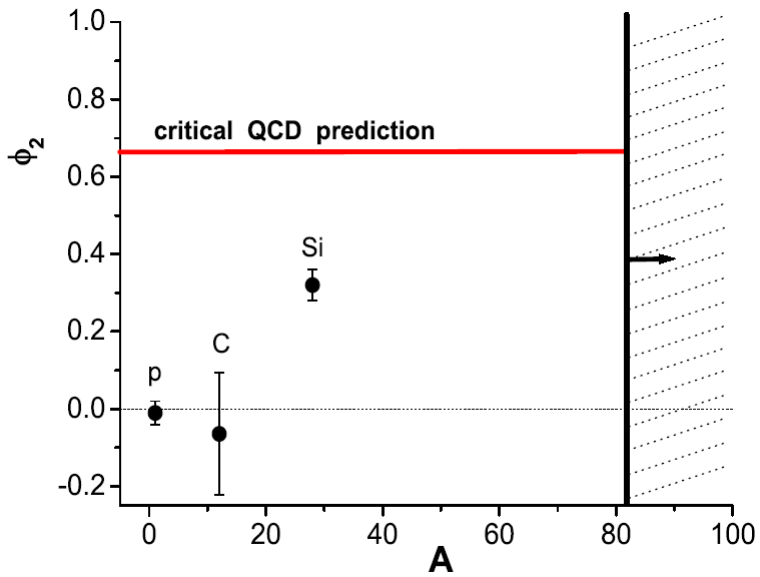
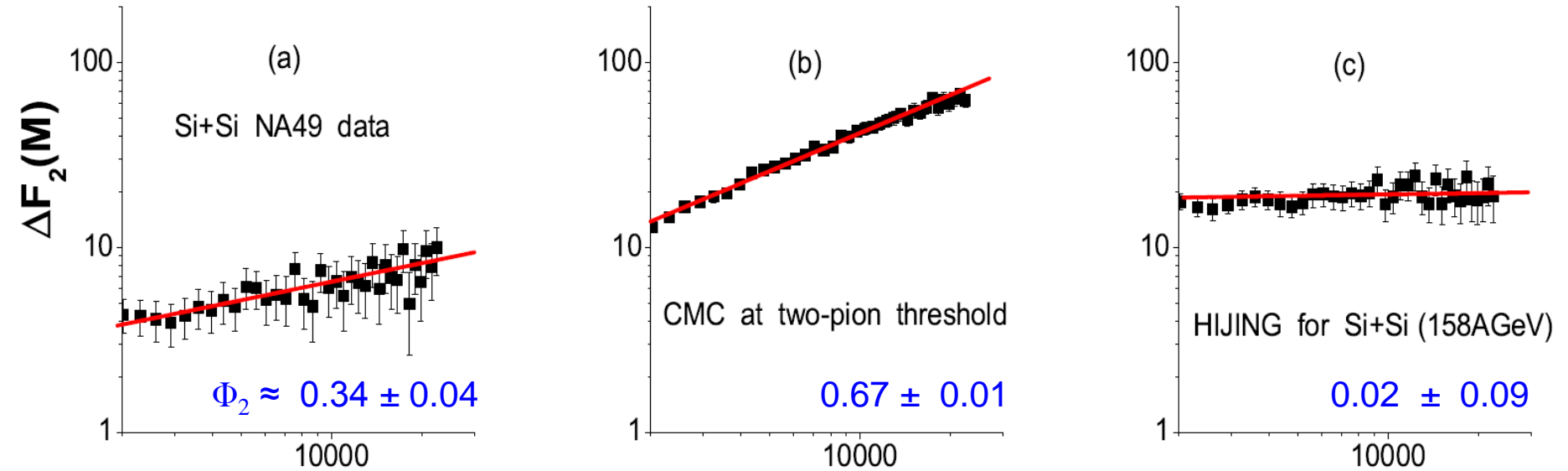
NA49 results for central collisions at 158A GeV:

T.Anticic et al, PRC81,064907(2010)



Combinatorial background too large

NA49 results on factorial moment ΔF_2 in central Si+Si collisions



$\pi^+\pi^-$ intermittency seen in central Si+Si collisions at 158A GeV

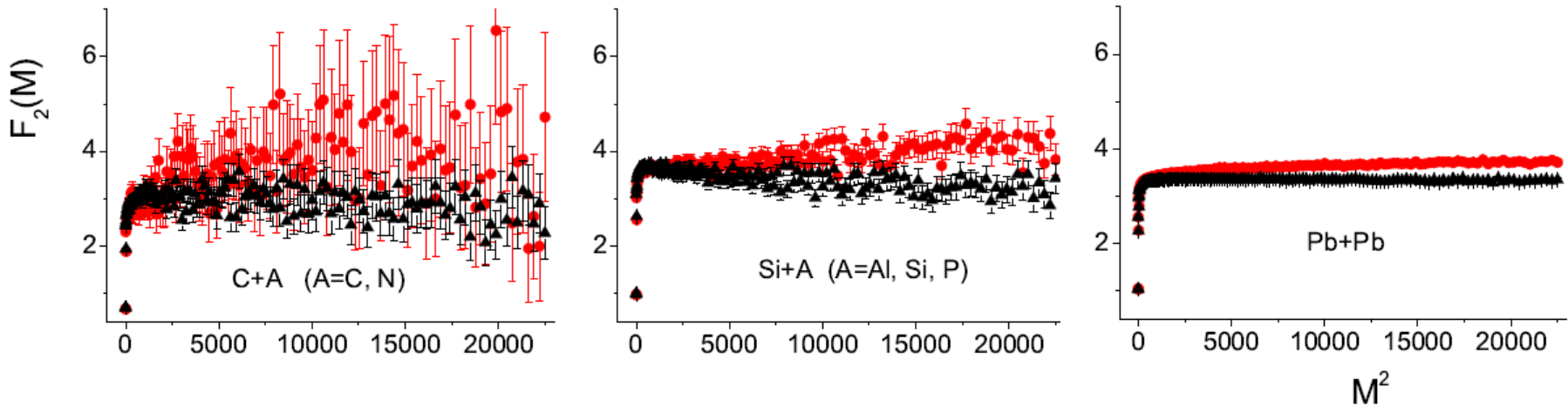


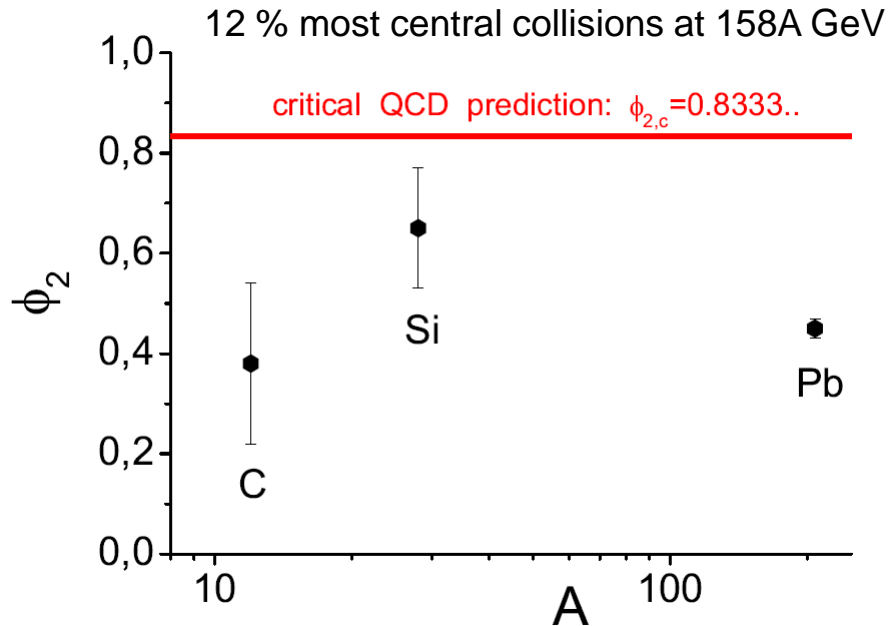
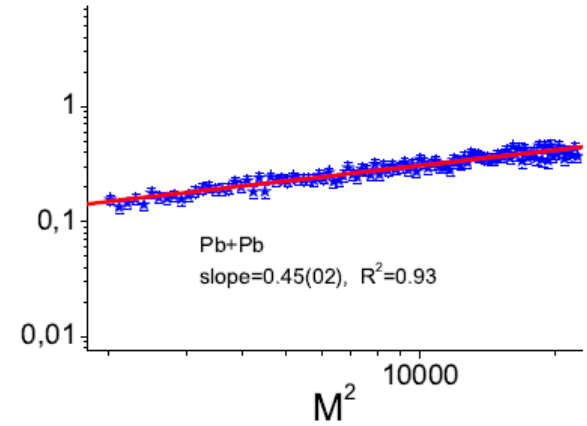
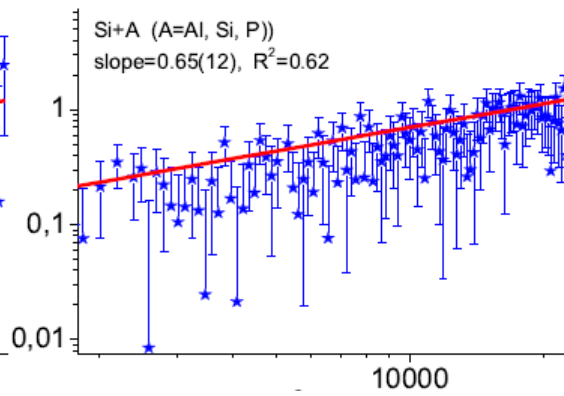
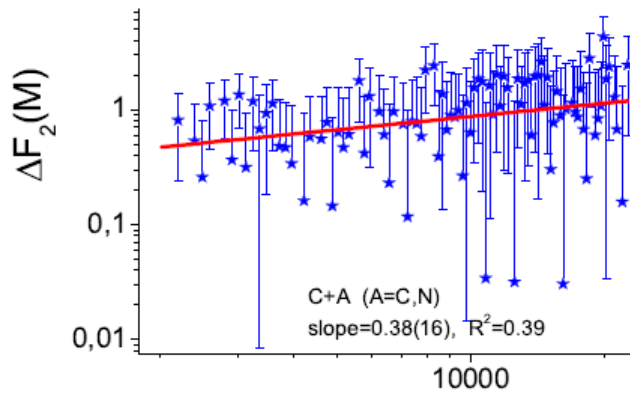
proton intermittency analysis (preliminary results)

(N.Davis, Univ.of Athens)

- protons identified by dE/dx measured in the TPCs
- selection by cuts in dE/dx such that purity $> 80\%$
- cms rapidity $|y_{cms}| < 0.75$

● NA49 data (centrality 0-12%)
▲ mixed events

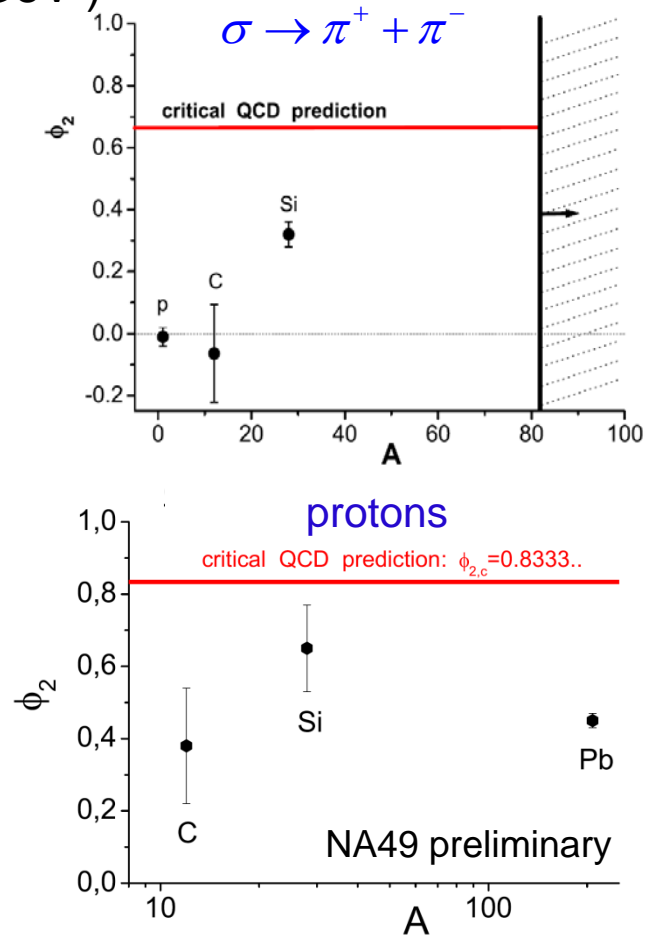
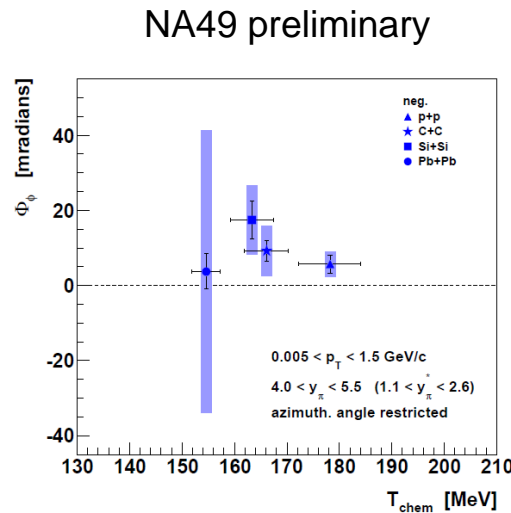
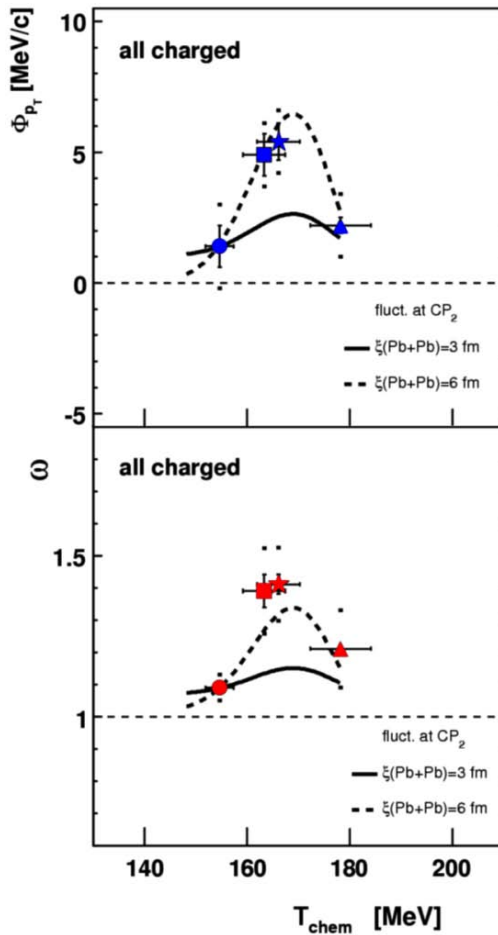




suggestive of maximum in
p intermittency for central
Si+Si collisions at 158A GeV

Conclusion from the critical point search in NA49

(central collisions at 158A GeV)



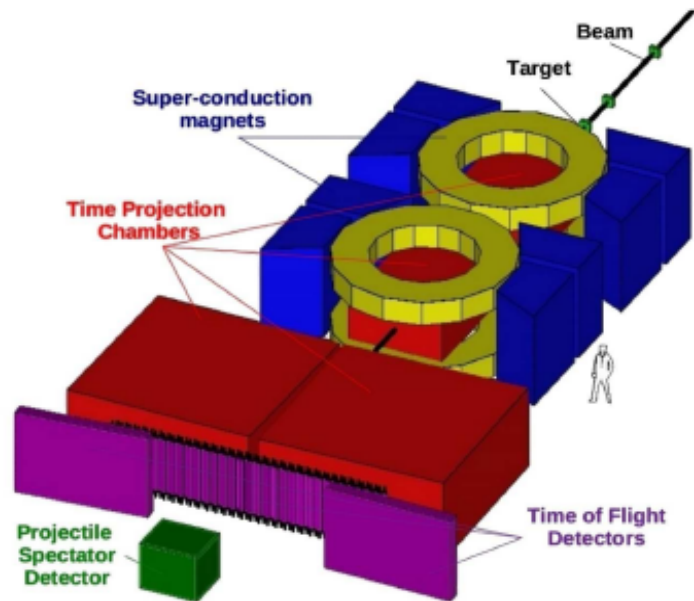
fluctuations of $\langle p_T \rangle$, N_{ch} , $\langle \phi \rangle$, intermittency of $\pi^+\pi^-$, p tend to a maximum in Si+Si collisions at 158A GeV

first hint of the hill of fluctuations ??

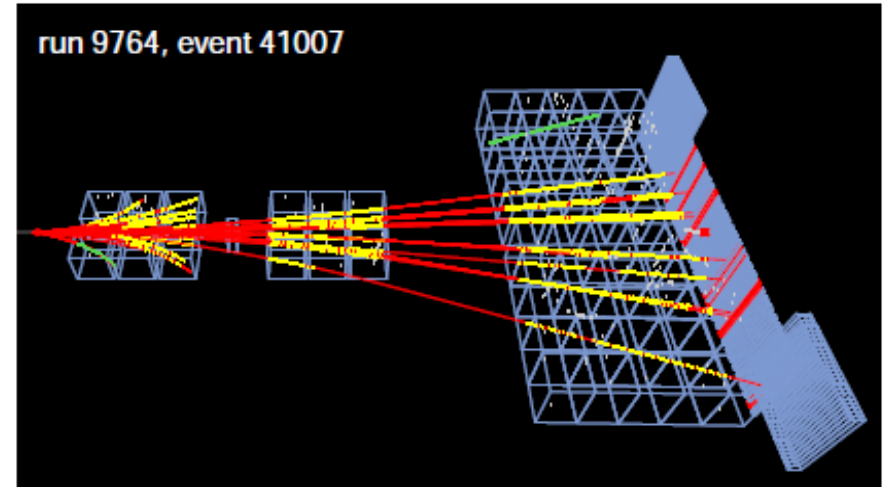


NA61/SHINE – successor and extension of NA49

(SHINE – SPS Heavy Ion and Neutrino Experiment)



π^- -C interaction at 350 GeV/c

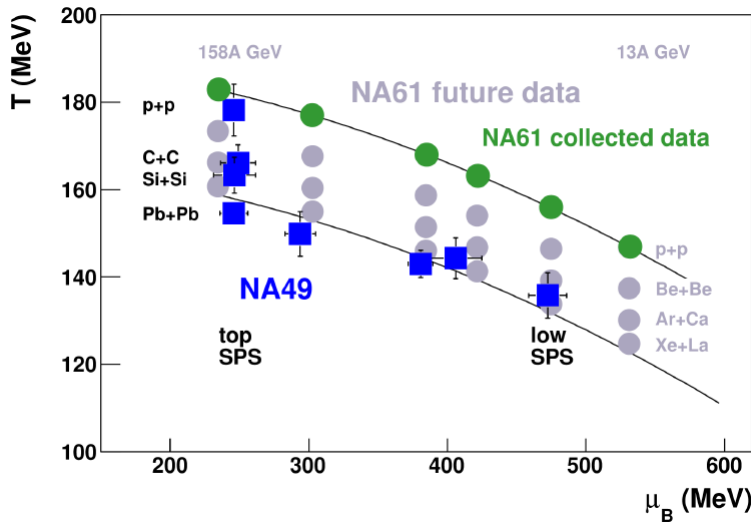


- study of the onset of deconfinement and search for the critical point
- precision particle production measurement for improving calculations of T2K neutrino beam and air shower properties (P. Auger, KASKADE expts.)
- study of nuclear modification factor and Cronin effect using p+p and p+Pb interactions with extended range in $p_T \leq 4.5$ GeV/c

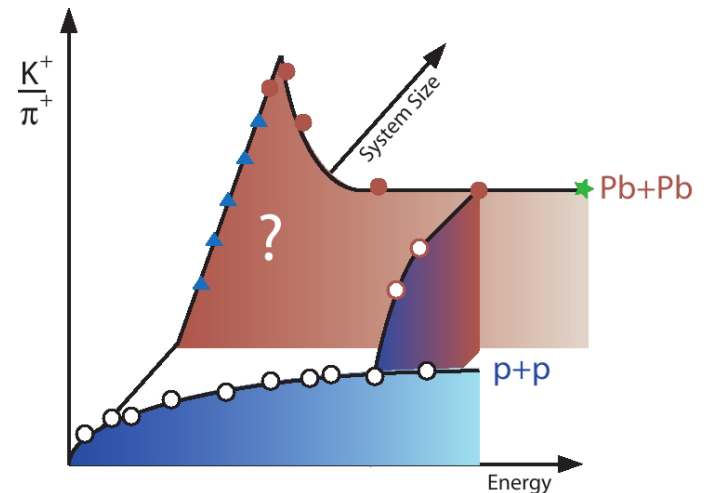
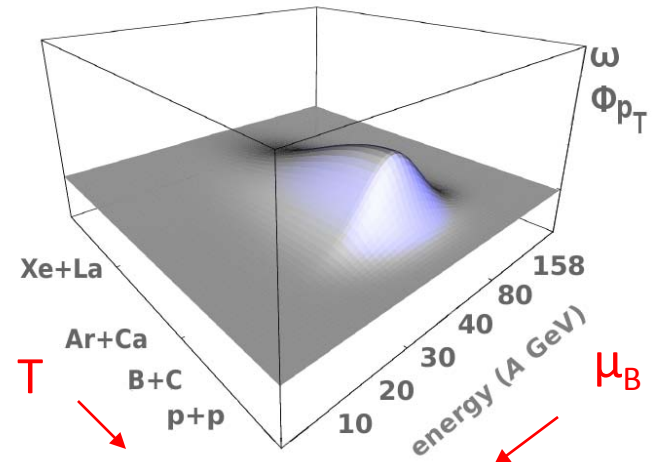


Ion physics program of NA61/SHINE: scan in energy and system size A

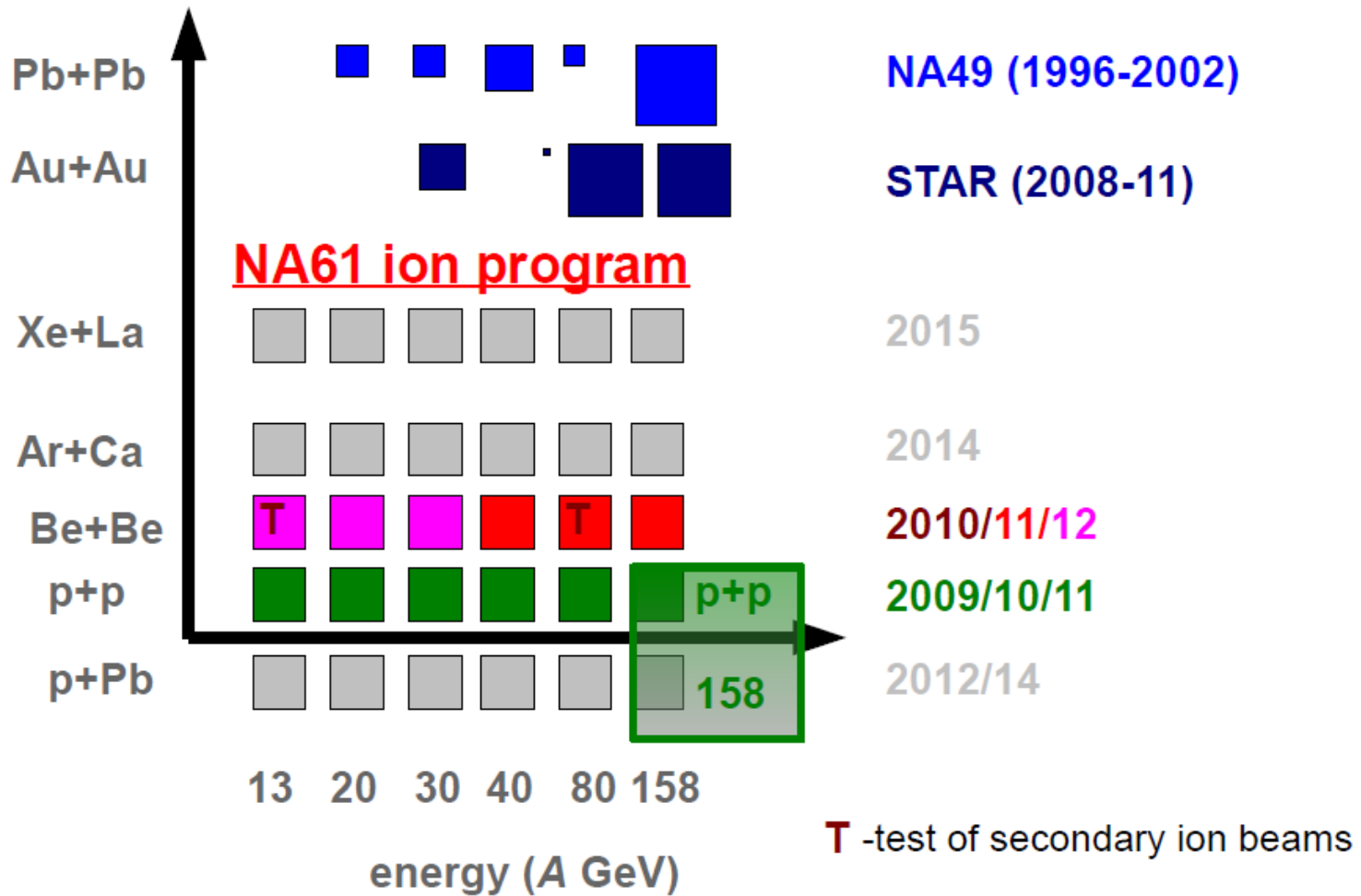
search for hill of fluctuations
as signature of critical point



study onset of deconfinement:
disappearance of horn etc.



Status and plans for ion collisions at SPS energies

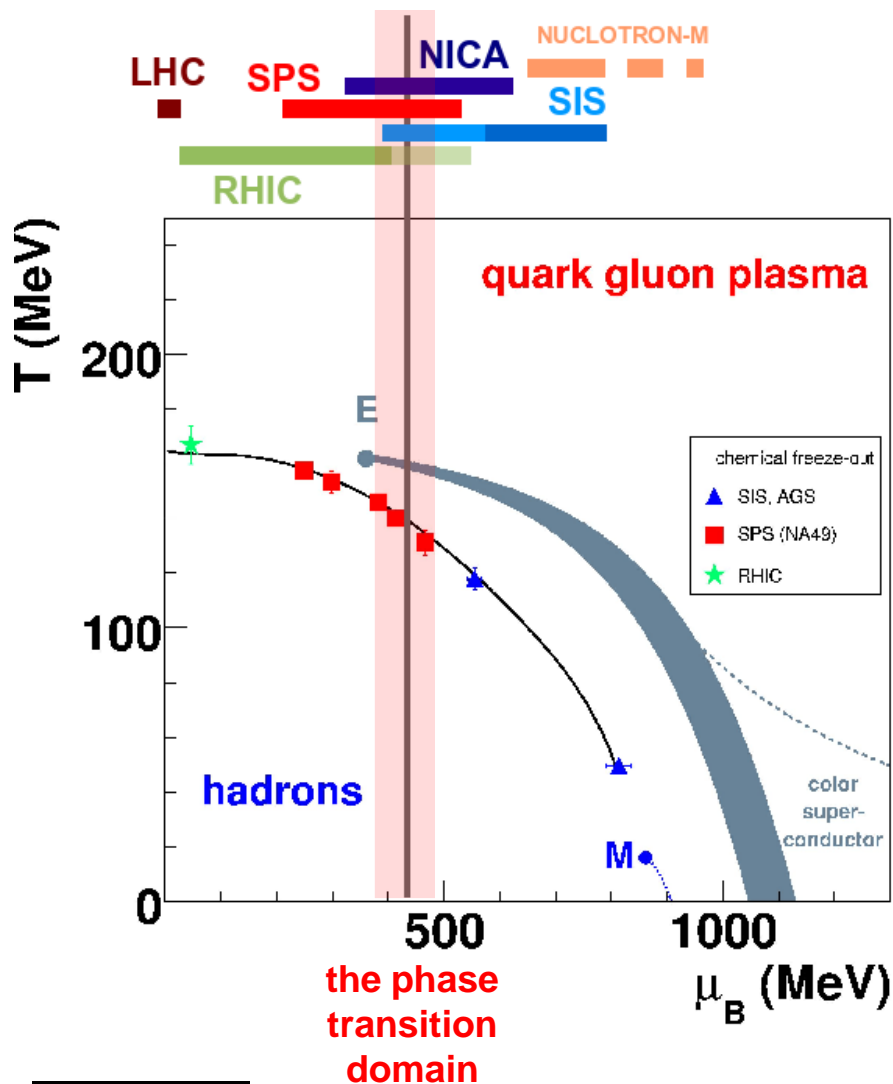


Landscape of experimental program on nucleus-nucleus reactions

Facility	SPS	RHIC	NUCLOTRON-M	NICA	SIS-100/300	LHC
Laboratory	CERN Geneva	BNL Brookhaven	JINR Dubna	JINR Dubna	FAIR GSI Darmstadt	CERN Geneva
Experiment	NA61/SHINE	STAR PHENIX	BM@N	MPD	HADES + CBM CBM	ALICE ATLAS CMS
Start of data taking	2009(11)	2010	2015	2017	2017/18 (2019/20)	2009
cms energy [GeV/(N+N)]	5.1 – 17.3	7.7 (5?) – 200	< ~ 3.5	4 – 11	2.3 – ~4.5 ~4.5 – ~8.5	up to 5500 14000 (p+p)
Physics	CP & OD	CP & OD	HDM	OD & HDM	HDM, OD & CP	PDM



QCD critical point searches – future experimental landscape



partly complementary programs

CERN SPS 2011 →

BNL RHIC 2010 →

DUBNA Nuclotron 2015

NICA 2017

GSI SIS-100 2017

strong points of NA61:

- tight constraint on spectators
- high event rate at all SPS energies
- flexibility to change A and energy
- overlap with AGS energy
- coverage of full forward hemisphere

strong points of BNL/STAR:

- full azimuthal acceptance
- acceptance unchanged with energy
- excellent TOF identification
- low track density



Conclusions

- evidence for onset of deconfinement at SPS energies – confirmed by STAR low energy scan at RHIC
- interpretation supported by LHC results
- discrepancies between NA49 and STAR results on particle ratio fluctuations need to be resolved
- 2D scan of fluctuations for critical point search in μ_B, T phase diagram started by NA49 and continuing in NA61/SHINE
- hints of a maximum of fluctuations for Si+Si at 158A GeV → strong motivation for NA61/SHINE program
- looking forward to:
comprehensive STAR results from RHIC low energy scan
future programs at NICA and FAIR

NA49:

78 physicists from 23 institutes and 12 countries:

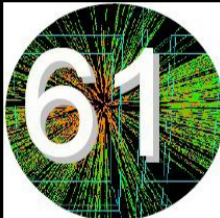
NIKHEF, Amsterdam, Netherlands
University of Athens, Athens, Greece
Comenius University, Bratislava, Slovenia
Eotvos Lorand University, Budapest, Hungary
KFKI IPNP, Budapest, Hungary
MIT, Cambridge, USA
INP, Cracow, Poland
Joint Institute for Nuclear Research, Dubna, Russia
GSI, Darmstadt, Germany
University of Frankfurt, Frankfurt, Germany
CERN, Geneva, Switzerland
Jan Kochanowski University, Kielce, Poland
University of Marburg, Marburg, Germany
MPI, Munich, Germany
Charles University, Prag, Czech Republic
University of Washington, Seattle, USA
Faculty of Physics, University of Sofia, Sofia, Bulgaria
Sofia University, Sofia, Bulgaria
INR&NE, BAS, Sofia, Bulgaria
State University of New York, Stony Brook, USA
Soltan Institute for Nuclear Studies, Warsaw, Poland
Warsaw University of Technology, Warsaw, Poland
University of Warsaw, Warsaw, Poland
Rudjer Boskovic Institute, Zagreb, Croatia



NA61:

134 physicists from 27 institutes and 15 countries:

University of Athens, Athens, Greece
University of Belgrade, Belgrade, Serbia
University of Bergen, Bergen, Norway
University of Bern, Bern, Switzerland
KFKI IPNP, Budapest, Hungary
Jagiellonian University, Cracow, Poland
Joint Institute for Nuclear Research, Dubna, Russia
Fachhochschule Frankfurt, Frankfurt, Germany
University of Frankfurt, Frankfurt, Germany
University of Geneva, Geneva, Switzerland
Forschungszentrum Karlsruhe, Karlsruhe, Germany
Institute of Physics, University of Silesia, Katowice, Poland
Jan Kochanowski University, Kielce, Poland
Institute for Nuclear Research, Moscow, Russia
University of Nova Gorica, Nova Gorica, Slovenia
LPNHE, Universites de Paris VI et VII, Paris, France
Faculty of Physics, University of Sofia, Sofia, Bulgaria
St. Petersburg State University, St. Petersburg, Russia
State University of New York, Stony Brook, USA
KEK, Tsukuba, Japan
Soltan Institute for Nuclear Studies, Warsaw, Poland
Warsaw University of Technology, Warsaw, Poland
University of Warsaw, Warsaw, Poland
Univeristy of Wroclaw, Wroclaw, Poland
Universidad Tecnica Federico Santa Maria, Valparaiso, Chile
Rudjer Boskovic Institute, Zagreb, Croatia
ETH Zurich, Zurich, Switzerland



Backup Slides

Calculate ν_{dyn} in NA49

$$\nu = \frac{\langle A^2 \rangle}{\langle A \rangle^2} + \frac{\langle B^2 \rangle}{\langle B \rangle^2} - 2 \frac{\langle AB \rangle}{\langle A \rangle \langle B \rangle}$$

The definition of ν_{dyn} assumes uncorrelated background

$$\nu_{\text{stat}} = \frac{1}{\langle A \rangle} + \frac{1}{\langle B \rangle} \qquad \nu_{\text{dyn}} = \nu - \nu_{\text{stat}}$$

To subtract the correlation present in mixed events, we instead define

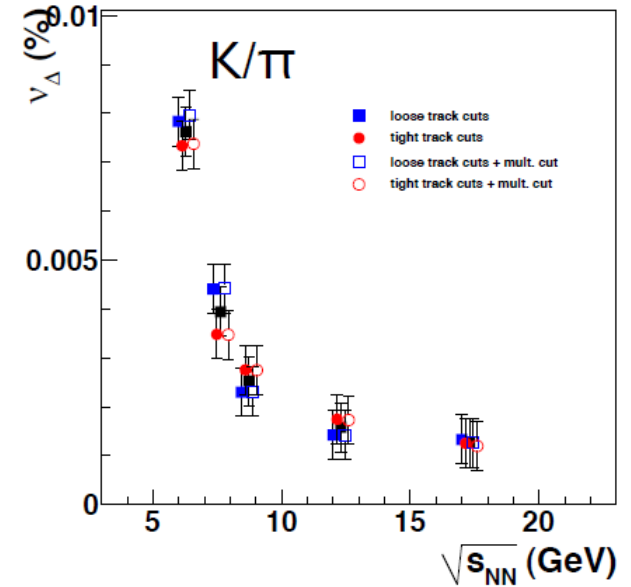
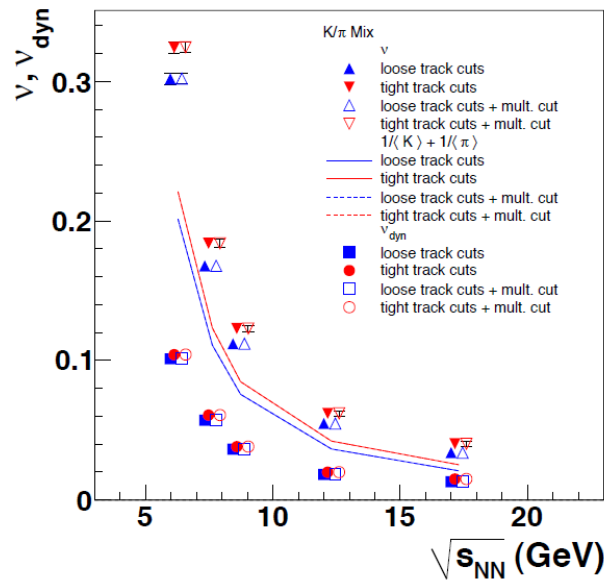
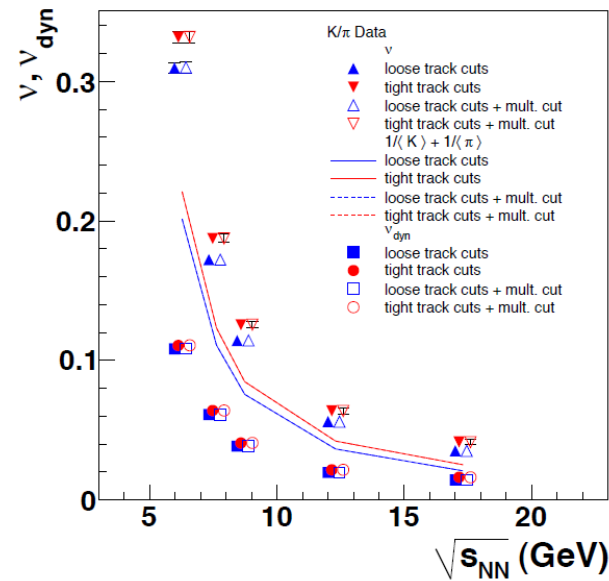
$$\nu_{\Delta} = \nu_{\text{data}} - \nu_{\text{mix}}$$

K/ π fluctuations

Results for v_{Δ}

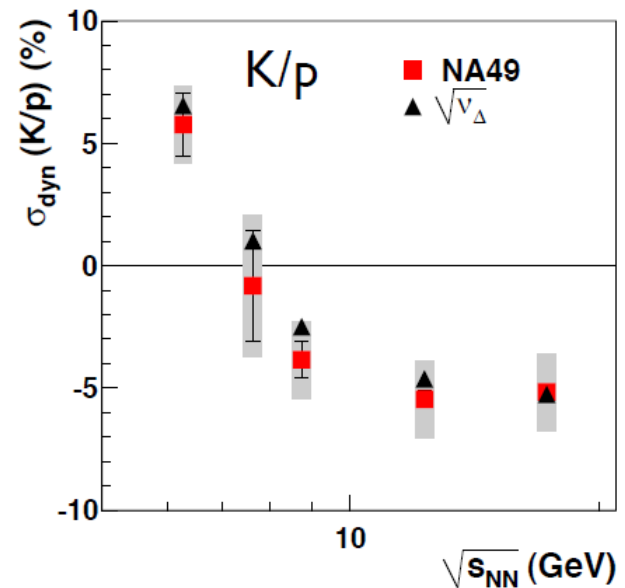
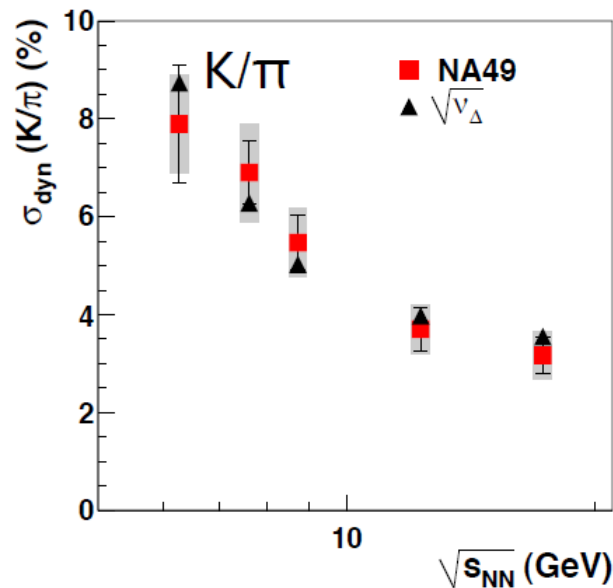
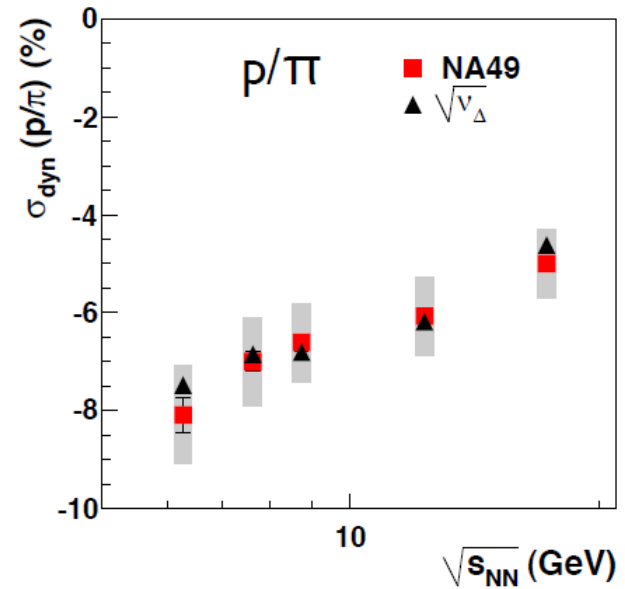
v and v_{dyn} (K, π) data

v and v_{dyn} (K, π) mix



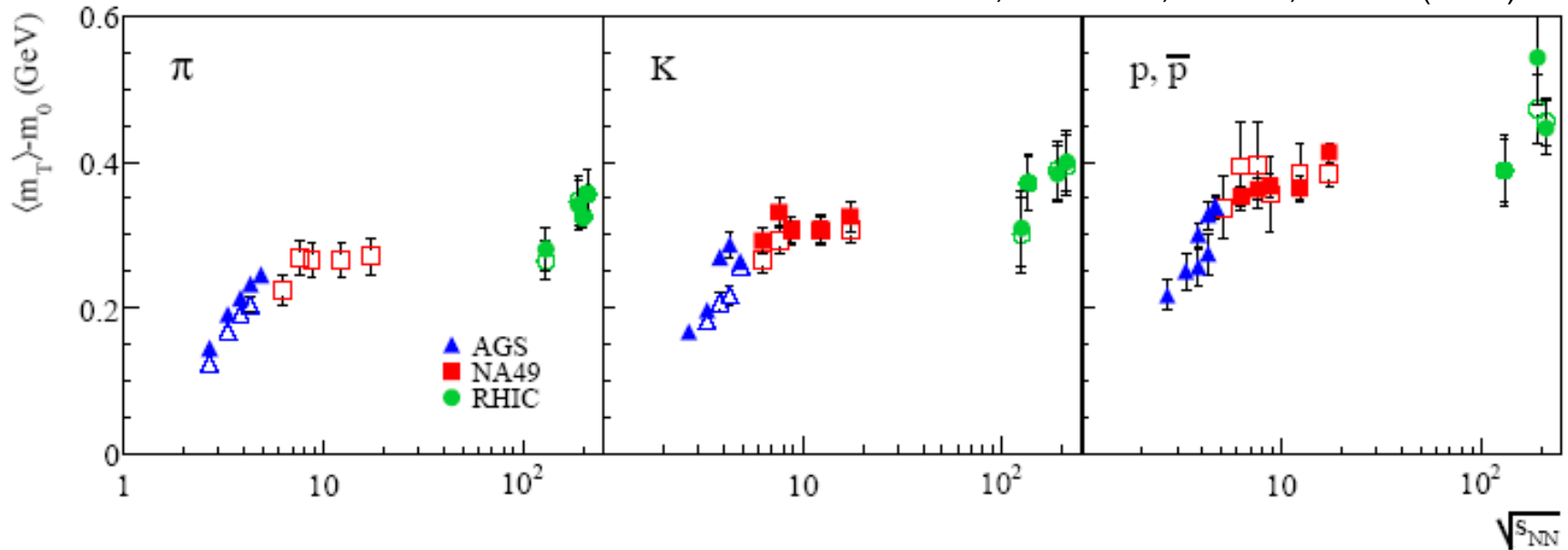
Calculate v_{dyn} in NA49

Compare to σ_{dyn} results



Plateau of average transverse mass

NA49, C. Alt et al., PRC77,024903 (2008)



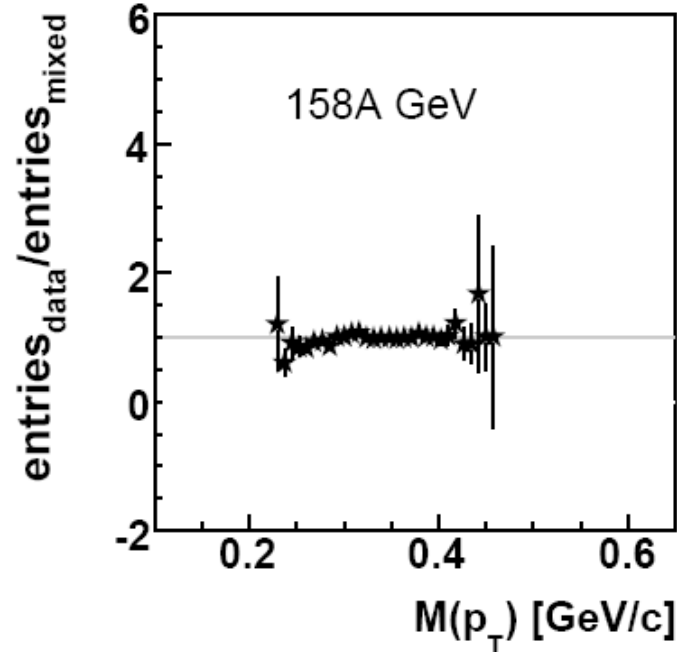
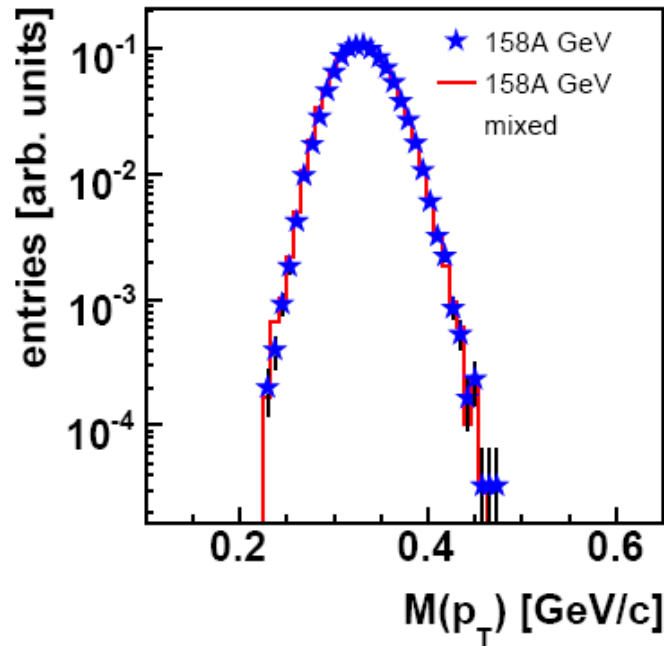
- Increase of $\langle m_T \rangle$ for abundant final state particles (π , K, p) slows sharply at the lowest SPS energy

- consistent with approximately constant pressure and temperature in a mixed phase system

(L.van Hove, PLB 89 (1982) 253; M.Gorenstein et al., PLB 567 (2003) 175)

fluctuations of average transverse momentum $\langle p_T \rangle$

5 % most central Pb+Pb collisions at 158A GeV:



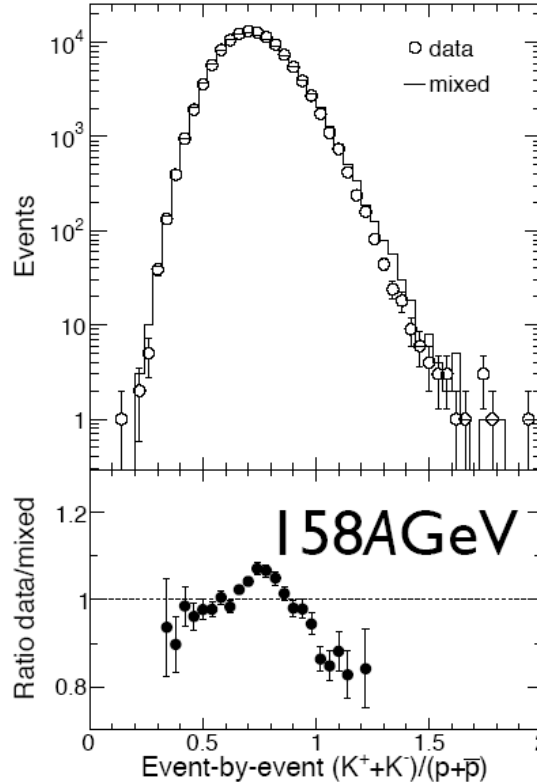
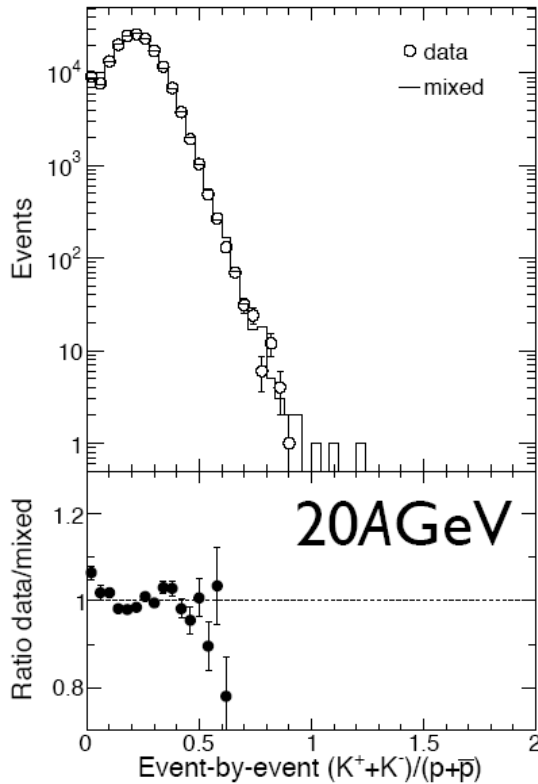
$$\Phi_{p_T} = \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\langle Z^2 \rangle}$$

$$z = p_T - \langle p_T \rangle \quad Z = \sum_{i=1}^N (p_T^i - \langle p_T \rangle)$$

- superposition: $\Phi(AA) = \Phi(NN)$
- uncorrelated particle emission: $\Phi = 0$
- insensitive to fluctuations of impact parameter (volume)

strongly intensive observable

Event-by-event particle ratio fluctuations (example K/p)



3.5 % most central
Pb+Pb collisions:

$$\sigma_{dyn} = \text{sign}(\sigma_{data}^2 - \sigma_{mix}^2) \sqrt{|\sigma_{data}^2 - \sigma_{mix}^2|}$$

- e-by-e fit of particle multiplicities required in NA49
- mixed events (no correlations) used as reference

$1/N_{part}$ dependence

Electric charge fluctuations

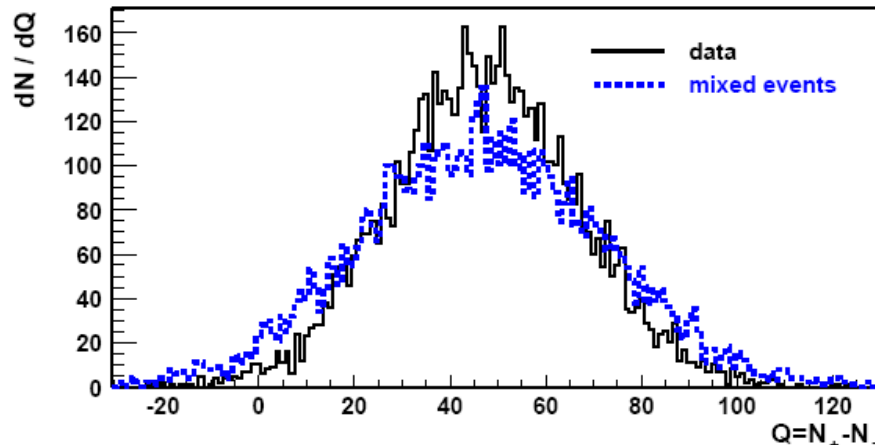
- smaller in a QGP than in a hadron gas

(Jeon,Koch,Asakawa,Heinz,Müller)

$$\Delta\Phi_q = \Phi_q - \Phi_{q,gcc}$$

PRC70,064903(2004)

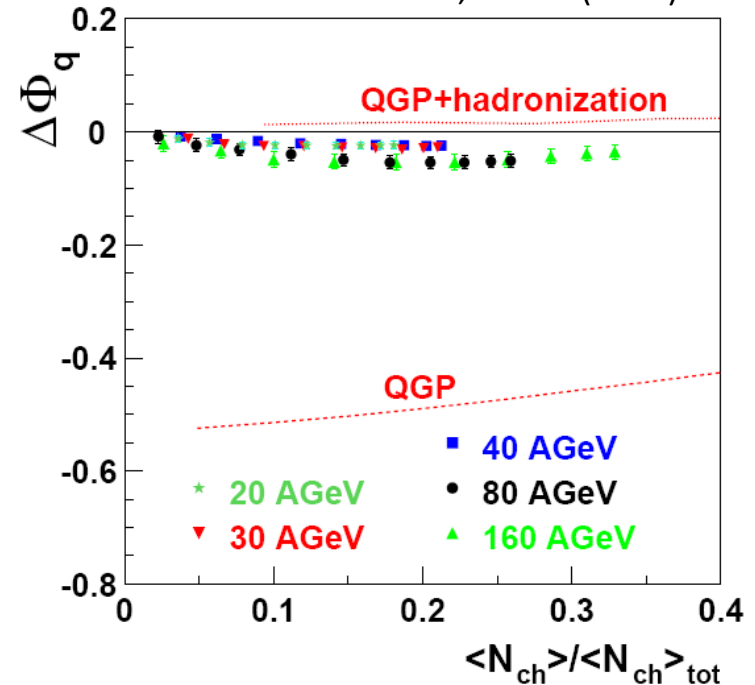
Central Pb+Pb collisions 158A GeV



Global charge conservation

$$\Phi_q = \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{Z^2}$$

$$z = q - \bar{q} \quad Z = \sum_{i=1}^N (q_i - \bar{q})$$



QGP signature probably erased by hadronisation (Bialas) or the effect of resonance decays (Zaraneek)

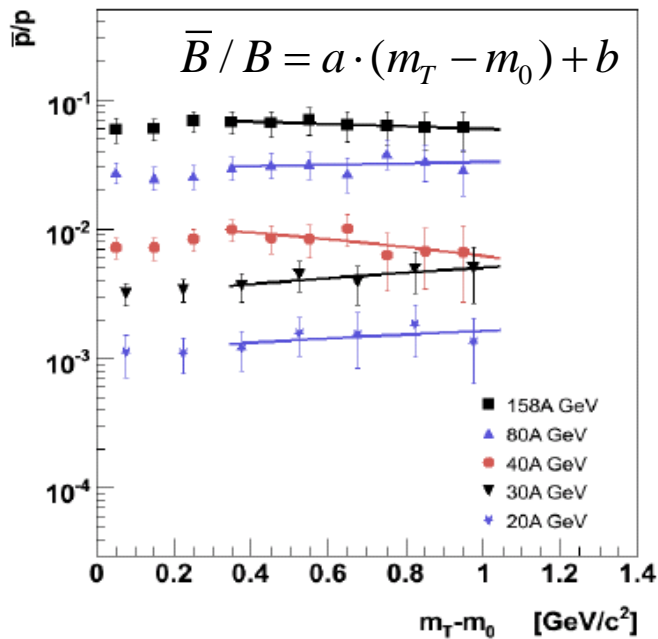
transverse mass spectra of baryons and anti-baryons

presence of critical point predicted to attract fireball evolution trajectory
 altering average freeze-out time $\rightarrow \beta_T$ dependence of anti-p/p ratio

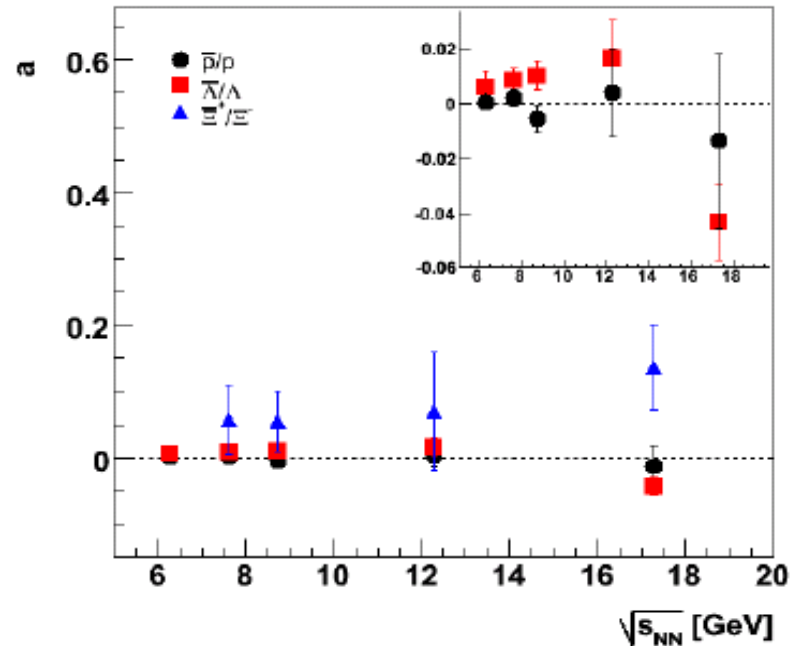
critical point: anti-p/p increases with m_T

annihilation: decreases

M.Asakawa et al.,PRL101,122302(2008)



similar data for Λ and Ξ



no significant change with \sqrt{s}
 of slope parameter a

no evidence for critical point effect

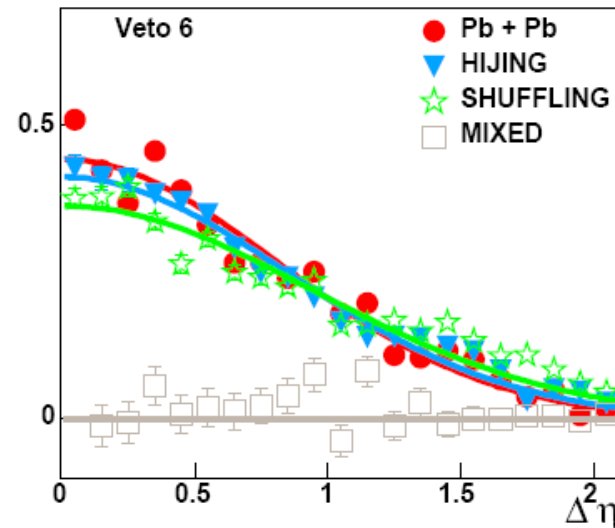
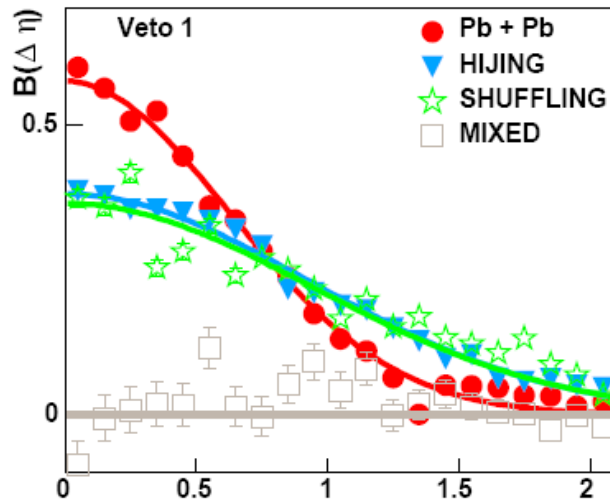
Balance Function: charge correlations in pseudo-rapidity

$$B(\delta\eta) = \frac{1}{2} \left(\frac{N_{(+-)}(\delta\eta) - N_{(--)}(\delta\eta)}{N_-} + \frac{N_{(-+)}(\delta\eta) - N_{(++)}(\delta\eta)}{N_+} \right)$$

narrowing of the balance function proposed as QGP signature
(delayed hadronisation due to phase coexistence)

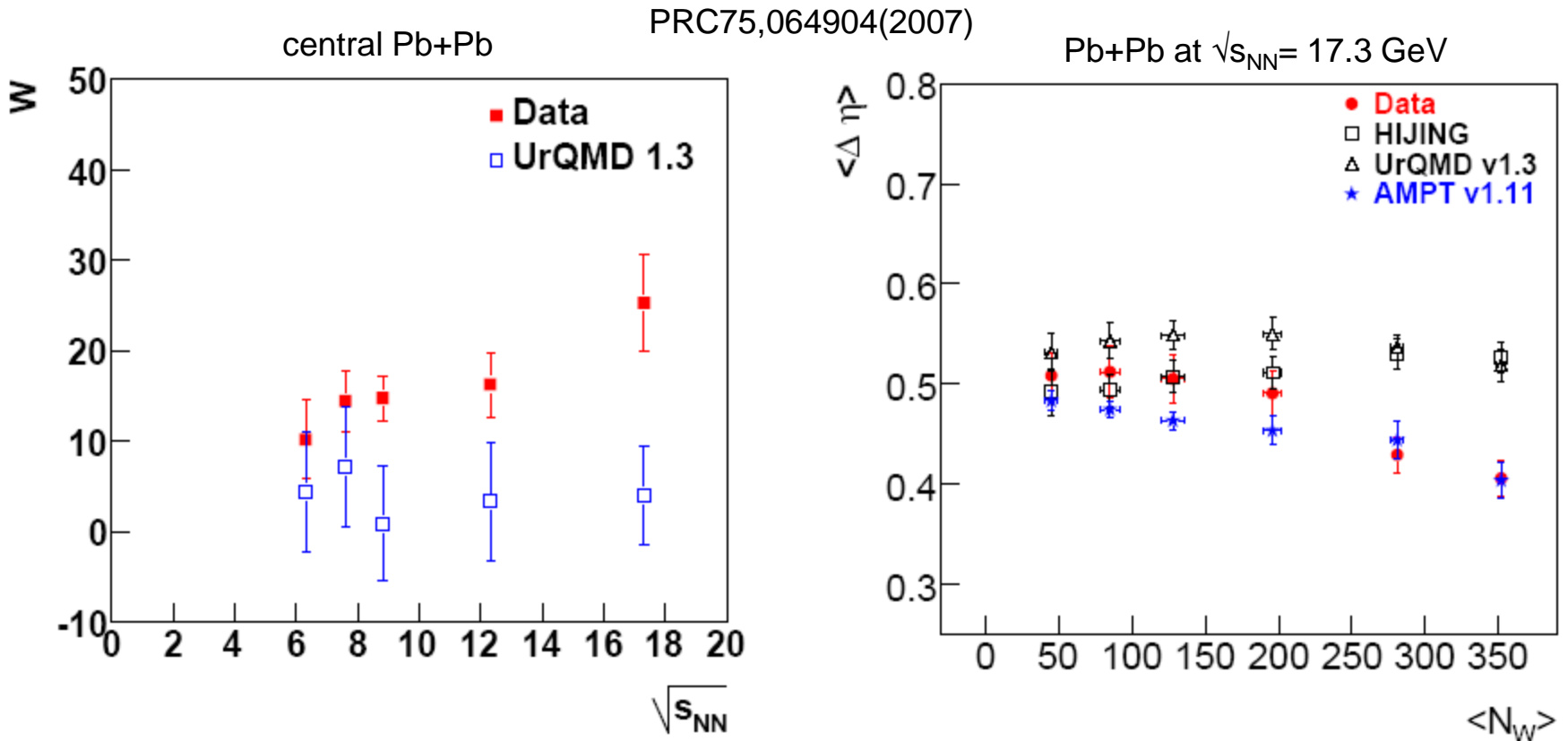
central

peripheral



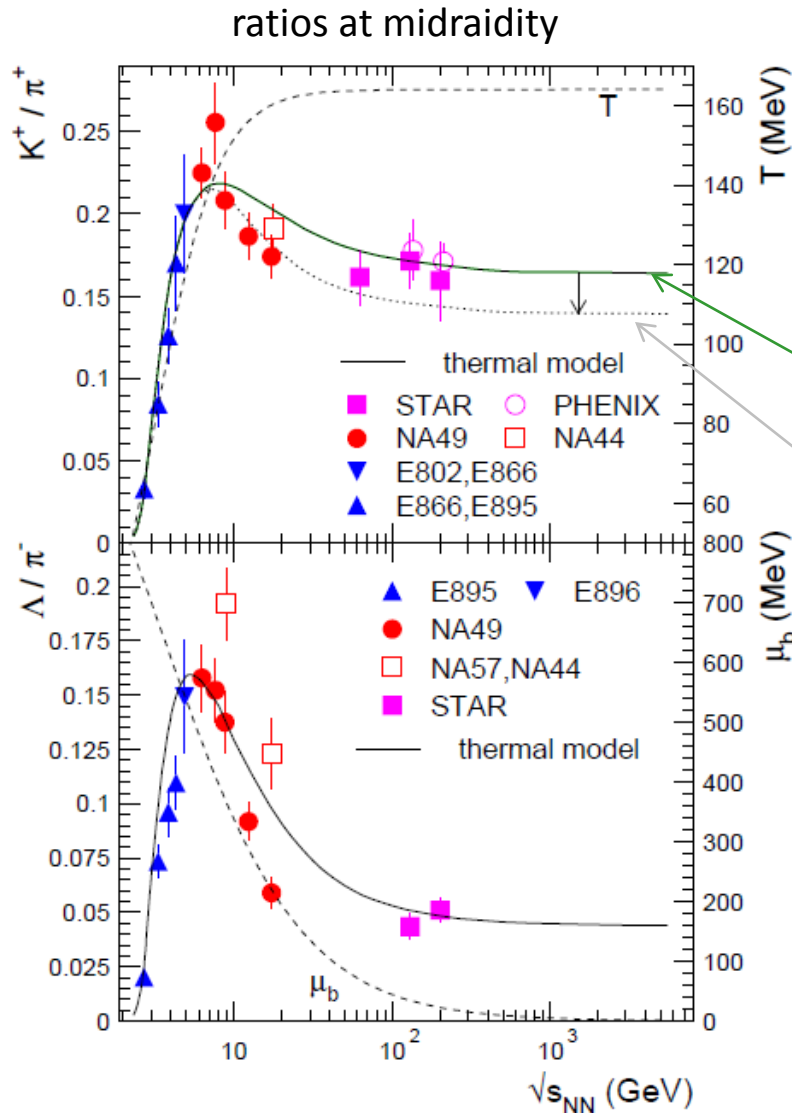
data compared to shuffled events: $W = (\langle \Delta\eta \rangle_{\text{shuff}} - \langle \Delta\eta \rangle_{\text{data}}) / \langle \Delta\eta \rangle_{\text{shuff}} \cdot 100$
(scrambling of rapidities, retention of global charge conservation)

BF: model comparisons at mid-rapidity



- no anomaly at SPS energy: effects due to local charge conservation and radial flow may dominate (Pratt, Bialas)
- microscopic model AMPT with deconfined phase reproduces BF narrowing

Statistical hadron gas model with additional assumptions



recent extension of statistical hadron gas model

full PDG high mass spectrum

Further extension with exponential mass spectrum

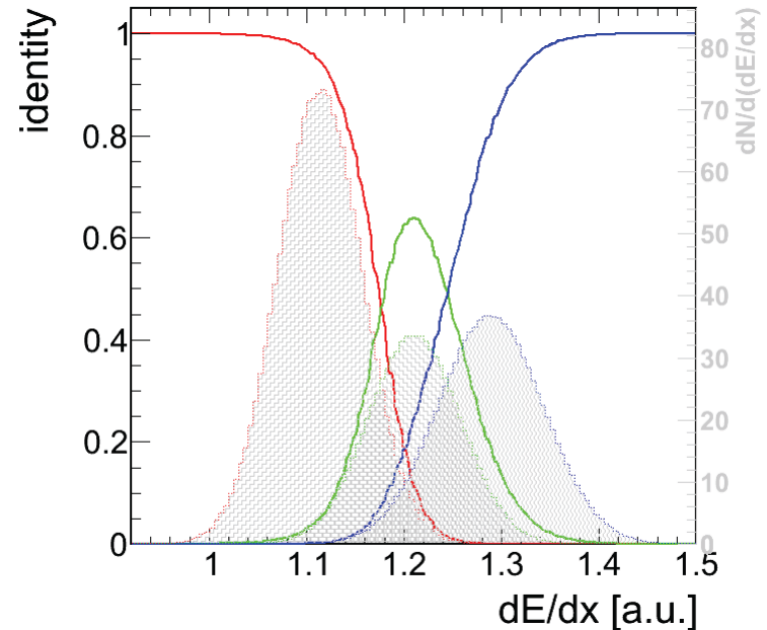
“Our results imply that hadronic observables near and above the ‘horn’ structure ... provide a link to the QCD phase transition”

deconvolution of resolution in case of probabilistic identification :

M.Gorenstein PRC84,024902(2011)

- inclusive $m = dE/dx$ distribution decomposed into contributions ρ_h of various hadrons of type h
- definition of identities:

$$w_h(m) = \frac{\rho_h(m)}{\sum_i \rho_i(m)} \quad , \quad \int \rho_h(m) dm = \langle N_h \rangle$$



the second moments $\langle N_{h,k} \rangle$ are obtained from a set of inhomogeneous linear equations (6 for 3 particle species) with coefficients (matrix A) and r.h.s terms (vector B) which are calculated from the identities and density functions $w_h(x)$, $\rho_h(x)$

$$\vec{N} = A^{-1} \vec{B}$$

the resulting $\langle N_{h,k} \rangle$ and $\langle N_h \rangle$ can then be used to calculate directly the fluctuation measures Ψ , σ_{dyn} , v_{dyn} , ...

new measure Ψ of particle ratio fluctuations

Gazdzicki et al. PRC83,054907(2011)

Ψ related to measure Φ_x , it has the same useful properties:

- strongly intensive measure, i.e. independent of N_W and its fluctuations
- no $1/N_W$ dilution
- vanishes in absence of correlations, e.g. for mixed events $\Psi = 0$

defined in terms of probabilistic identity measure:

$$\Psi_{w_h} = \frac{\langle Z^2 \rangle}{\langle N \rangle} - \overline{z^2} \quad z = w_{h,i} - \overline{w_h} \quad Z = \sum_{i=1}^n (w_{h,i} - \overline{w_h})$$

single particle

event quantity

unique identification: $w_{i,k} = \delta_{i,k}$

$$\Psi = \frac{1}{\langle N_a + N_b \rangle^3} \left[\langle N_a^2 \rangle \langle N_b \rangle^2 + \langle N_a \rangle^2 \langle N_b^2 \rangle - 2 \langle N_a \rangle \langle N_b \rangle \langle N_a N_b \rangle - \langle N_a \rangle^2 \langle N_b \rangle - \langle N_a \rangle \langle N_b^2 \rangle \right]$$

$$= \frac{\langle N_a \rangle^2 \langle N_b \rangle^2}{\langle N_a + N_b \rangle^3} v_{dyn} \approx \frac{\langle N_a \rangle^2 \langle N_b \rangle^2}{\langle N_a + N_b \rangle^3} \sigma_{dyn}^2$$