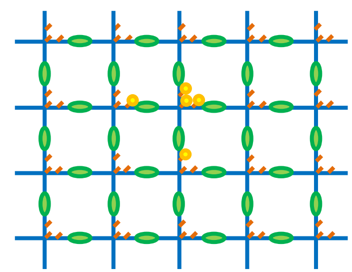
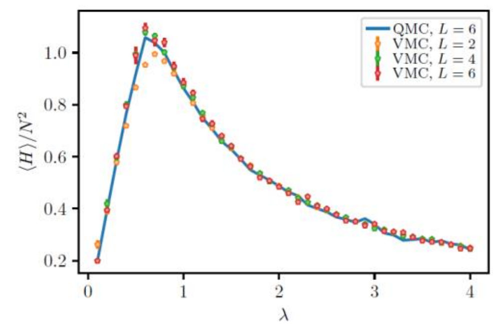
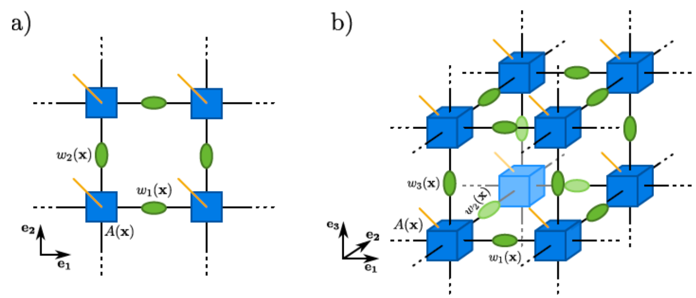
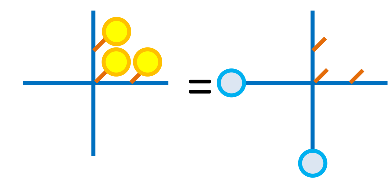
Tensor Network States for HEP









Erez Zohar

Racah Institute of Physics, Hebrew University of Jerusalem

Funded by the European Union

ERC Consolidator Grant
Oversign 2024-2029

Gauge Theories are interesting:

- **In the context of high energy physics: they are the standard model's description of the forces and interactions**

- **In the context of condensed matter physics: toric code, quantum double models, emergent/effective interactions in many models (High T_c?), …**

Gauge Theories are **challenging:**

- **Local symmetry** → many constraints
- Involve **non-perturbative physics**
  - **Confinement of quarks** → hadronic spectrum
  - **Exotic phases of QCD** (color superconductivity, quark-gluon plasma)

→ **Hard to treat experimentally** (strong forces)

→ **Hard to treat analytically** (non perturbative)

→ **Lattice Gauge Theory** (Wilson, Kogut-Susskind…)

→ Lattice regularization in a gauge invariant way

Conventional LGT techniques

- Discretization of both space and time
- Monte Carlo computations on a Wick-rotated, Euclidean lattice

$$\left\langle \hat{A}\left(\hat{\Phi}\right)\right\rangle = \frac{\int \mathcal{D}\phi A(\phi) e^{iS_M}}{\int \mathcal{D}\phi e^{iS_M}}$$

$$\underset{t\to -i\tau}{\longrightarrow} \frac{\int \mathcal{D}\phi A(\phi) e^{-S_E}}{\int \mathcal{D}\phi e^{-S_E}} \equiv \int \mathcal{D}\phi A\left(\phi\right) p\left(\phi\right)$$

- **Very (very) successful for many applications, e.g. the hadronic spectrum**
- **Problems:**
  - **Real-Time evolution:**
    - Not available in Wick rotated, Euclidean spacetimes, used in conventional Monte-Carlo path integral LGT calculations
  - **Sign problem:**
    - Appears in several scenarios with fermions (finite density), represented by Grassman variables in a Wick-rotated, Euclidean spacetime

Approach 1:
Quantum Computing (Z. Davoudi, Monday)

- **Need larger computational power? Use a quantum computer.**
  - Straight Forward way to formulate tough problems using well-defined modular building blocks (universality).
  - Applicable for a significant class of problems.
  - Supremacy and advantages already nicely demonstrated.
- **However:**
  - Large scale computers not yet available
  - Fault tolerant, non-noisy devices not yet available, only NISQ
  - Brute force? (not really: clever representations, digital quantum simulation)

Quantum Problem

→

Quantum Solution

Approach 2:
Quantum Simulation (M. Dalmonte, Tuesday)

- **Available, less restrictive quantum technology:**
  - No need to be universal.
  - No need to store memory or have long coherence times.
  - Can involve analog ingredients.
  - Can involve more “physical intuition” (highly subjective)
  - Requiring theory-experiment collaboration.
- **However:**
  - Still not necessarily large systems
  - Can’t do everything in a completely analog fashion, need to compromise on digital ingredients.
  - Requiring theory-experiment collaboration.

Quantum Problem → Quantum Simplification → Quantum Solution

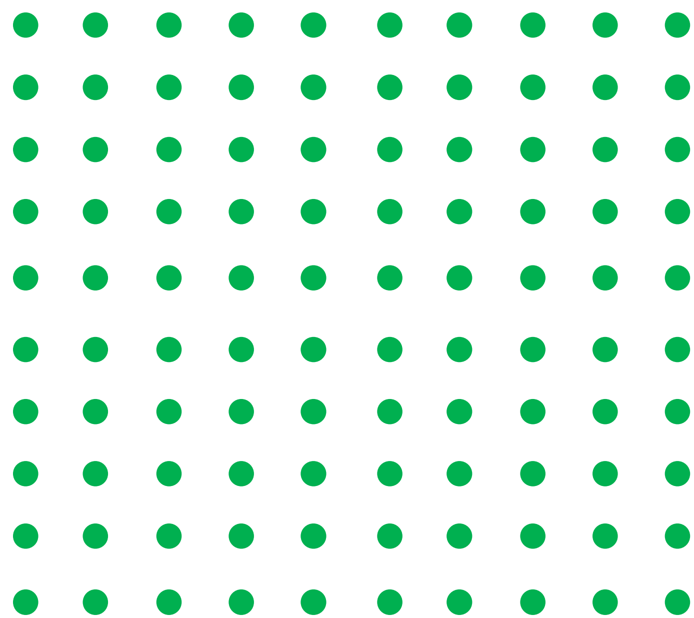
Approach 3:
Entanglement based classical simulation (today)

- **Advantages…**
- **Disadvantages…**
- Stay tuned to this talk.

Quantum Problem →
Quantum Simplification →
Classical Solution

Many-Body Area Law, Handwaving Formulation

"The ground state of a many-body Hamiltonian with local interactions (+ a few more assumptions) **obeys an bipartite entanglement entropy area law."**



See, e.g., Eisert, Cramer, Plenio, RMP 2010

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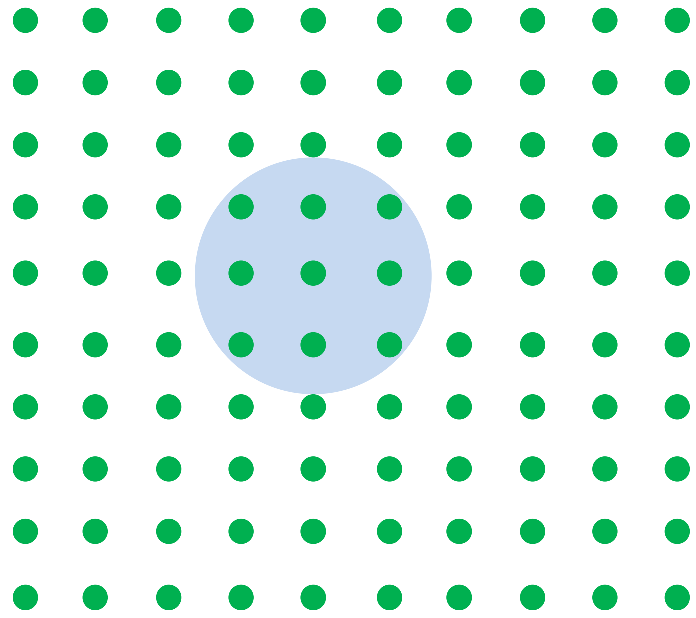

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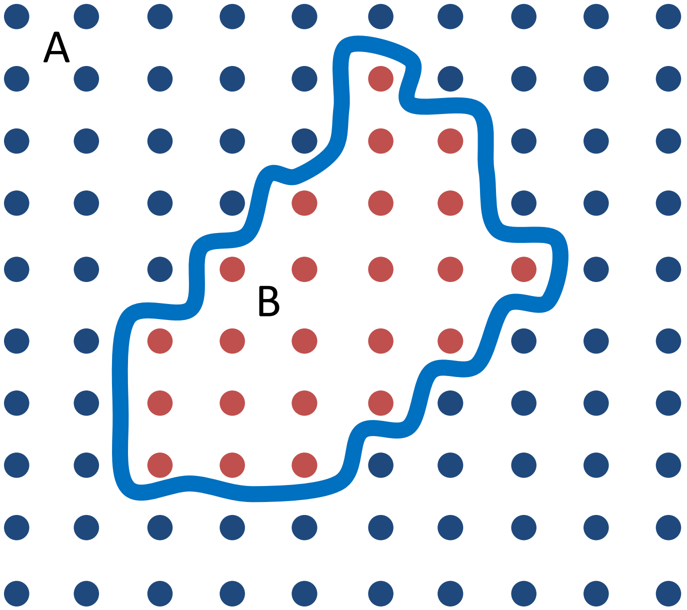

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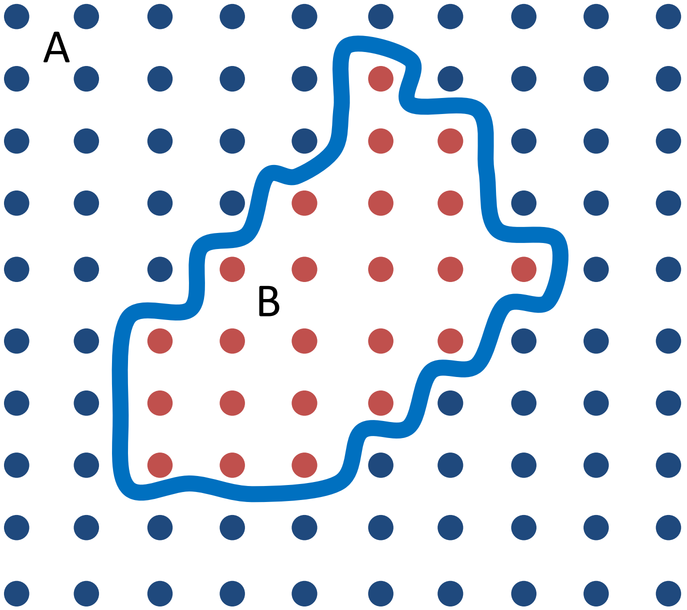
$$|\psi\rangle = \sum_{\mu} b_{\mu} |\mu\rangle_A |\mu\rangle_B$$

$$p_{\mu} = |b_{\mu}|^2$$

$$S = -\sum_{\mu} p_{\mu} \log p_{\mu}$$



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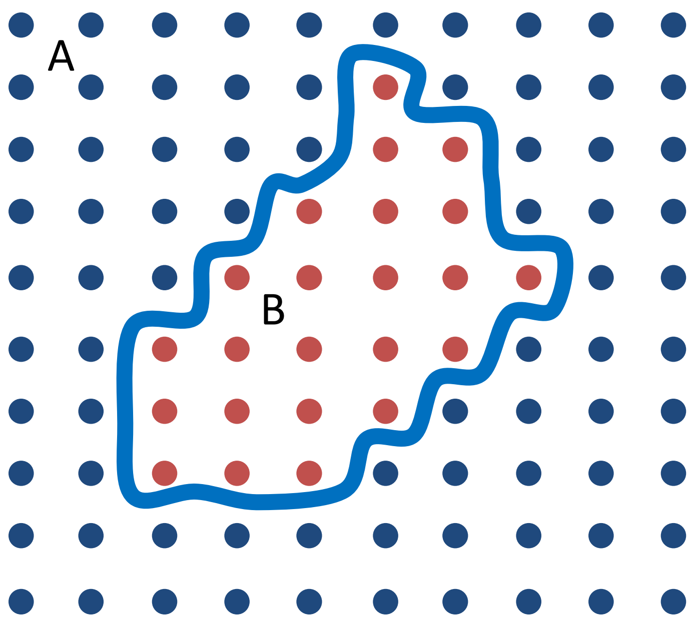
$$p_{\mu} = |b_{\mu}|^2$$

$$S = -\sum_{\mu} p_{\mu} \log p_{\mu}$$

$$S \propto \mathrm{Area}\,(\mathrm{Boundary})$$



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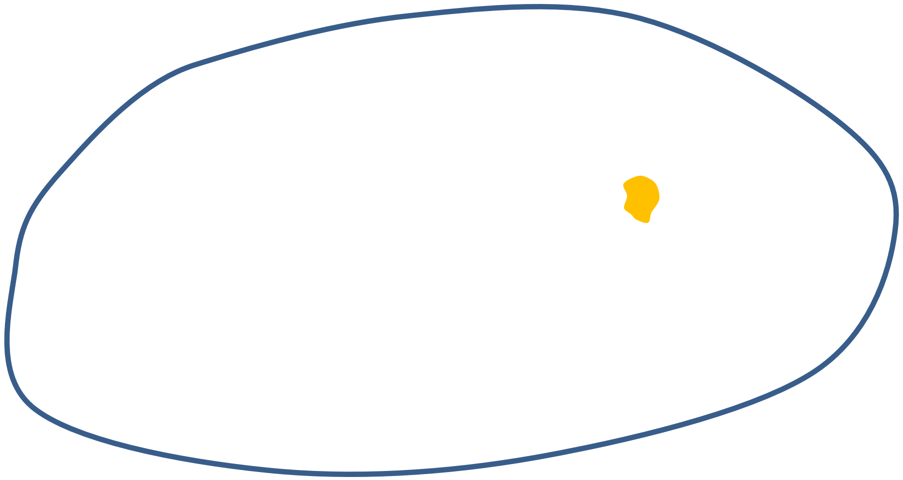
Many-Body Area Law, Handwaving Formulation

"The ground state of a many-body Hamiltonian with local interactions (+ a few more assumptions) **obeys an bipartite entanglement entropy area law."**

Physically relevant corner of the Hilbert space



See, e.g., Eisert, Cramer, Plenio, RMP 2010

Tensor Network States

- The **number of variables** needed to describe states of a many-body system **scales exponentially** with the system size. This makes it hard to simulate large systems (classically).

- **Tensor network states** are ansatz states for **many body systems**, mostly on a lattice, for either **analytical or numerical** studies, based on contractions of **local tensors that depend on few parameters**.

- In spite of their **simple description**, tensor network states describe and approximate **physically relevant states of many-body systems**.

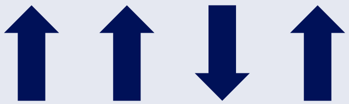
Relevant reviews:
Schollwöck, Ann. Phys. 2011
Orus, Ann. Phys. 2014
Cirac, Perez-Garcia, Schuch, Verstraete, RMP 2021

Tensor Networks: A motivation

A general superposition state

$$|\psi_0\rangle = \sum_{\{i\}} c_{i_1,\ldots i_N} |i_1 \ldots i_N\rangle$$

An example system: Ising spins



How to get to polynomial scaling?

Can we just skip the small coefficients?

$$c^{0,0,1,0,1} = 0.3623$$
$$c^{0,1,1,0,1} = 0.0003$$
$$c^{1,0,0,0,0} = -0.0004$$
$$c^{0,1,0,0,1} = 0.5203$$

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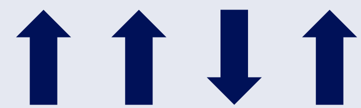

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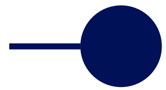
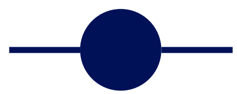
Tensor Networks Notation

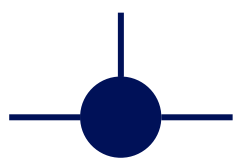


Vector

# of legs = rank of the object



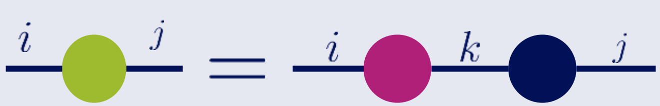
Matrix



Tensor

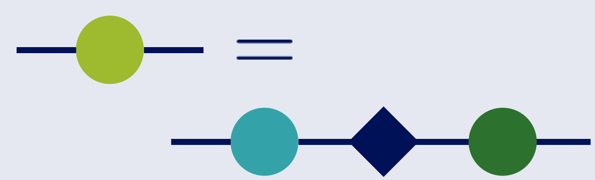
Matrix-Matrix Multiplication

$$C_{ij} = \sum_k A_{ik} B_{kj}$$



Singular Value Decomposition

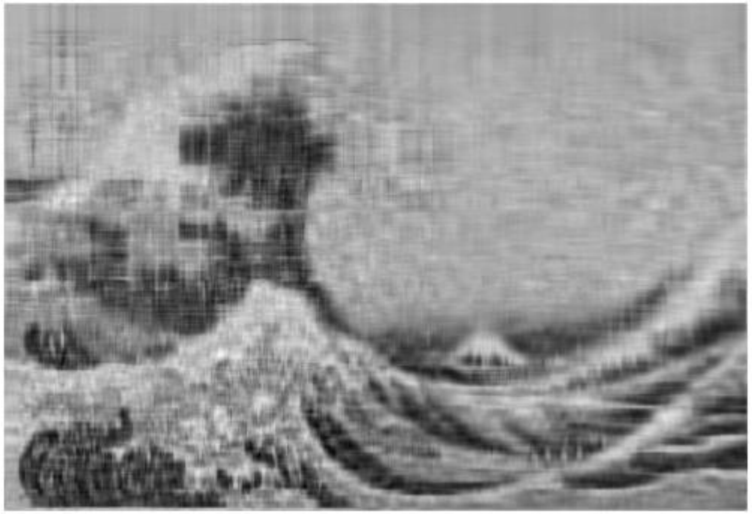
$$M = USV^{\dagger}$$



Through the SVD (and what Alice saw there)

Original Image

Truncated Image (20 SV)



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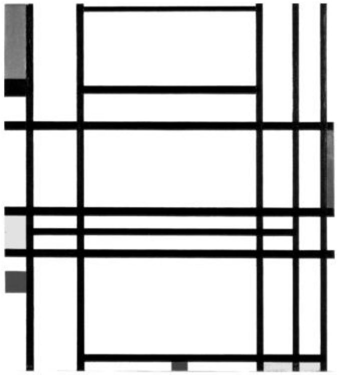
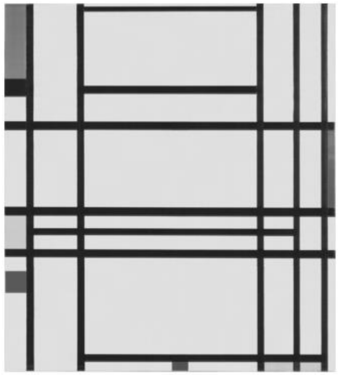
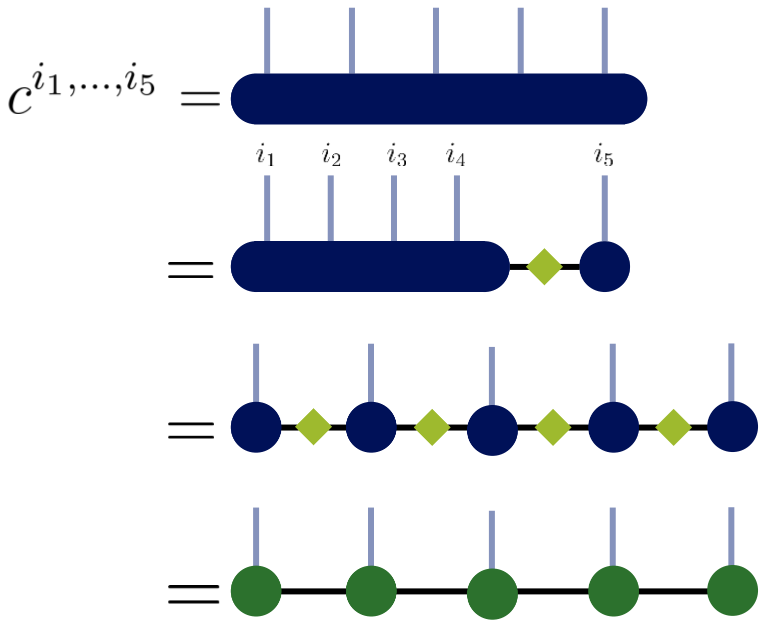


Truncated Image (20 SV)



Expressing a state by a matrix product



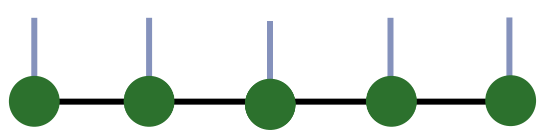
Bridgeman and Chubb (2017) 50 p. 223001
Schollwöck, Annals of Physics 326, 96–192 (2011).

Reducing to polynomially many parameters

Truncate to a virtual bond dimension D to reduce to polynomially many parameters.

$$|\psi\rangle = \sum_{\{i\}} \sum_{\{\alpha\}} A^{i_1}_{1,\alpha_1} A^{i_2}_{\alpha_1,\alpha_2} \cdots A^{i_N}_{\alpha_{N-1},1} |i_1, \cdots, i_N\rangle$$

$=$



A general superposition state

$$|\psi_0\rangle = \sum_{\{i\}} c_{i_1,\ldots i_N} |i_1 \ldots i_N\rangle$$

Matrix Product State

$$|\psi\rangle = \sum_{\{i\}} \sum_{\{\alpha\}} A^{i_1}_{1,\alpha_1} A^{i_2}_{\alpha_1,\alpha_2} \cdots A^{i_N}_{\alpha_{N-1},1} |i_1, \cdots, i_N\rangle$$

Notation

Physical index:	i_j
Virtual Index:	α_j
Tensors: D x D x d	$A^i_{\alpha,\beta}$

Take Home Message

The bond dimension controls the approximation.

Counting Parameters: what did we gain?

A general superposition state

$$|\psi_0\rangle = \sum_{\{i\}} c_{i_1,\dots i_N} |i_1 \dots i_N\rangle$$

Number of Parameters

$$d^N$$

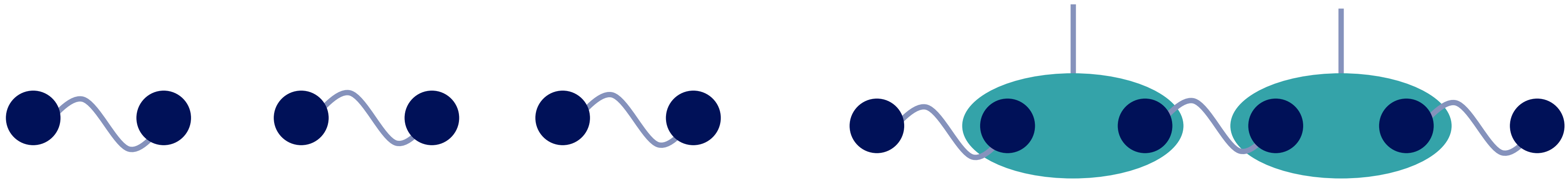
Matrix Product State

$$|\psi\rangle = \sum_{\{i\}} \sum_{\{\alpha\}} A^{i_1}_{1,\alpha_1} A^{i_2}_{\alpha_1,\alpha_2} \cdots A^{i_N}_{\alpha_{N-1},1} |i_1, \cdots, i_N\rangle$$

Number of Parameters

$$N(D \times D \times d)$$

MPS as Projected Entangled Pair States (PEPS)



Entangled Pairs

$$|\Phi\rangle = \sum_{j=0}^{D-1} |jj\rangle$$

Example: Bell state

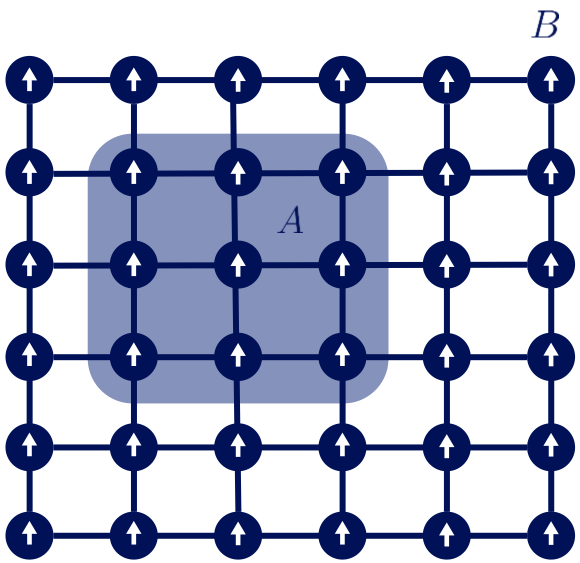
$$|\Phi\rangle = \frac{1}{\sqrt{2}} |00\rangle + |11\rangle$$

Projector

$$\omega = \sum_{i,\alpha,\beta} A^{i}_{\alpha,\beta} |i\rangle\langle\alpha\beta|$$

Affleck, I., Kennedy, T., Lieb, E. H. & Tasaki, H. Phys. Rev. Lett. 59, 799–802 (1987).

Physical relevance



How does the entanglement scale?

~~**Volume Law** $S \propto V$~~

Area Law $S \propto \partial V$

Tensor networks efficiently approximate…

…ground states of local, gapped Hamiltonians.

Hastings, Phys. Rev. B 76, 035114 (2007).

Arad, Kitaev, Landau, Vazirani, arXiv:1301.1162.

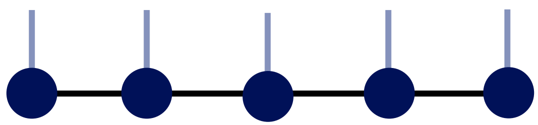
… states if their correlators decay quickly enough.

Brandao and Horodecki, Nature Phys **9**, 721 (2013).

… states with low entropy.

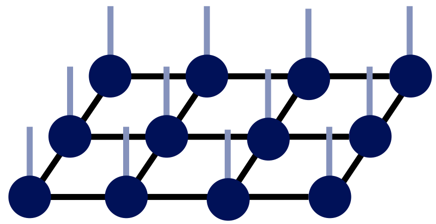
F. Verstraete and J. I. Cirac, Phys. Rev. B 73, (2006).

Different Tensor Network Types



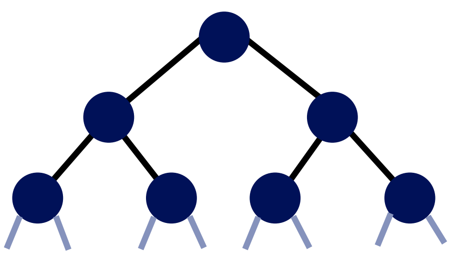
Matrix Product States (MPS)

M. Fannes, B. Nachtergaele, and R. F. Werner (1992) Commun.Math. Phys. 144 pp. 443–490



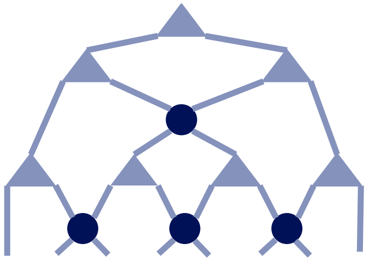
Projected Entangled Pair States (PEPS)

F. Verstraete and J. I. Cirac, arXiv:cond-mat/0407066.



Tree Tensor Network

Y.-Y. Shi, L.-M. Duan, and G. Vidal, Phys. Rev. A 74, 022320 (2006).



Multiscale Entanglement Renormalization Ansatz (MERA)

G. Vidal, Phys. Rev. Lett. 101, 110501 (2008).

Tensor Networks for Lattice Models in HEP

Variational Methods

Compute an upper bound to the energy

$$E_{\text{var}} = \min_{\theta} \langle \psi(\theta) | H | \psi(\theta) \rangle$$

Bañuls, Cichy, Rep. Prog. Phys. **83**, 024401 (2020).

Bañuls et al., Eur. Phys. J. D **74**, 165 (2020).

Renormalization Group Methods

Compute the partition sum

$$Z = \sum_{\{\sigma\}} e^{-\beta E(\sigma_1, \sigma_2, \ldots, \sigma_N)}$$

Meurice, Sakai, Unmuth-Yockey,
Rev. Mod. Phys. **94**, 025005 (2022).

Tensor Network States for LGTs

- **Real-Time evolution:**
  - Not available in Wick rotated, Euclidean spacetimes, used in conventional Monte-Carlo path integral LGT calculations
  - Calculations in **quantum Hilbert spaces**, where states evolve in real time, instead of in Wick-rotated statistical mechanics analogies.

- **Sign problem:**
  - Appears in several scenarios with fermions (finite density), represented by Grassman variables in a Wick-rotated, Euclidean spacetime
  - Calculations in **quantum Hilbert spaces**: fermions are fermions, no integration over time dimension. If the problem arises, it can be the result of using a particular method, nothing general.

See the review paper by Bañuls and Cichy, Rep. Prog. Phys. 83 024401 (2020)

Numerical Algorithms

Statics:

- **DMRG**: Density Matrix Renormalization group.

  Very successful for 1+1-d.
  (Very) challenging scaling to higher dimensions (best way to contract in more than 1d – nontrivial)

  White, 1992; Schollwöck, 2011

- **iPEPS:** infinite PEPS.

  Very powerful for infinite systems, also in 2+1-d.

  Jordan et al, 2008

Dynamics: for real / imaginary time evolution, mostly 1+1-d:

- **TEBD:** Time evolving block decimation.

  Vidal, 2007

- **TDVP:** Time dependent variational principle.

  Haegeman et al, 2011

MPS & LGT – Numerical Approach

- **Mostly in 1+1d**, combining MPS (Matrix Product States) with **DMRG (Density Matrix Renormalization Group)**; have been widely and successfully used for various many body models, mostly from condensed matter.

- **Very successfully applied to 1+1d lattice gauge theories**, including finite chemical potential and real time evolution (string breaking) for Abelian and non-Abelian theories
  - Bañuls, Cichy, Cirac, Kühn, Jansen, Saito…
  - Dalmonte, Montangero, Pichler, Rico, Silvi, Tschirsich, Zoller…
  - Buyens, Haegeman, Hebenstreit, van Acoleyen, Verschelde, Verstraete…
  - Borla, Moroz, Grusdt, Verresen… (rather more CM-like)

- High dimensional generalizations: challenging and demanding scaling; works nicely for ladders, cylinders etc.

See the review paper by Bañuls and Cichy, Rep. Prog. Phys. 83 024401 (2020) and refs. therein

LGT DMRG – Early Works

PHYSICAL REVIEW D **66**, 013002 (2002)

Density matrix renormalization group approach to the massive Schwinger model

T. M. R. Byrnes, P. Sriganesh, R. J. Bursill, and C. J. Hamer
School of Physics, The University of New South Wales, Sydney, NSW 2052, Australia
(Received 10 February 2002; published 1 July 2002)

The massive Schwinger model is studied using a density matrix renormalization group approach to the staggered lattice Hamiltonian version of the model. Lattice sizes up to 256 sites are calculated, and the estimates in the continuum limit are almost two orders of magnitude more accurate than previous calculations. Coleman's picture of "half-asymptotic" particles at a background field $\theta = \pi$ is confirmed. The predicted phase transition at finite fermion mass (m/g) is accurately located and demonstrated to belong in the 2D Ising universality class.

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PUBLISHED: *July 12, 2005*

Matrix product representation of gauge invariant states in a $\mathbb{Z}_2$ lattice gauge theory

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Upton, New York 11973, U.S.A.
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ABSTRACT: The Gauss law needs to be imposed on quantum states to guarantee gauge invariance when one studies gauge theory in hamiltonian formalism. In this work, we propose an efficient variational method based on the matrix product ansatz for a $\mathbb{Z}_2$ lattice gauge theory on a spatial ladder chain. Gauge invariant low-lying states are identified by evaluating expectation values of the Gauss law operator after numerical diagonalization of the gauge hamiltonian.

U(1) in 1+1 with MPS

The mass spectrum of the Schwinger model with matrix product states

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E-mail: banulsm@mpq.mpg.de, krzysztof.cichy@desy.de, ignacio.cirac@mpq.mpg.de, karl.jansen@desy.de

Abstract: We show the feasibility of tensor network solutions for lattice gauge theories in Hamiltonian formulation by applying matrix product states algorithms to the Schwinger model with zero and non-vanishing fermion mass. We introduce new techniques to compute excitations in a system with open boundary conditions, and to identify the states corresponding to low momentum and different quantum numbers in the continuum. For the ground state and both the vector and scalar mass gaps in the massive case, the MPS technique attains precisions comparable to the best results available from other techniques.

PRL 112, 201601 (2014) PHYSICAL REVIEW LETTERS week ending 23 MAY 2014

Tensor Networks for Lattice Gauge Theories and Atomic Quantum Simulation

E. Rico,[1] T. Pichler,[1] M. Dalmonte,[2,3] P. Zoller,[2,3] and S. Montangero[1]

[1] Institute for Quantum Information Processing & IQST, Ulm University, D-89069 Ulm, Germany
[2] Institute for Theoretical Physics, Innsbruck University, A-6020 Innsbruck, Austria
[3] Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, A-6020 Innsbruck, Austria

(Received 9 January 2014; revised manuscript received 8 April 2014; published 23 May 2014)

We show that gauge invariant quantum link models, Abelian and non-Abelian, can be exactly described in terms of tensor networks states. Quantum link models represent an ideal bridge between high-energy and cold atom physics, as they can be used in cold atoms in optical lattices to study lattice gauge theories. In this framework, we characterize the phase diagram of a $(1+1)$D quantum link version of the Schwinger model in an external classical background electric field: the quantum phase transition from a charge and parity ordered phase with nonzero electric flux to a disordered one with a net zero electric flux configuration is described by the Ising universality class.

PHYSICAL REVIEW A **90**, 042305 (2014)

Quantum simulation of the Schwinger model: A study of feasibility

Stefan Kühn,* J. Ignacio Cirac, and Mari-Carmen Bañuls

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We analyze some crucial questions regarding the practical feasibility of quantum simulation for lattice gauge models. Our analysis focuses on two models suitable for the quantum simulation of the Schwinger Hamiltonian, or QED in $1+1$ dimensions, which we investigate numerically using tensor networks. In particular, we explore the effect of representing the gauge degrees of freedom with finite-dimensional systems and show that the results converge rapidly; thus even with small dimensions it is possible to obtain a reasonable accuracy. We also discuss the time scales required for the adiabatic preparation of the interacting vacuum state and observe that for a suitable ramping of the interaction the required time is almost insensitive to the system size and the dimension of the physical systems. Finally, we address the possible presence of noninvariant terms in the Hamiltonian that is realized in the experiment and show that for low levels of noise it is still possible to achieve a good precision for some ground-state observables, even if the gauge symmetry is not exact in the implemented model.

PRL 113, 091601 (2014) PHYSICAL REVIEW LETTERS

Matrix Product States for Gauge Field Theories

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The matrix product state formalism is used to simulate Hamiltonian lattice gauge theories. To this end, we define matrix product state manifolds which are manifestly gauge invariant. As an application, we study $(1+1)$-dimensional one flavor quantum electrodynamics, also known as the massive Schwinger model, and are able to determine very accurately the ground-state properties and elementary one-particle excitations in the continuum limit. In particular, a novel particle excitation in the form of a heavy vector boson is uncovered, compatible with the strong coupling expansion in the continuum. We also study full quantum nonequilibrium dynamics by simulating the real-time evolution of the system induced by a quench in the form of a uniform background electric field.

PHYSICAL REVIEW D **94**, 085018 (2016)

Hamiltonian simulation of the Schwinger model at finite temperature

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Using matrix product operators, the Schwinger model is simulated in thermal equilibrium. The variational manifold of gauge-invariant matrix product operators is constructed to represent Gibbs states. As a first application, the chiral condensate in thermal equilibrium is computed, and agreement with earlier studies is found. Furthermore, as a new application, the Schwinger model is probed with a fractional charged static quark-antiquark pair separated infinitely far from each other. A critical temperature beyond which the string tension is exponentially suppressed is found and is in qualitative agreement with analytical studies in the strong coupling limit. Finally, the *CT* symmetry breaking is investigated, and our results strongly suggest that the symmetry is restored at any nonzero temperature.

PRL **118**, 071601 (2017) PHYSICAL REVIEW LETTERS wo
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Density Induced Phase Transitions in the Schwinger Model: A Study with Matrix Product States

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(Received 11 November 2016; published 17 February 2017)

We numerically study the zero temperature phase structure of the multiflavor Schwinger model at nonzero chemical potential. Using matrix product states, we reproduce analytical results for the phase structure for two flavors in the massless case and extend the computation to the massive case, where no analytical predictions are available. Our calculations allow us to locate phase transitions in the mass-chemical potential plane with great precision and provide a concrete example of tensor networks overcoming the sign problem in a lattice gauge theory calculation.

PHYSICAL REVIEW D **95**, 094509 (2017)

Finite-representation approximation of lattice gauge theories at the continuum limit with tensor networks

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(Received 9 March 2017; published 24 May 2017)

It has been established that matrix product states can be used to compute the ground state and single-particle excitations and their properties of lattice gauge theories at the continuum limit. However, by construction, in this formalism the Hilbert space of the gauge fields is truncated to a finite number of irreducible representations of the gauge group. We investigate quantitatively the influence of the truncation of the infinite number of representations in the Schwinger model, one-flavor QED_2, with a uniform electric background field. We compute the two-site reduced density matrix of the ground state and the weight of each of the representations. We find that this weight decays exponentially with the quadratic Casimir invariant of the representation which justifies the approach of truncating the Hilbert space of the gauge fields. Finally, we compute the single-particle spectrum of the model as a function of the electric background field.

PHYSICAL REVIEW D **93**, 094512 (2016)

Chiral condensate in the Schwinger model with matrix product operators

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(Received 29 March 2016; published 24 May 2016)

Tensor network (TN) methods, in particular the matrix product states (MPS) ansatz, have proven to be a useful tool in analyzing the properties of lattice gauge theories. They allow for a very good precision, much better than standard Monte Carlo (MC) techniques for the models that have been studied so far, due to the possibility of reaching much smaller lattice spacings. The real reason for the interest in the TN approach, however, is its ability, shown so far in several condensed matter models, to deal with theories which exhibit the notorious sign problem in MC simulations. This makes it prospective for dealing with the nonzero chemical potential in QCD and other lattice gauge theories, as well as with real-time simulations. In this paper, using matrix product operators, we extend our analysis of the Schwinger model at zero temperature to show the feasibility of this approach also at finite temperature. This is an important step on the way to deal with the sign problem of QCD. We analyze in detail the chiral symmetry breaking in the massless and massive cases and show that the method works very well and gives good control over a broad range of temperatures, essentially from zero to infinite temperature.

Real Time and String Breaking for U(1)

PHYSICAL REVIEW X **6,** 011023 (2016)

Real-Time Dynamics in U(1) Lattice Gauge Theories with Tensor Networks

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Tensor network algorithms provide a suitable route for tackling real-time-dependent problems in lattice gauge theories, enabling the investigation of out-of-equilibrium dynamics. We analyze a U(1) lattice gauge theory in $(1+1)$ dimensions in the presence of dynamical matter for different mass and electric-field couplings, a theory akin to quantum electrodynamics in one dimension, which displays string breaking: The confining string between charges can spontaneously break during quench experiments, giving rise to charge-anticharge pairs according to the Schwinger mechanism. We study the real-time spreading of excitations in the system by means of electric-field and particle fluctuations. We determine a dynamical state diagram for string breaking and quantitatively evaluate the time scales for mass production. We also show that the time evolution of the quantum correlations can be detected via bipartite von Neumann entropies, thus demonstrating that the Schwinger mechanism is tightly linked to entanglement spreading. To present a variety of possible applications of this simulation platform, we show how one could follow the real-time scattering processes between mesons and the creation of entanglement during scattering processes. Finally, we test the quality of quantum simulations of these dynamics, quantifying the role of possible imperfections in cold atoms, trapped ions, and superconducting circuit systems. Our results demonstrate how entanglement properties can be used to deepen our understanding of basic phenomena in the real-time dynamics of gauge theories such as string breaking and collisions.

PHYSICAL REVIEW X **6,** 041040 (2016)

Confinement and String Breaking for QED$_2$ in the Hamiltonian Picture

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The formalism of matrix product states is used to perform a numerical study of $(1+1)$-dimensional QED—also known as the (massive) Schwinger model—in the presence of an external static "quark" and "antiquark". We obtain a detailed picture of the transition from the confining state at short interquark distances to the broken-string "hadronized" state at large distances, and this for a wide range of couplings, recovering the predicted behavior both in the weak- and strong-coupling limit of the continuum theory. In addition to the relevant local observables like charge and electric field, we compute the (bipartite) entanglement entropy and show that subtraction of its vacuum value results in a UV-finite quantity. We find that both string formation and string breaking leave a clear imprint on the resulting entropy profile. Finally, we also study the case of fractional probe charges, simulating for the first time the phenomenon of partial string breaking.

PHYSICAL REVIEW D **96,** 114501 (2017)

Real-time simulation of the Schwinger effect with matrix product states

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Matrix Product States (MPS) are used for the simulation of the real-time dynamics induced by an electric quench on the vacuum state of the massive Schwinger model. For small quenches it is found that the obtained oscillatory behavior of local observables can be explained from the single-particle excitations of the quenched Hamiltonian. For large quenches damped oscillations are found and comparison of the late time behavior with the appropriate Gibbs states seems to give some evidence for the onset of thermalization. Finally, the MPS real-time simulations are compared with results from real-time lattice gauge theory which are expected to agree in the limit of large quenches.

Non Abelian (including dynamics)

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ABSTRACT: Using matrix product states, we explore numerically the phenomenology of string breaking in a non-Abelian lattice gauge theory, namely 1+1 dimensional SU(2). The technique allows us to study the static potential between external heavy charges, as traditionally explored by Monte Carlo simulations, but also to simulate the real-time dynamics of both static and dynamical fermions, as the latter are fully included in the formalism. We propose a number of observables that are sensitive to the presence or breaking of the flux string, and use them to detect and characterize the phenomenon in each of these setups.

PHYSICAL REVIEW X **7,** 041046 (2017)

Efficient Basis Formulation for (1 + 1)-Dimensional SU(2) Lattice Gauge Theory: Spectral Calculations with Matrix Product States

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We propose an explicit formulation of the physical subspace for a (1 + 1)-dimensional SU(2) lattice gauge theory, where the gauge degrees of freedom are integrated out. Our formulation is completely general, and might be potentially suited for the design of future quantum simulators. Additionally, it allows for addressing the theory numerically with matrix product states. We apply this technique to explore the spectral properties of the model and the effect of truncating the gauge degrees of freedom to a small finite dimension. In particular, we determine the scaling exponents for the vector mass. Furthermore, we also compute the entanglement entropy in the ground state and study its scaling towards the continuum limit.

Quantum
the open journal for quantum science
PAPERS PERSPECTI

Finite-density phase diagram of a $(1+1)-d$ non-abelian lattice gauge theory with tensor networks

Pietro Silvi[1,2], Enrique Rico[3], Marcello Dalmonte[4,5], Ferdinand Tschirsich[1], and Simone Montangero[1,6]

We investigate the finite-density phase diagram of a non-abelian $SU(2)$ lattice gauge theory in $(1+1)$-dimensions using tensor network methods. We numerically characterise the phase diagram as a function of the matter filling and of the matter-field coupling, identifying different phases, some of them appearing only at finite densities. For weak matter-field coupling we find a meson BCS liquid phase, which is confirmed by second-order analytical perturbation theory. At unit filling and for strong coupling, the system undergoes a phase transition to a charge density wave of single-site (spin-0) mesons via spontaneous chiral symmetry breaking. At finite densities, the chiral symmetry is restored almost everywhere, and the meson BCS liquid becomes a simple liquid at strong couplings, with the exception of filling two-thirds, where a charge density wave of mesons spreading over neighbouring sites appears. Finally, we identify two tri-critical points between the chiral and the two liquid phases which are compatible with a $SU(2)_2$ Wess-Zumino-Novikov-Witten model. Here we do not perform the continuum limit but we explicitly address the global $U(1)$ charge conservation symmetry.

Newer Works

SciPost SciPost Phys. 6, 028 (2019)

Phase diagram and conformal string excitations of square ice using gauge invariant matrix product states

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Abstract

We investigate the ground state phase diagram of square ice — a U(1) lattice gauge theory in two spatial dimensions — using gauge invariant tensor network techniques. By correlation function, Wilson loop, and entanglement diagnostics, we characterize its phases and the transitions between them, finding good agreement with previous studies. We study the entanglement properties of string excitations on top of the ground state, and provide direct evidence of the fact that the latter are described by a conformal field theory. Our results pave the way to the application of tensor network methods to confining, two-dimensional lattice gauge theories, to investigate their phase diagrams and low-lying excitations.

PHYSICAL REVIEW D **104**, 114501 (2021)

Entanglement generation in (1+1)D QED scattering processes

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We study real-time meson-meson scattering processes in (1 + 1)-dimensional QED by means of tensor networks. We prepare initial meson wave packets with given momentum and position introducing an approximation based on the free fermions model. Then, we compute the dynamics of two initially separated colliding mesons, observing a rich phenomenology as the interaction strength and the initial states are varied in the weak and intermediate coupling regimes. Finally, we consider elastic collisions and measure some scattering amplitudes as well as the entanglement generated by the process. Remarkably, we identify two different regimes for the asymptotic entanglement between the outgoing mesons: it is perturbatively small below a threshold coupling, past which its growth as a function of the coupling abruptly accelerates.

PHYSICAL REVIEW RESEARCH **7**, 013322 (2025)

Quantum many-body scarring in a non-Abelian lattice gauge theory

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Quantum many-body scarring (QMBS) is an intriguing mechanism of weak ergodicity breaking that has recently spurred significant attention. Particularly prominent in Abelian lattice gauge theories (LGTs), an open question is whether QMBS nontrivially arises in non-Abelian LGTs. Here, we present evidence of robust QMBS in a non-Abelian SU(2) LGT with dynamical matter. Starting in product states that require little experimental overhead, we show that prominent QMBS arises for certain quenches, facilitated through meson and baryon-antibaryon excitations, highlighting its non-Abelian nature. The uncovered scarred dynamics manifests as long-lived coherent oscillations in experimentally accessible local observables as well as prominent revivals in the state fidelity. Our findings bring QMBS to the realm of non-Abelian LGTs, highlighting the intimate connection between scarring and gauge symmetry, and are amenable for observation in a recently proposed trapped-ion qudit quantum computer.

Disorder-Free Localization and Fragmentation in a Non-Abelian Lattice Gauge Theory

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(Dated: May 9, 2025)

We investigate how isolated quantum many-body systems equilibrate when quenched far from equilibrium under non-Abelian gauge-symmetry constraints. By encoding gauge superselection sectors into static SU(2) background charges, we map out the dynamical phase diagram of a 1 + 1D SU(2) lattice gauge theory with dynamical matter. We uncover three distinct regimes: (i) an ergodic phase, (ii) a fragmented phase that is nonthermal but delocalized, and (iii) a disorder-free many-body localized regime. In the latter, a superposition of superselection sectors retains spatial matter inhomogeneities in time, as confirmed by distinctive temporal scalings of entropy. We highlight the non-Abelian nature of these phases and argue for potential realizations on qudit processors.

Newer works

Parton Distribution Functions in the Schwinger Model with Tensor Networks

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Parton distribution functions (PDFs) describe universal properties of bound states and allow us to calculate scattering amplitudes in processes with large momentum transfer. Calculating PDFs involves the evaluation of matrix elements with a Wilson line in a light-cone direction. In contrast to Monte Carlo methods in Euclidean spacetime, these matrix elements can be directly calculated in Minkowski-space using the Hamiltonian formalism. The necessary spatial- and time-evolution can be efficiently applied using established tensor network methods. We present PDFs in the Schwinger model calculated with matrix product states.

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PHYSICAL REVIEW D **111,** 014504 (2025)

Real-time scattering in the lattice Schwinger model

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Tensor network methods have demonstrated their suitability for the study of equilibrium properties of lattice gauge theories, even close to the continuum limit. We use them in an out-of-equilibrium scenario, much less explored so far, by simulating the real-time collisions of composite mesons in the lattice Schwinger model. Constructing wave-packets of vector mesons at different incoming momenta, we observe the opening of the inelastic channel in which two heavier mesons are produced and identify the momentum threshold. To detect the products of the collision in the strong coupling regime we propose local quantitites that could be measured in current quantum simulation platforms.

Parton Distribution Functions in the Schwinger model from Tensor Network States

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(Dated: April 19, 2025)

Parton distribution functions (PDFs) describe the inner, non-perturbative structure of hadrons. Their computation involves matrix elements with a Wilson line along a direction on the light cone, posing significant challenges in Euclidean lattice calculations, where the time direction is not directly accessible. We propose implementing the light-front Wilson line within the Hamiltonian formalism using tensor network techniques. The approach is demonstrated in the massive Schwinger model (quantum electrodynamics in 1+1 dimensions), a toy model that shares key features with quantum chromodynamics. We present accurate continuum results for the fermion PDF of the vector meson at varying fermion masses, obtained from first principle calculations directly in Minkowski space. Our strategy also provides a useful path for quantum simulations and quantum computing.

PACS numbers: 11.15.Ha,12.38.Gc,12.15Ff

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- Tagliacozzo, Vidal, **Entanglement renormalization and gauge symmetry**, PRB 2011
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- Tagliacozzo, Celi, Lewenstein, **Tensor Networks for Lattice Gauge Theories with Continuous Groups**, PRX 2014
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  - Gauge field Hilbert space = Matter Hilbert Space (Higgs-like theories)

- Zohar, Burrello, **Building projected entangled pair states with a local gauge symmetry**, NJP 2016
  - Different Gauge Field Hilbert spaces, allowing for fermionic constructions (matching the standard model content)

Hamiltonian LGT TNs in 2+1d and more

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- Zapp and Orus, **Tensor network simulation of QED on infinite lattices: learning from (1+1)d, and prospects for (2+1)d**, PRD 2014
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- Felser, Silvi, Collura, Montangero, **Two-dimensional quantum-link lattice Quantum Electrodynamics at finite density**, PRX 2020
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- Emonts, Bañuls, Cirac, Zohar, **Variational Monte Carlo simulation with tensor networks of a pure gauge Z3 theory in 2+1d**, PRD 2020
- Emonts, Kelman, Borla, Moroz, Gazit, Zohar, **Finding the ground state of a lattice gauge theory with fermionic tensor networks: A 2+1-D Z2 demonstration**, PRD 2023
- Kelman, Borla, Elyovich, Gomelski, Roose, Emonts, Zohar, **Gauged Gaussian PEPS - A High Dimensional Tensor Network Formulation for Lattice Gauge Theories**, PRD 2024

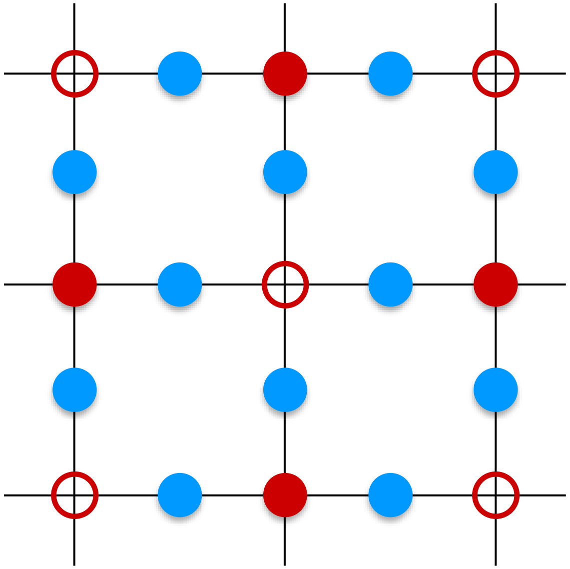
The LGT Hilbert Space

- **The lattice is spatial**: time is a continuous, real coordinate.

- **Matter particles** (fermions) – on the **vertices**.

- **Gauge fields** – on the lattice's **links**

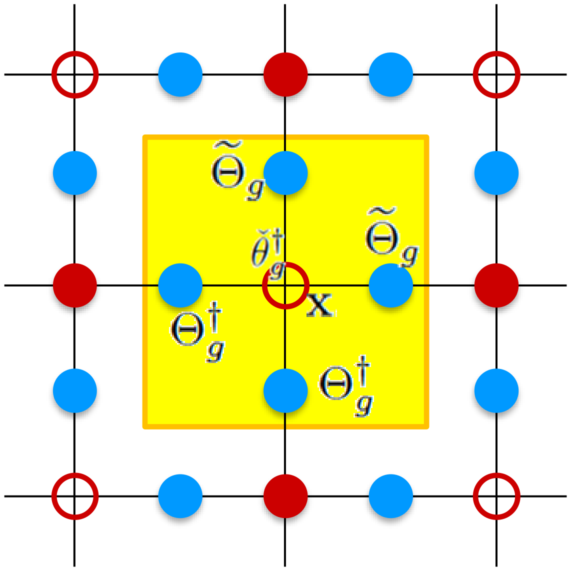
- Hamiltonian picture → Hilbert space
  → Natural way to describe constraints



Gauge Transformations

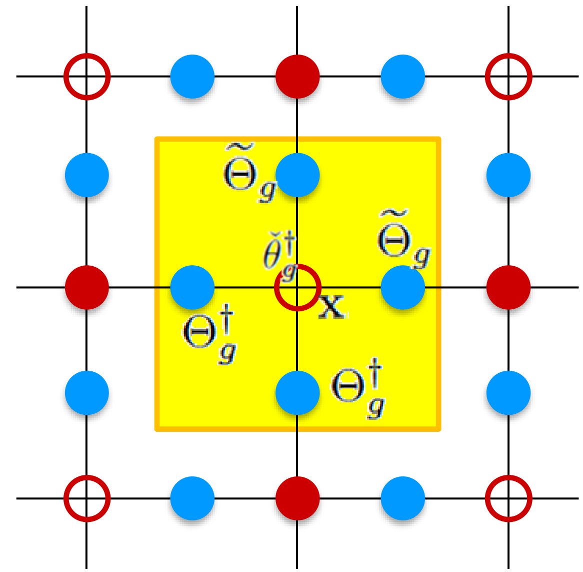
- Act on both the matter and gauge degrees of freedom.
- **Local** : a unique transformation (depending on a unique element of the **gauge group**) may be chosen for each site
- The states are **invariant under each local transformation separately**.

$$\hat{\Theta}_g(\mathbf{x}) = \prod_{k=1\ldots d}\left(\widetilde{\Theta}_g(\mathbf{x},k)\,\Theta_g^\dagger\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right)\breve{\theta}_g^\dagger(\mathbf{x})$$



Symmetry → Conserved Charge

– **Transformation rules on the links**



$$\{|g\rangle\}_{g\in G}$$

$$\Theta_g |h\rangle = \left|hg^{-1}\right\rangle \quad \Theta_g = e^{i\phi_a(g)R_a}$$

$$\widetilde{\Theta}_g |h\rangle = \left|g^{-1}h\right\rangle \quad \widetilde{\Theta}_g = e^{i\phi_a(g)L_a}$$

– **Gauge Transformations:**

$$\hat{\Theta}_g(\mathbf{x}) = \prod_{k=1\ldots d}\left(\widetilde{\Theta}_g(\mathbf{x},k)\,\Theta_g^\dagger\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right)\breve{\theta}_g^\dagger(\mathbf{x})$$

$$\hat{\Theta}_g(\mathbf{x})|\Psi\rangle = |\Psi\rangle \quad \forall \mathbf{x}, g$$

– **Generators → Gauss law , left and right E fields:**

$$G_a(\mathbf{x}) = \sum_{k=1\ldots d}\left(L_a(\mathbf{x},k) - R_a\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right) - Q_a(\mathbf{x})$$

$$G_a(\mathbf{x})|\Psi\rangle = 0 \quad [G_a(\mathbf{x}), H] = 0 \quad \forall \mathbf{x}, a$$

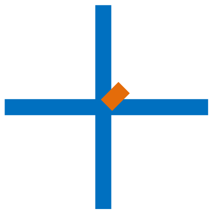
PEPS

- **Projected Entangled Pair States**: a particular tensor network construction, that
  - Allows to **encode and treat symmetries** in a very natural way.
  - Has, by construction, a **bipartite entanglement area law**, and therefore is suitable for describing "physically relevant" states.
  - Offers new approaches for the **study of phase diagrams and other properties of many body systems**.

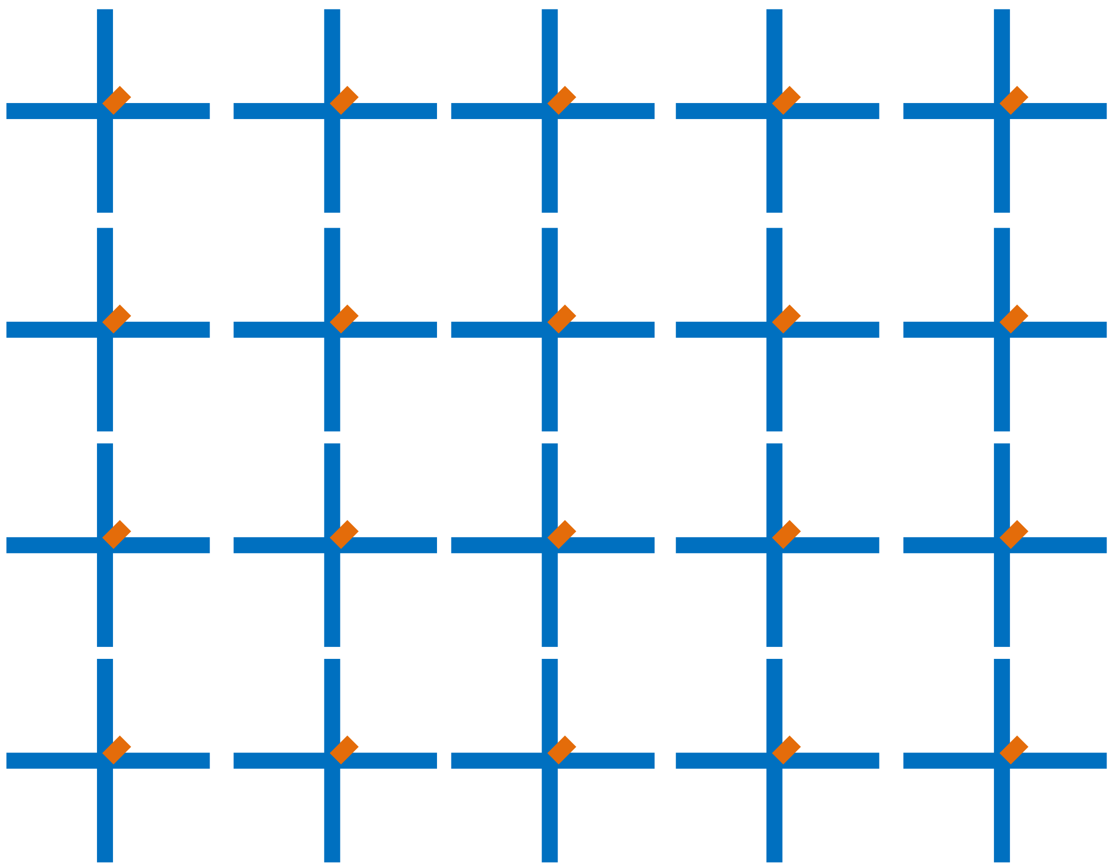
- In 1 space dimension – **MPS (Matrix Product States)**

PEPS

- Constructed out of local ingredients that include **physical** and **auxiliary** degrees of freedom.

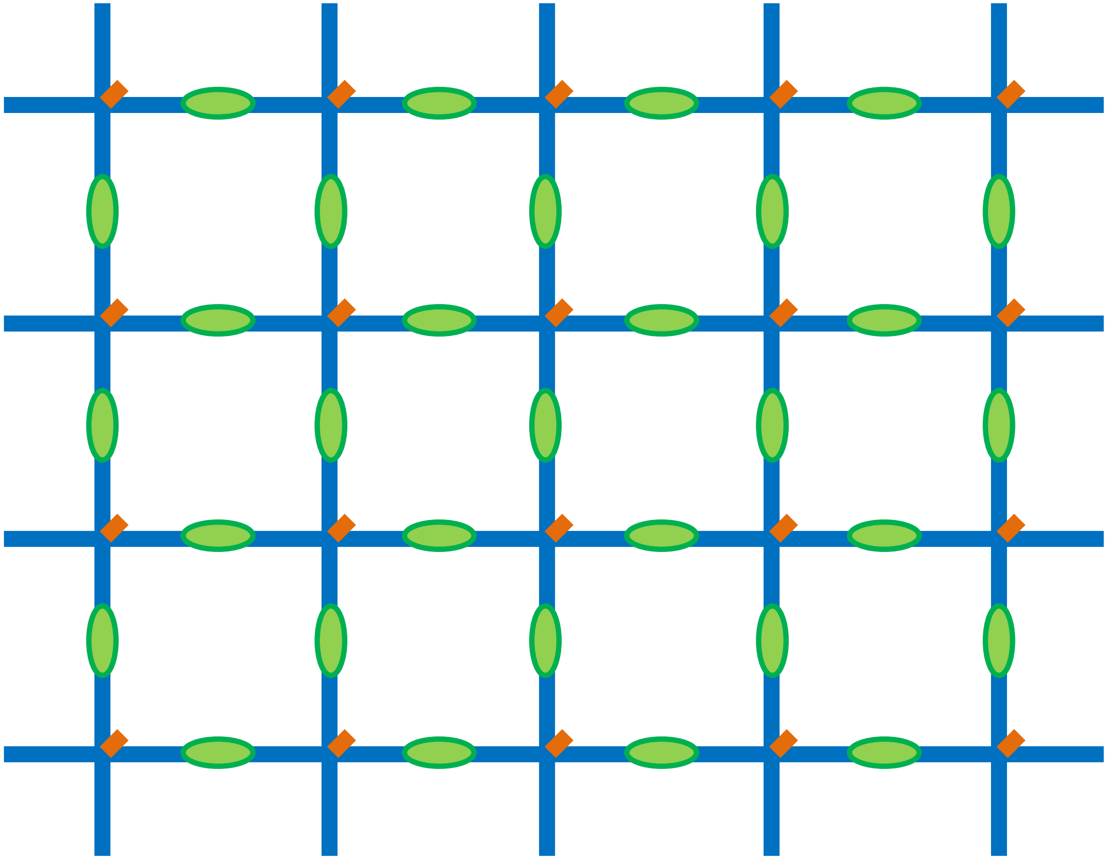


$$A(\mathbf{x})\,|\Omega\rangle$$



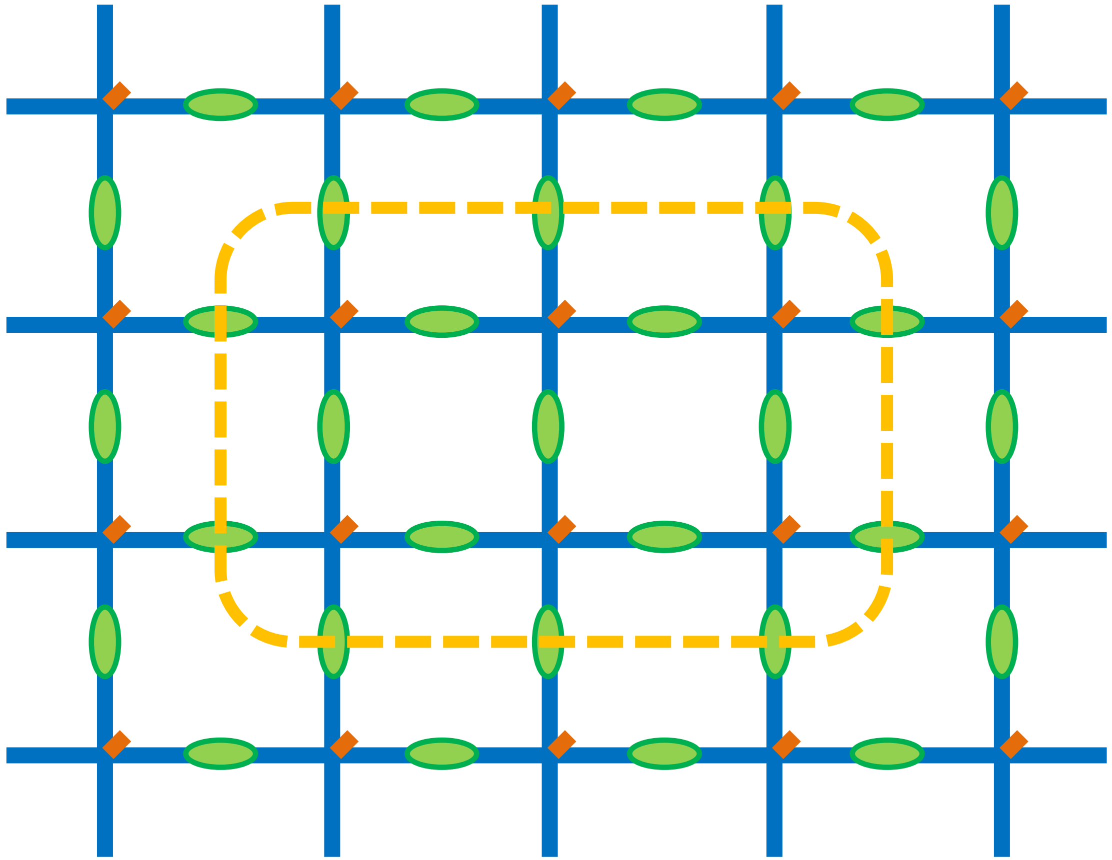
$$\prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$

- A **physical** only state is obtained out of projecting pairs of **auxiliary** degrees of freedom, on the two sides of a link, onto maximally entangled states.

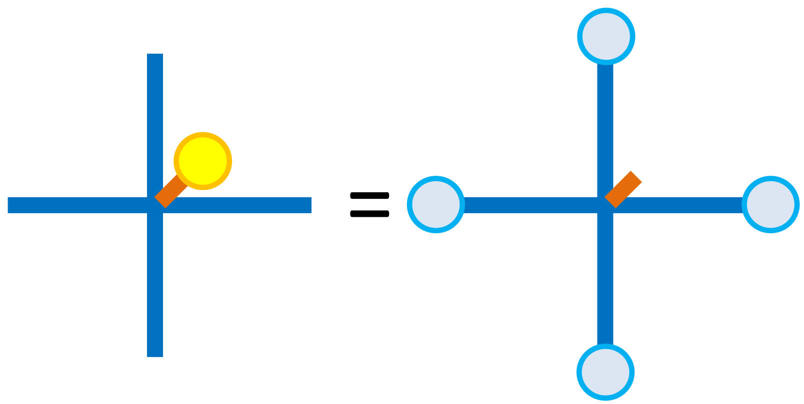


$$\left|\psi_0\right\rangle = \left\langle \Omega_v \right| \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} A\left(\mathbf{x}\right) \left|\Omega\right\rangle$$

- An entanglement area law is satisfied by construction.

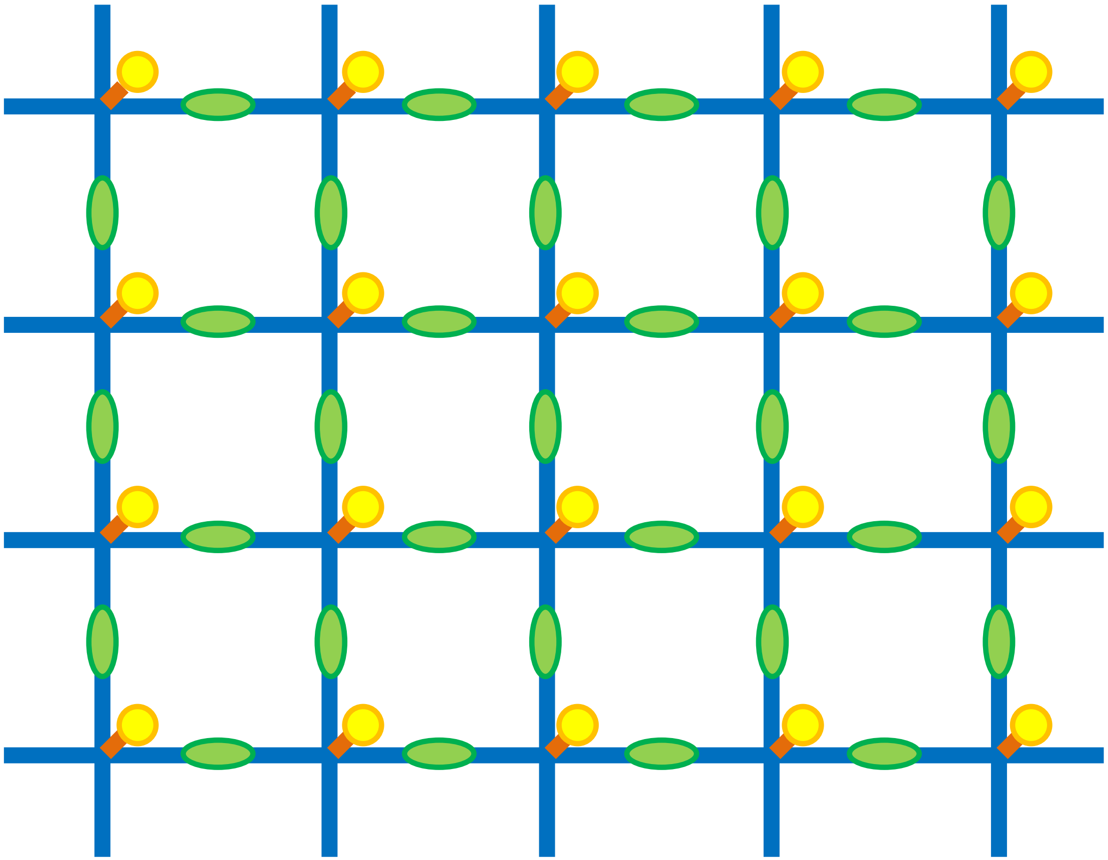


- Demanding global symmetry:
  - Acting with a group transformation on the **physical** degrees of freedom is equivalent to acting on the **auxiliary** ones.

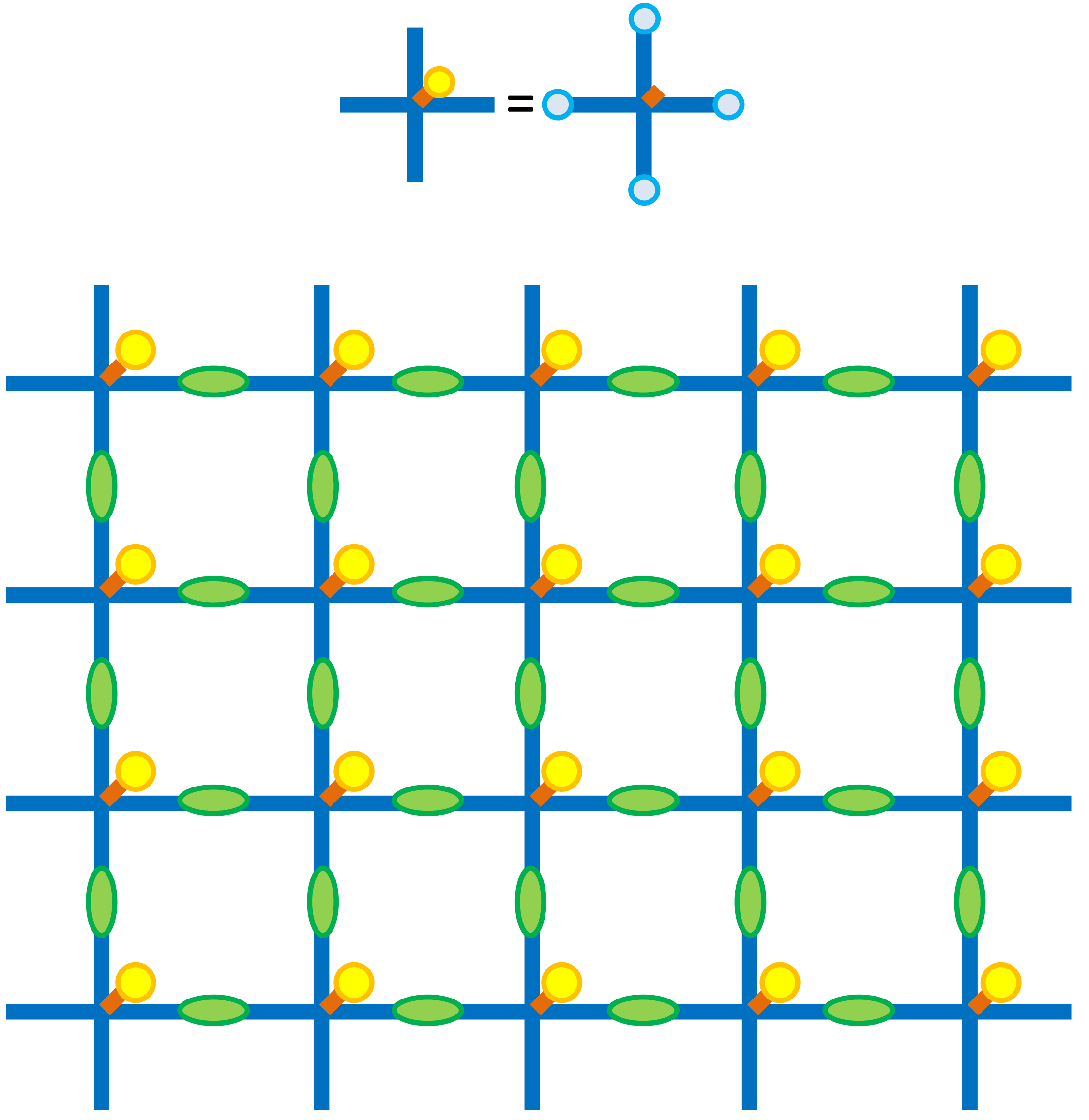


  - Projectors are invariant under group actions from both sides.

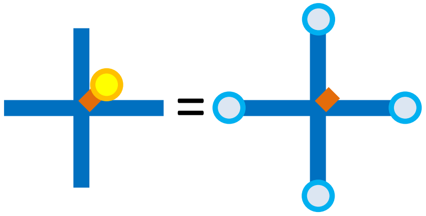
Global Transformation: $e^{i\Lambda\sum_{\mathbf{x}} Q(\mathbf{x})} |\psi_0\rangle$

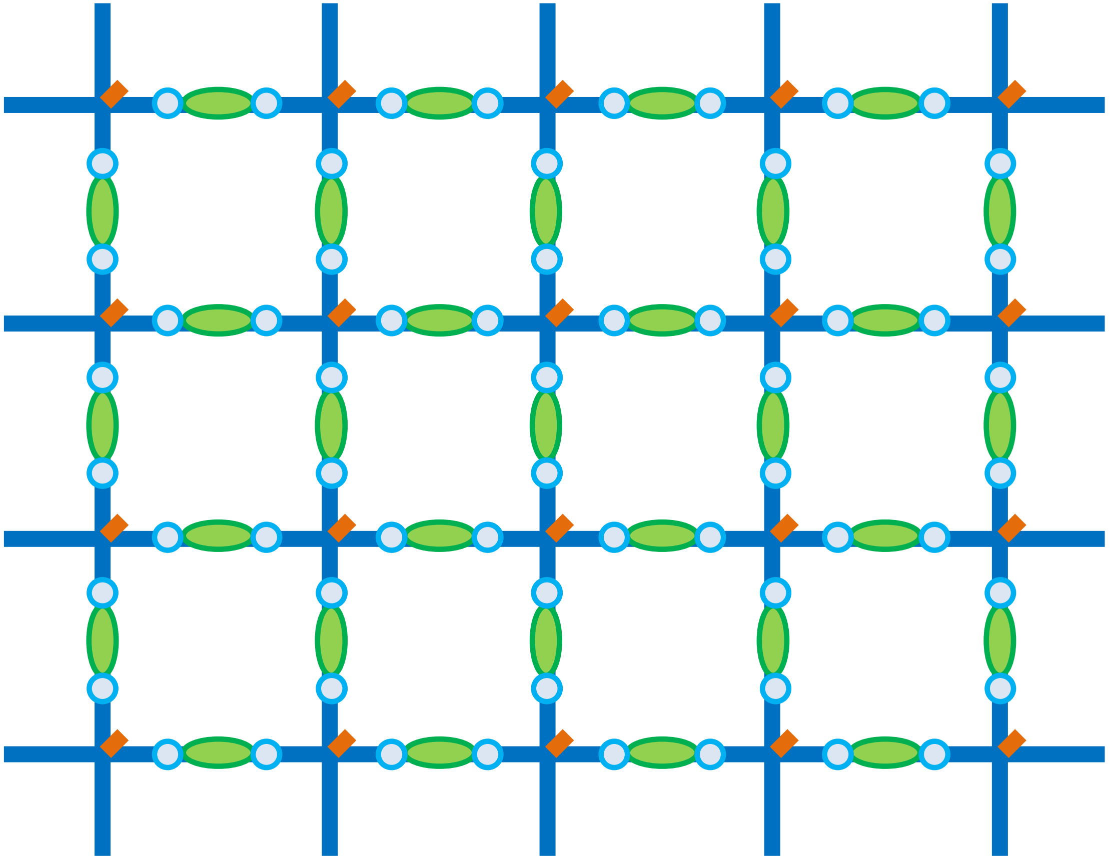


$$|\psi_0\rangle = \langle \Omega_v| \prod_{\ell} \omega_{\ell} \prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$

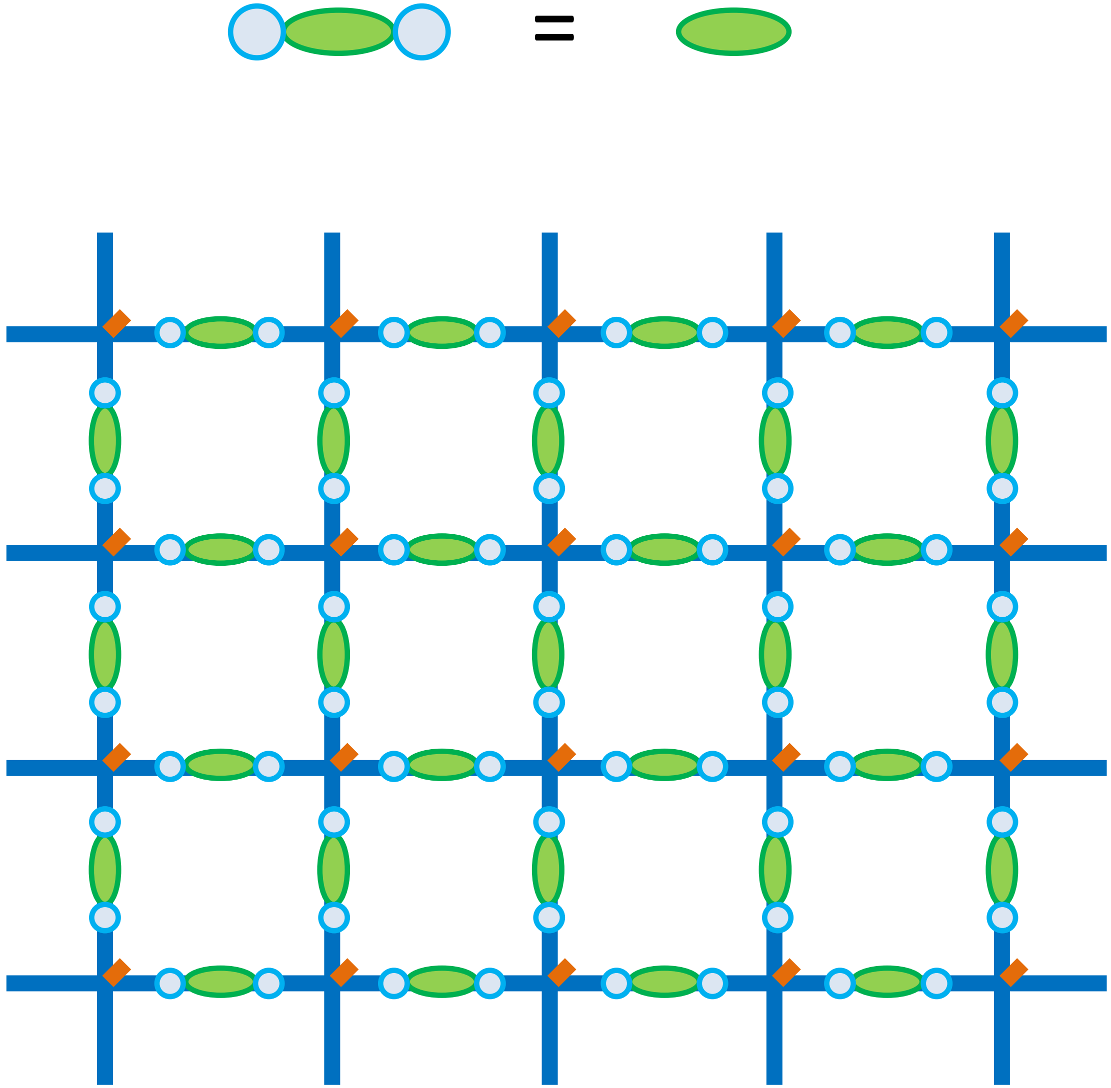


$$|\psi_0\rangle = \langle \Omega_v| \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$

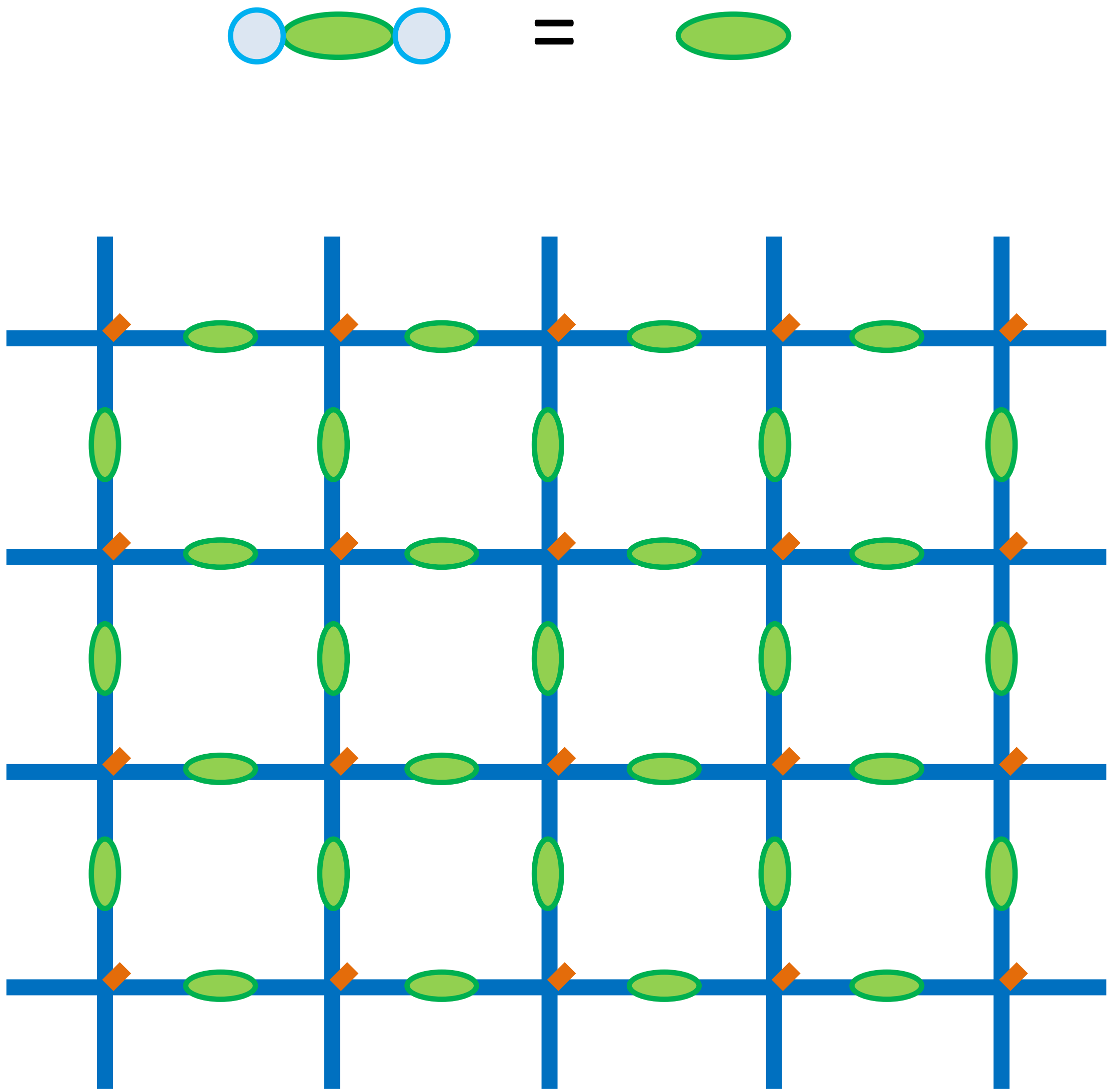




$$|\psi_0\rangle = \langle \Omega_v| \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$

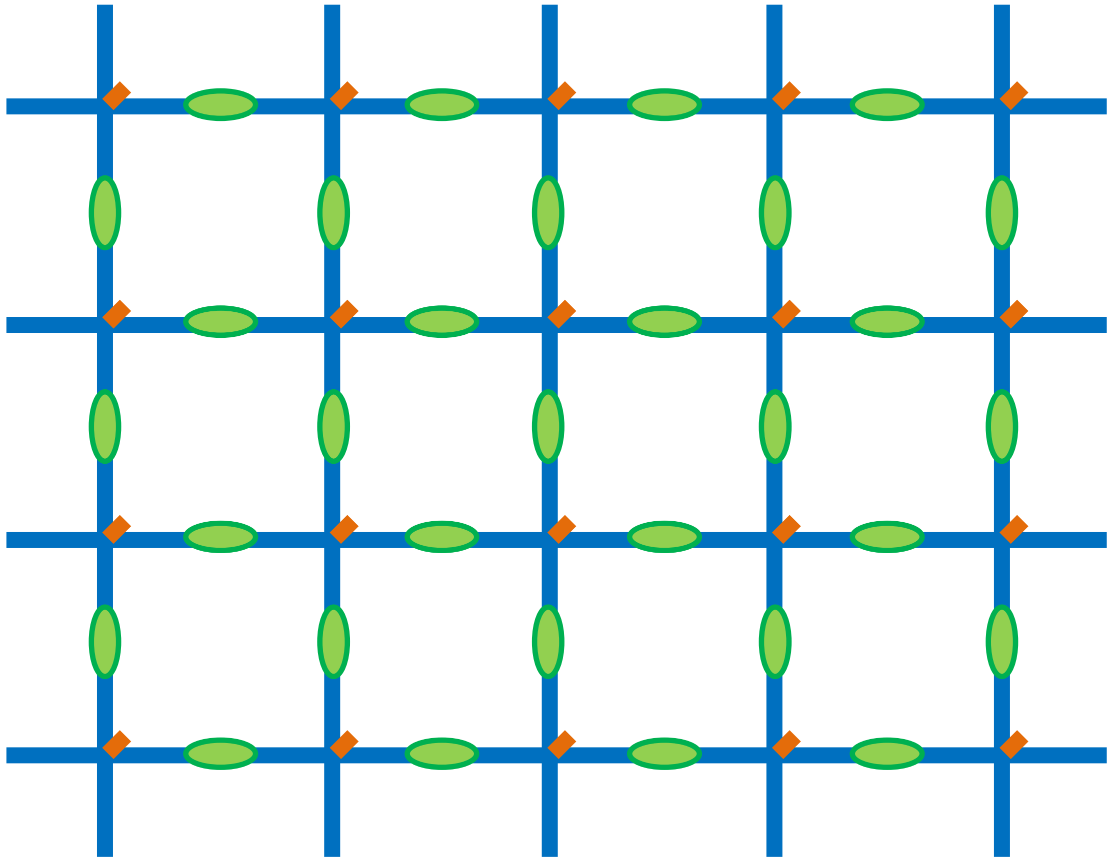


$$|\psi_0\rangle = \langle \Omega_v| \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$



$$|\psi_0\rangle = \langle \Omega_v| \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$

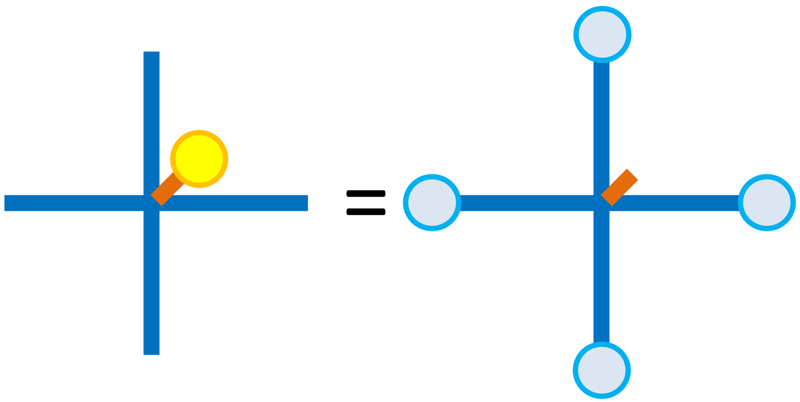
Global Symmetry: $e^{i\Lambda \sum_{\mathbf{x}} Q(\mathbf{x})} |\psi_0\rangle = |\psi_0\rangle$



$$|\psi_0\rangle = \langle \Omega_v| \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$

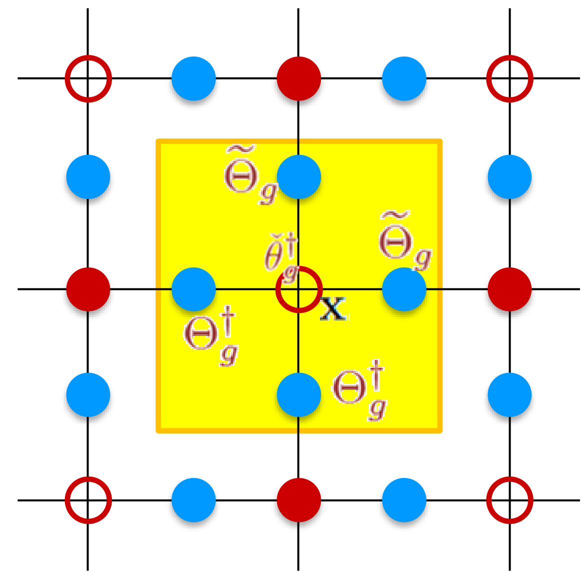
Virtual vs. Physical Gauge Invariance

Virtual - PEPS



Physical charge, but auxiliary electric fields: local symmetry exists, but it is auxiliary/virtual. The physical symmetry is global, after the bonds projection.

Physical – LGT states



$$\hat{\Theta}_g(\mathbf{x}) = \prod_{k=1\ldots d} \left(\widetilde{\Theta}_g(\mathbf{x},k)\, \Theta_g^\dagger\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right) \breve{\theta}_g^\dagger(\mathbf{x})$$

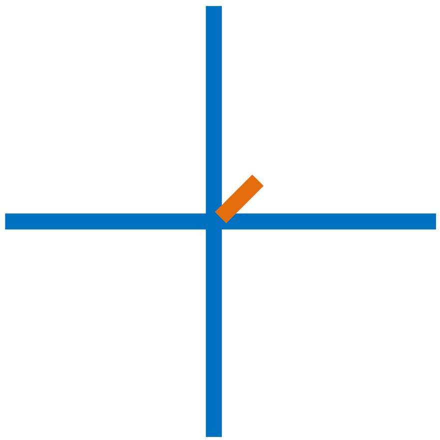
$$\hat{\Theta}_g(\mathbf{x})\left|\Psi\right\rangle = \left|\Psi\right\rangle \quad \forall \mathbf{x}, g$$

Fundamental analogy between PEPS and LGTs - making PEPS a suitable ansatz

Gauging the PEPS: minimal coupling of a state

- Lift the virtual symmetry to be physical:
  Lift the global symmetry to a local one.

$$|\psi_0\rangle = \langle \Omega_v | \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$



E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

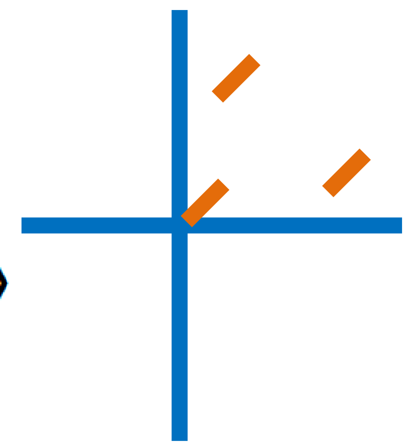
Gauging the PEPS: minimal coupling of a state

- Lift the virtual symmetry to be physical:
  The global to local.
- Step 1: Introduce gauge field Hilbert spaces on the links. Add (by a tensor product) the gauge field singlet states:

$$\left|\psi_0\right\rangle = \left\langle \Omega_v \right| \prod_{\ell} \omega_{\ell} \prod_{\mathbf{x}} A\left(\mathbf{x}\right) \left|\Omega\right\rangle$$

$$\downarrow$$

$$\left\langle \Omega_v \right| \prod_{\ell} \omega_{\ell} \prod_{\mathbf{x}} A\left(\mathbf{x}\right) \left|0\right\rangle_{\mathbf{x},1} \left|0\right\rangle_{\mathbf{x},2} \left|\Omega\right\rangle$$



E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

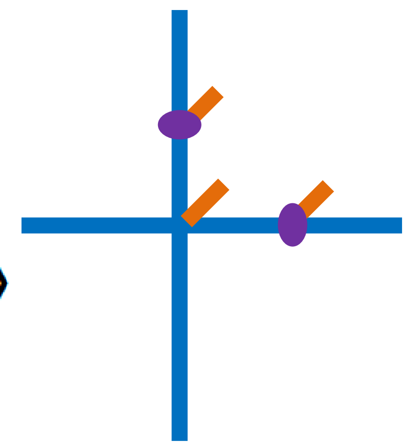
Gauging the PEPS: minimal coupling of a state

- Lift the **virtual** symmetry to be **physical**:
  The **global** to **local**.
- Step 2: Entangle the **auxiliary degrees** on the outgoing links with the **gauge fields**, by a unitary **gauging transformation** (map the auxiliary electric field information to the physical one).

$$|\psi_0\rangle = \langle \Omega_v| \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$

$$\downarrow$$

$$\langle \Omega_v| \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} A(\mathbf{x}) |0\rangle_{\mathbf{x},1} |0\rangle_{\mathbf{x},2} |\Omega\rangle$$



$$\downarrow$$

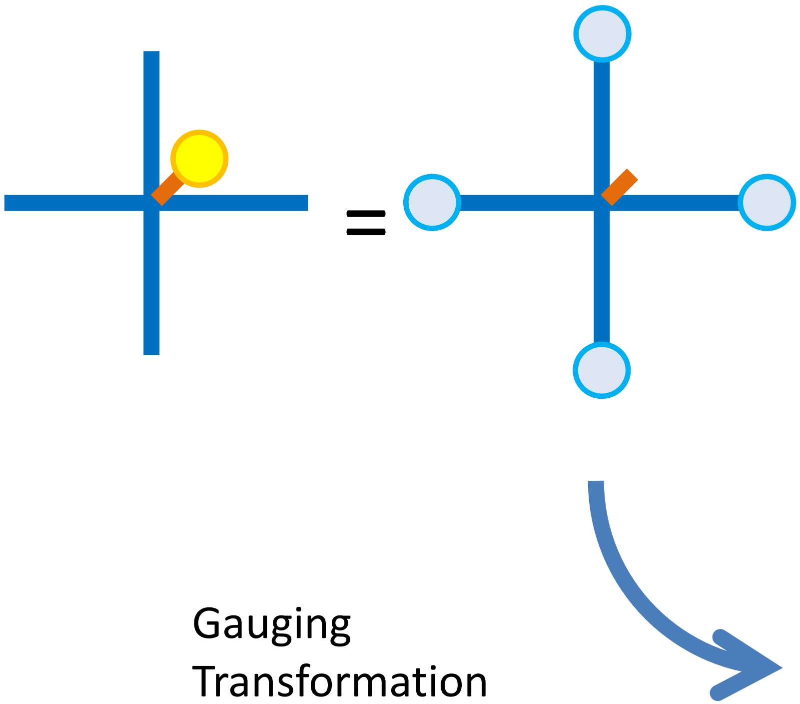
$$|\psi\rangle = \langle \Omega_v| \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} \mathcal{U}_G(\mathbf{x},1)\, \mathcal{U}_G(\mathbf{x},2) A(\mathbf{x}) |0\rangle_{\mathbf{x},1} |0\rangle_{\mathbf{x},2} |\Omega\rangle$$

The "initial" gauge field state may be replaced with something richer, as long as it is pure gauge invariant
Kelman, Borla, Gomelski, Elyovich, Roose, Emonts, **Zohar**, Phys. Rev. D 2024 110, 054511 (2024)

E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

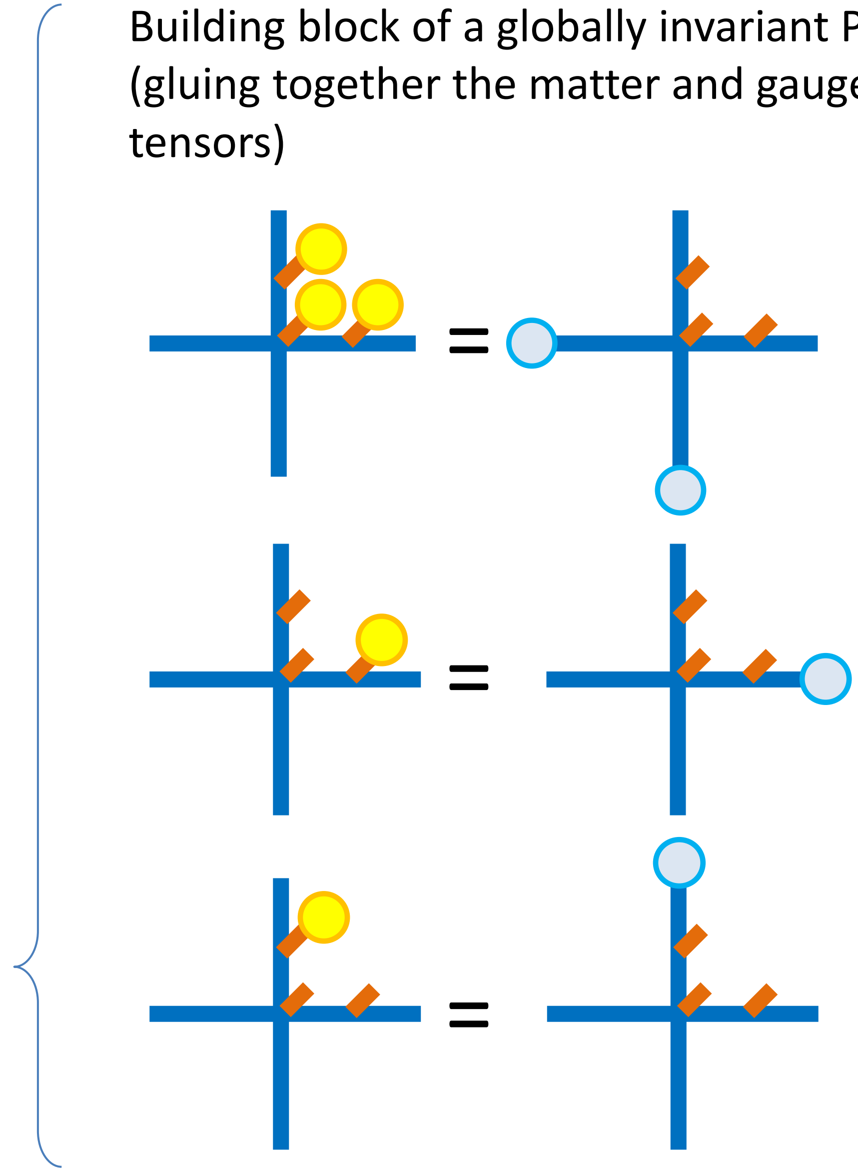
Gauging the PEPS: minimal coupling of a state

Building block of a globally invariant PEPS



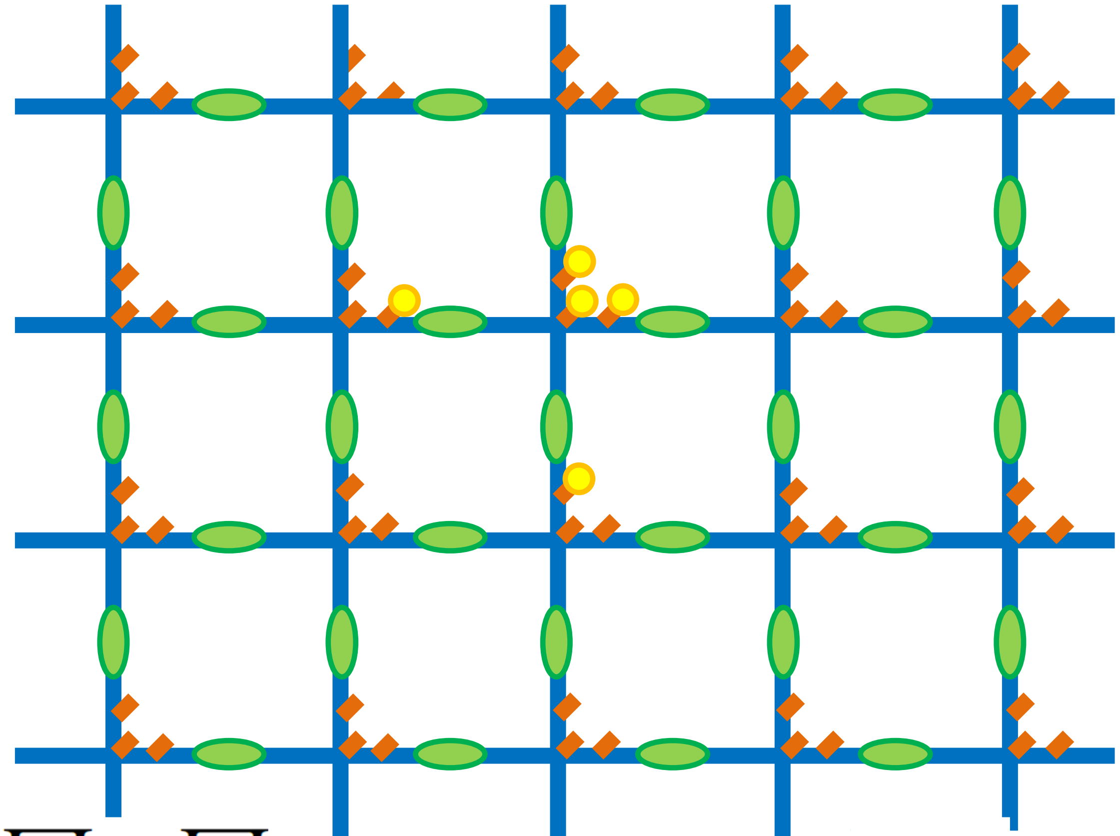
Gauging Transformation

Building block of a globally invariant PEPS (gluing together the matter and gauge field tensors)



E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

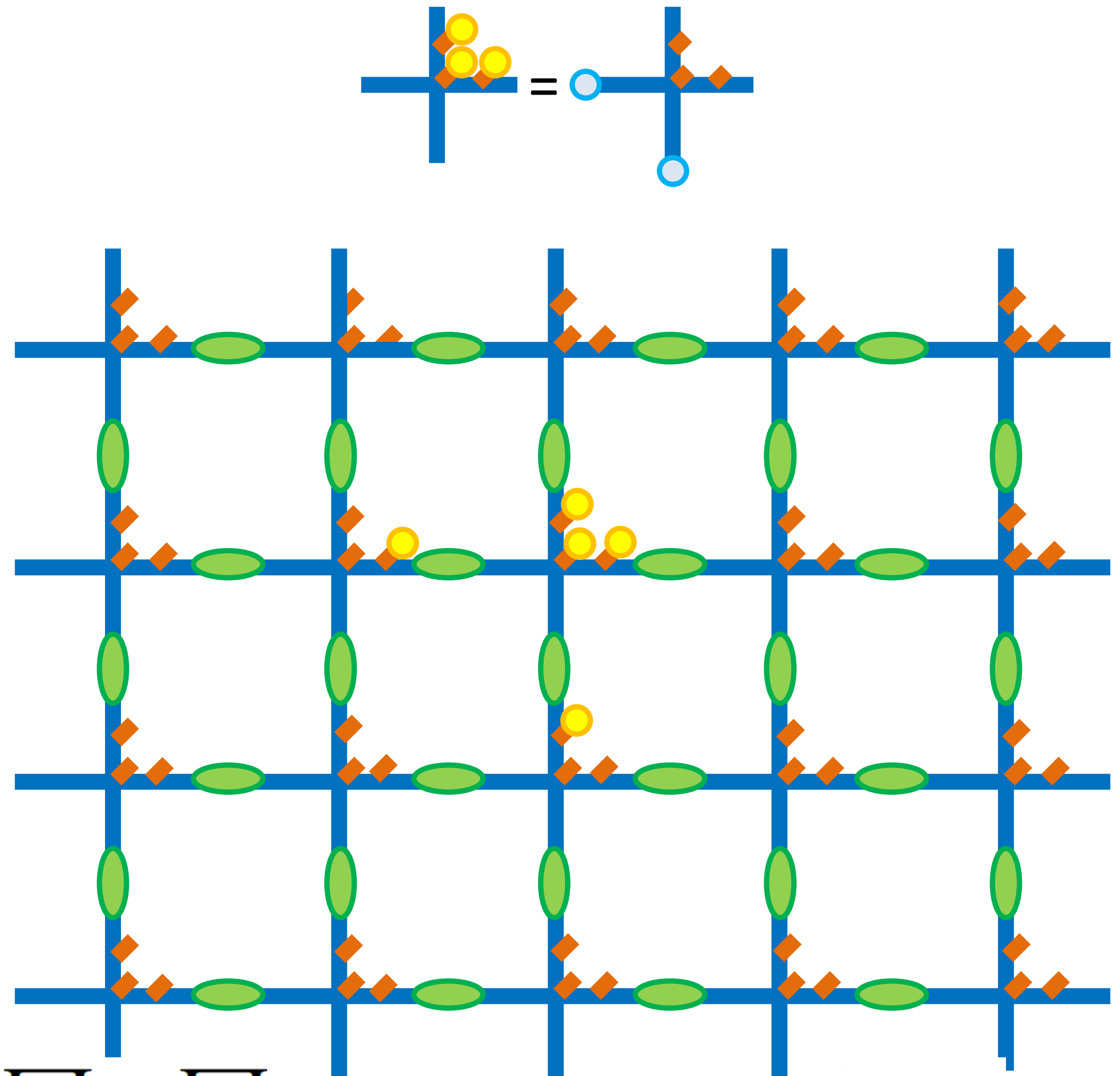
Local Transformation: $e^{i\Lambda\mathcal{G}(\mathbf{x}_0)}\left|\psi\right\rangle$



$$\left|\psi\right\rangle = \left\langle\Omega_v\right|\prod_{\ell}\omega_{\ell}\prod_{\mathbf{x}}\mathcal{U}_G\left(\mathbf{x},1\right)\mathcal{U}_G\left(\mathbf{x},2\right)A\left(\mathbf{x}\right)\left|0\right\rangle_{\mathbf{x},1}\left|0\right\rangle_{\mathbf{x},2}\left|\Omega\right\rangle$$

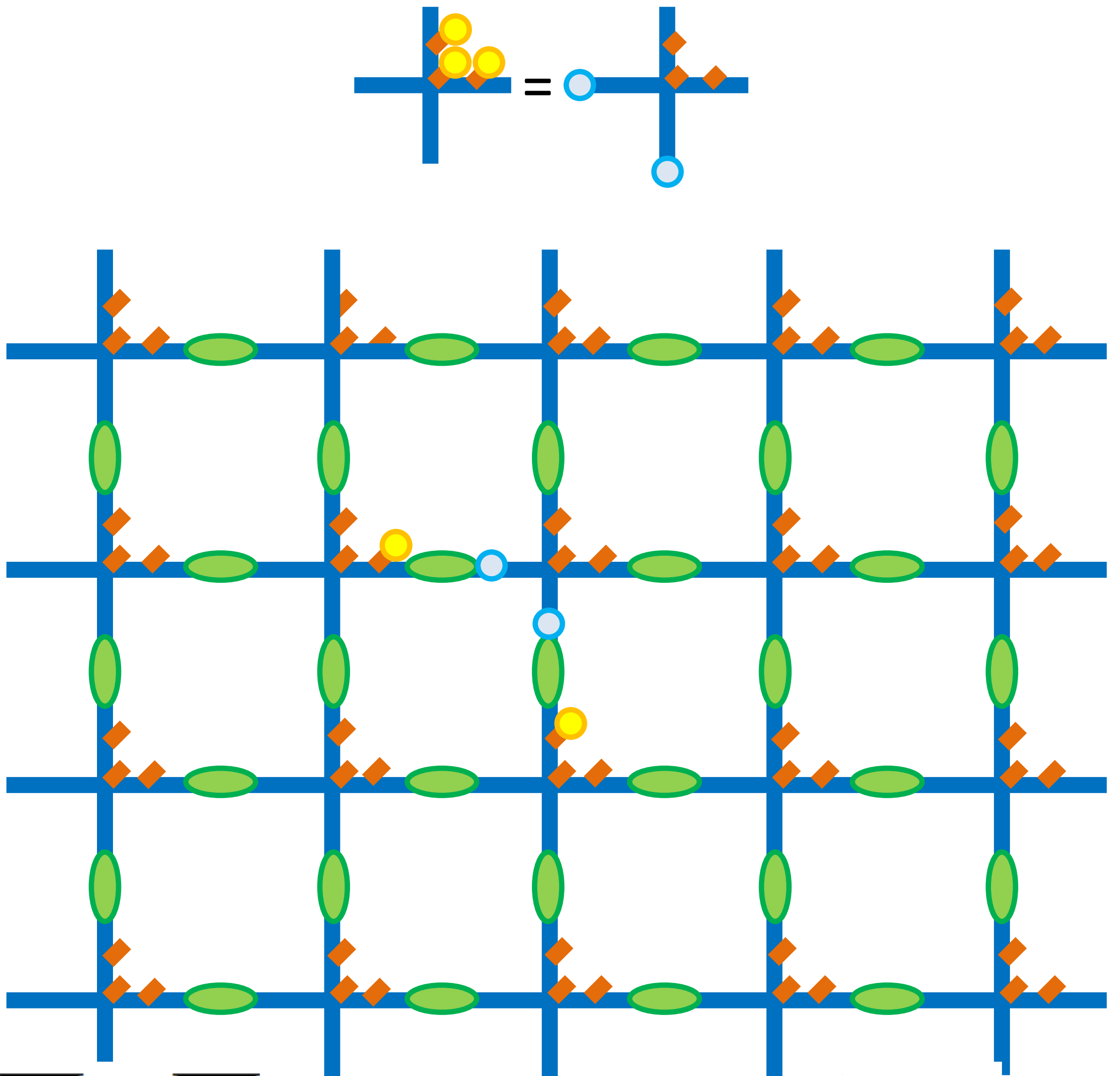
E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)



$$|\psi\rangle = \langle \Omega_v | \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} \mathcal{U}_G(\mathbf{x},1)\, \mathcal{U}_G(\mathbf{x},2) A(\mathbf{x}) |0\rangle_{\mathbf{x},1} |0\rangle_{\mathbf{x},2} |\Omega\rangle$$

E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)



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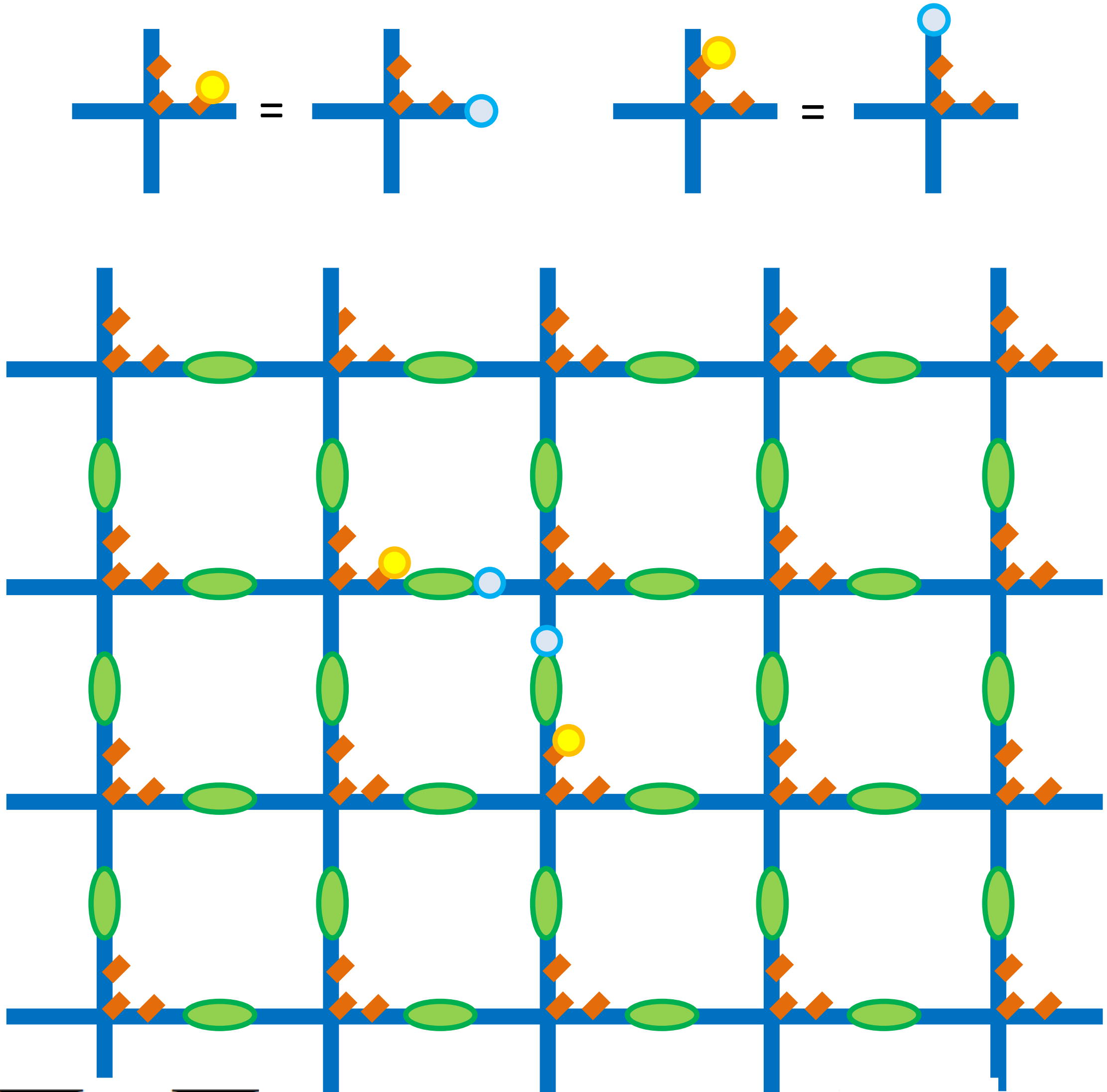
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)



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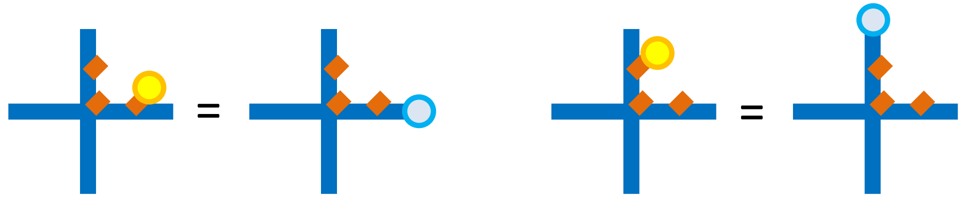
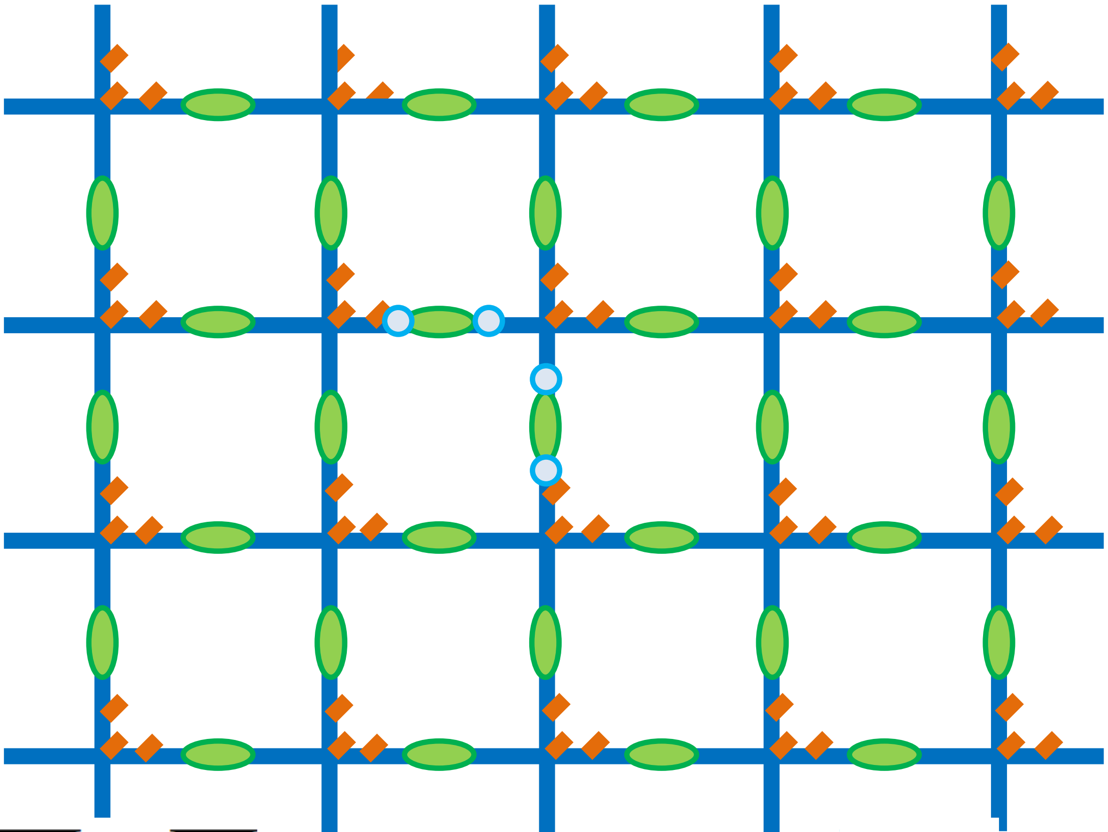
E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)





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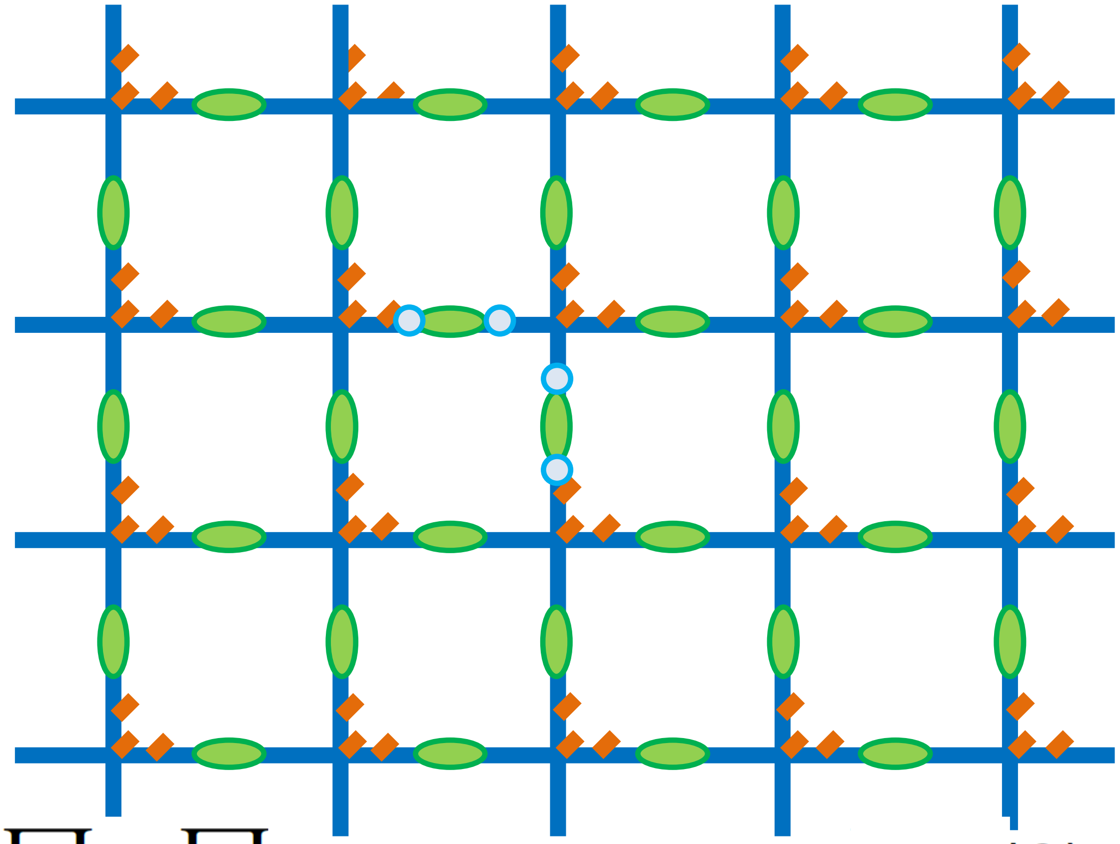
E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)



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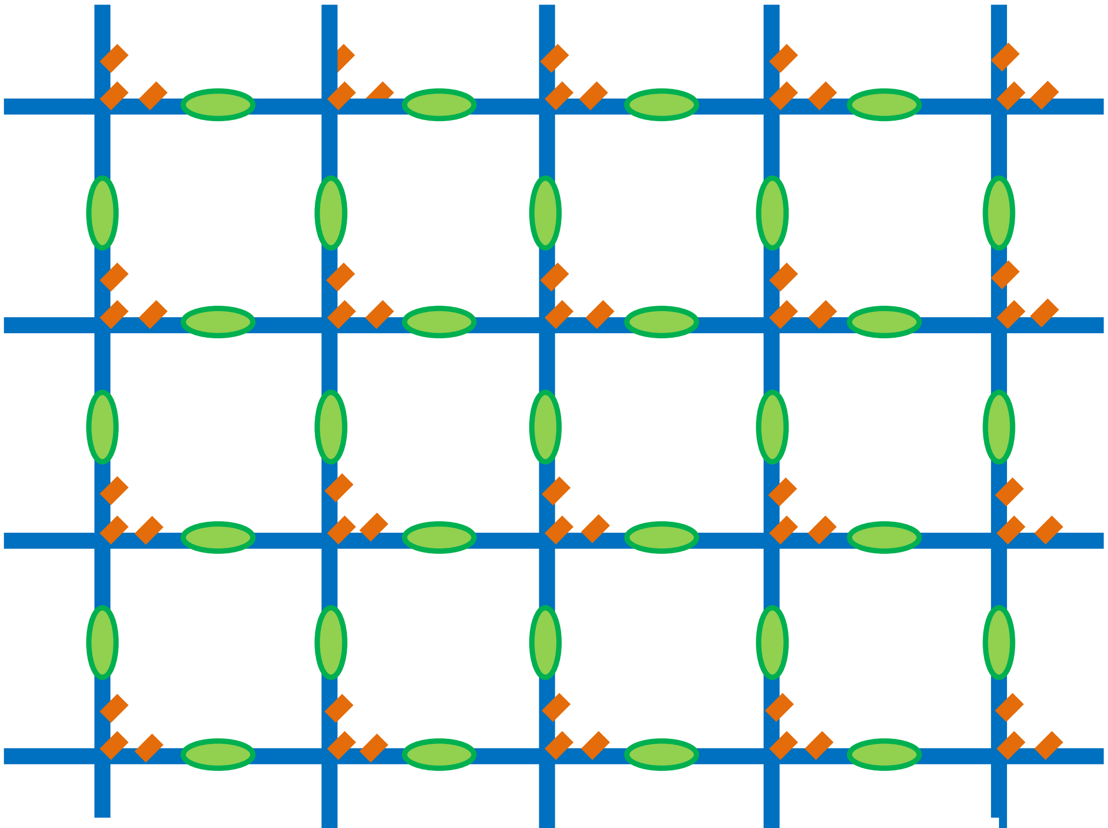
E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Local Symmetry: $e^{i\Lambda\mathcal{G}(\mathbf{x}_0)}\left|\psi\right\rangle = \left|\psi\right\rangle$

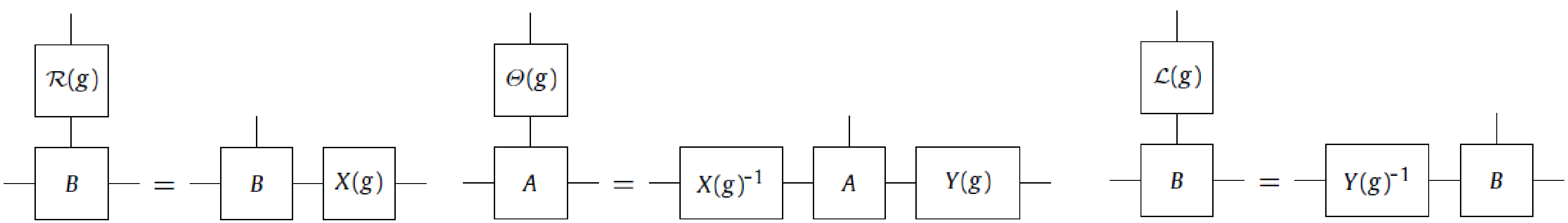


$$\left|\psi\right\rangle = \left\langle\Omega_v\right| \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} \mathcal{U}_G\left(\mathbf{x},1\right) \mathcal{U}_G\left(\mathbf{x},2\right) A\left(\mathbf{x}\right) \left|0\right\rangle_{\mathbf{x},1} \left|0\right\rangle_{\mathbf{x},2} \left|\Omega\right\rangle$$

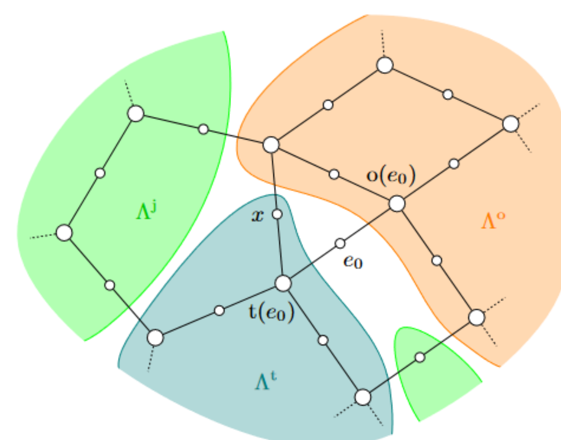
E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Is this the only way to gauge?

- We have shown that the current gauging mechanism produces LGT state with proper gauge invariance. But does this cover all the options for gauge invariant PEPS? YES!
  - Rigorously proven for MPS (PEPS in 1d) that once simple physical and mathematical properties (injectivity etc) are satisfied, gauge invariance implies the described PEPS structure.
    - Kull, Molnar, **Zohar**, Cirac, Ann. Phys 386, 199-241 (2017)



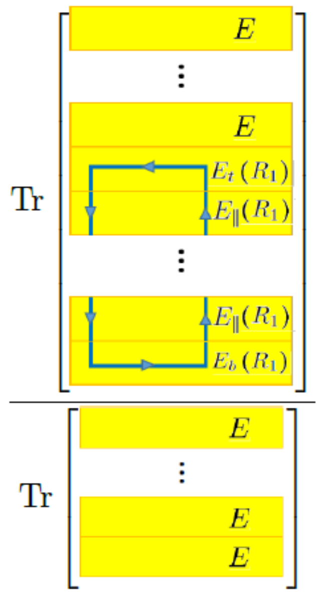
  - Recently completed a proof of the theorem for higher dimensions and arbitrary geometries.
    - Blanik, Garre-Rubio, Molnar, **Zohar** – arxiv:2410.18947!



What can the local tensors tell us about physics?

- Wilson loops:

$$\langle W(R_1, R_2)\rangle = $$



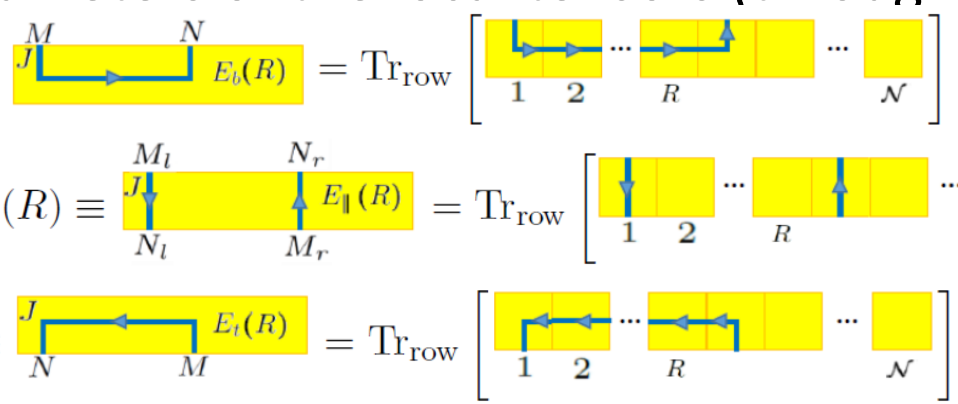
$$= \frac{\mathrm{Tr}\left[\hat{E}_b(R_1)\,\hat{E}_\parallel^{R_2-1}(R_1)\,\hat{E}_t(R_1)\,\hat{E}^{\mathcal{N}-R_2-1}\right]}{\mathrm{Tr}\left[E^{\mathcal{N}}\right]}$$

- One can formulate conditions for perimeter / area law, based on parameters of the local tensors (through transfer operators)

$$\left[\hat{E}_b\right]^J_{MN}(R) \equiv \quad = \mathrm{Tr}_{\mathrm{row}}\left[\quad\right]$$

$$\left[\hat{E}_\parallel\right]^J_{M_l N_l M_r N_r}(R) \equiv \quad = \mathrm{Tr}_{\mathrm{row}}\left[\quad\right]$$

$$\left[\hat{E}_t\right]^J_{MN}(R) \equiv \quad = \mathrm{Tr}_{\mathrm{row}}\left[\quad\right]$$



→ Highest eigenvalue proportional to *R* – (or simply *R* dependent) necessary condition for an area law (other conditions required!)

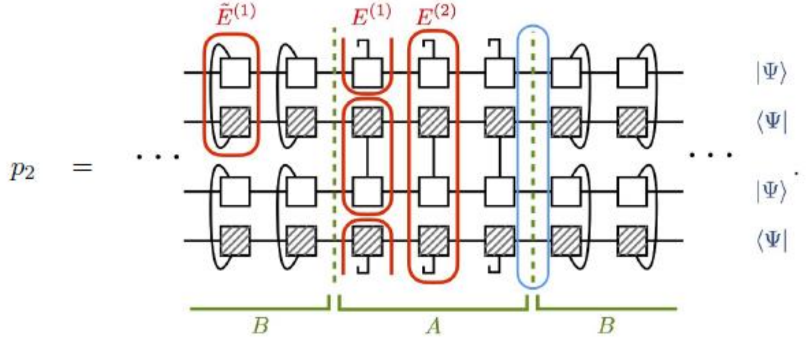
E. Zohar, Phys. Rev. Research 3, 033179 (2021)

Entanglement!

- Gauge symmetry → superselection sectors → one can introduce the and discuss **superselection resolved entanglement**, generalizing **symmetry resolved entanglement**

N. Feldman, **J. Knaute**, **E. Zohar** and M. Goldstein, JHEP05 083 (2024)

- The special construction allows for an easy computation of Renyi entropies.



J. Knaute, M. Feuerstein, E. Zohar, JHEP02 174 (2024)

Locally gauge invariant fermionic PEPS

- We We wish to describe PEPS of fermionic matter coupled to dynamical gauge fields.

- Starting point – **Gaussian fermionic PEPS** with a global symmetry.
  - **Gaussian states** – ground states of quadratic Hamiltonians, completely described by their covariance matrix. Very easy to handle analytically with the use of the Gaussian formalism.
  - **Fermionic PEPS** – defined with fermionic creation operators acting on the Fock vacuum. Easy to parameterize if they are Gaussian (Kraus, Schuch, Verstraete, Cirac, PRA 2011)

E. Zohar, M. Burrello, T.B. Wahl, and J.I. Cirac, Ann. Phys. 363, 385-439 (2015)
E. Zohar, T.B. Wahl, M. Burrello, and J.I. Cirac, Ann. Phys. 374, 84-137 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)
A. Kelman, U. Borla, I. Gomelski, J. Elyovich, G. Roose, P. Emonts, **E. Zohar**, Phys. Rev. D 2024 110, 054511 (2024)

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- **Start with these, then make the symmetry local and add the gauge field.**

E. Zohar, M. Burrello, T.B. Wahl, and J.I. Cirac, Ann. Phys. 363, 385-439 (2015)
E. Zohar, T.B. Wahl, M. Burrello, and J.I. Cirac, Ann. Phys. 374, 84-137 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)
A. Kelman, U. Borla, I. Gomelski, J. Elyovich, G. Roose, P. Emonts, **E. Zohar**, Phys. Rev. D 2024 110, 054511 (2024)

Locally gauge invariant fermionic PEPS

- We We wish to describe PEPS of fermionic matter coupled to dynamical gauge fields.

- Starting point – **Gaussian fermionic PEPS** with a global symmetry.
  - **Gaussian states** – ground states of quadratic Hamiltonians, completely described by their covariance matrix. Very easy to handle analytically with the use of the Gaussian formalism.
  - **Fermionic PEPS** – defined with fermionic creation operators acting on the Fock vacuum. Easy to parameterize if they are Gaussian (Kraus, Schuch, Verstraete, Cirac, PRA 2011)

- **Start with these, then make the symmetry local and add the gauge field.** Similar to **minimal coupling**: **Gauge a free matter state → obtain an interacting matter-gauge field state** without introducing further parameters.

E. Zohar, M. Burrello, T.B. Wahl, and J.I. Cirac, Ann. Phys. 363, 385-439 (2015)

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Magnetic Basis

- The physical Hilbert space: $\mathcal{H}_{\text{phys}} = \mathcal{H}_{\text{phys}}\left(\left\{q\left(\mathbf{x}\right)\right\}\right) \subset \mathcal{H}_{\text{gauge}} \times \mathcal{H}_{\text{matter}}$

- Gauge field configuration states:

$$\left|\mathcal{G}\right\rangle = \bigotimes_{\mathbf{x},k} \left|g\left(\mathbf{x},k\right)\right\rangle$$
$$\mathcal{DG} = \bigotimes_{\mathbf{x},k} dg\left(\mathbf{x},k\right)$$

- **General gauge invariant state**:

$$\left|\psi\right\rangle = \int \mathcal{DG}\left|\mathcal{G}\right\rangle_{\text{Gauge}} \left|\psi\left(\mathcal{G}\right)\right\rangle_{\text{Matter}}$$

Where $\left|\psi\left(\mathcal{G}\right)\right\rangle$ represents matter coupled to an external (classical) gauge field $\mathcal{G}$.

- E.g. for *U*(1): $\left|\Phi\right\rangle = \bigotimes_{\mathbf{x},k} \left|\phi\left(\mathbf{x},k\right)\right\rangle$

$$\mathcal{D}\Phi = \bigotimes_{\mathbf{x},k} d\phi\left(\mathbf{x},k\right)$$
$$\left|\psi\right\rangle = \int \mathcal{D}\Phi\left|\Phi\right\rangle_{\text{Gauge}} \left|\psi\left(\Phi\right)\right\rangle_{\text{Matter}}$$

Monte Carlo with gauged Gaussian fPEPS

- Expressing our states in the **magnetic basis** that allows us to perform **efficient Monte-Carlo calculations**

$$|\Psi\rangle = \int \mathcal{DG} \, |\mathcal{G}\rangle \, |\psi(\mathcal{G})\rangle$$

  - $|\mathcal{G}\rangle$ is a **fixed configuration state of the gauge field on the links**.

$$|\mathcal{G}\rangle \equiv \bigotimes_{\mathbf{x},k} |g(\mathbf{x},k)\rangle \qquad \mathcal{DG} = \prod_{\mathbf{x},k} dg(\mathbf{x},k)$$

$$\langle \mathcal{G}' | \mathcal{G} \rangle = \delta(\mathcal{G}', \mathcal{G})$$

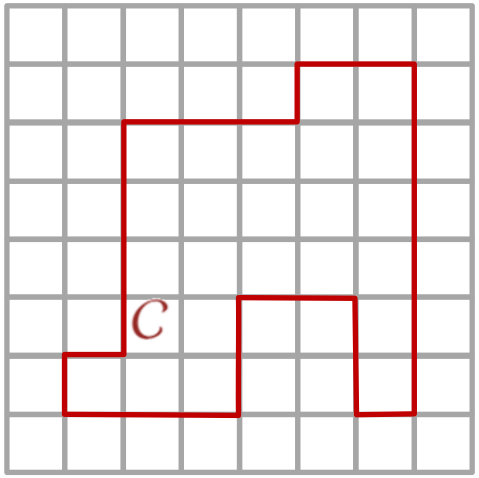
  - $|\psi(\mathcal{G})\rangle$ is a **fermionic Gaussian state**, representing **fermions coupled to a static, background gauge field** $\mathcal{G}$.

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Monte Carlo with gauged Gaussian fPEPS

- Wilson Loops:

$$W(C) = \mathrm{Tr}\left(\prod_{\{\mathbf{x},k\}\in C} U(\mathbf{x},k)\right)$$



Large rectangles –
detecting confinement of static charges (Wilson, 1974)

- The gauge configuration states are eigenstates thereof:

$$W(C)\left|\mathcal{G}\right\rangle = \mathrm{Tr}\left(\prod_{\{\boldsymbol{x},k\}\in C} D\left(g\left(\boldsymbol{x},k\right)\right)\right)\left|\mathcal{G}\right\rangle$$

Monte Carlo with gauged Gaussian fPEPS

- Wilson Loops: $$W(C) = \mathrm{Tr}\left(\prod_{\{\mathbf{x},k\}\in C} U(\mathbf{x},k)\right)$$

  - exp. value for $|\Psi\rangle = \int \mathcal{DG} |\mathcal{G}\rangle |\psi(\mathcal{G})\rangle$:

$$\langle W\rangle = \frac{\int \mathcal{DG}\,\mathrm{Tr}\left(\prod_{\{\boldsymbol{x},k\}\in C} D\left(g\left(\boldsymbol{x},k\right)\right)\right) \langle\psi(\mathcal{G})|\psi(\mathcal{G})\rangle}{\int \mathcal{DG}\,\langle\psi(\mathcal{G})|\psi(\mathcal{G})\rangle}$$

- The function

$$p(\mathcal{G}) = \frac{\langle\psi(\mathcal{G})|\psi(\mathcal{G})\rangle}{\int \mathcal{DG}'\,\langle\psi(\mathcal{G}')|\psi(\mathcal{G}')\rangle}$$

is a probability density.

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Monte Carlo with gauged Gaussian fPEPS

- Wilson Loops: $$W(C) = \mathrm{Tr}\left(\prod_{\{\mathbf{x},k\}\in C} U(\mathbf{x},k)\right)$$

  - exp. value for $|\Psi\rangle = \int \mathcal{D}\mathcal{G} \, |\mathcal{G}\rangle \, |\psi(\mathcal{G})\rangle$:

$$\langle W \rangle = \frac{\int \mathcal{D}\mathcal{G} \mathrm{Tr}\left(\prod_{\{\boldsymbol{x},k\}\in C} D(g(\boldsymbol{x},k))\right) \langle \psi(\mathcal{G}) | \psi(\mathcal{G}) \rangle}{\int \mathcal{D}\mathcal{G} \langle \psi(\mathcal{G}) | \psi(\mathcal{G}) \rangle}$$

- **The fermionic calculation is easy, through the gaussian formalism: very efficient**
- **No sign problem: the probability density is obtained from a norm of a state, and thus is real and positive.**

→ **Monte Carlo integration!**

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Monte Carlo with gauged Gaussian fPEPS

- The method is **extendable to further physical observables**, always involving the **probability density function**

$$p(\mathcal{G}) = \frac{\langle \psi(\mathcal{G}) | \psi(\mathcal{G}) \rangle}{\int \mathcal{D}\mathcal{G}' \langle \psi(\mathcal{G}') | \psi(\mathcal{G}') \rangle}$$

and possibly elements of the **covariance matrix of the Gaussian state** $|\psi(\mathcal{G})\rangle$, which could be calculated very efficiently.

 – For example, mesonic operators $M(\mathbf{x},\mathbf{y},C) = \psi^\dagger(\mathbf{x}) \prod_{\ell \in C} U(\ell) \psi(\mathbf{y})$

$$\langle M(\mathbf{x},\mathbf{y},C) \rangle = \int \mathcal{D}\Phi\, p(\Phi)\, e^{i\sum_{\ell \in C} \phi(\ell)} \frac{\langle \psi(\Phi) | \psi^\dagger(\mathbf{x}) \psi(\mathbf{y}) | \psi(\Phi) \rangle}{\langle \psi(\Phi) | \psi(\Phi) \rangle}$$

 (given for U(1) for simplicity).

- **It is possible to contract gauged Gaussian fPEPS beyond 1+1d, and without the sign problem of conventional LGT methods (it is not a Euclidean path integral).**

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Variational Monte Carlo

- We have a family of gauge invariant quantum states, depending on a finite set of parameters, allowing for efficient computation of any observable.
- In particular, we can compute the expectation value of the Hamiltonian while varying the parameters → variational search for the ground state.
- For a given state, compute relevant observables and identify the physics, based on the Hamiltonian parameters, and even prepare a phase diagram.

More in Patrick Emonts' talk

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Kelman, Borla, Gomelski, Elyovich, Roose**,** Emonts**, Zohar**, Phys. Rev. D 2024 110, 054511 (2024)

Current State of the Ansatz State

- Generalized formulation, including:
  - Flavor physics
  - Gauge fixing
  - Separation of longitudinal and transversal degrees of freedom
  - Improved sampling procedures

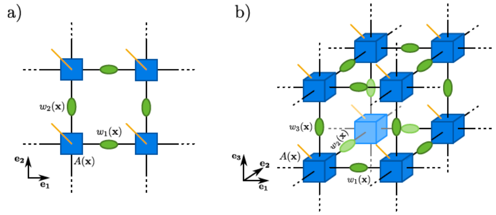
Kelman, Borla, Gomelski, Elyovich, Roose, Emonts, **Zohar**, Phys. Rev. D 2024 110, 054511 (2024)





- Straight forward generalization of the pre-gauging state to 3+1-d, for various lattice fermions prescriptions (naïve, staggered, Wilson).

  Gauging can be done exactly the same, as well as Monte-Carlo sampling; the dependence on the dimension is indirect, only through the number of links to be integrated.



Emonts, **Zohar**, Phys. Rev. D 108, 014514 (2023)

Does it capture all LGT ground states?

- Conventional LGT Hamiltonian: $$H = \underbrace{\int \mathcal{DG} \left|\mathcal{G}\right\rangle \left\langle\mathcal{G}\right| \otimes H_0\left(\mathcal{G}\right)}_{H_1} + \underbrace{H_{\mathrm{E}} \otimes \mathbb{I}}_{H_2}$$

  H_1: Diagonal in the group element basis; quadratic in the fermionic part

  H_2: Diagonal in the irrep basis, identity on fermions

- Imaginary time evolution, trotterized: $$e^{-\beta H} \approx \left(e^{-\frac{\beta}{N}H_1} e^{-\frac{\beta}{N}H_2}\right)^N$$

  $e^{-\frac{\beta}{N}H_1}$: Preserves Gaussianity: Maps GGFPEPS to themselves

  $e^{-\frac{\beta}{N}H_2}$: Doesn't preserve Gaussianity – Maps GGFPEPS to superpositions thereof

Therefore: **A gauged superposition of Gaussian fermionic PEPS describes exactly the ground state of a lattice gauge theory**

Roose, Zohar, arxiv:2412.01737

→ Suggest this state as a new Ansatz, still efficient when few Gaussians are present (inspired by Bravyi, Comm. Math. Phys 356, 451; 2017)

→ Few Gaussians are expected for states that are only non-Gaussian in a small patch of space. For example, bound states on top of the strongly interacting vacuum.

Future : Time Evolution With Gauged Gaussian PEPS

- If we expand our states in Gaussian ingredients, the time evolution should be very easy too (**not using tensor network algorithms**, again).
- Inspiration:

PHYSICAL REVIEW RESEARCH **2**, 043145 (2020)

Real-time dynamics in 2+1D compact QED using complex periodic Gaussian states

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We introduce a class of variational states to study ground-state properties and real-time dynamics in $(2+1)$-dimensional compact QED. These are based on complex Gaussian states which are made periodic to account for the compact nature of the $U(1)$ gauge field. Since the evaluation of expectation values involves infinite sums, we present an approximation scheme for the whole variational manifold. We calculate the ground-state energy density for lattice sizes up to 20×20 and extrapolate to the thermodynamic limit for the whole coupling region. Additionally, we study the string tension both by fitting the potential between two static charges and by fitting the exponential decay of spatial Wilson loops. As the ansatz does not require a truncation in the local Hilbert spaces, we analyze truncation effects which are present in other approaches. The variational states are benchmarked against exact solutions known for the one plaquette case and exact diagonalization results for a $\mathbb{Z}_3$ lattice gauge theory. Using the time-dependent variational principle, we study real-time dynamics after various global quenches, e.g., the time evolution of a strongly confined electric field between two charges after a quench to the weak-coupling regime. Up to the points where finite-size effects start to play a role, we observe equilibrating behavior.

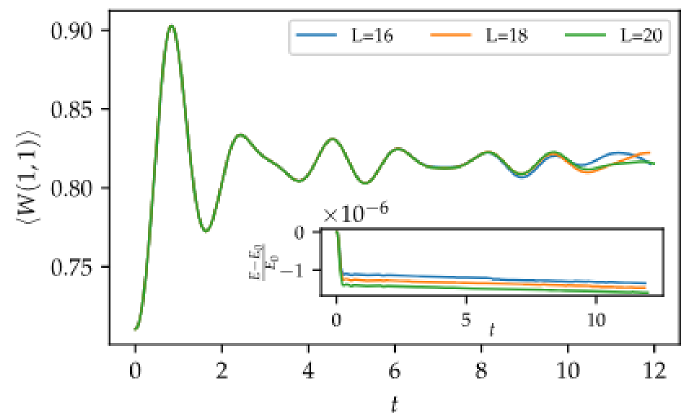


FIG. 11. Variational time evolution after a quench from $g^2 = 0.6$ to $g^2 = 0.3$ for lattice sizes of 16×16, 18×18, and 20×20. The inset shows the relative error in energy E with respect to the initial energy E_0 after the quench.

as well as many other recent works remarkably extending the Gaussian formalism (Shi, Demler and Cirac; Bravyi; Tagliacozzo …)

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

In particular (work in progress):

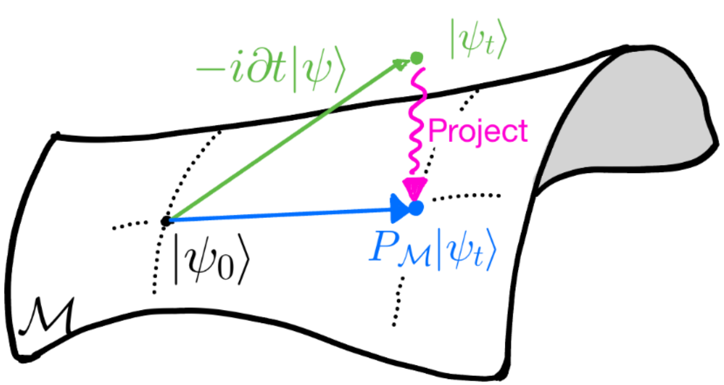
- Similarly to the imaginary time evolution, for real time:

$$e^{-iHT} \approx \left(e^{-\frac{iT}{N}H_1} e^{-\frac{iT}{N}H_2}\right)^N$$

Preserves Gaussianity: Maps GGFPEPS to themselves

Doesn't preserve Gaussianity – Maps GGFPEPS to superpositions thereof

- Therefore, if we project onto the manifold of fixed degree after every time step, corresponding to an effective nonlinear Schrödinger equation, as in the TDVP algorithm, and many recent extensions of Gaussian states:



$$-i\frac{\partial}{\partial t}|\psi\rangle = P_{T_{\mathcal{M}(|\psi\rangle)}} H |\psi\rangle$$

For idealized scenarios the degree of the targeted manifold may remain small.

→ Oscillations of quark anti-quark pairs

→ String breaking

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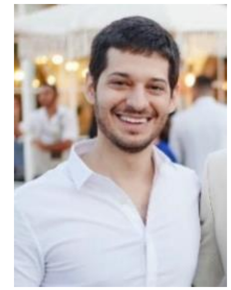

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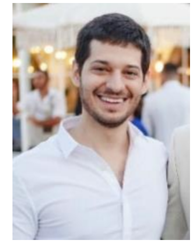

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