相対論的電磁流体とその周辺

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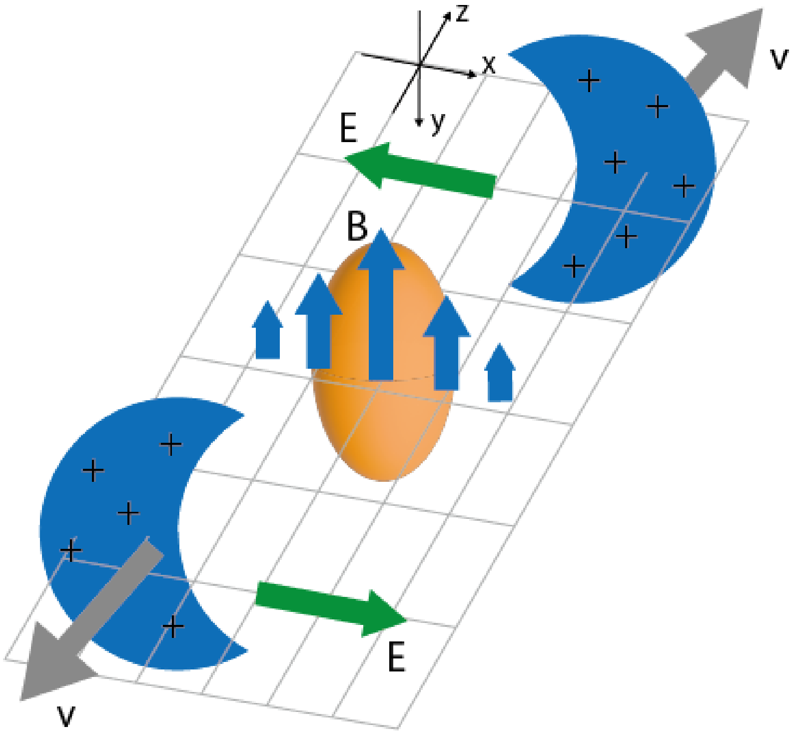
March 1, 2025@Go-Forward 研究会、長崎

Electromagnetic Fields in Heavy Ion Collisions ?

- **Strong Electromagnetic field ?**
  - Au + Au ($\sqrt{s_{NN}} = 200$ GeV) : 10^{14} T ~10 m_π^2
  - Pb + Pb ($\sqrt{s_{NN}} = 2.76$ TeV) : 10^{15} T



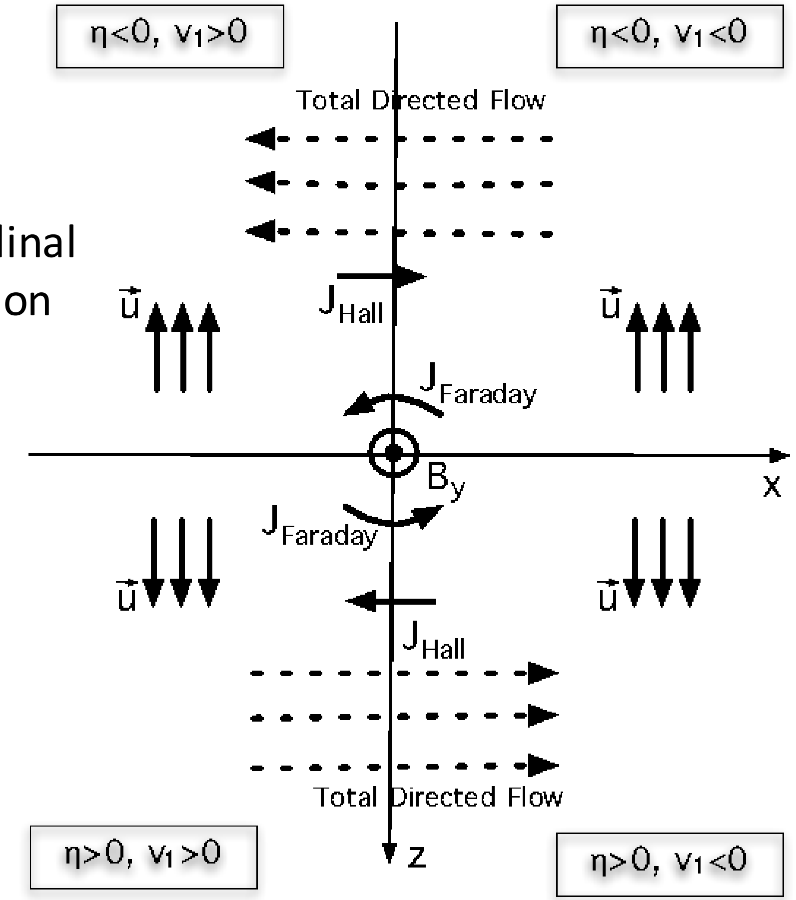
Charge Dependent Flow @ STAR

- **Charge dependent directed flow**

STAR, PRX14,011-28(2024)

$$v_1 \sim \langle \cos(\phi - \Psi_R) \rangle$$

$$\Delta v_1 \sim v_1(h^+) - v_1(h^-)$$



Gursoy et al, PRC89,054905(2014)

EM field effect

- Hall effect $\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$

  Positive Δv_1

- Faraday effect $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

  Negative Δv_1

- Coulomb effect produced by spectators

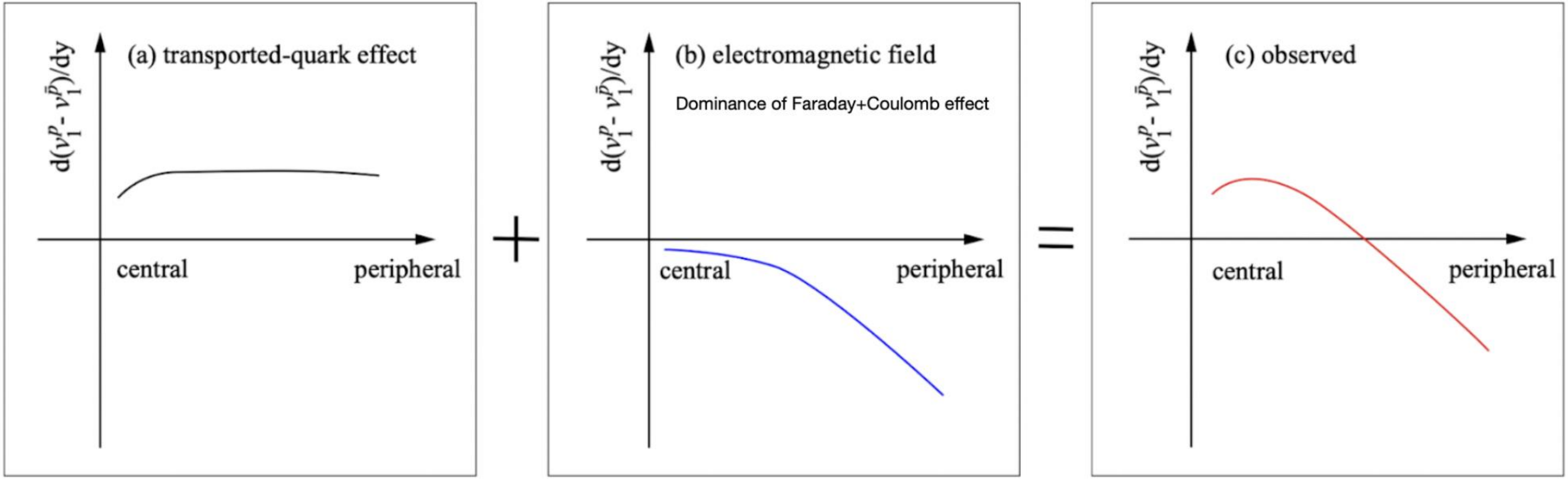
  Negative Δv_1

+ non EM field effect (transported quark effect)
  u and d quarks transported from incoming nuclei

  Positive Δv_1 for proton

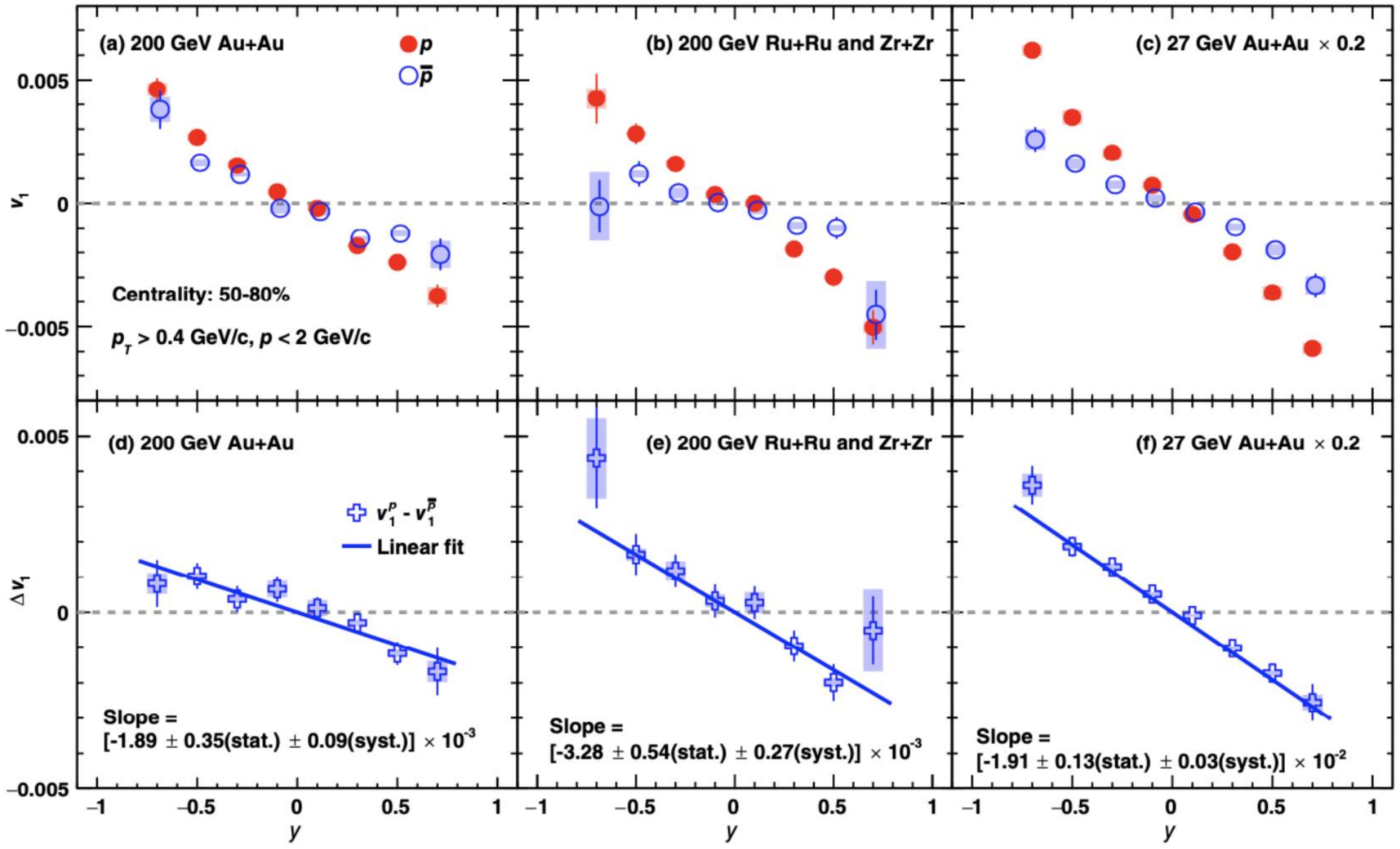
Expectation

STAR, PRX14,011-28(2024)



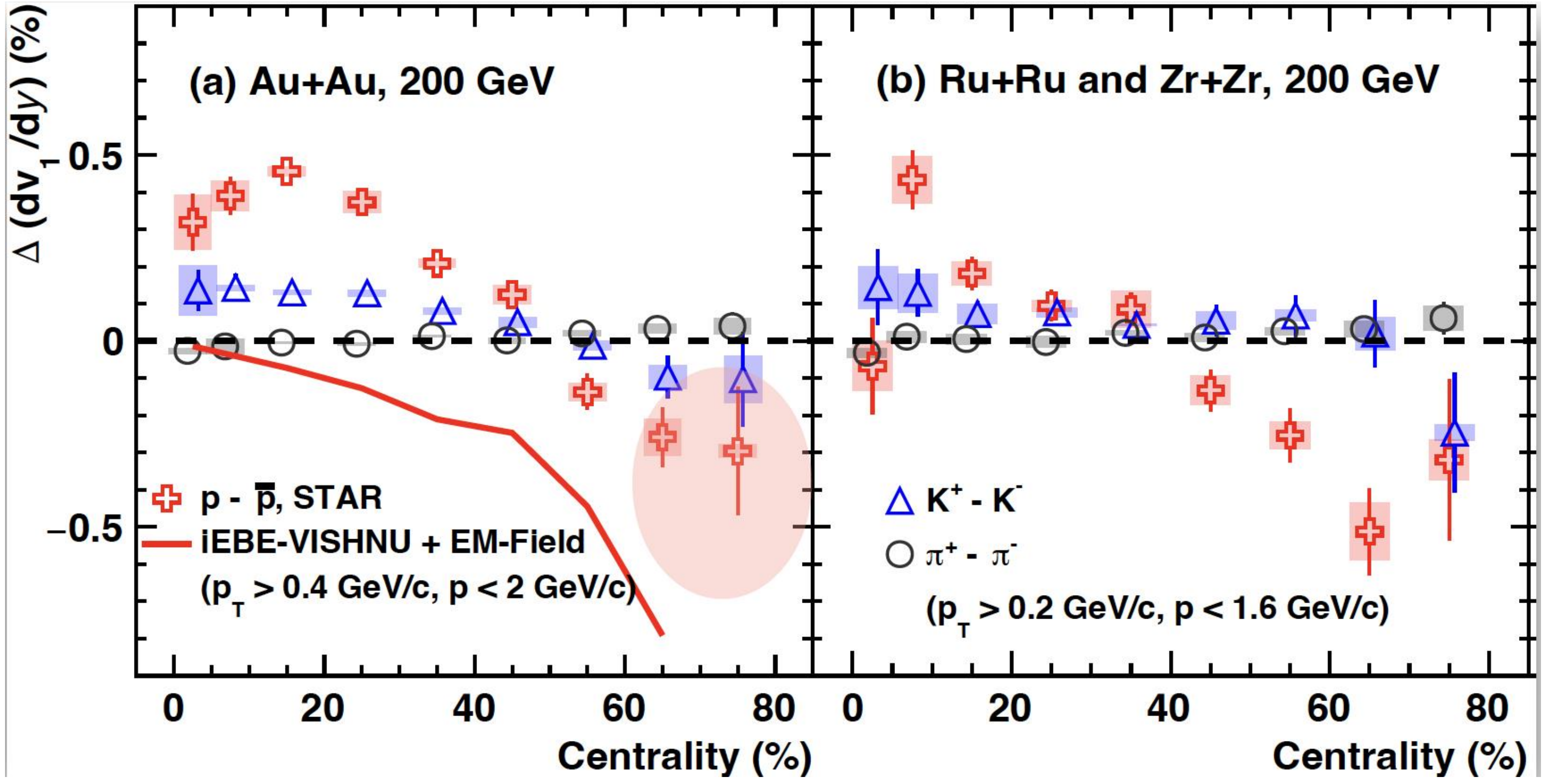
Charge Dependent Flow

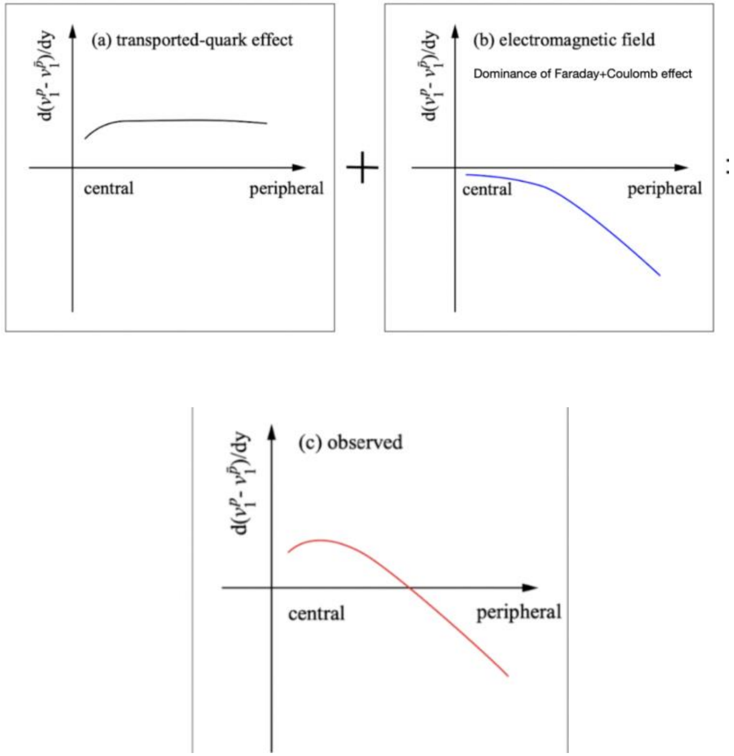
STAR, PRX14,011-28(2024)



Charge Dependent Flow

STAR, PRX14,011-28(2024)





Electromagnetic Fields and Property of QGP

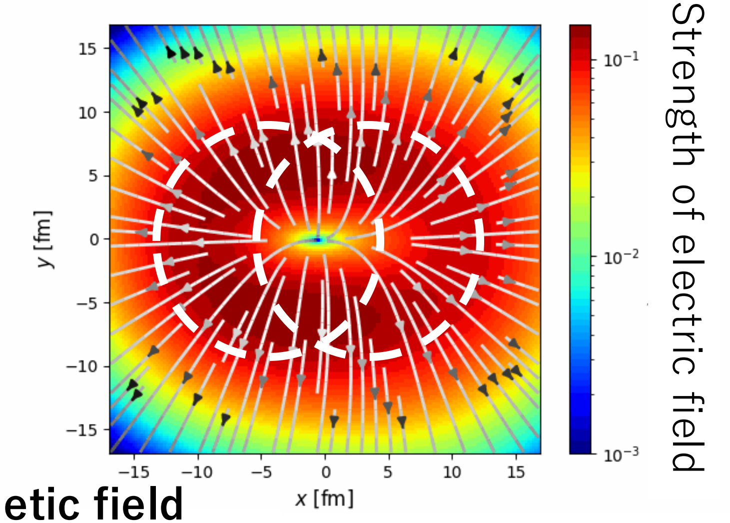
- **Electric Conductivity**
  - **Dissipation of electric field**
    - Ampere's law： $\partial_t \vec{E} - \nabla \times \vec{B} = -\vec{j}$

$\vec{B}$: magnetic field
$\vec{E}$: electric field

Ohm's law makes electric field dissipate

➡ Dissipated energy to fluid（medium）

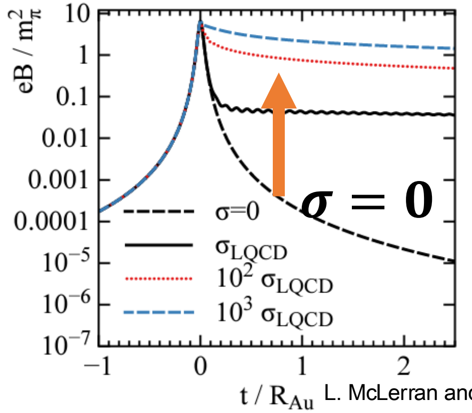
Electric field



  - **Charge is induced by electric field**
    - Induced charge depends on charge conductivity
  - **Dissipation of magnetic field**

Charge conductivity of QGP

Magnetic field



$\sigma \neq 0$

Suppresses of dissipation

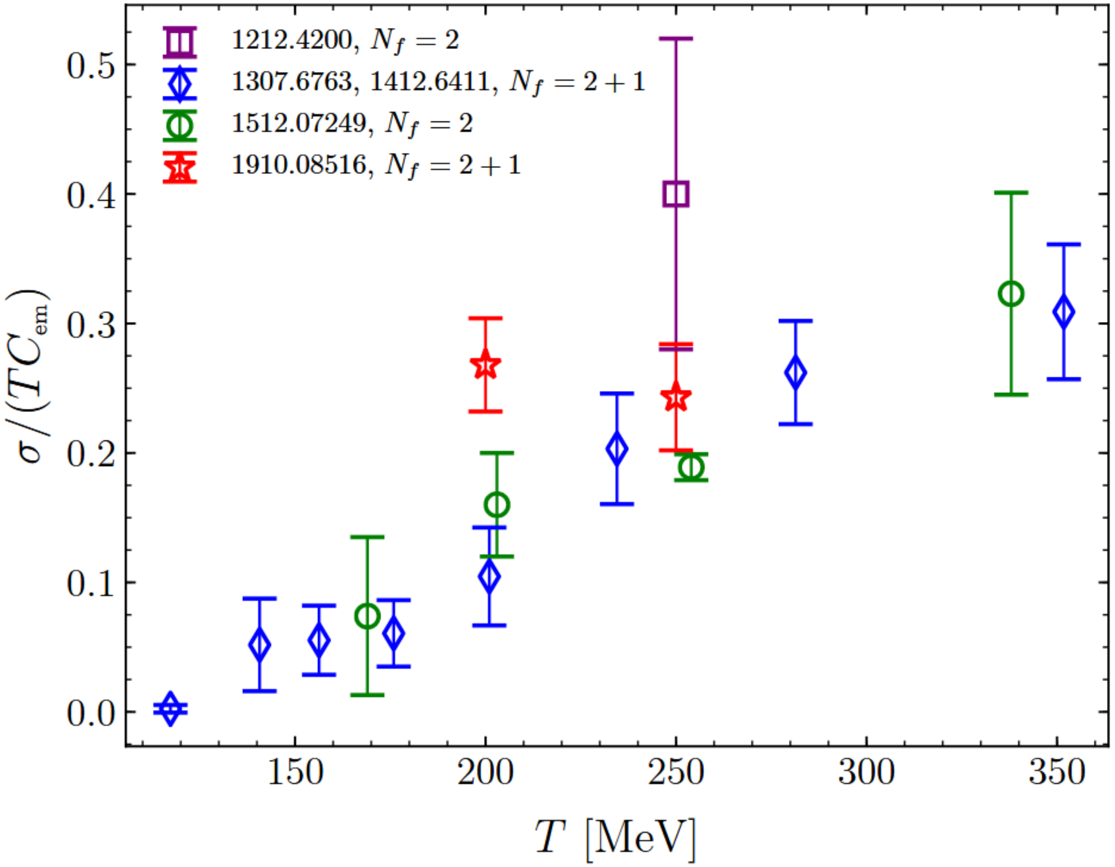
Electric field is dissipated.

L. McLerran and V. Skokov, Nucl. Phys. A 929 (2014), 184-190

⬅ **dissipation of electromagnetic fields and charge distribution QGP**

Electric Conductivity of QCD Matter

- **Lattice QCD**



Aarts, Nikolaev, EPJ.A 57, 118 (2021); 2008.12326 [hep-lat]

Electric Conductivity on the Lattice

$$\sigma = \frac{1}{6}\frac{\partial}{\omega}\left(\int d^4x e^{i\omega t}\langle[j_\mu^{\rm em}(t,x), j_\mu^{\rm em}(0,0)]\rangle\right)|_{\omega=0}$$

Uses linear-response theory (Kubo formula)
Low energy limit of the electromagnetic spectral function

- Does not include external magnetic field effects
- Uses approximately realistic pion mass
- General agreement among results using a variety of methods and parameters

Relativistic resistive magnetohydrodynamics

Understanding of QGP Property

Charge conductivity of QGP from analysis of high-energy heavy-ion collisions

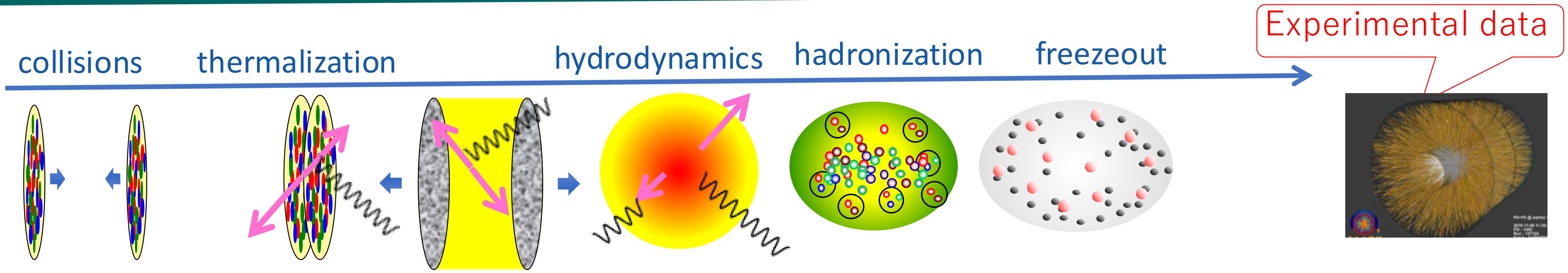
Physical property	Observables	Quantitative analysis
Charge conductivity	Charge dependent flow, EM probes..	Just started
Shear viscosity	Azumithal anisotoropy v_n	○
Bulk viscosity	P_T distributions	○
Diffusion coefficient	Jet energy loss	○

Charge dependent directed flow
Asymmetic collisions → i.e., Hirono, Hongo, and Hirano, PRC 90, 021903 (2014).
Symmetric collisions

Proposed EM observables
Dileptons → i.e., Akamatsu, Hamagaki, Hatsuda, and Hirano, PRC 85, 054903 (2012).
Photons → i.e., Sun and Yan, PRC 109, 034917 (2024).

Construction of relativistic resistive magnetohydrodynamics

Space-Time Evolution of HIC



Initial condition | Hydrodynamics | Final state interactions

+ EM fields

Initial condition | Magnetohydrodynamics | Final state interactions

Initial Conditions

- **Smoothed initial conditions**
- **Fluctuating initial conditions**
- **Glasma initial condition**

Smoothed Initial Condition

Tuchin, Phys.Rev.C88,024911(2013)

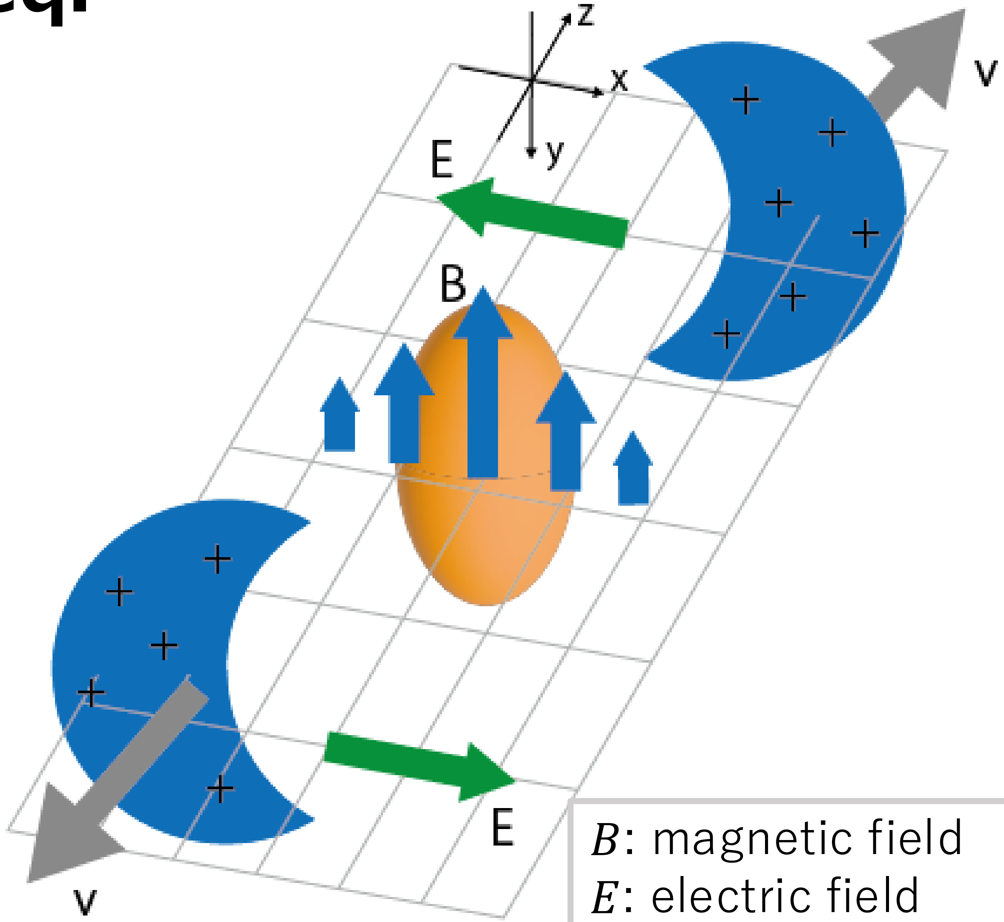
■Asymptotic solution of Maxwell eq.

➤Electromagnetic field made by point charge moving in the longitudinal axis

- Proton distribution in nucleus : uniform sphere
- Constant charge conductivity (σ =0.023 fm^{-1})

$$\nabla \cdot \boldsymbol{B} = 0\,, \quad \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}\,,$$
$$\nabla \cdot \boldsymbol{D} = e\delta(z - vt)\delta(\boldsymbol{b})\,,$$
$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} + \sigma \boldsymbol{E} + ev\hat{z}\delta(z - vt)\delta(\boldsymbol{b})$$

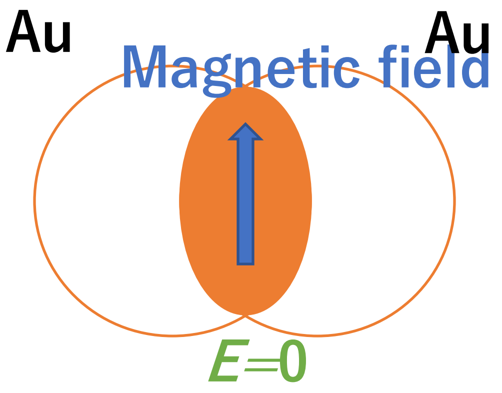
Integration of the asymptotic solutions over the charge distribution inside of nucleus



B: magnetic field
E: electric field
v: velocity of heavy ion

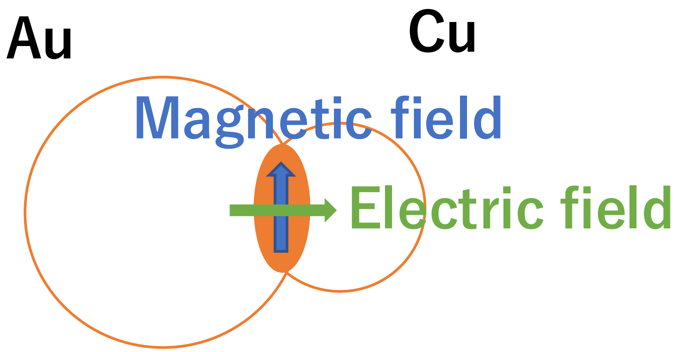
Electromagnetic Field in Symmetric and Asymmetric Systems

■**Au-Au collisions**



- Magnetic field
  - Strong magnetic field
- Electric field
  - No electric field

■**Cu-Au collisions**



- Magnetic field
  - Strong magnetic field
- Electric field
  - $E \neq 0$ **due to the asymmetry of the charge distribution**

Hirono, Hongo, Hirano

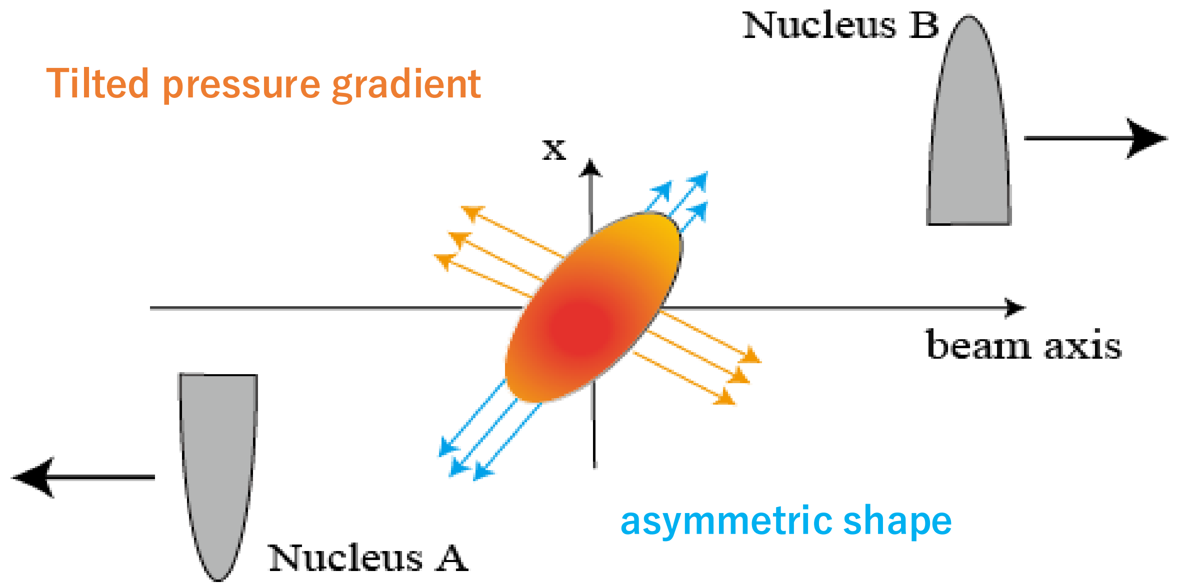
Initial Condition：QGP Medium

■ Tilted Glauber model

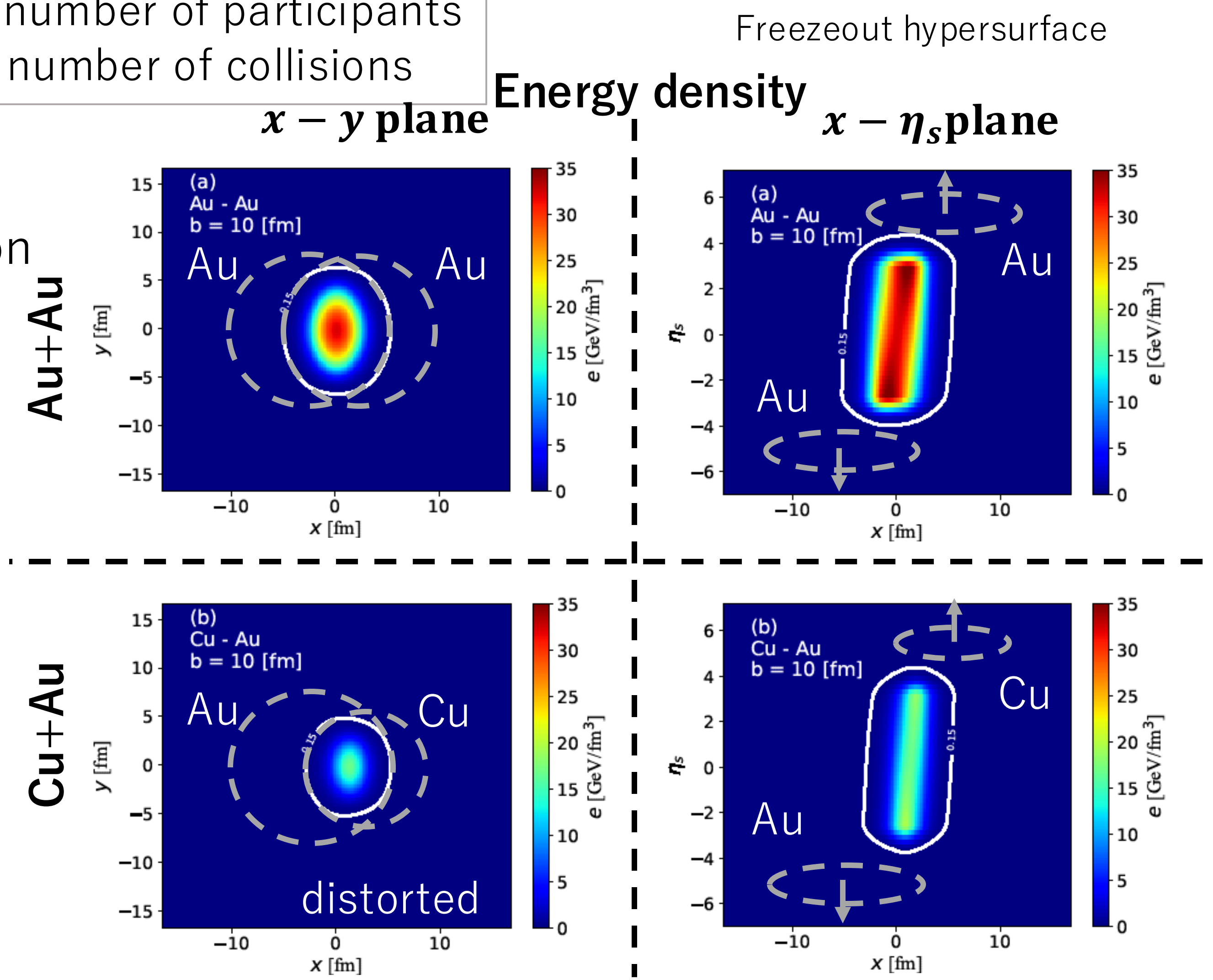
n_p : number of participants
n_c : number of collisions

- Energy density is scaled by n_p and n_c
- Tilted distribution in the longitudinal direction

For directed flow v_1



Bozek, et al, Phys. Rev. C 81, 054902(2010)



Initial Condition：Electromagnetic Fields ($\eta_s = 0$)

Tuchin, Phys.Rev.C88,024911(2013)

■**Au+Au**

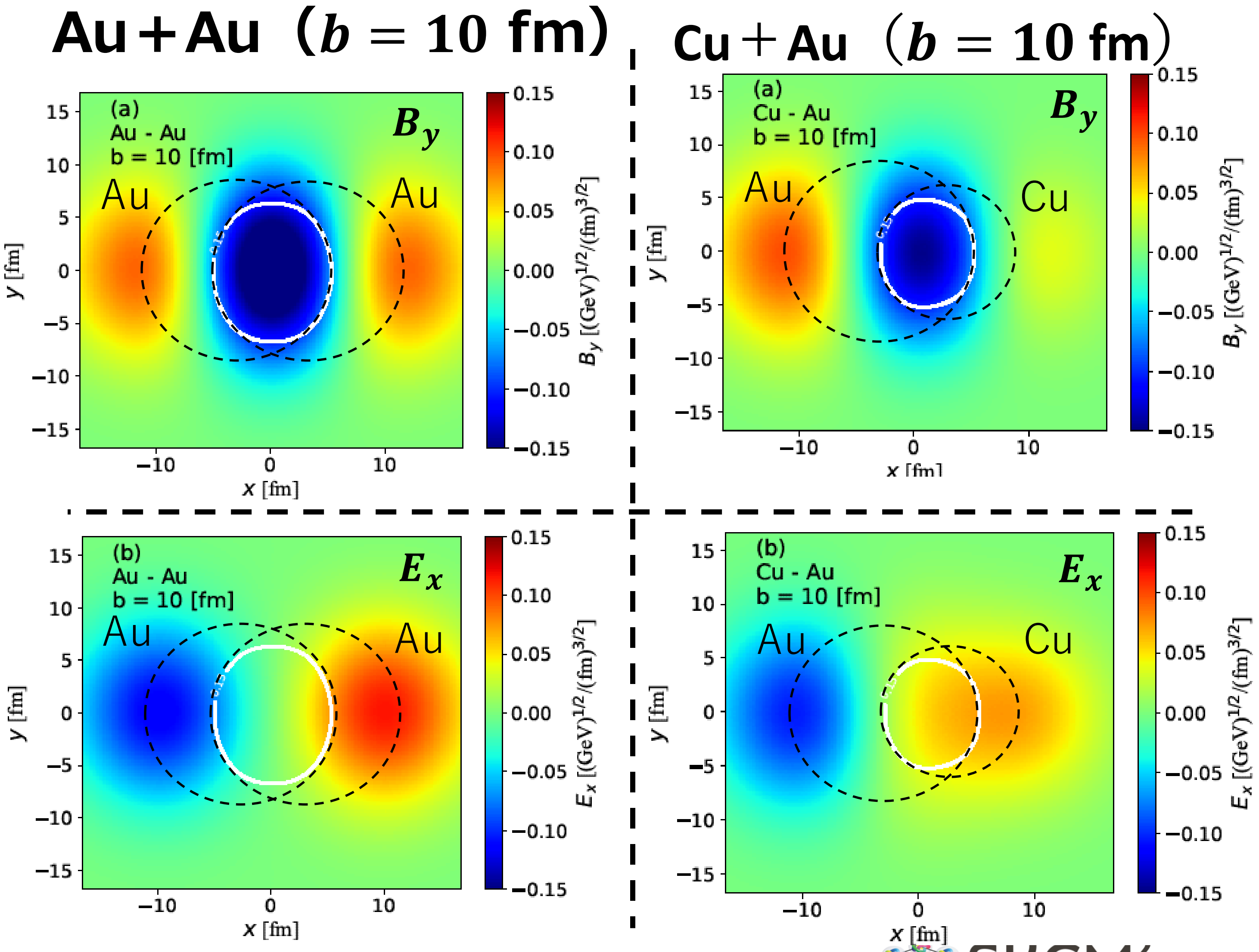
➢Strong magnetic fields in QGP

➢Electric field ～0 in QGP

■**Cu+Au**

➢Strong magnetic field in QGP

➢Finite electric field in QGP



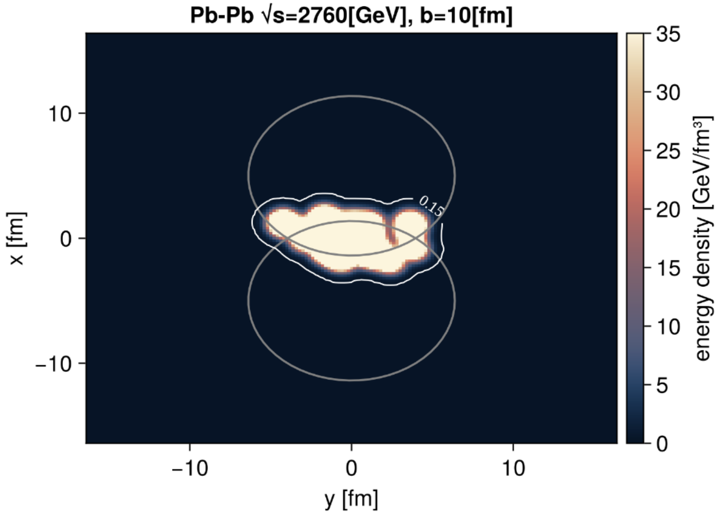
Freezeout hypersurface

Initial Conditions

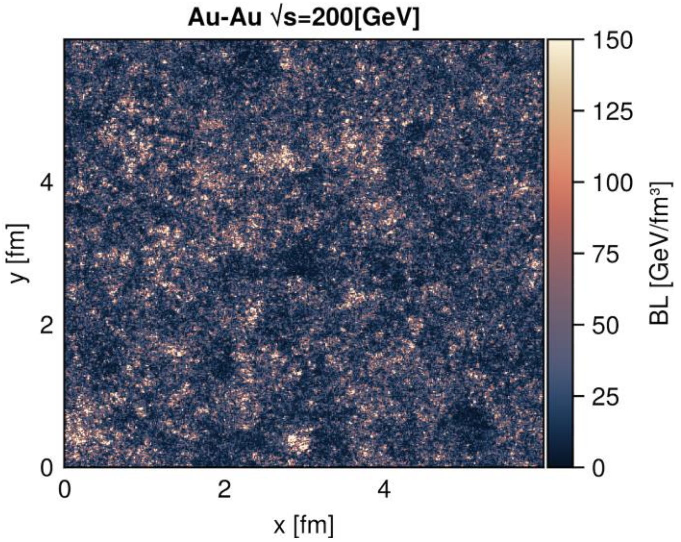
- **Smoothed initial conditions**

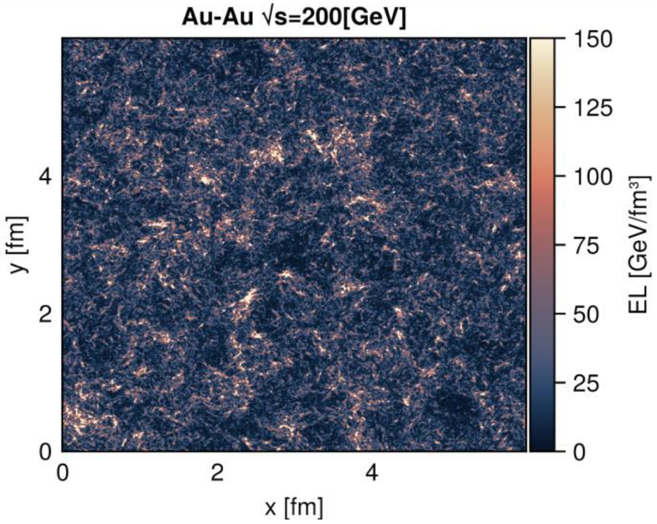
- **Fluctuating initial conditions**
  - Smaller B field, Larger E field

- **Glasma initial condition**
  - Benoit-san's talk



Benoit





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Magnetohydrodynamics

- **Magnetohydrodynamics**
  - Conservation law

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\nabla_\mu T_m^{\mu\nu} = -J_\mu F^{\mu\nu}$$

Energy momentum tensor

$$T^{\mu\nu} = T_m^{\mu\nu} + T_f^{\mu\nu}$$

matter

$$T_m^{\mu\nu} = (e+p)u^\mu u^\nu + pg^{\mu\nu}$$

Maxwell eq.

$$\nabla_\mu F^{\mu\nu} = -J^\nu$$

EM field

$$T_f^{\mu\nu} = F^{\mu\lambda}F_\lambda^\nu - \frac{1}{4}g^{\mu\nu}F^{\lambda\kappa}F_{\lambda\kappa};$$

$$\nabla_\mu T_f^{\mu\nu} = J_\mu F^{\mu\nu}$$

  - Ohm's law *Blackman and Field, PRL71,3481(1993)*

$$J^\mu = \sigma F^{\mu\nu}u^\nu + qu^\mu$$

Electrical conductivity

$$q = -J^\mu u_\mu$$

Magnetohydrodynamics

- **Magnetohydrodynamics**
  - Conservation law

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\nabla_\mu T^{\mu\nu}_m = -J_\mu F^{\mu\nu}$$

Energy momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_m + T^{\mu\nu}_f$$

matter

$$T^{\mu\nu}_m = (e+p)u^\mu u^\nu + pg^{\mu\nu}$$

+viscosity, *Hattori..*

Maxwell eq.

$$\nabla_\mu F^{\mu\nu} = -J^\nu$$

EM field

$$T^{\mu\nu}_f = F^{\mu\lambda}F^\nu_\lambda - \frac{1}{4}g^{\mu\nu}F^{\lambda\kappa}F_{\lambda\kappa};$$

$$\nabla_\mu T^{\mu\nu}_f = J_\mu F^{\mu\nu}$$

- Ohm's law *Blackman and Field, PRL71,3481(1993)*

$$J^\mu = \sigma F^{\mu\nu}u^\nu + qu^\mu$$

$$q = -J^\mu u_\mu$$

Electrical conductivity

Charge diffusion, *Dash et al., PRD107,056003(2023)*

Magnetohydrodynamics

- **RRMHD equation**
  - ➢ Conservation law + Maxwell eq. + Ohm's law

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda$$

$$J^\mu = J_c^\mu + q u^\mu$$

e: energy density
p: pressure
$p_{em} = (E^2 + B^2)/2$

$$\varepsilon = (e+p)\gamma^2 - p + p_{em}$$
$$m^i = (e+p)\gamma^2 v^i + \epsilon^{ijk} B_j E_k$$
$$\Pi^{ij} = (e+p)\gamma^2 v^i v^j + (p + p_{em}) g^{ij} - E^i E^j - B^i B^j$$

Energy Conservation

$$\partial_t \varepsilon + \nabla \cdot m = 0$$

Momentum conservation

$$\partial_t m^i + \nabla \cdot \Pi^{\mathrm{i}} = 0$$

Faraday's law

$$\partial_t \vec{B} + \nabla \times \vec{E} = 0$$

Ohm's law

$$\vec{J} = q\vec{v} + \underline{\sigma\gamma[\vec{E} + \vec{v} \times \vec{B} - (\vec{v} \cdot \vec{E})\vec{v}]}$$
$$=: \vec{J}_c$$

Ampere's law

$$\partial_t \vec{E} - \nabla \times \vec{B} = -\vec{J}$$

$$\partial_t \vec{E} - \nabla \times \vec{B} = q\vec{v}$$

$$\partial_t \vec{E} = \vec{J}_c$$

operator splitting

- Integration with quasi-analytic solutions

$$\vec{E}_\perp = -\vec{v} \times \vec{B} + \left(E_\perp^0 + \vec{v} \times \vec{B}\right) \exp(-\sigma\gamma t)$$
$$\vec{E}_\parallel = E_\parallel^0 \exp(-\sigma t/\gamma)$$

Benoit, Miyoshi, C. N., and Takahashi, ArXiv: 2502.04611
Nakamura, Miyoshi, C. N. and Takahashi, Phys. Rev. C 107, (2023) 014901
Nakamura, Miyoshi, C. N. and Takahashi, Eur.Phys.J.C 83 (2023) 3, 229.
Nakamura, Miyoshi, C. N. and Takahashi, Phys. Rev. C 107 (2023) 3, 034912

RRMHD Equation in Milne Coordinates

New

- **Milne coordinates**
  - **Expanding systems in the longitudinal direction (τ, x, y, η_s)**
    - Strong expansion in the longitudinal direction is effectively included.
    - Number of grid of fluid is saved.

$$\tau = \sqrt{t^2 - z^2}$$
$$\eta_s = \frac{1}{2}\ln\frac{t+z}{t-z}$$

RRMHD Equation

$$\partial_\tau(\tau U) + \partial_i(\tau F^i) = \tau S$$

$$U = \begin{pmatrix} D \\ m_j \\ \varepsilon \\ B^j \\ E^j \\ q \end{pmatrix}, \mathrm{F^i} = \begin{pmatrix} Dv^i \\ \Pi^{ji} \\ m^i \\ \varepsilon^{jik}E_k \\ \varepsilon^{jik}B_k \\ J^i \end{pmatrix}, S = \begin{pmatrix} 0 \\ \frac{1}{2}T^{ik}\partial_j g_{ik} \\ -\frac{1}{2}T^{ik}\partial_0 g_{ik} \\ 0 \\ J_c^i \\ 0 \end{pmatrix}$$

The first RRMHD code in Milne coordinates

Validation of the Code

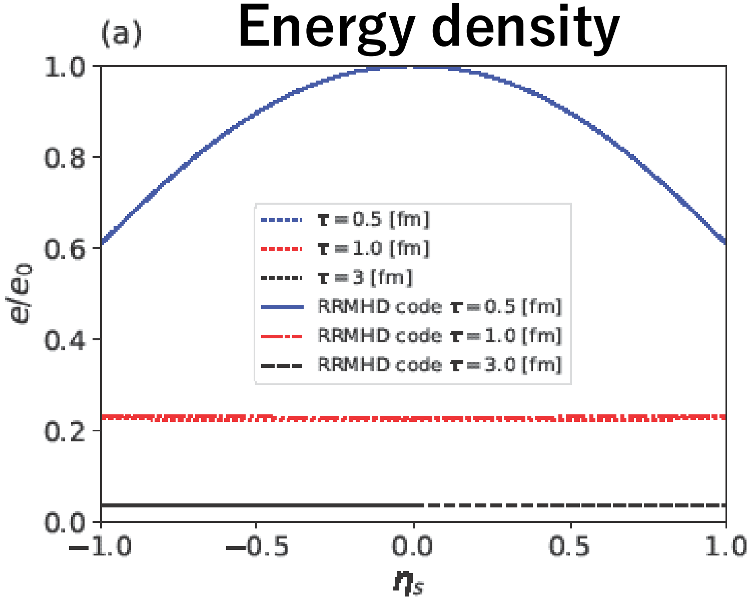
- **RRMHD in the Milne coordinates**

Nakamura, Miyoshi, CN and Takahashi, Eur.Phys.J.C 83 (2023) 3, 229.

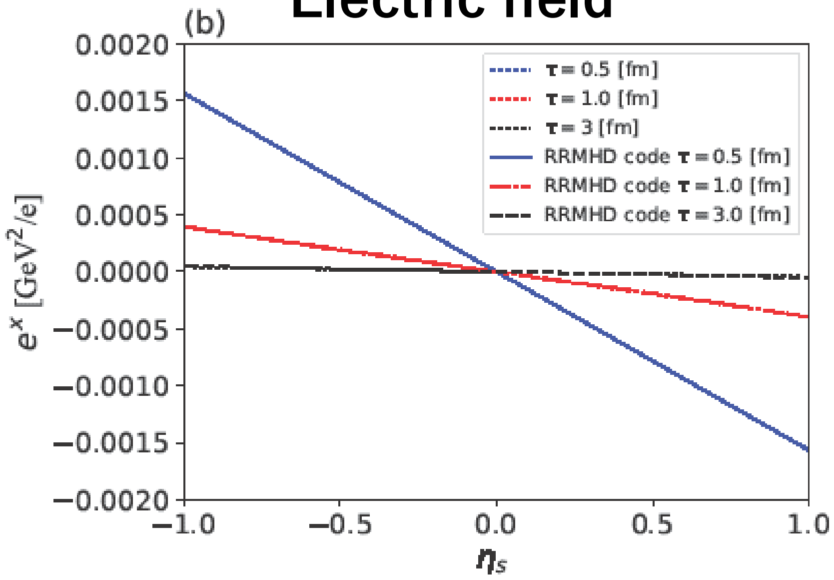
New Test Problem

- **(1+1) dimensional expansion system** $u^\mu = (\cosh Y, 0, 0, \sinh Y)$
  - Comparison between quasi-analytical solution and RRMHD simulation

Energy density



Electric field



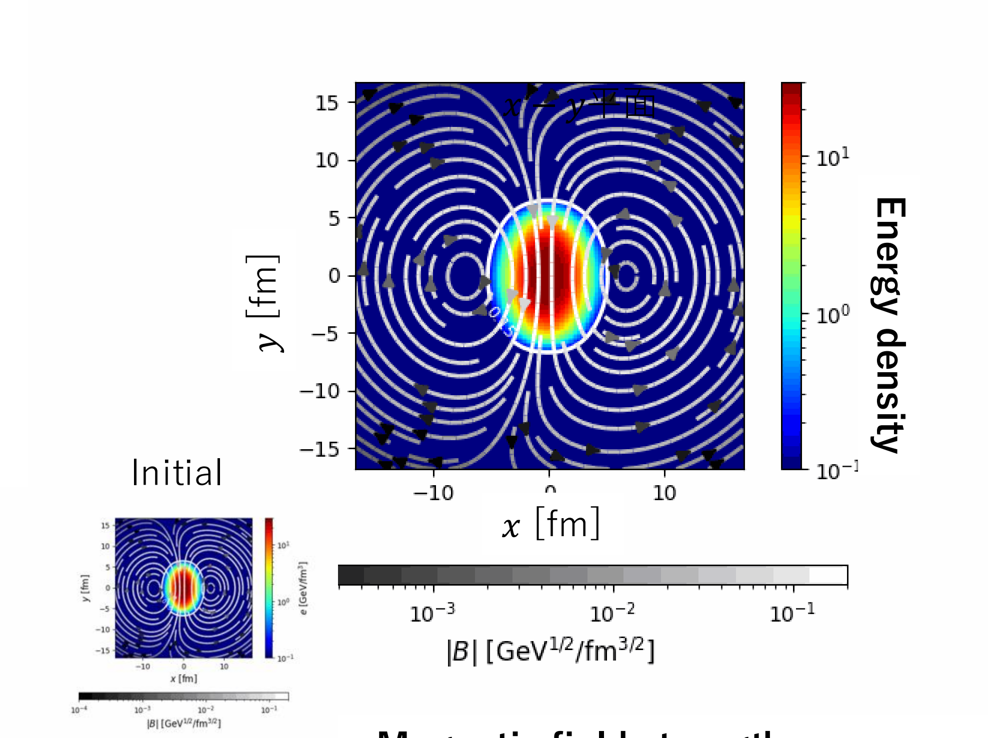
Solid line : RRMHD code
Dashed line: quasi-analytical solution

➡ **Application to Heavy Ion Collisions**

Space-time Evolution

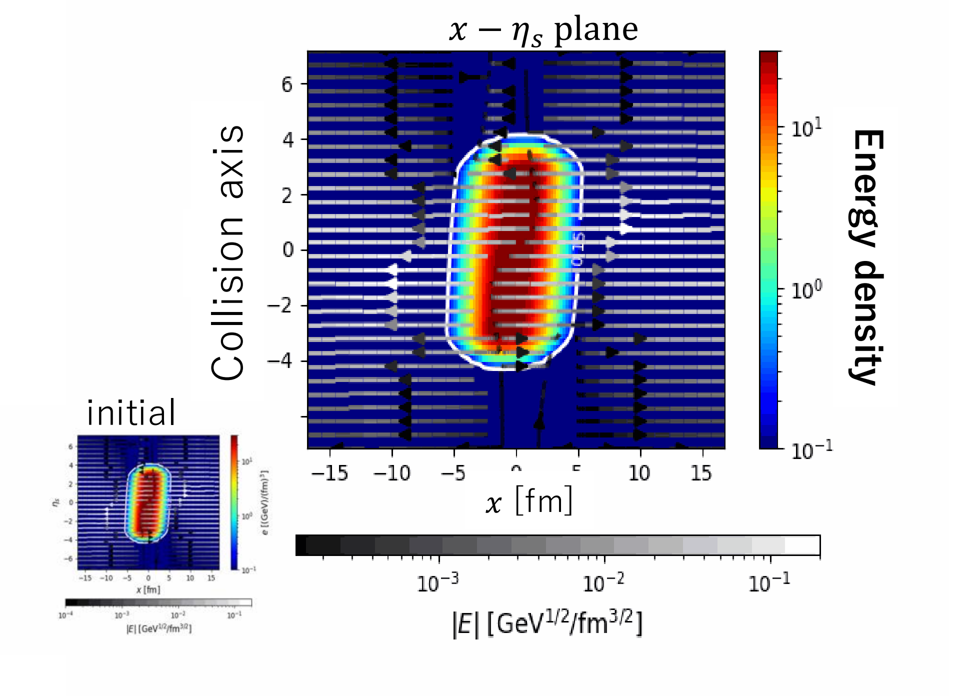
Nakamura, Miyoshi, CN and Takahashi, PRC 107, no.1, 014901 (2023)

Au+Au collision system



Magnetic field strength

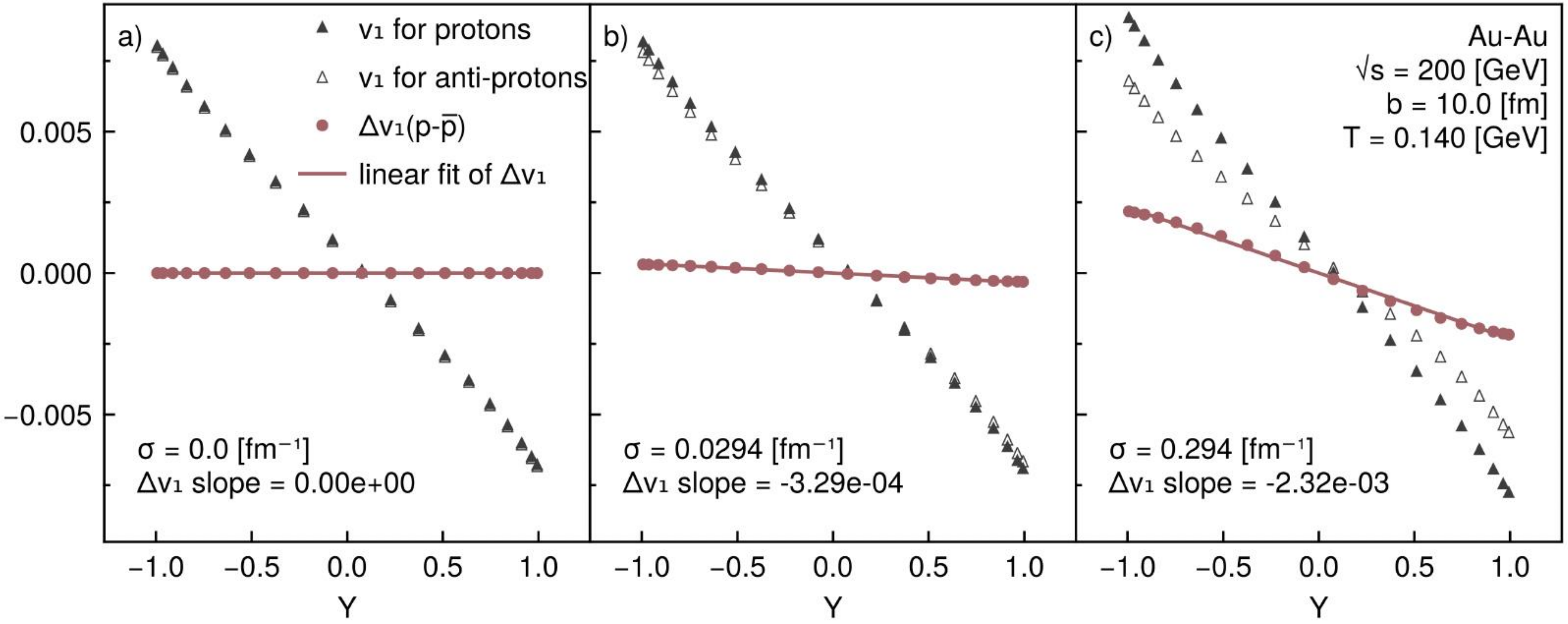
First calculation in HIC with RRMHD code



Electric field strength

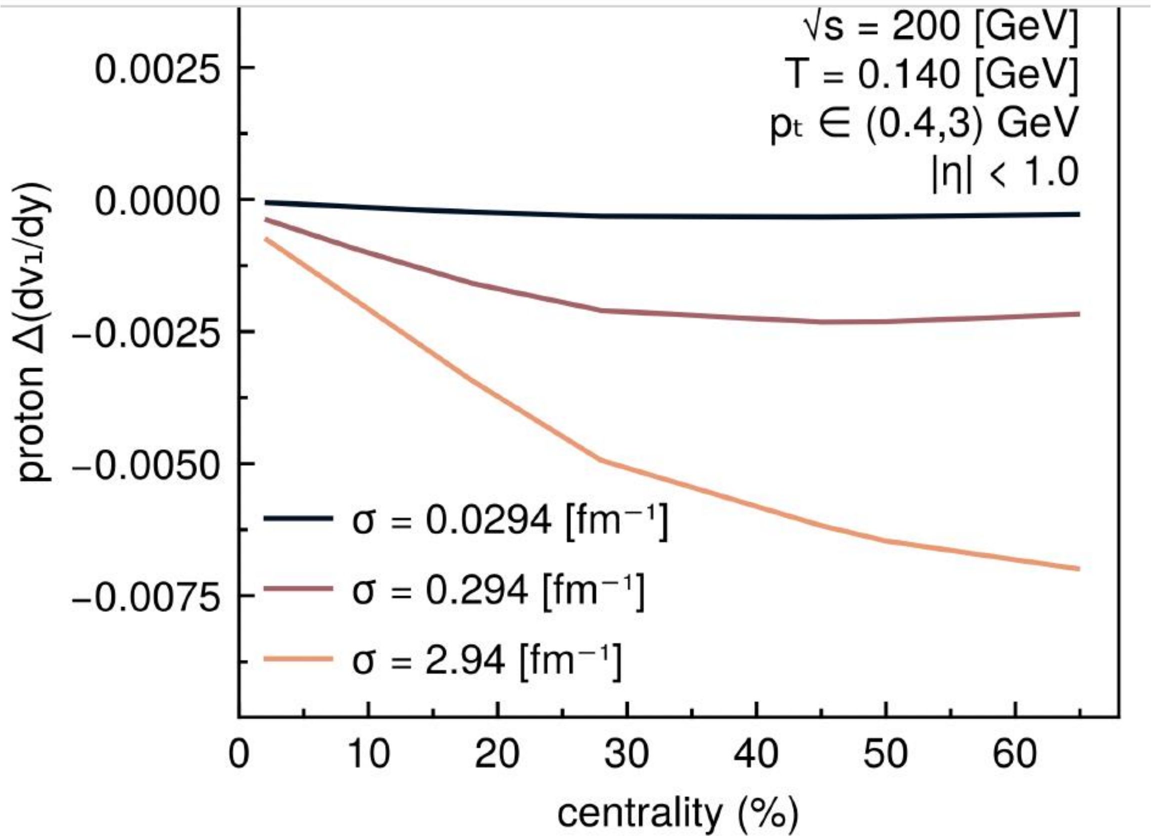
Analysis of Heavy Ion Collisions

Charge Dependent Directed Flow

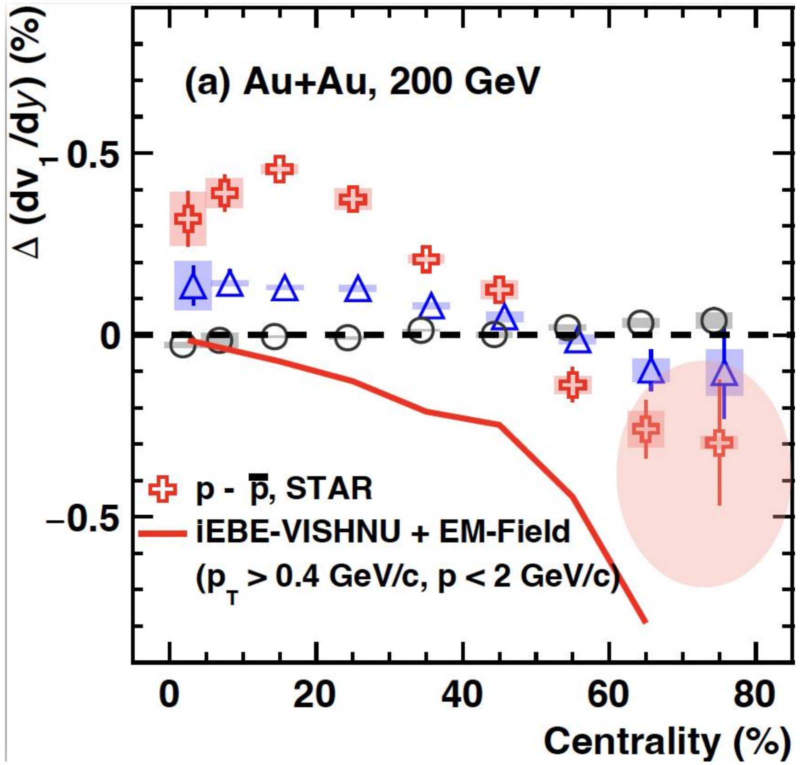


Benoit, Miyoshi, C. N., and Takahashi, ArXiv: 2502.04611

Charge Dependent Flow



Benoit, Miyoshi, C. N., and Takahashi, ArXiv: 2502.04611



Caveat:

- No baryon current
- Initial condition
- EoS
- Final state interactions

Electromagnetic Dissipation for QGP Photon

Benoit

- **Electromagnetic fields inside QGP**
  - EM fields penetrating QGP drive charge carriers out-of-equilibrium

$$J^\mu = qu^\mu + \sigma F^{\mu\nu} u_\nu$$

First order dissipation from the EM fields

  - Taking the Boltzmann equation in the relaxation time application

$$k^\mu \partial_\mu f_a + \underline{eQ_a F^{\mu\nu} k_\mu \frac{\partial f_a}{\partial k^\nu}} = -\frac{k^\mu u_\mu}{\tau_R} \delta f_{a,EM}^{(n)}$$

Sun and Yan, PRC 109, 034917 (2024).

Vlasov term for the external EM fields

Order "n" corrections
to the quark distribution function

$$\delta f_{a,EM}^{(1)}(X,k) = -\frac{-f_{a,eq}(1-f_{a,eq})}{T\chi_{el} k^\mu u_\mu} e\sigma Q_a e^\mu k_\mu$$

Electric conductivity of QGP from
Landau matching with the current

EM fields in the fluid rest frame

$$e^\mu = (\gamma v_k E^k, \gamma E^i + \gamma \epsilon^{ijk} v_j B_k)$$

Electromagnetic Dissipation for QGP Photon

Benoit

- **Electromagnetic fields inside QGP**
  - The fluid + EM field contributions from hydrodynamics
  - All of those values can be calculated self-consistently using relativistic resistive magneto-hydrodynamics (RRHMD)

Temperature and four velocity

$$\delta f^{(1)}_{a,EM}(X,k) = -\frac{-f_{a,eq}(1-f_{a,eq})}{T\chi_{el}k^\mu u_\mu} e\sigma Q_a e^\mu k_\mu$$

Electric susceptibility of QGP

$$\chi_{a,el} = -\frac{1}{3}\int \frac{d\vec{p}}{(2\pi)^3 E_p}(p^\sigma p^\nu \Delta_{\sigma\nu})\frac{-f_{a,eq}(1-f_{a,eq})}{p^\mu u_\mu}$$

Spacetime dependent EM fields in QGP medium

$$e^\mu = (\gamma v_k E^k, \gamma E^i + \gamma \epsilon^{ijk} v_j B_k)$$

Photon production from QGP and EM fields

Benoit

- **Rate of QGP photon production should be increased by the EM fields**

$$E_k \frac{d\mathcal{R}}{d^3\vec{k}} = E_k \frac{d\mathcal{R}}{d^3\vec{k}}^{\mathrm{QGP}} + E_k \frac{d\mathcal{R}}{d^3\vec{k}}^{\mathrm{EM}}$$

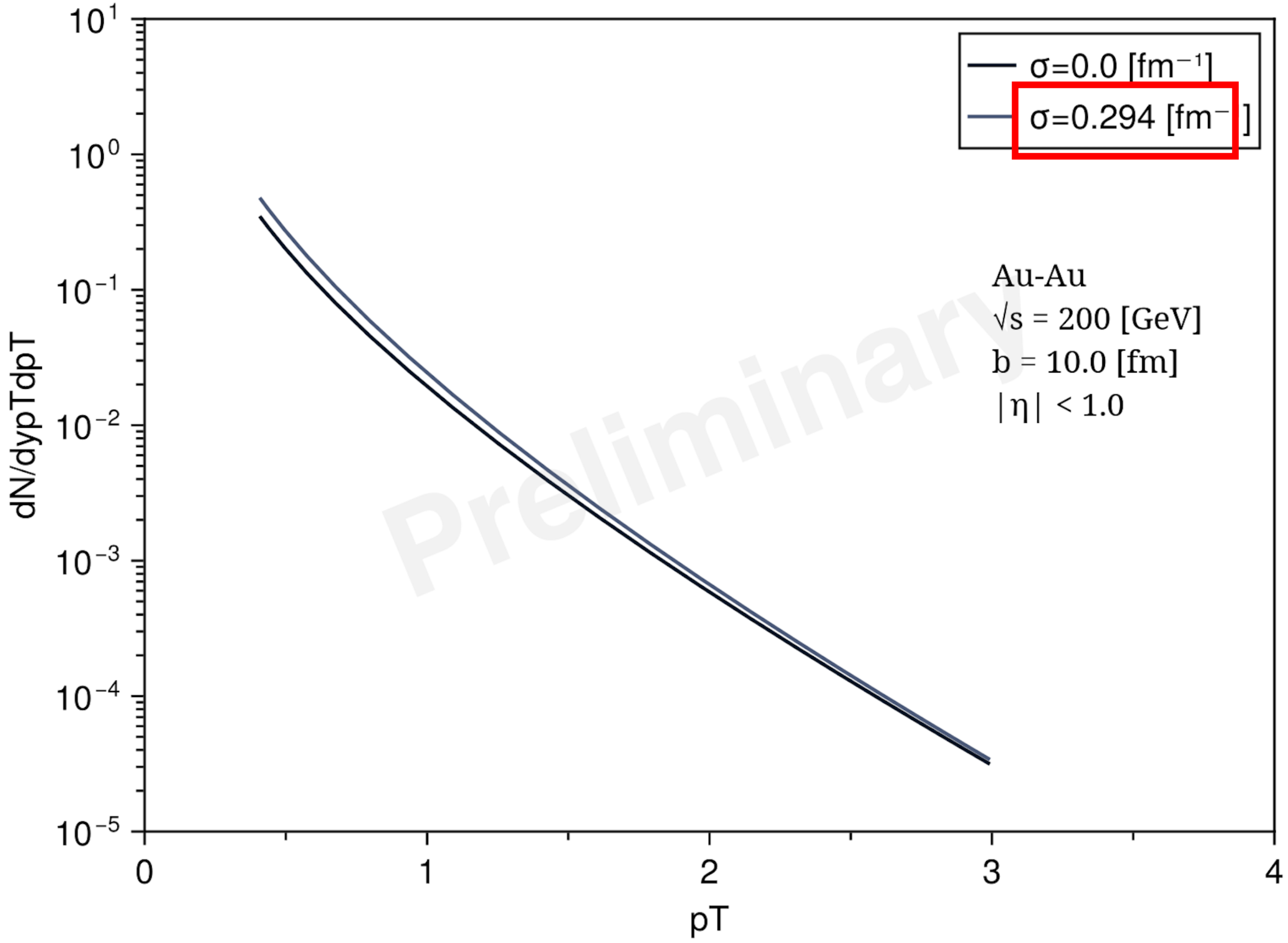
$$E_k \frac{d\mathcal{R}}{d^3\vec{k}}^{\mathrm{EM}} \sim C\alpha_s \alpha_{\mathrm{EM}} \mathcal{I} \mathcal{L}_c \sum_a \delta f^{(1)}_{a,\mathrm{EM}}(X,k)$$

We focus on effect of EM dissipation

We neglect viscous dissipation effect

P_T Spectra of Direct Photon

Benoit



$$E_k \frac{d\mathcal{R}}{d^3\vec{k}}^{\mathrm{EM}} \sim C\alpha_s\alpha_{\mathrm{EM}}\mathcal{I}\mathcal{L}_c \sum_a \delta f^{(1)}_{a,\mathrm{EM}}(X,k)$$

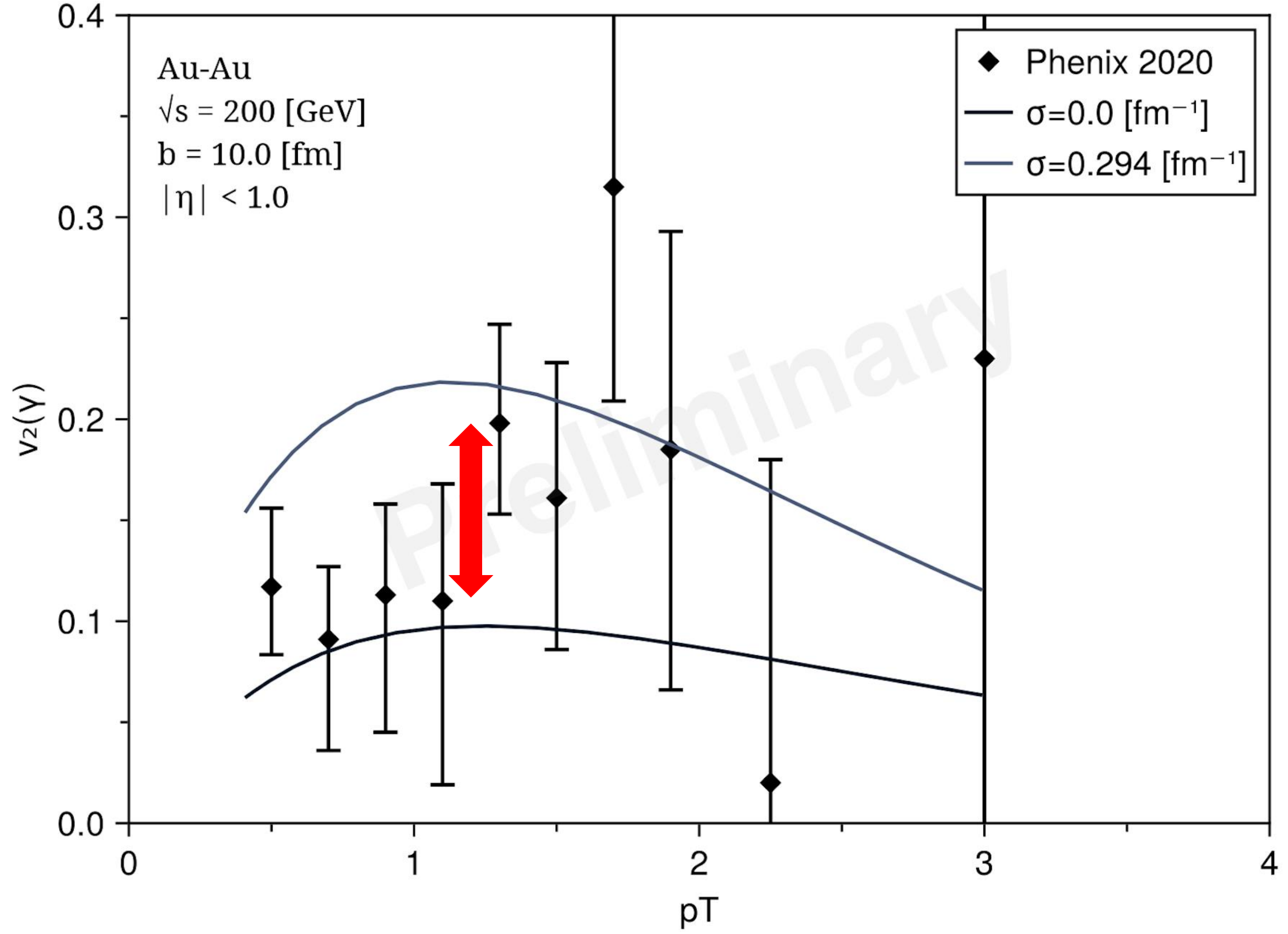
From Lattice QCD

$\sigma = 0.029$ [fm^{-1}]

Small contribution to P_T spectra

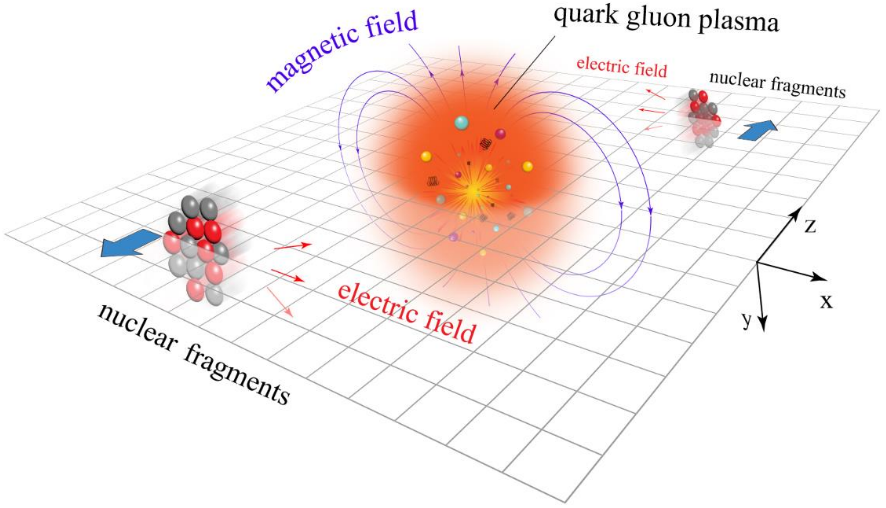
Elliptic Flow of Direct Photon

Benoit



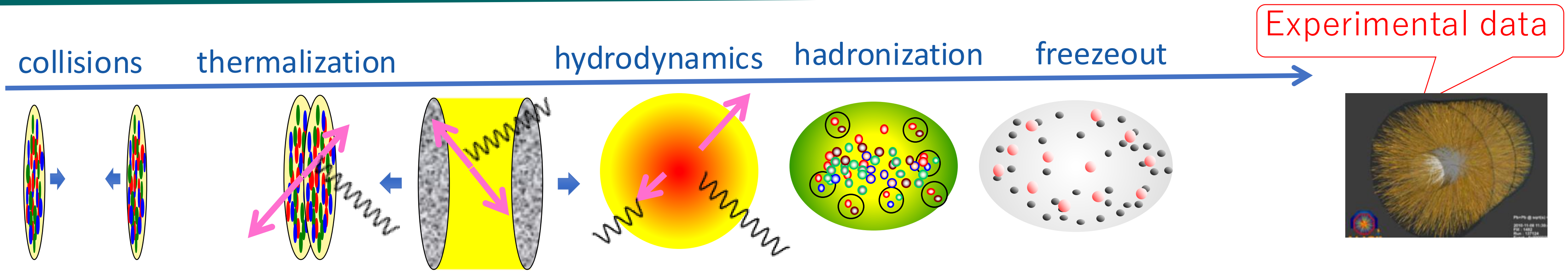
Large enhancement is observed.

$$v_2(\gamma) \equiv \frac{v_0 v_2 + v_0^{EM} v_2^{EM}}{v_0 + v_0^{EM}}$$



Since largest magnetic field has an elliptic orientation, a larger impact from the EM corrections on elliptic flow appears.

Summary



Initial condition

Event-by-event fluctuation
Glasma

Causality

Magnetohydrodynamics

Baryon current
EoS
viscosity

Final state interactions

Chiral Magnetohydrodynamics

Charge Dependence of Δv_1 : Au + Au

- $\Delta v_1 = v_1^{\pi^+}(\eta) - v_1^{\pi^-}(\eta)$
  - Clear dependence of charge conductivity
    - Proportion to electric conductivity
    - Negative charge induced in the opposite direction of fluid flow

      suppression of v_1 of negative charge
  - Δv_1 with finite σ is consistent with STAR data
    - $\sigma = 0.0058\ \mathrm{fm}^{-1}$

      ex. $\sigma_{LQCD} = 0.023\ \mathrm{fm}^{-1}$

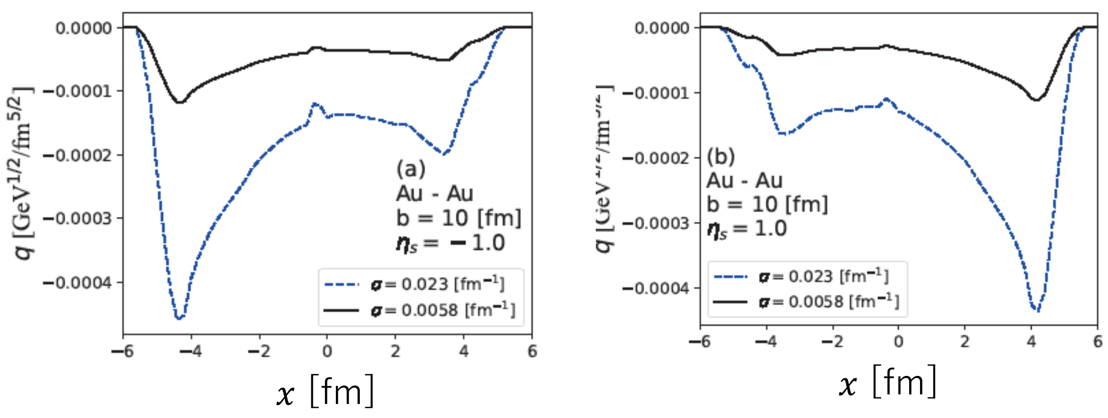
      from lattice QCD

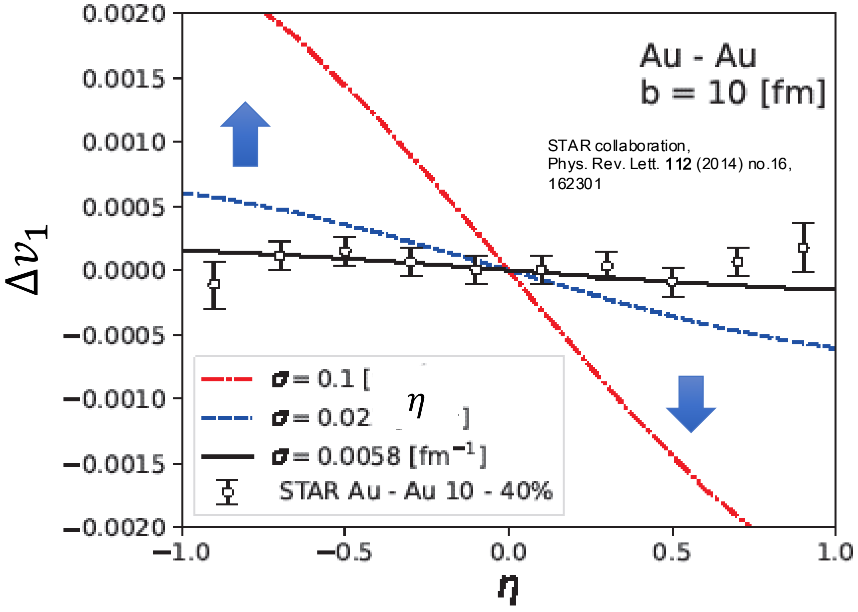
      Gert Aarts, et al.

      Phys. Rev. Lett., 99:022002, 2007.

✓ **QGP electrical conductivity from high-precision measurement of Δv_1**

Charge distribution on freezeout hypersurface





$\sigma = 0.1$ fm^{-1}

$\sigma = 0.023$ fm^{-1}

$\sigma = 0.0058$ fm^{-1}

○ : STAR data

$$\eta = \frac{1}{2}\ln\frac{|p| + p_z}{|p| - p_z}$$

Charge Dependence of Δv_1 ：Cu + Au

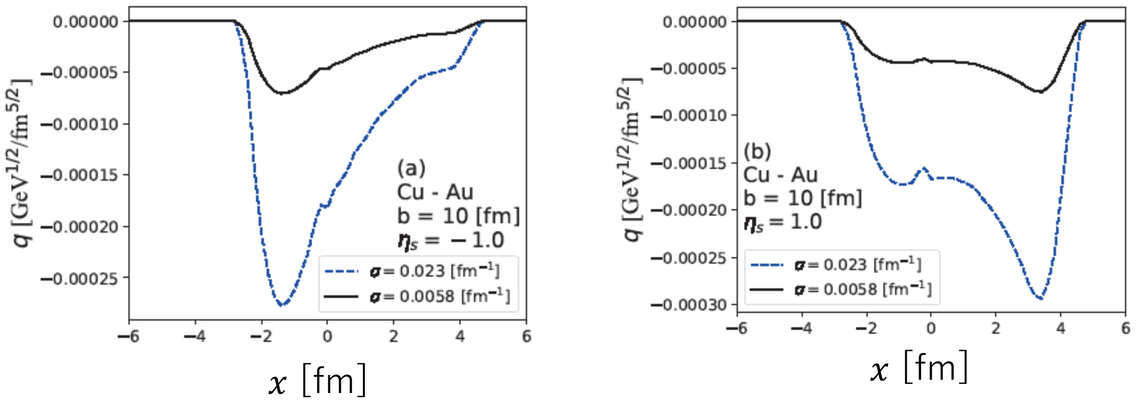
Nakamura, Miyoshi, CN and Takahashi, Phys. Rev. C 107 (2023) 3, 034912

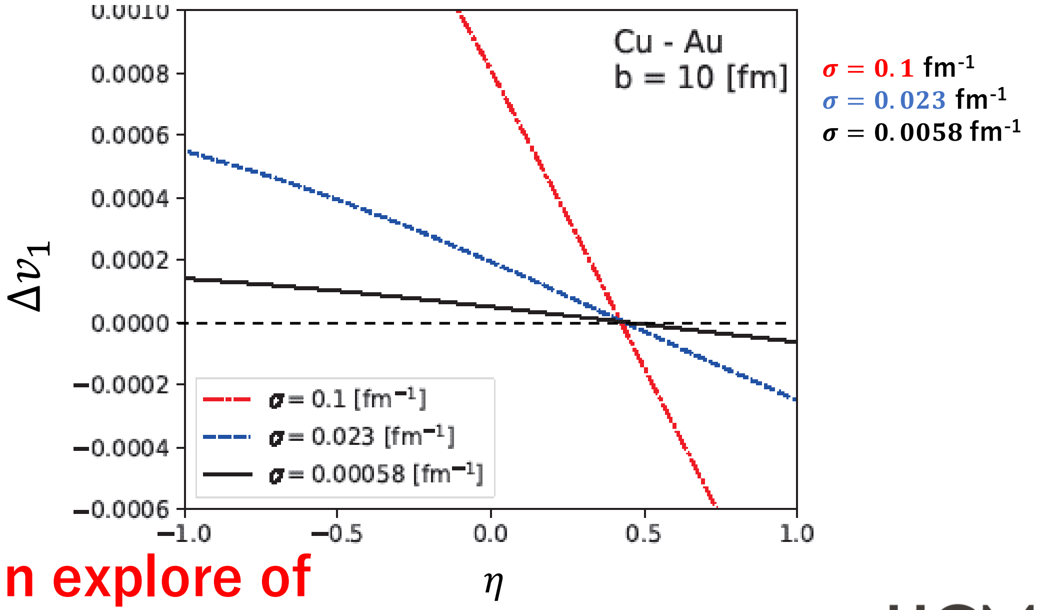
- $\Delta v_1 = v_1^{\pi^+}(\eta) - v_1^{\pi^-}(\eta)$
  - **Electric field created by initial condition**
    - Δv_1 is finite at $\eta = 0$
    - Asymmetry structure to $\eta = 0$
  - **Proportion to electric conductivity**
    - Δv_1 vanishes at $\eta = 0.5$.

✓ **Electrical conductivity <- Δv_1 at $\eta = 0$**

✓ **Initial electrical field from η dependence of Δv_1**

Charge distribution on freezeout hypersurface





Asymmetric system has advantage in explore of QGP electrical conductivity.