Long-range interacting quantum systems: a bird's eye view

CERN, Geneva, 27/05/2025

Nicolò Defenu

The tin can phone

Your voice makes the bottom of the can vibrate.

↓

The vibrations of the can's bottom are transferred to the string

↓

The sound propagates along the string as vibrations

↓

The vibrations reach the second can

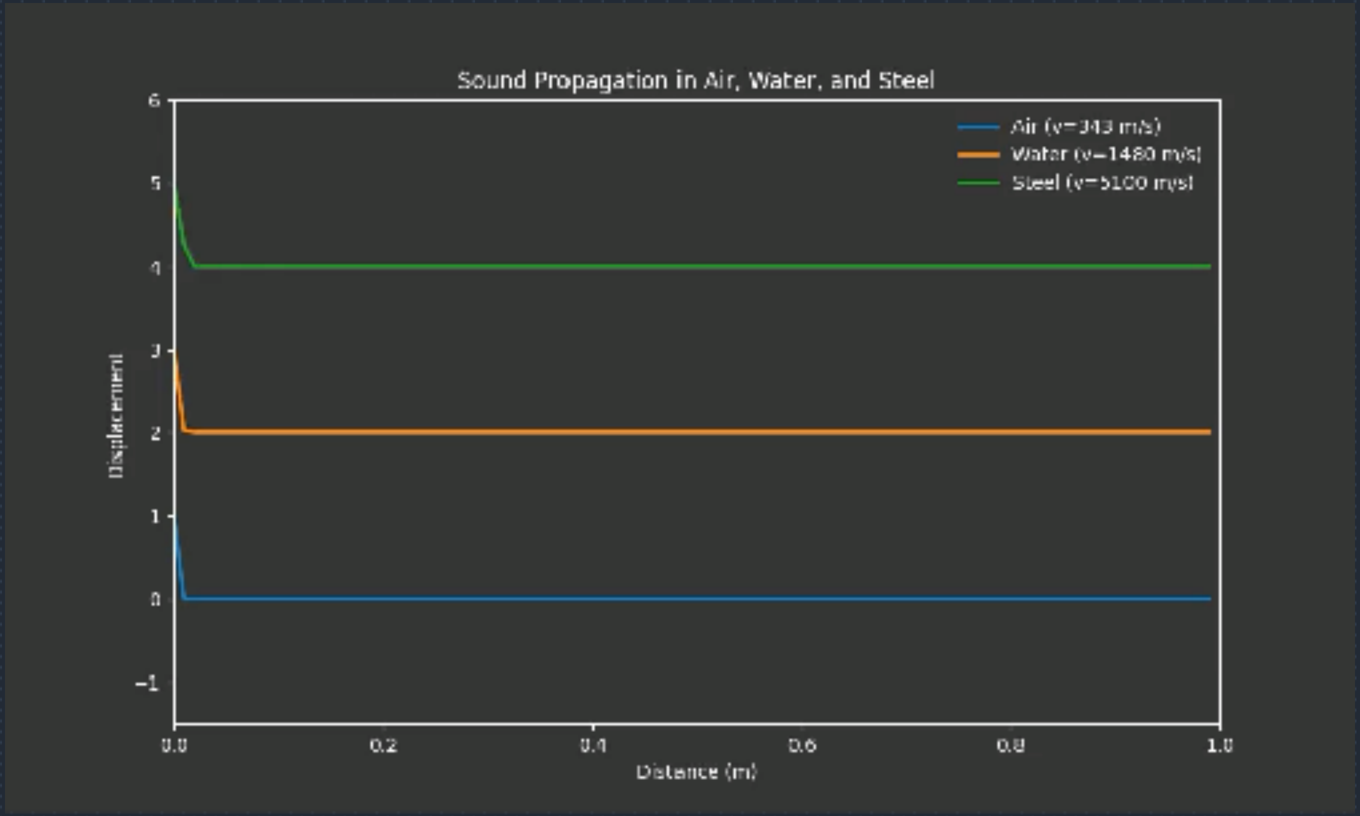
↓

The vibrating bottom of the second can creates sound waves

A fast tin-can phone

How to optimise this "technology"?

Increase the sound velocity? → Change the material



The fastest tin-can phone

Sound is a universal phenomenon $\omega(k) \approx v\,k$ with $\partial_k \omega(k) = v = \sqrt{\frac{K}{\rho}}$

K is the bulk modulus and ρ is the density of the material

Finite sound velocity is a consequence of locality of the interactions

$$H_{\text{osc}} = \sum_i \frac{p_i^2}{2m} + \sum_{j>i} K_{ij} x_i x_j$$

Local

$$\lim_{|i-j|\to\infty} K_{ij} < \exp(-\lambda\,|\,i-j\,|)$$

Non-local

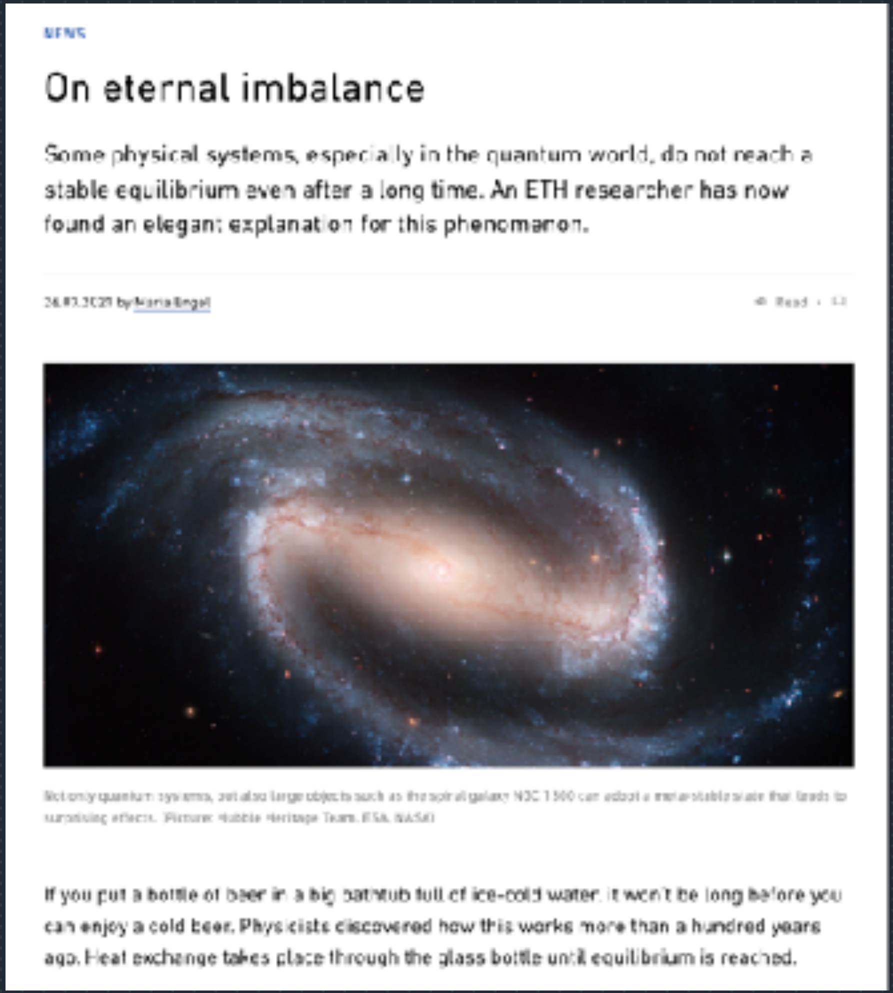
$$\lim_{|i-j|\to\infty} K_{ij} \sim \frac{1}{|\,i-j\,|^{\alpha}}$$

Local or long-range

Long-range potentials are quite common in master programs

$$V_G(r) = \frac{GM}{r}$$

- Often responsible for fundamental phenomena.
- Do not come often around in condensed matter.
- May be engineered in quantum simulators.



NEWS

On eternal imbalance

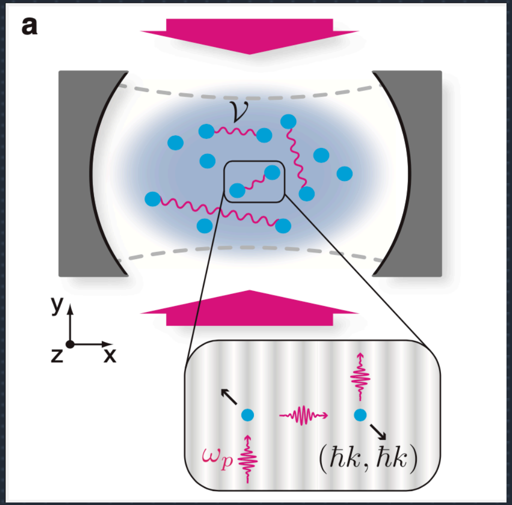
Some physical systems, especially in the quantum world, do not reach a stable equilibrium even after a long time. An ETH researcher has now found an elegant explanation for this phenomenon.

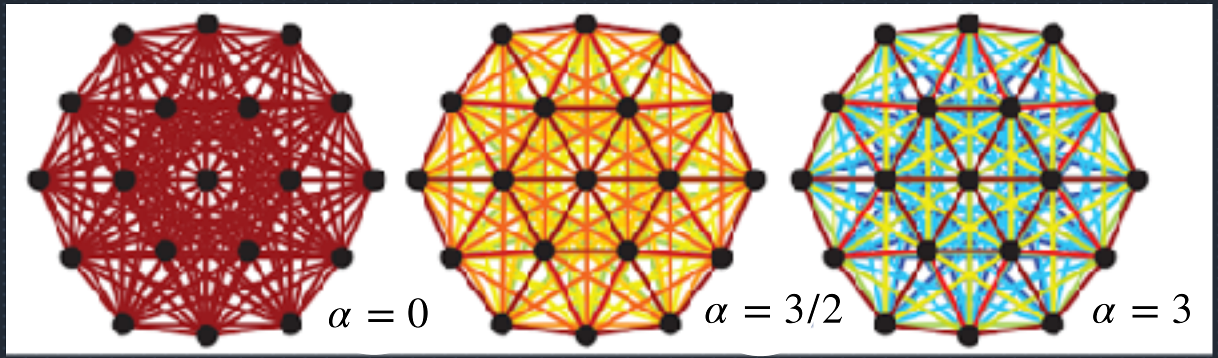
Not only quantum systems, but also large objects such as the spiral galaxy NGC 1300 can adopt a meta-stable state that leads to surprising effects. (Picture: Hubble Heritage Team, ESA, NASA)

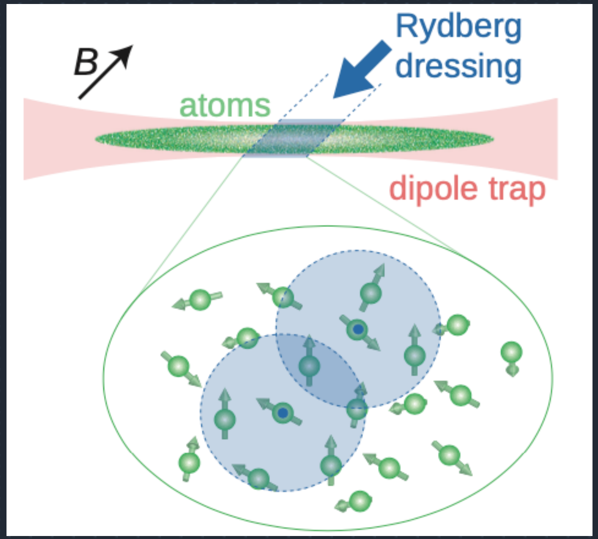
If you put a bottle of beer in a big bathtub full of ice-cold water, it won't be long before you can enjoy a cold beer. Physicists discovered how this works more than a hundred years ago. Heat exchange takes place through the glass bottle until equilibrium is reached.

Long-range quantum simulators

$$V(r) \sim r^{-\alpha}$$







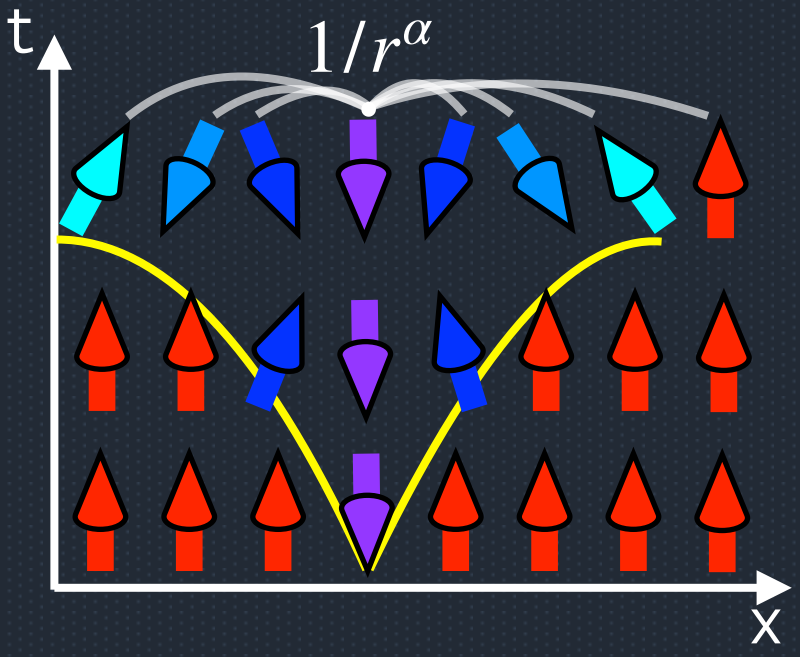
α

0 $\quad d \quad$ $d+2$

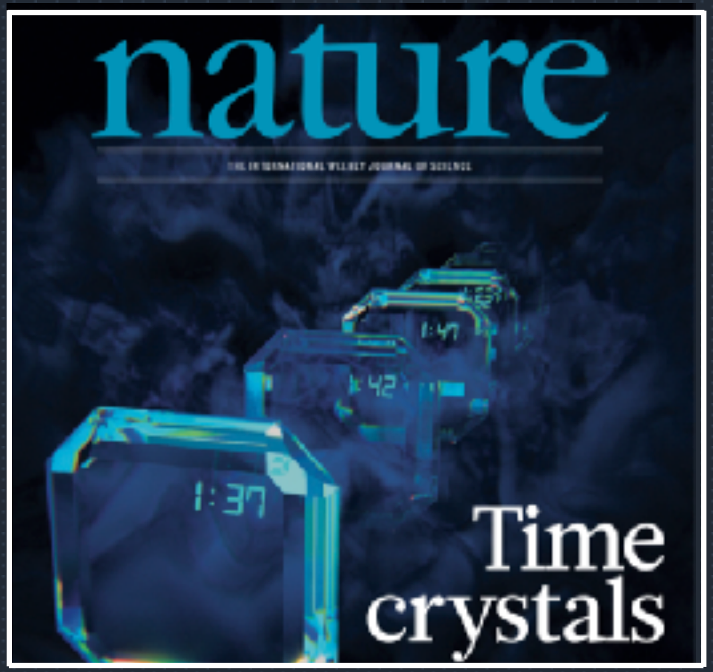
"Long-range interacting quantum systems", ND et al. *Rev. Mod. Phys.* **95**, 035002 (2023).

Novel Dynamical Phenomena

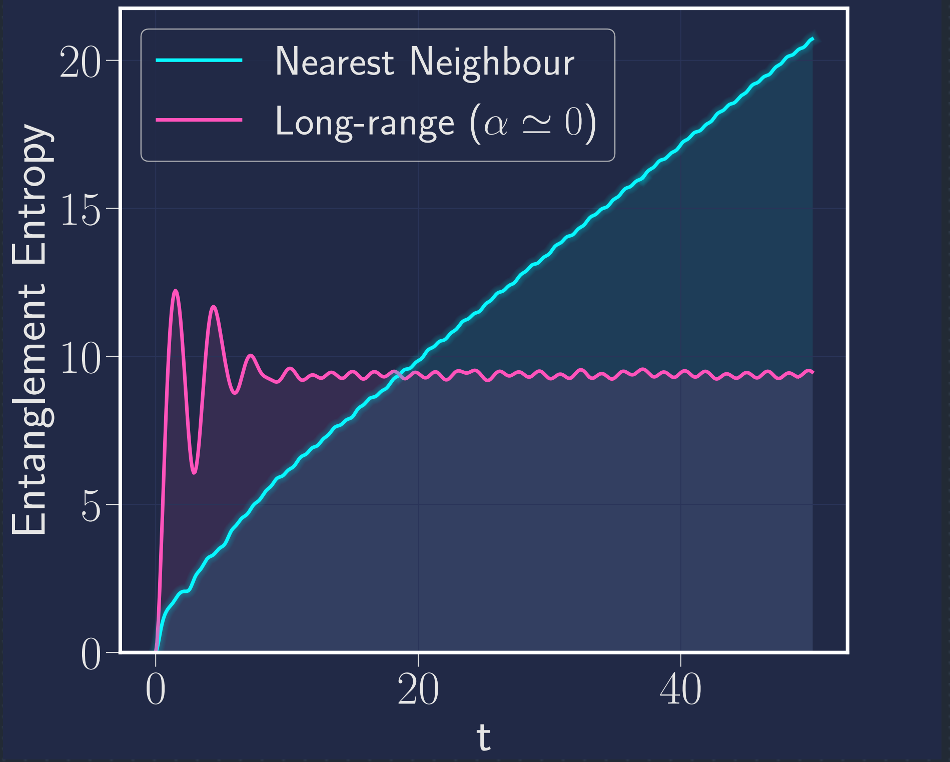
Fast Signal Spreading



Time Crystals



Non-ergodic states



P. Richerme, et al., Nature 511, 198 (2014).

J. Zhang, et al., Nature 543, 164 (2017).

A new world, full of treasures

Quantum Finite Range

Entanglement Scaling in Quantum Circuits

Quantum Flat $(\alpha = 0)$

Terra Incognita $0 < \alpha < \infty$

Open Dynamics of Quasi-Stationary States

Tuneable defect formation

Non-ergodicity

Classical Finite Range

Classical Long-Range

Long-range quantum Ising model

$$H = -\sum_{l<j} J_{ij}\left(\sigma_l^x\sigma_j^x + \sigma_l^y\sigma_j^y + \sigma_l^z\sigma_j^z\right)$$

Long-range interactions: $J_{ij} = \frac{1}{N_\alpha}\frac{1}{|i-j|^\alpha}$ $\quad N_\alpha^{-1} \approx \begin{cases} (1-\alpha)2^{(1-\alpha)}N^{\alpha-1} & \text{if } \alpha < 1 \\ 1/\log(N) & \text{if } \alpha = 1 \\ 1/\zeta(\alpha) & \text{if } \alpha > 1. \end{cases}$

$T \ll J \Rightarrow |0\rangle \equiv \Pi_i|\uparrow\rangle_i$ or $\Pi_i|\downarrow\rangle_i$ and $\lim_{|i-j|\to\infty}\left\langle \sigma_i^z\sigma_j^z\right\rangle \propto N_0^2$

$T \gg J \Rightarrow |0\rangle \equiv \Pi_i|\rightarrow\rangle_i$ with $|\rightarrow\rangle \equiv |\uparrow\rangle + |\downarrow\rangle$ and $\lim_{|i-j|\to\infty}\left\langle \sigma_i^z\sigma_j^z\right\rangle \propto e^{-\frac{|i-j|}{\xi}}$

What is the condition for $T_c > 0$

Spin-wave approximation

$$H = -\sum_{l<j} J_{ij}\left(\sigma_l^x\sigma_j^x + \sigma_l^y\sigma_j^y + \sigma_l^z\sigma_j^z\right)$$

Holstein-Primakoff transformation

$$\sigma_\ell^x \simeq \frac{a_\ell + a_\ell^\dagger}{2} \qquad \sigma_\ell^y \simeq \frac{a_\ell - a_\ell^\dagger}{2i} \qquad \sigma_\ell^z = \frac{1}{2} - a_\ell^\dagger a_\ell$$

$$H = -\sum_{l<j} J_{ij}(a_i^\dagger a_j + h.c.) + h\sum_j a_j^\dagger a_j,$$

1) Auerbach A 1998 Interacting Electrons and Quantum Magnetism

Long-range interactions

$$J_k = \sum_{\ell=1}^{N} \frac{\cos(k\ell)}{\ell^{\alpha}} \approx \mathrm{Li}_\alpha\left(e^{ik}\right) + \mathrm{Li}_\alpha\left(e^{-ik}\right)$$

$$\lim_{k\to 0} J_k = h_c + c\,k^{\sigma} + O(k^2), \qquad \sigma = \alpha - 1$$

$$\rho(\omega) = \int \frac{dk}{2\pi}\delta\left(\omega - J_k\right) \longrightarrow \lim_{\omega\to 0} \rho(\omega) \propto \omega^{\frac{d_s}{2}-1}$$

The Spectral Dimension

The spectral dimension determines the Mermin-Wagner Theorem

Spontaneous symmetry breaking of continuous symmetries only depends on

$$d_s = \frac{2d}{\sigma}$$

Finite temperature SSB for $d_s > 2$

What about discrete symmetries?

1) R. Burioni, D. Cassi, *Phys. Rev. Lett.* ***76***, *1091 (1996).*
2) R. Burioni, D. Cassi, A. Vezzani, *Phys. Rev.* **E *60***, *1500 (1999).*

The Phase Diagram

$$H = -\sum_{l<j} J_{ij}\left(\sigma_l^x\sigma_j^x + \sigma_l^y\sigma_j^y + \sigma_l^z\sigma_j^z\right)$$

p^{σ}

$$\eta_{\mathrm{lr}} = 2 - \sigma$$

$p^{2-\eta}$

1) J. Sak, Phys. Rev. B 8, 281 (1973).

Particle production from quantum vacuum

$$H_I = \sum_{ij} \frac{J_{ij}}{2} \sigma_i^z \sigma_j^z - h(t) \sum_i \sigma_i^x$$

$$\lambda(t) = h(t) - h_c \approx \delta t$$

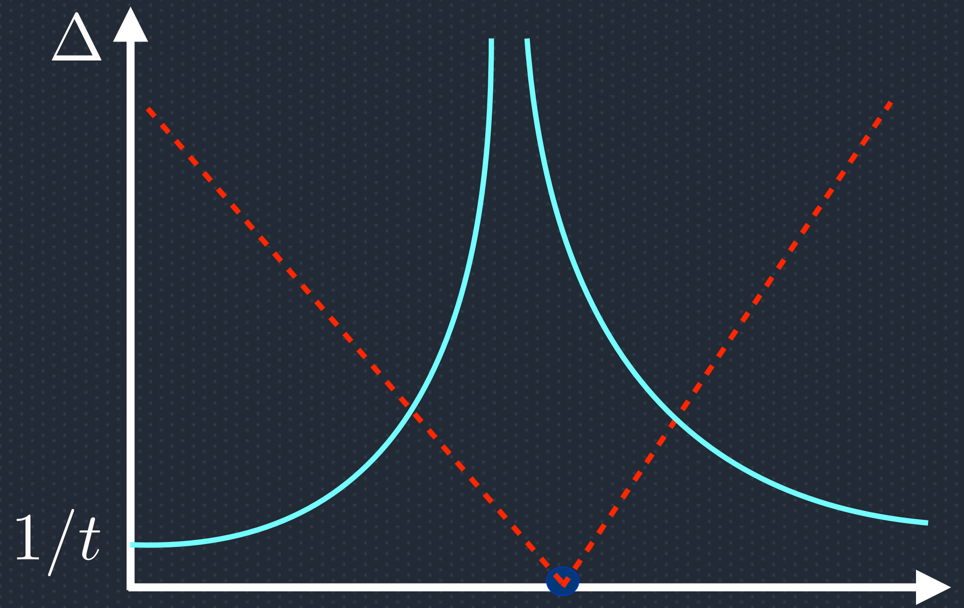
$$t \in [-1/\delta, t_f]$$

$\delta \to 0$

Kibble-Zurek

Equilibration time

$\tau_{eq} \propto \Delta^{-1}$



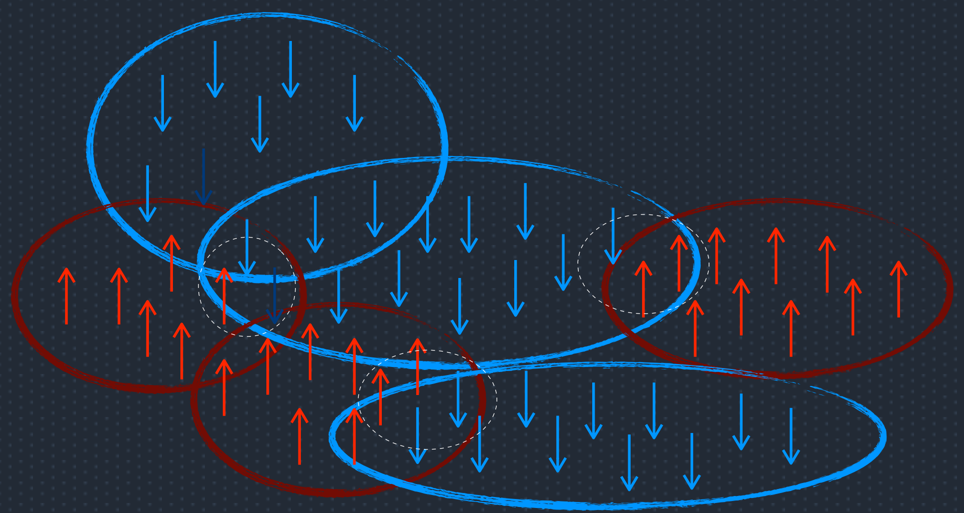
Time-Dependent control parameter

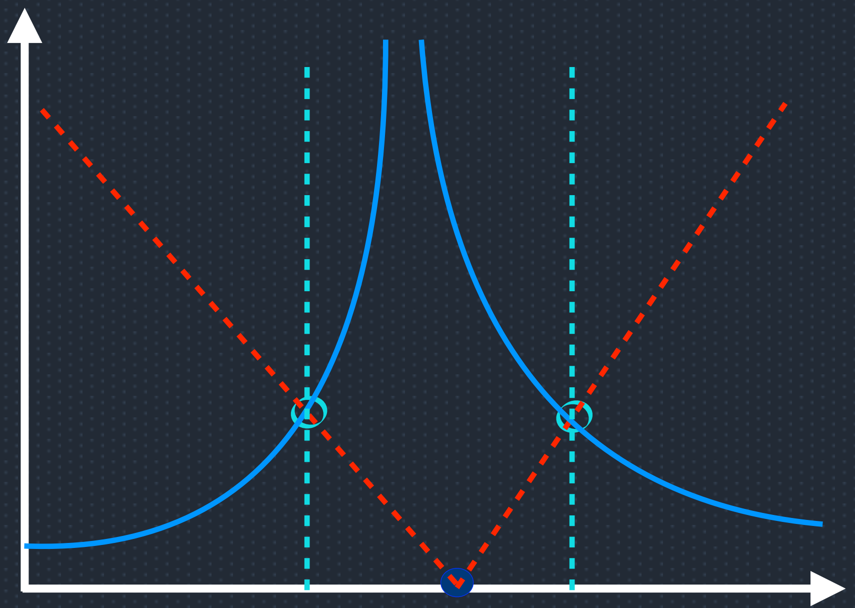
$$H_I = \sum_{ij} \frac{J_{ij}}{2} \sigma_i^z \sigma_j^z - h(t) \sum_i \sigma_i^x$$

Adiabatic condition: $$\frac{\partial \Delta(t)}{\partial t} \ll \Delta(t)^2$$

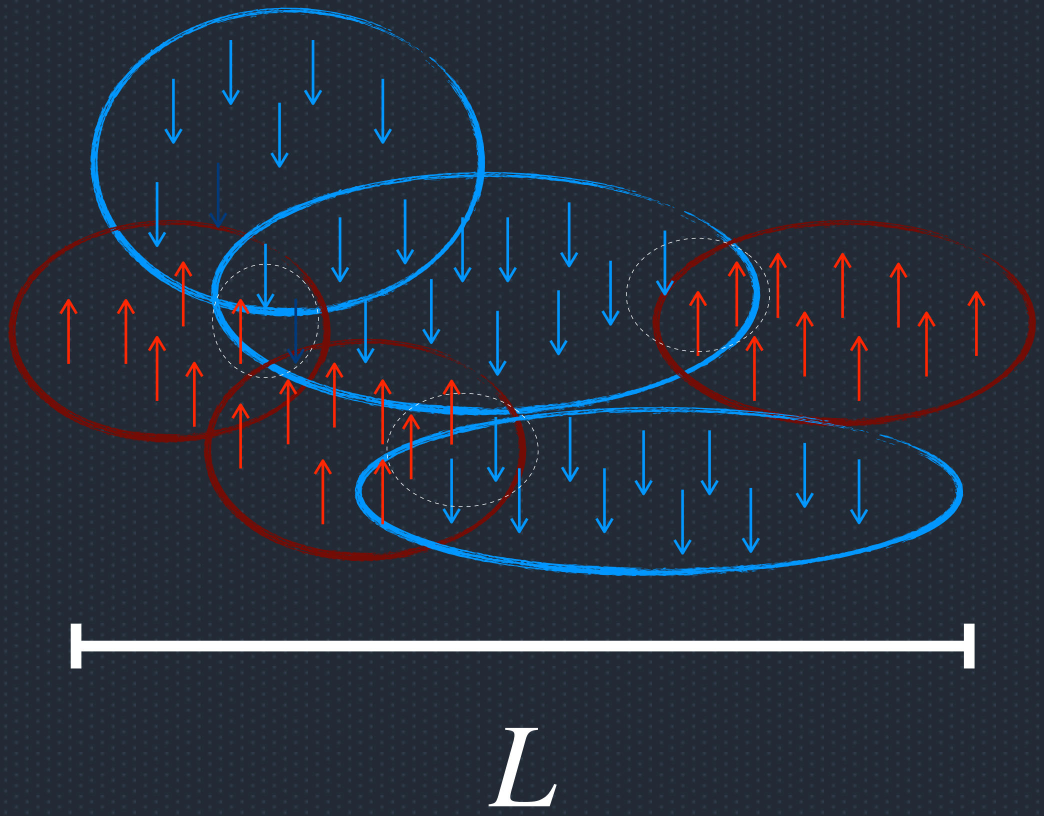
$$\xi_{\text{freeze}} \propto \delta^{-\frac{\nu}{1+z\nu}}$$

$$t_{\text{freeze}} \propto \delta^{-\frac{z\nu}{1+z\nu}}$$





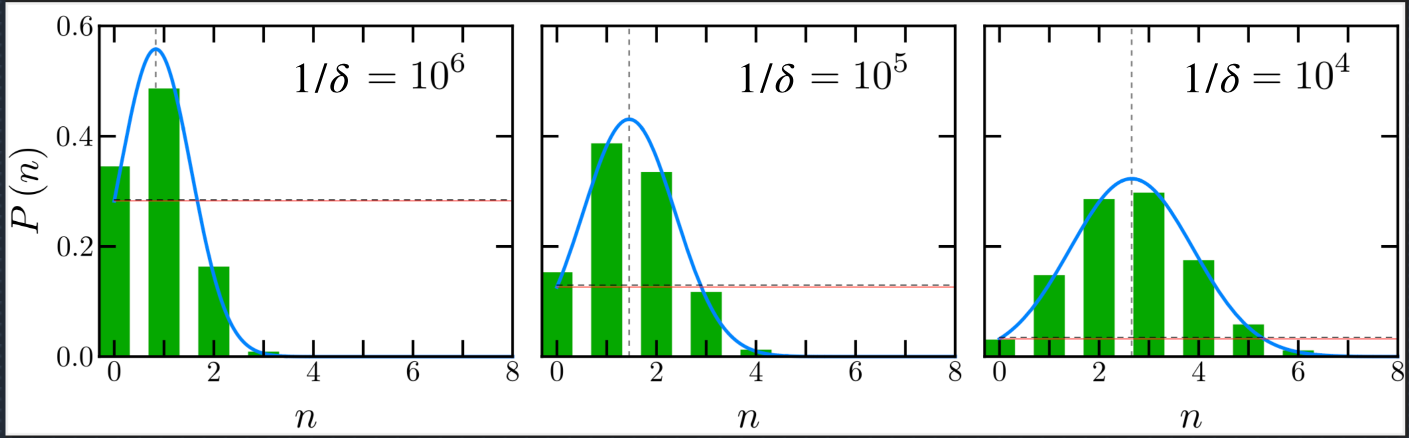
Full counting defect statistics



$$\mathcal{N} \propto \left(\frac{L}{\xi_{\text{freeze}}} \right)^d$$

Probability to generate n defects

$$P(n) \approx B(\mathcal{N}, n, p) = \binom{\mathcal{N}}{n} p^n (1-p)^n$$



Fernando J. Gómez-Ruiz, Jack J. Mayo, and Adolfo del Campo, Phys. Rev. Lett. 124, 240602 (2020)

Smitha Vishveshwara,Physics 13, 98 (2020)

Lipkin-Meshkov-Glick (LMG) Model

$$H = -\frac{J}{N}\sum_{ij}\sigma_i^z\sigma_j^z - h\sum_i \sigma_i^x = -\frac{J}{2N}S_x^2 - 2hS_z + \frac{J}{2}$$

$$H - E_{\mathrm{MF}} = \Delta(t)a^\dagger a$$

$$\lim_{t\to+\infty}(E(t) - E_{eq}) = Q \propto \delta^\theta$$

Controversy

Finite Size

$$\theta = 1/2z = 3/2$$

- Phys. Rev. B 78, 104426 (2008).
- Phys. Rev. Lett. 112, 030403 (2014).

Scaling Argument

$$\theta = z\nu/(1+z\nu) = 1/3$$

- Phys. Rev. Lett. 115, 180404 (2015).

Experiment: Klinder et al, Proc. Natl. Acad. Sci. **112**, 3290 (2015).

Theory: Defenu et al, Phys. Rev. Lett. **121**, 240403 (2018).

Scalar QED

Klein-Gordon Equation:

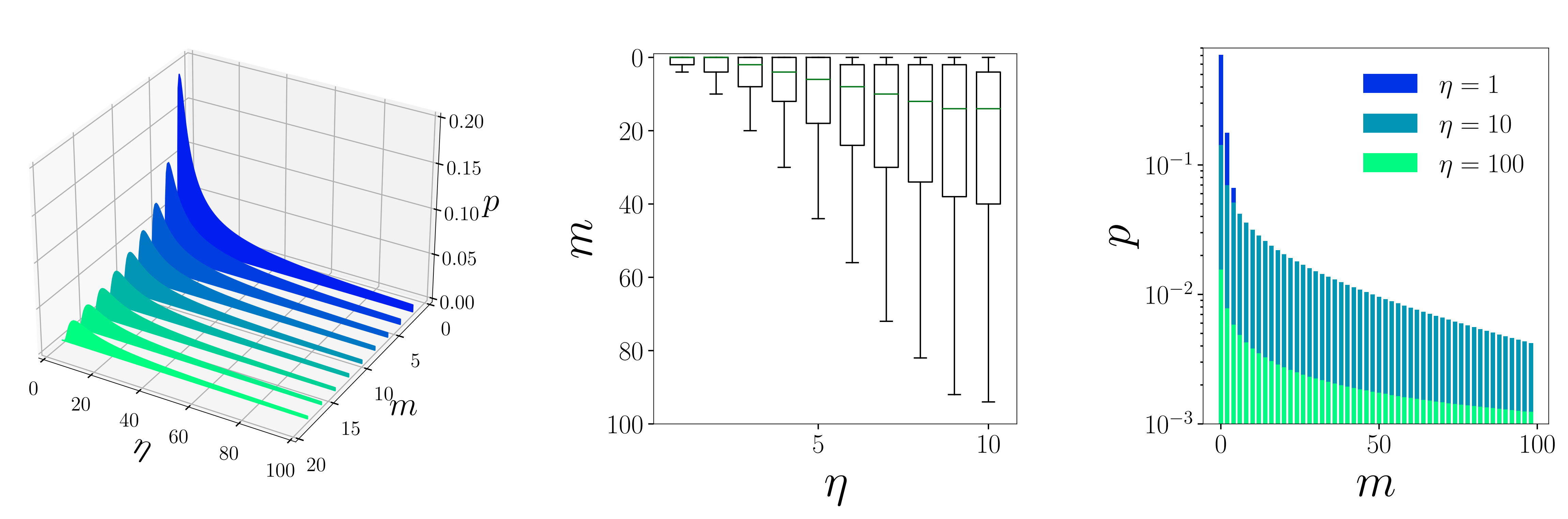
$$\left(D_\mu^2 + m^2\right)\Psi = 0$$

Adiabatic States	Exact States
$\psi_{n,0}(x) = \frac{1}{\sqrt{2^n n!}}\left(\frac{\omega_t}{\pi}\right)^{\frac{1}{4}} e^{-\omega_t \frac{x^2}{2}} H_n\left(x\sqrt{\omega_t}\right)$	$\psi_n(x,t) = \frac{1}{\sqrt{2^n n!}}\left(\frac{1}{2\pi\xi^2(t)}\right)^{\frac{4}{4}} e^{-\Omega(t)\frac{x^2}{2}} H_n\left(\frac{x}{\sqrt{2}\xi(t)}\right) e^{-i\left(n+\frac{1}{2}\right)\lambda(t)}$
$\Omega(t) = -i\frac{\dot{\xi}(t)}{\xi(t)} + \frac{1}{2\xi^2(t)}$ ✓	$\lambda(t) = \int^t \frac{dt'}{2\xi(t)^2}$ ✗

$$\omega(t)^2 = \Delta(t)^2 = m^2$$

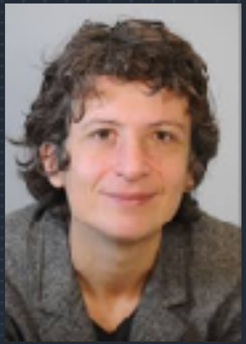
Negative Binomial Distribution

$$\mathscr{P}(m) = \binom{m/2+k-1}{m/2} \sin\left(\frac{\pi}{2+\eta}\right)^{2k} \cos\left(\frac{\pi}{2+\eta}\right)^{m}$$



S. Gherardini, L. Buffoni, ND, *Phys. Rev. Lett.* **133**, 113401 (2024)

Collaborators and funding



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Thank you