Overview on progress of Scattering Amplitudes

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- Conceptually, it leads beautiful mathematic structure of scattering amplitudes.
- Practically, it makes some previous impossible calculations trivial, in particular precision calculations in QCD.

Feynman diagram

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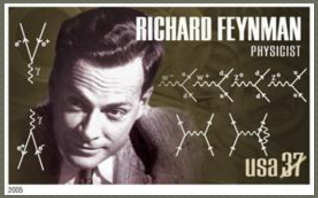

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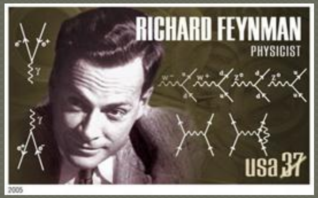

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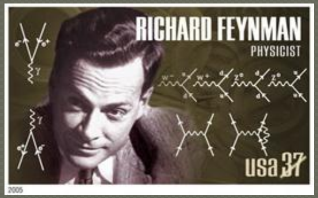

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- Fast-growing of # of Feynman diagrams:

$n_g =$	3	4	5	6	7	...
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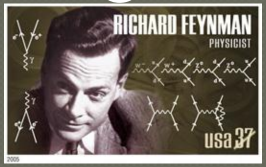

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- Fast-growing of Feynman diagrams:

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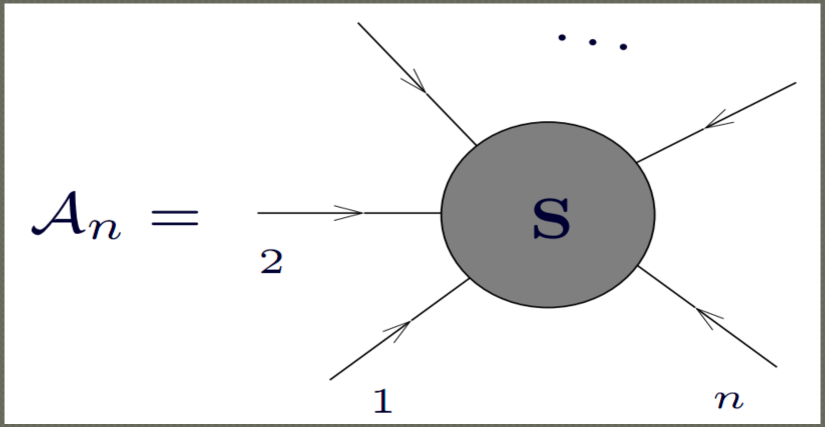
- Very complicated Feynman diagram calculations lead to very simple results.

Scattering Amplitudes



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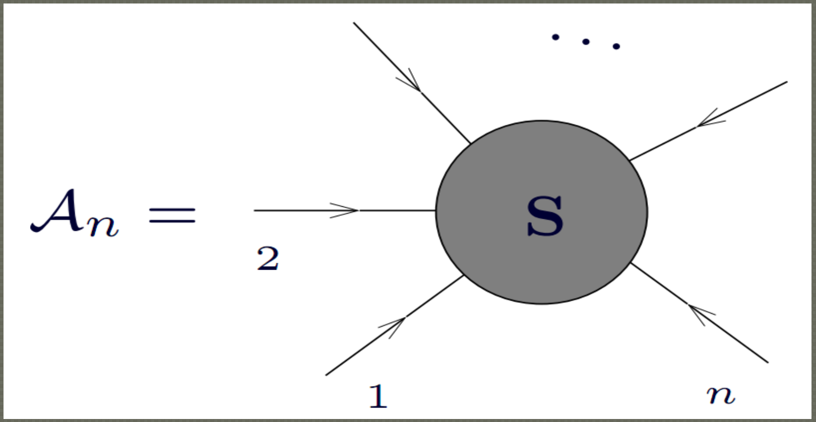

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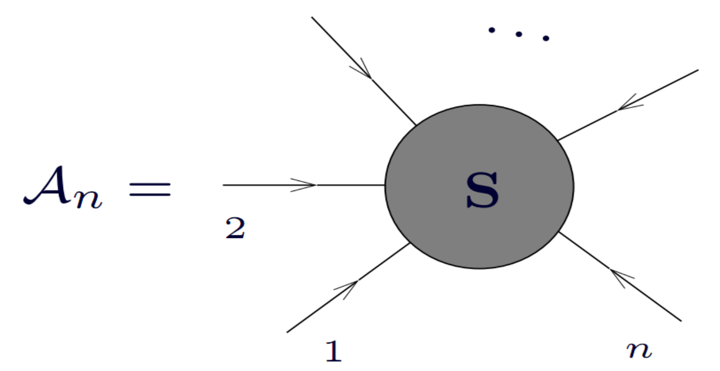

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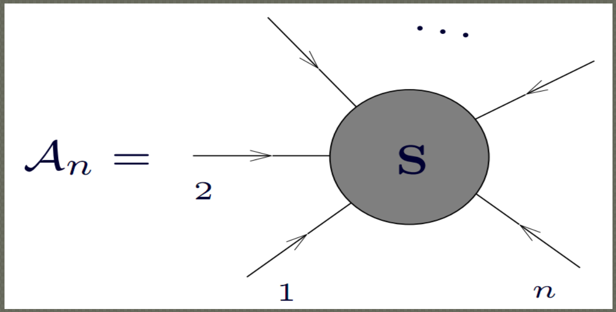

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- NMHV, NNMHV, and so on.

Introduction to notation

- Only color-ordered partial amplitudes will be considered,

$$\mathcal{A}(p_1, p_2, \ldots, p_n) = \sum_{\text{permutation}} \mathrm{Tr}(T^{a_1} T^{a_2} \ldots T^{a_n}) A_n(p_1, p_2, \ldots, p_n)$$

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- Spinor helicity formalism: $p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$ by using the on-shell condition $p^2 = 0.$
- Lorentz invariants are defined as

$$\langle i\, j \rangle = \epsilon_{ab} \lambda_i^a \lambda_j^b, \; [i\, j] = \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}_i^{\dot{a}} \tilde{\lambda}_j^{\dot{b}}.$$

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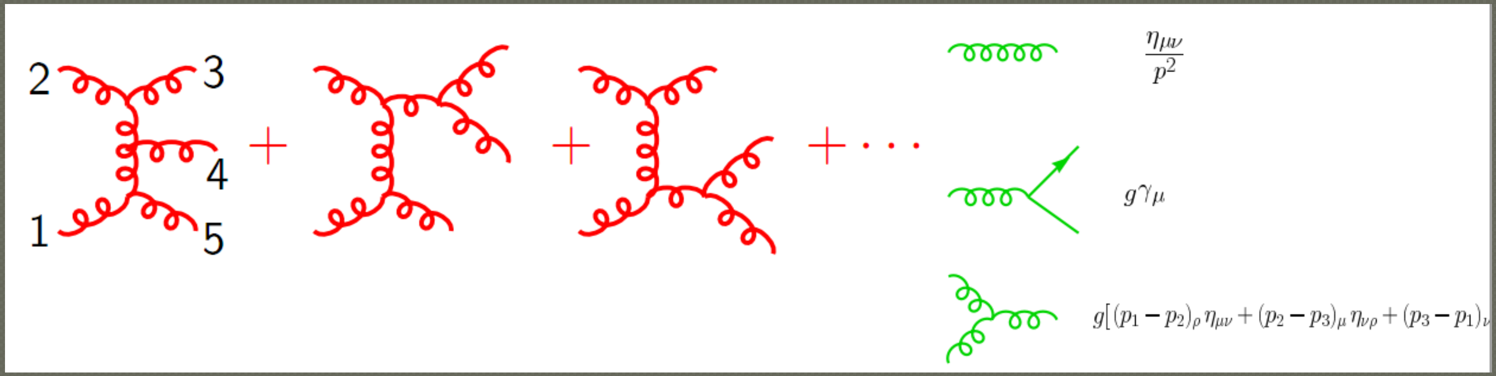

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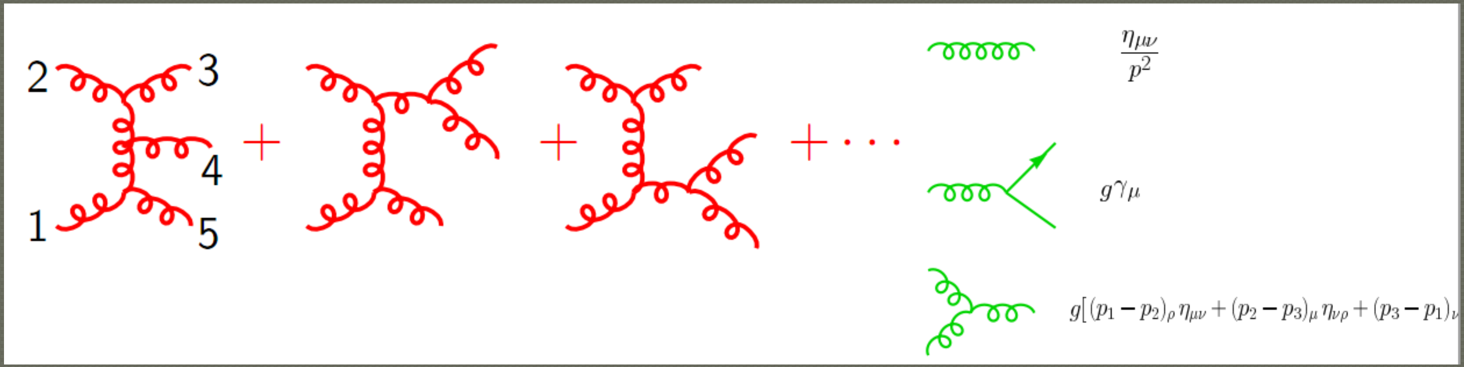

- The result obtained from traditional methods

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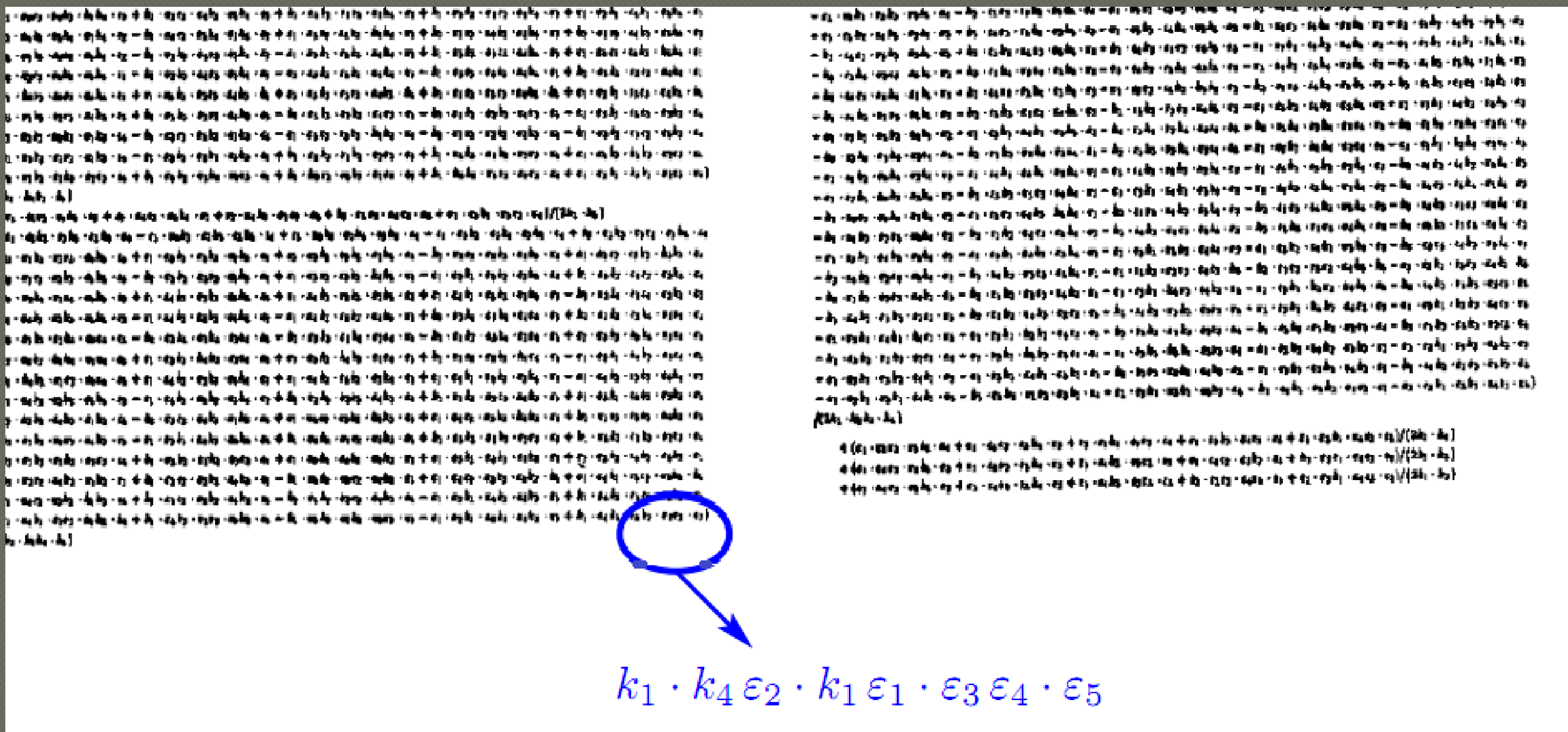

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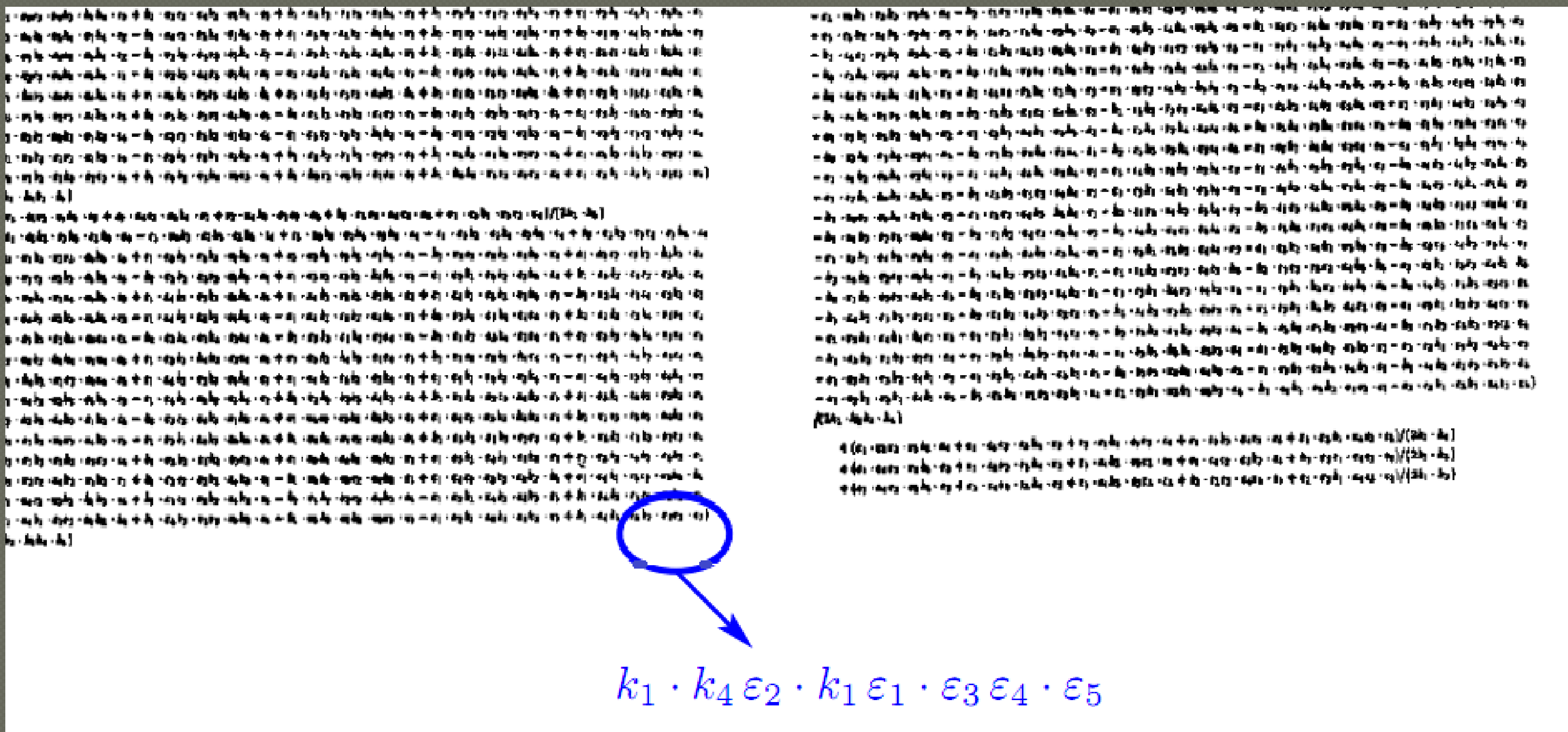

- The following contains all the physical content as the above formula:

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$$A_5(1^{\pm},2^+,3^+,4^+,5^+) = 0$$

$$A_5(1^-,2^-,3^+,4^+,5^+) = i\frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,1\rangle}$$

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$$\mathcal{A}_5(1,2,3,4,5) = \sum_{\text{perms}} \mathrm{Tr}(T^{a_1}T^{a_2}T^{a_3}T^{a_4}T^{a_5})\, A_5(1^-, 2^-, 3^+, 4^+, 5^+)$$

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- QCD amplitudes can be decomposed into simpler ones

$$A^{QCD} = A^{N=4} - 4A^{N=1 \text{ chiral}} + A^{N=0 \text{ or scalar}}$$

BCFW recursion relations

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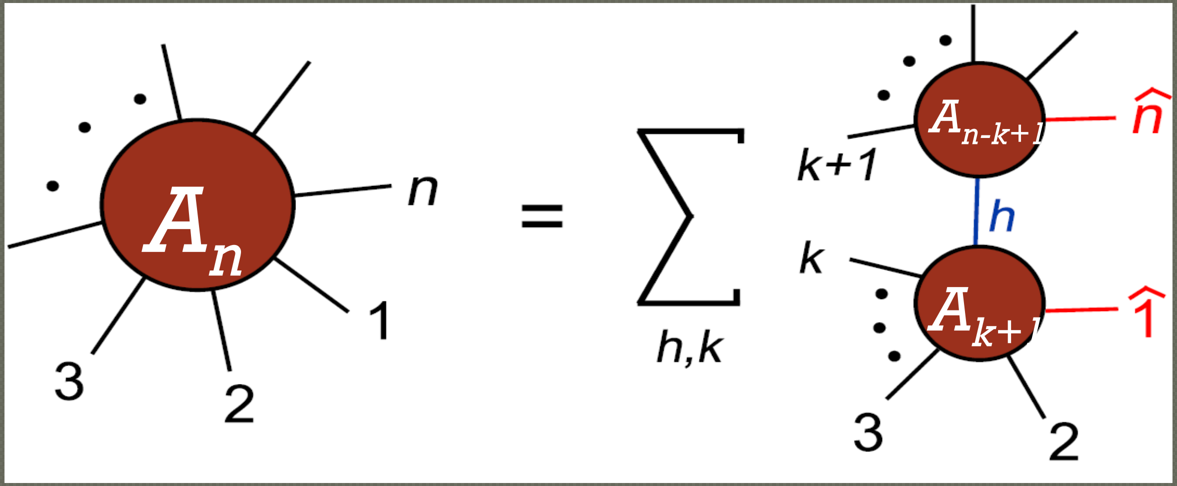

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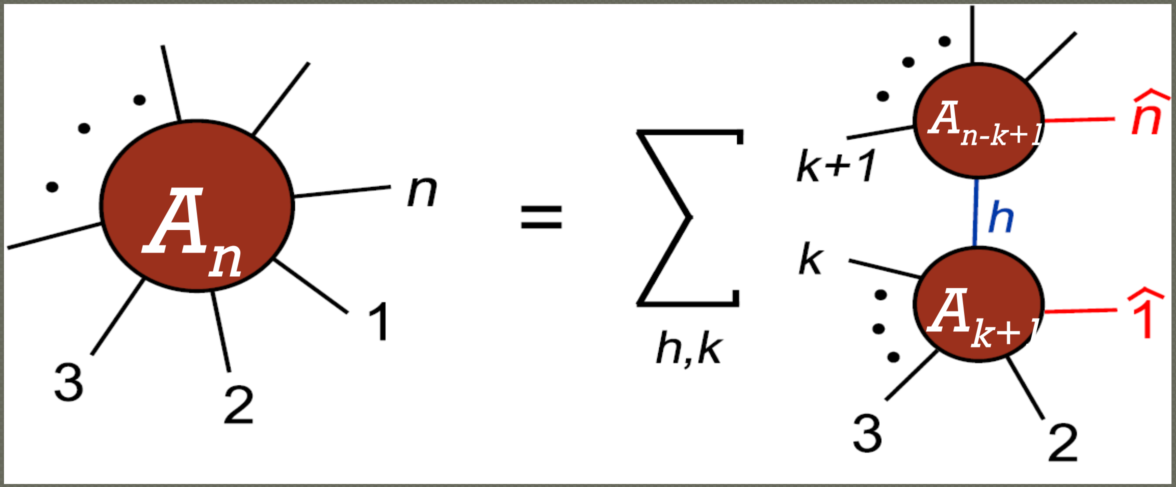

- Recursion is great, having solution is even better.

Solutions to BCFW

- First non-trivial case, MHV amplitude [Parke Taylor,86']

$$A_n^{\mathrm{MHV}}(\lambda_i, \tilde{\lambda}_i, \eta_i) = \delta^4\Big(\sum_i p_i\Big)\delta^8\Big(\sum_i \lambda_i\eta_i\Big)\frac{1}{\langle 1\, 2\rangle \cdots \langle n\, 1\rangle}.$$

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- N=8 SUGRA was also solved similarly [Drummond, Spradlin, Volovich, CW, '09]

BCFW at loop-level

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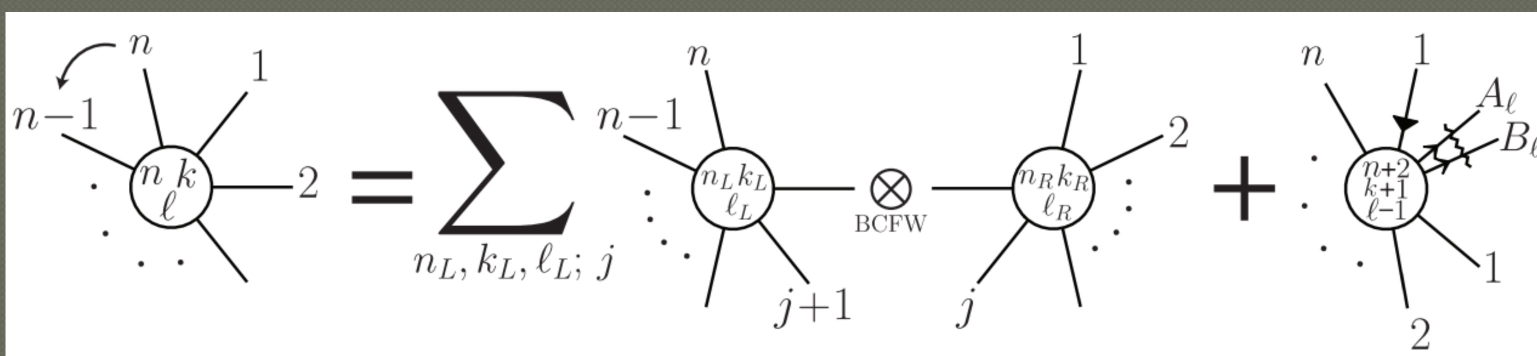

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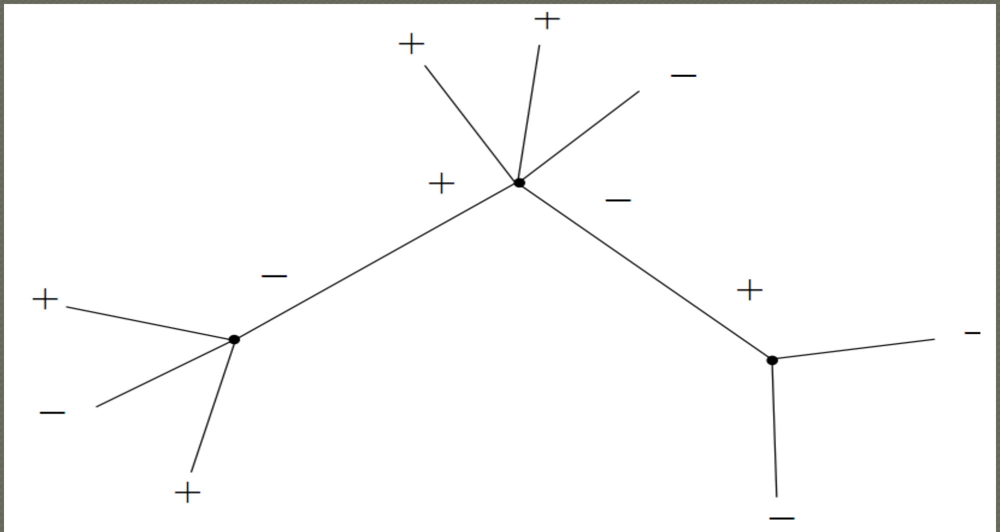

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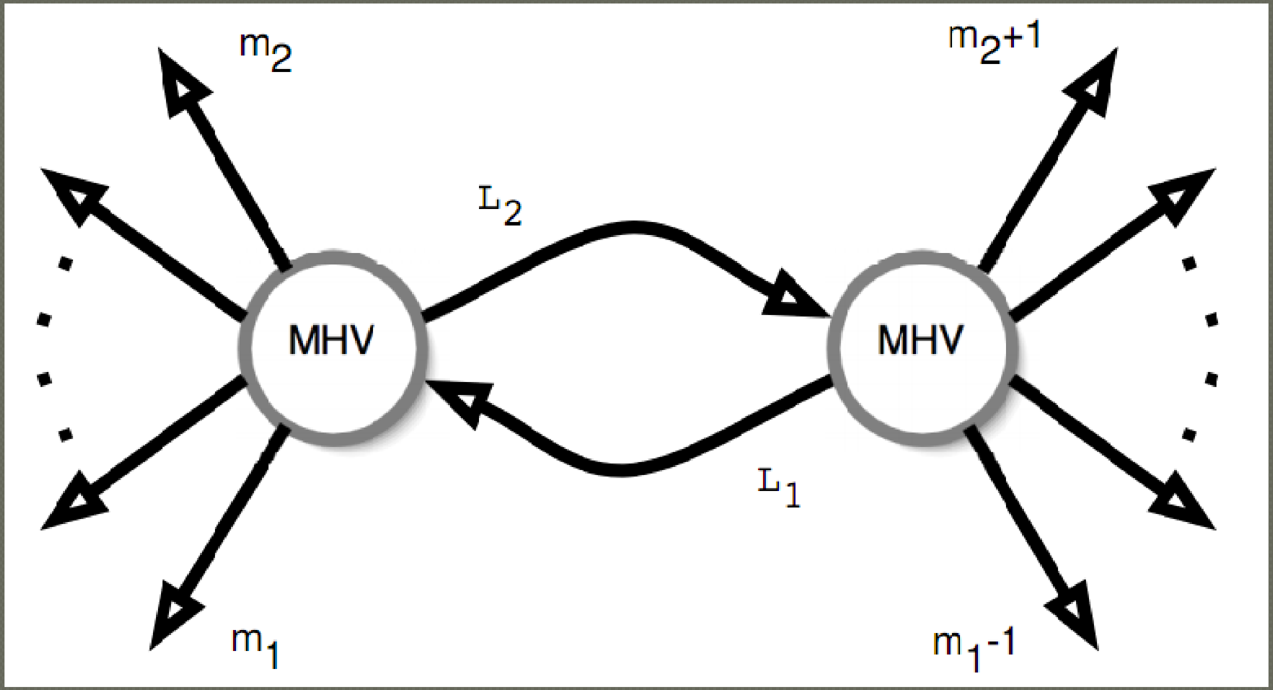

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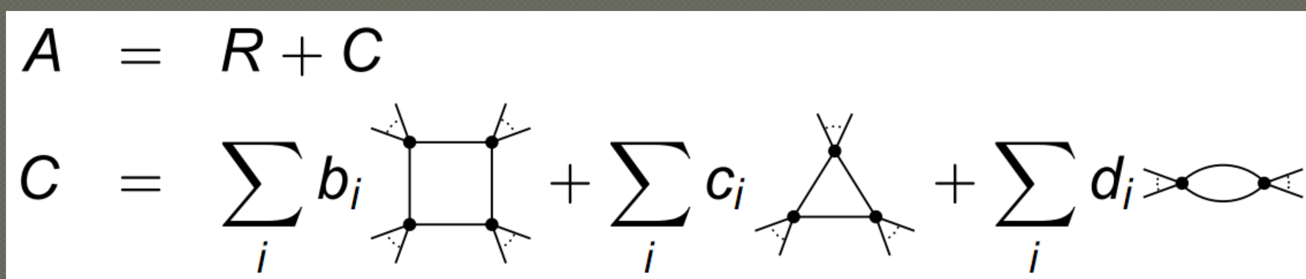

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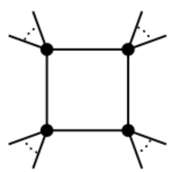
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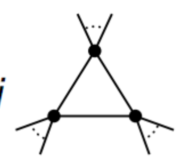
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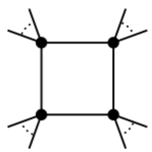
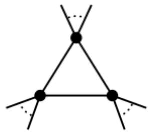
$$C = \sum_i b_i$$  $$+ \sum_i c_i$$  $$+ \sum_i d_i$$ 

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- The rational part can be compute by recursion relations.

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$$P_i = x_i - x_{i+1}, Q_i = \theta_i - \theta_{i+1}$$

- Conformal symmetry and dual conformal symmetry comprise two lowest levels of a Yangian symmetry.

[Drummond, et al, 08'][Drummond et al, 09'][Arkani-Hamed, 10']

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- All-loop amplitudes can be written as [Bern, Dixon, Smirnov, 05'] BDS ansatz + Remainder function.

Wilson loop/Correlation/Amplitude

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- It helps computing the scattering amplitudes: OPE of Wilson loop [Alday, Gaiotto, Maldacena, Sever, Vieria]

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- Twistor string theory and Grassmannian formulation are closely related to each other. [Bourjaily, Trnka, Volovich, CW, 10']

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- Much more to uncover.

Thank you!