

# Mid-circuit measurements in quantum simulations of lattice gauge theories: from the quantum Zeno effect to error correction

Matteo Wauters

CERN, 27<sup>th</sup> May 2025



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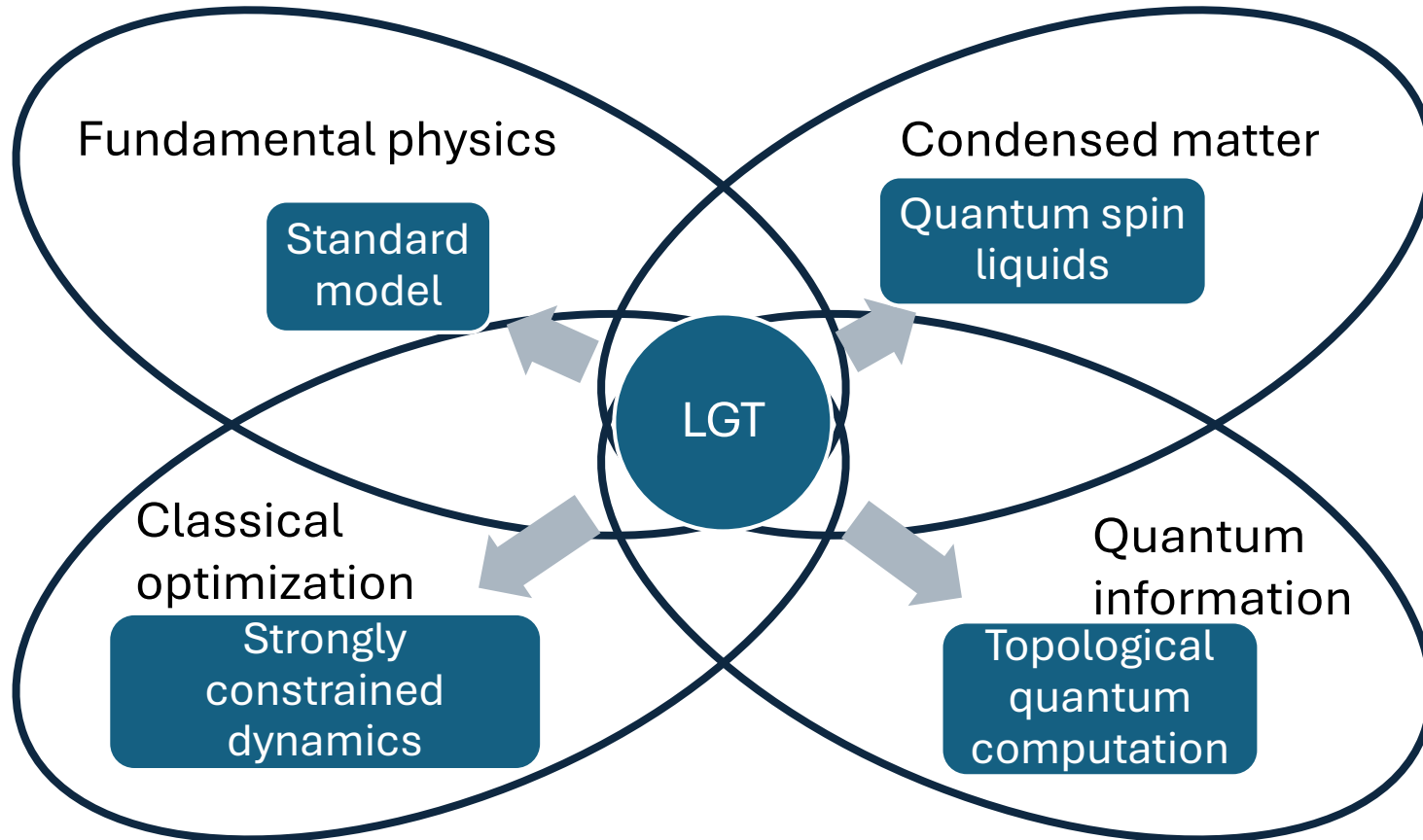


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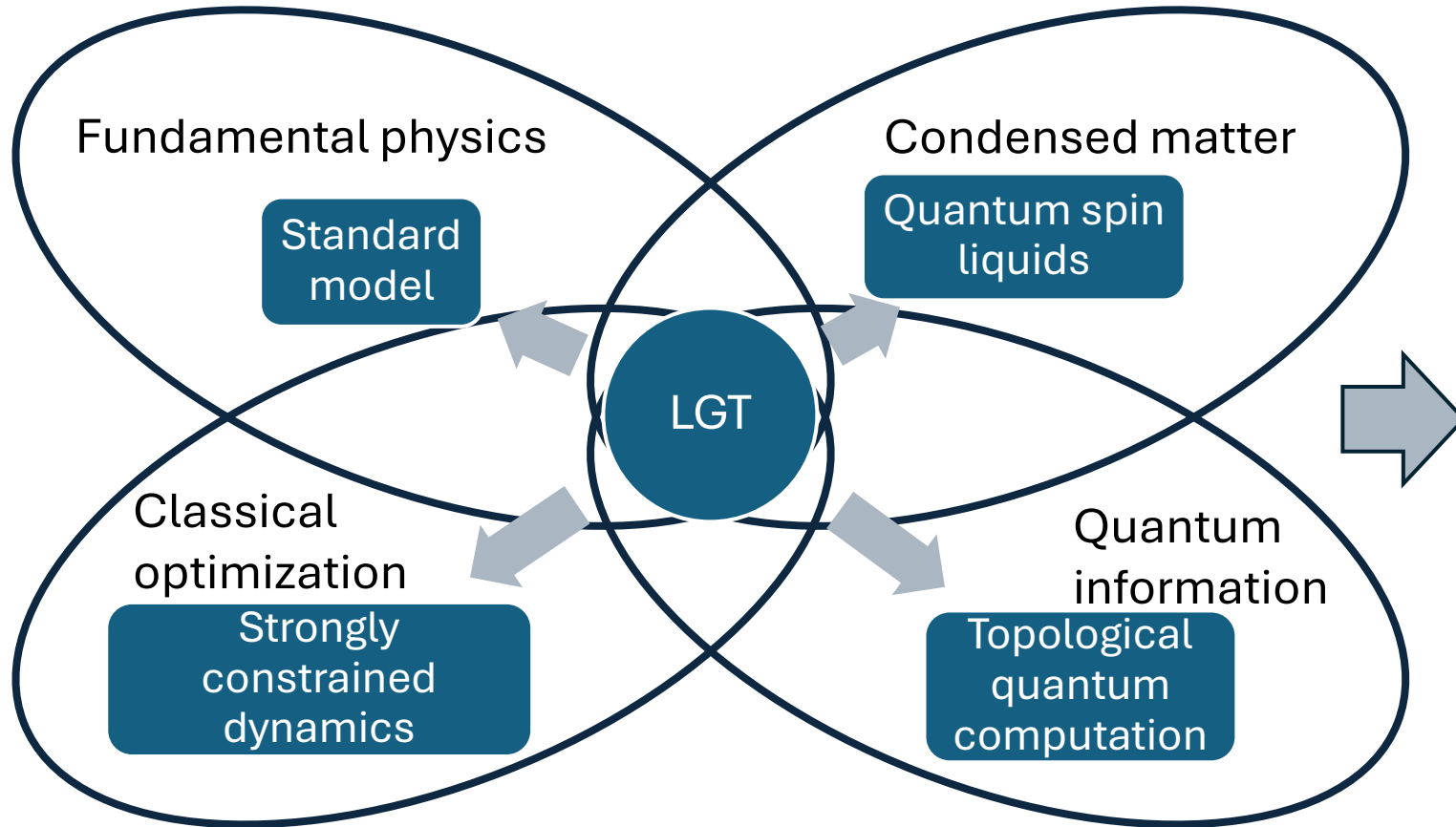


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# Quantum simulations of lattice gauge theories



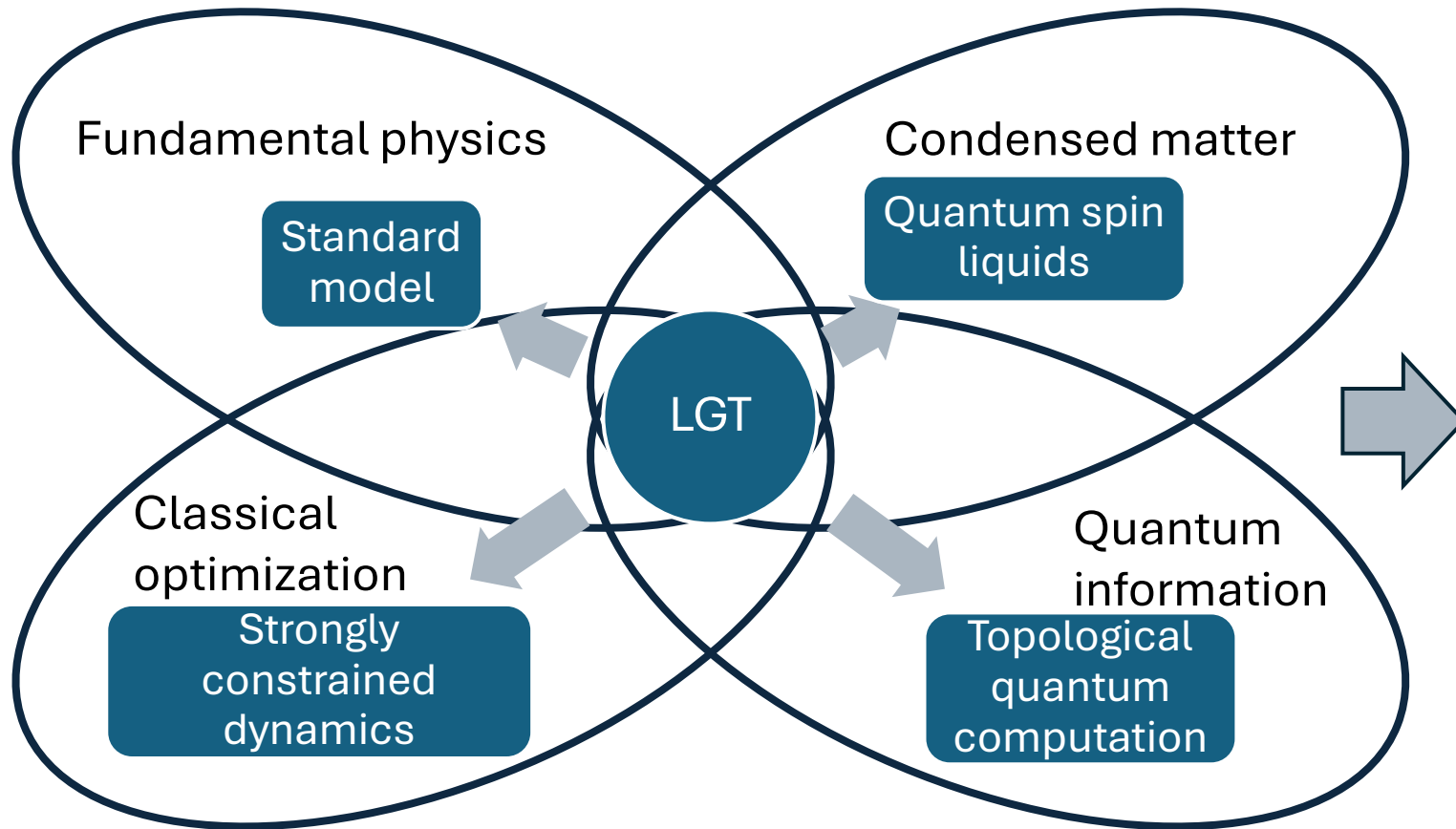
# Quantum simulations of lattice gauge theories



Quantum simulations of many-body physics

1. Go beyond classical methods
2. Hardware and software benchmark
3. Synthetic quantum systems

# Quantum simulations of **monitored** lattice gauge theories



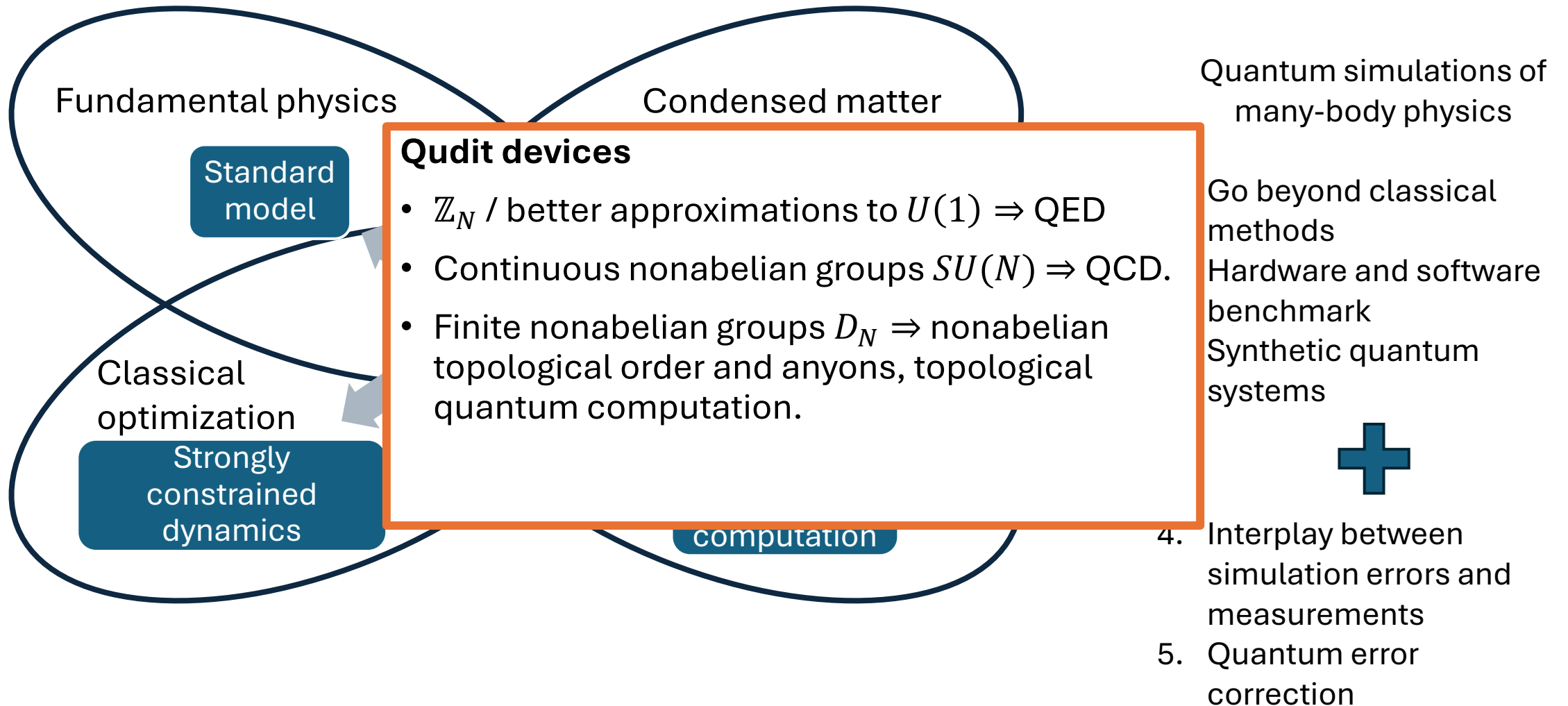
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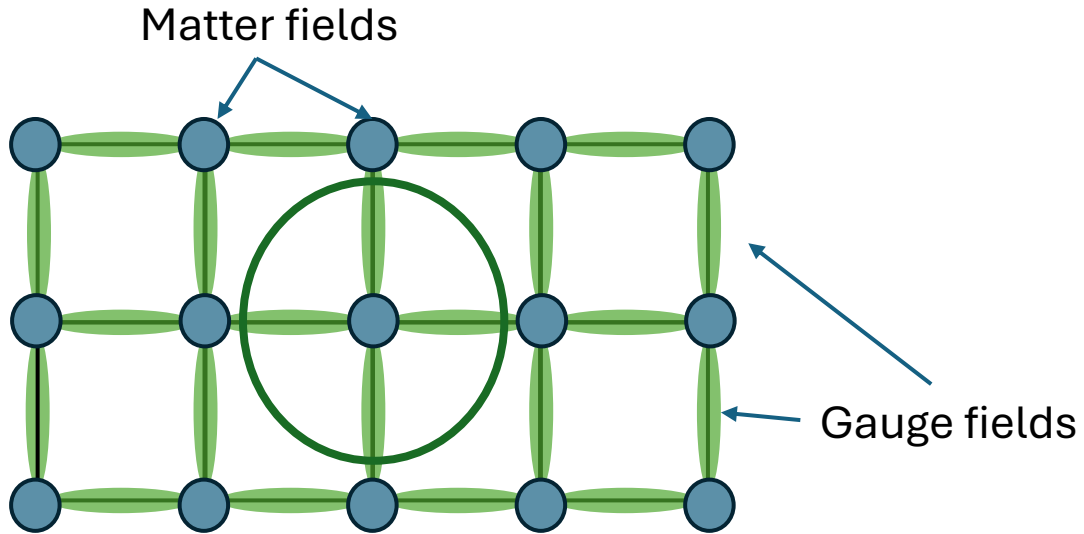


4. Interplay between simulation errors and measurements
5. Quantum error correction

# Quantum simulations of **monitored** lattice gauge theories



# Quantum simulations of lattice gauge theories



Local symmetry

$$U(1): \nabla \cdot \mathbf{E} = -\rho$$

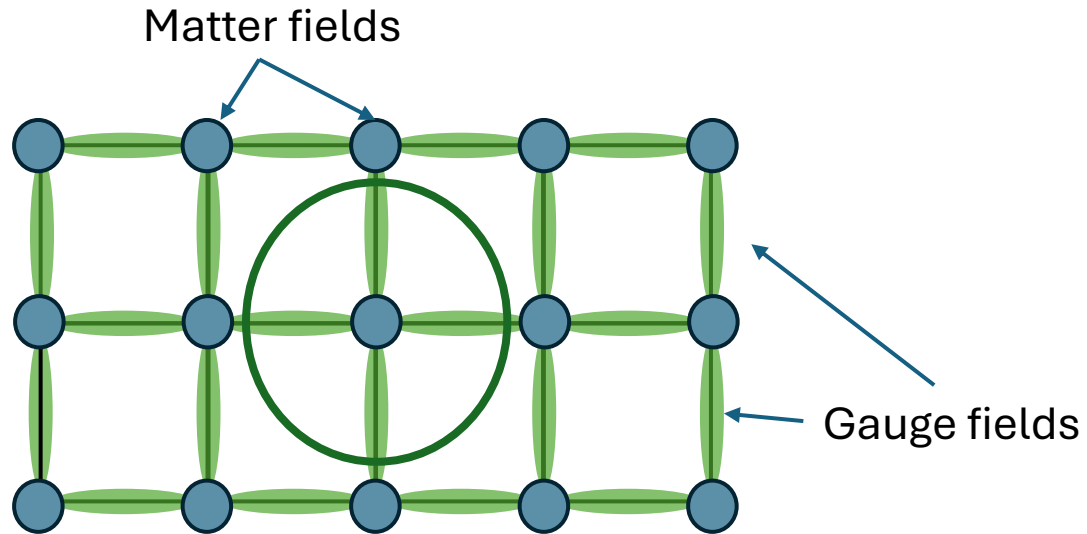
$$\text{Lattice} \Rightarrow \sum_k (E_{\mathbf{r}+\mathbf{e}_k} - E_{\mathbf{r}-\mathbf{e}_k}) + \psi_{\mathbf{r}}^\dagger \psi_{\mathbf{r}} = 0$$

Gauge group  $G$

$$\Theta_{\mathbf{r}}(g) H \Theta_{\mathbf{r}}^\dagger(g) = H \quad \forall \mathbf{r} \in \mathbb{Z}^d, \forall g \in G$$

**Extensive number of symmetries**

# Quantum simulations of lattice gauge theories



## Local symmetries to restrict to physical states?

Do: smaller Hilbert space, possibly nonlocal Hamiltonian, multi-body interactions

Don't: larger Hilbert space, local Hamiltonian, gauge symmetry breaking

### Local symmetry

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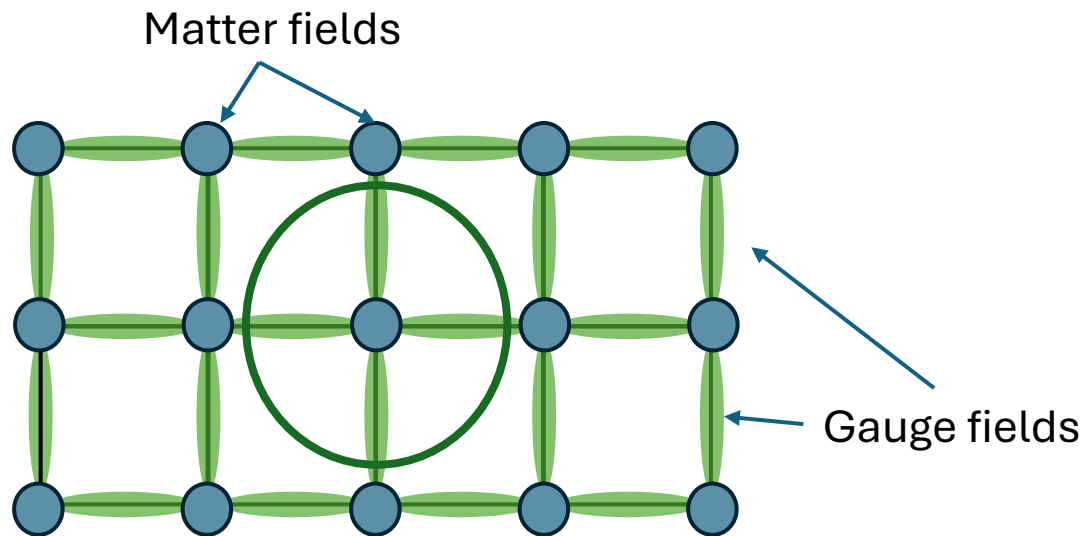
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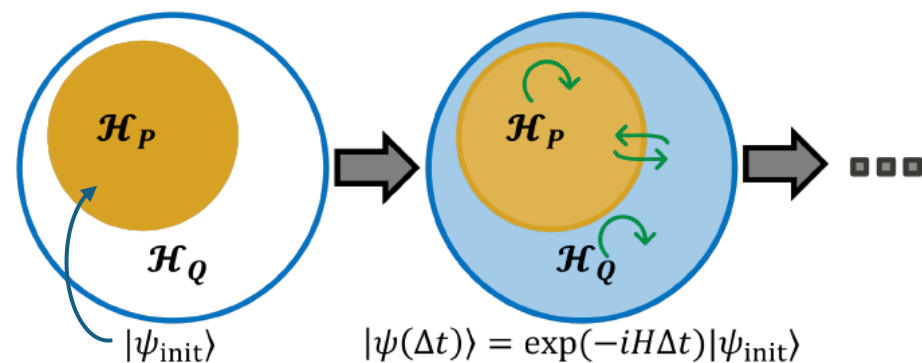
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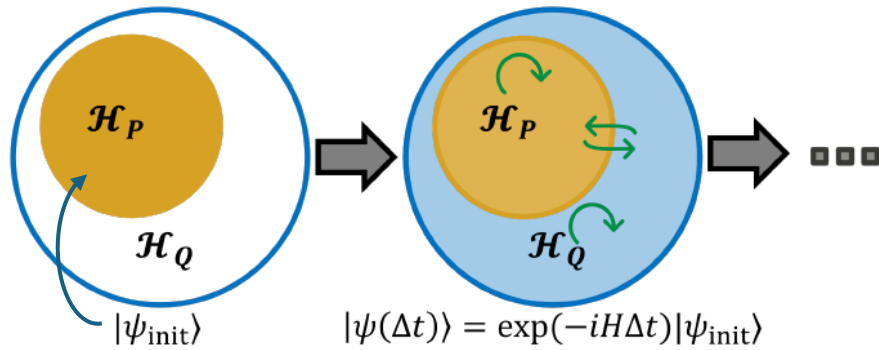
Do: smaller Hilbert space, possibly nonlocal Hamiltonian, multi-body interactions

Don't: larger Hilbert space, local Hamiltonian, gauge symmetry breaking

$$H = \begin{bmatrix} \text{Phys} & 0 \\ 0 & \text{non Phys} \end{bmatrix} \xrightarrow{+\text{errors}} \tilde{H} = \begin{bmatrix} \text{Phys} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \text{non Phys} \end{bmatrix}$$



# Quantum simulations of lattice gauge theories



## How do we detect and suppress errors?

Qubits, abelian LGT:

Post-selection

Engineered dissipation

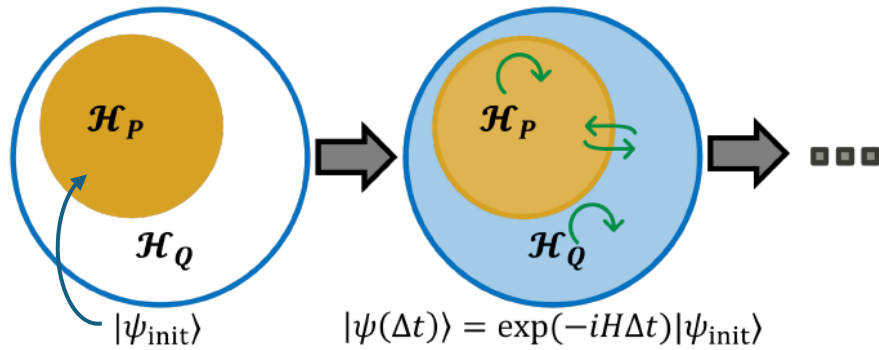
Effective Hamiltonian

Nonabelian LGT:

**Local symmetry generators do not commute**

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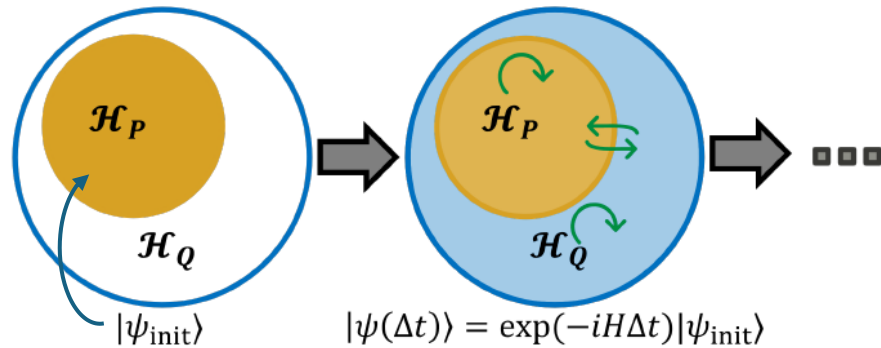
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- Works with any error channel
- Building block for error correction



1. **Dynamical post-selection**
2. **Post-processed symmetry verification**

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## Monitored LGT

- ? Interplay between unitary dynamics and measurements
- ? Trajectory statistics with different measurement schemes
- ? Conditional operations  $\Rightarrow$  towards error-corrected simulations
- ? Measurement-based control for qudits

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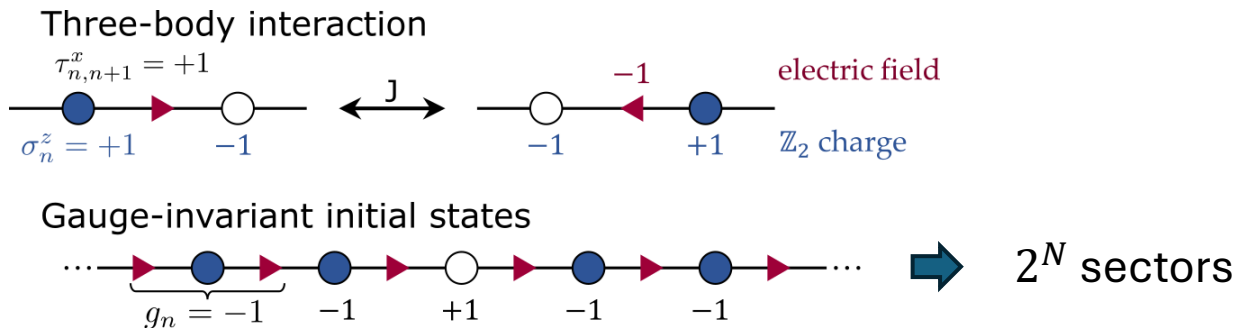
1. **Dynamical post-selection**
2. **Post-processed symmetry verification**

# Dynamical post-selection: warmup on $\mathbb{Z}_2$

Z2 LGT in one dimension (Fermionic matter + JW)

$$\hat{H}_0 = -J \sum_n (\sigma_n^+ \tau_{n,n+1}^z \sigma_n^- + \text{h.c.}) + \mu \sum_n (-1)^n \sigma_n^z - f \sum_n \tau_{n,n+1}^x$$

Local gauge charge  $\hat{G}_n = -\tau_{n-1,n}^x \sigma_n^z \tau_{n,n+1}^x \quad [\hat{H}_0, \hat{G}_n] = 0 \quad \forall n$

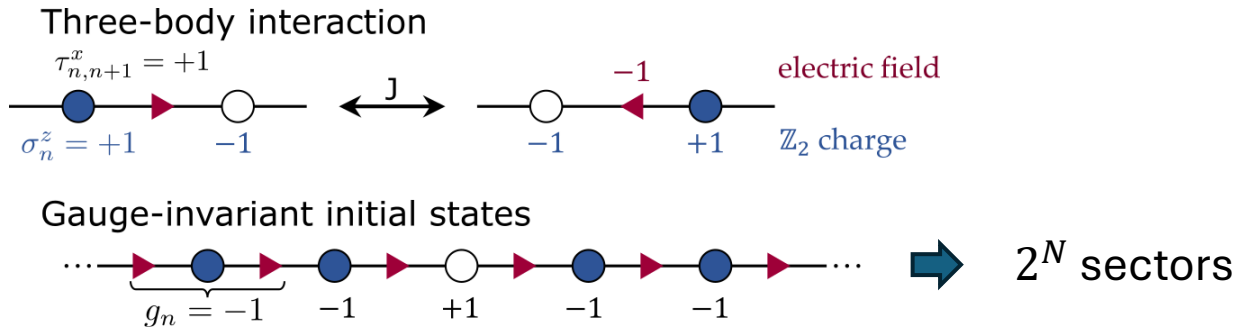


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Gauge-symmetry breaking coherent error

$$\hat{H}_{\text{err}} = \lambda \sum_n (\sigma_n^+ \sigma_{n+1}^- + \text{h.c.}) + \lambda \sum_n \tau_{n,n+1}^z$$

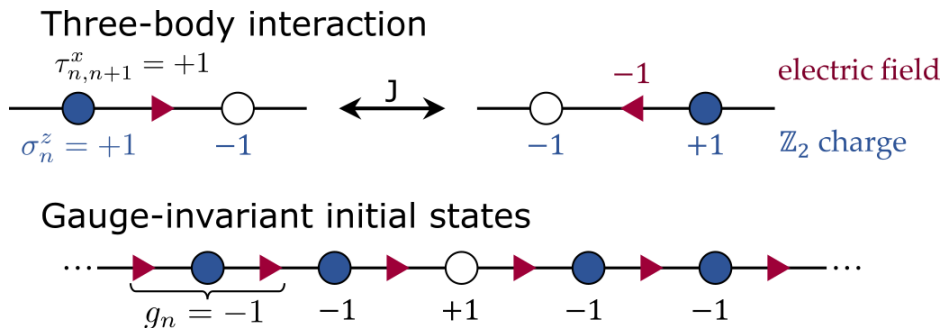
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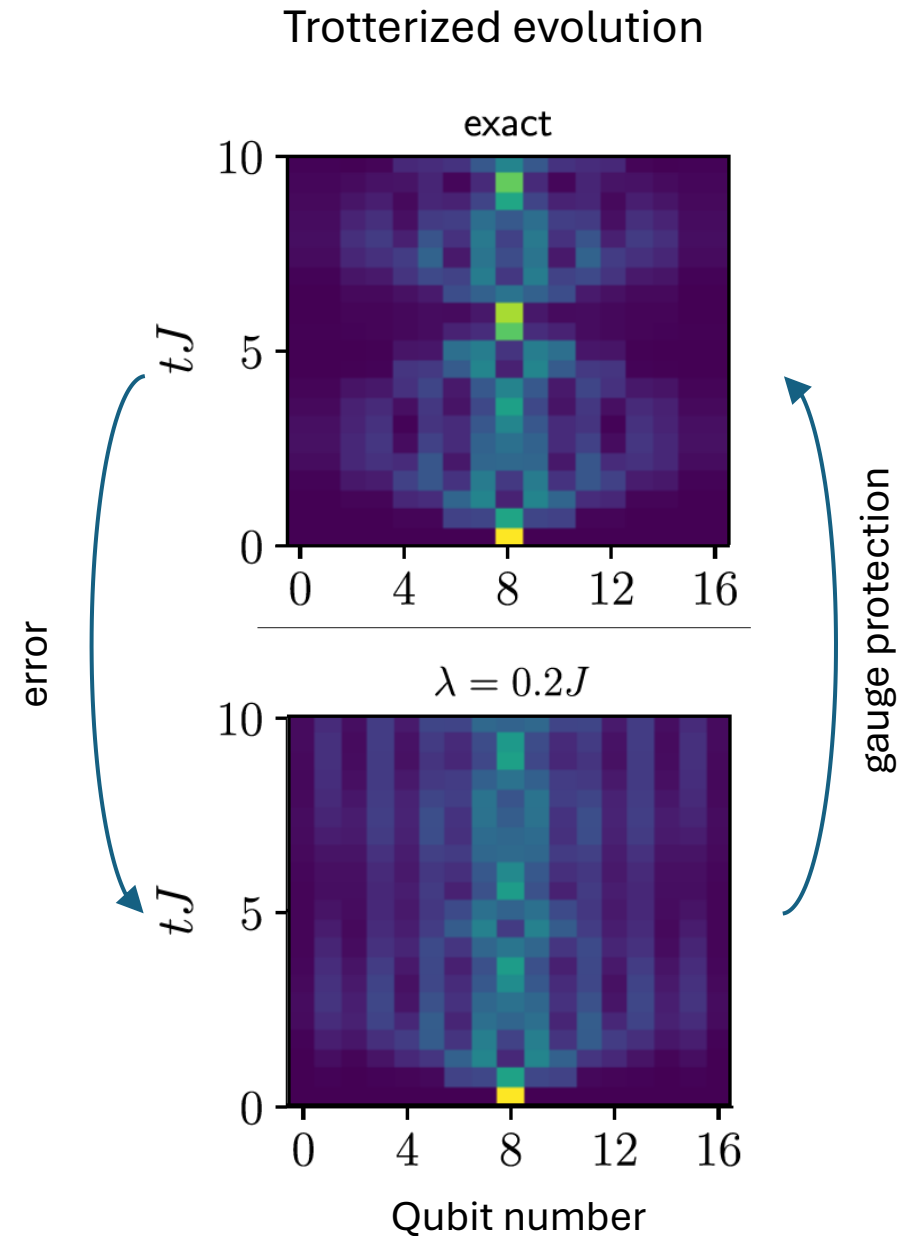
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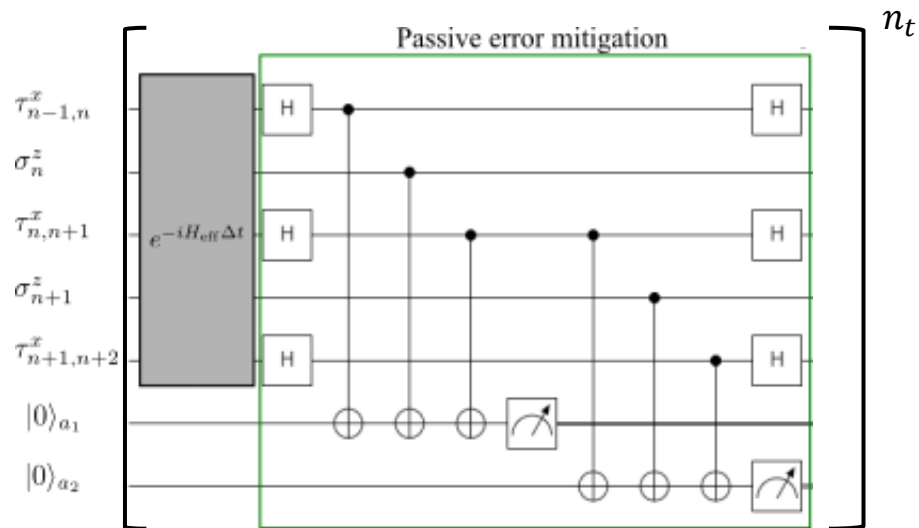
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# Measurement-induced gauge protection in digital quantum simulations

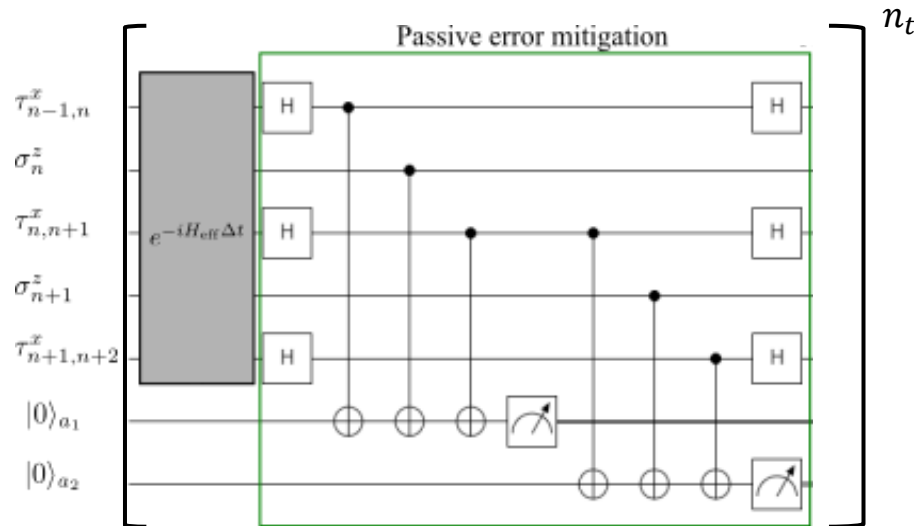
## Dynamical post-selection approach



- At each Trotter step  $\hat{G}_n$  is encoded in an ancillary qubit
- Requires  $O(N)$  ancilla qubits
- No reset of  $|a\rangle$

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$$|\psi(t)\rangle = |\psi\rangle_{\text{phys}} \Rightarrow |\psi(t + \Delta t)\rangle = \alpha|\psi\rangle_{\text{phys}} + \beta|\psi\rangle_{\text{np}}$$

Coupling with the ancilla  $|a\rangle$

$$|\psi, a\rangle = \alpha|0\rangle_a|\psi\rangle_{\text{phys}} + \beta|1\rangle_b|\psi\rangle_{\text{np}}$$

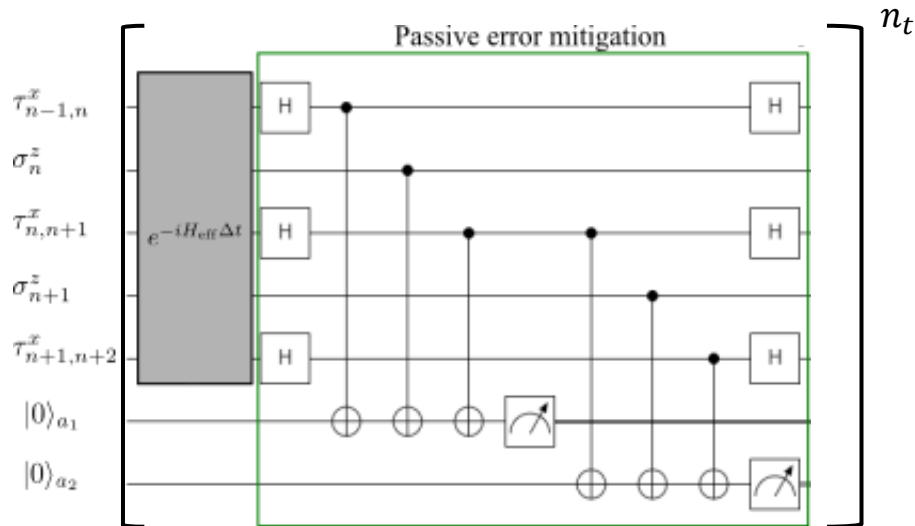
Measure  $|a\rangle$

$$|\tilde{\psi}(t + \Delta t)\rangle = \begin{cases} |\psi\rangle_{\text{phys}} & \text{with probability } \alpha^2 \rightarrow \text{keep} \\ |\psi\rangle_{\text{np}} & \text{with probability } \beta^2 \rightarrow \text{discard} \end{cases}$$

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Equation for the density matrix in the “continuous time limit”

$$\dot{\rho} = -i\hbar[\hat{H}, \rho] + \frac{1}{2\tau} \sum_n \hat{G}_n \rho \hat{G}_n^\dagger - \frac{1}{2} \{ \hat{G}_n^\dagger \hat{G}_n, \rho \} = \mathcal{L}\rho$$

Time between measurements

# Measurement-induced gauge protection in digital quantum simulations

$$\dot{\rho} = -i\hbar[\hat{H}, \rho] + \gamma \sum_n \hat{G}_n \rho \hat{G}_n^\dagger - \frac{1}{2} \{ \hat{G}_n^\dagger \hat{G}_n, \rho \} = \mathcal{L}\rho$$

Possible implementations:

- Engineered dissipation [1]
- Random gauge transformations [2]
- Continuous measurements
- Continuous limit for DPS

[1] Stannigel et al. PRL **112** (2014)

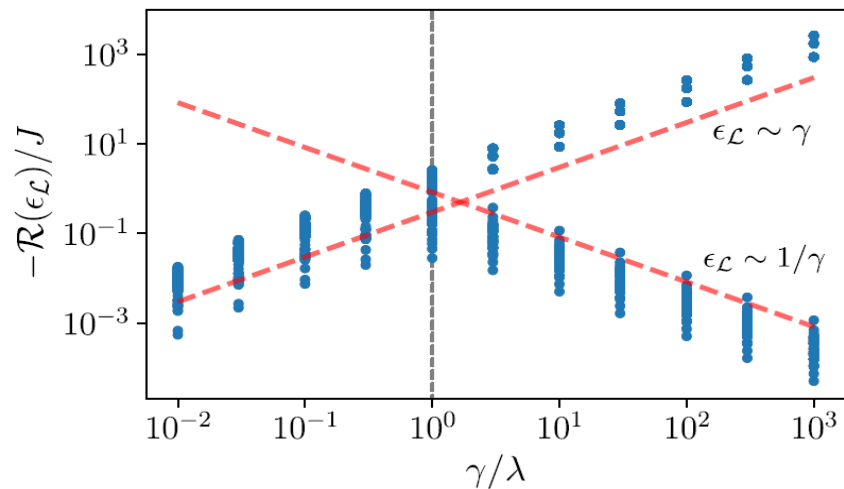
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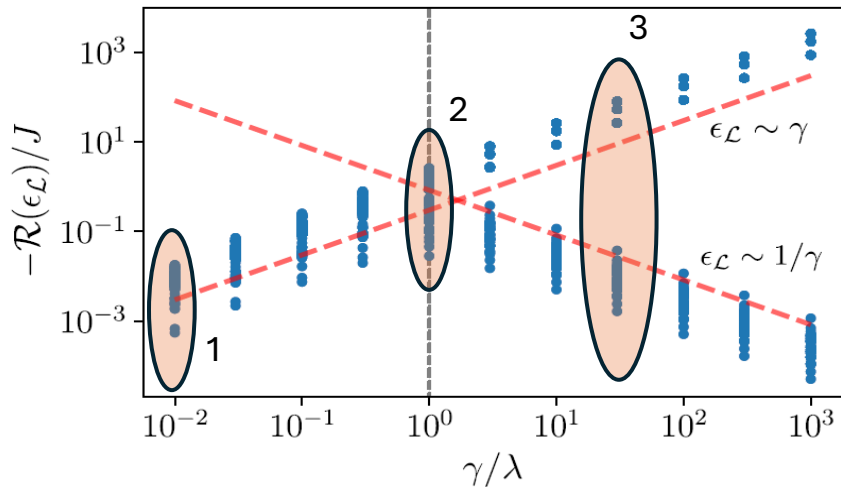
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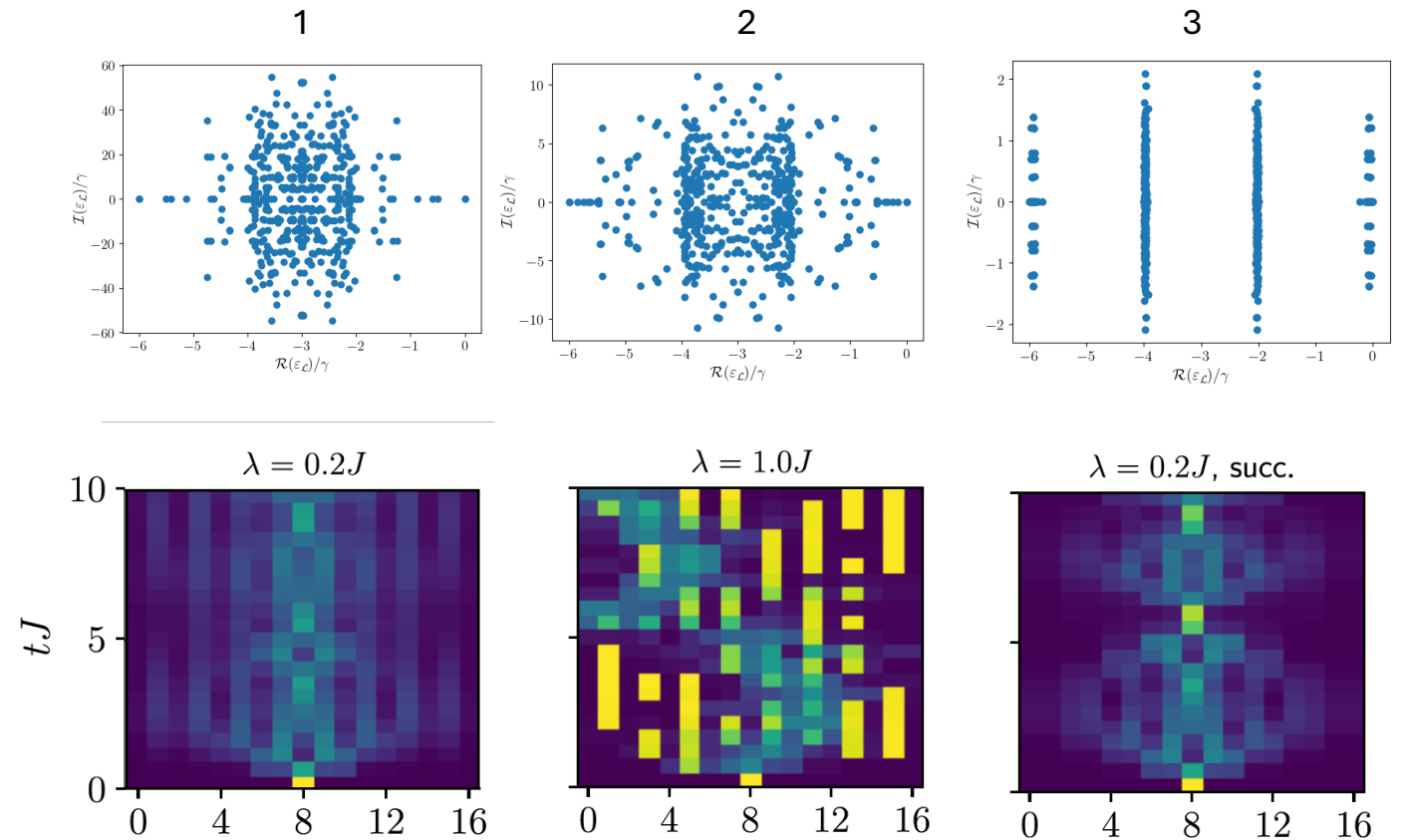
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**Quantum Zeno transition between protected and chaotic phases**



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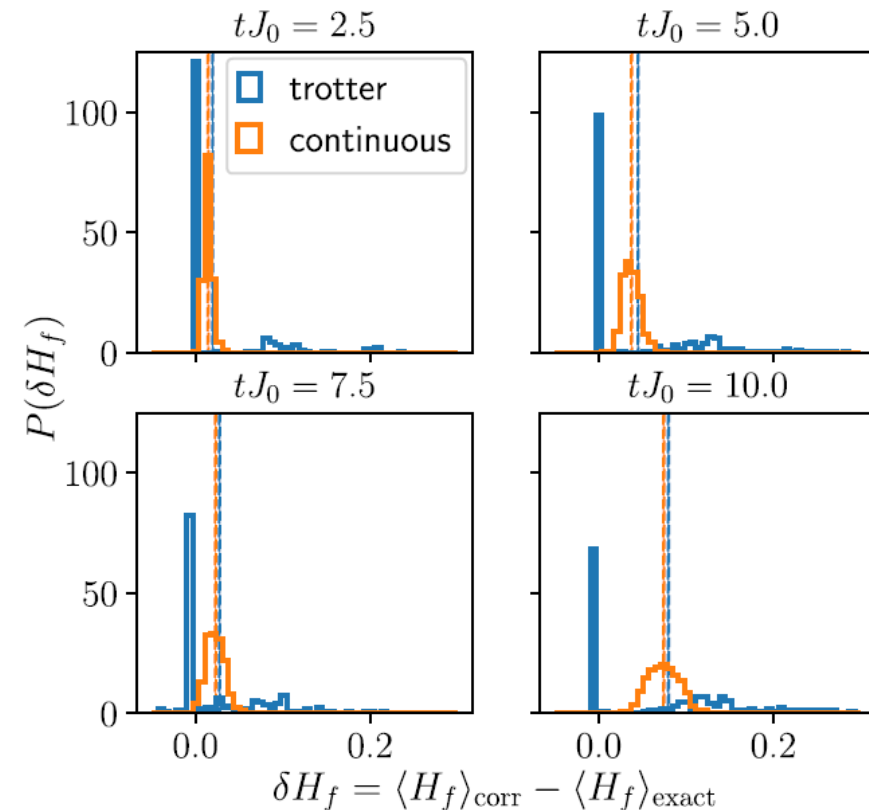
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**Quantum Zeno transition between protected (QZP) and chaotic phases**

**Importance of unraveling: same ensemble average, different stochastic trajectories**



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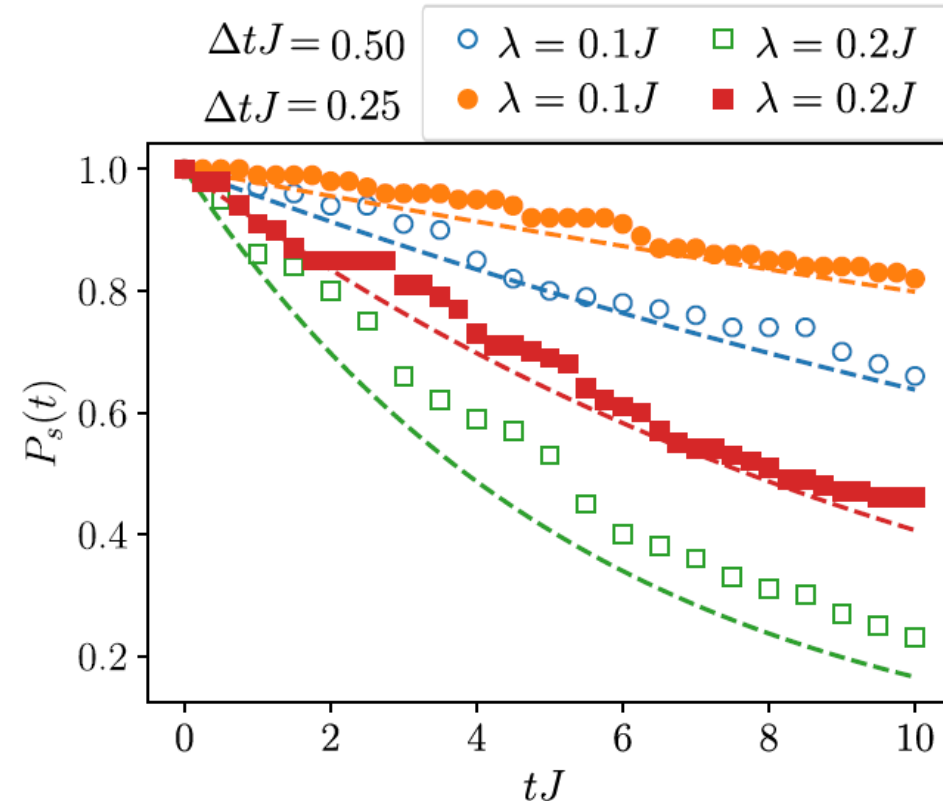
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# Nonabelian LGT: benchmark on $D_3$

## $D_3$ gauge symmetry group

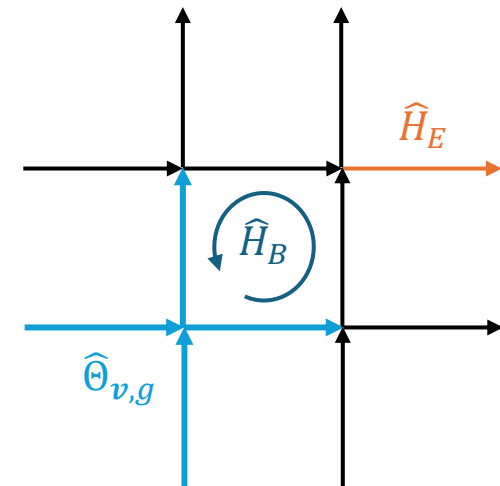
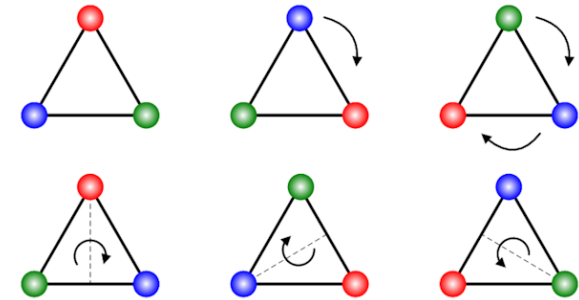
- Smallest discrete nonabelian group  $\Rightarrow$  “fits” Ca trapped ion qudit platform\*
- Topological order and universal anyons

2+1 dimensional square lattice , pure gauge:

$$\hat{H}_0 = \boxed{-\frac{1}{g^2} \sum_p \mathcal{R} \left[ \text{Tr} \left( \hat{U}_{p_1}^j \hat{U}_{p_2}^j \hat{U}_{p_3}^{j\dagger} \hat{U}_{p_4}^{j\dagger} \right) \right]} + \hat{H}_E$$

$\hat{H}_B$

$$[\hat{H}_B, \hat{H}_E] \neq 0$$



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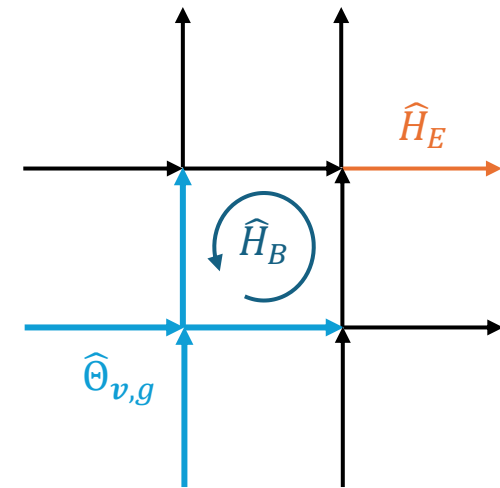
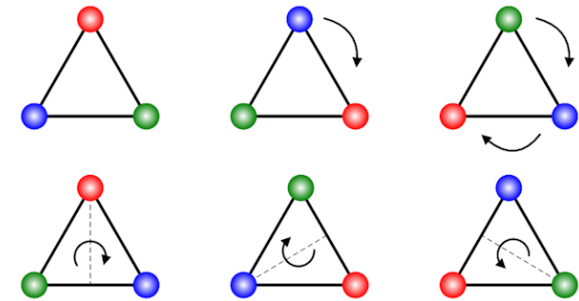
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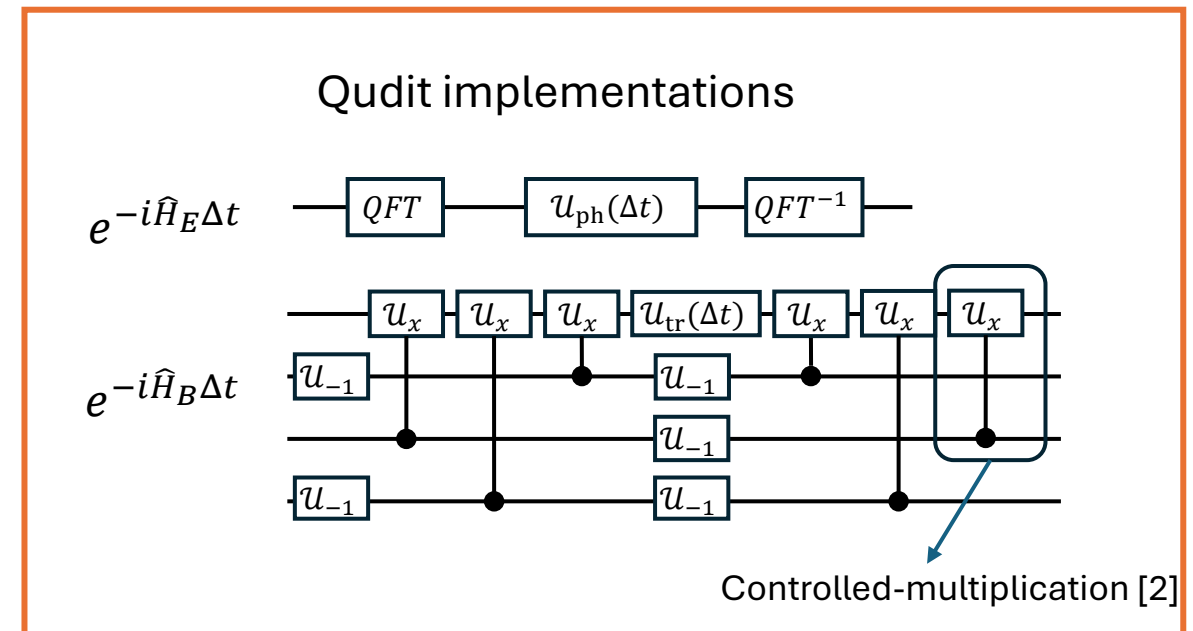
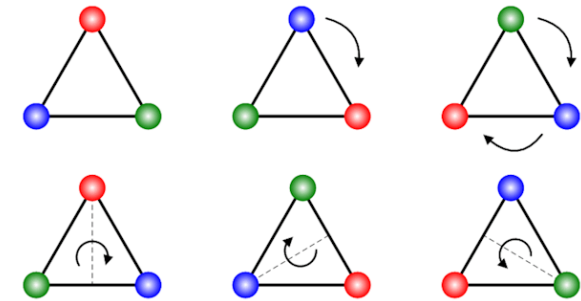
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[1] Ringbauer et al, Nat Phys 18 (2022)

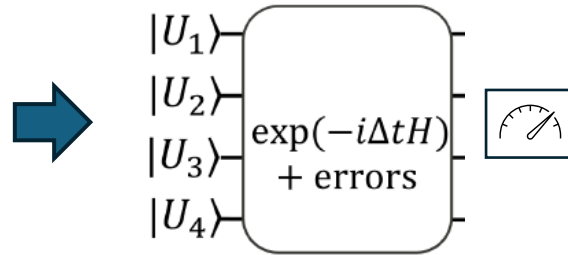
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## $D_3$ Dynamical post-selection

What do we measure?

Computational basis

- ✓ Plaquette operator
- ✗ Local gauge charge



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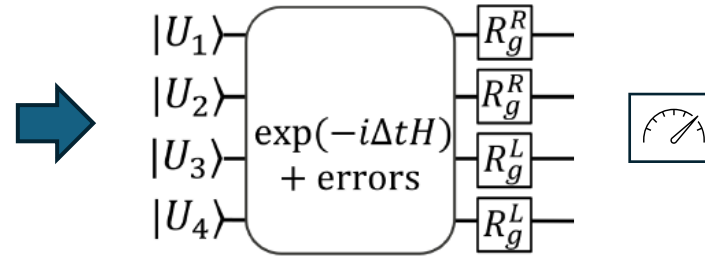
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Eigenbasis of  $\hat{\Theta}_{v,g}$

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- ✗ Gauge transformations do not commute
- ✓ Local gauge charge



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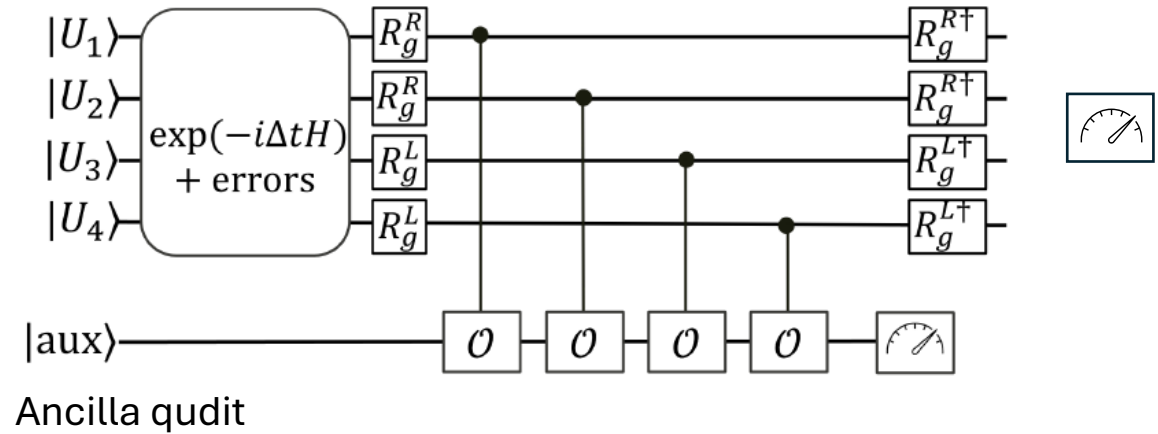
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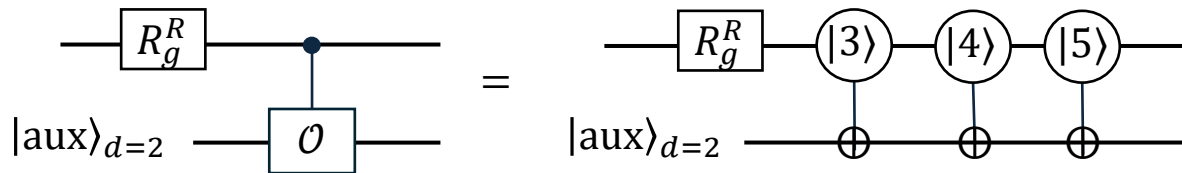
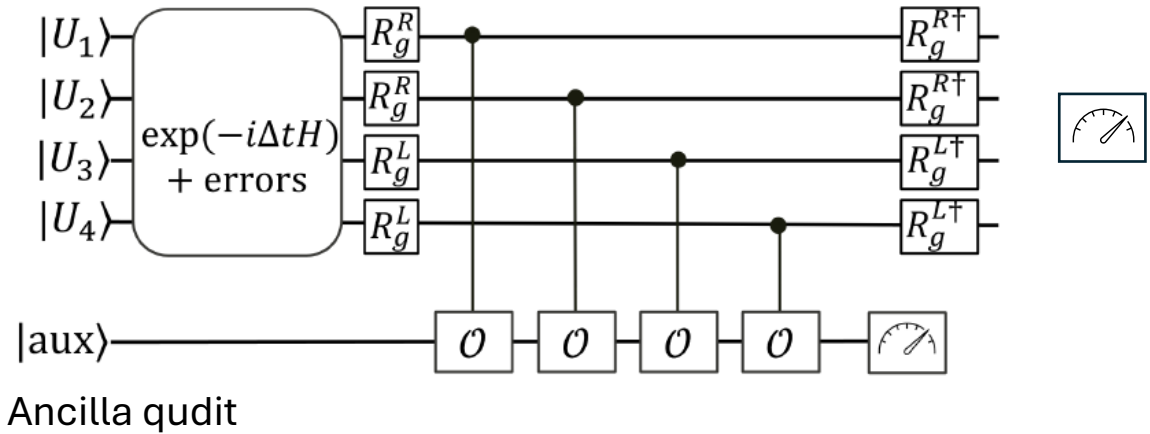
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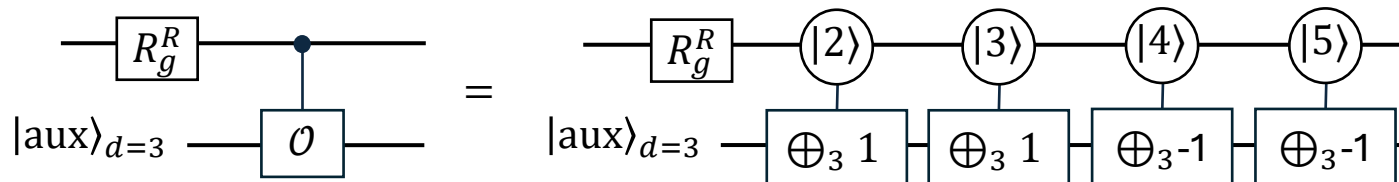
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Digital post-selection



$g$  is a reflection

$$R_g^{R\dagger} \hat{\Theta}_g^R R_g^R = \text{diag}(1, 1, 1, -1, -1, -1)$$



$g$  is a rotation

$$R_g^{R\dagger} \hat{\Theta}_g^R R_g^R = \text{diag}(1, 1, e^{i2\pi/3}, e^{i2\pi/3}, e^{-i2\pi/3}, e^{-i2\pi/3})$$

# $D_3$ post-Processed Symmetry Verification (PSV)

$\Pi_s$  = projector on gauge-symmetric sector

$\rho$  = density matrix after noisy evolution

$$\rho_s = \frac{\Pi_s \rho \Pi_s}{\text{Tr}[\Pi_s \rho]}$$

Symmetry-projected expectation value of gauge-invariant observable

$$\text{Tr}(O \rho_s) = \frac{\text{Tr}[O \Pi_s \rho \Pi_s]}{\text{Tr}[\Pi_s \rho]} = \frac{\text{Tr}[O_s \rho]}{\text{Tr}[\Pi_s \rho]}, \quad \text{with } O_s = \Pi_s O \Pi_s = \Pi_s O$$

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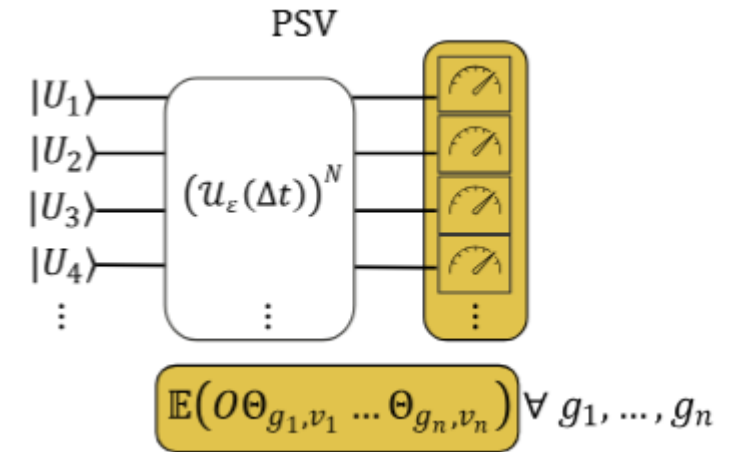
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Effective group symmetrization  
by averaging over multiple  
observables

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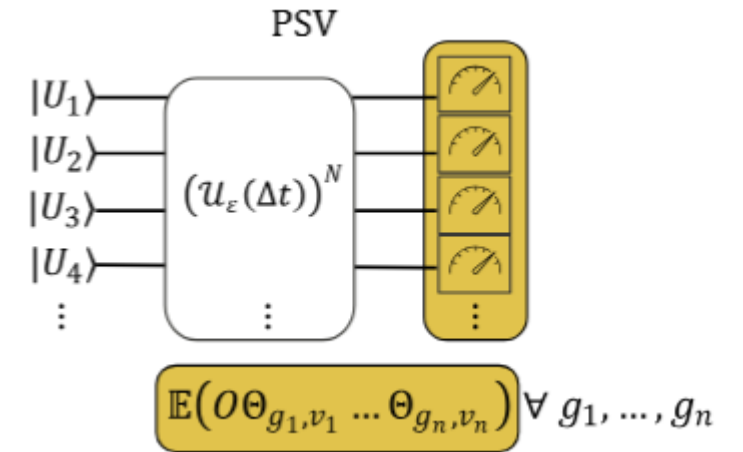
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Ugly sum with exponentially many terms



Effective group symmetrization by averaging over multiple observables

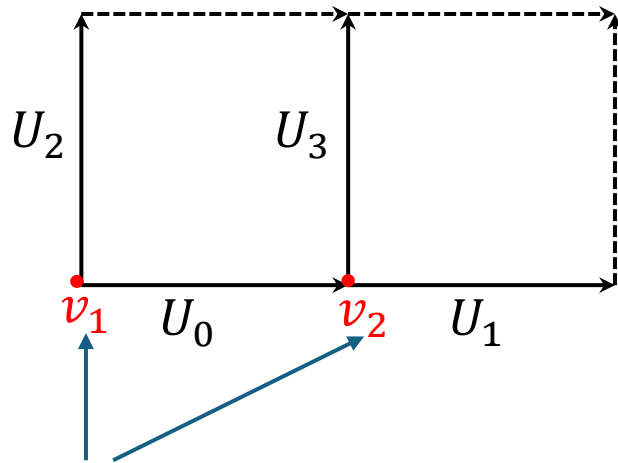
Operator dressed with local gauge transformation

# Numerical Results

Two plaquettes with PBC

$$\dim \mathcal{H}_{\text{tot}} = 6^4 = 1296$$

$$\dim \mathcal{H}_{\text{phys}} = 49$$



Two vertices

$$\widehat{\Theta}_{v_1, g} = \widehat{\Theta}_{0, g}^L \widehat{\Theta}_{2, g}^L \widehat{\Theta}_{0, g}^R \widehat{\Theta}_{1, g}^R$$

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Noise model: random unitaries close to the identity

$$|\psi(t)\rangle = \exp(-iHt) |\psi(0)\rangle$$

$$\cong \left[ \mathcal{U}_{\varepsilon(\gamma)} \exp\left(-\frac{iH_E t}{N}\right) \exp\left(-\frac{iH_B t}{N}\right) \right]^N |\psi(0)\rangle$$

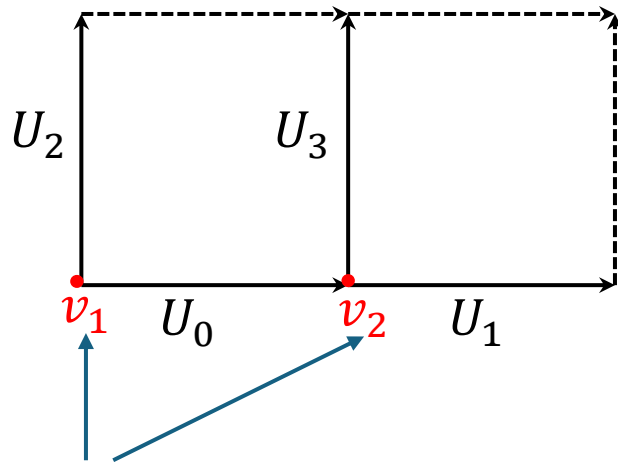
$$\widehat{H}_0 = -\frac{1}{g^2} \sum_p \mathcal{R} \left[ \text{Tr} \left( \widehat{U}_{p_1}^j \widehat{U}_{p_2}^j \widehat{U}_{p_3}^{j\dagger} \widehat{U}_{p_4}^{j\dagger} \right) \right] + \widehat{H}_E$$

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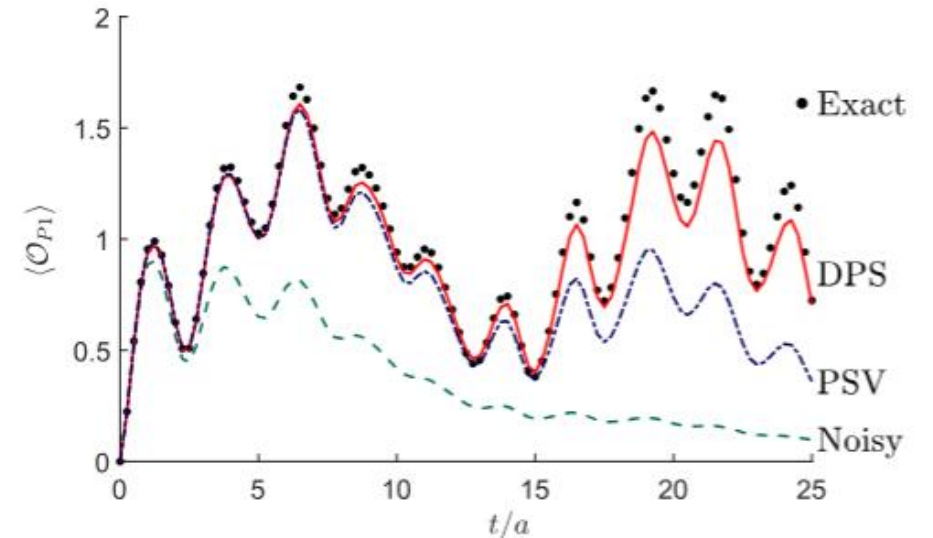
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Quench protocol:

- **DPS:** Each trotter step, measure **one** local charge
- **PSV:** 16 independent observables to sample

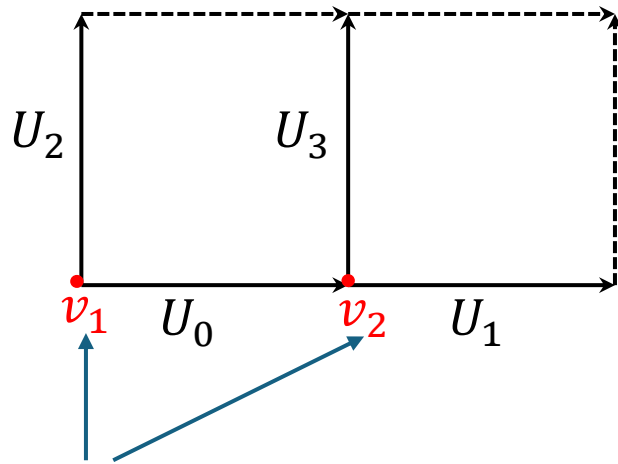


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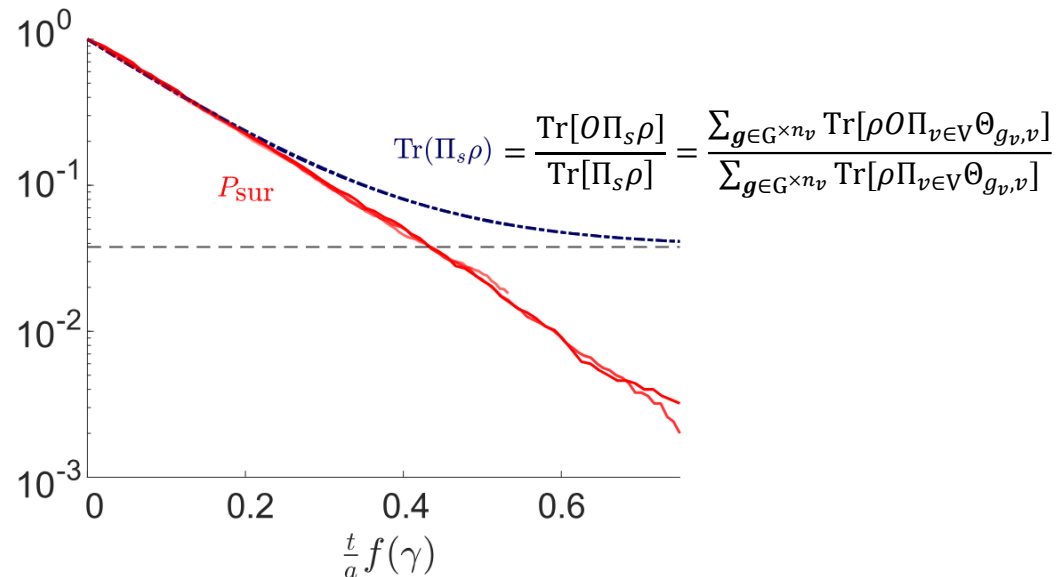
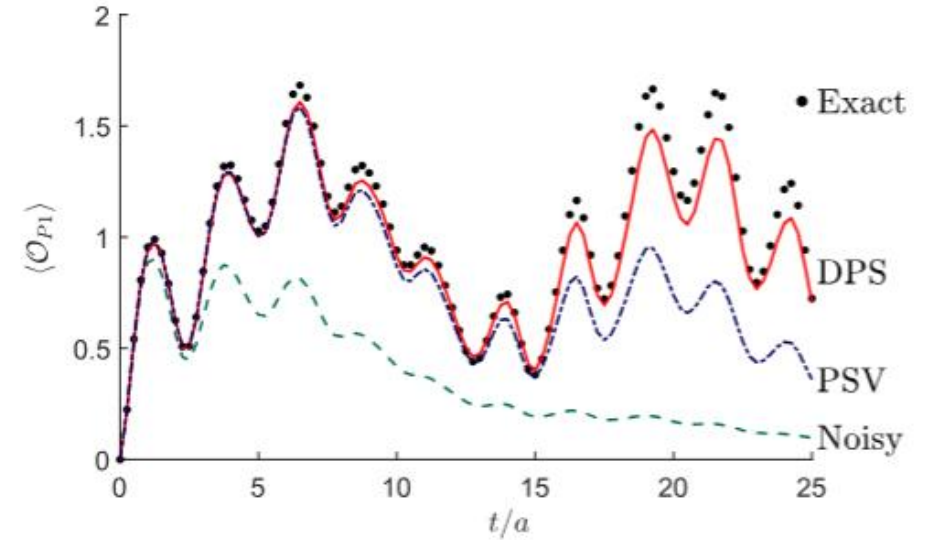
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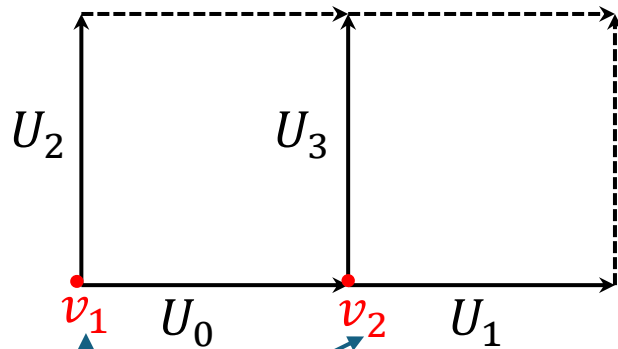


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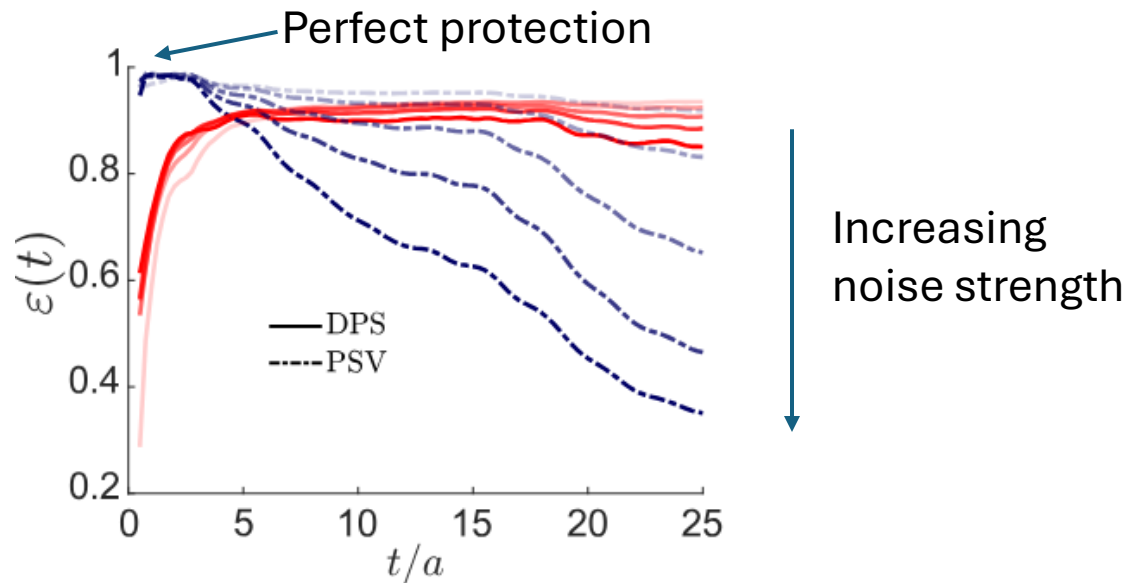
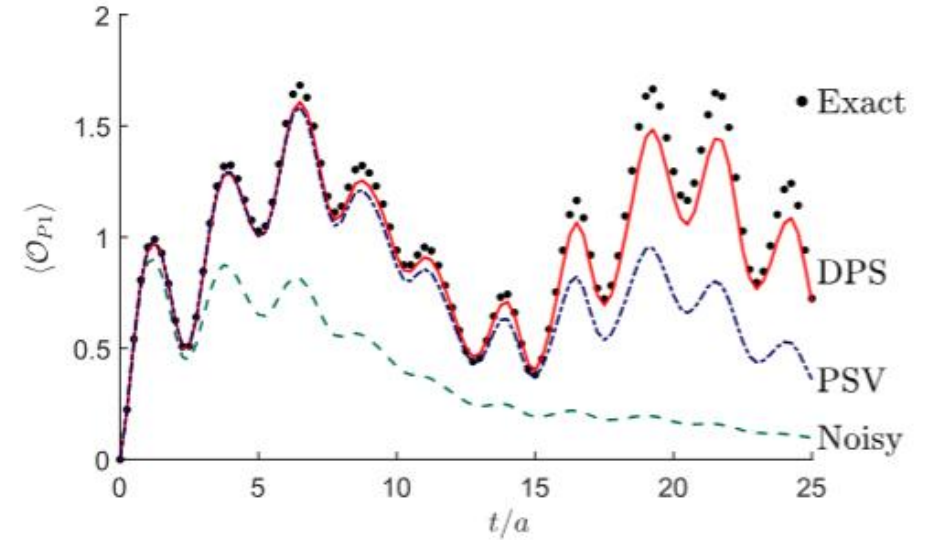
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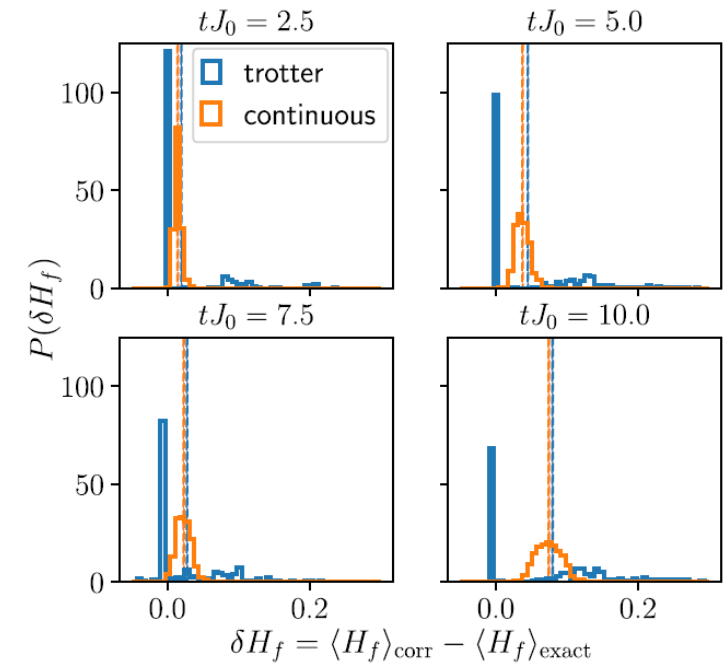
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# Conclusions & outlook

## Take home messages

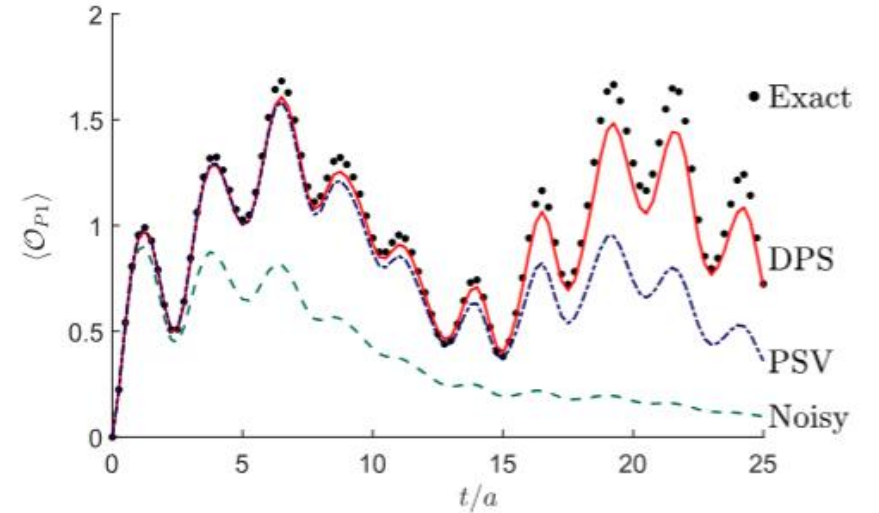
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# Conclusions & outlook

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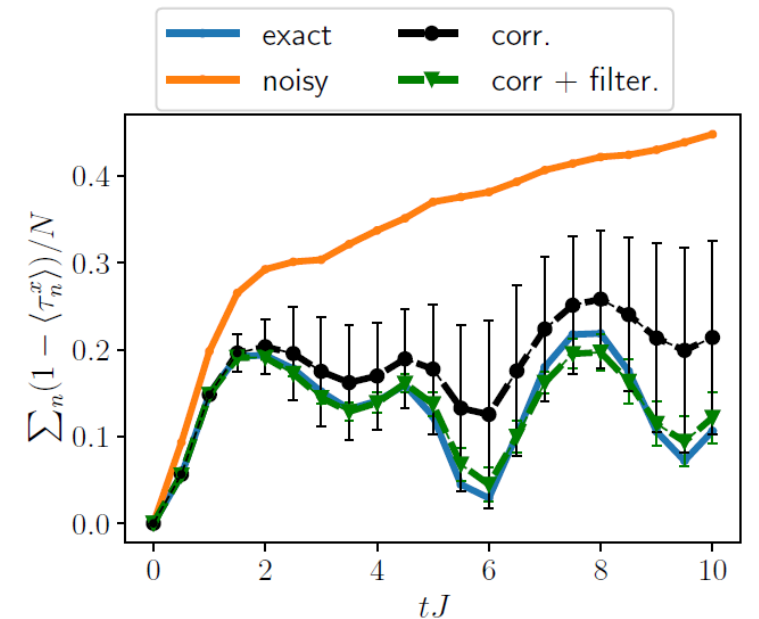
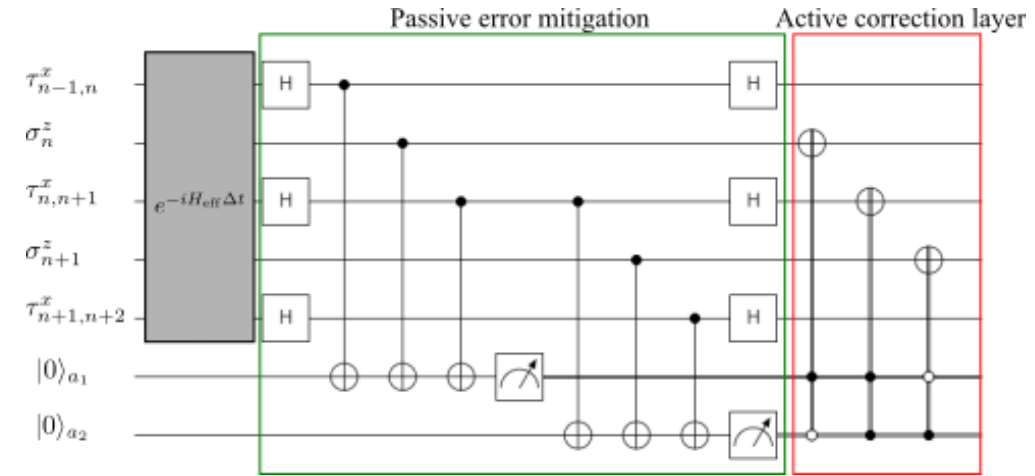
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- $\mathbb{Z}_2$  LGT  $\Rightarrow$  DPS+error correction



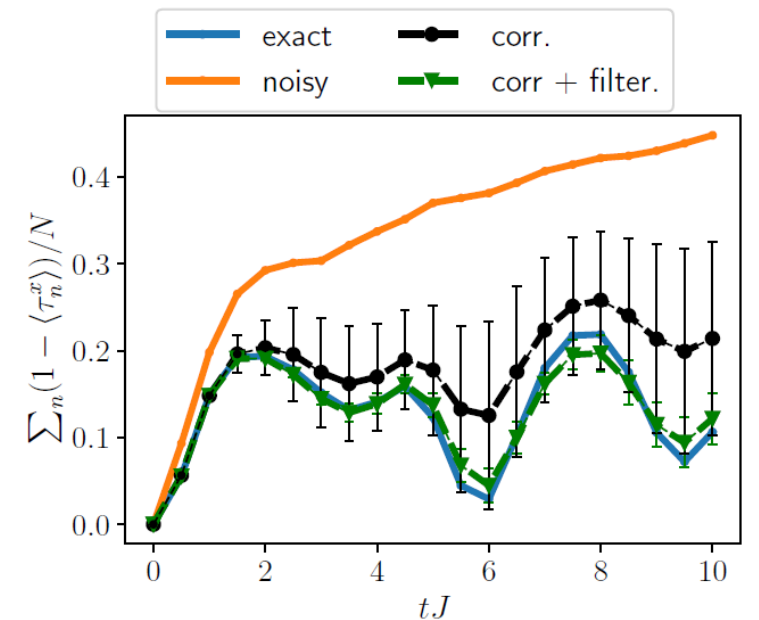
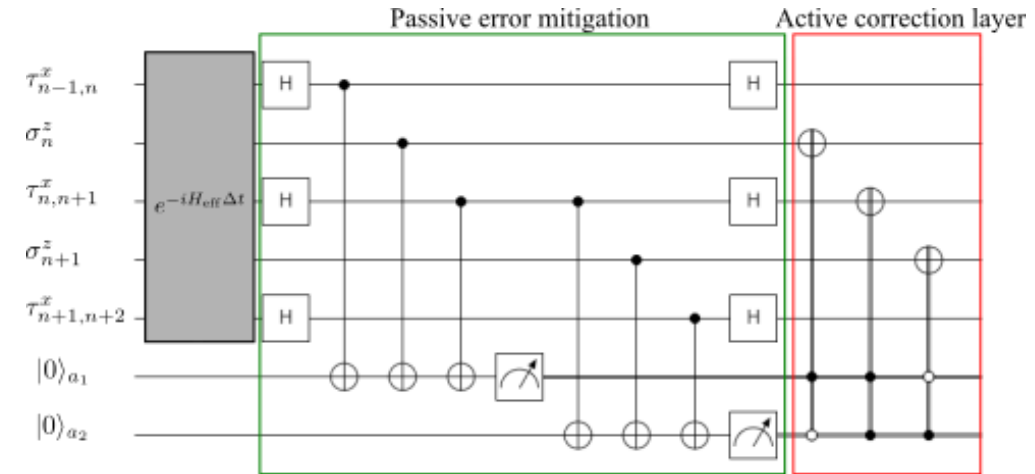
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## What's next?

- Measurement-induced chaos / chaos-integrability transition in the trajectory statistics
- Adapt to specific LGT model and hardware
- Generalize of PSV in qudits + workable approximations

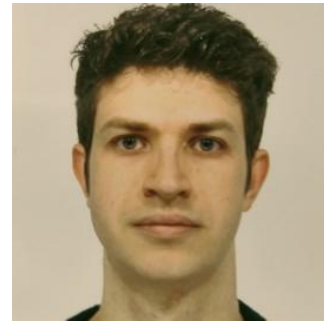


# Acknowledgements

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Edoardo Ballini



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Julius Mildenberger



**MW**, E. Ballini, A Biella, P Hauke Phys. Rev. B 111 (2025)

E Ballini, J Mildenberger, **MW**, P Hauke arxiv: 2412.07844



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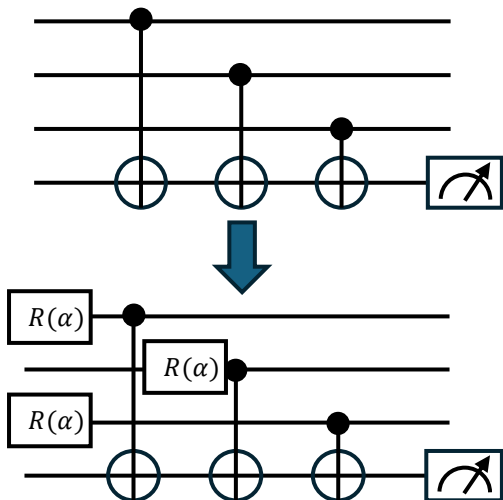
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di Fisica Nucleare  
**TIFPA**  
Trento  
Institute for  
Fundamental  
Physics and  
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# Noisy measurements

Imperfect coupling between computational qubits and ancilla

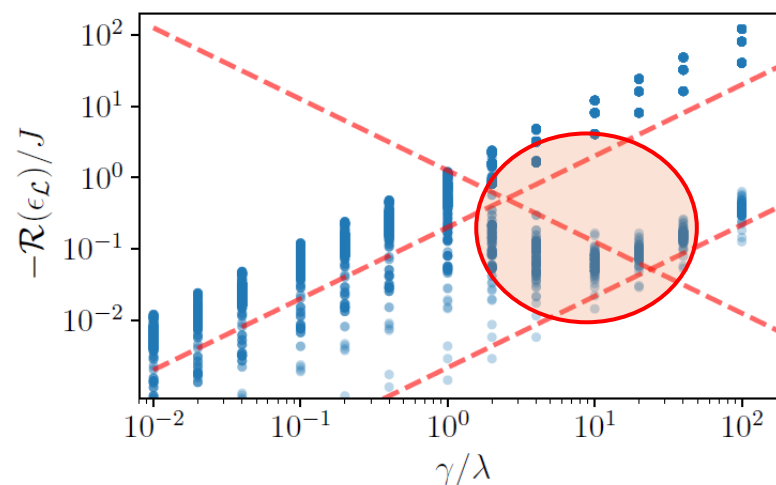


$$R(\alpha) = e^{-i\alpha_z Z} e^{-i\alpha_x X}, \quad \alpha_{x/z} \in \left[-\frac{\eta}{2}, \frac{\eta}{2}\right]$$

$$\text{Measurement fidelity } \mathcal{F}(\eta) \approx 1 - 3\eta^2$$

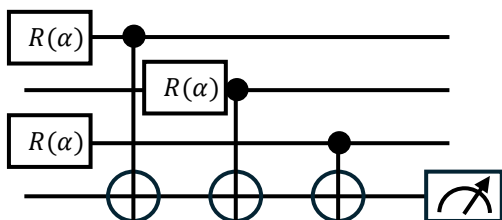
- ? Does the Zeno gauge-protection survive?
- ? Is there any advantage in DPS?

$$\mathcal{L}[\rho] \rightarrow \mathcal{L}_{\mathcal{F}=1}[\rho] + \gamma\eta^2 \sum_l \sum_{C=X,Z} C_l \rho C_l^\dagger - \frac{1}{2} \{C_l^\dagger C_l, \rho\}$$

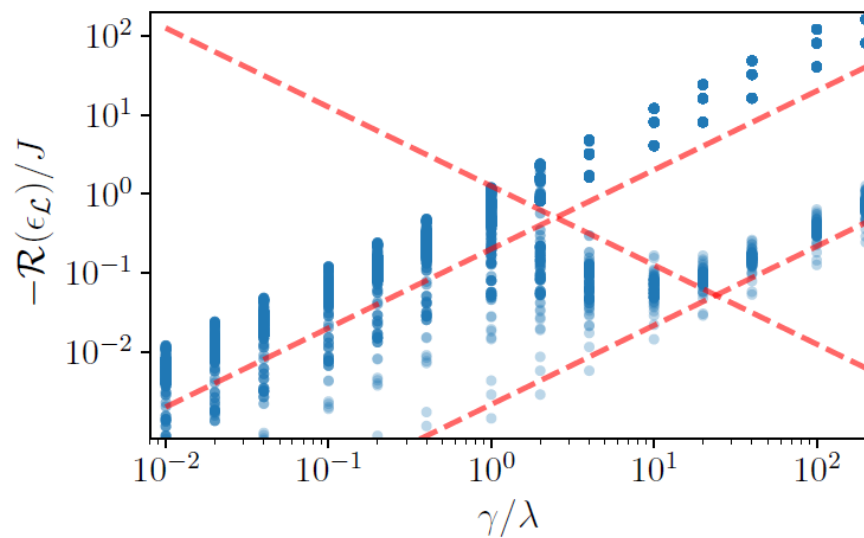


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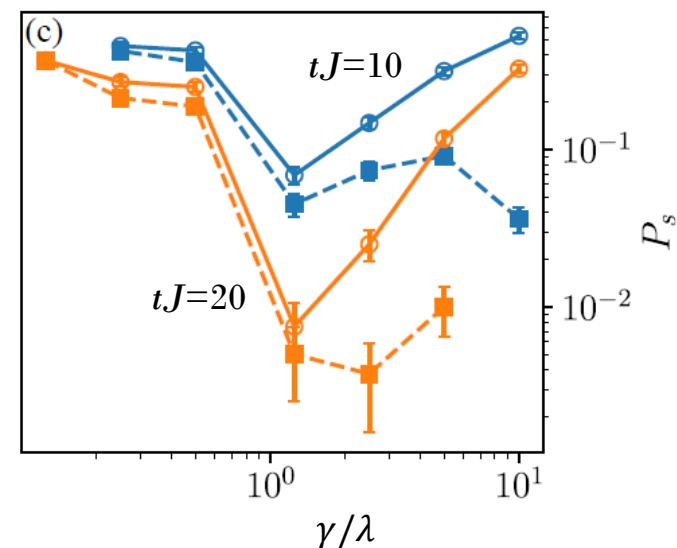
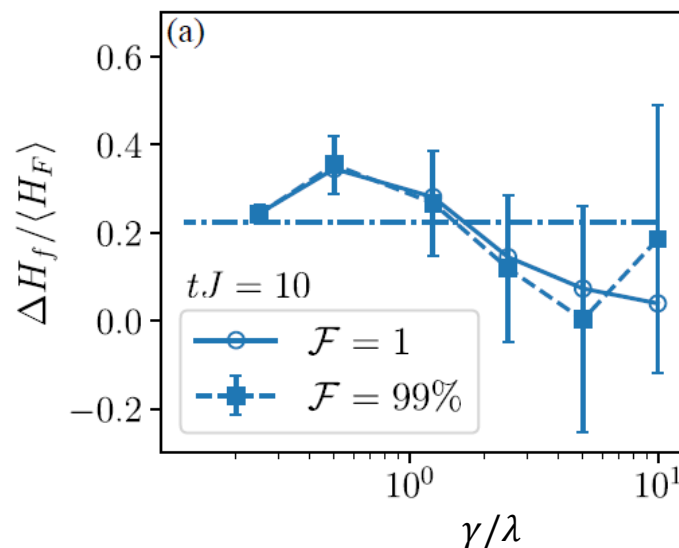


Liouvillian spectrum



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Trotterized dynamics with measurement frequency  $1/\tau$ ,  $\tau = m\Delta t$

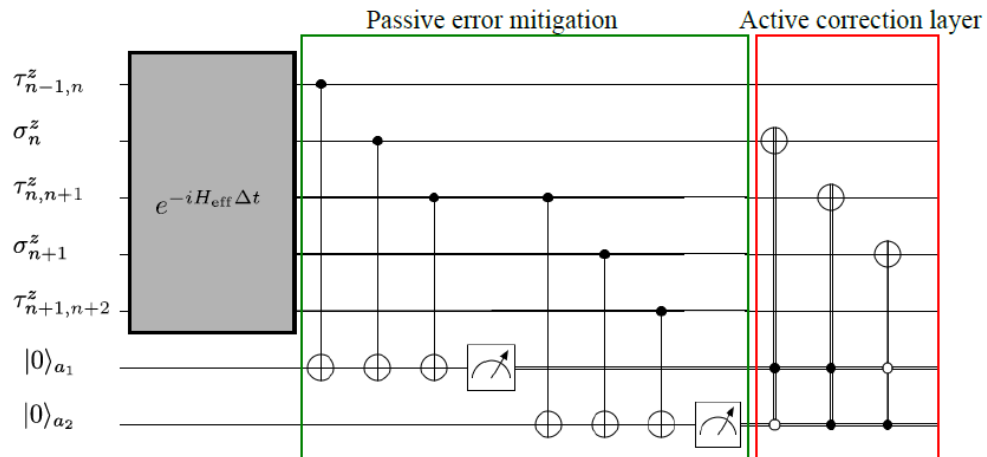
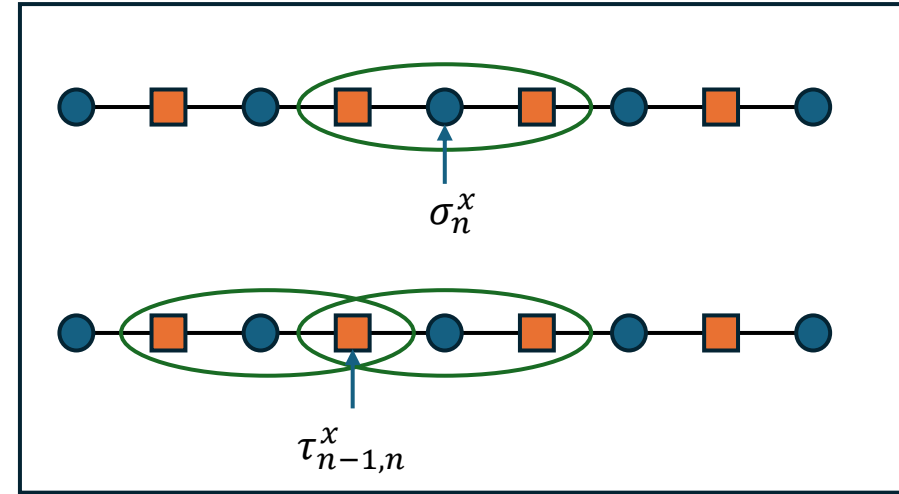


# Towards error correction

Add bit-flip ( $X$ ) error channel

$$\rho(t + \Delta t) = (1 - Lp_{\text{err}})U(\Delta t)\rho(t)U^\dagger(\Delta t) + p_{\text{err}} \sum_n U(\Delta t)X_n\rho(t)X_nU^\dagger(\Delta t)$$

$p_{\text{err}}$  = bit-flip error probability per Trotter step

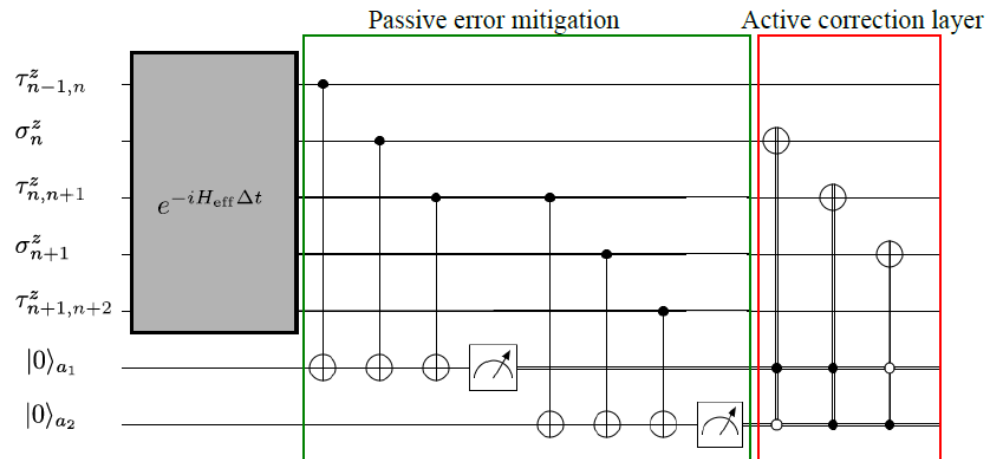
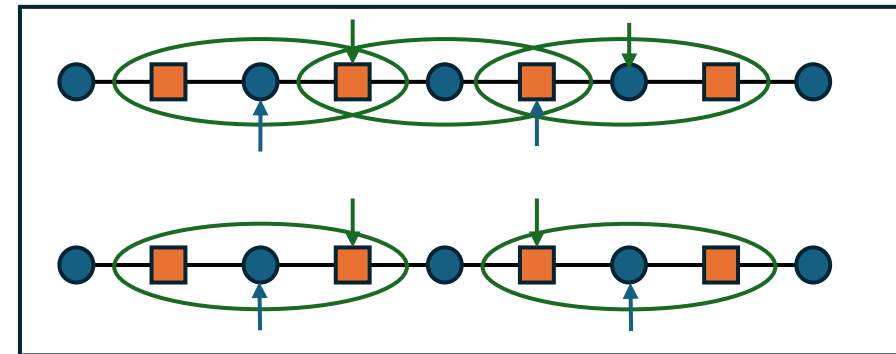
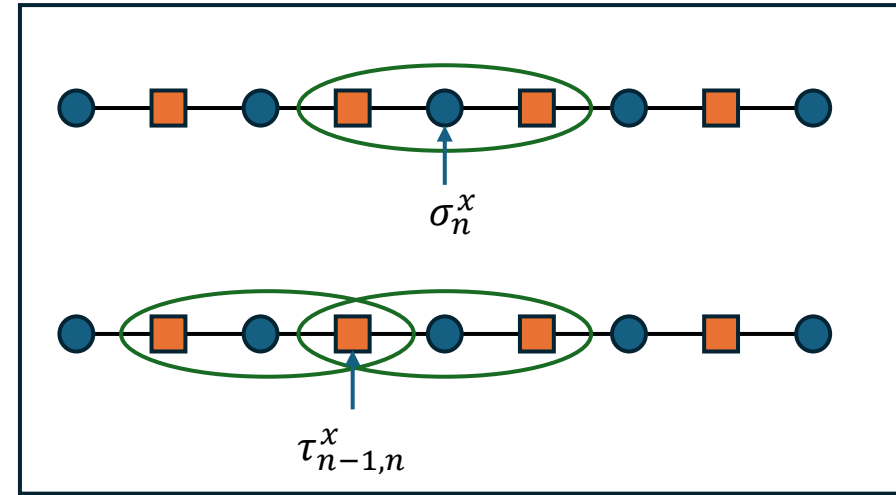


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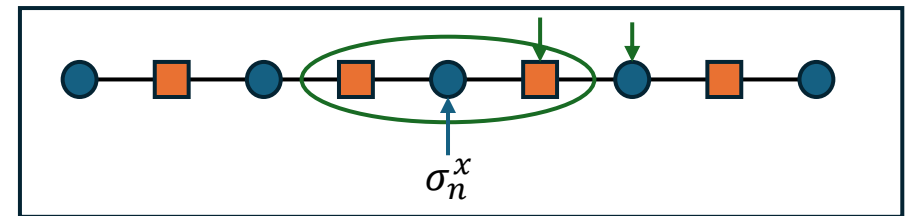
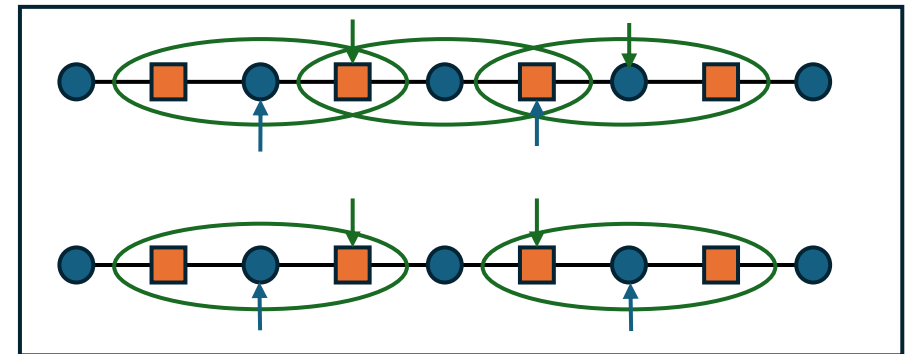
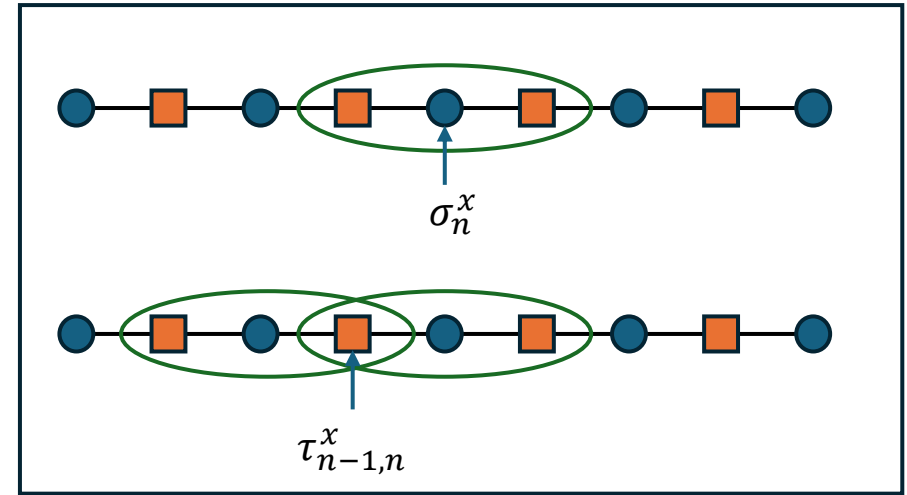
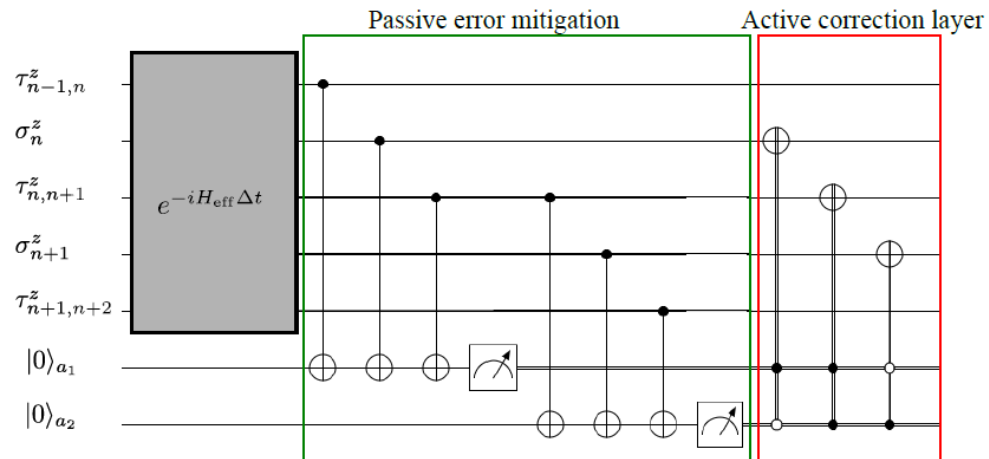


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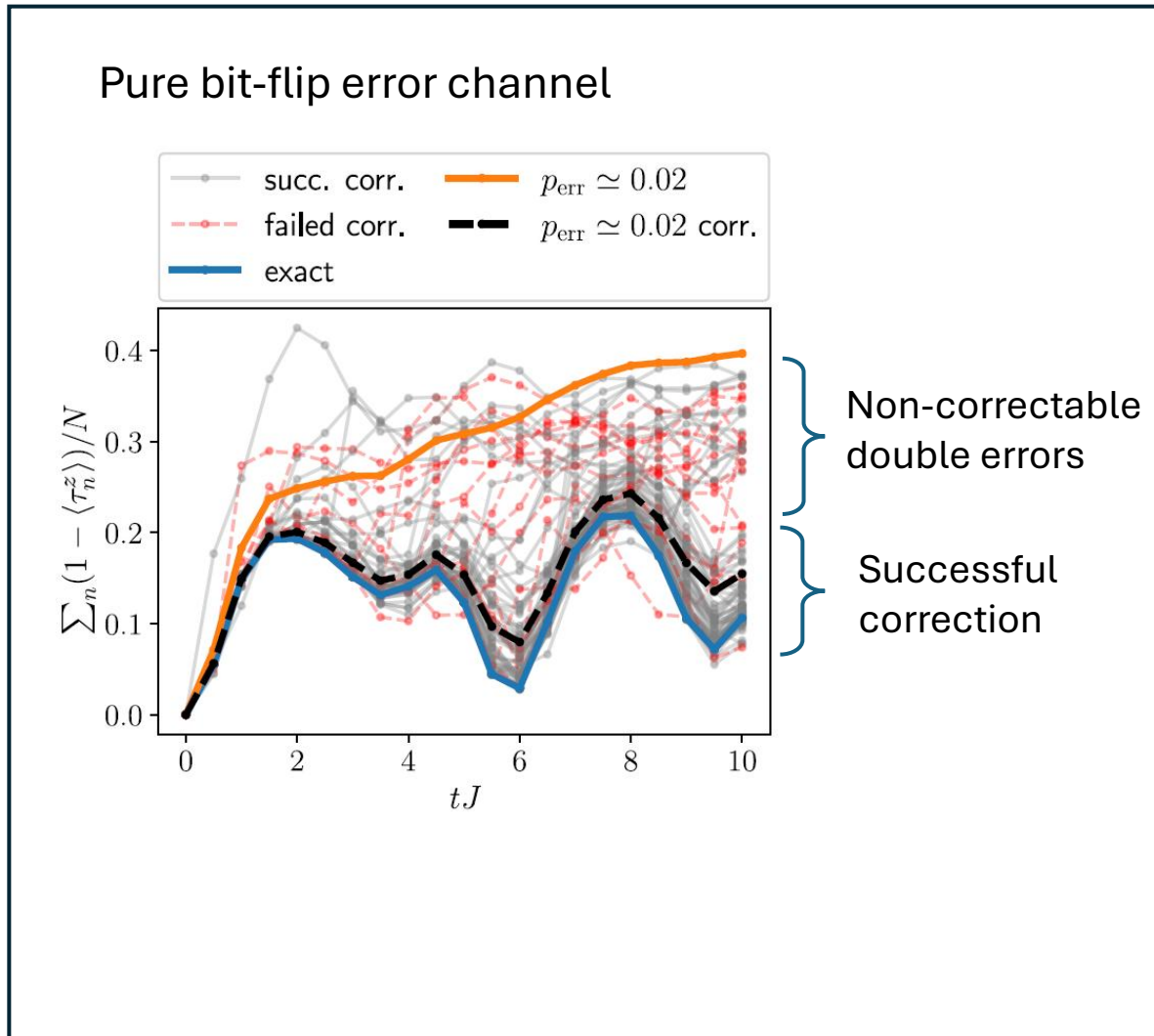
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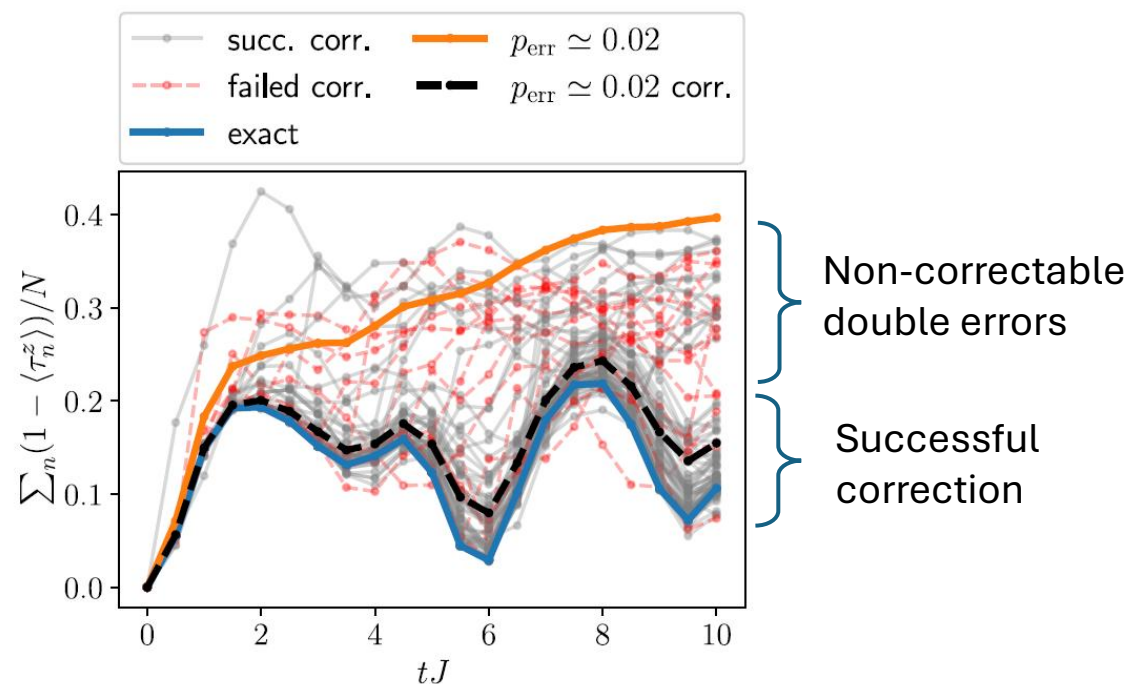


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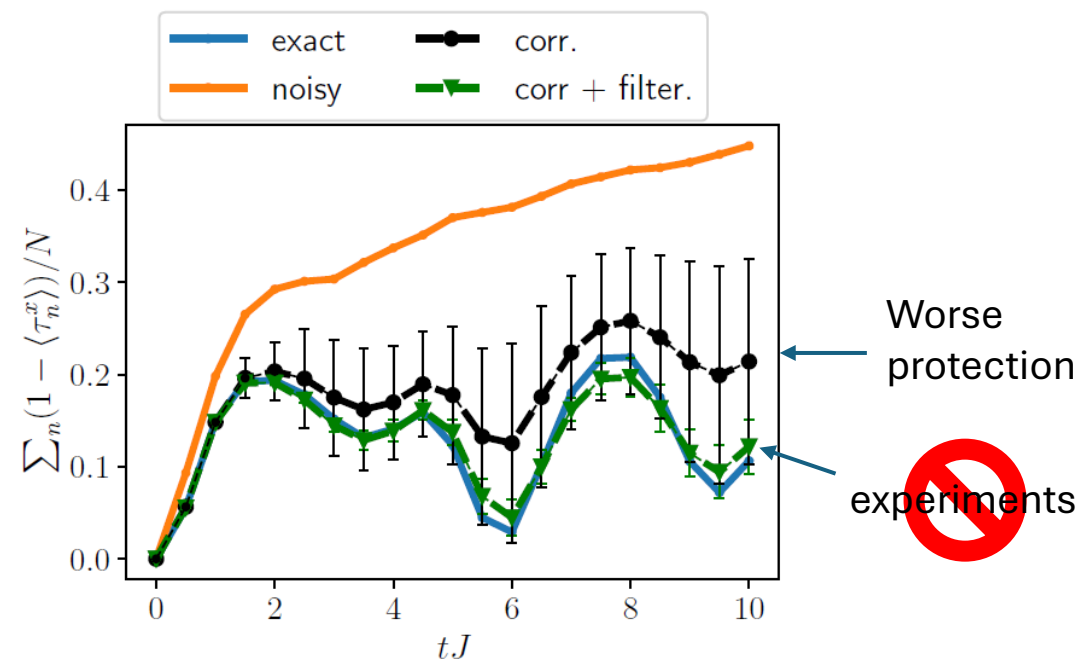


# Towards error correction

Pure bit-flip error channel



Bit flip + coherent error



Quantum Zeno effect suppresses coherent gauge drift.  
 Conditional operations correct incoherent bit flips.

# Measurement-induced gauge protection in digital quantum simulations

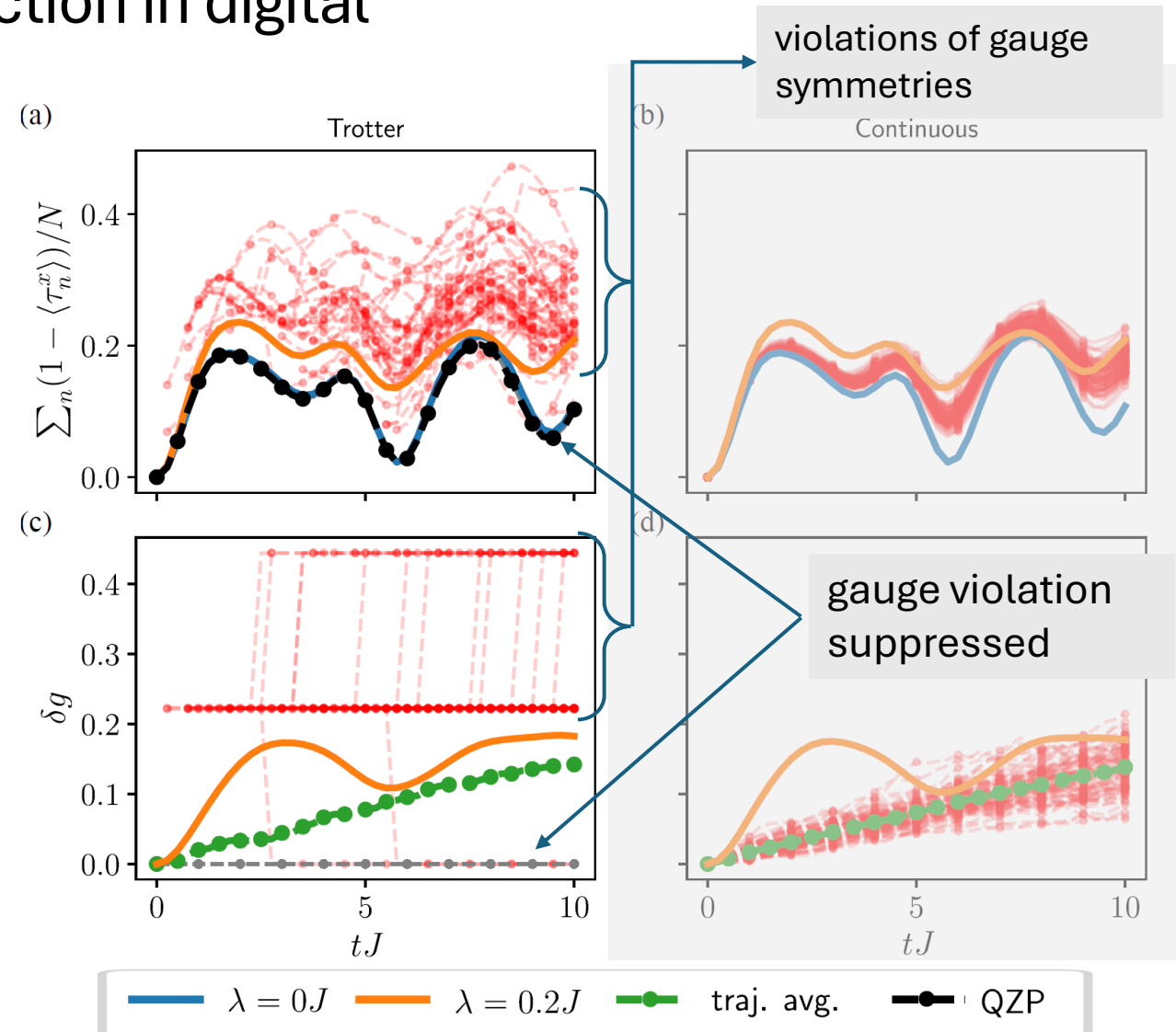
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Possible implementations:

- Engineered dissipation [1]
- Random gage transformations [2]
- Continuous measurements
- Continuous limit for DPS

**Quantum Zeno transition between protected (QZP) and chaotic phases**

**Importance of unraveling: same ensemble average, different stochastic trajectories**



[1] Stannigel et al. PRL **112** (2014)

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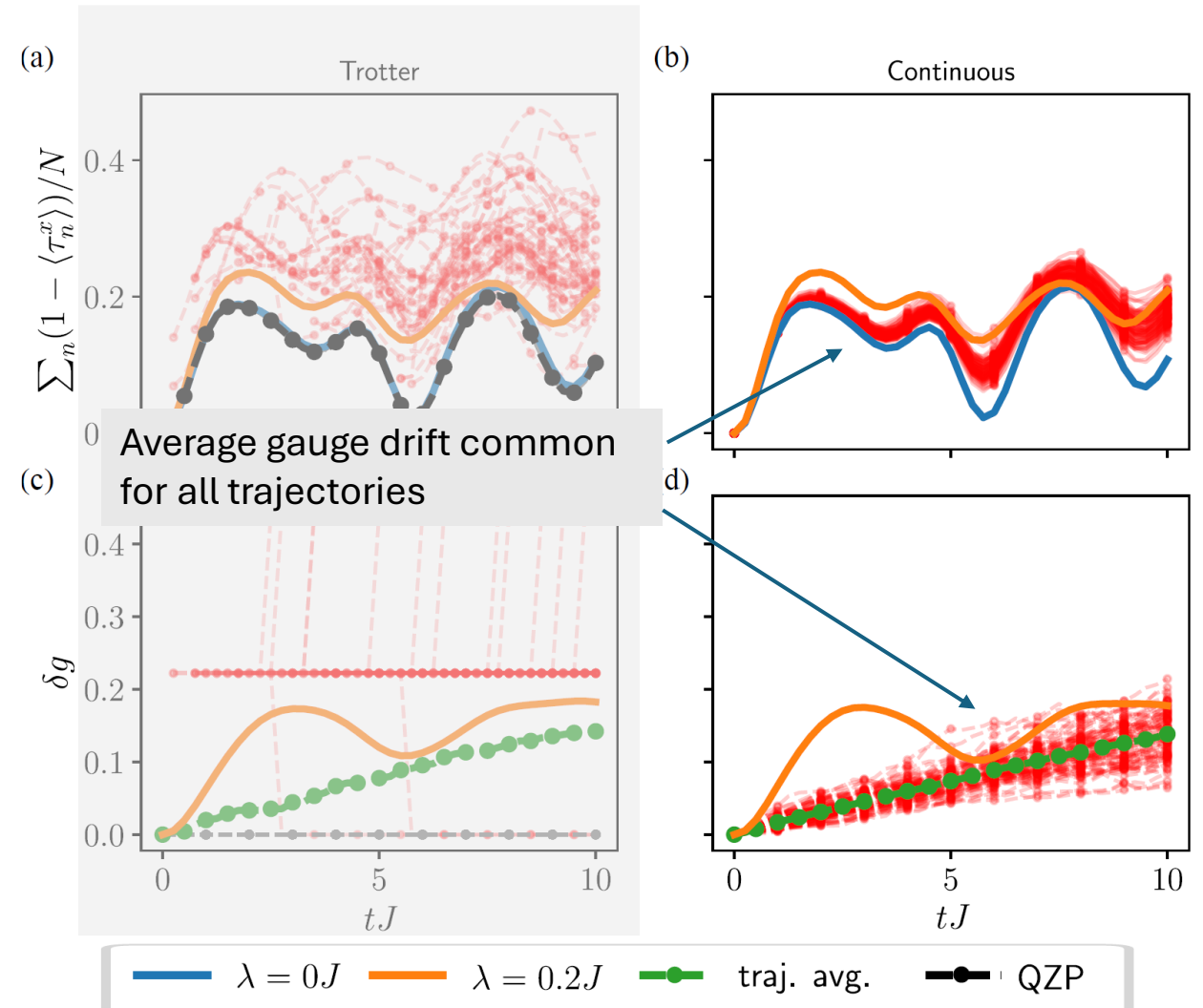
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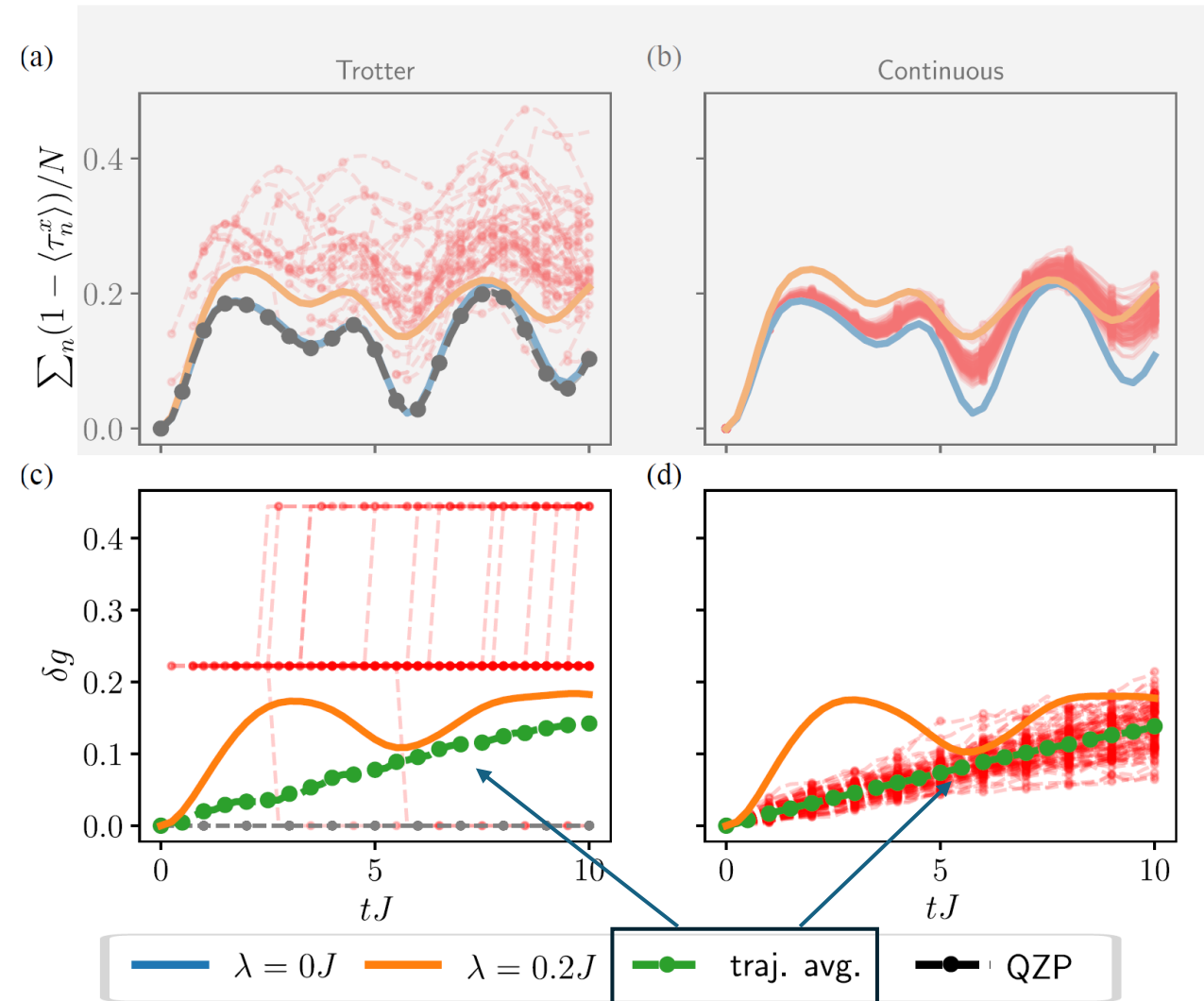
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