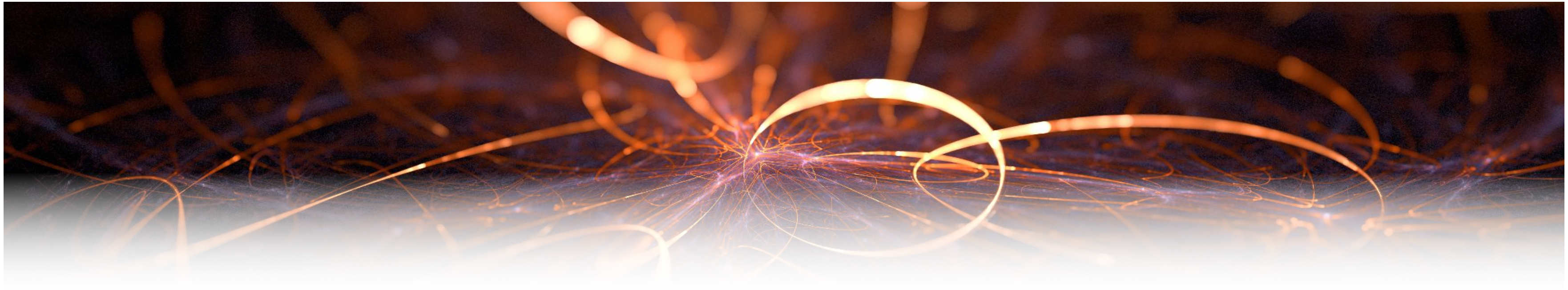
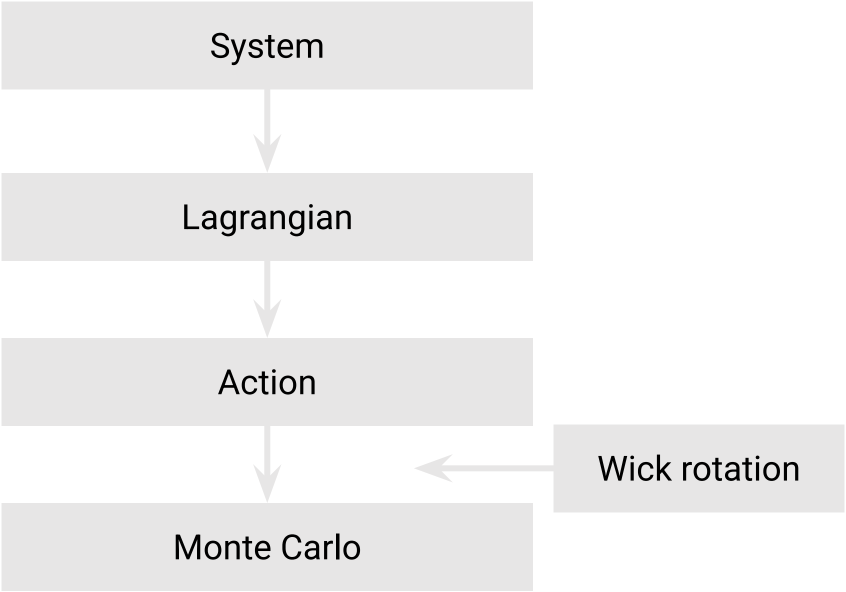


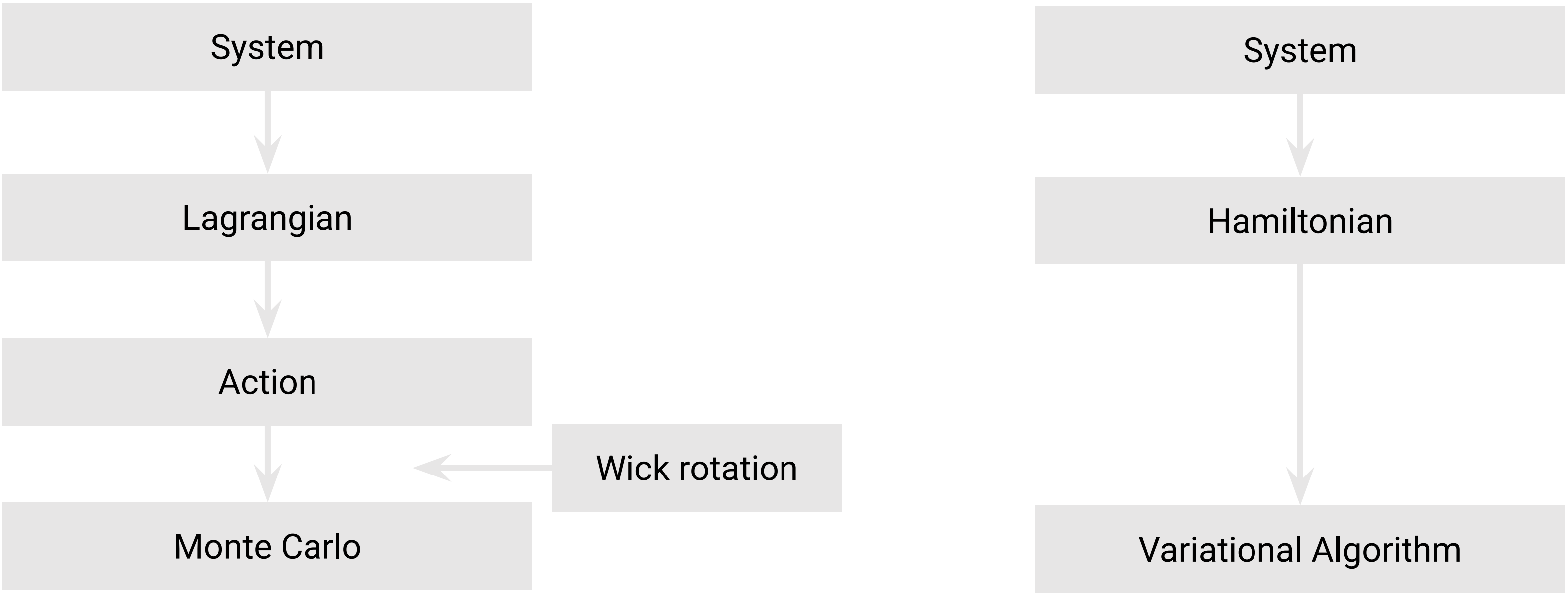
Projected Entangled Pair States for Lattice Gauge Theories with Dynamical Fermions

Patrick Emonts | 28.05.2025 | Real-time and out-of-equilibrium dynamics in the quantum HEP era | CERN

The usual pipeline



Our Approach



Mari Carmen Bañuls and Krzysztof Cichy (2020) Rep. Prog. Phys. 83 p. 024401;
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Kenneth G. Wilson (1974) Phys. Rev. D 10 pp. 2445–2459

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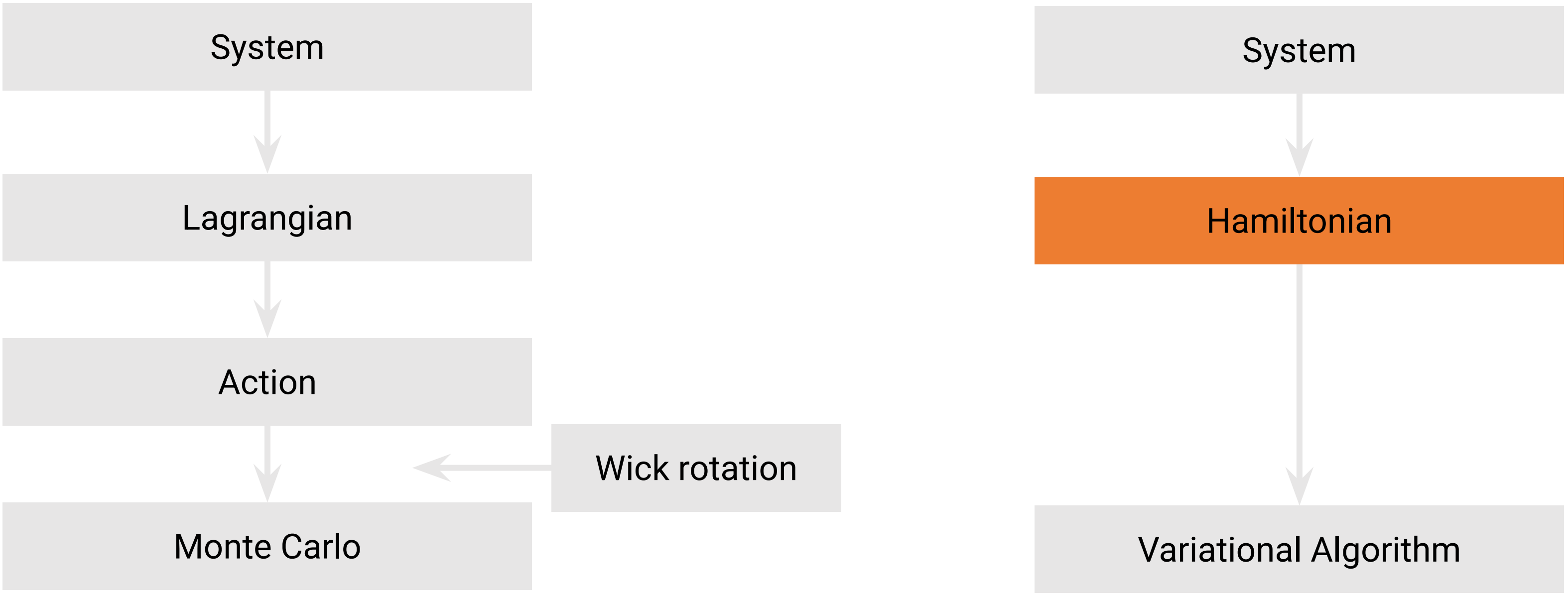

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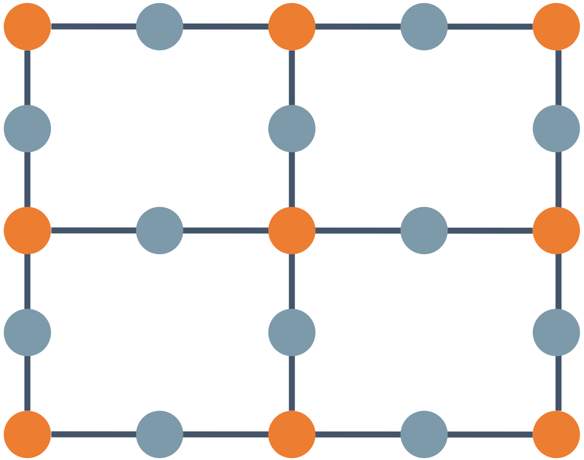
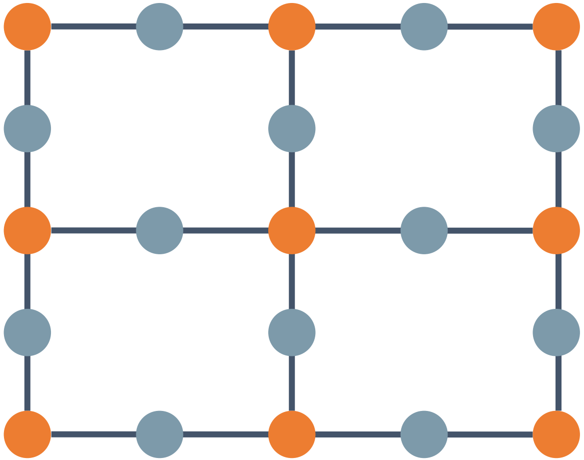
Hilbert spaces and Lattices



Hilbert space

$$\mathcal{H} \subset \mathcal{H}_{\text{gauge fields}} \otimes \mathcal{H}_{\text{fermions}}$$

Hilbert spaces and Lattices



Hilbert space

$$\mathcal{H} \subset \mathcal{H}_{\text{gauge fields}} \otimes \mathcal{H}_{\text{fermions}}$$

A general state

$$|\Psi\rangle = \int D\mathcal{G} |\mathcal{G}\rangle |\psi_F(\mathcal{G})\rangle$$

with $$D\mathcal{G} = \prod_{x,k} dg(x,k)$$

Which way to go?

Quantum Computation/
Quantum Simulation

Classical Simulation

C. W. Bauer, Z. Davoudi, N. Klco, and M. J. Savage, Nat Rev Phys 5, 420 (2023).
M. C. Bañuls and K. Cichy, Rep. Prog. Phys. **83**, 024401 (2020).
M. C. Bañuls et al., Eur. Phys. J. D 74, 165 (2020).

Which way to go?

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M. C. Bañuls et al., Eur. Phys. J. D 74, 165 (2020).

Computing an Expectation Value

Assume that the observable acts only on the gauge field and is diagonal in the group element basis:

$$\langle\Psi|O|\Psi\rangle = \frac{\langle\Psi|O|\Psi\rangle}{\langle\Psi|\Psi\rangle}$$

$$= \frac{\int D\mathcal{G}\langle\mathcal{G}|O|\mathcal{G}\rangle\langle\psi_F(\mathcal{G})|\psi_F(\mathcal{G})\rangle}{\int \mathcal{G}'\langle\psi_F(\mathcal{G}')|\psi_F(\mathcal{G}')\rangle}$$

Computing an Expectation Value

Assume that the observable acts only on the gauge field and is diagonal in the group element basis:

$$\langle \Psi | O | \Psi \rangle = \frac{\langle \Psi | O | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

$$= \frac{\int D\mathcal{G} \langle \mathcal{G} | O | \mathcal{G} \rangle \langle \psi_F(\mathcal{G}) | \psi_F(\mathcal{G}) \rangle}{\int \mathcal{G}' \langle \psi_F(\mathcal{G}') | \psi_F(\mathcal{G}') \rangle}$$

$$= \int D\mathcal{G} \mathcal{F}_O(\mathcal{G}) p(\mathcal{G})$$

with $$p(\mathcal{G}) = \frac{\langle \psi_F(\mathcal{G}) | \psi_F(\mathcal{G}) \rangle}{\int D\mathcal{G}' \langle \psi_F(\mathcal{G}') | \psi_F(\mathcal{G}') \rangle}$$

A wishlist

General State Formulation

$$|\Psi\rangle = \int D\mathcal{G} |\mathcal{G}\rangle |\psi_F(\mathcal{G})\rangle$$

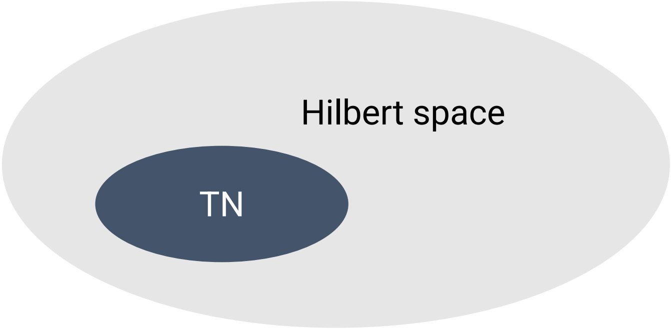
ToDo List

1. How do we construct $|\psi_F(\mathcal{G})\rangle$?
2. How do we efficiently calculate the expectation values?
3. Are those states useful?

Finding an Ansatz

Idea

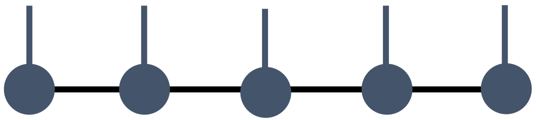
Use an Ansatz with polynomially many parameters although the Hilbert space has exponentially many states



We explore only a small part of the Hilbert space

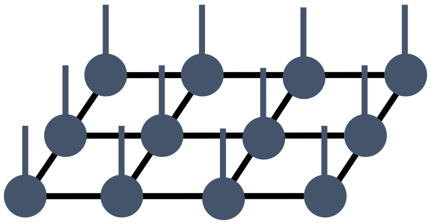
J. I. Cirac, D. Pérez-García, N. Schuch, and F. Verstraete, Rev. Mod. Phys. 93, 045003 (2021).
M. Fannes, B. Nachtergaele, and R. F. Werner (1992) Commun.Math. Phys. 144 pp. 443–490

Different Families of Tensor Networks



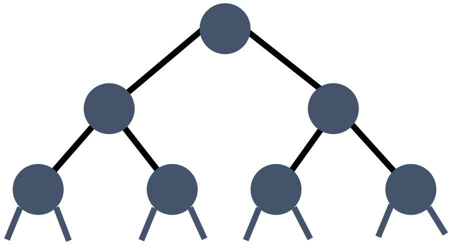
Matrix Product States (MPS)

M. Fannes, B. Nachtergaele, and R. F. Werner (1992)
Commun.Math. Phys. 144 pp. 443–490



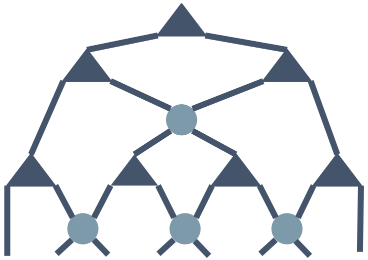
Projected Entangled Pair States (PEPS)

F. Verstraete and J. I. Cirac, arXiv:cond-mat/0407066.



Tree Tensor Network

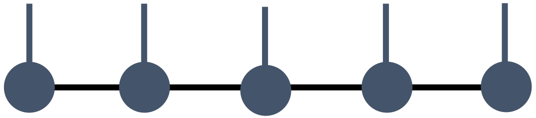
Y.-Y. Shi, L.-M. Duan, and G. Vidal, Phys. Rev. A 74, 022320 (2006).
T. Felser, S. Notarnicola, and S. Montangero, Phys. Rev. Lett. 126, 170603 (2021).



Multiscale Entanglement Renormalization Ansatz (MERA)

G. Vidal, Phys. Rev. Lett. 101, 110501 (2008).

Different Families of Tensor Networks



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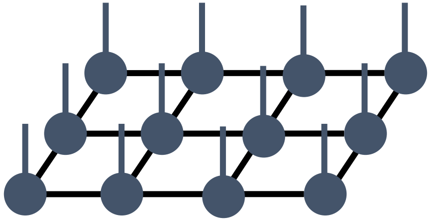

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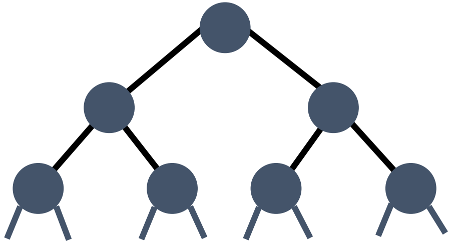

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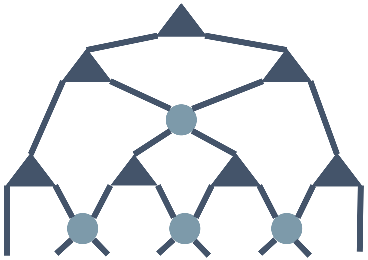

Multiscale Entanglement Renormalization Ansatz (MERA)

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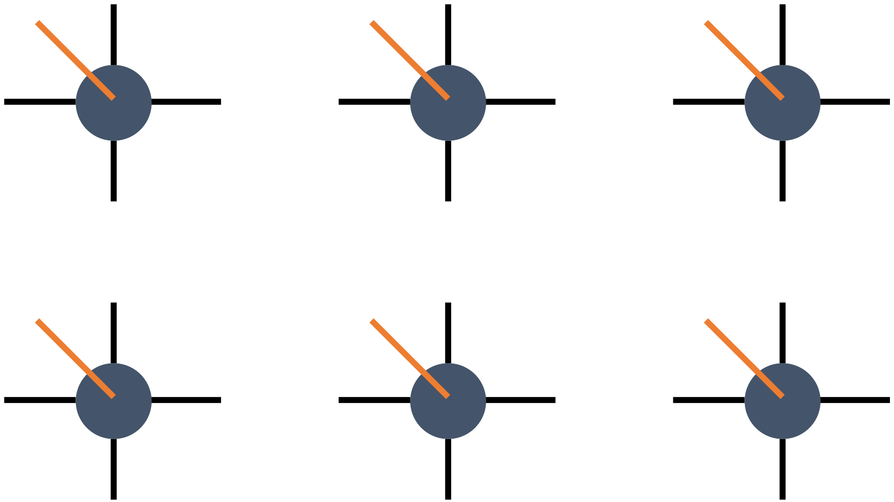
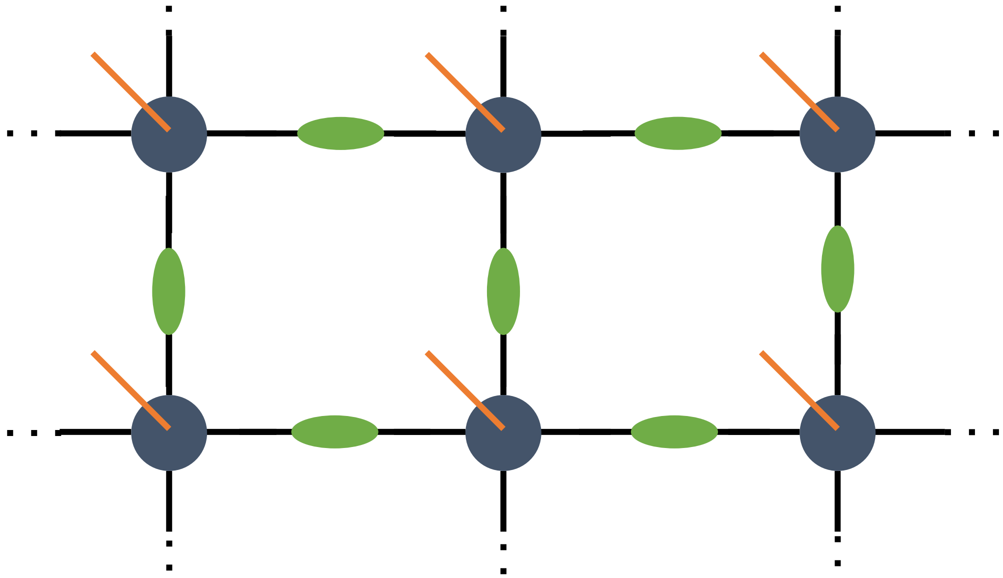
Building a State – GGPEPS



Construction

$$\prod_{\boldsymbol{x}} \mathcal{A}(\boldsymbol{x})|\Omega\rangle$$

Building a state



Construction

$$|\psi_0\rangle = \langle\Omega_v| \prod_{\ell} \omega_\ell \prod_{\boldsymbol{x}} \mathcal{A}(\boldsymbol{x}) |\Omega\rangle$$

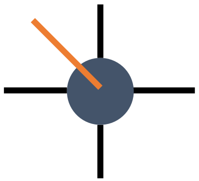
Local Gauge Invariance

Goal

Couple the gauge field to the state such that it is locally invariant under gauge transformations.

Gauging Procedure

$$|\psi_0\rangle = \langle \Omega_v | \prod_\ell \omega_\ell \prod_x \mathcal{A}(x) | \Omega \rangle$$



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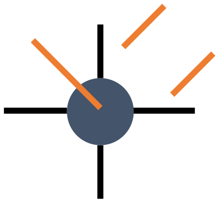
$$\rightarrow \langle\Omega_v| \prod_\ell \omega_\ell \prod_x \mathcal{A}(x)|0\rangle_{x,1}|0\rangle_{x,2}|\Omega\rangle$$



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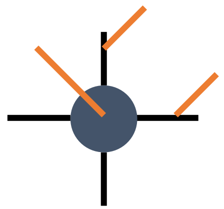
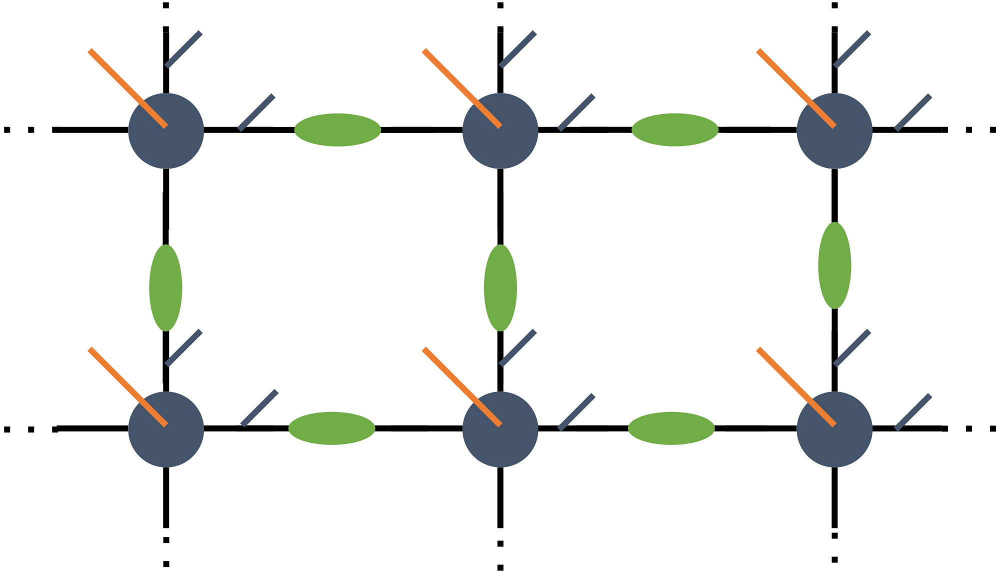
$$\to \langle\Omega_v| \prod_\ell \omega_\ell \prod_x \mathcal{A}(x)|0\rangle_{x,1}|0\rangle_{x,2}|\Omega\rangle$$

$$|\psi\rangle = \langle\Omega_v| \prod_\ell \omega_\ell \prod_x \mathcal{U}_G(x,1)\mathcal{U}_G(x,2) \prod_x \mathcal{A}(x)|0\rangle_{x,1}|0\rangle_{x,2}|\Omega\rangle$$



Building a State – GGPEPS



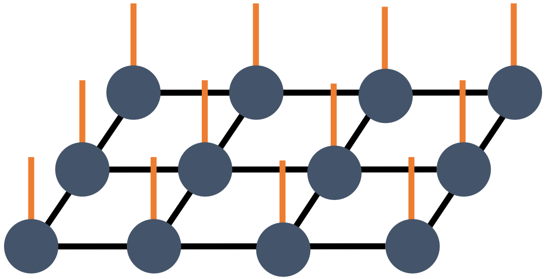
Construction

$$|\psi_F(\mathcal{G})\rangle = \langle\Omega_v| \prod_{\ell} \omega_\ell \prod_{\ell} U_\ell(\mathcal{G}) \prod_{\boldsymbol{x}} \mathcal{A}(\boldsymbol{x})|\Omega\rangle$$

How to compute an expectation value?

ToDo List

1. How do we construct $|\psi_F(\mathcal{G})\rangle$? ✓
2. How do we efficiently calculate the observables?
3. Are those states useful?

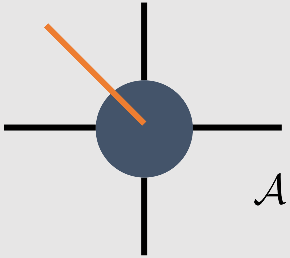


Contracting PEPS is hard!

N. Schuch, M. M. Wolf, F. Verstraete, and J. I. Cirac, Phys. Rev. Lett. **98**, 140506 (2007).

A Gauged Gaussian PEPS

Tensor



$$\mathcal{A}(\boldsymbol{x}) = \exp(T_{ij} a_i^\dagger(\boldsymbol{x}) b_j^\dagger(\boldsymbol{x}))$$

Projector

$$\omega(\boldsymbol{x}, k) = \omega_k(\boldsymbol{x}) \Omega_k(\boldsymbol{x}) \omega_k^\dagger(\boldsymbol{x})$$

$$\omega_0(\boldsymbol{x}) = \exp(l_+^\dagger(\boldsymbol{x} + \boldsymbol{e}_1) r_-^\dagger(\boldsymbol{x})) \exp(l_-^\dagger(\boldsymbol{x} + \boldsymbol{e}_1) r_+^\dagger(\boldsymbol{x}))$$

Use covariance matrices instead

Covariance Matrices

Goal

Estimate the numerator

$$p(\mathcal{G}) = \frac{\langle \psi_F(\mathcal{G}) | \psi_F(\mathcal{G}) \rangle}{\int D\mathcal{G}' \langle \psi_F(\mathcal{G}') | \psi_F(\mathcal{G}') \rangle}$$

Sergey Bravyi (2005) Quantum Inf. and Comp. 5 pp. 216–238

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Estimate the numerator

$$p(\mathcal{G}) = \frac{\langle \psi_F(\mathcal{G}) | \psi_F(\mathcal{G}) \rangle}{\int D\mathcal{G}' \langle \psi_F(\mathcal{G}') | \psi_F(\mathcal{G}') \rangle}$$

The Covariance Matrix

$$\Gamma_{a,b} = \frac{i}{2} \langle [\gamma_a, \gamma_b] \rangle = \frac{i}{2} \frac{\langle \Phi | [\gamma_a, \gamma_b] | \Phi \rangle}{\langle \Phi | \Phi \rangle}$$

Covariance Matrices for GGPEPS

$$|\psi_F(\mathcal{G})\rangle = \langle \Omega_v | \underbrace{\prod_{\ell} \omega_\ell \prod_{\ell} U_\ell(\mathcal{G})}_{\Gamma_{\text{in}}(\mathcal{G})} \underbrace{\prod_{\boldsymbol{x}} \mathcal{A}(\boldsymbol{x}) | \Omega \rangle}_{\Gamma_M}$$

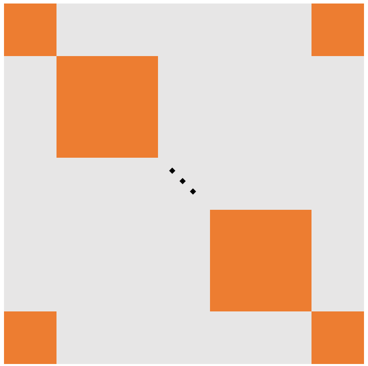
Sergey Bravyi (2005) Quantum Inf. and Comp. 5 pp. 216–238

Computing the Norm

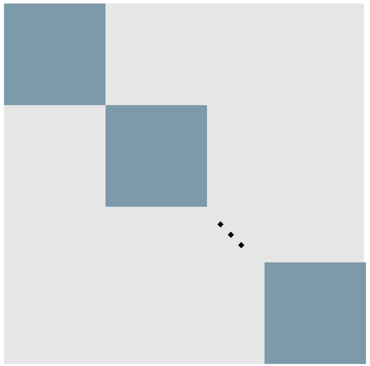
Covariance Matrices for GGPEPS

$$|\psi_F(\mathcal{G})\rangle = \langle\Omega_v| \underbrace{\prod_\ell \omega_\ell \prod_\ell U_\ell(\mathcal{G})}_{\Gamma_{\text{in}}(\mathcal{G})} \underbrace{\prod_{\boldsymbol{x}} \mathcal{A}(\boldsymbol{x})|\Omega\rangle}_{\Gamma_M}$$

$\Gamma_{\text{in}}(\mathcal{G})$



Γ_M



The Norm

$$|\psi_F(\mathcal{G})|^2 = \sqrt{\det\left(\frac{1 - \Gamma_{\text{in}}(\mathcal{G})M_D}{2}\right)}$$

The MC Algorithm

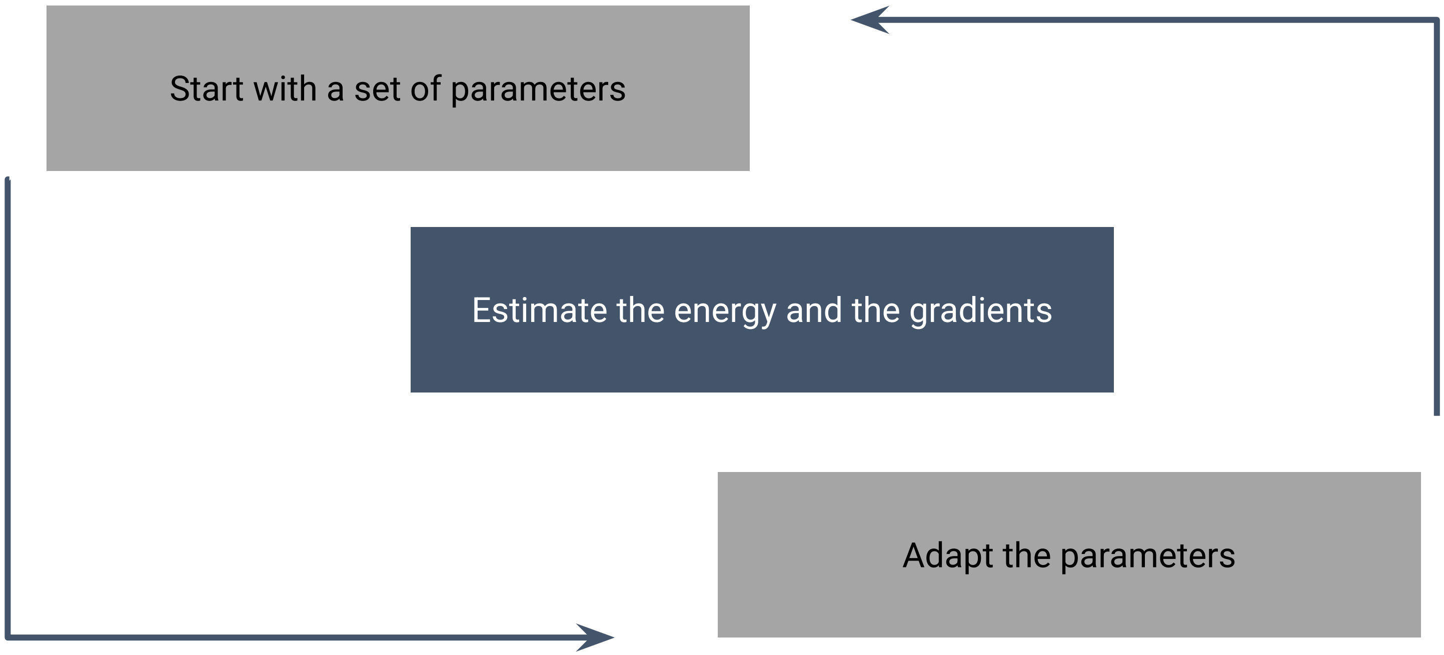
Draw a new gauge field configuration

Build the state $|\psi_F(\mathcal{G})\rangle$

Calculate the acceptance probability by computing $\langle\psi_F(\mathcal{G})|\psi_F(\mathcal{G})\rangle$

Accept or Decline

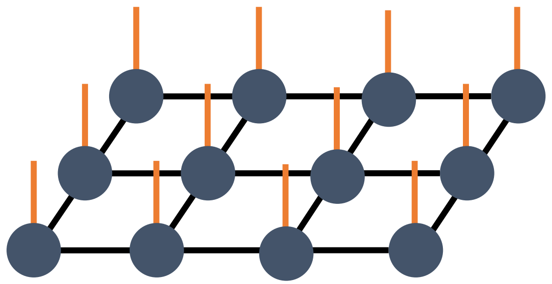
The Variational MC Algorithm



How to compute an expectation value?

ToDo List

1. How do we construct $|\psi_F(\mathcal{G})\rangle$? ✓
2. How do we efficiently calculate the observables? ✓
3. Are those states useful?



GGPEPS can be contracted via covariance matrices

N. Schuch, M. M. Wolf, F. Verstraete, and J. I. Cirac, Phys. Rev. Lett. **98**, 140506 (2007).

Let's consider a Z2 LGT with Fermions

Kogut Susskind Hamiltonian

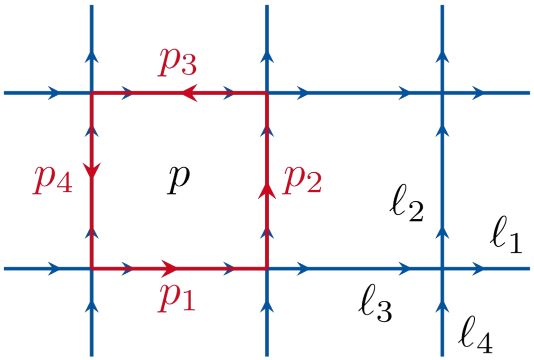
$$H = g_{\mathrm{E}} H_{\mathrm{E}} + g_{\mathrm{B}} H_{\mathrm{B}} + g_{\mathrm{I}} H_{\mathrm{I}} + g_{\mathrm{M}} H_{\mathrm{M}}$$

$$H_{\mathrm{E}} = \sum_{\ell} 2\left[1 - \sigma_{\ell}^{z}\right]$$

$$H_{\mathrm{B}} = \sum_{p} \left[1 - \sigma_{p_1}^{x} \sigma_{p_2}^{x} \sigma_{p_3}^{x} \sigma_{p_4}^{x}\right]$$

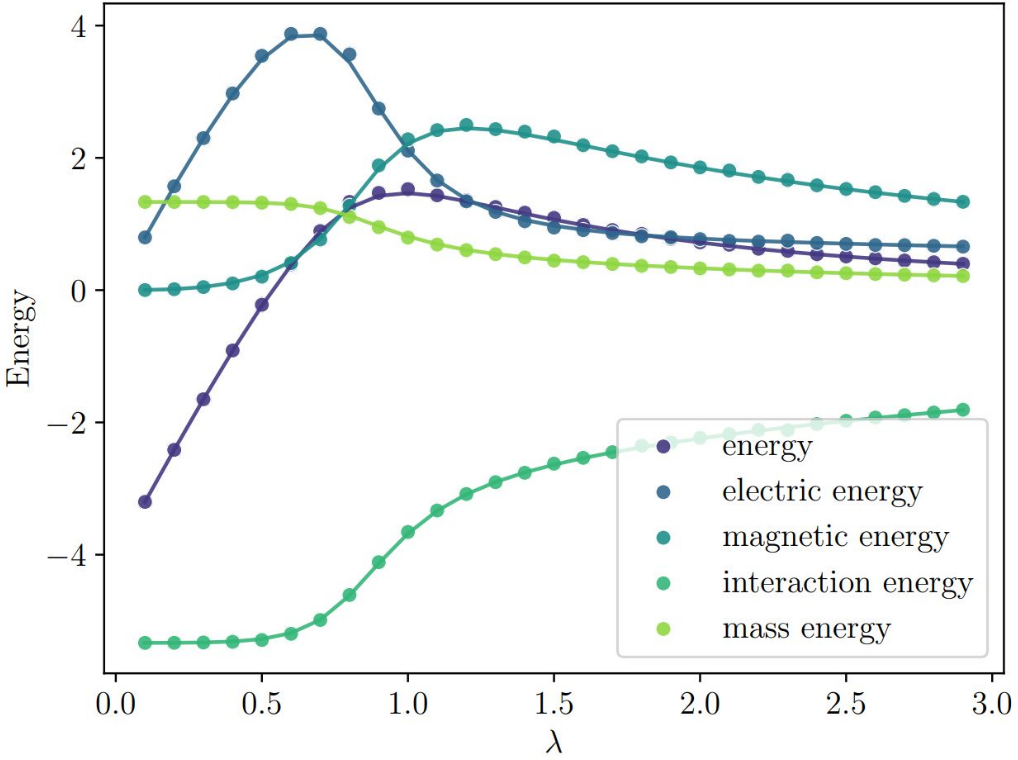
$$H_{\mathrm{M}} = \sum_{x} \left(\frac{1}{2} + (-1)^{x} \psi^{\dagger}(x)\psi(x)\right)$$

$$H_{I} = \sum_{x} \left(\psi^{\dagger}(x) U(x,1) \psi(x + e_1) + h.c\right) + H_{I,2}$$



John Kogut and Leonard Susskind (1975) Phys. Rev. D 11 pp. 395–408
D. Horn, M. Weinstein, and S. Yankielowicz (1979) Phys. Rev. D 19 pp. 3715–3731

Results – 2x2 Benchmark



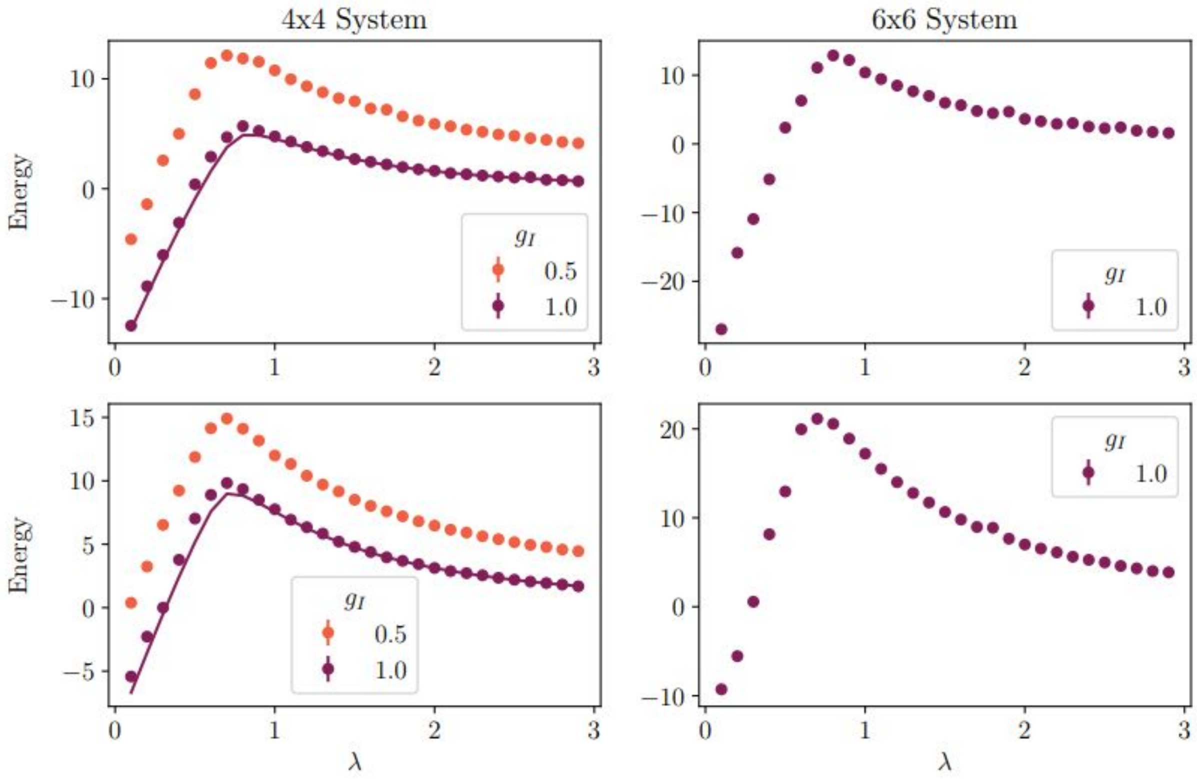
$$g_E = \lambda$$
$$g_I = 1.0$$
$$g_M = 1.0$$
$$g_B = 1/\lambda$$

A. Kelman, U. Borla, P. Emonts, and E. Zohar, arXiv:2412.16951.

Results – Monte Carlo Sampling

$g_{\mathrm{M}} = 0.0$

$g_{\mathrm{M}} = 1.0$

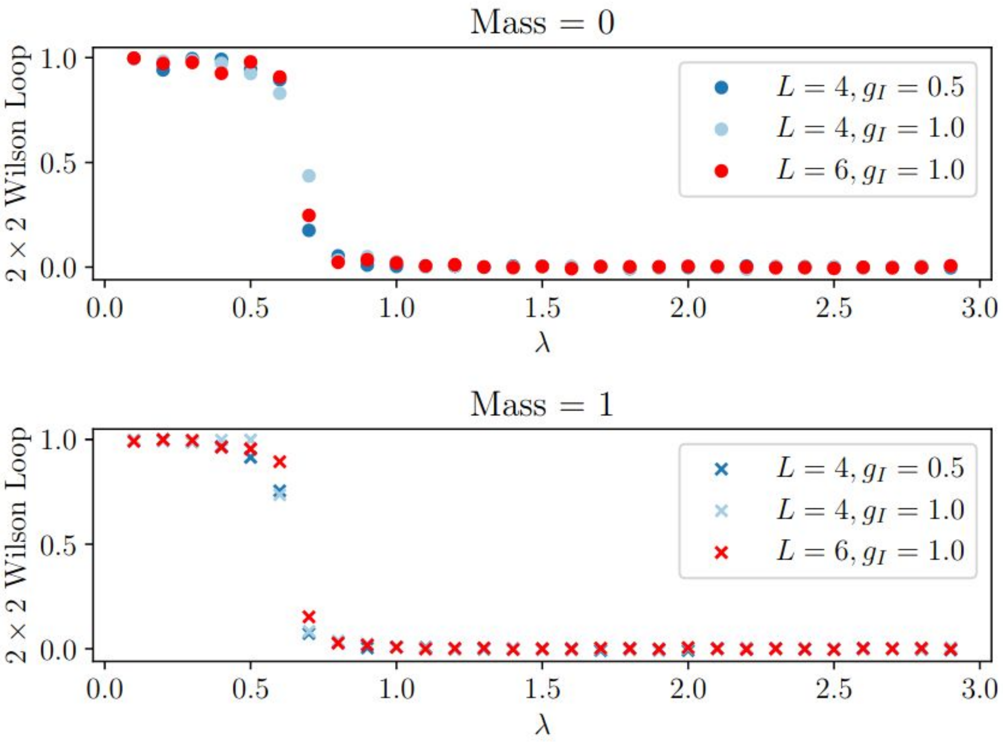


$g_{\mathrm{E}} = \lambda$

$g_{\mathrm{B}} = 1/\lambda$

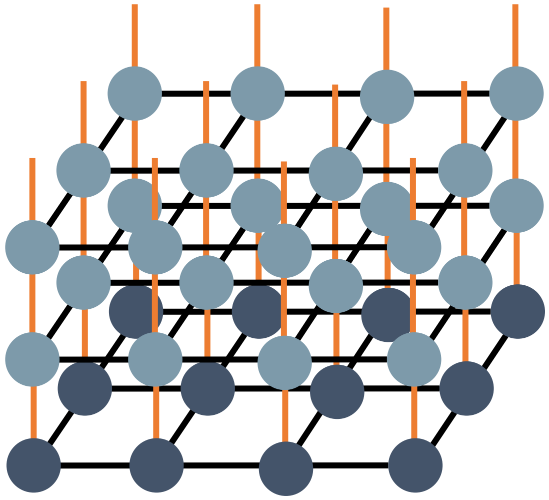
A. Kelman, U. Borla, P. Emonts, and E. Zohar, arXiv:2412.16951.

Results – Resolving Phases



A. Kelman, U. Borla, P. Emonts, and E. Zohar, arXiv:2412.16951.

Some Thoughts on Time Evolution



Trotterization

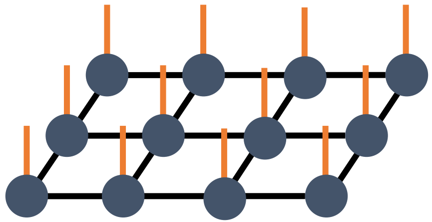
$$e^{-iHT} \approx \left(e^{-iH_1 t} e^{-iH_2 t}\right)^N$$

Time Evolution (TDVP)

$$-i\frac{\partial}{\partial t}|\psi\rangle = P_{T_{\mathcal{M}}(|\psi\rangle)} H |\psi\rangle$$

Summary

How do we construct $|\psi_F(\mathcal{G})\rangle$?

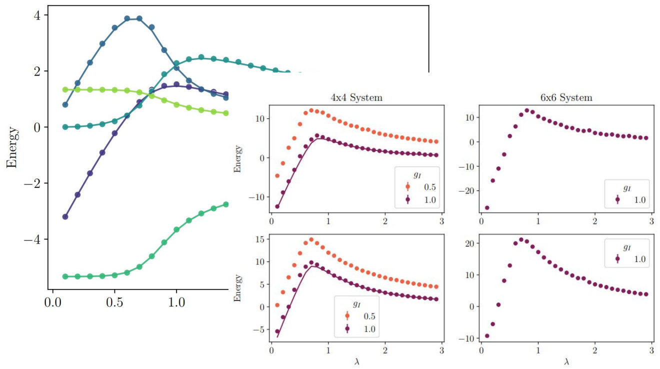


GGPEPS

How do we efficiently calculate the expectation values?

Covariance Matrices and Variational Monte Carlo Sampling

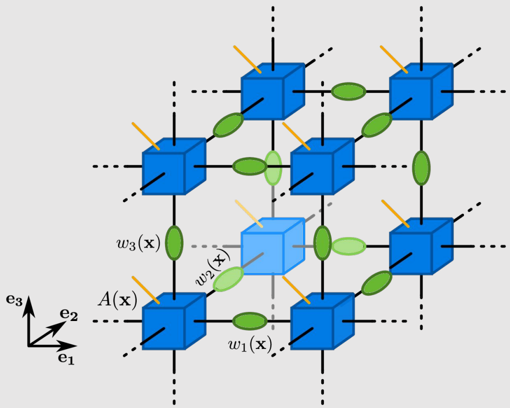
Are those states useful?



A. Kelman, U. Borla, P. Emonts, and E. Zohar, arXiv:2412.16951.
A. Kelman, U. Borla, I. Gomelski, J. Elyovich, G. Roose, P. Emonts, and E. Zohar, Phys. Rev. D 110, 054511 (2024).

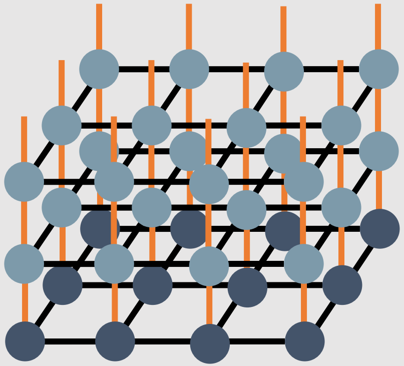
What are the next steps?

Three spatial dimensions?



P. Emonts and E. Zohar,. Phys. Rev. D 108, 014514 (2023).

Time Evolution?



Non-Abelian Gauge Groups?

D_6

$SU(2)$

$SU(3)$

E. Zohar and J. I. Cirac, Phys. Rev. D 97, 034510 (2018).

Science is a team effort

Ariel Kelman

Umberto Borla

Itay Gomelski

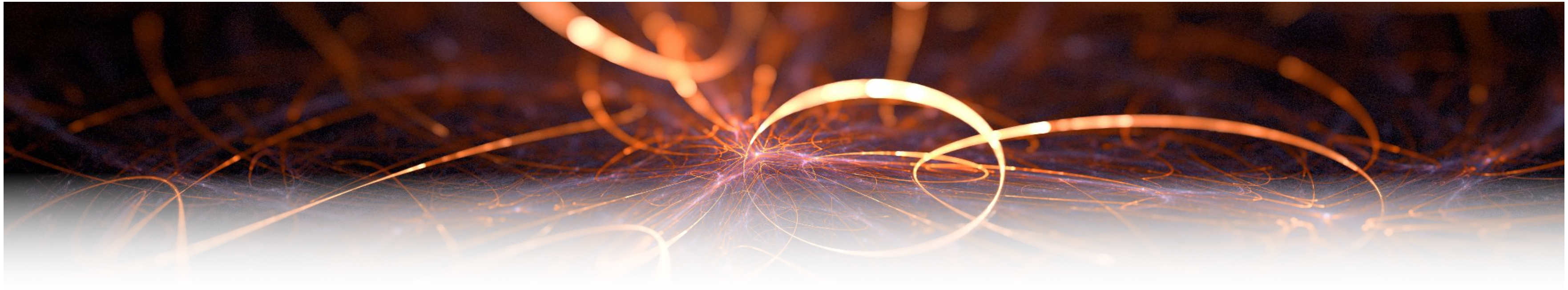
Jonathan Elyovich

Gertian Roose

Erez Zohar

QPhoton C3S Center

NWO



Projected Entangled Pair States for Lattice Gauge Theories with Dynamical Fermions

Patrick Emonts | 28.05.2025 | Real-time and out-of-equilibrium dynamics in the quantum HEP era | CERN