Thermalization in ϕ^4 theory via quantum simulation

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In collaboration with Iván Cuntín and Bin Wu

May 26, 2025, CERN, “Real-time and out-of-equilibrium dynamics”

Funded by the European Union

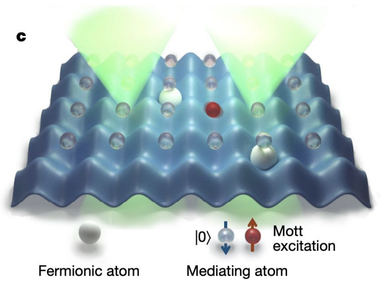
Why bother with quantum simulation

- Real-time evolution may face sign problems under various scenarios

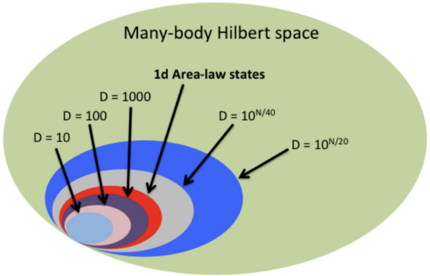
- Hilbert space dimensionality grows exponentially with particle numbers, surpass current computing resources

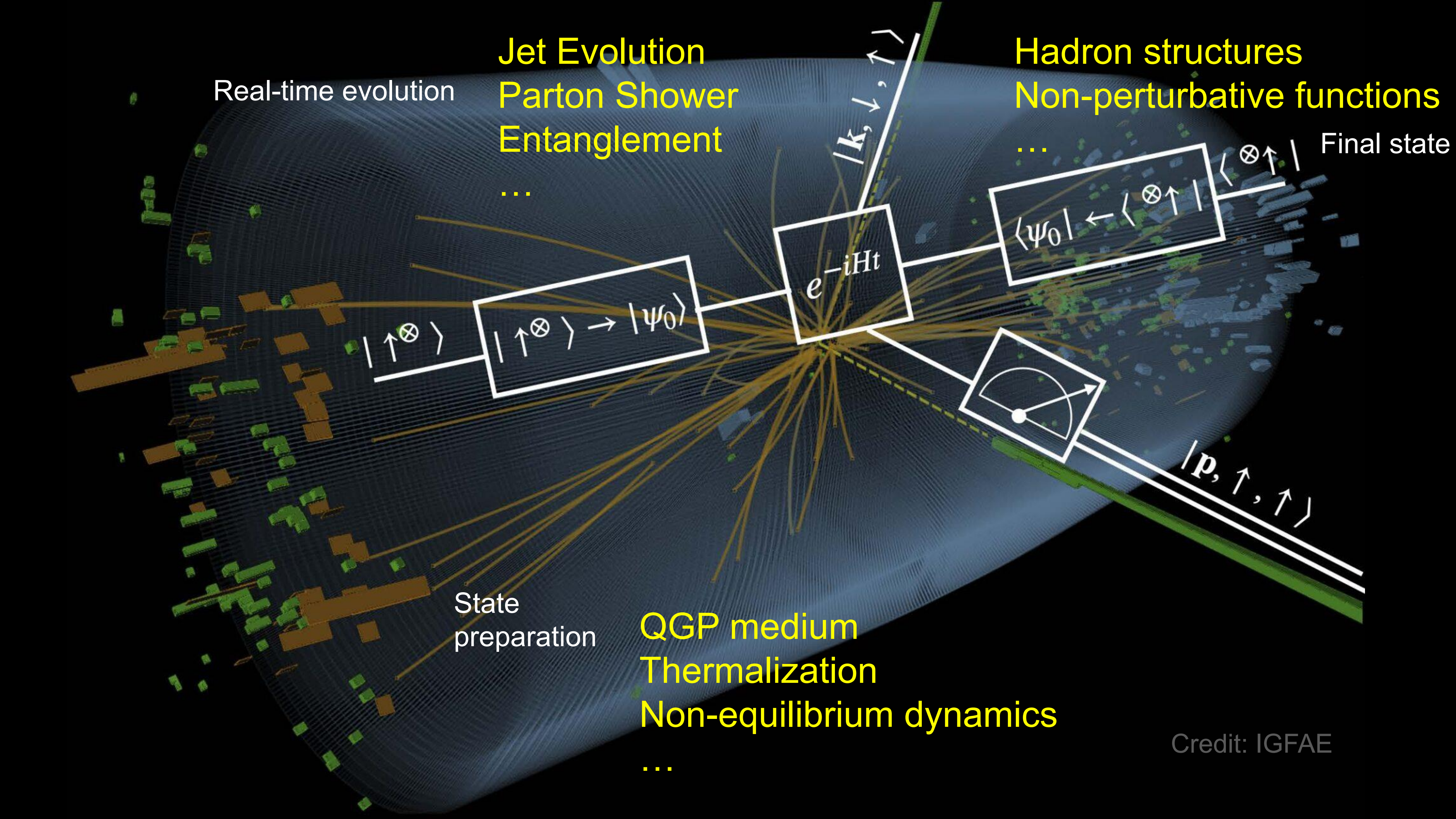
- Hamiltonian truncation limits physical understanding compared with direct simulation

Quantum computing (in a broader sense) may potentially help to mitigate some



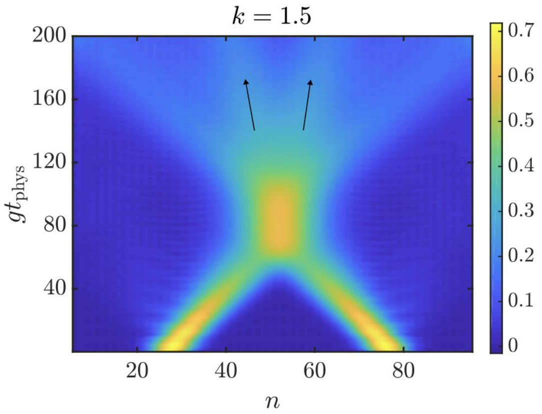
$S \sim \log D$




Real-time evolution
Jet Evolution
Parton Shower
Entanglement
...
Hadron structures
Non-perturbative functions
...
Final state
$|\uparrow^{\otimes}\rangle$
$|\uparrow^{\otimes}\rangle \rightarrow |\psi_0\rangle$
e^{-iHt}
$\langle\psi_0| \leftarrow \langle\otimes\uparrow|$
$\langle\otimes\uparrow|$
$|\mathbf{k}, \downarrow, \uparrow\rangle$
$|\mathbf{p}, \uparrow, \uparrow\rangle$
State
preparation
QGP medium
Thermalization
Non-equilibrium dynamics
...
Credit: IGFAE

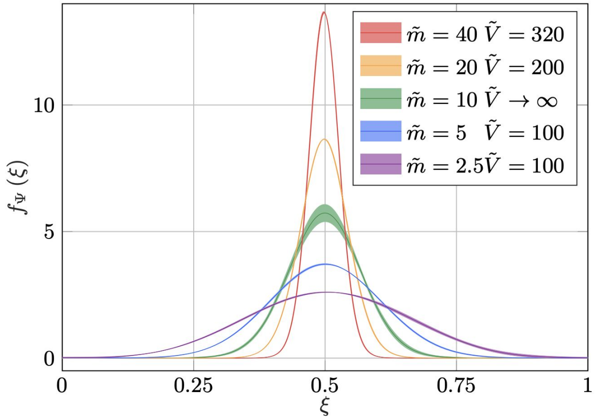
Many interesting QIS to HEP

Meson inelastic scattering



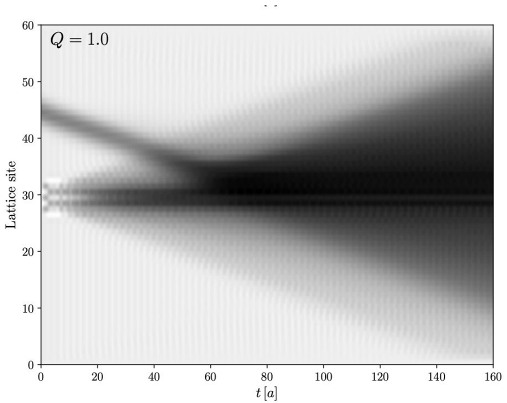
Papaefstathiou, Knolle, and Banuls, 2402.18429

Parton distribution function



Banuls et al, 2504.07508
(See Manuel's talk on Monday)

Energy loss with nuclear matter



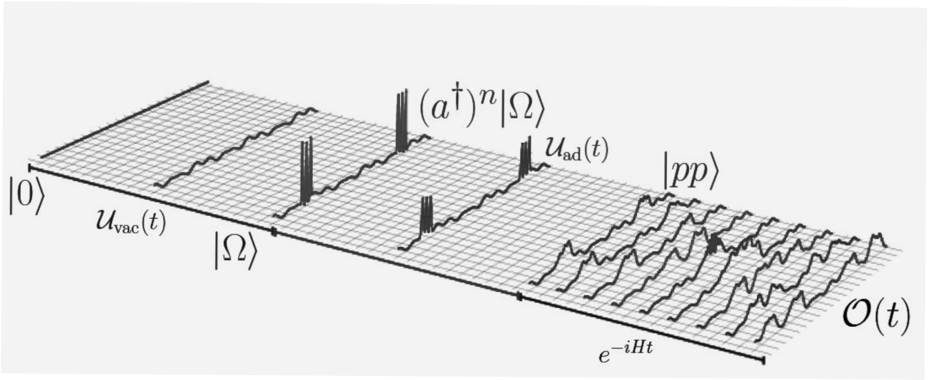
Barata and Rico, 2502.17558

Prototypical task

Quantum simulation of quantum field theory = perform “**ideal experiments**” on quantum computer

Jordan, Lee, & Preskill,
1111.3633, 1401.7115, 1703.00454

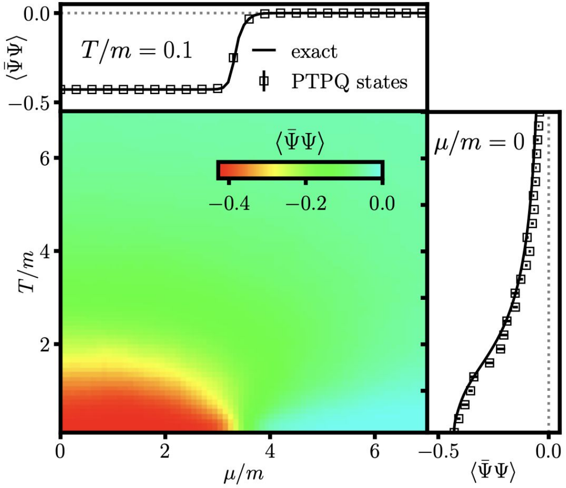
- Prepare initial state in quantum computer
- Evolve state forward in time using Hamiltonian, for some specified time interval
- Measure observables by simulating measurement performed in idealized lab



Today, I will talk about thermal medium prep and its dynamics

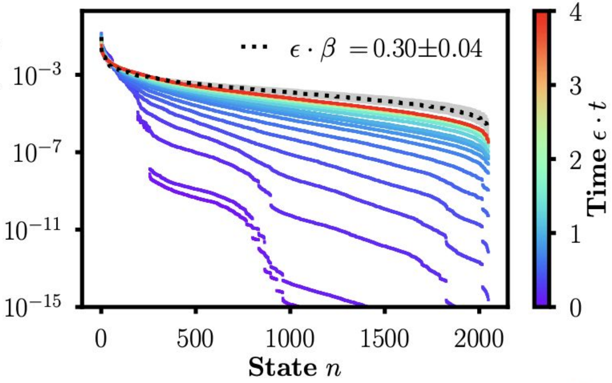
Recent thermal applications

Thermal state prep
for chiral phase diagram



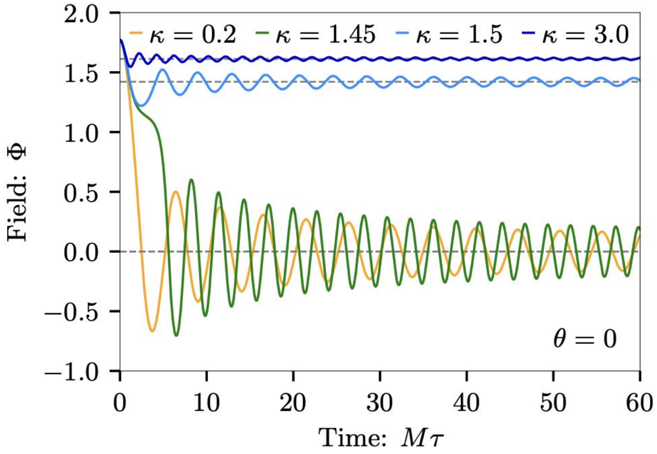
Davoudi, Mueller, Powers, 2107.11416
See Zohreh’s talk on Monday
(also recent work on nonequilibrium
2404.02965)

Entanglement structure
towards thermalization



Mueller, Zache, Ott, 2107.11416
See Torsten’s talk on Monday

Damped oscillation in Bosonization



Batini, et al, 2406.04789
See Laura’s talk on Monday

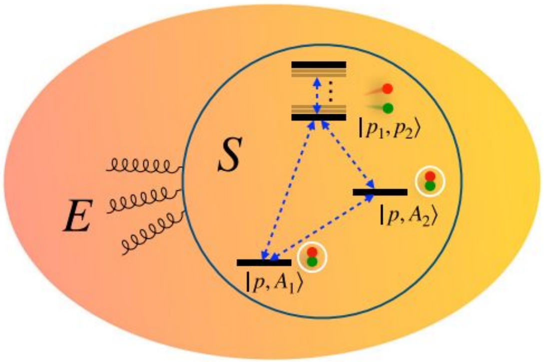
Open quantum system approach

Simulating hard probes (quarkonium, jets, etc) in QGP as open system via Lindblad equation

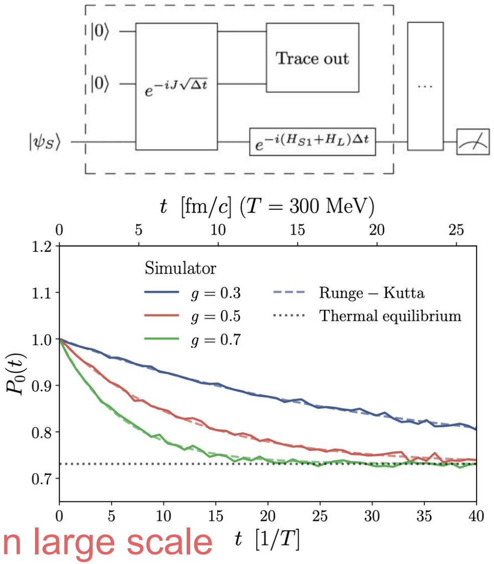
$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_S(t) = -i\left[H_{S1}(t) + H_L, \rho_S(t)\right] + \sum_{j=1}^{m}\left(L_j\rho_S(t)L_j^\dagger - \frac{1}{2}\left\{L_j^\dagger L_j, \rho_S(t)\right\}\right)$$

Blaizot & Escobedo, 1711.10812, 1803.07996

Cleve & Wang, 1612.09512



Jong et al, 2010.03571, 2106.08394



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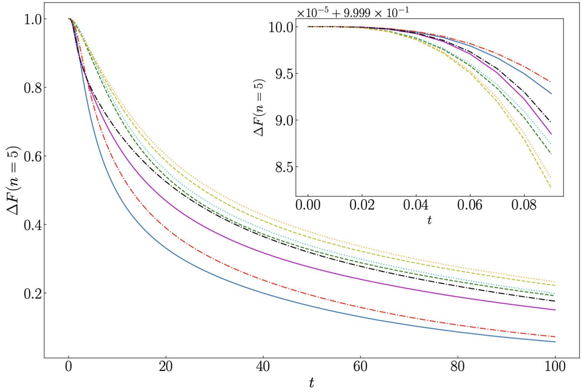


Angelides et al, 2501.13675
See TN approach in Jansen's talk on Monday

Simplifying assumptions, expensive in large scale

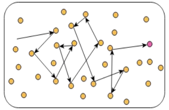
Quantum search algorithm to speed up expectation

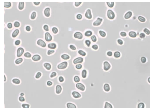
One example: heavy quark thermalization via Langevin equation

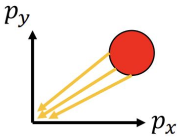
$$dx_i = \frac{p_i}{E(\vec{p})}dt, \quad i = x, y, z,$$

$$dp_i = -A(\vec{x}, \vec{p}, t)p_i dt + \sigma_{ij}(\vec{x}, \vec{p}, t)dW_j,$$

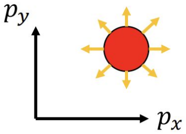
Drag coefficient　　　Stochastic Wiener process





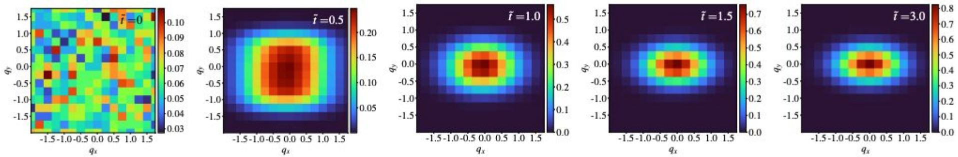


Drag: Energy loss

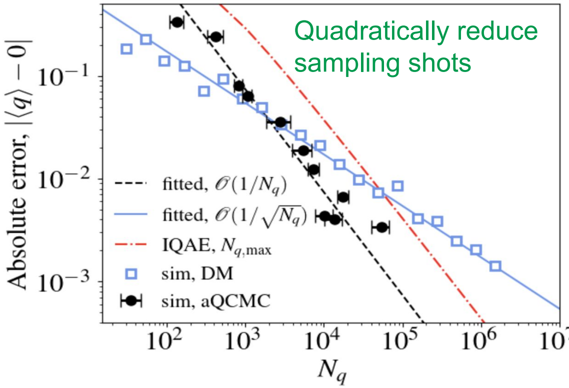


Diffusion: Momentum broadening

Use Grover-like operator to accelerate the extraction of **expectation** at final step by a square root over classical MC



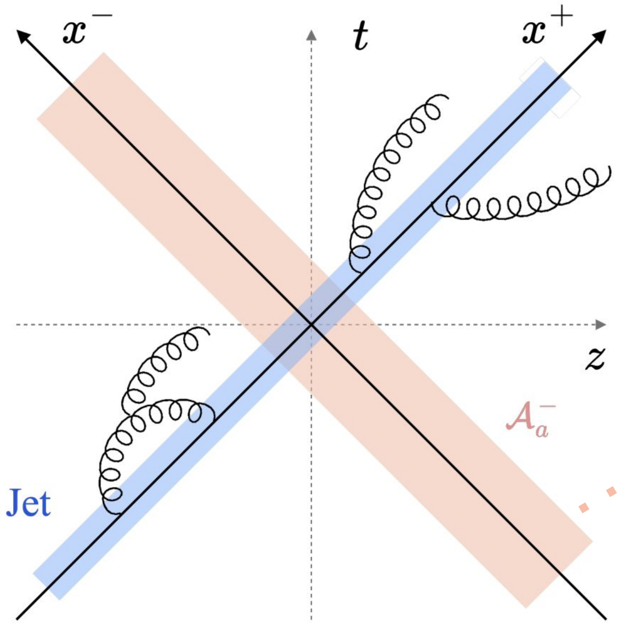
Du and WQ, 2312.16294
Similar ideas in quantum walk
2109.13975, 2207.10694, …



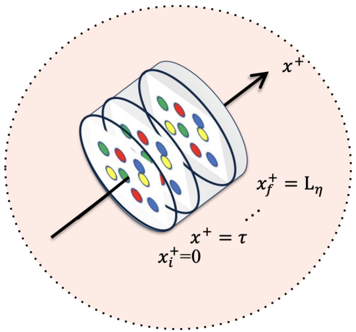
Limited application, based on arithmetic operations

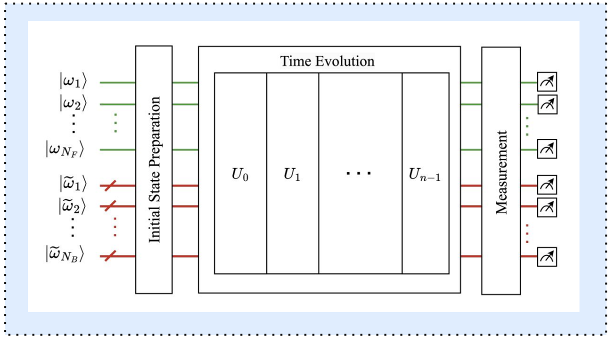
Light-front QCD Hamiltonian + Classical background field

Li et al, 2002.09757, 2107.02225, 2305.12490, …
Barata, Salgado, 2104.04661
Barata, Du, Li, WQ, Salgado, 2208.06750 & 2307.01792
WQ, Li, Kreshchuk, Salgado, 2411.09762



$|q\rangle + |qg\rangle + |qgg\rangle$





$$\langle\!\langle \rho_a(x^+,\boldsymbol{x})\rho_b(y^+,\boldsymbol{y})\rangle\!\rangle = g^2\mu^2\delta_{ab}\delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\delta(x^+-y^+)$$

$$(m_g^2 - \nabla_\perp^2)\mathcal{A}_a^-(x^+,\boldsymbol{x}) = \rho_a(x^+,\boldsymbol{x})$$

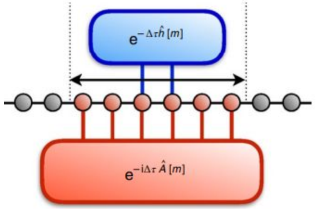
See Meijian's talk on Tuesday about in-medium jet quenching

Universal framework to simulate (3+1)-d QCD jet probe evolution in medium in real-time… can we improve it?

Quantum strategies to prepare thermal medium

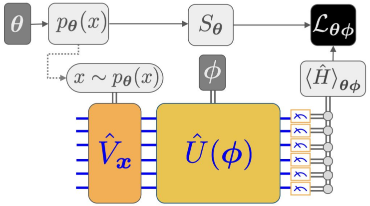
For its non-unitary nature, thermal medium preparation is hard. Nonetheless, several options available

$$|\psi(\beta)\rangle = \exp(-\beta H)\,|\psi_0\rangle$$



Quantum imaginary time

Motta et al, 1901.07653
Davoudi et al, 2208.13112



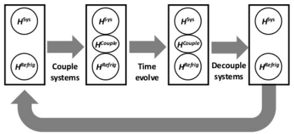
Variational quantum thermalizer

Verdon et al, 1910.02071

$$|\mathrm{TFD}\rangle \equiv \frac{1}{\sqrt{Z}}\sum_n e^{-\beta E_n/2}\,|n\rangle_L \otimes |n^*\rangle_R$$

Thermofield double states

Cottrell et al, 1811.11528



Quantum fridge

Ball and Cohen, 2212.06730

. . .

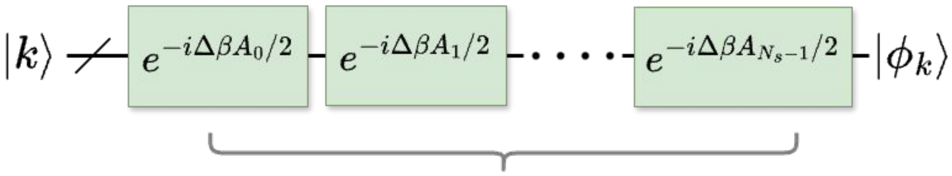
Quantum imaginary time evolution

Motta et al, 1901.07653

Key idea: trotterize the non-unitary evolution by replacing imaginary time with real time evolution

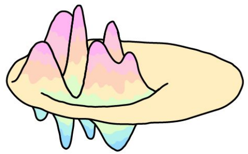
$$|\psi(\beta/2)\rangle = e^{-\beta\hat{H}/2}|\psi_0\rangle \approx \prod_{i=1}^{N_s} e^{-\Delta\beta\hat{H}/2}|\psi_0\rangle$$

$$|\psi_{i+1}\rangle = e^{-\Delta\beta\hat{H}/2}|\psi_i\rangle = e^{-i\Delta\beta\hat{A}_i/2}|\psi_i\rangle\sqrt{c_i(\Delta\beta)} + \mathcal{O}(\Delta\beta^2)$$

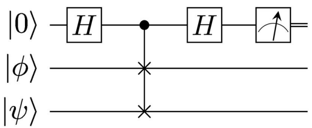


$N_s = \beta/\Delta\beta$ trotter steps using quantum gates

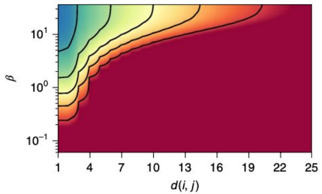
Practical advantage:



Exact, free from barren plateaus



No ancilla qubits



Correlation saturation

Recent QITE applied to field theories

Davoudi et al, 2208.13112 (Lattice gauge theory)
Czajka et al, 2112.03944 (Chemical potentials)
Pedersen et al, 2311.11616 (Schwinger topological)
...

Hastings and Koma, 0507008

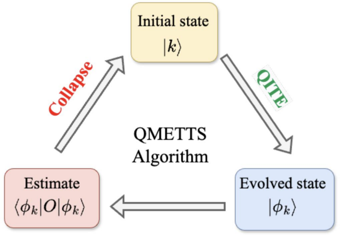
Fermion and boson system at equilibrium

WQ and Wu, 2404.07912
Cuntin, WQ, Wu, 2411.19601

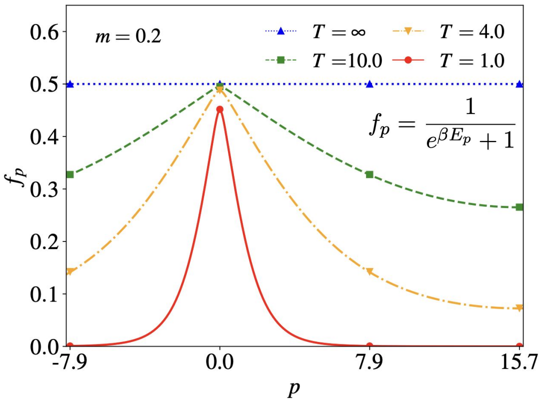
For example, use QITE to prepare thermal states of 1+1 Fermion system

$$\mathcal{L} = \frac{1}{2}\bar{\psi}(i\partial\!\!\!/ - m)\psi - \mathcal{H}_I(\psi)$$

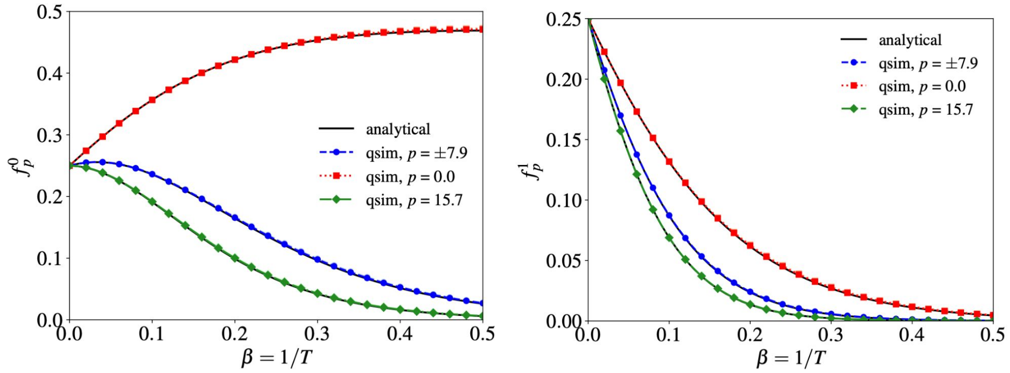
$$\langle \hat{O} \rangle_\beta \equiv Z_\beta^{-1}\mathrm{Tr}[e^{-\beta \hat{H}}\hat{O}]$$



Quantum Metropolis



Free theory



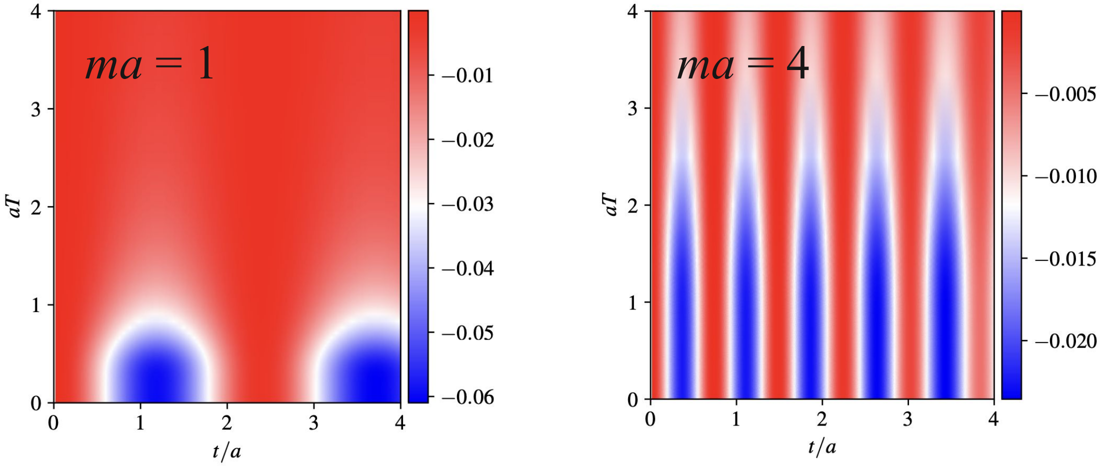
Interaction theory with homogenous background field

Extending to QED2 at finite temperature

Schwinger model (1+1 QED) with quenched evolution (imbalance of chirality) at finite temperature to study dynamics of charge operators => explain Chiral Magnetic Effect (CME)

CME in 1+1, Kharzeev, Yee, 1012.6026
Ikeda, Kang, Kharzeev, WQ, Zhao, 2407.21496

CME in 1+1 = generation of electric current



Thermal damping controlled by the mass

Real & **Imag** evolution simultaneously!

$$\langle O(t)\rangle_\beta = \frac{\mathrm{Tr}\left(e^{-\beta H}O(t)\right)}{\mathrm{Tr}\left(e^{-\beta H}\right)}$$

Vector current

$$J(x) \equiv \bar{\psi}(x)\gamma^1\psi(x)$$

Real-time evolution of out-of-equilibrium dynamics

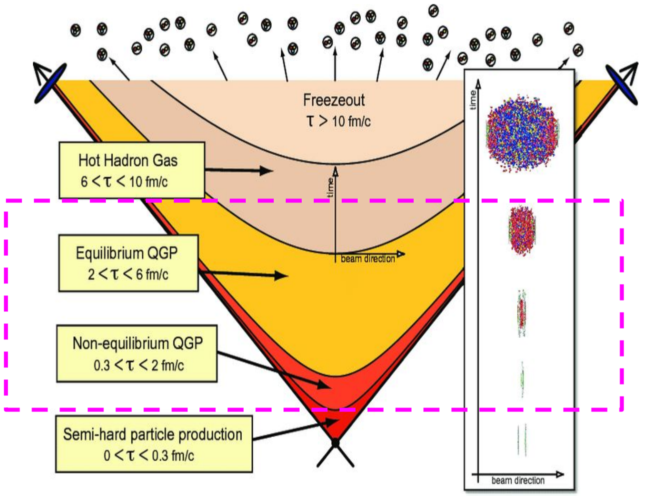
Berges, 2312.10673
Zhou et al, 2107.13563
Rigol, Dunjko, Olshanii, 0708.1324

Our ultimate goal is studying **non-equilibrium effects** directly **toward thermalization with various initial conditions**, closely related to what happens in heavy-ion collisions

One expects universal form of thermalization. The question remains whether isolated system can thermalize and under what conditions.

We are interested in ϕ^4 theory as it is one of the simplest field theory.

Quantum simulation, unlike conventional approaches, captures **all non-perturbative** contributions, including entanglement between modes.



The ϕ^4 theory

Klco and Savage, 1808.10378
Macridin et al, 1802.07347
Hardy et al, 2407.13819

The lagrangian and Hamiltonian density

$$\mathcal{L} = \frac{1}{2}[\partial_\mu \phi \partial^\mu \phi - m\phi^2] - \frac{\lambda}{4!}\phi^4$$

$$\mathcal{H} = T^{00} = \frac{1}{2}\pi^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{\lambda}{4!}\phi^4$$

Lattice Hamiltonian

$$H_{\text{lat}} = a^d \sum_{n=0}^{N-1} \left[\frac{1}{2}\pi_n^2 + \frac{1}{2}m^2\phi_n^2 + \frac{1}{2}(\nabla\phi)_n^2 + \frac{\lambda}{4!}\phi_n^4\right]$$

$$\Omega_x \equiv \{0, a, \cdots, (N-1)a\}$$

$$\Omega_p \equiv \begin{cases} \{(\frac{-N+1}{2})\Delta p, \cdots, (\frac{N-1}{2})\Delta p\} & \text{if } N \text{ is odd} \\ \{(\frac{-N+2}{2})\Delta p, \cdots, (\frac{N}{2})\Delta p\} & \text{if } N \text{ is even} \end{cases}$$

1. Field Operator (FO) Basis: spatial lattice sites => Eigenstate of ϕ fields

2. Harmonic Oscillator (HO) Basis: particle quanta (p) => Eigenstate of occupancy/mode

The ϕ^4 theory

Klco and Savage, 1808.10378
Macridin et al, 1802.07347
Hardy et al, 2407.13819

FO Basis

$$\phi_n \left|\varphi_\alpha\right\rangle = \varphi_\alpha \left|\varphi_\alpha\right\rangle$$

$$\alpha = 0, 1, ..., N_\varphi - 1 \qquad N_\varphi = 2^{n_q}$$

$$\varphi_\alpha = -\varphi_{\max} + \alpha\Delta\varphi \in [-\varphi_{\max},\ \varphi_{\max}]$$

$$H_{\text{f.o.}} = H_{\text{lat}} = a\sum_{n=0}^{N-1}\left[\frac{1}{2}\pi_n^2 + \frac{1}{2}m^2\phi_n^2 + \frac{1}{2}(\nabla\phi)_n^2 + \frac{\lambda}{4!}\phi_n^4\right] \qquad \pi_n = \mathcal{F}_n \phi_n \mathcal{F}_n^{-1}$$

Pros: most terms, especially interaction, are diagonal, so easy to simulate

Cons: sensitive to ϕ field discretizations (expensive qubit cost)

The ϕ^4 theory

Klco and Savage, 1808.10378
Macridin et al, 1802.07347
Hardy et al, 2407.13819

HO Basis:

$$\phi_n = \frac{1}{aN}\sum_p \frac{1}{\sqrt{2E_p}}\left(a_p e^{ipan} + a_p^\dagger e^{-ipan}\right)$$

$$a_p^\dagger \to \sum_{n=0}^{2^{n_q}-2} \sqrt{n+1}\,|n+1\rangle\langle n|$$

$$H_{\text{h.o.}} = \sum_p E_p\left(a_p^\dagger a_p + \frac{1}{2}\right) + \frac{\lambda}{4!N^3}\sum_{p_1,p_2,p_3,p_4}\left(\prod_{i=1}^{4}\frac{1}{4\sqrt{E_{p_i}}}\right)\delta_{p_1+p_2+p_3+p_4,0}$$
$$\left(a_{p_1}+a^\dagger_{-p_1}\right)\left(a_{p_2}+a^\dagger_{-p_2}\right)\left(a_{p_3}+a^\dagger_{-p_3}\right)\left(a_{p_4}+a^\dagger_{-p_4}\right)$$

$$E_p = \sqrt{m^2 + \frac{4}{a^2}\sin^2\left(\frac{ap}{2}\right)}$$

Pros: free Hamiltonian diagonal, small qubits needed per mode

Cons: interaction is non-trivial, overall need a lot of CNOT gates

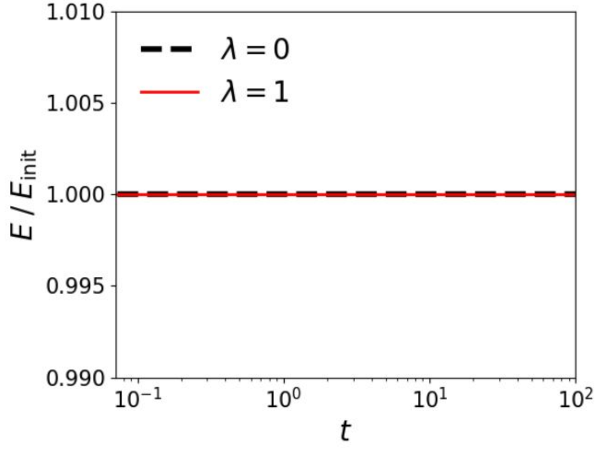
HO Results in 0+1

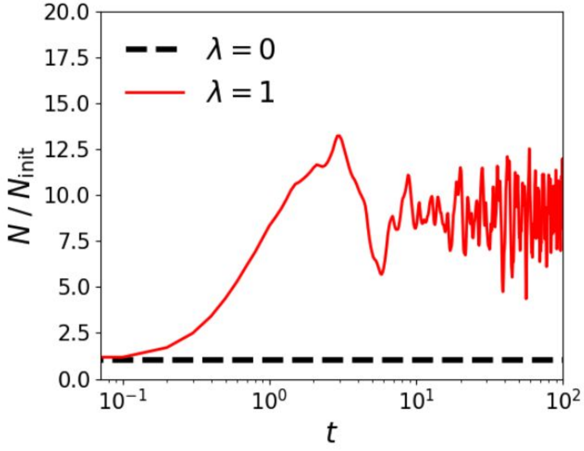
$$H = \frac{1}{2}\pi^2 + \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$$

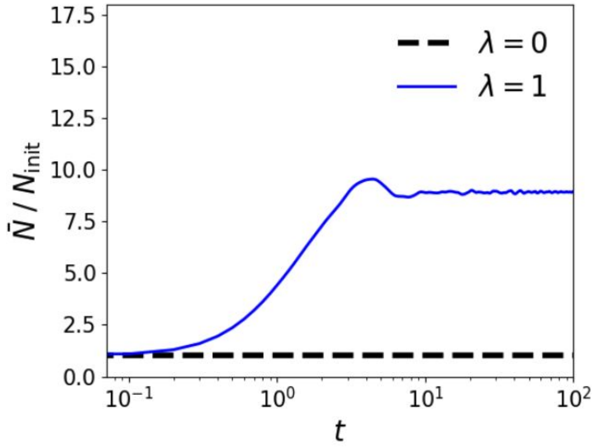
$$N = a^\dagger a$$

$$\langle \bar{O} \rangle \equiv \lim_{t\to\infty} \frac{1}{t}\int_0^t O(t')\,\mathrm{d}t' \approx \frac{\Delta t}{t}\sum_{t'=0}^{t} O(t')$$

System starts with some fixed occupancy mode |n>





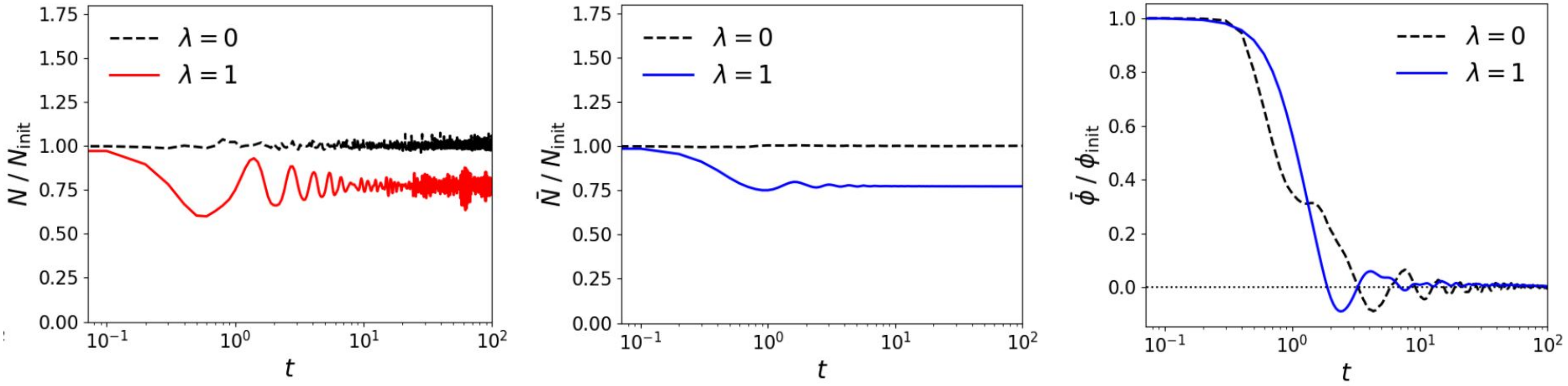


* only trotter-free results presented in this talk

FO Results in 0+1

$$H = \frac{1}{2}\pi^2 + \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4 \qquad N = a^\dagger a \qquad \langle \bar{O} \rangle \equiv \lim_{t\to\infty} \frac{1}{t}\int_0^t O(t')\,\mathrm{d}t' \approx \frac{\Delta t}{t}\sum_{t'=0}^{t} O(t')$$

System starts with some fixed ϕ field



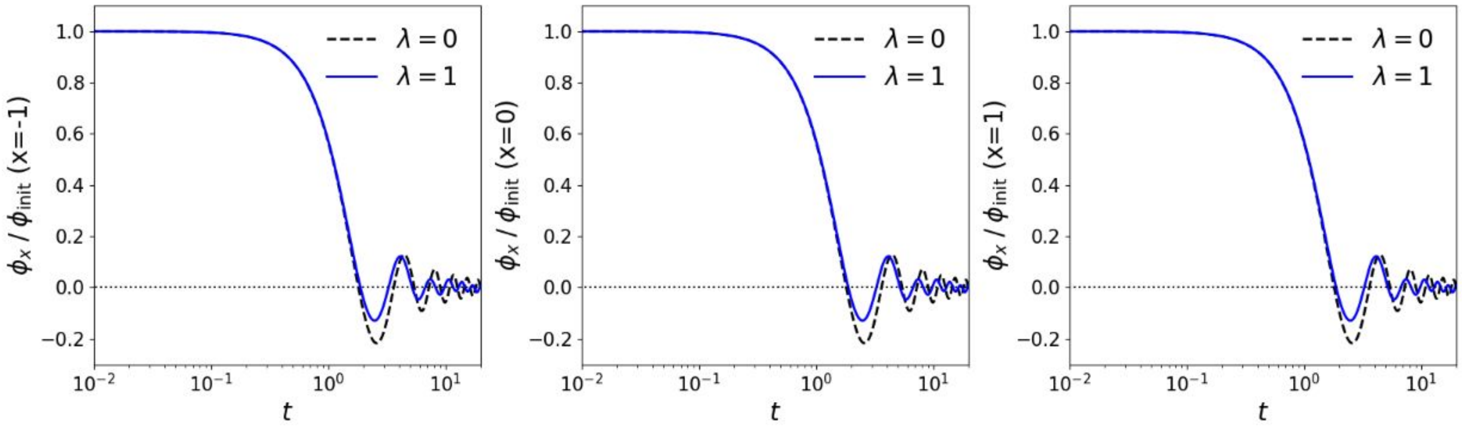
insufficient ϕ discretizations

HO Results in 1+1

More interesting study 1+1 with different momenta…

In HO basis, we can also start with some uniform $\phi_x = \phi_0$ field by diagonalization to extract the initial state.

Preliminary results
(9 qubits)



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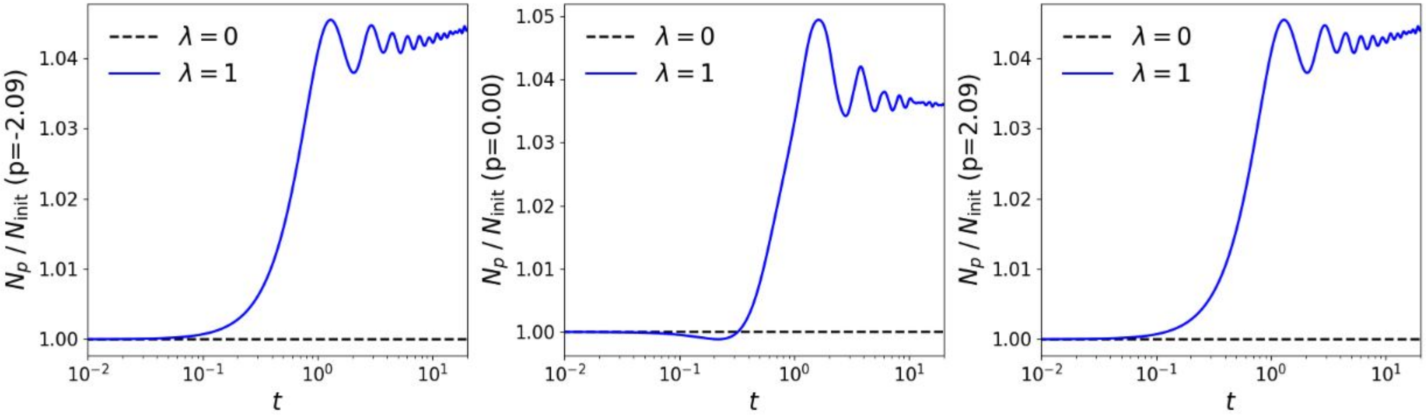
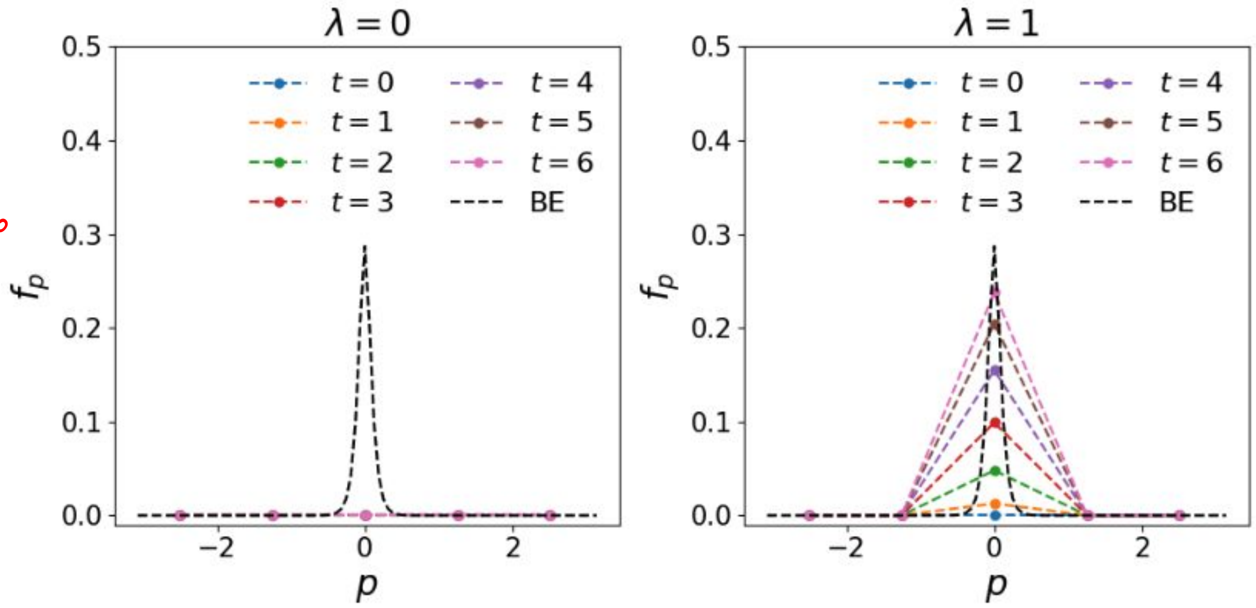
Preliminary results
(9 qubits)



HO Results in 1+1

One can also start with some uniform energy eigenstate, to see build-up toward Bose Einstein distribution

Preliminary results
(10 qubits)



Summary and outlook

- Quantum computing technology is available today and developing fast for HEP.
- Several quantum strategies offer advantages to prepare thermal medium of the system.
- Quantum simulation helps to understand real-time evolution of non-equilibrium dynamics relevant to Heavy-ion collisions.
- Larger scale simulation under development (esp for FO). It remains interesting to explore other options such as tensor networks, continuous-variable simulation etc.

Thanks for your attention!

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