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Lund, May 13th, 2025

Spin-dependent hadronization in Pythia

Albi Kerbizi
Lund University and INFN Trieste

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LUND UNIVERSITY

INFN Trieste
Istituto Nazionale di Fisica Nucleare

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Spin-dependent hadronization in Pythia

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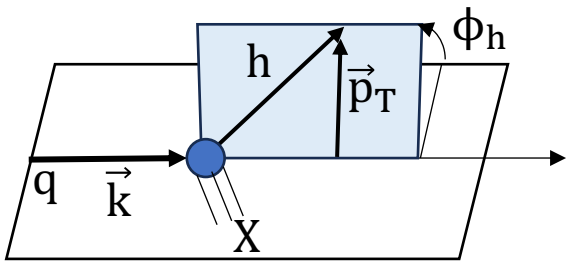
Motivation
- spin effects in hadronization are a beautiful challenge!
- connected to the partonic spin structure of nucleons
- there is data DIS (COMPASS, HERMES, JLAB)
  e^+e^- (BELLE, BABAR, BESIII)
  pp (RHIC)
- new experiments forseen EIC, JLAB22, LHCSpin

Spin effects in hadronization

- **Collins effect** in the fragmentation of transversely polarized quarks

Collins, NPB 396, 161 (1993)

$$z = p^+/k^+$$



$$D_{q^\uparrow \to h+X}(z, \vec{p}_T) = D_{1q}^h(z, p_T^2)$$

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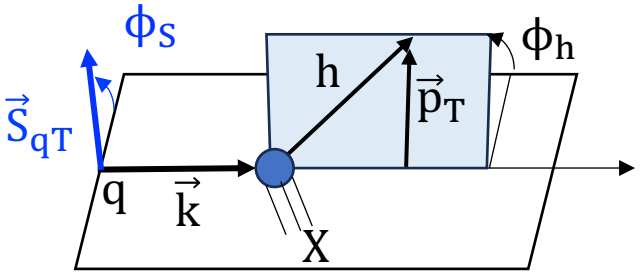


Collins fragmentation function (FF)

$$D_{q^\uparrow \to h+X}(z, \vec{p}_T) = D_{1q}^h(z, p_T^2) + \frac{H_{1q}^{h\perp}(z, p_T^2)}{zM_h} \vec{S}_{qT} \cdot \hat{k} \times \vec{p}_T$$

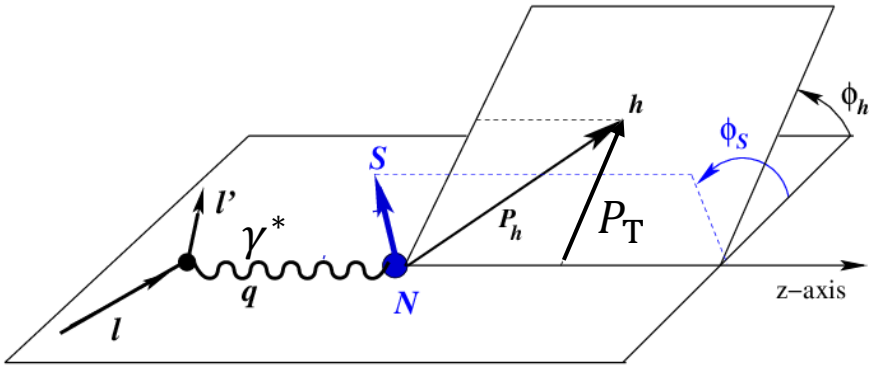
$$\propto 1 - \frac{H_{1q}^{h\perp}}{zM_h D_{1q}^h} |\vec{S}_{qT}| |\vec{p}_T| \sin(\phi_h - \phi_{S_{qT}})$$

- Asymmetric azimuthal distribution
  sensitive to the quark transverse polarization → quark polarimetry!

- Appears in different processes:
  - semi-inclusive DIS (SIDIS)
  - $e^+e^- \to$ hadrons
  - $pp \to hX$

Collins effect in SIDIS: the Collins asymmetry

- Asymmetry in the azimuthal distribution of the observed hadrons in DIS



$$\phi_{Coll} = \phi_h + \phi_S - \pi$$

$$d\sigma^h \propto 1 + D_{NN}\,|\vec{S}_T| A_{Coll} \sin\phi_{Coll} + \cdots$$

$$\mathbf{A_{Coll}} \simeq \frac{\sum_q e_q^2 h_1^q \otimes H_{1q}^{\perp h}}{\sum_q e_q^2 f_1^q \otimes D_{1q}^h}$$

Transversity PDF h_1^q → transverse spin of quarks in a transversely polarized nucleon

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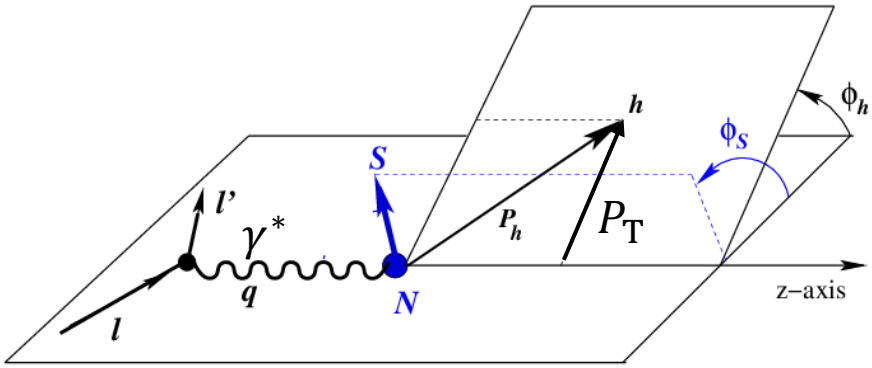

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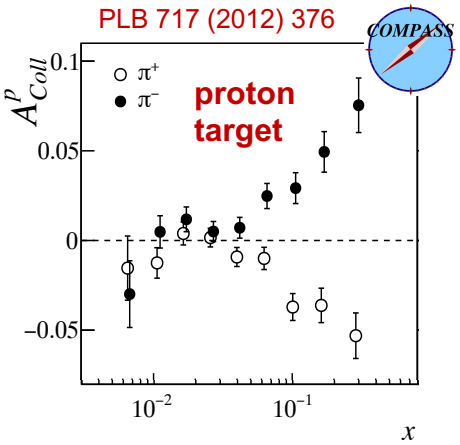
- A_{Coll} measured by different experiments, e.g. COMPASS with a p target



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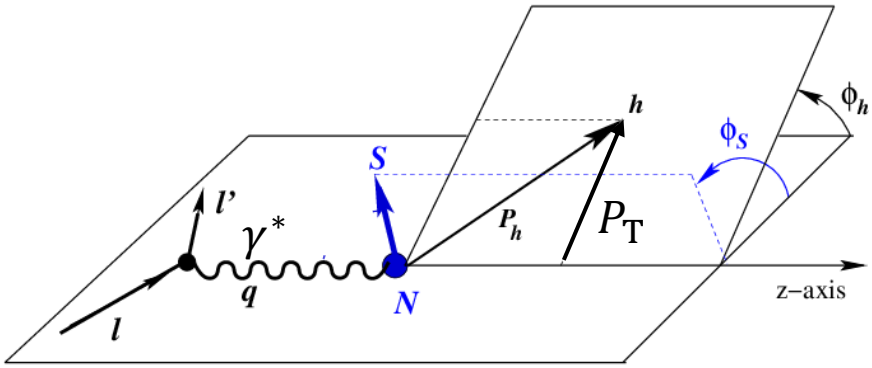

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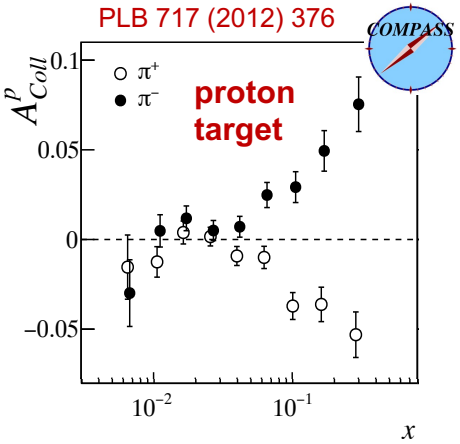
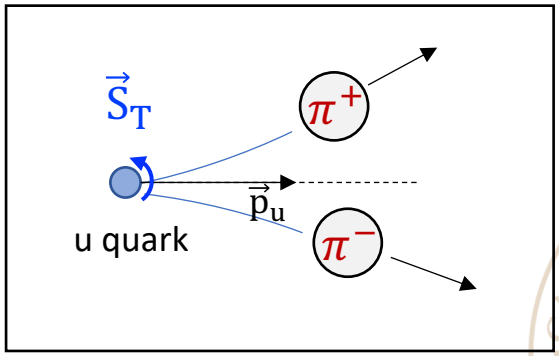
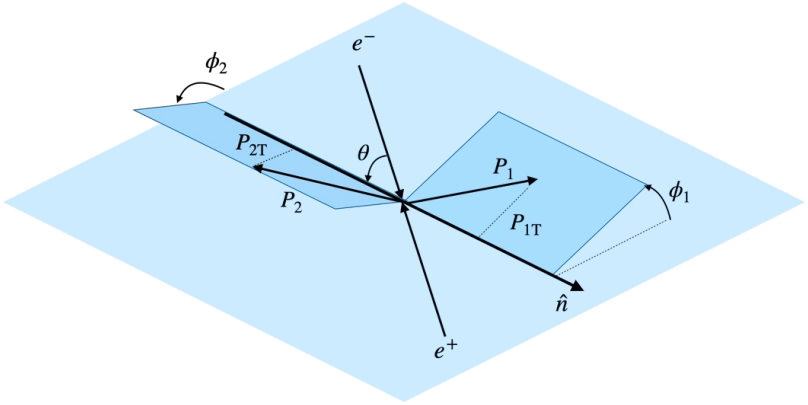


$$A_{Coll}^p(x) \simeq \frac{h_1^u}{f_1^u} \times \frac{H_{1q}^{\perp h}}{D_{1q}^h}$$



Collins effect in e^+e^-: the Collins asymmetry

- Azimuthal anisotropy in the azimuthal distribution of back-to-back hadrons in e^+e^-



$$d\sigma^{h_1 h_2} \propto 1 + \hat{a}_{NN}(\theta)\, A_{12} \cos(\phi_1 + \phi_2) + \cdots$$

$$A_{12} \simeq \frac{\sum_q e_q^2 H_{1q}^{\perp h_1} \times H_{1q}^{\perp h_2}}{\sum_q e_q^2 D_{1q}^{h_1} \times D_{1q}^{h_2}}$$

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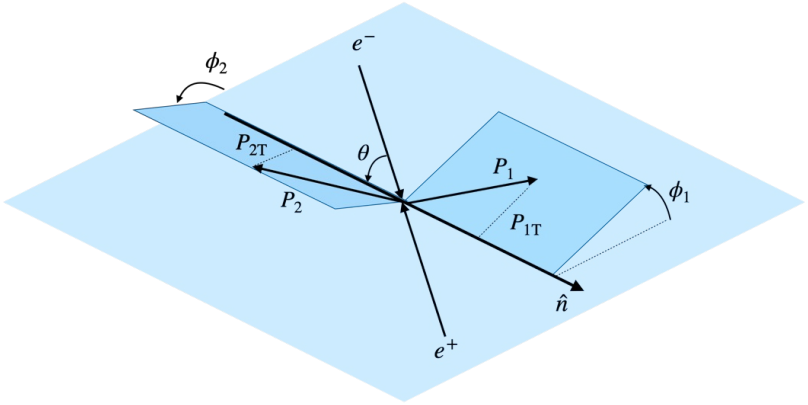

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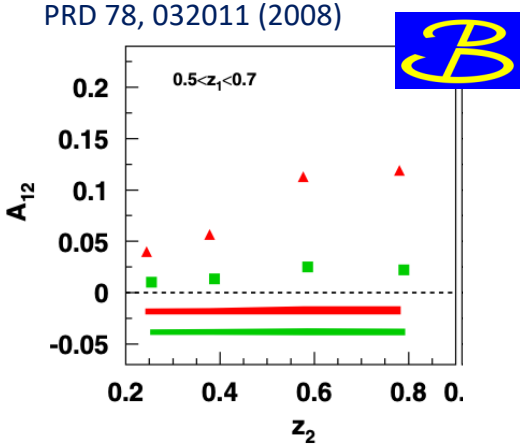
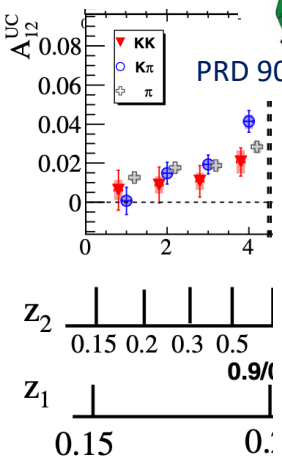
- Measured by BELLE, BABAR, BESIII

PRD 78, 032011 (2008)



PRD 90, 052003 (2014)



- Independent confirmation that Collins FF ≠0
- Source of information on Collins FF

Other interesting spin effects not mentioned

- interference FF
- jet handedness ($G_1^{\perp}$ FF)
- spontaneous polarization of hyperons (polarizing FF)
- jet functions
- …

Here focus on the Collins effect

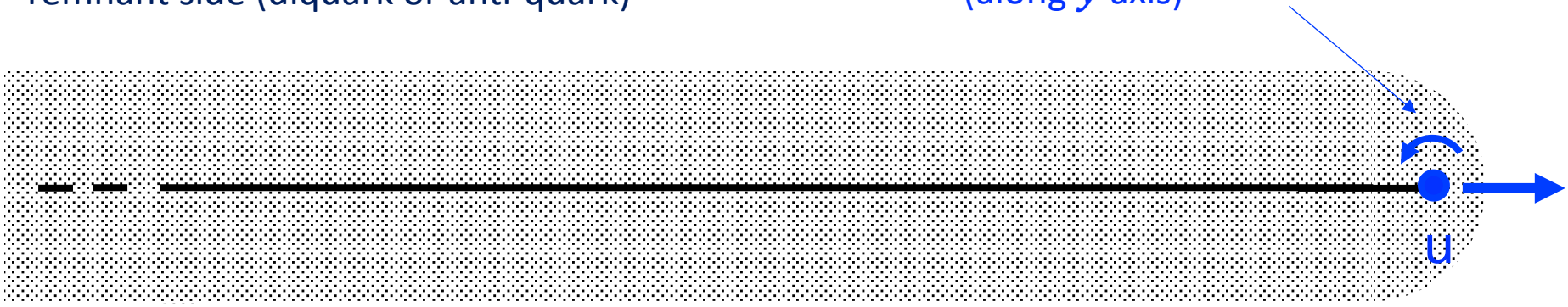
- model and simulations (parton showers OFF)

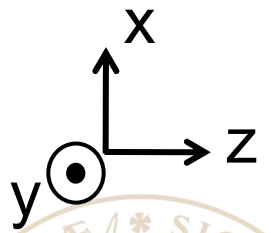
"classical" string + $^3P_0$

X. Artru [2009] arXiv:1001.1061

← remnant side (diquark or anti-quark)

transversely polarized
(along $\hat{y}$ axis)





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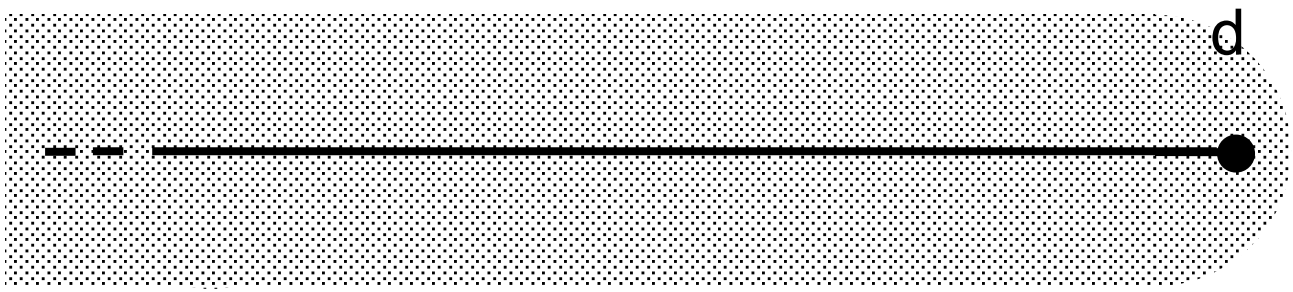

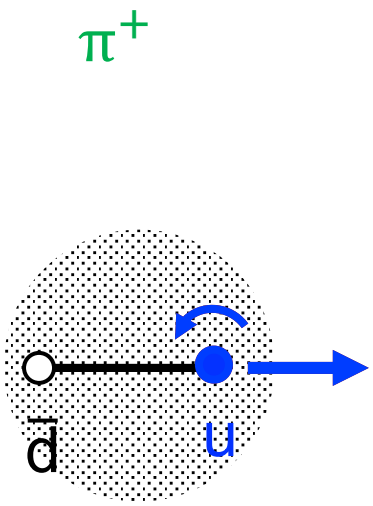

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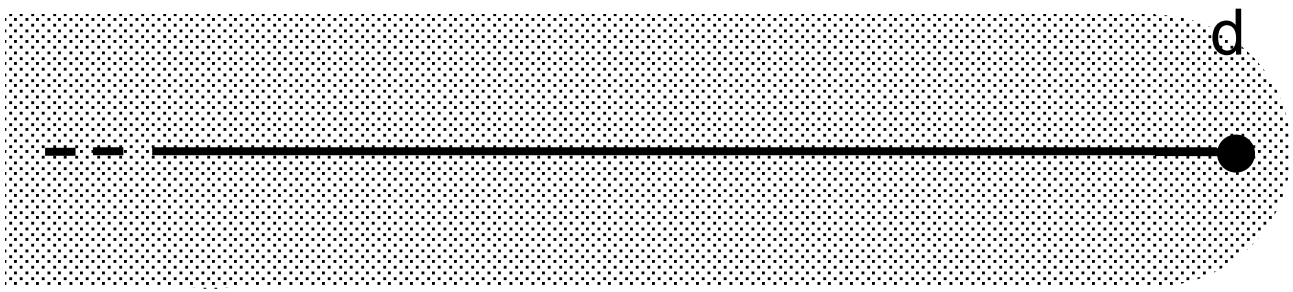
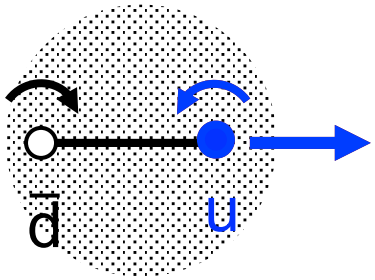


π^+



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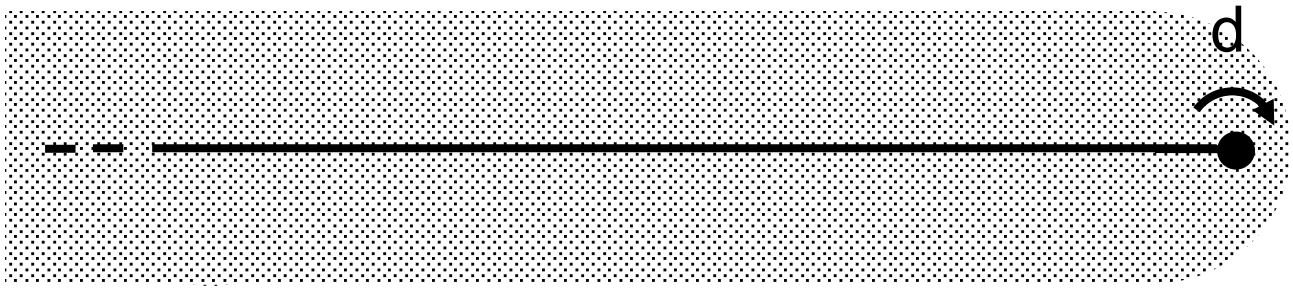

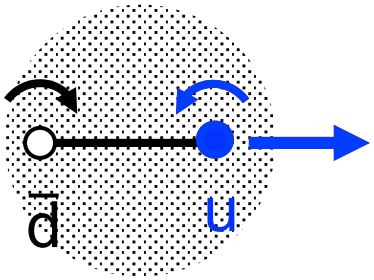

$^3P_0$ state
L=1, S=1, J=L+S=0
$J^{PC} = 0^{++}$ → vacuum QN

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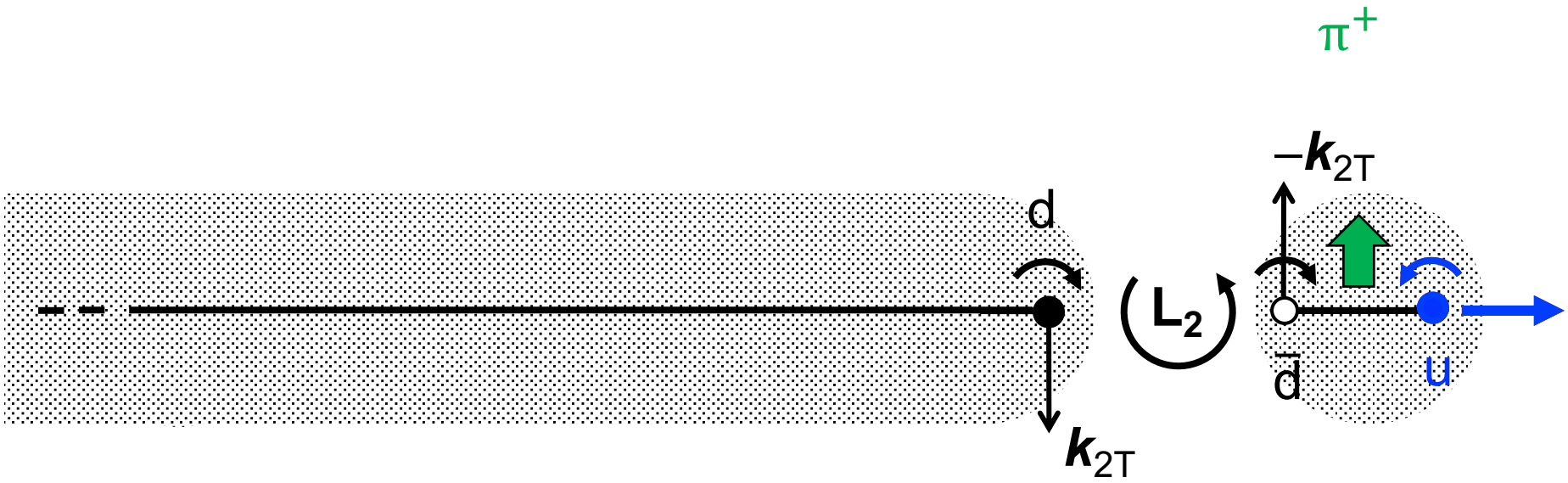

$^3P_0$ state
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h_1

See also B. Andersson, G. Gustafson and G. Ingelman, PLB 85 (1979) 417-420

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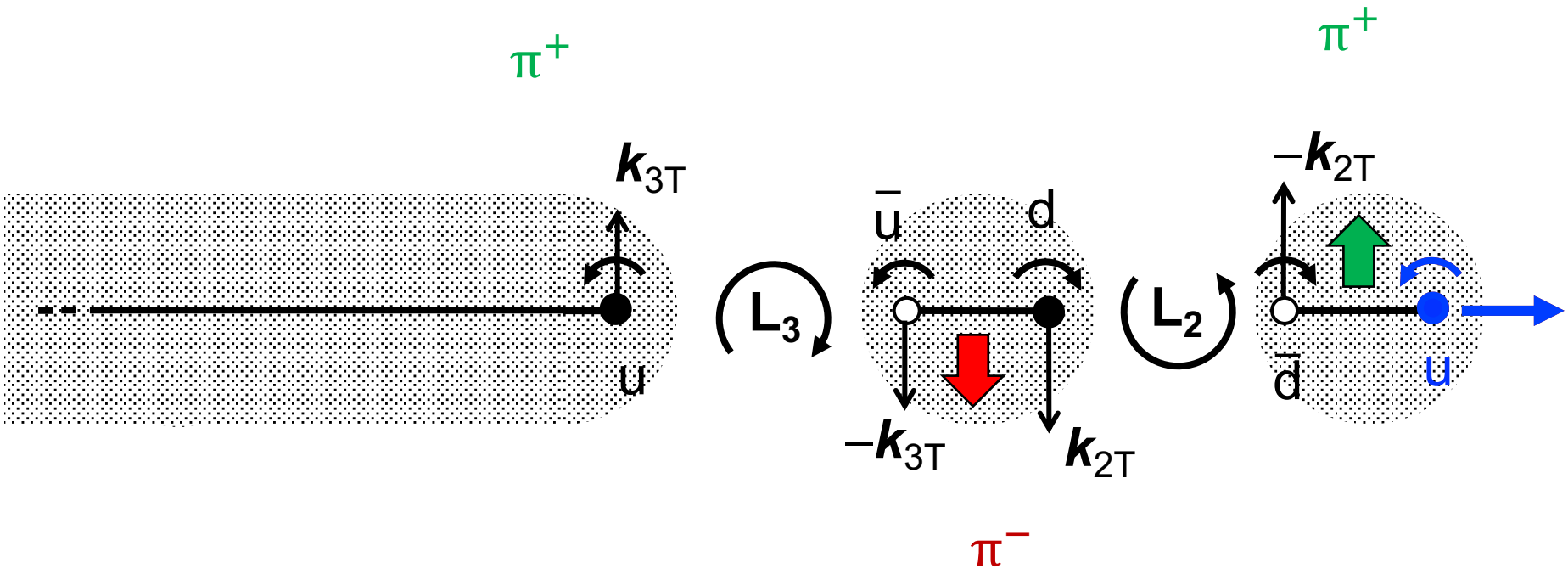

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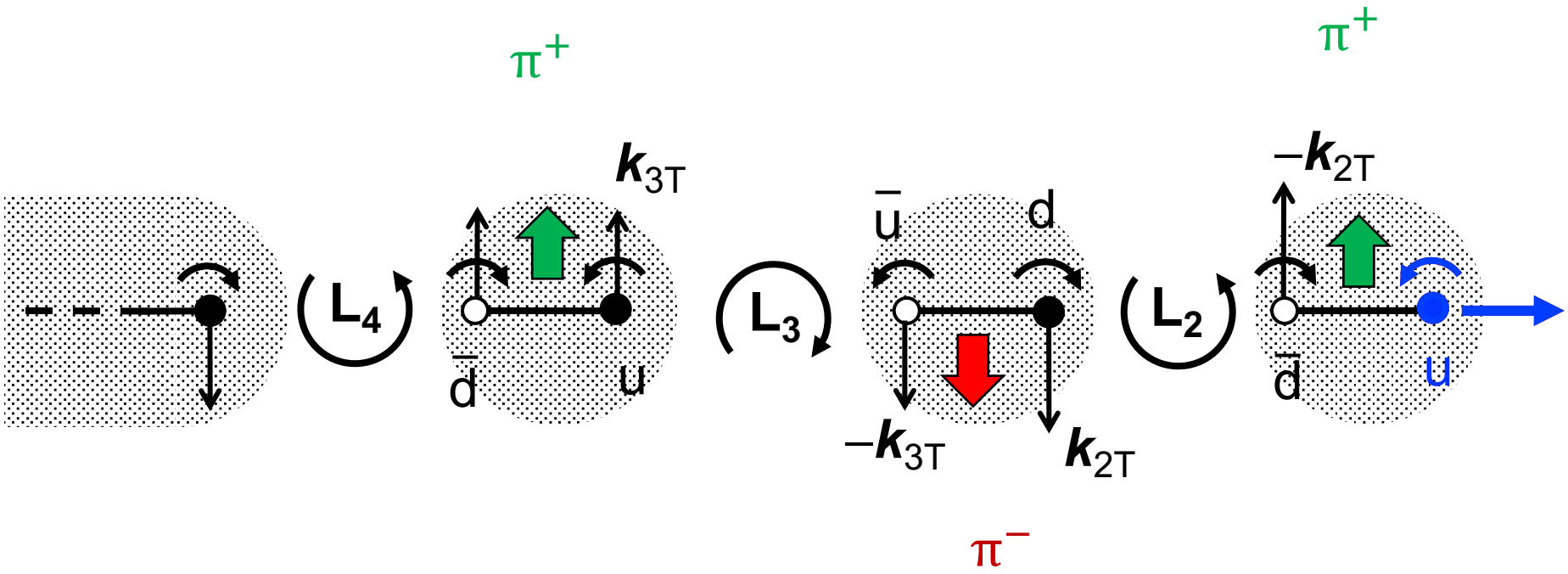

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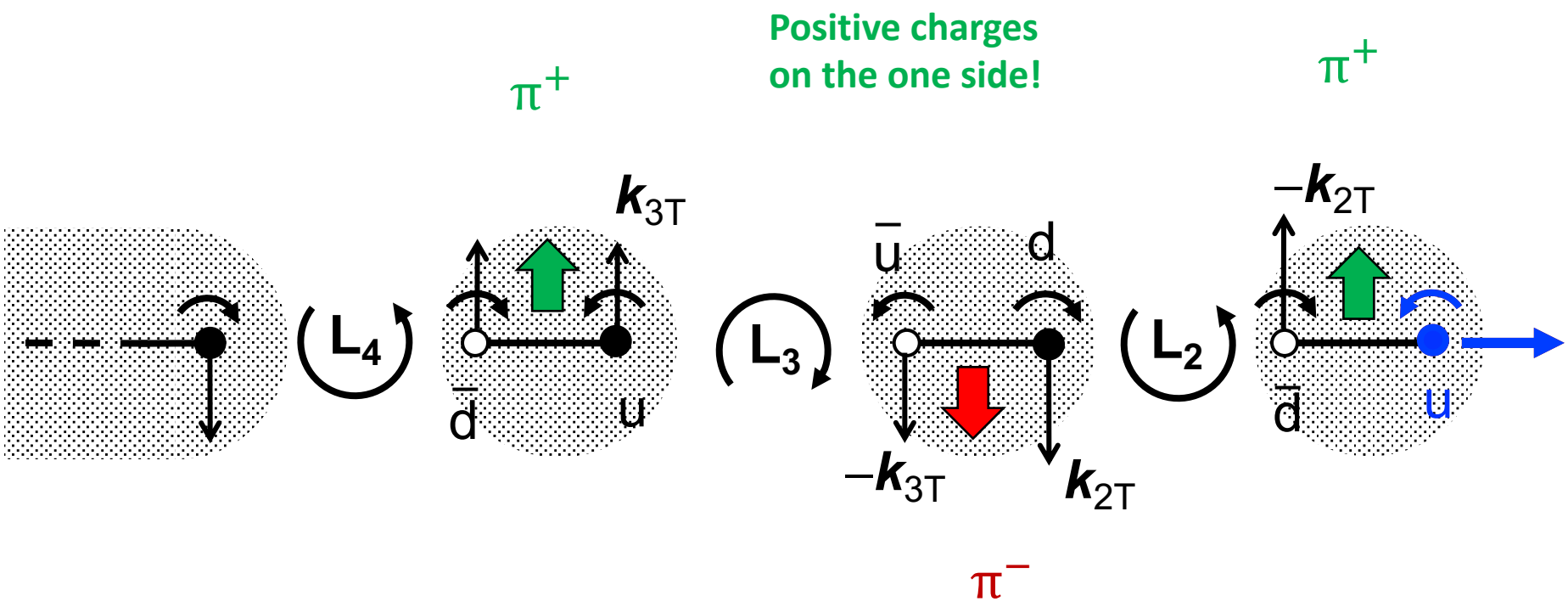

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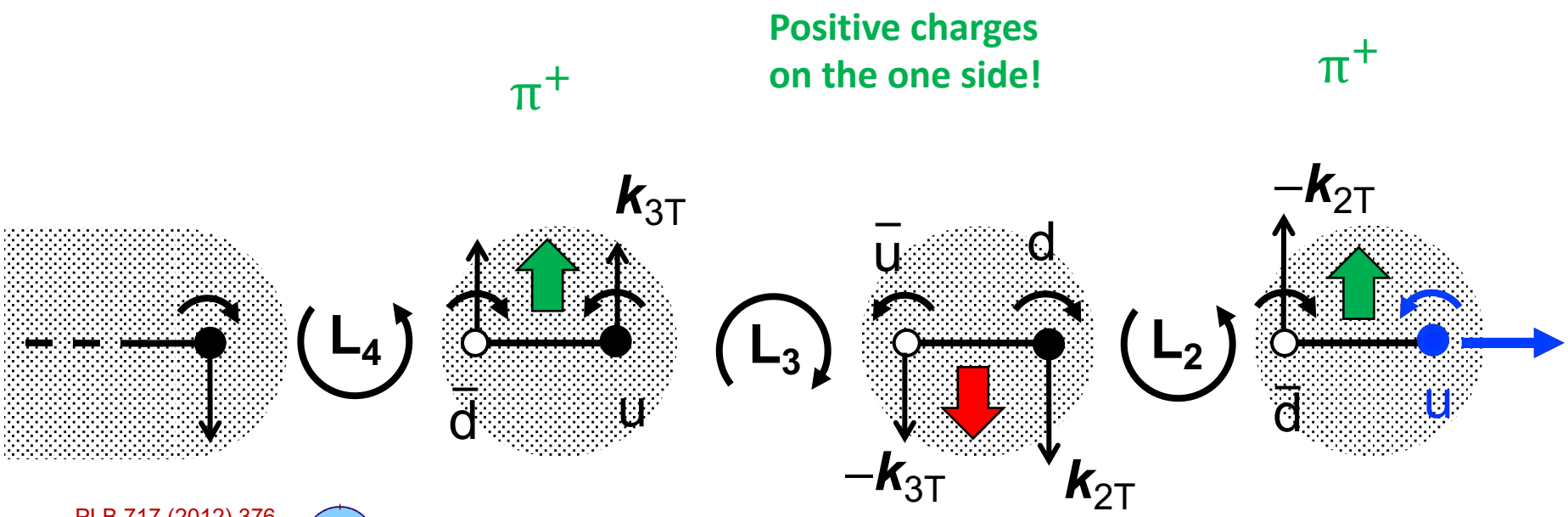
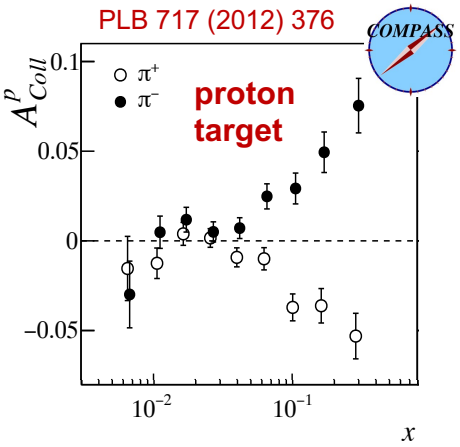
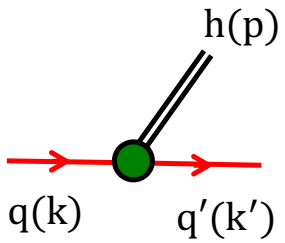




"quantum" string + $^3P_0$

Quark splitting amplitude: Lund string fragmentation ⊗ spin



Pythia

$$T_{q',h,q} \propto C_{q',h,q}\; D_h(M^2) \left(\frac{1-Z_+}{\varepsilon_h^2}\right)^{\frac{a}{2}} \exp\left[-\frac{b_L \varepsilon_h^2}{2Z_+}\right] N_a^{-\frac{1}{2}}(\varepsilon_h^2)\, e^{-\frac{b_T k'^2_T}{2}}$$

AK, Artru, Martin, PRD 104, 114038 (2021)

"quantum" string + $^3P_0$

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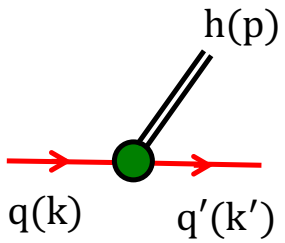

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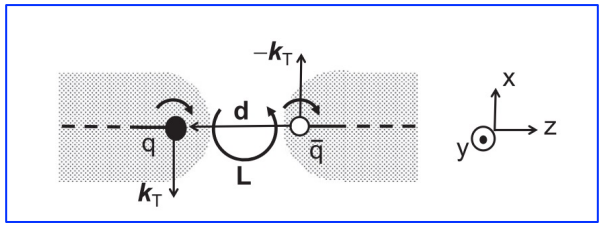
$$\otimes\ [\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T] \qquad \Gamma_{h,s_h}$$

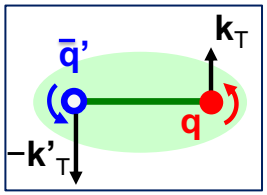
Coupling to PSM, VM

$\mu = [0.42 + i0.76]\ \mathrm{GeV}/c^2$

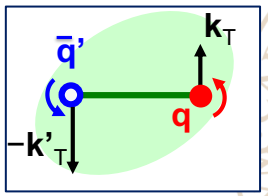
from e^+e^-
«by eye» tuning

$^3P_0$





f_L L pol.



θ_{LT} LT interf.

AK, Artru, Martin, PRD 104, 114038 (2021)

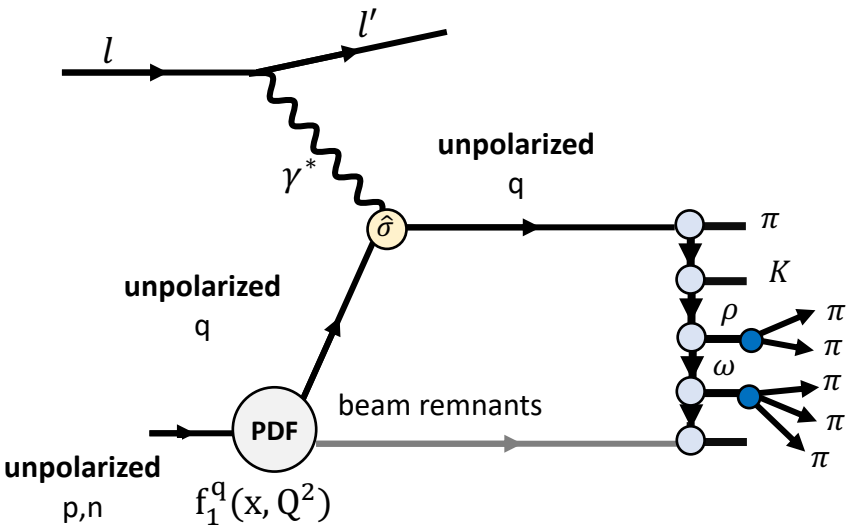
Implementation of spin effects in Pythia for DIS

StringSpinner

AK, L. Lönnblad, CPC **272** (2022) 108234;
CPC **292** (2023) 108886

Spin in Pythia for DIS: StringSpinner

- Pythia sets up PDFs, generates hard scattering and ties string
- Trace endpoint quark polarization starting from target
- Apply rules of the string+$^3P_0$ model in hadronization



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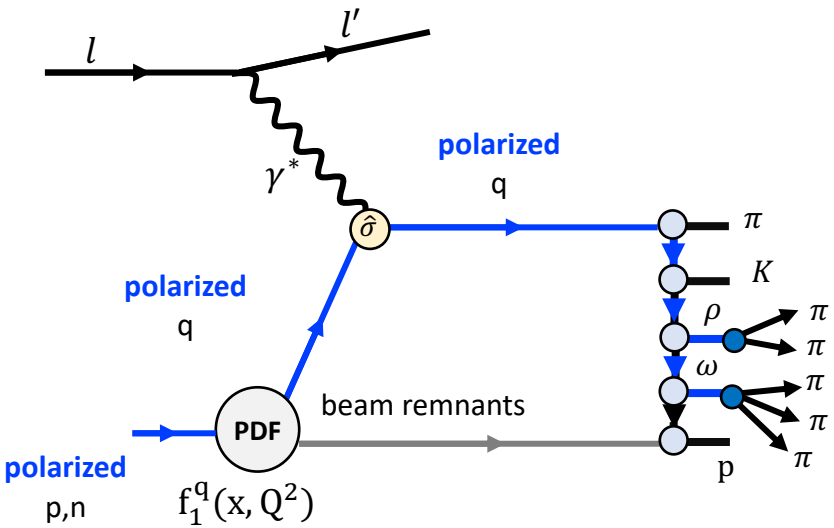

Transversity $h_1^q(x)$
or arbitrary polarization of quarks

AK, L. Lönnblad, CPC **272** (2022) 108234;
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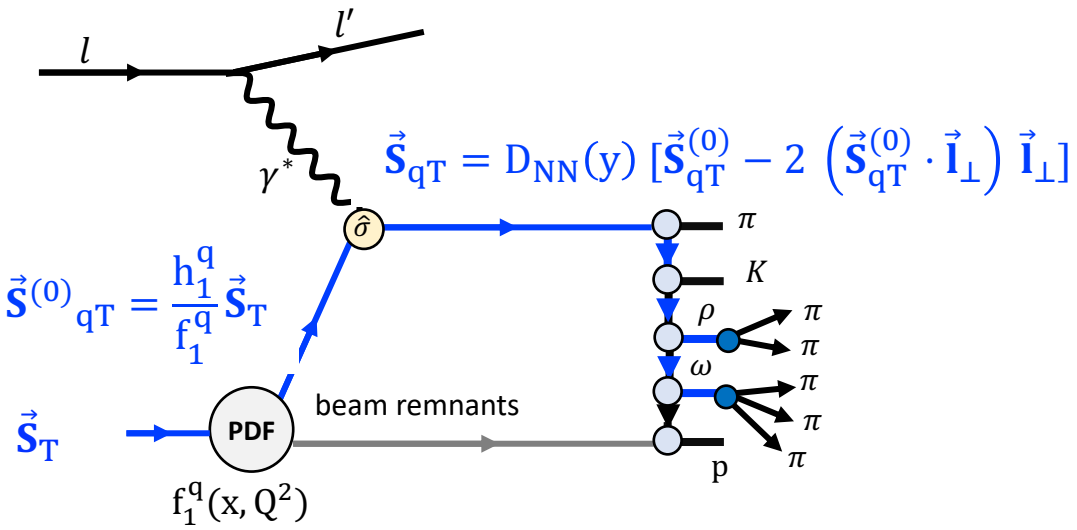

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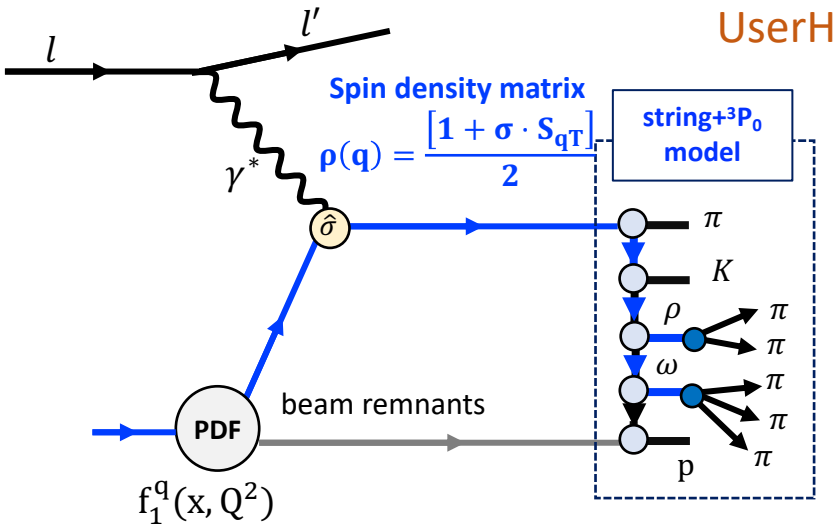

UserHooks:: setStringEnds

Transversity $h_1^q(x)$

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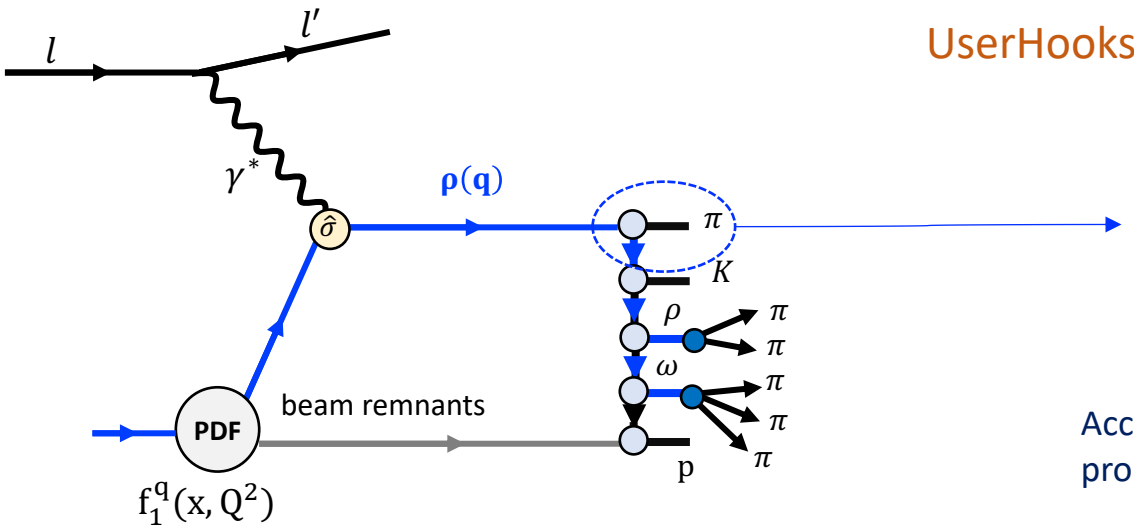
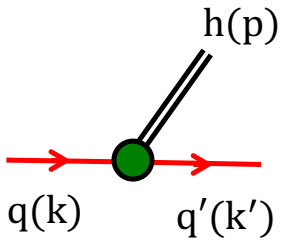


UserHooks:: doVetoFragmentation

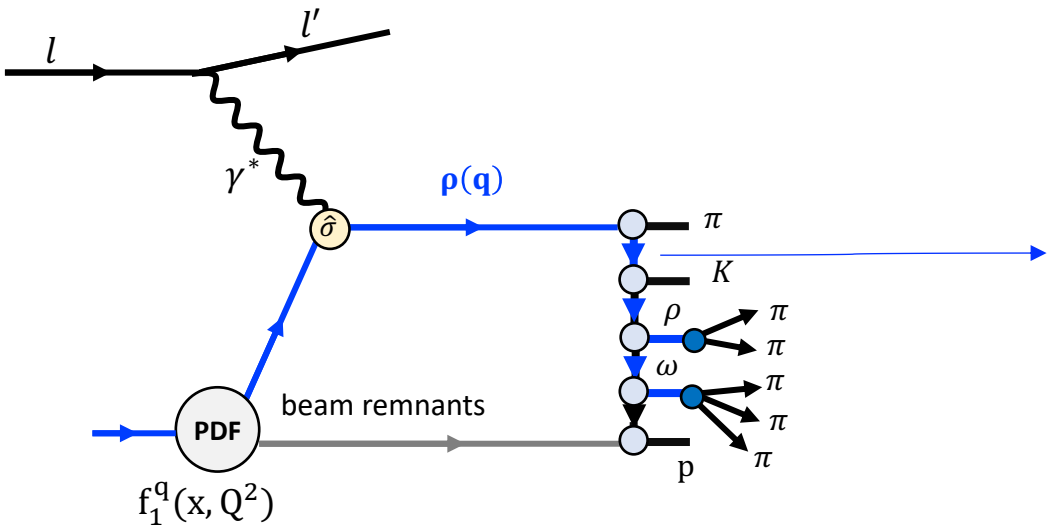


Accept splitting with probability

$$w = \frac{dP_{spin}(q \to h + q')}{dP_{no\ spin}(q \to h + q')} = \frac{Tr_{q'}\ T_{q',h,q}\ \rho(q) T^{\dagger}_{q',h,q}}{\frac{1}{2} Tr_{q'}\ T_{q',h,q}\ T^{\dagger}_{q',h,q}}$$

Spin in Pythia for DIS: StringSpinner

- Pythia sets up PDFs, generates hard scattering and ties string
- Trace endpoint quark polarization starting from target
- Apply string+$^3P_0$

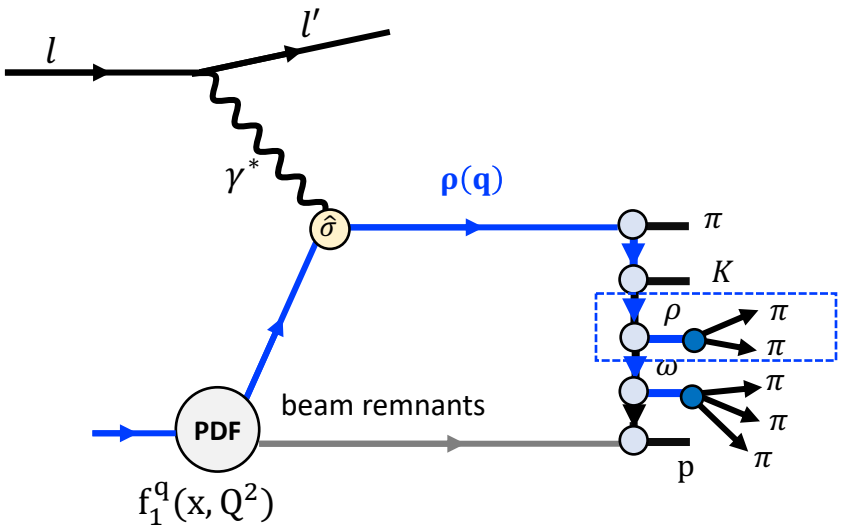


Evaluate spin density matrix of next quark q'

$$\rho(q') \propto T_{q',h,q}\, \rho(q)\, T^{\dagger}_{q',h,q}$$

Spin in Pythia for DIS: StringSpinner

- Pythia sets up PDFs, generates hard scattering and ties string
- Trace endpoint quark polarization starting from target
- Apply string+$^3P_0$



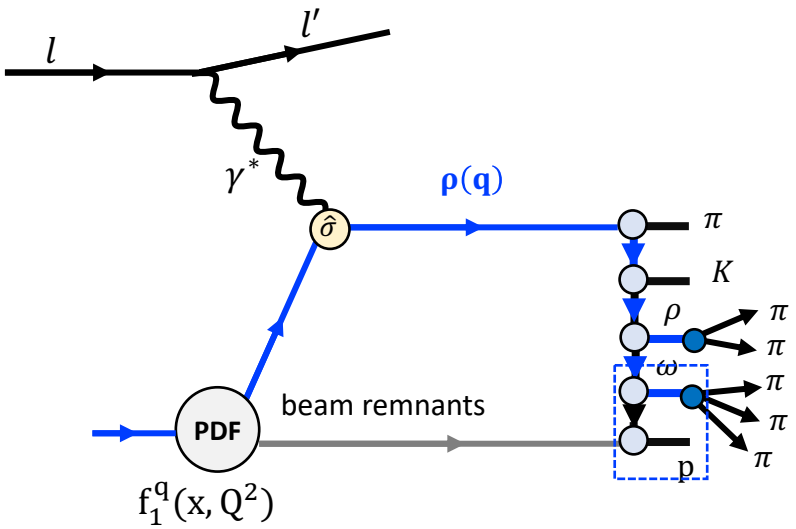
DecayHandler class

branching ratios, decay channel from Pythia
angular distributions externally

Collins-Knowles recipe
i) calculate $\rho(h)$
ii) decay h polarized
iii) return a decay matrix D

Spin in Pythia for DIS: StringSpinner

- Pythia sets up PDFs, generates hard scattering and ties string
- Trace endpoint quark polarization starting from target
- Apply string+$^3P_0$



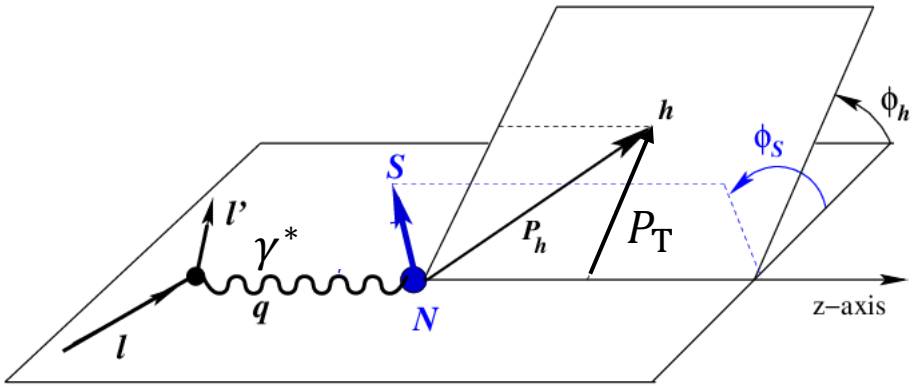
UserHooks:: doVetoFragmentation

for the final two hadrons

Results for SIDIS

COMPASS 160 GeV muons off a proton target

HERMES 27 GeV electrons off a proton target



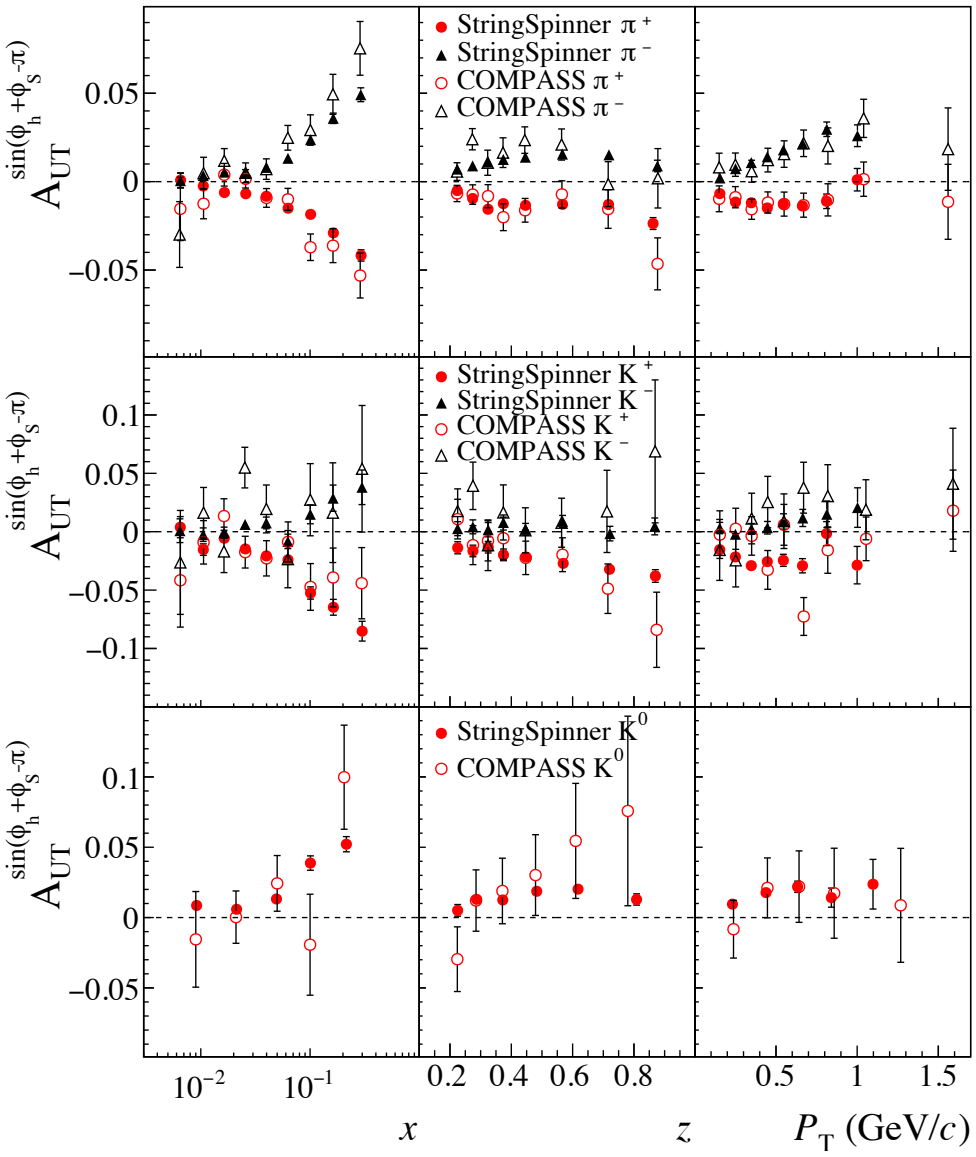
Taken from AK, Lönnblad CPC **292** (2023) 108886

VM polarization parameters

f_L 0.93

θ_{LT} 0.0 Different values for e^+e^-

Collins asymmetries for π and K

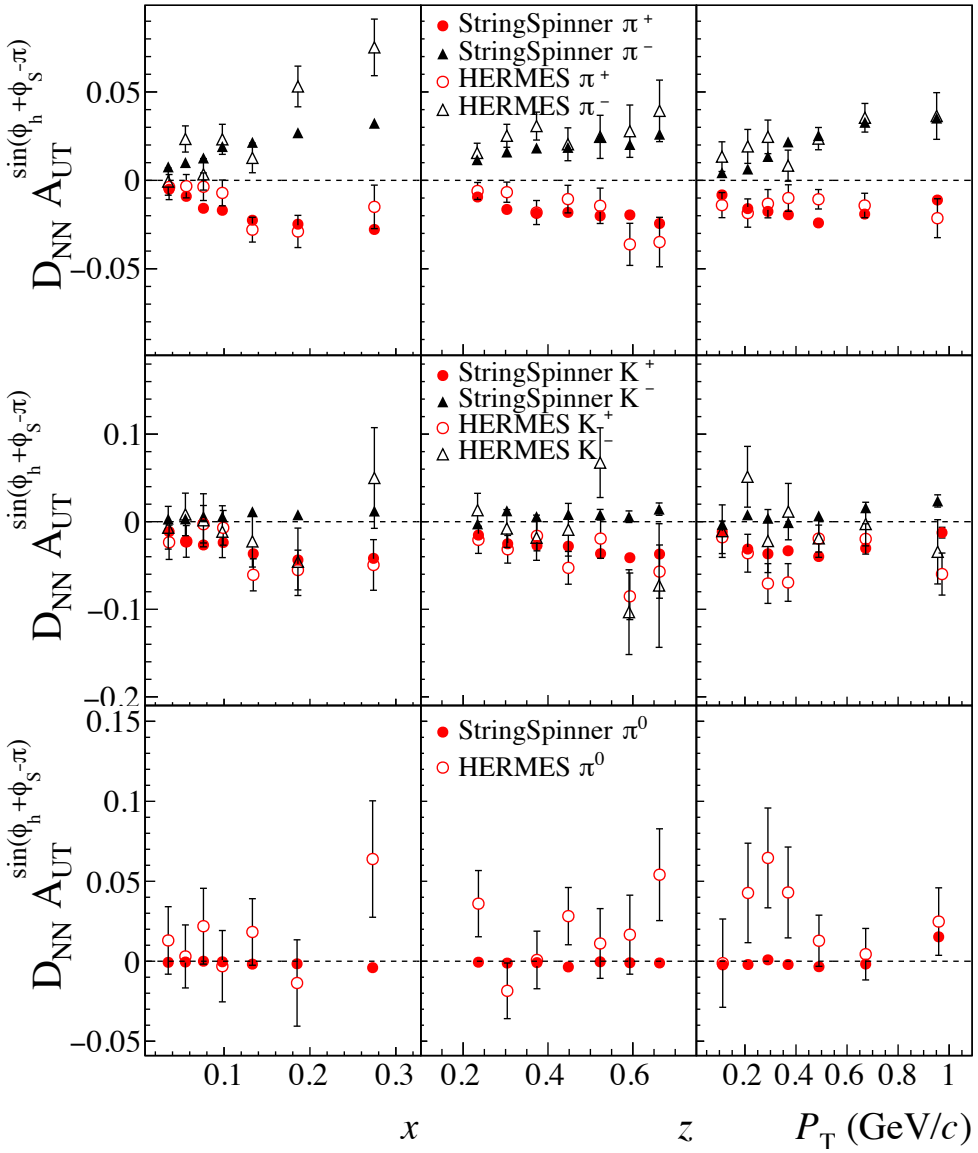


PLB 717 (2012) 376

Comparison with COMPASS

160 GeV muons off transversely polarized protons

Collins asymmetries for π and K



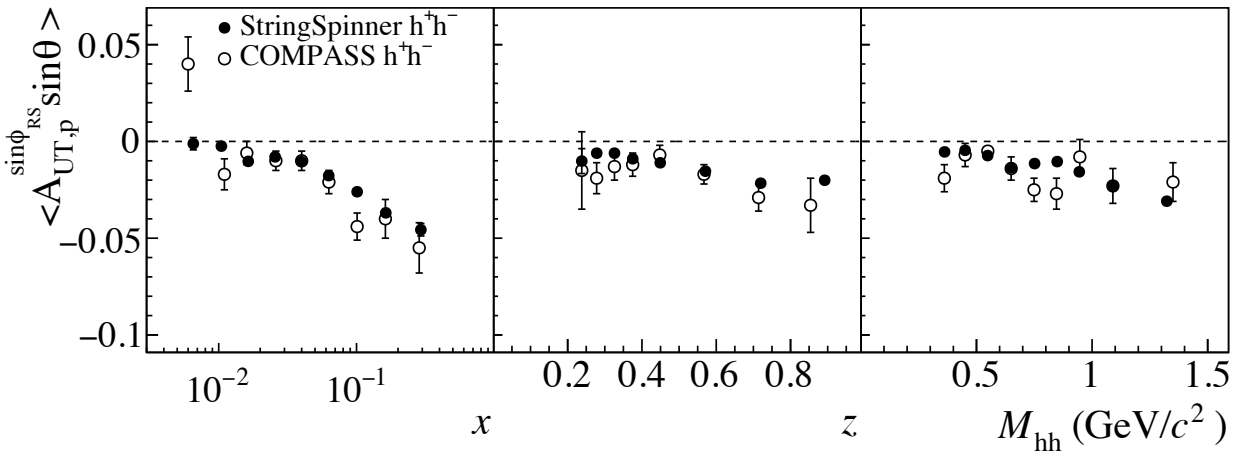
PLB693 (2010) 11

Comparison with HERMES

~27 GeV electrons off protons

Dihadron asymmetries for h^+h^- (interference FF)

PLB 736 (2014) 124

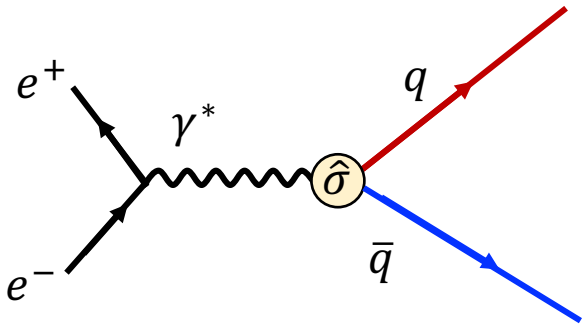


Same mechanism reproduces also 2h asymmetries

Implementation of spin effects in Pythia for e^+e^-

AK, X. Artru, PRD 109 (2024) 5, 05402
AK, L. Lönnblad, A. Martin, PRD 110 (2024) 7, 074029

Recursive recipe for e^+e^-



Steps:

1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition

Recursive recipe for e^+e^-

Steps:

1. Hard scattering

2. Joint spin density matrix
3. Hadron emission from q
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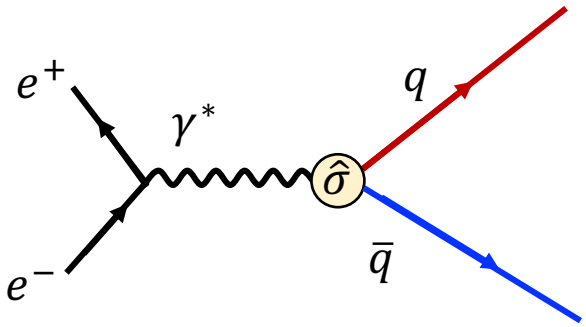
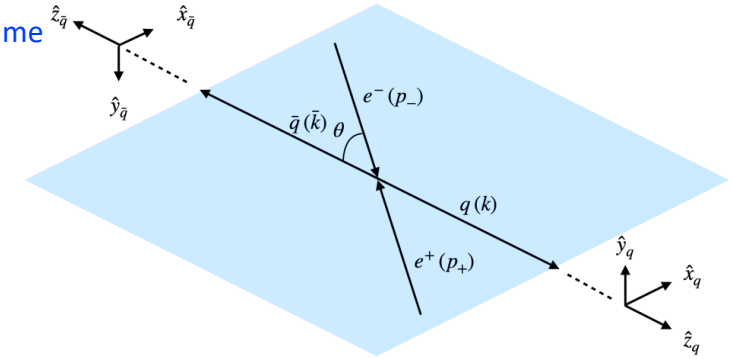
6. Exit condition



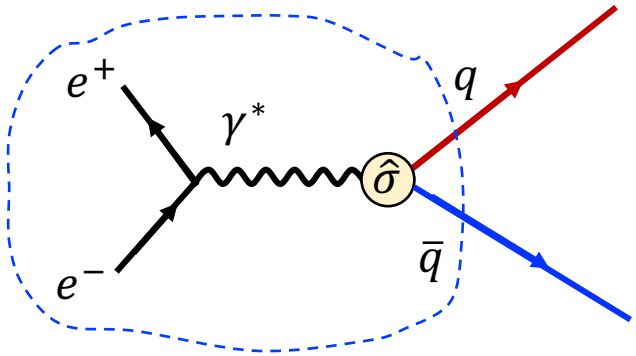
Set up the scattering $e^+e^- \to q\bar{q}$ in the c.m.s

generate the quark flavors and kinematics using differential cross section



Recursive recipe for e^+e^-

Steps:
1. Hard scattering
2. **Joint spin density matrix**
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition



$\rho(q,\bar{q})$

- Set up the joint spin density matrix of the $q\bar{q}$ pair

$$\rho(q,\bar{q}) = C^{q\bar{q}}_{\alpha\beta} \; \sigma^{\alpha}_{q} \otimes \sigma^{\beta}_{\bar{q}}$$

$C^{q\bar{q}}_{\alpha\beta}$: correlation coefficients

$\sigma^{\alpha}_{q} \otimes \sigma^{\beta}_{\bar{q}}$: Pauli matrices along QHF and AHF

$\alpha = 0, x_q, y_q, z_q$

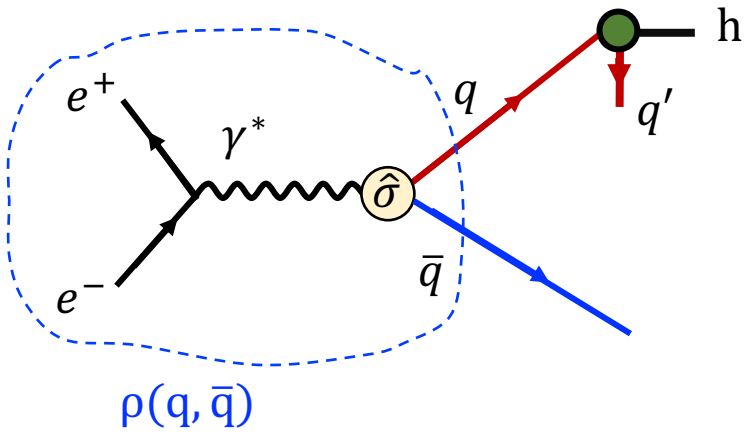
$\beta = 0, x_{\bar{q}}, y_{\bar{q}}, z_{\bar{q}}$

For γ^* exchange

$$\rho(q,\bar{q}) \propto 1_q \otimes 1_{\bar{q}} - \sigma^z_q \otimes \sigma^z_{\bar{q}} + \frac{\sin^2\theta}{1+\cos^2\theta}\left[\sigma^x_q \otimes \sigma^x_{\bar{q}} + \sigma^y_q \otimes \sigma^y_{\bar{q}}\right]$$

Recursive recipe for e^+e^-

Steps:
1. Hard scattering
2. Joint spin density matrix
3. **Hadron emission from q**
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition



❑ Emit the first hadron using the splitting function
(emission probability density)

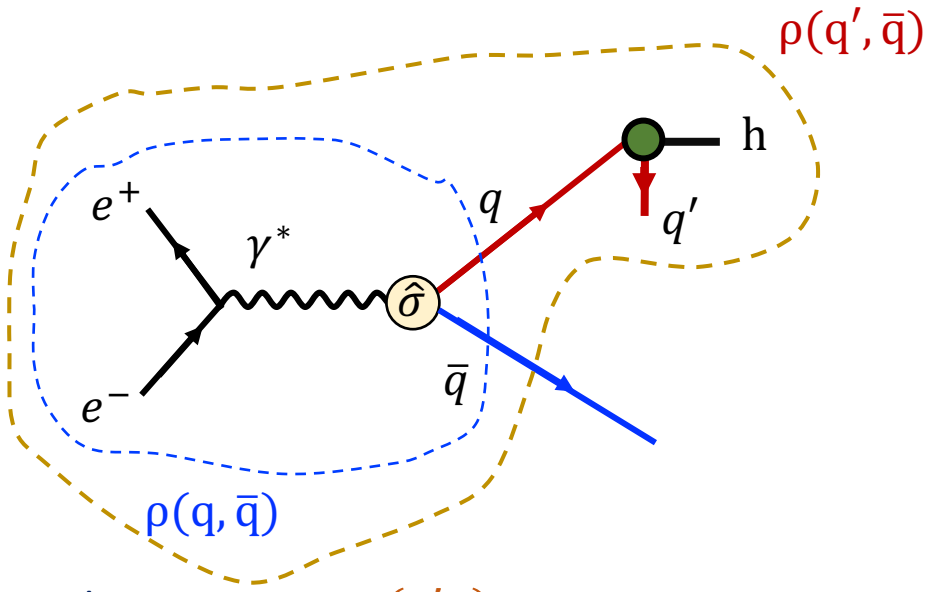
$$\frac{dP(q \to h + q'; q\bar{q})}{dZ_+ Z_+^{-1} d^2p_T} = Tr_{q'\bar{q}} \mathbf{T}_{q',h,q}\, \rho(q,\bar{q})\, \mathbf{T}^{\dagger}_{q',h,q} = F_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T, C^{q\bar{q}})$$

$$\mathbf{T}_{q',h,q} \equiv T_{q',h,q} \otimes 1_{\bar{q}}$$

in the QHF

❑ For VM emission need also to handle the polarized decay
→ backup

Recursive recipe for e^+e^-



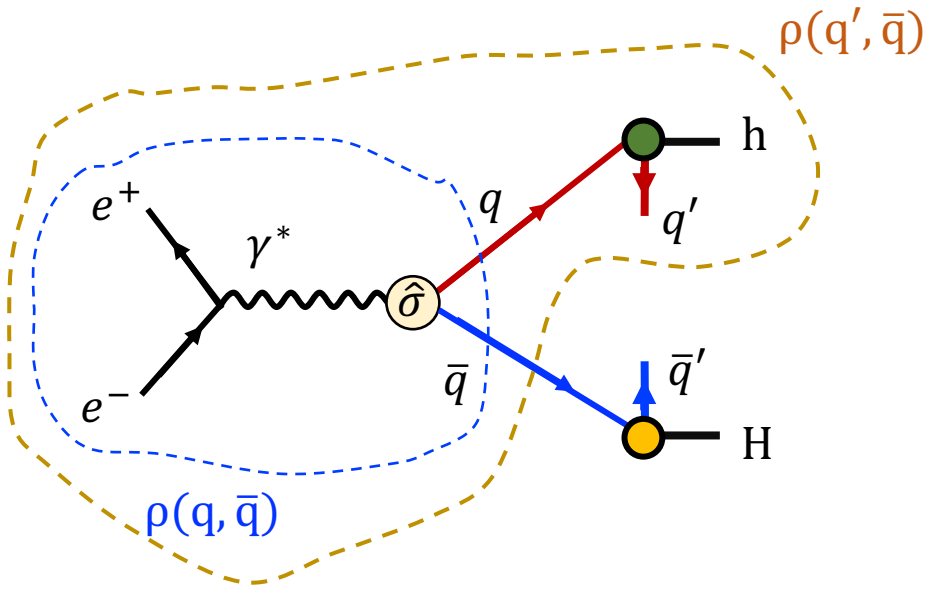
Steps:
1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. **Update density matrix**
5. Hadron emission from $\bar{q}$
6. Exit condition

- Evaluate the spin density matrix $\rho(q'\bar{q})$

$$\rho(q',\bar{q}) = \mathbf{T}_{q',h,q}\ \ \rho(q,\bar{q})\ \ \mathbf{T}^{\dagger}_{q',h,q}$$

includes the information on the emission of h

Recursive recipe for e^+e^-



Steps:
1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
5. **Hadron emission from $\bar{q}$**
6. Exit condition

- Emit a hadron from the $\bar{q}$ side using the splitting function

Depend on the azimuthal angle h

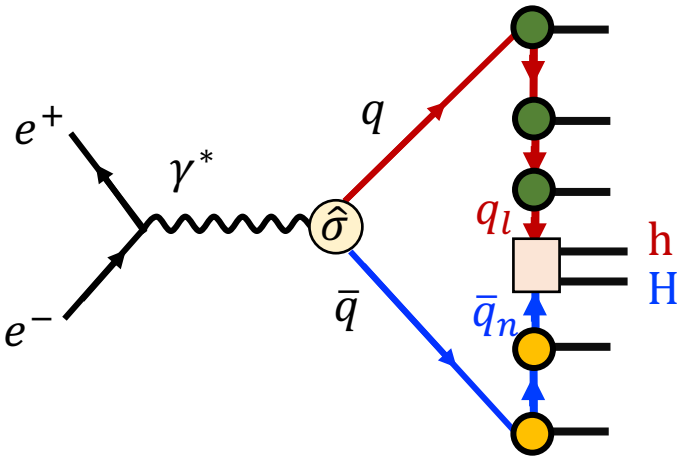
$$\frac{dP(\bar{q} \to H + \bar{q}'; q'\bar{q})}{dZ_Z_^{-1} d^2P_T} = \mathrm{Tr}_{q'\bar{q}'} \mathbf{T}_{\bar{q}',H,\bar{q}}\, \rho(q', \bar{q})\, \mathbf{T}^{\dagger}_{\bar{q}',H,\bar{q}} = F_{\bar{q}',H,\bar{q}}\left(Z_, P_T; \bar{k}_T, C^{q'\bar{q}}\right)$$

Expressed in the AHF

conditional probability of emitting H, having emitted h
→ correlations between the transverse momenta

[Collins NPB, 304:794–804, 1988, Knowles NPB, 310:571–588, 1988]

Recursive recipe for e^+e^-



Steps:
1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. **Exit condition**

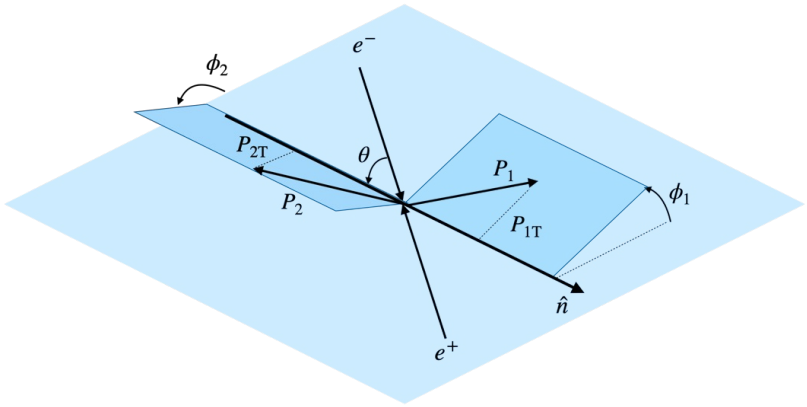
[AK, X. Artru, PRD 109 (2024) 5, 054029]

- Iterate until the exit condition is called and the last quark pair is hadronized
  more details in PRD 109 (2024) 5, 054029

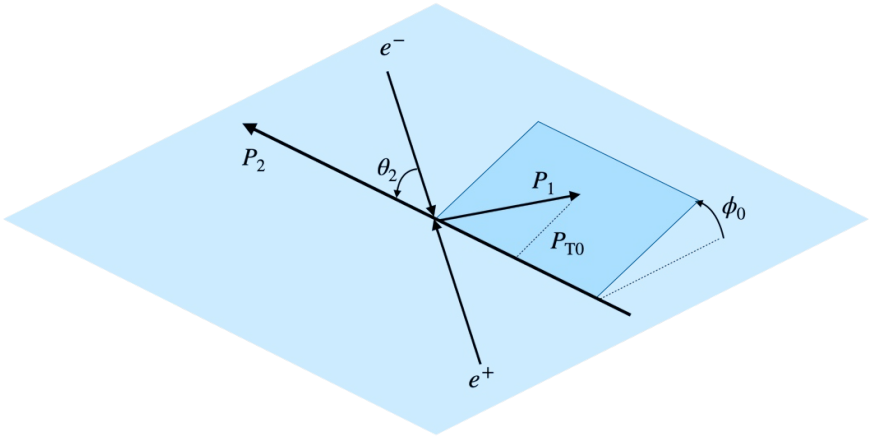
Results for e^+e^- annihilation

BELLE, BABAR $\quad \sqrt{s} \simeq 10.6$ GeV

«thrust axis» method
Collins asym. A_{12}



«hadronic plane» method
Collins asym. A_0



Taken from AK, L. Lönnblad, A. Martin, PRD 110 (2024) 7, 074029

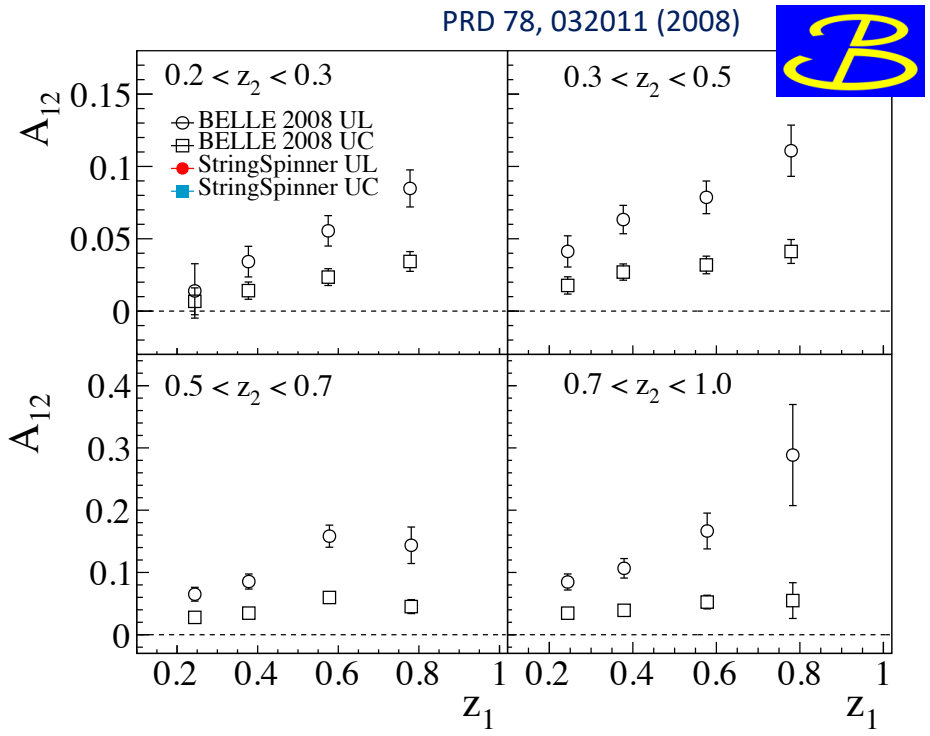
VM polarization parameters

f_L	0.12
θ_{LT}	-0.65

OK also for SIDIS

A_{12} asymmetry for charged π π pairs

PRD 78, 032011 (2008)



$$z_i = \frac{2E_{h_i}}{\sqrt{s}}$$

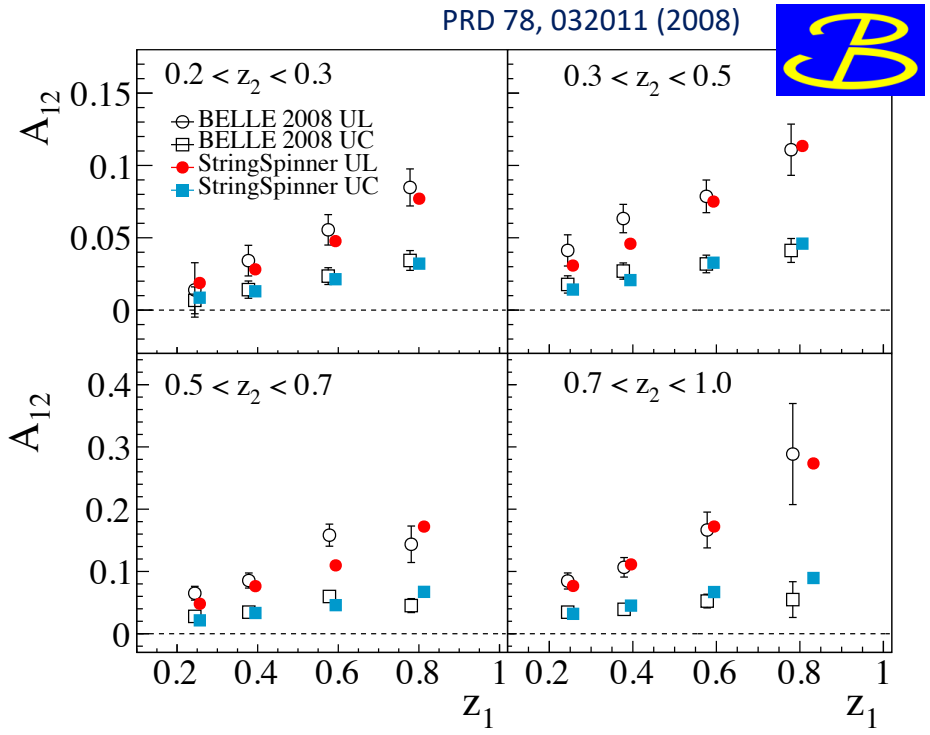
Belle asymmetries corrected for thrust smearing

Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5\ \text{GeV}$$

A_{12} asymmetry for charged π π pairs

PRD 78, 032011 (2008)



$$z_i = \frac{2E_{h_i}}{\sqrt{s}}$$

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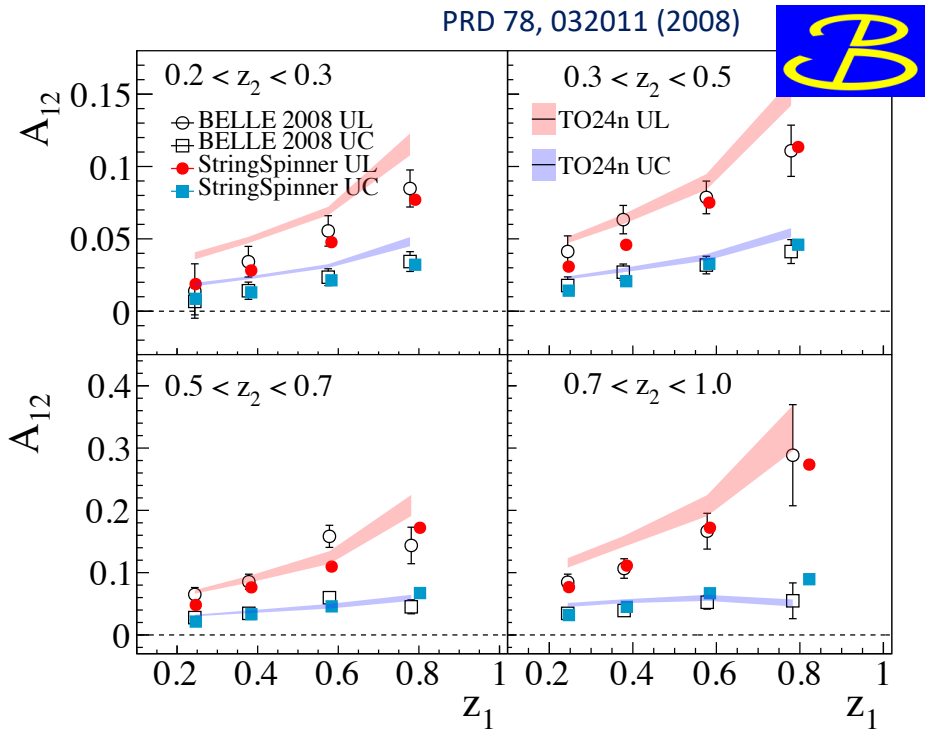
StringSpinner reproduces trend and size

A_{12} asymmetry for charged π π pairs

PRD 78, 032011 (2008)



Bands from M. Boglione, C. Flore (2024, private comm.)
fit of SIDIS, e+e- and pp data

$$z_i = \frac{2E_{h_i}}{\sqrt{s}}$$

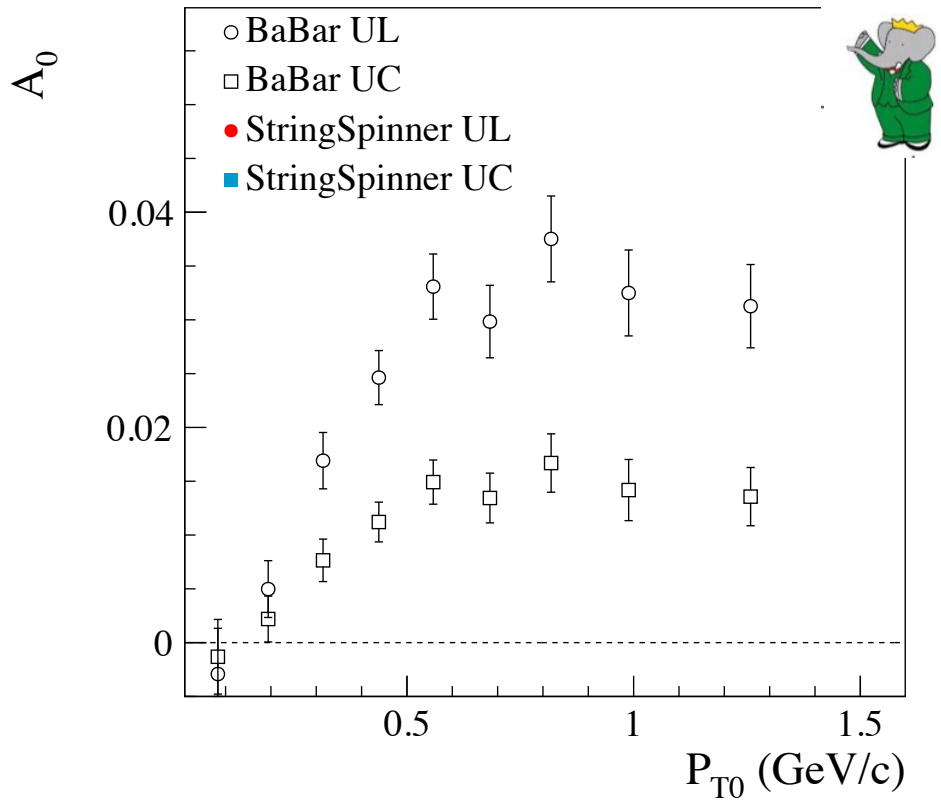
Belle asymmetries corrected for thrust smearing
Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5\ \text{GeV}$$

StringSpinner reproduces trend and size

A_0 asymmetry for π π pairs: P_{T0} dependence

PRD 90, 052003 (2014)

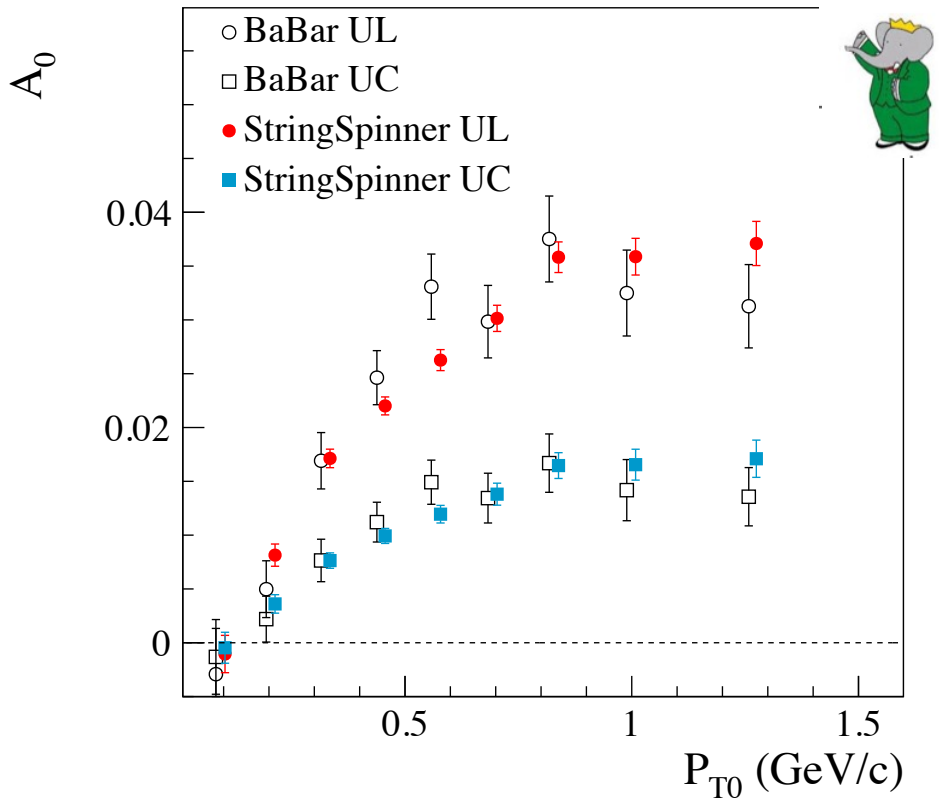


Cuts:

$T > 0.8, z > 0.15, Q_T < 3.5\text{GeV}$

A_0 asymmetry for π π pairs: P_{T0} dependence

PRD 90, 052003 (2014)



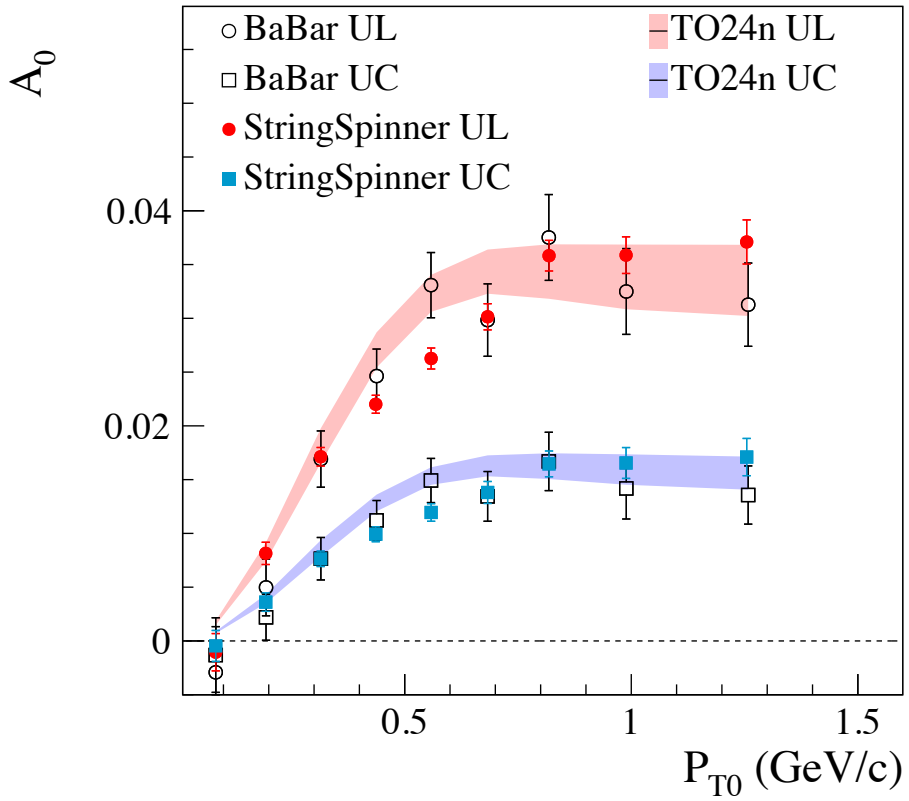
Cuts:

$T > 0.8, z > 0.15, Q_T < 3.5\text{GeV}$

Transverse-momentum dependence reproduced by string+$^3P_0$!

A_0 asymmetry for π π pairs: P_{T0} dependence

PRD 90, 052003 (2014)



Bands from M. Boglione, C. Flore (2024, private comm.)
fit of SIDIS, e+e- and pp data

Cuts:
$T > 0.8, z > 0.15, Q_T < 3.5\text{GeV}$

Transverse-momentum dependence reproduced by string+$^3P_0$!

Conclusions

- We are trying to systematically implement spin effects in string fragmentation string+$^3P_0$ model
- Encouraging results for DIS and e^+e^- (StringSpinner)
- More developments foreseen
  - baryon production (ongoing)
  - connection of string+$^3P_0$ to FSR (planned, to start soon)
- Spin in hadronization still restricted to a small community but will be very relevant for EIC

looking forward to see how far we can go
with Pythia and string+$^3P_0$ model!!

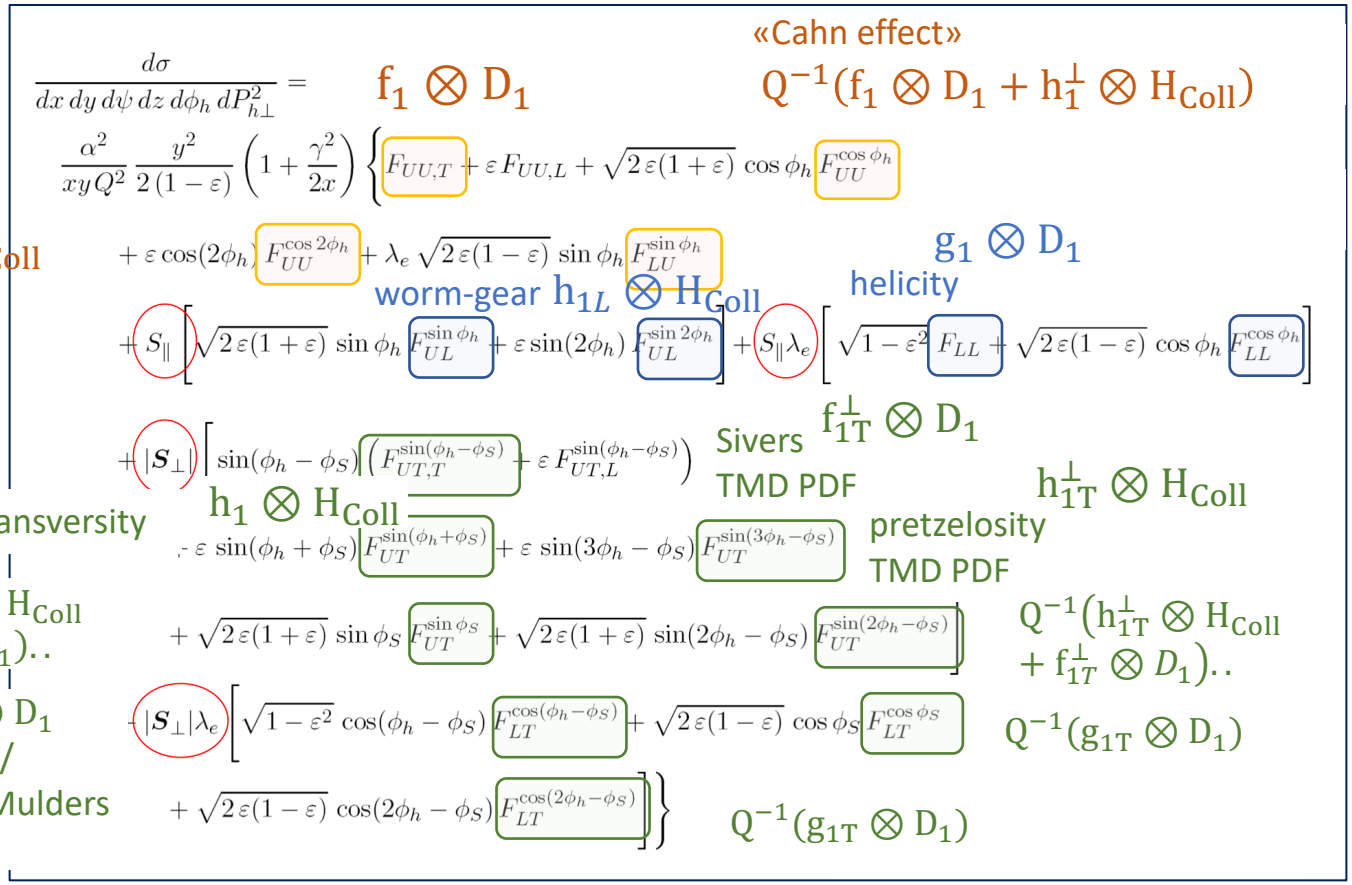
Semi-inclusive deep inelastic scattering

~ spin independent part

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T} + \; \mathrm{f_1} \otimes \mathrm{D_1}$$

Semi-inclusive deep inelastic scattering

what we are missing by neglecting spin!!



Much more than f_1^q is needed to describe the nucleon structure...

TMDs in Pythia?

Collins effect and quark polarimetry in polarized SIDIS

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \qquad Q^{-1}(h_1^\perp \otimes H_{Coll})$$

$$\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h}$$

$h_1^\perp \otimes H_{Coll}$

$$+\varepsilon\cos(2\phi_h)F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h F_{LU}^{\sin\phi_h}$$

$h_{1L} \otimes H_{Coll}$

$$+S_\parallel\Big[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)F_{UL}^{\sin 2\phi_h}\Big]+S_\parallel\lambda_e\Big[\sqrt{1-\varepsilon^2}F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h F_{LL}^{\cos\phi_h}\Big]$$

$$+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\Big(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big)$$

$h_1 \otimes H_{Coll}$

$h_{1T}^\perp \otimes H_{Coll}$

pretzelosity TMD PDF

$$+\varepsilon\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$Q^{-1}(h_1 \otimes H_{Coll}+..)$

$Q^{-1}(h_{1T}^\perp \otimes H_{Coll}+..)..$

$$+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S F_{UT}^{\sin\phi_S}+\sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$

$$+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}$$

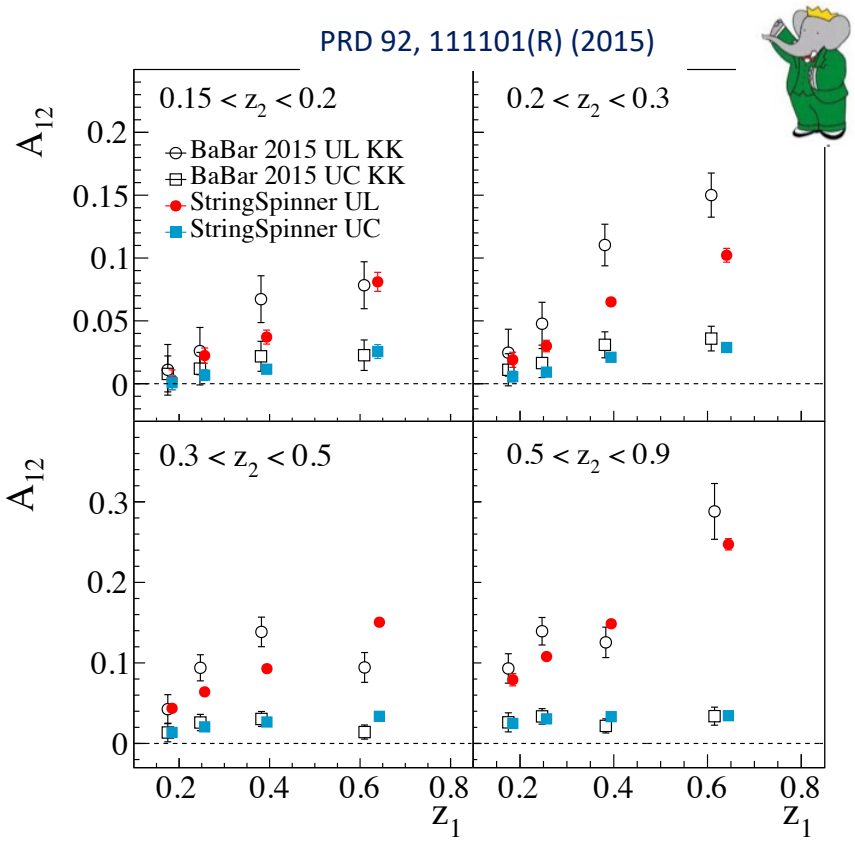
$$+\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}$$

Where spin in hadronization is required…

Backup

A_{12} asymmetry for charged KK pairs

PRD 92, 111101(R) (2015)



Corrected for thrust smearing

Cuts

$$T > 0.8, z > 0.15, Q_T < 3.5\ \text{GeV}, \alpha_O < \pi/4$$

A_{12}^{UC} much smaller than A_{12}^{UL} at large z

reproduced by string+$^3P_0$