Probing the spatial distribution of k-vectors *in situ* with Bose-Einstein condensate

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General context: determination of the fine-structure constant and the photon recoil measurement

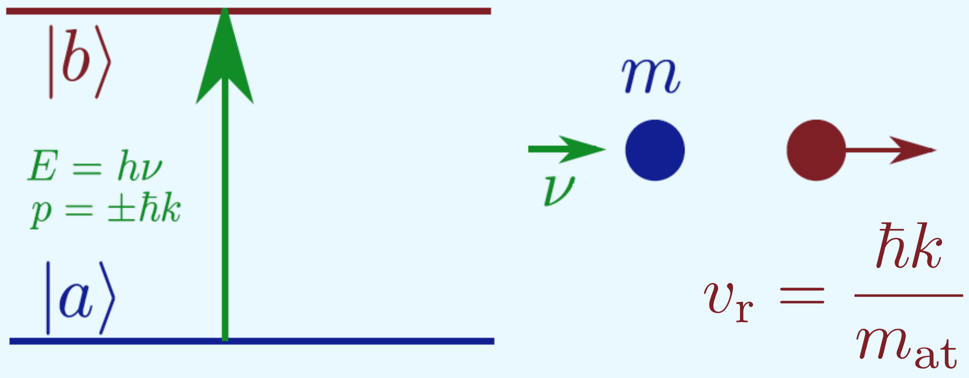
- Hydrogen atom: $hcR_\infty = \frac{1}{2} m_\mathrm{e} \alpha^2 c^2$

$$\alpha^2 = \frac{2R_\infty}{c}\frac{h}{m_\mathrm{e}} = \frac{2R_\infty}{c}\frac{A_\mathrm{r}(\mathrm{at})}{A_\mathrm{r}(\mathrm{e})}\frac{h}{m_\mathrm{at}}$$

- Limitation: $\frac{h}{m_\mathrm{at}}$ (or absolute atomic mass in the new SI)

Measured quantity	Relative uncertainty
Rydberg constant	1.9×10^{-12}
A_r (e)	1.8×10^{-11}
Ar(^{87}Rb)	7.0×10^{-11}

- G. Audi et al., 2014 Nuclear Data Sheets 120, 1-5 (2014)
- S. Sturm et al. Nature 506, 476-470 (2014),
- E. Tiesinga et al., Rev. Mod. Phys. 93, 025010



$k = 2\pi/\lambda$: wave vector

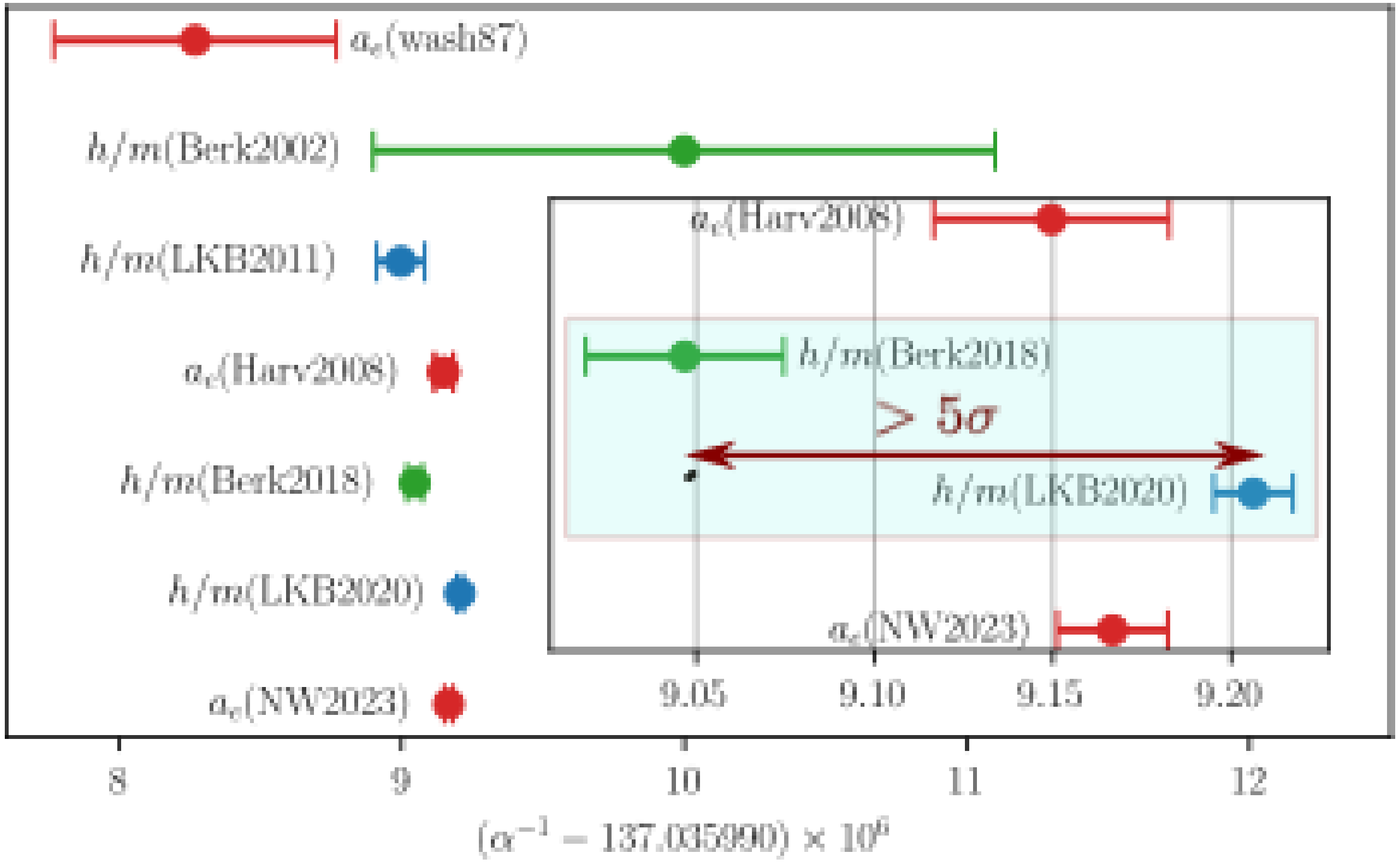
$v_r = 5.9$ mm/s for ^{87}Rb and 3.5 mm/s for ^{133}Cs

Most accurate determinations of α

L. Morel et al., Nature 588, 61-68 (2020)

$$\alpha^{-1} = 137.035999206(11)$$

- Relative uncertainty of 8.1×10^{-11}
- Statistical uncertainty of 4.3×10^{-11} on 48h integration time
- New systematic effects were considered



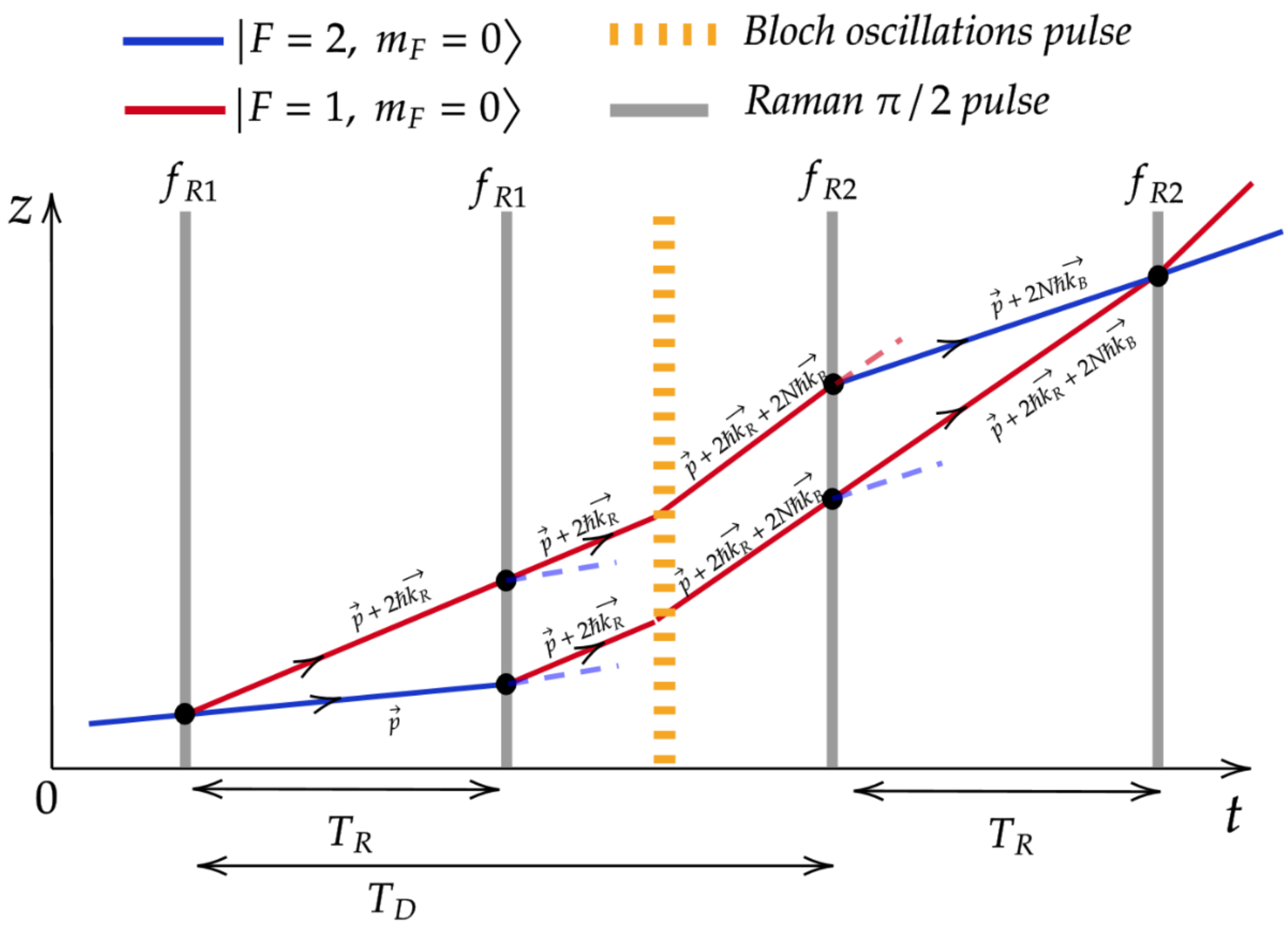
- 5.4 σ discrepancy with caesium recoil measurement

Errors budget 2020

L. Morel, Z. Yao, P. Cladé and S. Guellati-Khelifa, Nature 588, 61-68 (2020)

Source	Correction $[10^{-11}]$	Relative uncertainty $[10^{-11}]$
Gravity gradient	-0.6	0.1
Alignment of the beams	0.5	0.5
Coriolis acceleration		1.2
Frequencies of the lasers		0.3
Wave front curvature	0.6	0.3
Wave front distortion	3.9	1.9
Gouy phase	108.2	5.4
Residual Raman phase shift	2.3	2.3
Index of refraction	0	< 0.1
Internal interaction	0	< 0.1
Light shift (two-photon transition)	-11.0	2.3
Second order Zeeman effect		0.1
Phase shifts in Raman phase lock loop	-39.8	0.6
Global systematic effects	64.2	6.8
Statistical uncertainty		2.4
Relative mass of ^{87}Rb [16] : $86.909\,180\,531\,0(60)$		3.5
Relative mass of the electron [14] : $5.485\,799\,090\,65(16) \cdot 10^{-4}$		1.5
Rydberg constant [14] : $10\,973\,731.568\,160(21)\mathrm{m}^{-1}$		0.1
Total: $\alpha^{-1} = 137.035\,999\,206(11)$		8.1

Principle of recoil measurement

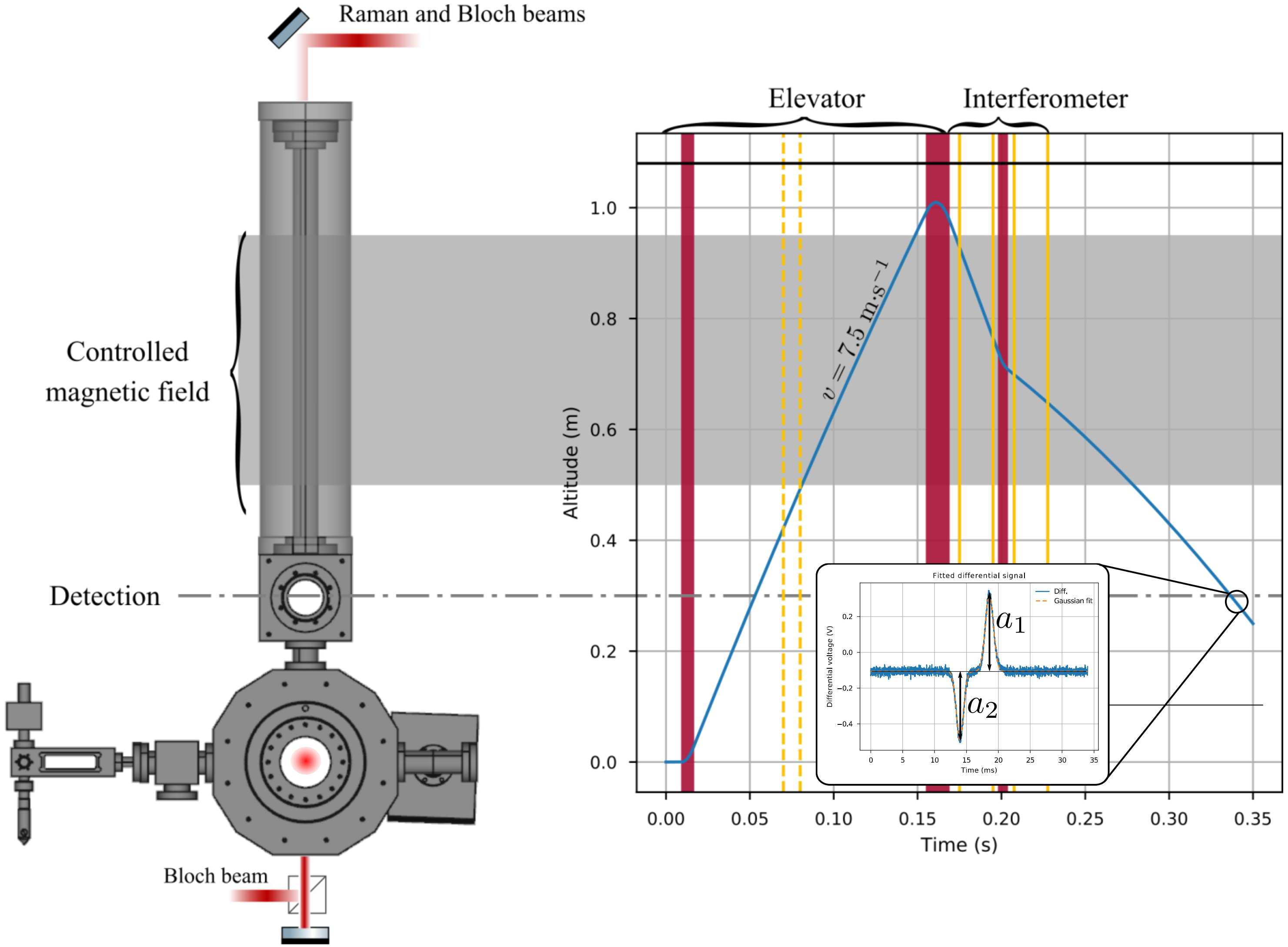


- Coherent acceleration: Bloch oscillations in an accelerated optical lattice:

$$\Delta v = N v_r \quad (N = 1000)$$

$$\Delta\Phi_{at} = \frac{\hbar}{m}(2N k_B k_R T_R)$$

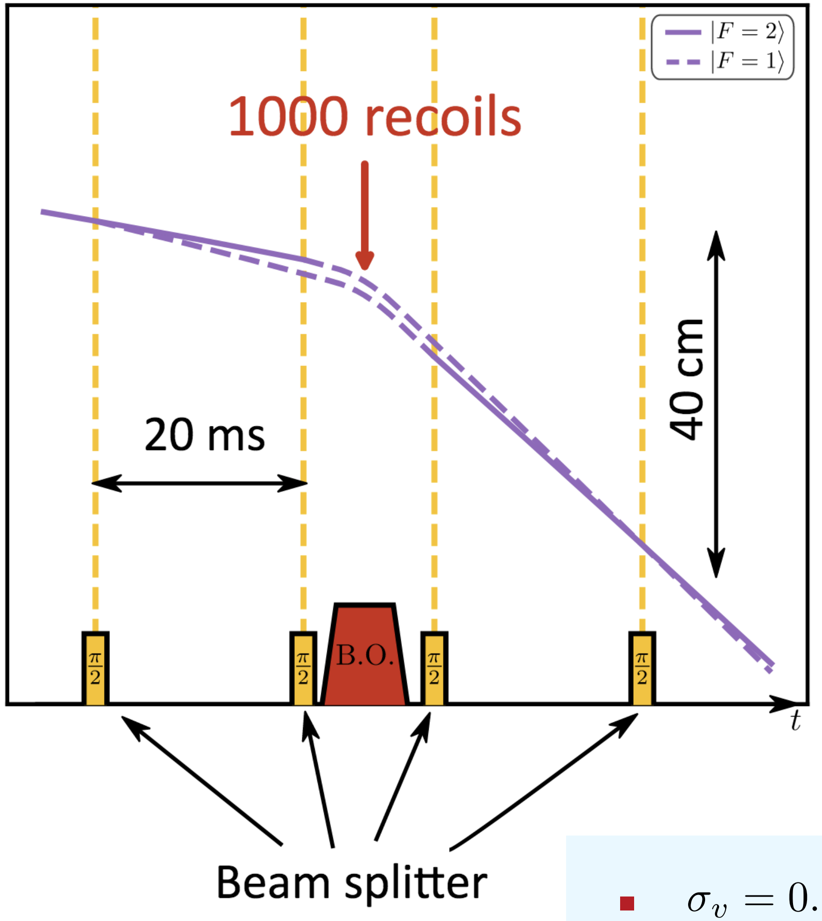
Experimental procedure

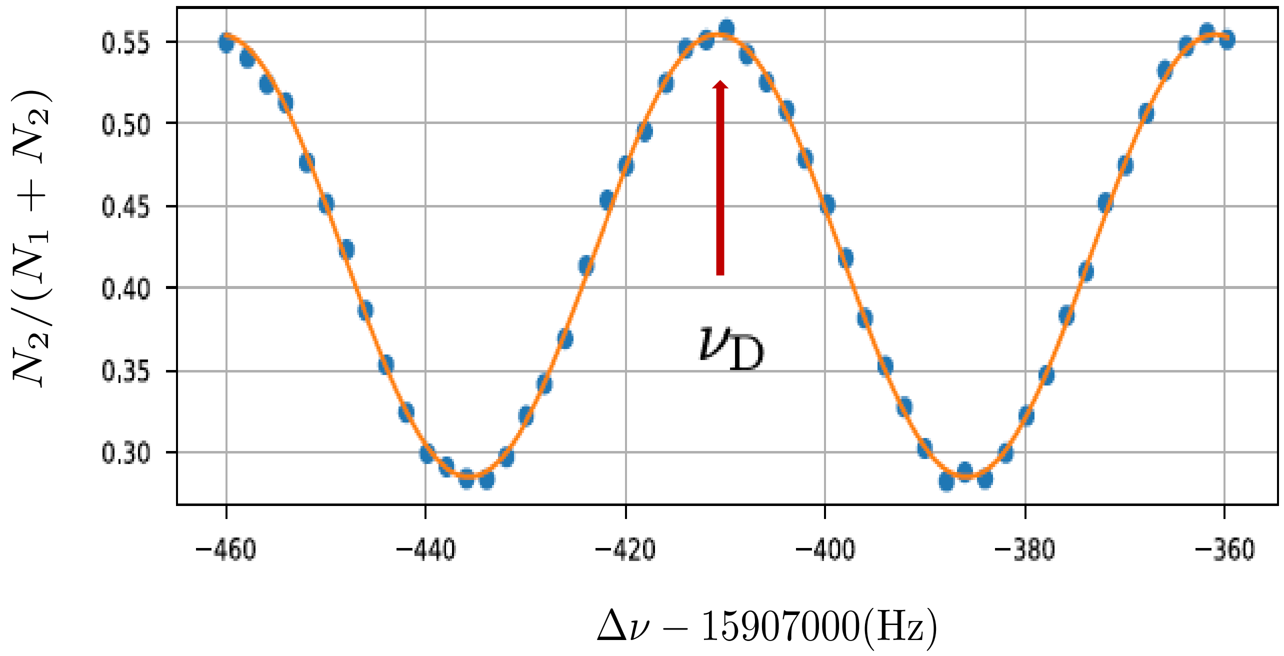


$$a_1 \propto N_1$$
$$a_2 \propto N_2 \longrightarrow P_2 = \frac{N2}{N_1+N_2}$$

Sensitivity of the atom interferometer

- We scan the phase of the laser to compensate (probe) the atomic phase: $\Delta\Phi_L = \nu_D T_R$





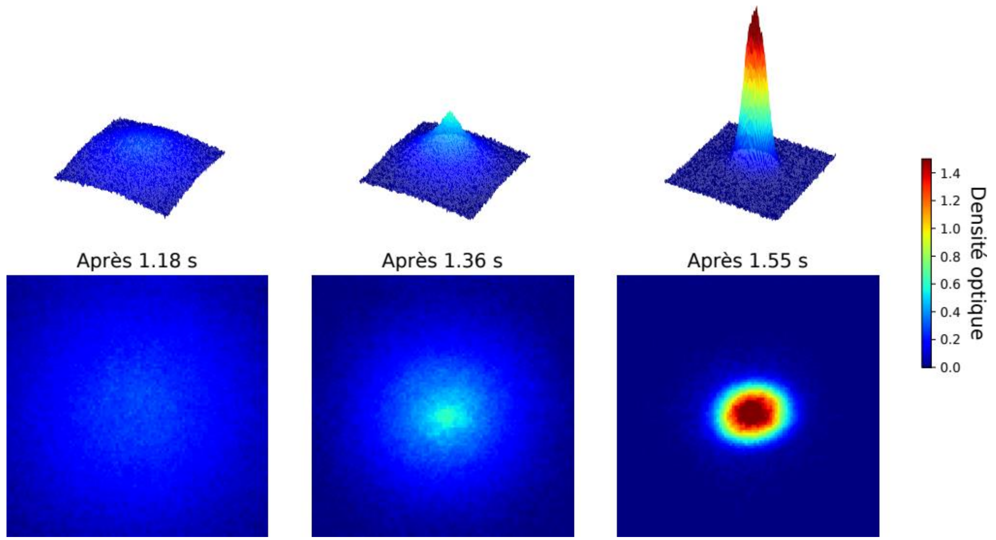
$$\Delta\Phi = T_R \left(2N k_R k_B \frac{\hbar}{m} - 2\pi\nu_D \right)$$

- $\sigma_\nu = 0.047\text{Hz} \longrightarrow 3\times10^{-6}\nu_r \longrightarrow 20\ \text{nm/s} \longrightarrow 3\times10^{-9}$ on h/m (1min)
- 10^5 atoms detected $\sigma_\Phi = 2\sigma_{QSL}$, limited by vibrations

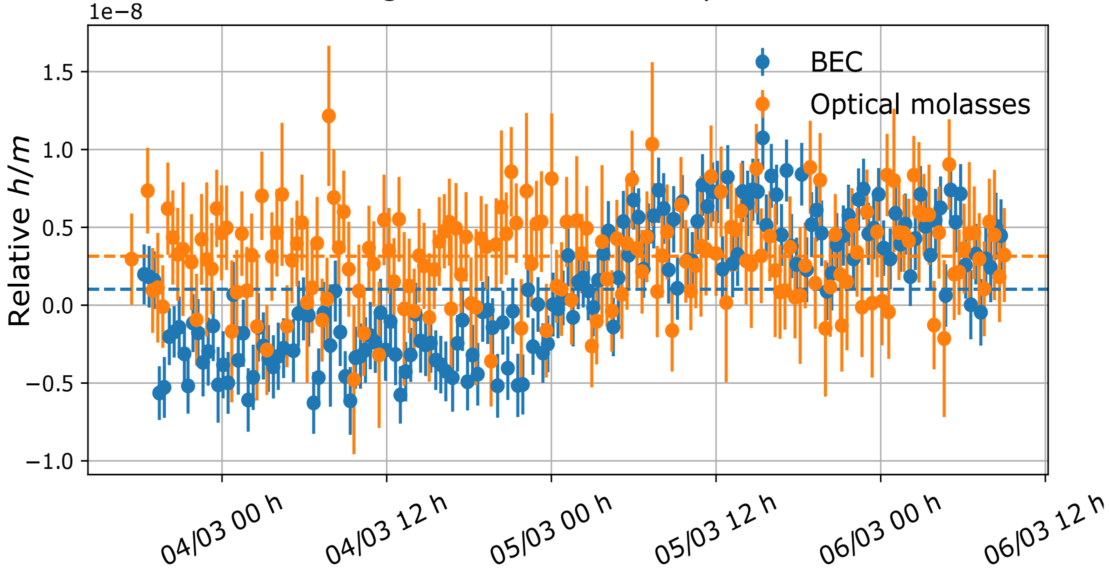
Laboratoire Kastler Brossel
Physique quantique et applications

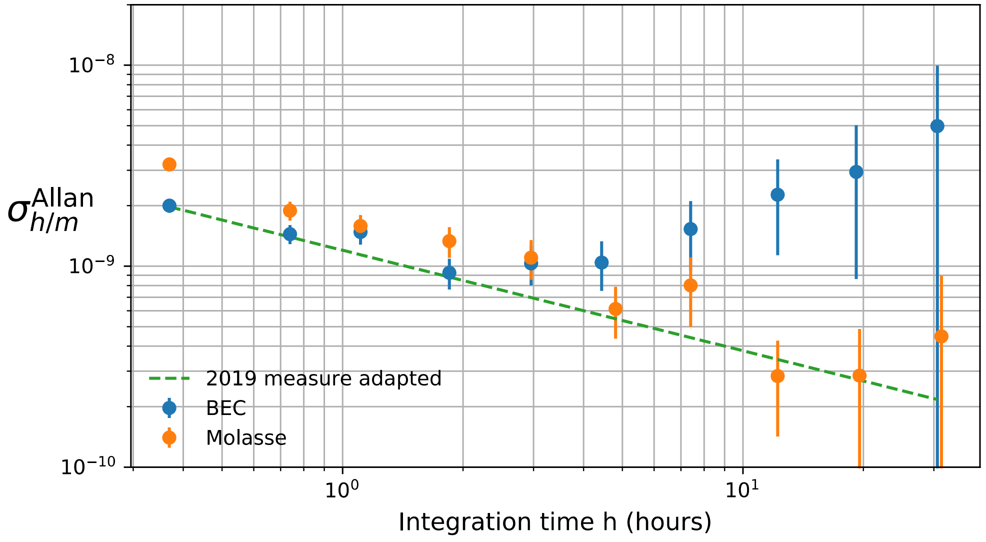
Preliminary measurement with Bose-Einstein condensate

BEC = 2x10^5 atoms@ 100 nK in F=1 m$_F$=0 in 3.6 s



- Measurements alternating between BEC and optical molasses





- Temporal fluctuation observed with the BEC (parasitic interference in the new optical setup),
- Not observed with the optical molasses

➢ Use BEC as probe to measure the spatial distribution of k-vectors and the intensity profile of laser beams in situ

Wave vector correction relative to a plane wave

$$2\pi\nu_D = \frac{\hbar}{m} N_{\rm B} \left(\vec{k}_{\rm B2} - \vec{k}_{\rm B1}\right) \cdot \left(\vec{k}_{\rm R2} - \vec{k}_{\rm R1}\right)$$

$$k_z = \hbar k_0 \left(1 + \vec{\kappa} \cdot \vec{u}_z\right)$$

k_0 is the wave vector of a plane wave and $\vec{u}_z$ is the propagation axis

$$2\pi\nu_D \simeq 4 N_{\rm B} \frac{\hbar}{m} k_{0\rm B} k_{0\rm R} \left(1 + \frac{1}{2} \vec{\kappa} \cdot \vec{u}_z\right)$$

$$\text{where } \vec{\kappa} = \left(\vec{\kappa}_{\rm B2} - \vec{\kappa}_{\rm B1} + \vec{\kappa}_{\rm R2} - \vec{\kappa}_{\rm R1}\right)$$

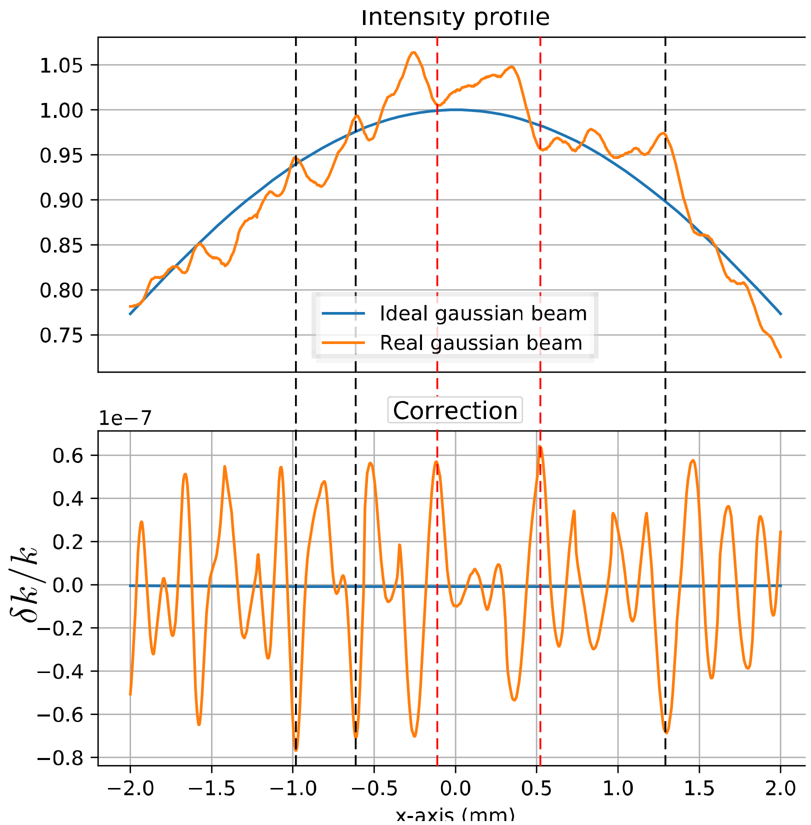
- Measurement of frequency ν_D directly provides the correction χ_z assuming that h/m is known

Recoil velocity of an atom in a distorted wavefront

$$E(\vec{r},t) = E_0(\vec{r},t)\ \mathrm{e}^{i(kz+\phi(\vec{r}))}; \quad k = \frac{2\pi\nu}{c}$$

- Momentum correction due to transverse phase and amplitude fluctuations (in paraxial approximation):

$$\kappa_z = -\frac{1}{2k_0^2}\left|\left|\vec{\nabla}_\perp \phi(\vec{r})\right|\right|^2 + \frac{1}{4k_0^2}\frac{\Delta_\perp I(\vec{r})}{I(\vec{r})},$$

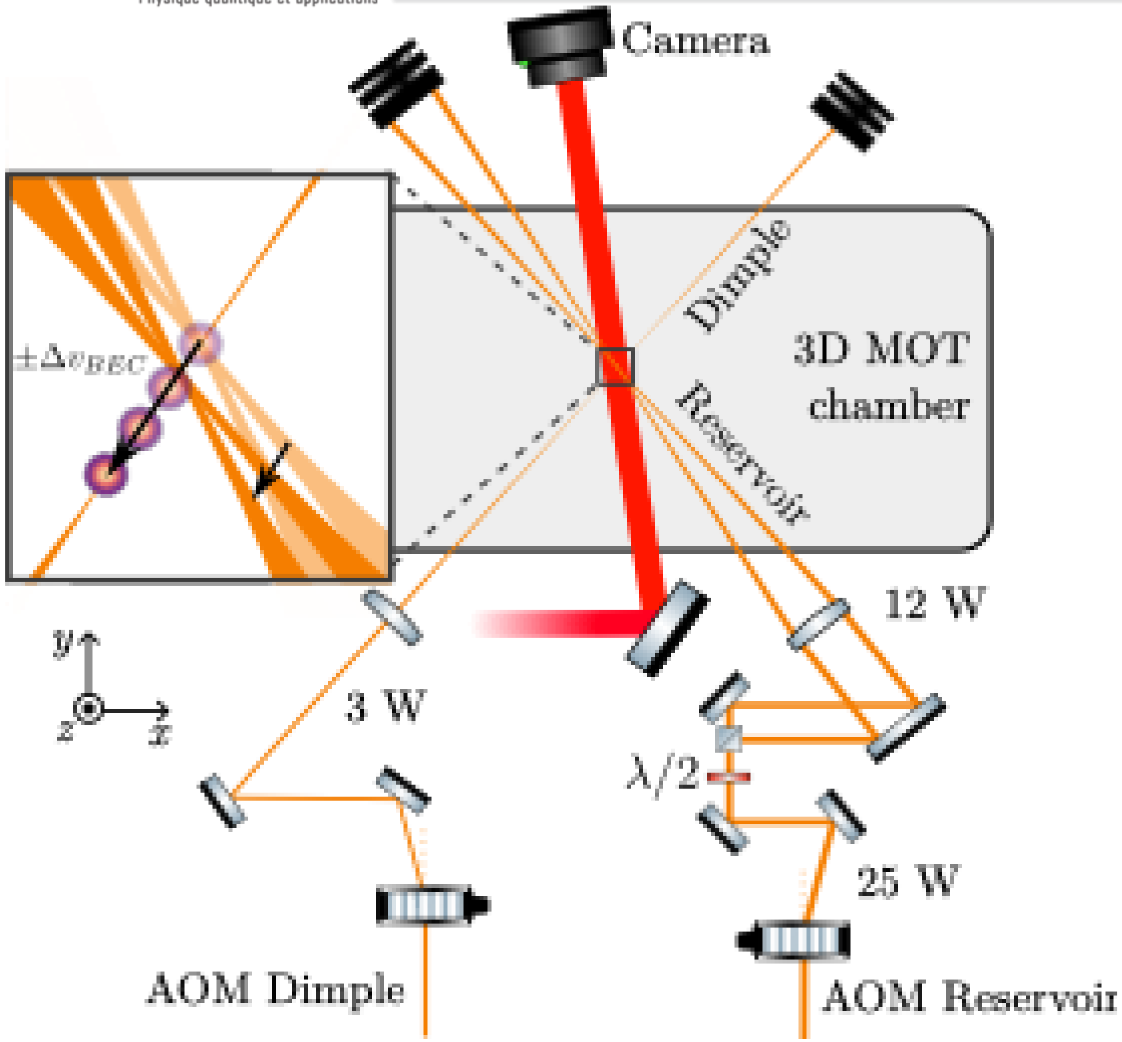


Correlation between the wavevector correction and efficiency of the recoil transfer (Bloch oscillations) *P* (I)

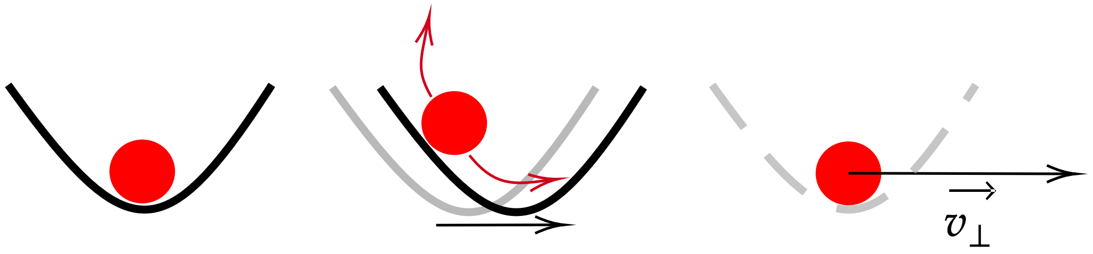
$$\langle \delta k \rangle = \frac{\langle \delta k\ P(I)\rangle}{\langle P(I)\rangle}$$

S. Bade et al., Phys. Rev. Lett. **121**, 073603 (2018)

How to move the BEC ?



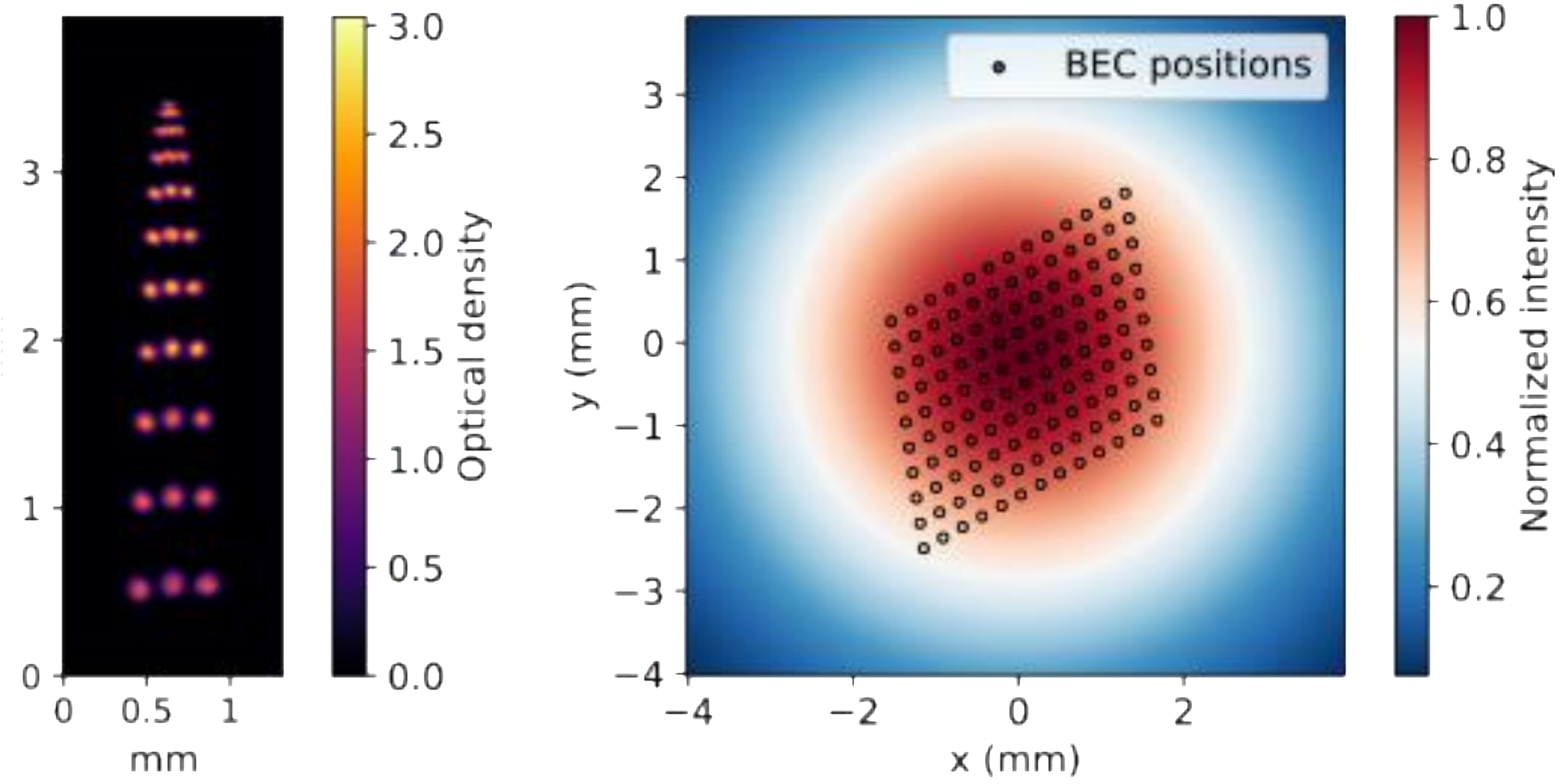
- Frequencies of the two AOMs (Reservoir/Dimple) are rapidly shifted by 1 MHz, allowing displacement of the center of the trap.



- After 10 ms, the dipole laser beams are switched off, leaving the BEC to move with an initial transverse velocity

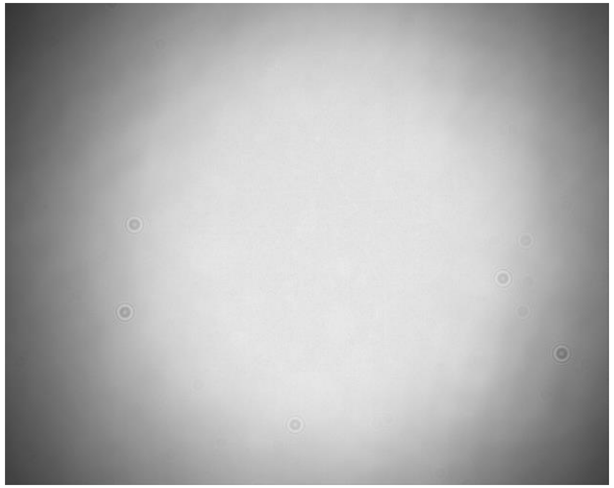
Control and calibration of transverse velocity

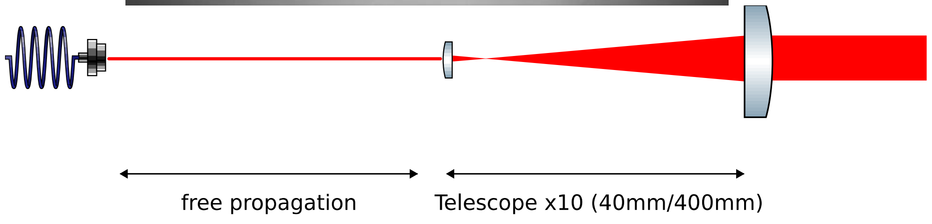
- We calibrate the BEC velocity by tracking the cloud trajectory using absorption imaging
- Maximum velocity of 10 mm/s along both x and y directions responds to a displacement of nearly 2 mm.
- The RMS cloud size of 350 µm.

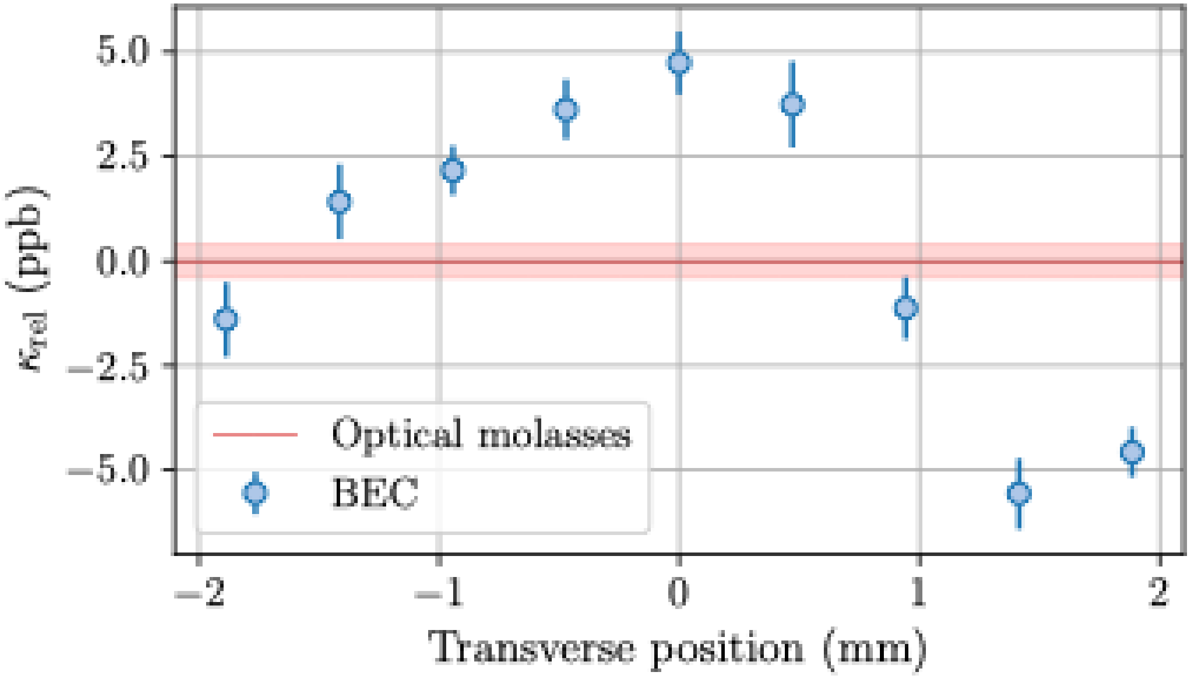


Measurement with a gaussian beam

- Clean beam

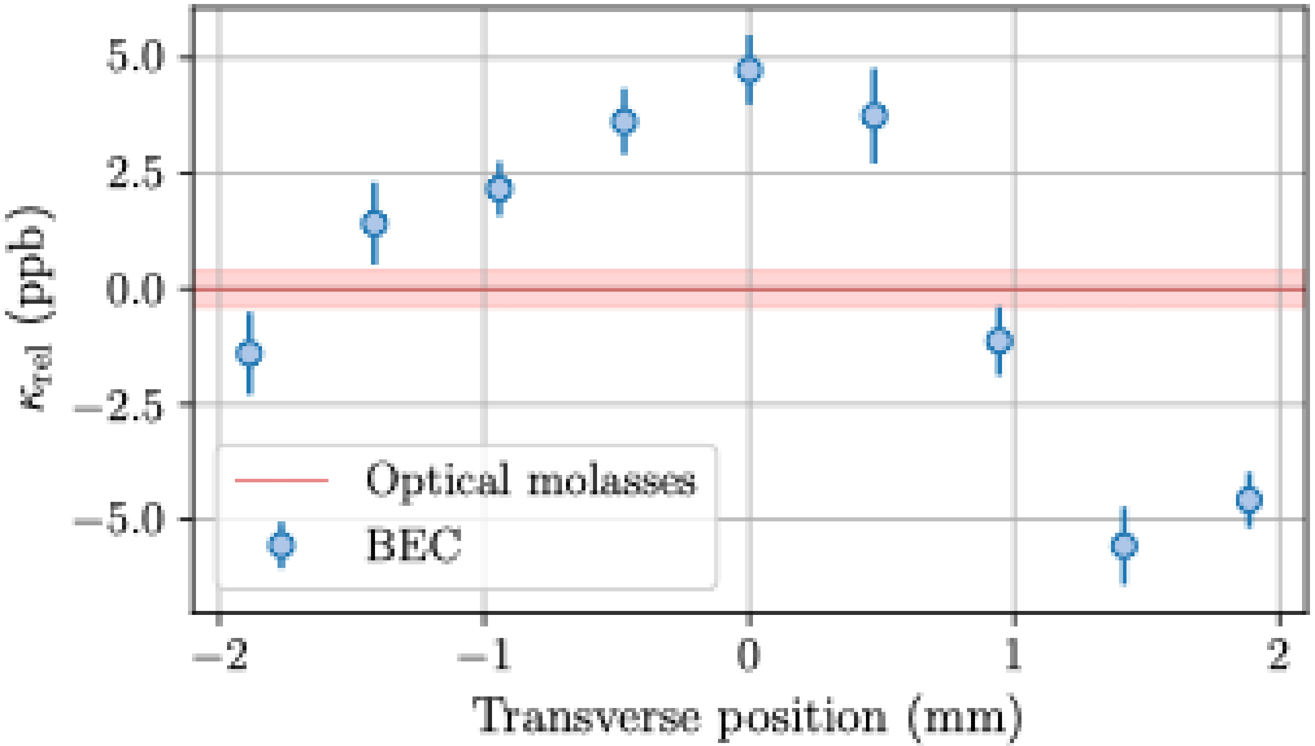






- Typical uncertainty on ν_D is 80 mHz (2.8×10^{-9})
- Each data point represents an average 18 such measurements statistical uncertainty of 6×10^{-10} on κ
- Full data were acquired over 117 hours, using BEC and optical molasses alternately.
- We use as reference value the average of 53 values with optical molasses ($\sigma_r = 3.8 \times 10^{-10}$ with $\chi^2 = 1.2$)

Measurement with a gaussian beam



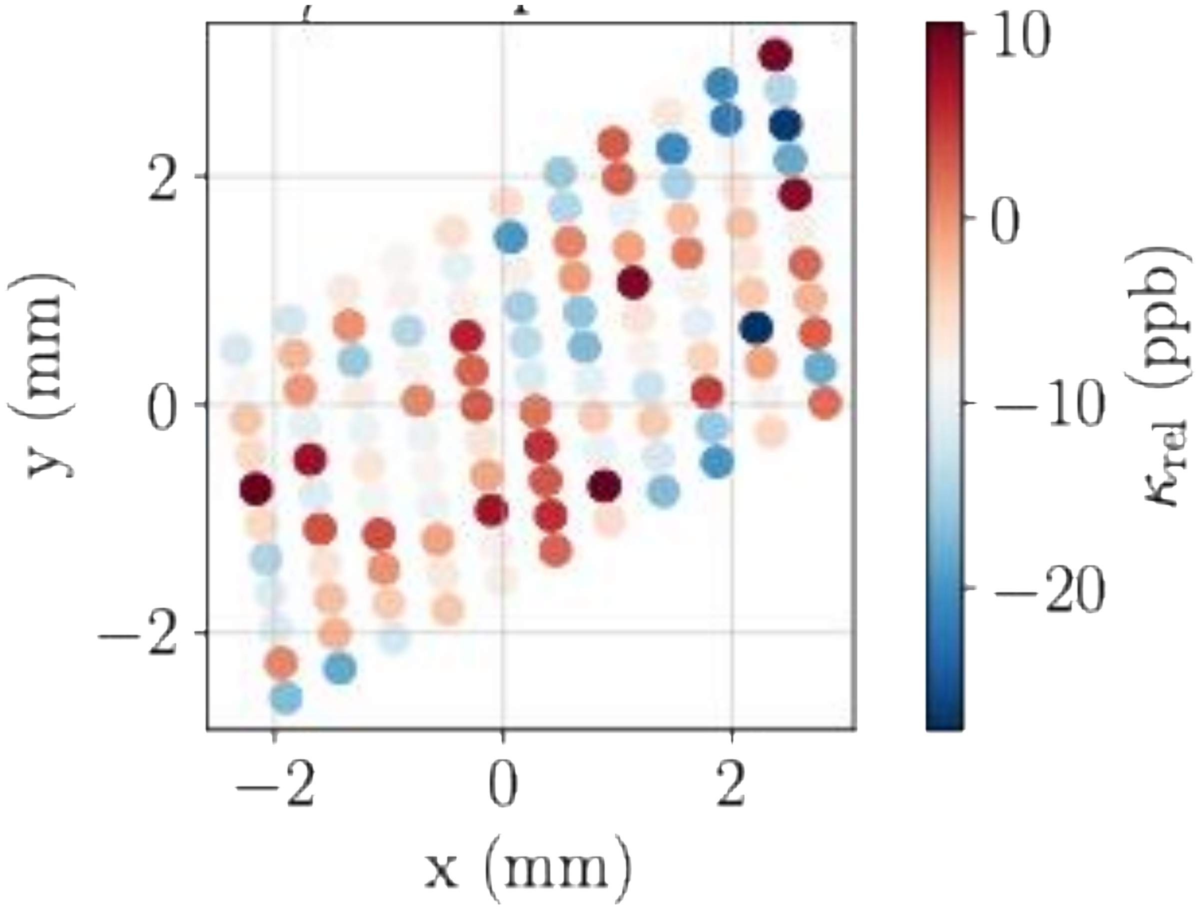
Gaussian beam:

$$\kappa_z = -\frac{2}{k^2 w^2}\left(1 - \frac{r^2}{w^2}\right)$$

A displacements in the range r = 0 to 2 mm with *waist*=5 mm, corresponds to a variation in $\kappa_z \approx$ 2x10−10

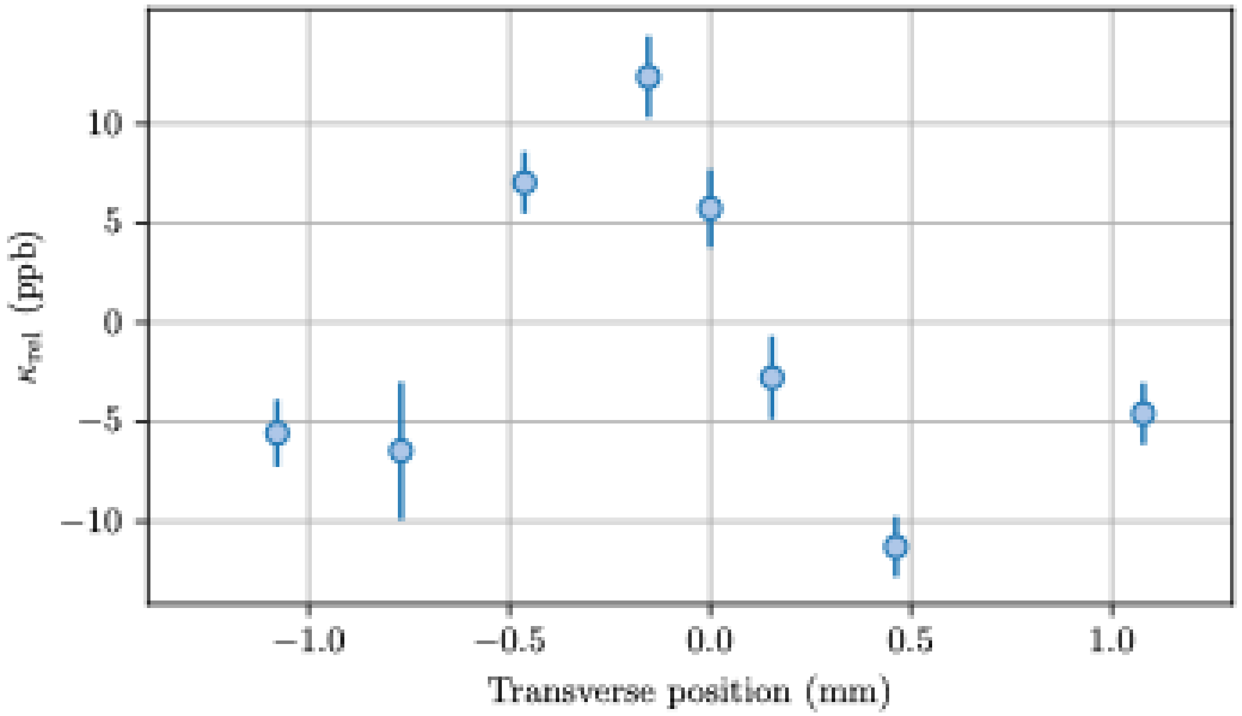
➢ The beam distortions exceed those expected from a simple Gaussian model.

Measurement with a Gaussian beam on a 2 x 2 mm^2 grid

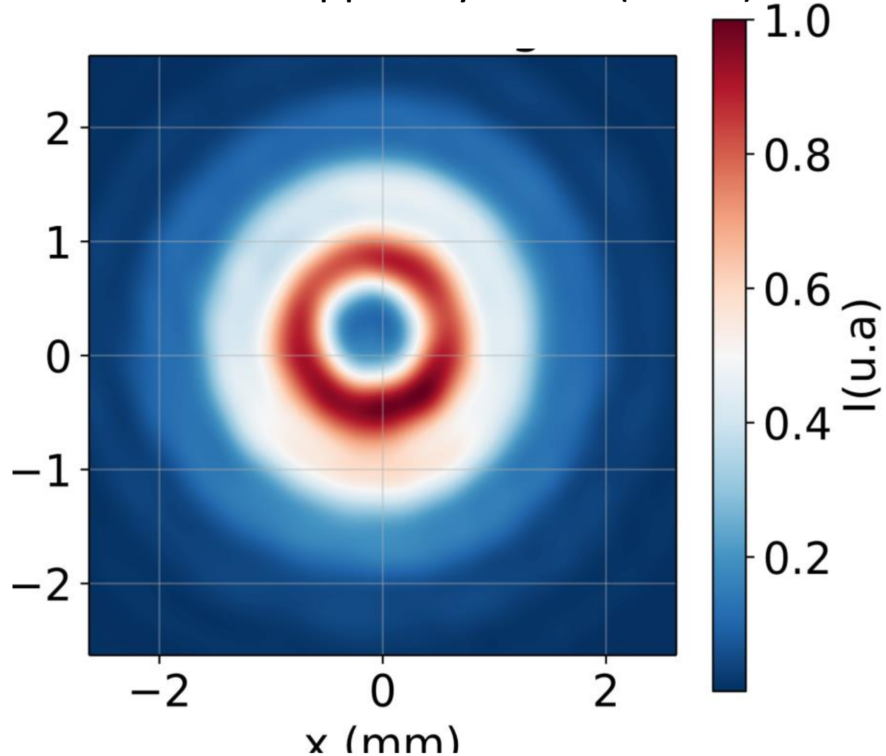


- Measurement using a 2D map on 121 positions centered around our beam.
- The average value is in good agreement with the value obtained using an optical molasse

Measurement with clipped beam



- Beam clipped by an iris (4 mm)



- We observe locally an "extra recoil" where the photon recoil exceeds the nominal value $h\nu/c$

Model and Monte Carlo simulations

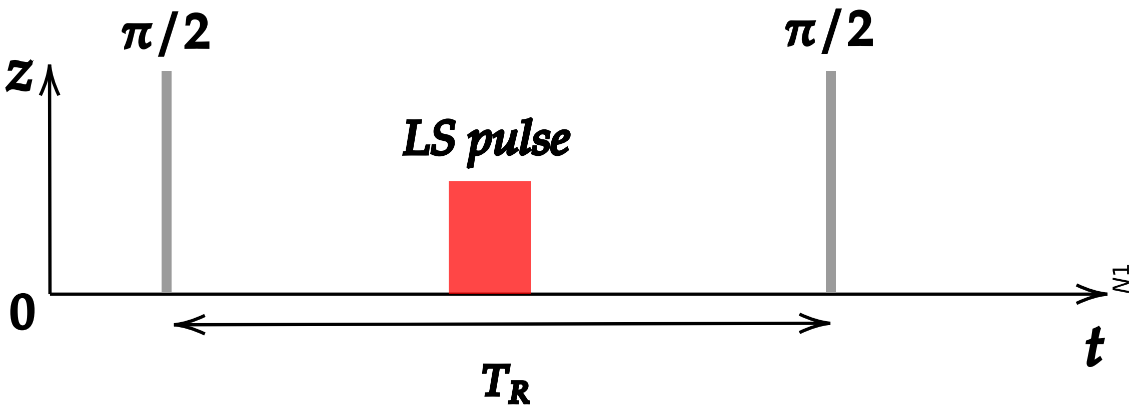
- **Monte-Carlo approach**: Simulate many classical atom trajectories with initial position/velocity dispersion, compute the interferometer phase shift and probability amplitude for each, then average over the atomic cloud.

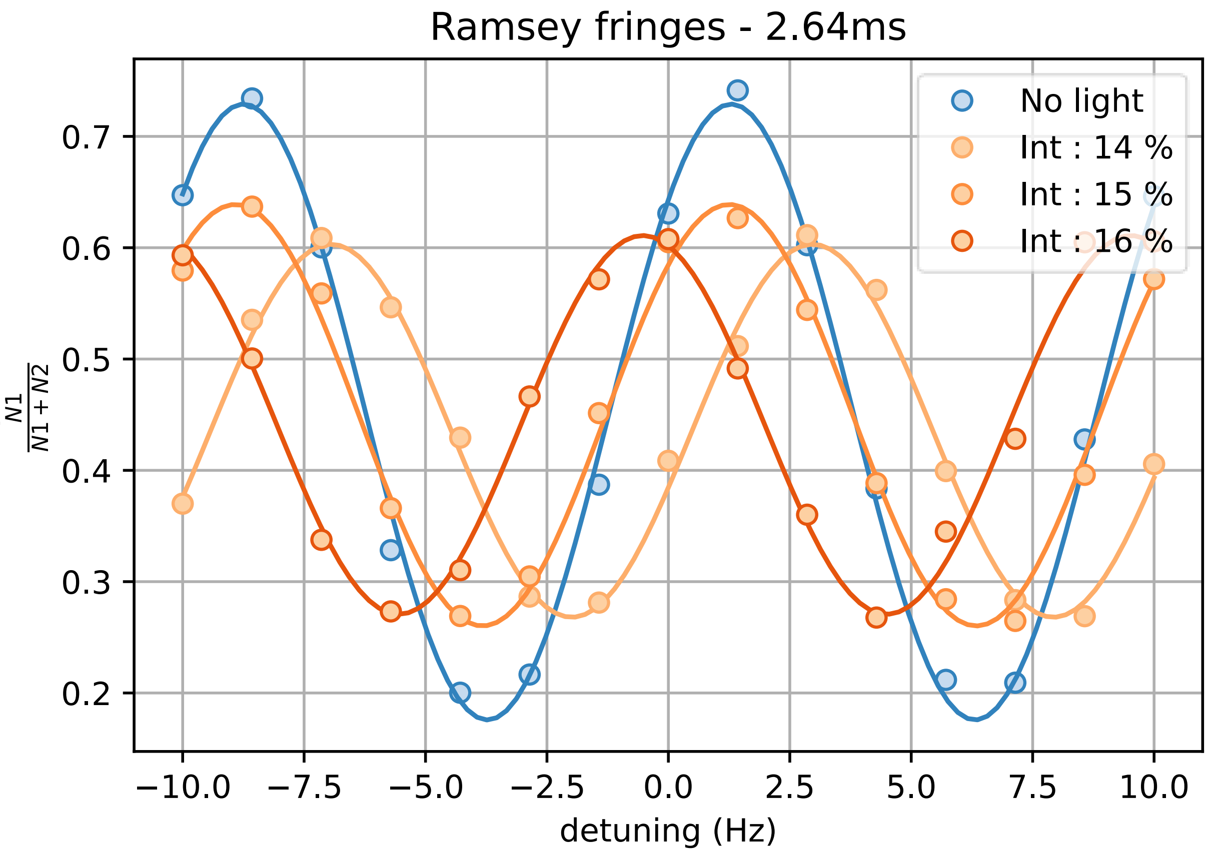
- **Evaluate the laplacian term** at the position of the atoms, by numerically propagating a truncated Gaussian beam

➢ Need to know the intensity profile of laser beams

Measurement of intensity probe of the beam *in-situ*

- Ramsey sequence with a « the upward Bloch beam pulse » switched on in between the two π/2 pulses
- It induces a differential light shift proportional to the intensity

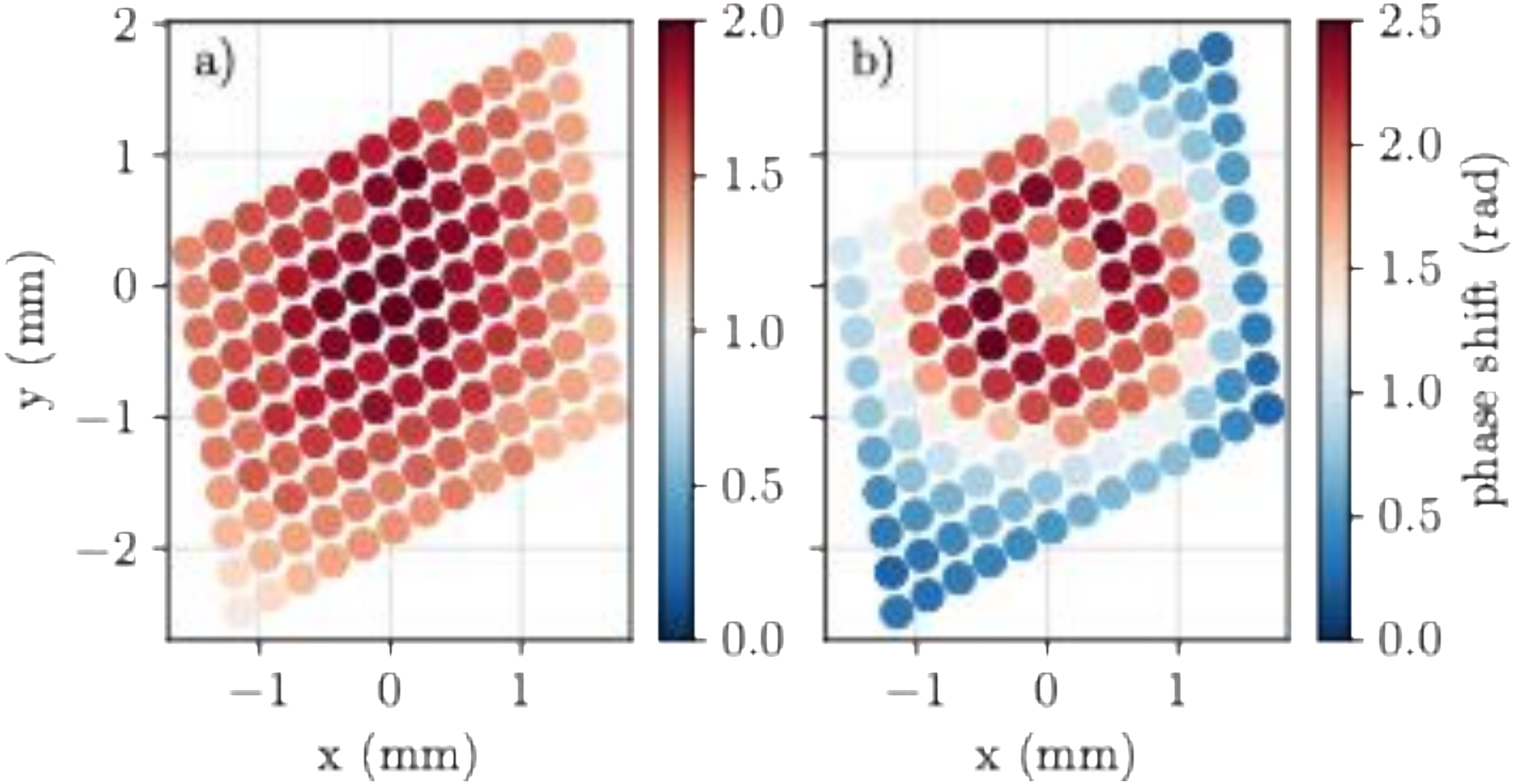




Reconstruction of beam intensity profile in situ

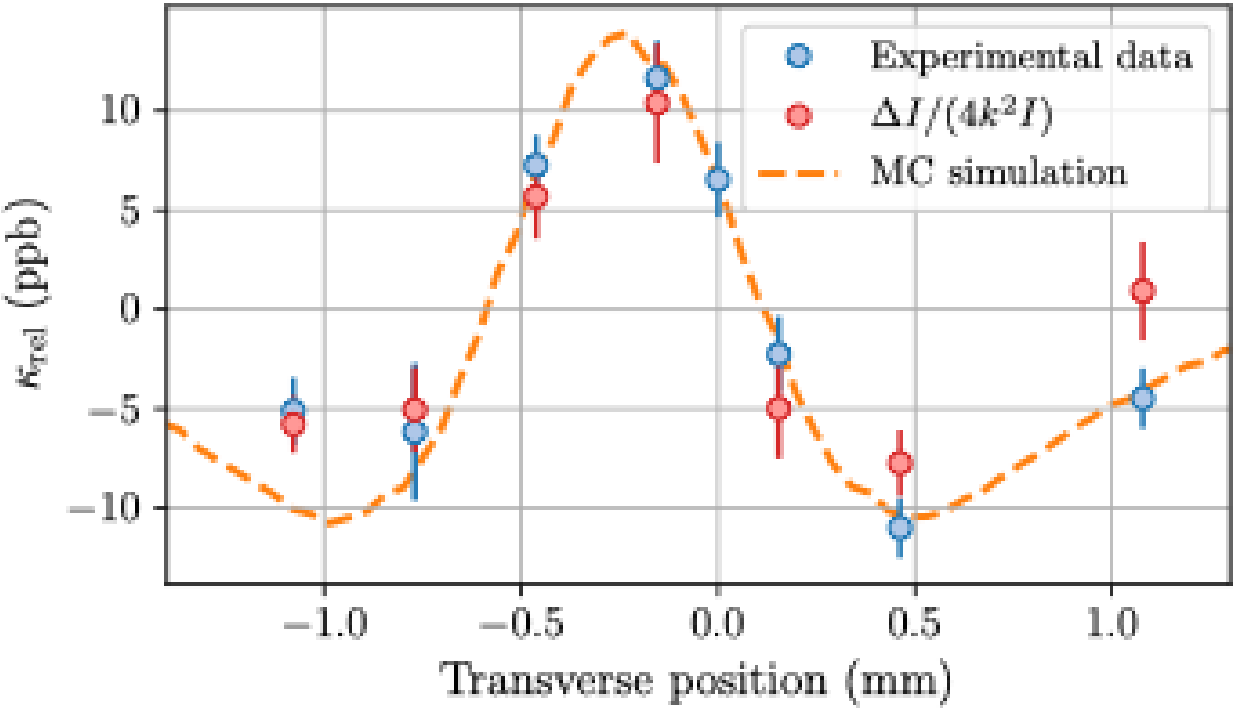
Intensity profile of the upward Bloch beam

- Clean beam
- Clipped beam



Each data point corresponds to the phase shift induced by light shift which is proportional to the intensity.

Measurement with clipped beam: Experiment/simulations

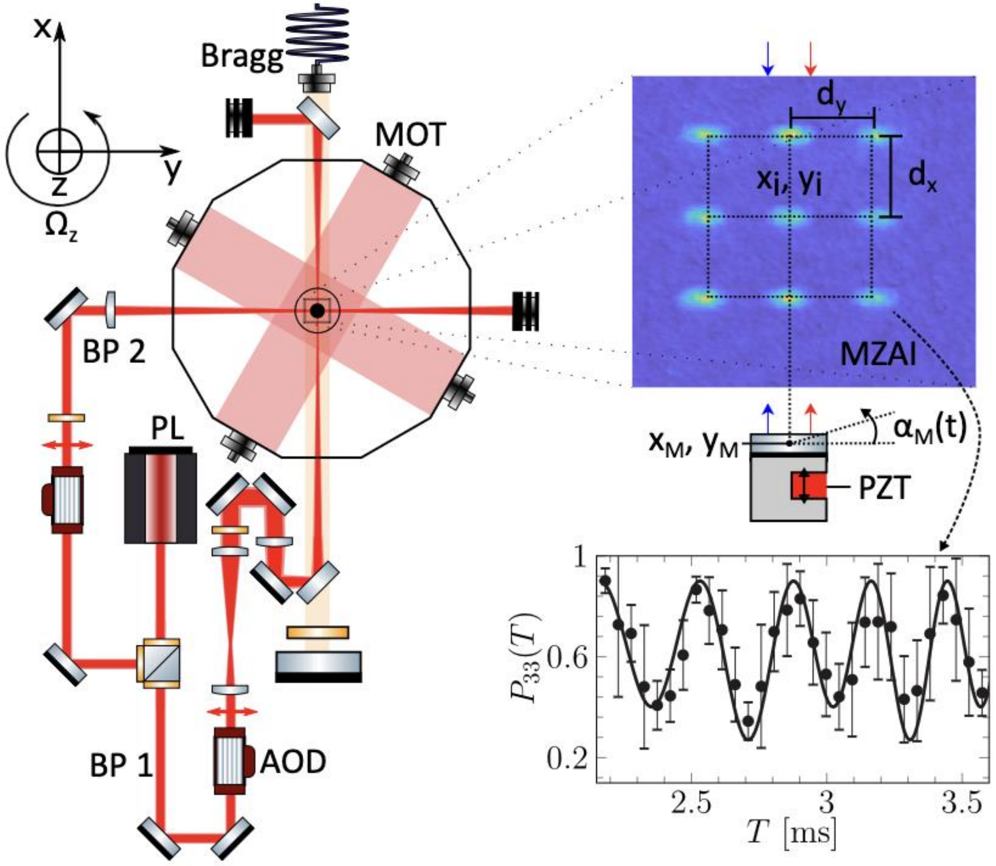


Around each point, we recorded the intensity on a 3x3 matrix of adjacent points and calculated the Laplacian of the intensity.

Limitations/solutions

- Statistical uncertainty of our measurements is limited by the time required to scan the full beam profile.

- A 2D matter wave array enables simultaneous probing of the beam cross-section at multiple positions, improving statistics and reducing the effects of long-term experimental drift



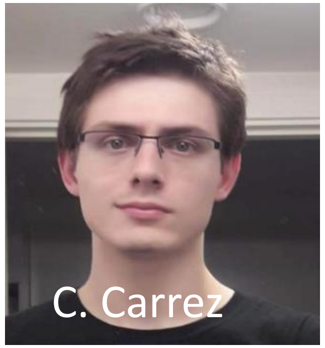
K. Stolzenberg et al, Phys. Rev. Lett. **134**, 143601 (2025)

Summary and Outlook

- We have demonstrated a new method for measuring the spatial distribution of the k-vectors and the laser intensity profile in-situ

- Developed a comprehensive numerical simulation and a simple model that is in good agreement with experimental data

- Outlook:
  - Implement a 2D lattice of BECs for simultaneous measurements at multiple positions
  - Improve the evaluation of corrections related to laser beam profiles.

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