Quantum Gravity:

Where do we stand?

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CERN Theory Division, 21 September 2011

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Einstein equations *tie together matter and geometry*:

$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{classical?}} = \underbrace{\kappa T_{\mu\nu}}_{\text{quantum?}}$$

$\rightarrow$ mathematical and conceptual inconsistencies?

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- Singularities in General Relativity (GR)
  - – Black holes: gravitational collapse generically unavoidable
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- Structure of space-time at the smallest distances?

‘Smallest distance’: Planck scales $\ell_P \sim 10^{-33} cm$ and $t_P = 10^{-43} sec$

[Planck mass: $M_P = 10^{19}\,\mathrm{GeV} \sim 10^{-5} g$]

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And: is the 'geometrization' of matter an unavoidable prerequisite for consistent quantization of gravity?

A Basic Fact

Perturbative quantum gravity is non-renormalizable

$$\Gamma^{(2)}_{div} = \frac{1}{\varepsilon}\frac{209}{2880}\frac{1}{(16\pi^2)^2}\int dV C_{\mu\nu\rho\sigma}C^{\rho\sigma\lambda\tau}C_{\lambda\tau}{}^{\mu\nu}$$

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This dichotomy has led to ...

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In other words: Is Quantum Gravity merely the quantization of Einstein Gravity or is it something altogether different?

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- Other:

  – Path integrals: Euclidean, Lorentzian, matrix models,...
  – Discrete quantum gravity (I): spin foams, group field theory... [→ D. Oriti]
  – Discrete quantum gravity (II): Causal dynamical triangulations [→ J. Ambjorn]
  – Non-commutative geometry and non-commutative space-time
  – Asymptotic safety and RG fixed points [→ D. Litim]
  – Emergent (quantum) gravity [→ P. Horava, E. Verlinde]
  – Causal histories, cellular automata, ...
  – .....

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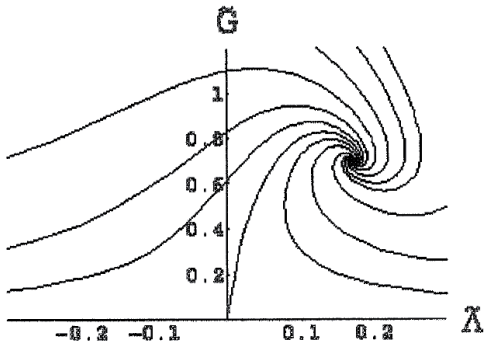

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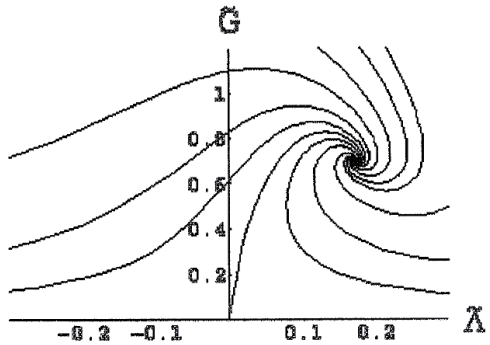

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Questions (from a skeptic):

- How to consistently truncate RG flow to a finite-dimensional subspace of couplings?
- How can RG flow be reconciled with general covariance?
- How can unitarity be maintained with higher order couplings $\propto \prod_{m,n} D^m R^n$?
- SM Landau poles must also be taken care of by RG fixed point!
- The acid test: RG evolution of 2-loop counterterm? [J. Distler]

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Supersymmetric extended objects

- No point-like interactions $\rightarrow$ no UV singularities?
- IIA/IIB und heterotic superstrings ($D = 10$)
- Supermembranes and M(atrix)-Theory ($D = 11$)

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- Main question: What is String Theory?

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- A Lie algebraic mechanism for the 'de-emergence' of space(-time) at the cosmological singularity?

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Just in case LHC keeps not finding any new fundamental spin-$\frac{1}{2}$ fermions:
$N = 8$ SUGRA does have the 'right' number of spin-$\frac{1}{2}$ fermions ($48 = 3 \times 16$)!
(after supersymmetry breaking and conversion of eight fermions into Goldstinos)

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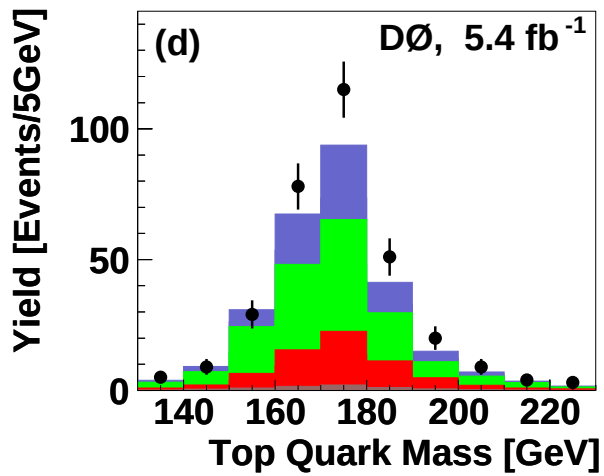
Of course, everyone agrees on this *desideratum*, but ....



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- Dynamics defined by constraints (via shift and lapse).

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Hope: can resolve classical singularities if wave functional $\Psi[g]$ smears over singular geometries (similar in spirit to resolution of Coulomb singularity for hydrogen atom)

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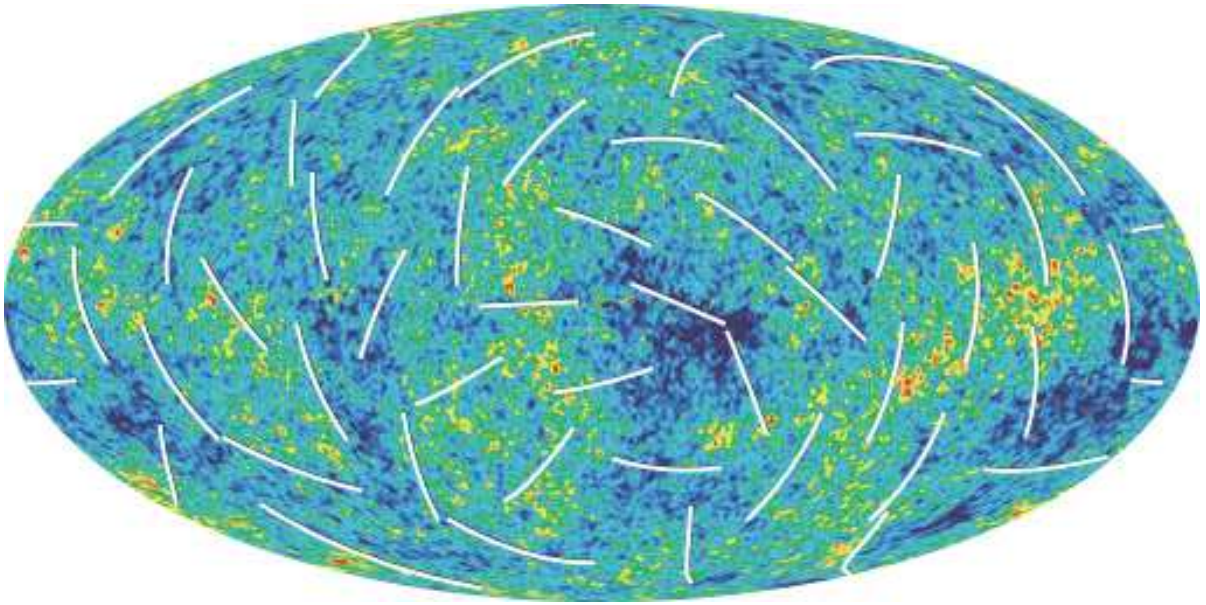
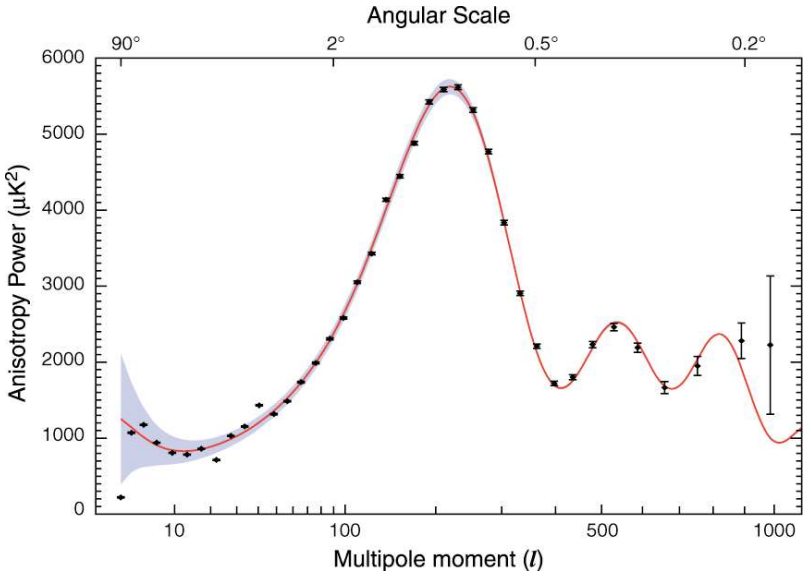
- Conceptual and interpretational problems
  - Physical interpretation of ‘wave function of the universe’ $\Psi[g]$?
  - Quantum theory in the cosmological context: Copenhagen vs. many worlds?
  - Decoherence and the emergence of a classical (FRW or de Sitter) universe?
  - Origin of time and time arrow?

Who collapsed this wave function?





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- Ways out?
  - – Ignore difficulties and proceed heuristically?
  - – Simplify WDW equation: mini-superspace and midi-superspace?
  - – 'change variables': metric $g_{ij} \rightarrow$ (Ashtekar) connection $A_i{}^a$ ($D = 4$ and $D = 3$ only)
  - – Spin networks and spin foams: a discrete structure at the Planck scale?

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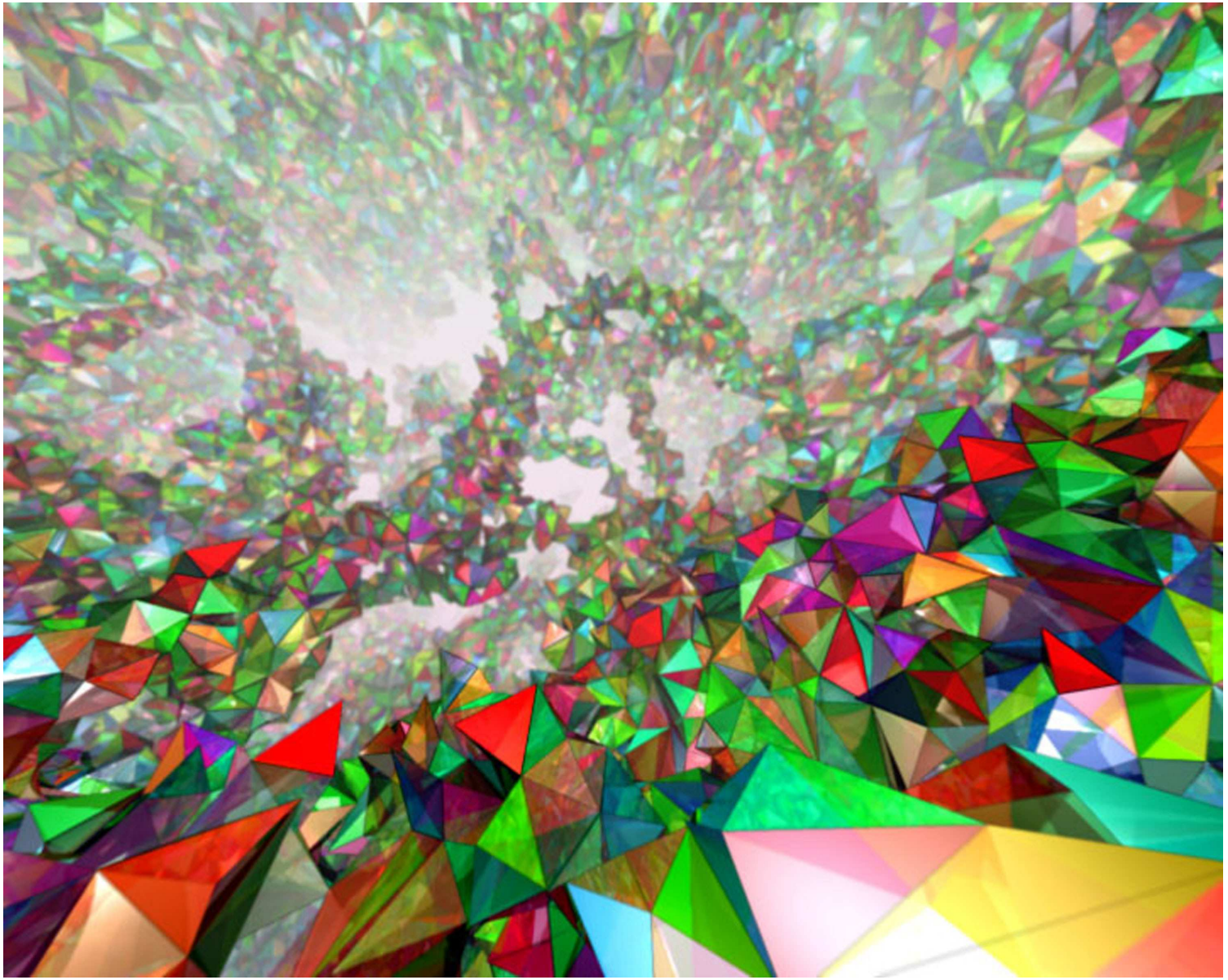
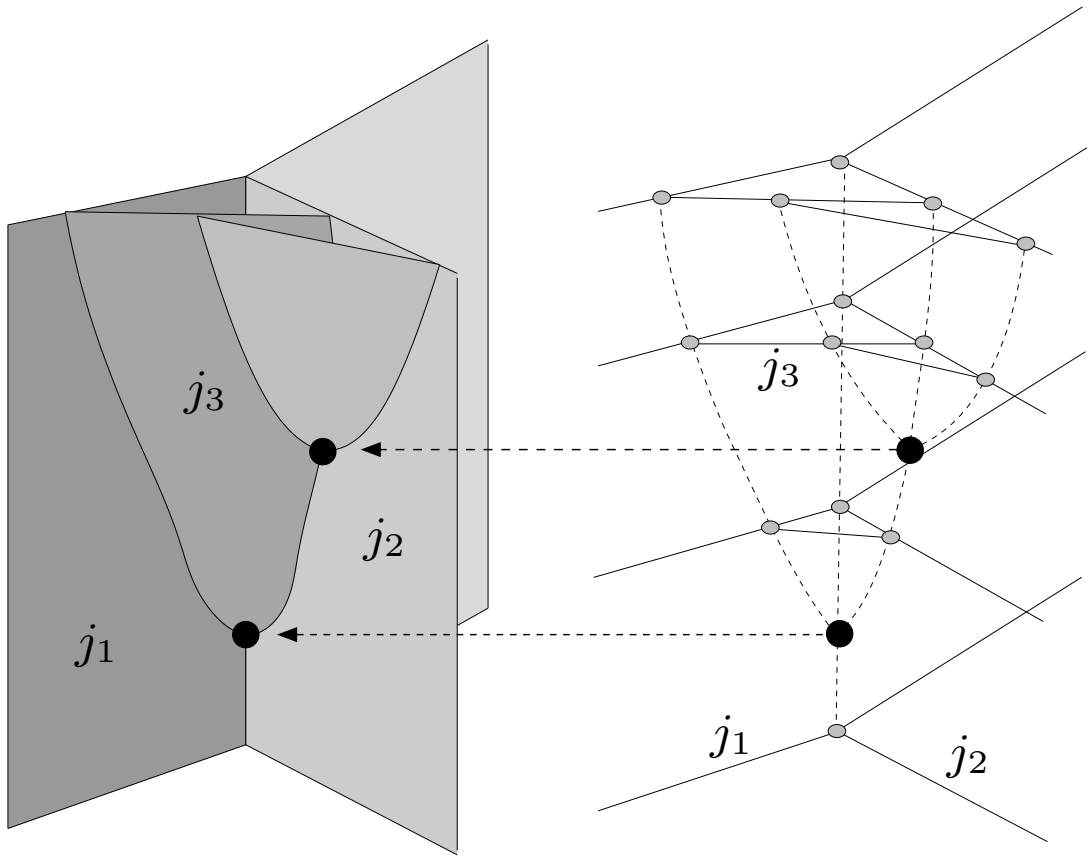
New features: *non-separable* (kinematical) Hilbert space $\Rightarrow$ operators not weakly continuous, no UV divergences, no negative norm states? BUT: semi-classical limit?

Quantum Geometry according to LQG



Spin Foams

Heuristically: Spin foam = evolution of spin network in ‘time’.



$\rightarrow$ spin labels now attached to faces of simplicial complex.

Spin Foam Models

Dynamics defined via generalized *spin state sum* model

$$Z_\phi = \sum_{\text{spins } \{j\}} \prod_{f,e,v} A_f(\{j\})\ A_e(\{j\})\ A_v(\{j\})$$

with amplitudes for faces f, edges e and vertices v.

$\rightarrow$ a novel way of defining models of lattice gravity!

Main question: *real* or *regularized* quantum space-time?

Further technical and conceptual issues:

- Riemannian $SO(4) \cong SO(3) \times SO(3)$ vs. Lorentzian $SO(1,3) \cong SL(2,\mathbb{C})$?
- Oscillatory or Wick rotated path integral: $\exp(iS)$ vs. $\exp(-S)$?
- Emergence of classical gravity (Newton's law) at long distances?

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- However: without a tight theoretical framework there are almost unlimited possibilities for such 'predictions'!

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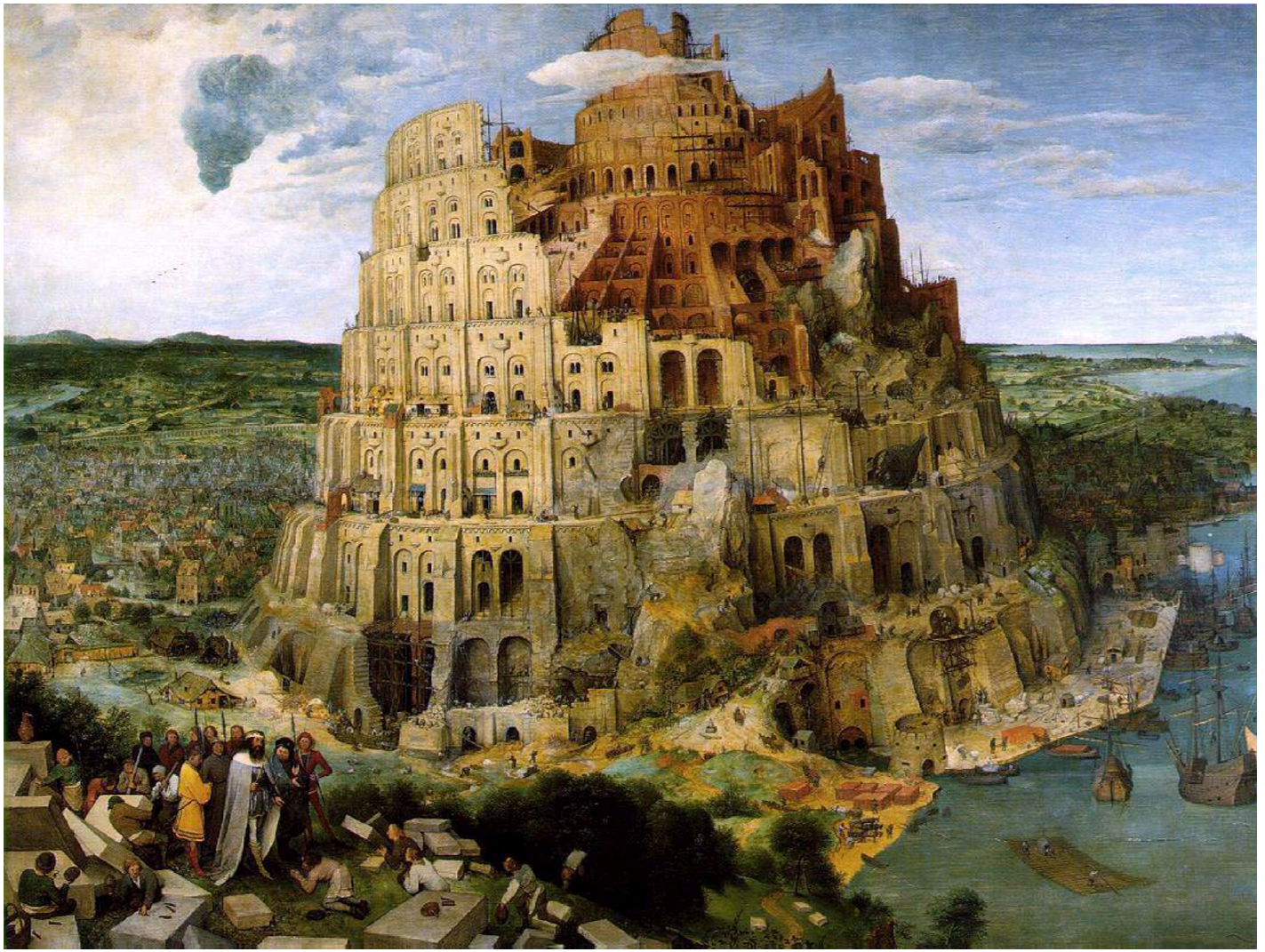
- So better start looking for *inconsistencies*!
- Otherwise *ansätze* may remain 'fantasy' [G.W. Gibbons]

The future of Quantum Gravity?



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- $\rightarrow$ an unobstructed view of Planck scale physics?

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- To discriminate between numerous different ansätze and ideas, need to rely more on Occam's razor!

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