Warped extra dimensions

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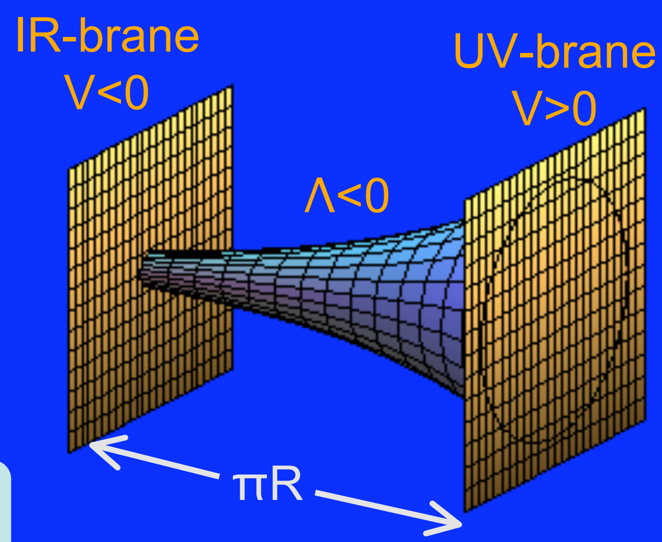
Hierarchies from Warping

[Randall, Sundrum 1999]

- 4D und 5D cosmological constants curve the extra dimension
- metric: Anti-de-Sitter (AdS) space (AdS curvature $k \sim M_5 \sim M_P$)

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$



warp factor ⟶ all scales red shifted:

↓

localized gravity

$$M_{\text{eff}} = e^{-ky} M_P$$

$R \sim 11k^{-1}$: $M_{\text{eff}}(\pi R) \sim$ TeV ⟶ ~~hierarchy problem~~

KK Gravitons

- wave functions: sin, cos ⟶ Bessel functions
- KK states are localized (IR-brane):

  ⟶ mass splitting ~ TeV -->

  ⟶ couplings ~ 1/TeV (instead of $1/M_P$!)

  ⟶ spin-2 resonances at colliders

  ⟶ m_1 > ~ TeV, depending on k/M_{Pl}

  ⟶ no astrophysical constraints like in ADD

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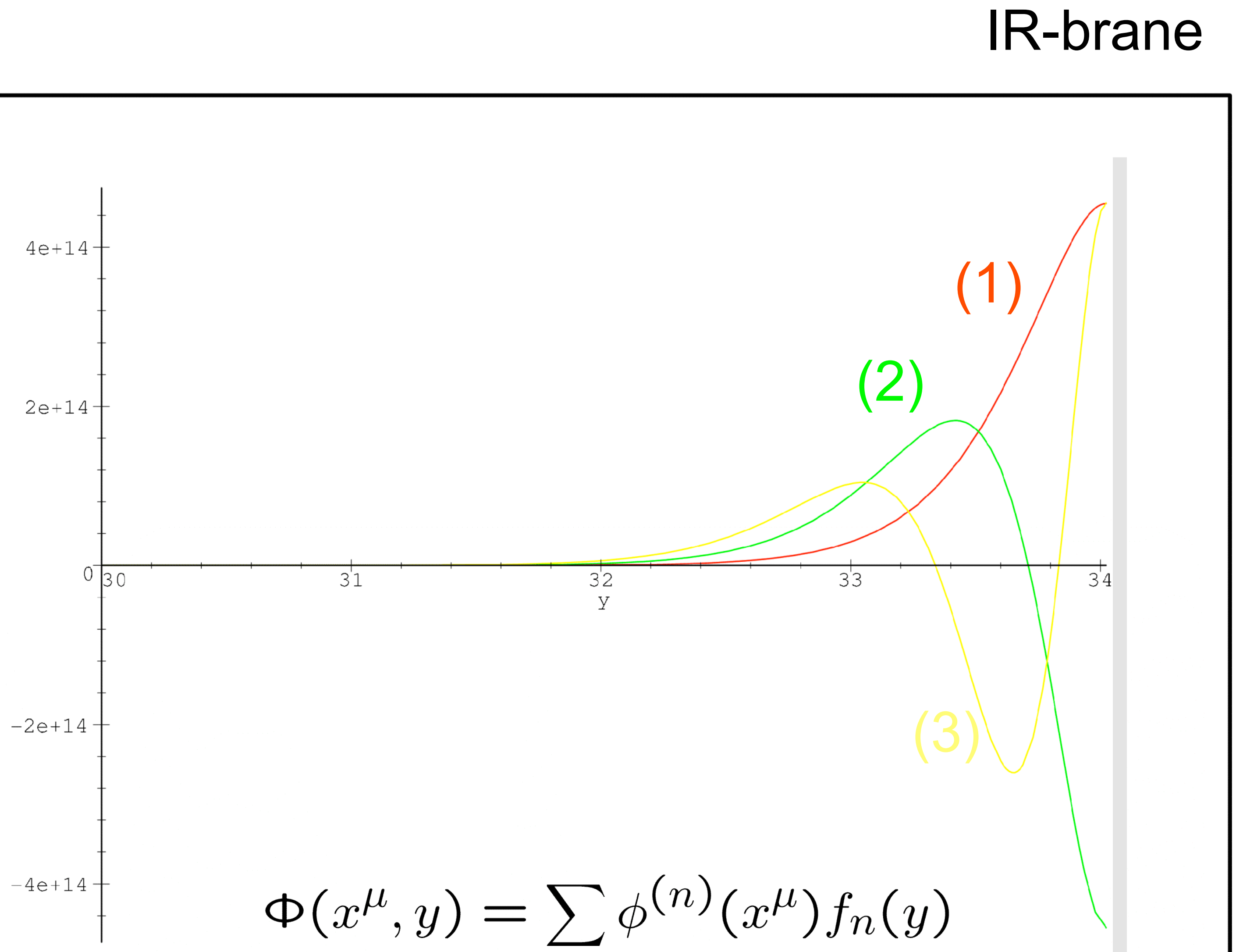
Graviton wave functions



$$\Phi(x^\mu, y) = \sum_n \phi^{(n)}(x^\mu) f_n(y)$$

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[Davoudiasl, Hewett, Rizzo
hep-ph/0006041]

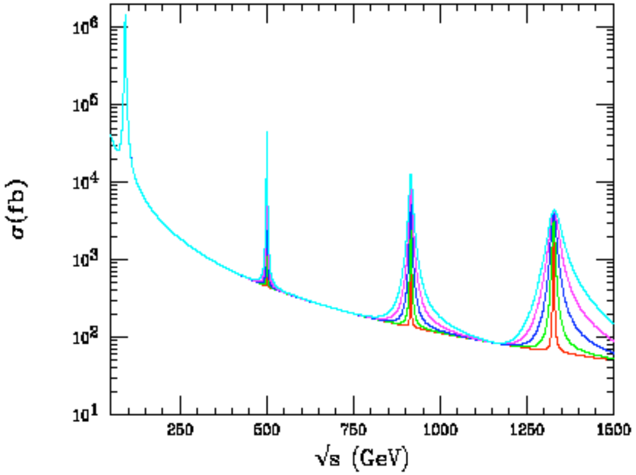


Figure 4: The cross section for $e^+e^- \to \mu^+\mu^-$ including the exchange of a KK tower of gravitons in the Randall-Sundrum model with $m_1 = 500$ GeV. The curves correspond to $k/\overline{M}_{Pl} =$ in the range $0.01 - 0.05$.

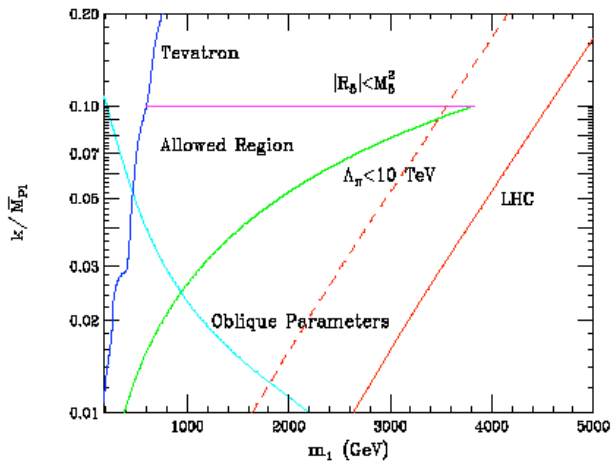
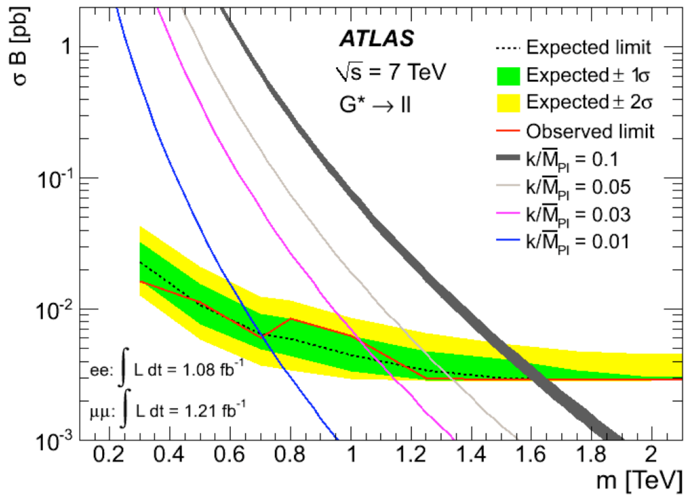


Figure 6: Summary of experimental and theoretical constraints on the Randall-Sundrum model in the two-parameter plane $k/\overline{M}_{Pl} - m_1$, for the case where the Standard Model fields are constrained to the TeV-brane. The allowed region lies in the center as indicated. The LHC sensitivity to graviton resonances in the Drell-Yan channel is represented by the diagonal dashed and solid curves, corresponding to 10 and 100 fb^{-1} of integrated luminosity, respectively. From (38).



	Z'_{E_6} Models						G^*			
Model/Coupling	Z'_ψ	Z'_N	Z'_η	Z'_I	Z'_S	Z'_χ	0.01	0.03	0.05	0.1
Mass limit [TeV]	1.49	1.52	1.54	1.56	1.60	1.64	0.71	1.03	1.33	1.63

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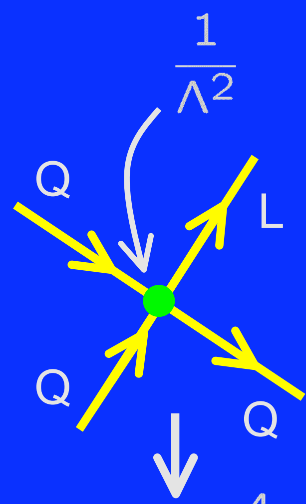
Problems

- the SM is an effective theory with cut-off Λ
- all mass scales get rescaled!

⟶ Λ ~ TeV

⟶ generic four-fermion operators suppressed only by 1/TeV

$$\frac{1}{M_5^3}\bar{\psi}\psi\bar{\psi}\psi \longrightarrow \frac{1}{(\mathrm{TeV})^2}\bar{\psi}\psi\bar{\psi}\psi$$



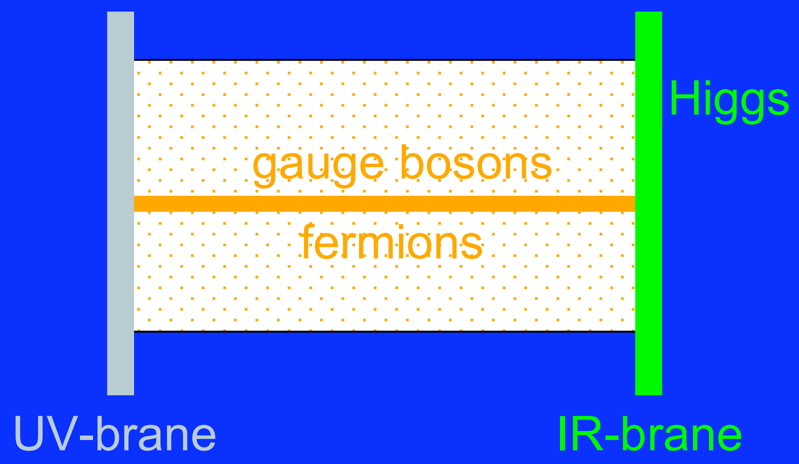
$$\tau_P \sim \frac{\Lambda^4}{m_P^5}$$

⟶ proton decay (QQQL: >10^{15} GeV)
flavor violation (DSDS: >10^{6} GeV)
neutrino masses (HHLL: >10^{14} GeV)
…
} ?

⟶ symmetries??

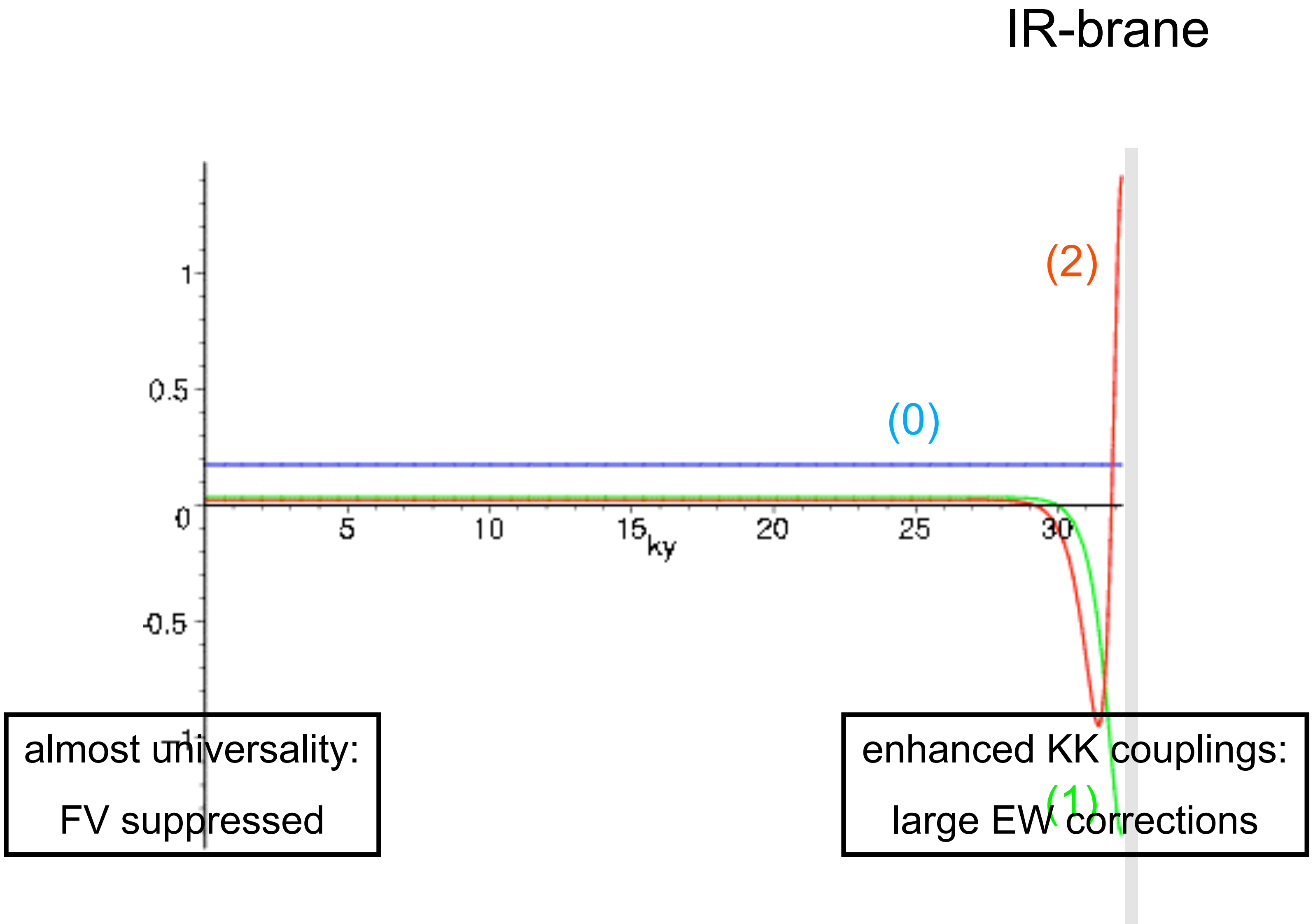
Way out: SM in the bulk

- cut-off is y-dependent: $\Lambda = e^{-ky} M_{\mathrm{P}}$
- electroweak scale ⟷ Higgs: IR-brane
- gauge bosons, fermions: bulk



⟶ allows cut-off Λ >> TeV !

Main properties of gauge bosons:



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Electroweak constraints

- W and Z mix with their KK excitations → SM mass relations modified at tree-level $(\sim M_W^4/M_{KK}^2)$
  M_W somewhat enhanced (contribution to the ρ parameter)
  → $M_{KK} > 10$ TeV
  (compared to an LHC reach of 5 TeV?)

- bounds can be relaxed to a few TeV by introducing
  a) brane kinetic terms [Carena, Ponton, Tait, Wagner]
  b) left-right symmetric gauge group [Agashe, Delgado, May, Sundrum]
  c) bulk Higgs (soft wall?) [Cabrer, Gersdorff, Quiros]
  d) more than 5 dimensions [Archer, SH]

More than 5 dimensions

To resolve hierarchy problem would like $M_{\rm fund} \sim m_{\rm fund}^{\rm Higgs}$. Given a $4+1+\delta$ dimensional space;

$$ds^2 = a^2(r)\eta_{\mu\nu}dx^\mu dx^\nu - b^2(r)dr^2 - c^2(r)d\Omega_\delta^2$$

where $d\Omega_\delta^2 = \gamma_{ij}d\phi^i d\phi^j$ and $i,j = 1\ldots\delta$. In the 4D effective theory, parameters modified by two effects:

Volume Effects

Fields propagating in the bulk (e.g. gravity) have couplings scaled by volume of extra dimensions. (ADD Model)(Arkani-Hamed, Dimopoulos, Dvali '98)(Antoniadis, Arkani-Hamed, Dimopoulos, Dvali '98).

$$M_{\rm P}^2 \sim \int d^{\delta+1}x\, a^2 b c^\delta \sqrt{\gamma}\, M_{\rm Fund}^{\delta+3}.$$

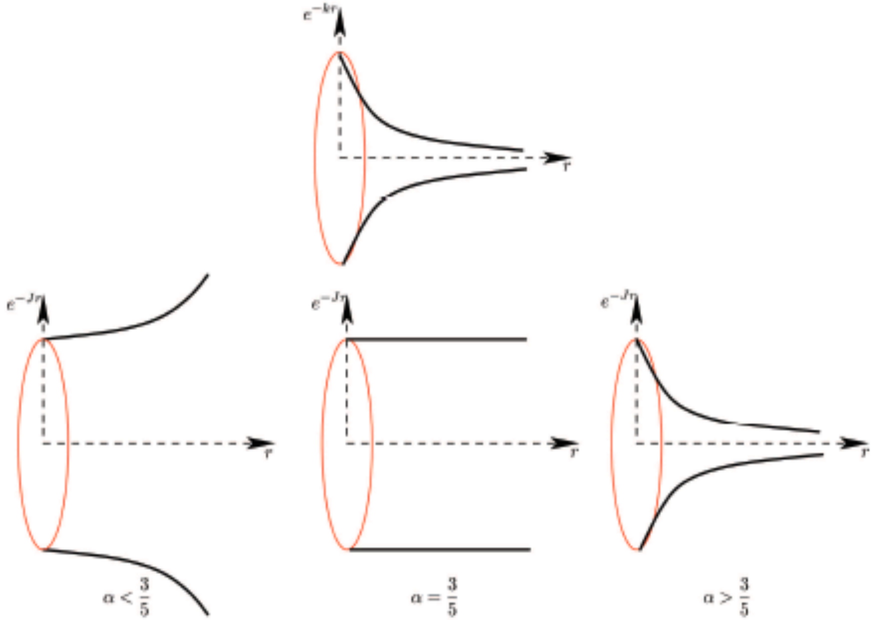
Warping

Fields localised on the branes have masses suppressed by gravitational redshifting. (RS Model)(Randall, Sundrum '99)

$$m_{\rm 4D}^2 = a^2(r_{\rm ir})m_{\rm fund}^2$$

If $M_{\rm P} \sim M_{\rm fund}$ then a warp factor $a(r_{\rm ir})^{-1} \equiv \Omega \sim 10^{15}$ would resolve hierarchy

Three classes of solution:



Effective Planck mass:

$$M_P^2 = M_{\rm Fund}^{3+\delta} \frac{(1 - e^{-(2k+\delta J)R})(R_\theta)^\delta}{(2k + \delta J)}.$$

Hence if $2k + \delta J > 0$ and $k \sim J \sim R_\theta^{-1} \sim M_{\rm Fund} \sim M_P$ then $\Omega \sim 10^{15}$ would resolve hierarchy problem.

KK decomposition of Gauge fields

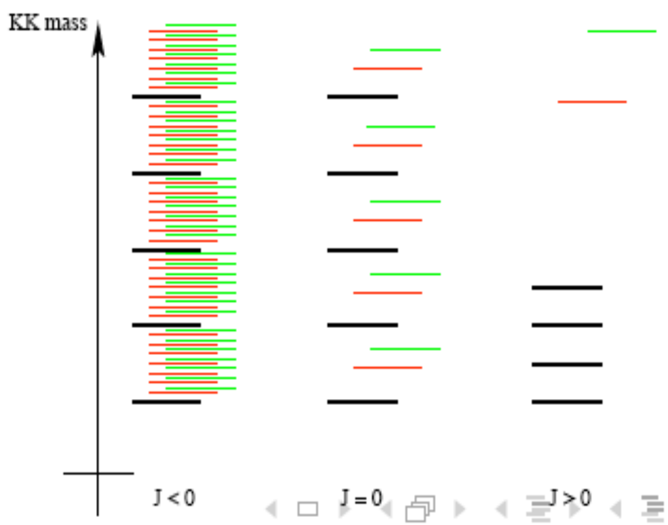
Assuming δ extra dimensions are toroidal i.e $\partial_i^2\Theta_n = -\frac{l_i^2}{R_\theta^2}\Theta_n$, the gauge profiles given by:

$$f_n'' - (2k+\delta J)f_n' - \sum_{i=1}^{\delta} e^{2Jr}\frac{l_i^2}{R_\theta^2}f_n + e^{2kr}m_n^2 f_n = 0.$$

substituting $x_n^2 = \frac{e^{2kr}m_n^2 - e^{2Jr}\sum_{i=1}^{\delta}\frac{l_i^2}{R_\theta^2}}{\gamma^2}$ such that $x_n' = \gamma x_n$ then;

$$f_n(x) = N x_n^{\frac{1}{2}\frac{2k+\delta J}{\gamma}}\left(\mathbf{J}_{-\frac{1}{2}\frac{2k+\delta J}{\gamma}}(x_n) + \beta \mathbf{Y}_{-\frac{1}{2}\frac{2k+\delta J}{\gamma}}(x_n)\right)$$

$$m_n \sim X_n \frac{\sqrt{\gamma^2 + \frac{e^{2JR}}{R_\theta^2}\sum_i^\delta l_i^2}}{e^{kR}}$$

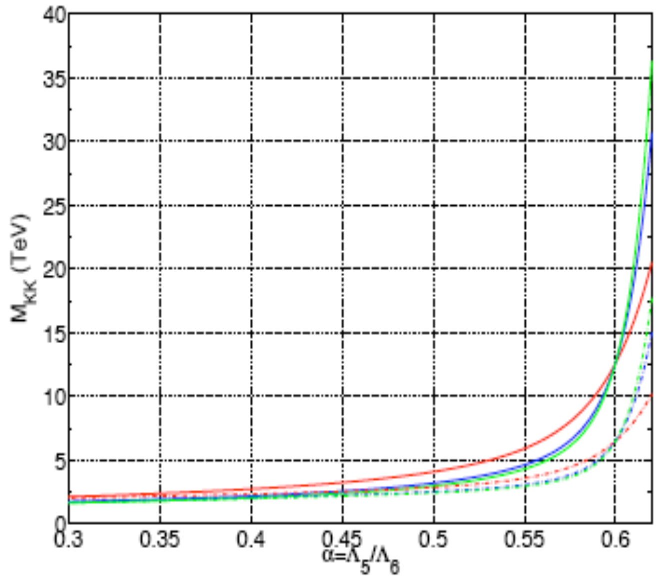


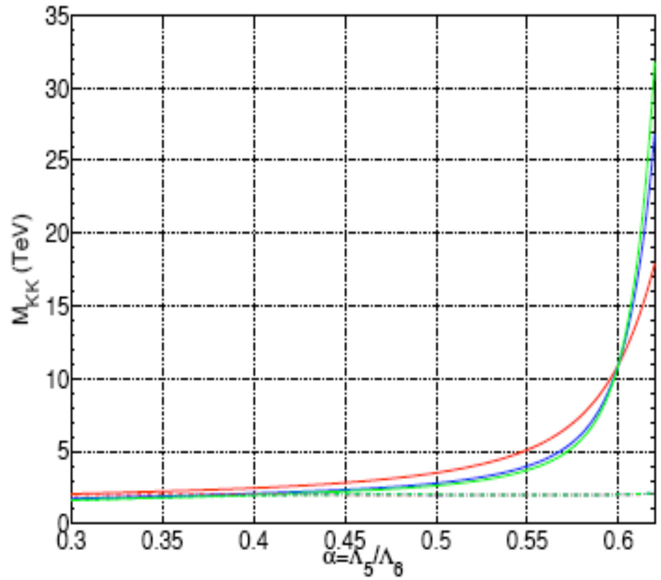
where

$X_n \sim \mathcal{O}(1)$.

Electroweak constraints

[Archer, SH]





- Lower bound on $M_{KK} = \frac{k}{\Omega}$ from s_Z^2 constraint for two models, bulk $SU(2) \times U(1)$ gauge symmetry (left) and bulk $SU(2)_R \times SU(2)_L \times U(1)$ custodial symmetry (right). With the fermions localised on the IR brane (solid lines), UV brane (dashed lines).
- Constraints for $\alpha > 0.6$ not plotted since KK modes strongly coupled so tree level analysis not valid.
- Overall lower bound corresponding to $F_n^2,\ F_n F_\psi \sim 1$ is

$$M_{KK} \gtrsim 2 - 2.5\ \text{TeV}$$

Fermion mass hierarchy

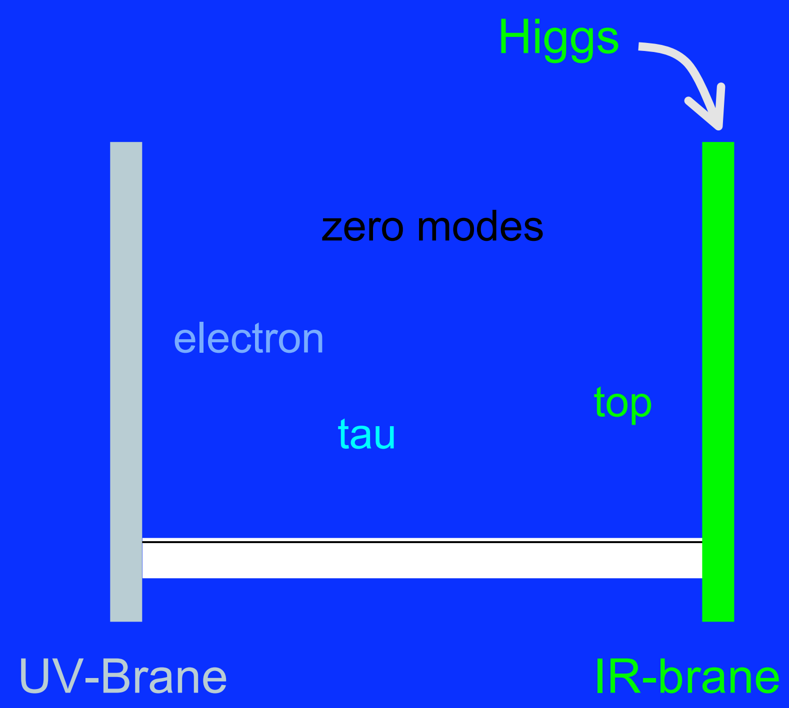
[Gherghetta, Pomarol; S.H., Shafi]

- zero modes: shape depends on 5D Dirac mass c

$$f_0 \sim e^{(1/2-c)ky}$$

c>½ ⟶ UV-brane

c<½ ⟶ IR-brane



- Yukawas $\lambda^{(5)}$ to the Higgs

- 5D geography: overlap

light (heavy) fermions ⟷ UV (IR)-brane

hierarchical fermion masses with $\lambda^{(5)} \sim 1$

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Flavor violation

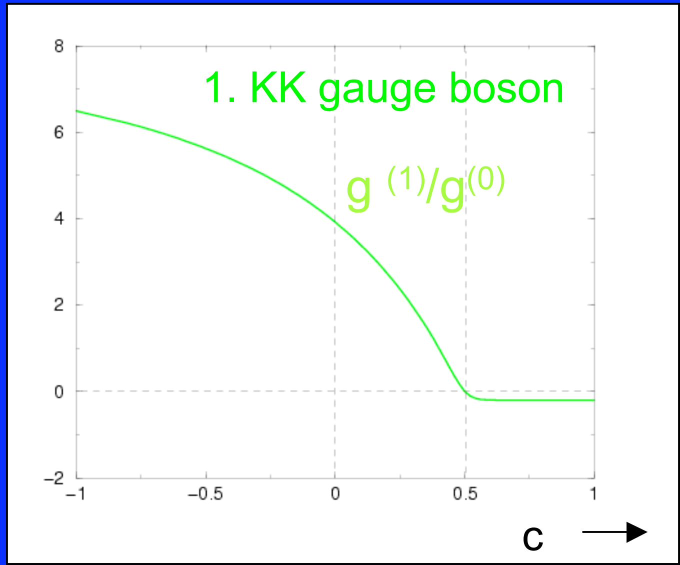
- KK gauge bosons couple non-universally, so that $g=\mathrm{diag}(g_e,g_\mu,g_\tau)$
- in the mass basis $\psi \longrightarrow U\psi$

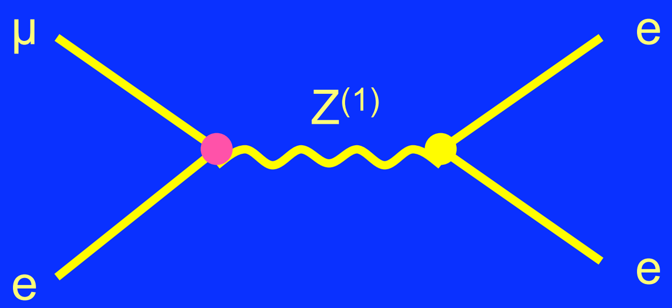
$$G = U^\dagger g U$$

flavor non-diagonal couplings

- but: couplings almost universal for c>1/2 where the light fermions are localized

"warped GIM mechanism"





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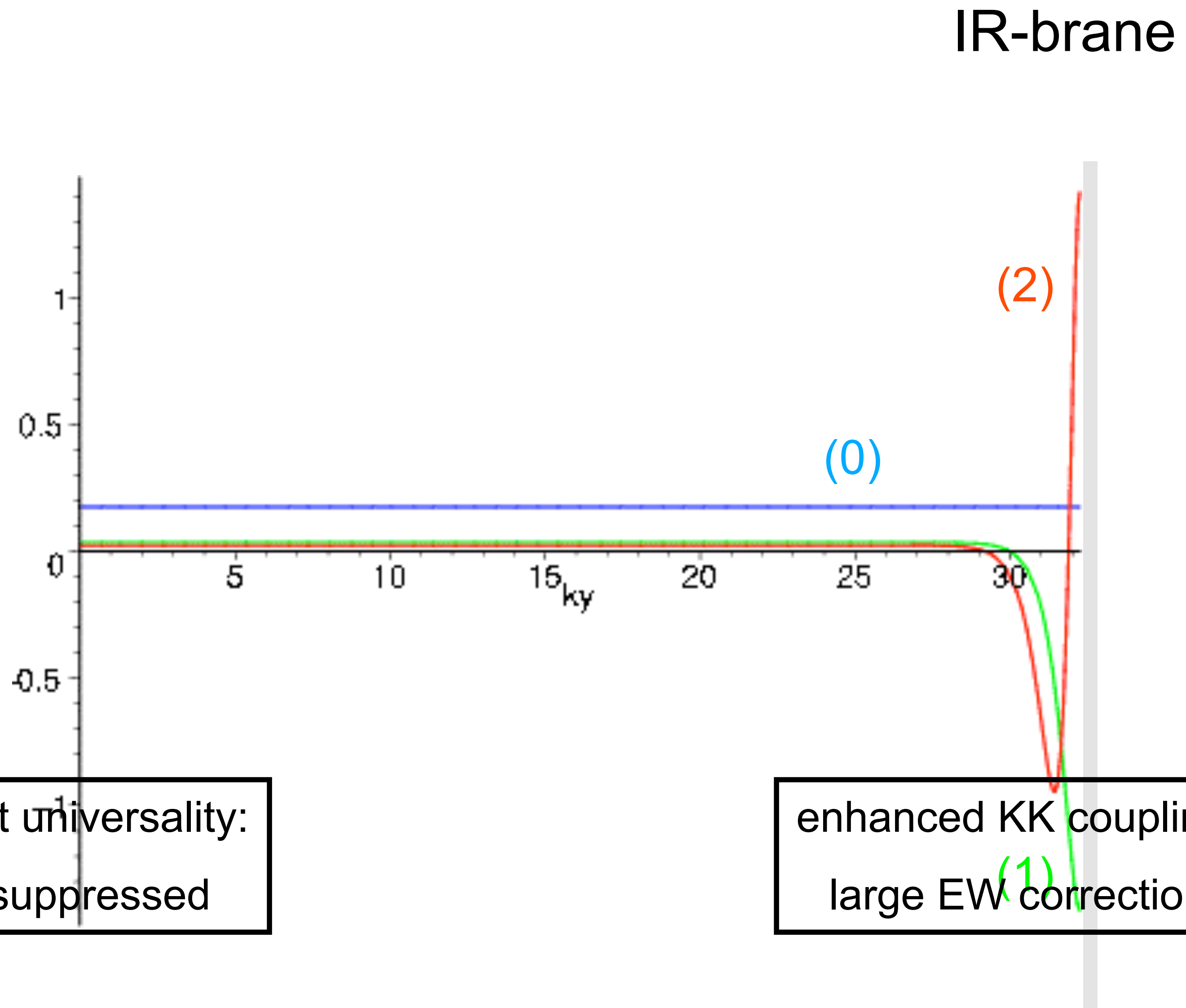
Main properties of gauge bosons:



almost universality:

FV suppressed

enhanced KK couplings:

large EW corrections

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Flavor violation: examples

- strongest constraints from CP violation in the Kaon system ϵ_K: m_{KK} > ~30 TeV [Bauer, Casagrande, Haisch, Neubert 2009]

note: bounds are much weaker than for flat extra dimensions, where m_{KK} > 1000 TeV! [Delgado, Pomarol, Quiros 1999]

- lepton flavor violation: $BR(\mu \to eee) \sim 10^{-13}$ [Huber 2003]

  (exp. bound: $< 1 \times 10^{-12}$)

  $BR(\mu \to e\gamma) \sim 10^{-16}$ (exp. bound: $< 1.1 \times 10^{-11}$)

  muon electron conversion $BR(\mu N \to eN) \sim 10^{-15}$

  exp. bound: $< 1 \times 10^{-13}$ (but plans to reach $\sim 10^{-17}$)

  → observable??

Soft-Wall Extra Dimension

[Batell, Gherghetta, Sword '08]

- Different phenomenology can be found by considering what is known as a "soft wall" extra dimension.
- The soft wall is realised by removing the IR brane and replacing it with a smooth space time cutoff generated by a dilaton field $\Phi(y)$. The action is given by

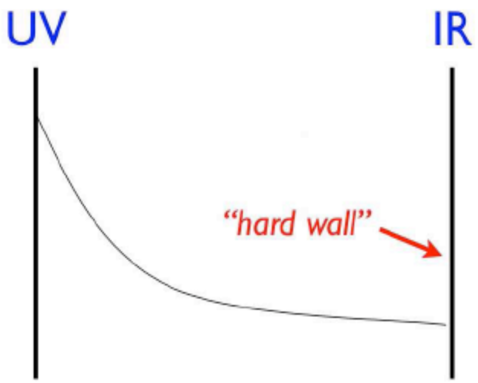
$$S = \int d^4x \int dy \sqrt{g}\, e^{-\Phi(y)}\, \mathcal{L}.$$

$$\Phi(y) = (\mu y)^2$$

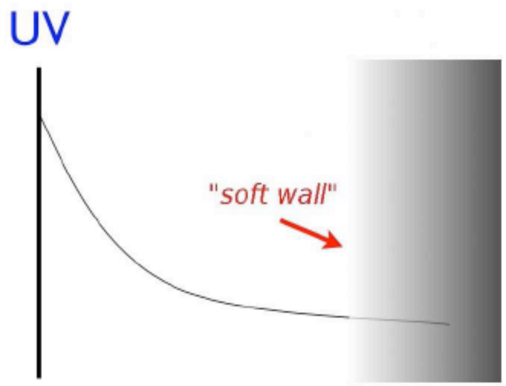
Mass scales:

AdS curvature k

KK scale μ



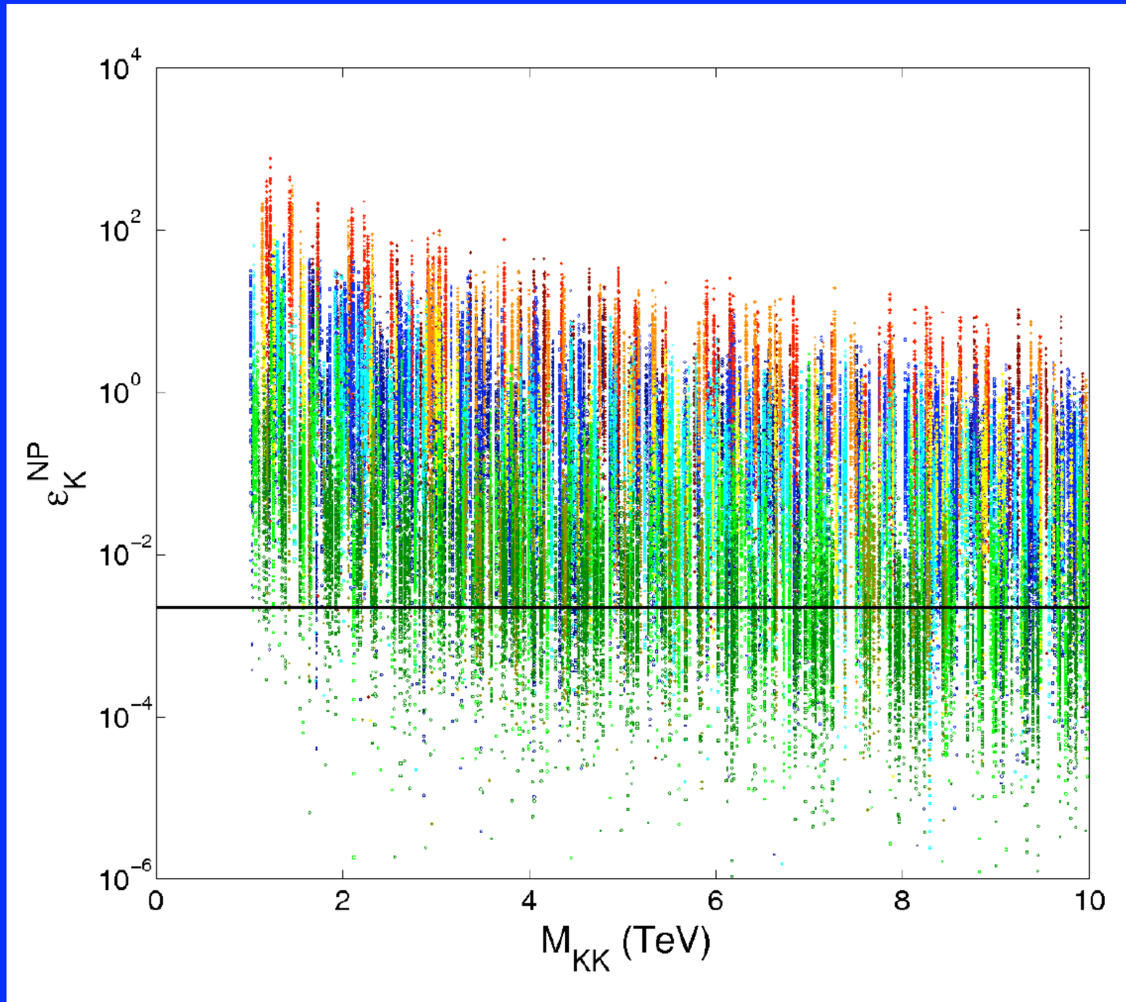
Randall-Sundrum, $m_n^2 \sim n^2$.



Soft-wall model, $m_n^2 \sim n$.

- In the soft wall model the Higgs must necessarily propagate in the bulk.
- It has been found that the soft-wall model is less constrained by EW precision observables than RS. (m_{KK}~2 TeV, Falkowski, Perez-Victoria '08)

Constraints from CP violation the kaon sector: much relaxed!



[Archer, SH, Jaeger 2011]

Conclusions

Models with warped geometry provide a rich phenomenology:

new resonances (KK gravitons etc.) at colliders

fermion mass geography

main constraints: electroweak (RS: > ~10 TeV → few TeV)

flavor (RS: > ~30 TeV → few TeV)

results depend sensitively on variants of the model: more than 5D?

soft wall?

Warped models may be dual to strongly coupled 4D models

Cosmology: no immediate dark matter candidate

holographic phase transition

KK Gravitons

- wave functions: sin, cos ⟶ Bessel functions
- KK states are localized (IR-brane):

  ⟶ mass splitting ~ TeV -->

  ⟶ couplings ~ 1/TeV
  (instead of $1/M_P$!)

  ⟶ spin-2 resonances at
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  ⟶ $m_1 > 0.2$-1.5 TeV, depending on k/M_{Pl}

  ⟶ no astrophysical constraints like in ADD

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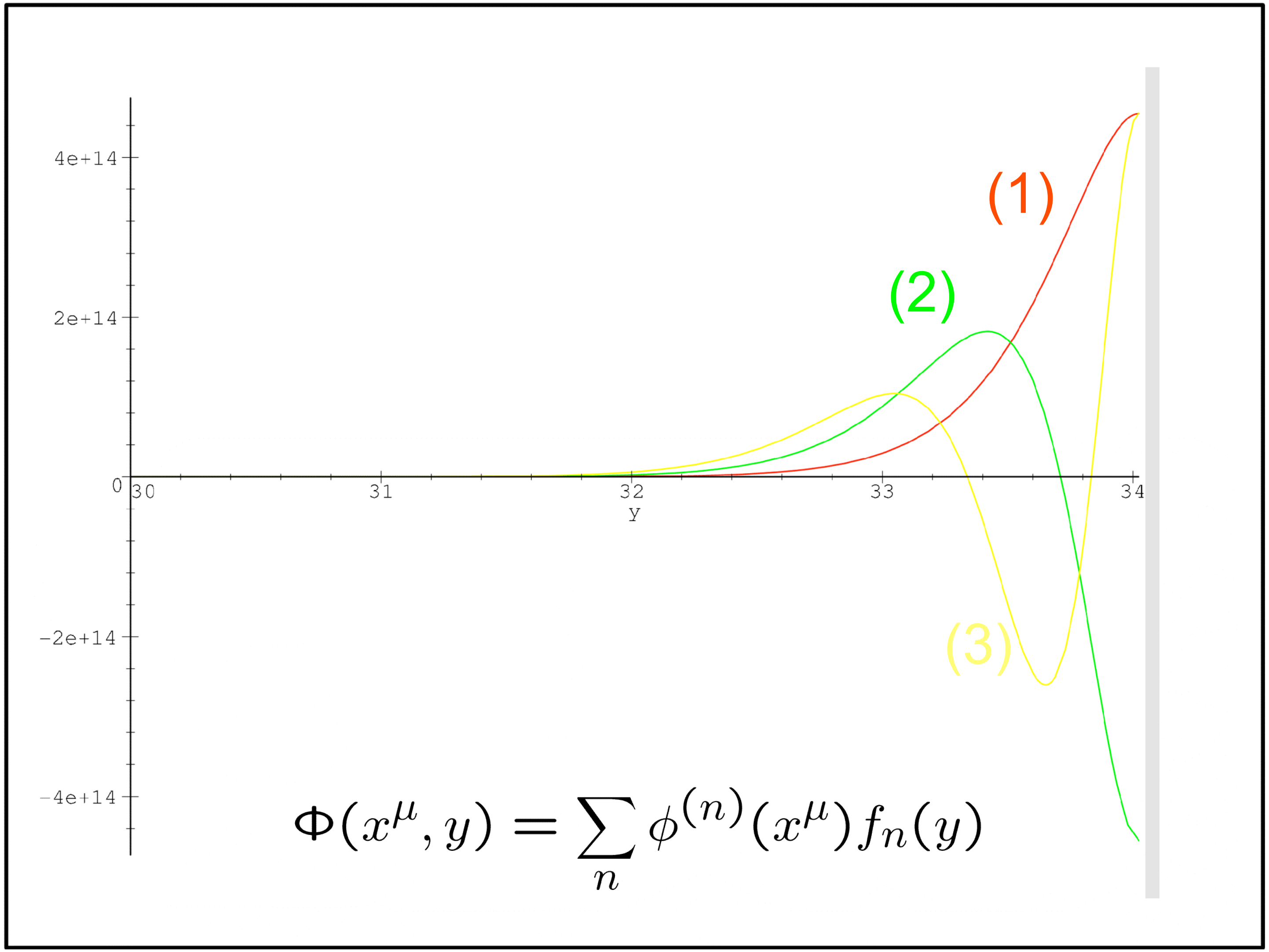
Graviton wave functions



$$\Phi(x^\mu, y) = \sum_n \phi^{(n)}(x^\mu) f_n(y)$$

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The Klebanov-Strassler solution

10-dimensional metric:

$$ds_{10}^2 = h^{-\frac{1}{2}}(\tau)\eta_{\mu\nu}dx^\mu dx^\nu - h^{\frac{1}{2}}(\tau)ds_6^2$$

$$\begin{aligned} h(\tau) &= 2^{\frac{2}{3}}(g_s M\alpha')^2 \epsilon^{\frac{-8}{3}} I(\tau) \\ I(\tau) &= \int_\tau^\infty dx \frac{x\coth x - 1}{\sinh^2 x}(\sinh(2x) - 2x)^{\frac{1}{3}}. \end{aligned}$$

“internal 5+1 dimensional metric:

$$ds_6^2 \approx \frac{1}{2}\epsilon^{\frac{4}{3}} K(\tau)\left[\frac{d\tau^2}{3K^3(\tau)} + \cosh^2(\frac{\tau}{2})d\Omega_3^2 + \sinh^2(\frac{\tau}{2})d\Omega_2^2\right]$$

$$K(\tau) = \frac{(\sinh(2\tau) - 2\tau)^{\frac{1}{3}}}{2^{\frac{1}{3}}\sinh(\tau)}.$$

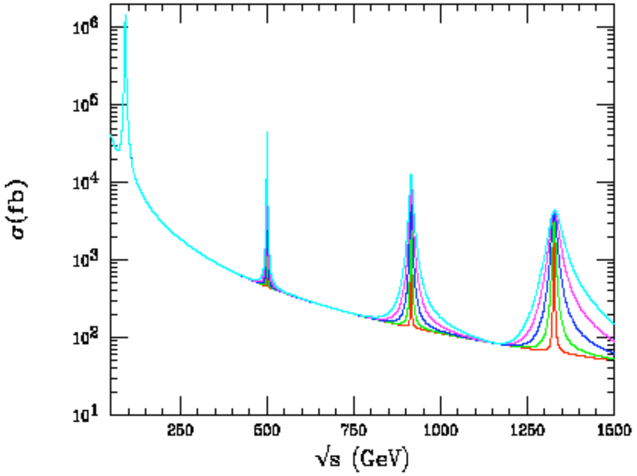

Figure 4: The cross section for $e^+e^- \to \mu^+\mu^-$ including the exchange of a KK tower of gravitons in the Randall-Sundrum model with $m_1 = 500$ GeV. The curves correspond to $k/\overline{M}_{Pl} =$ in the range $0.01 - 0.05$.

[Davoudiasl, Hewett, Rizzo hep-ph/0006041]

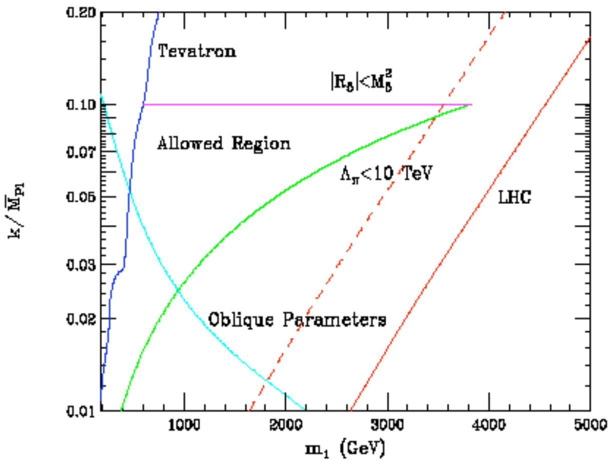

Figure 6: Summary of experimental and theoretical constraints on the Randall-Sundrum model in the two-parameter plane $k/\overline{M}_{Pl} - m_1$, for the case where the Standard Model fields are constrained to the TeV-brane. The allowed region lies in the center as indicated. The LHC sensitivity to graviton resonances in the Drell-Yan channel is represented by the diagonal dashed and solid curves, corresponding to 10 and 100 fb^{-1} of integrated luminosity, respectively.

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Bulk fields

Allowing the SM fields to propagate in the bulk allows for tree level corrections to EWO's dominated by the exchange of KK gauge fields. Would like to know size of these corrections.

Perform usual KK decomposition

$$\boxed{A_\mu = \sum_n A_\mu^{(n)}(x^\mu) f_n(r)\Theta_n(\phi_1, \ldots, \phi_\delta)}$$

such that

$$\int d^{1+\delta}x\; bc^\delta \sqrt{\gamma} f_n f_m \Theta_n \Theta_m = \delta_{nm},$$

where

$$f_n'' + \frac{(a^2 b^{-1} c^\delta)'}{(a^2 b^{-1} c^\delta)} f_n' - \frac{b^2}{c^2}\alpha_n f_n + \frac{b^2}{a^2} m_n^2 f_n = 0 \qquad -\frac{1}{\sqrt{\gamma}}\partial_{\phi_i}(\sqrt{\gamma}\gamma^{ij}\partial_{\phi_j}\Theta_n) = \alpha_n \Theta_n.$$

Since Θ_n will typically be a sum over eigenfunctions of α_n there is now large degeneracy corresponding to the δ extra KK towers. But since $\alpha_n \geqslant 0$ first KK mode always corresponds to $\alpha_n = 0$.

The Model

Here in order to investigate $D > 5$ dimensions consider simple bottom up extension of RS model. Very much toy model since warped only w.r.t one dimension r. (Kogan, Mouslopoulos, Papazoglou, Ross '01)

$$S = \int d^{5+\delta}x\sqrt{-G}\left[\Lambda - \frac{1}{2}M_{\rm Fund}^{3+\delta}R\right] + \int d^4x\sqrt{-g_{ir}}\left[\mathcal{L}_{ir} + V_{ir}\right] + \int d^4x\sqrt{-g_{uv}}\left[\mathcal{L}_{uv} + V_{uv}\right]$$

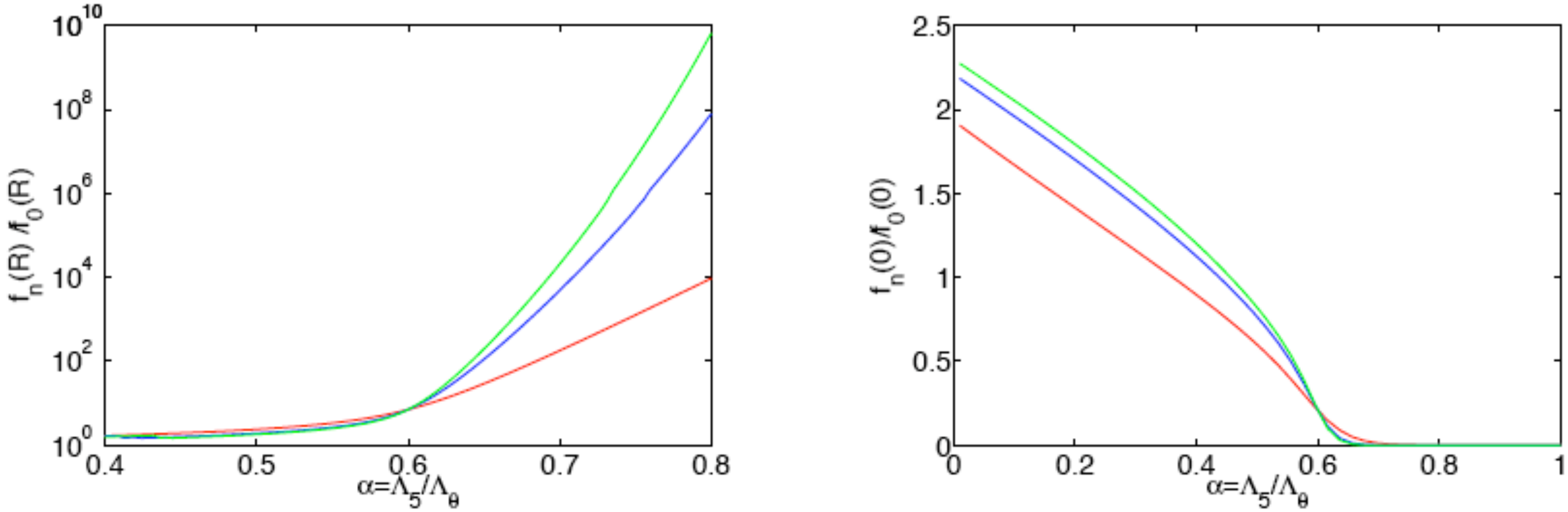
with a bulk cosmological constant

$$\Lambda = \begin{pmatrix} \Lambda\eta_{\mu\nu} & & & & \\ & \Lambda_5 & & & \\ & & \Lambda_\theta & & \\ & & & \ddots & \\ & & & & \Lambda_\theta \end{pmatrix} \qquad \text{and define} \qquad \alpha \equiv \frac{\Lambda_5}{\Lambda_\theta}.$$

The Einstein equations then admit the solution

$$\boxed{ds^2 = e^{-2kr}\eta_{\mu\nu}dx^\mu dx^\nu - dr^2 - \sum_{i=1}^{\delta} e^{-2Jr}d\theta_i^2}$$

The Relative Gauge Coupling



- The relative coupling of gauge fields in 6D (red), 8D (blue) and 10D (green) with particles localised on the IR brane (left) and UV brane (right).
- $F_n \longrightarrow 1$ for $J < 0$ and becomes large for $J > 0$.
- Conversely if fermions localised on UV brane, $F_\psi^{(n)}$ small for $J > 0$ but larger for $J < 0$.

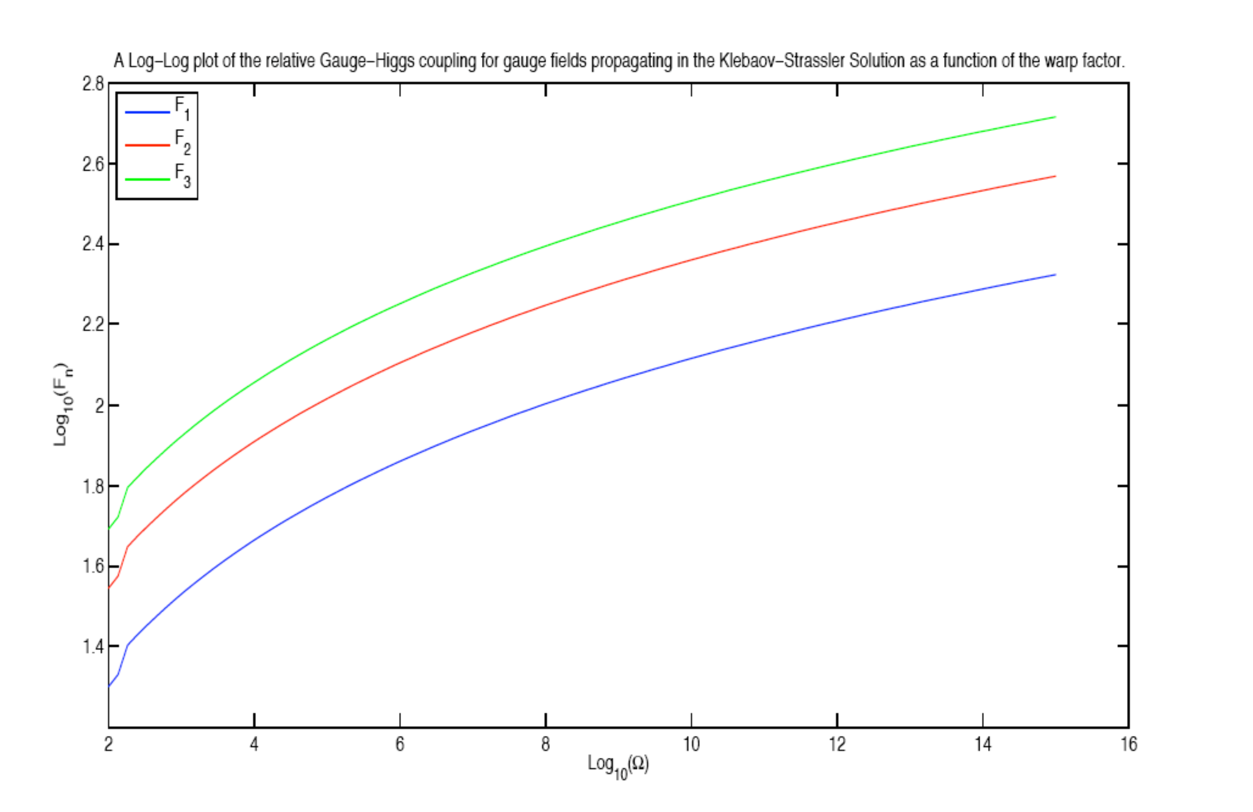
Electroweak constraints

- Typically tightest constraint comes from weak mixing angle:

$$s_Z^2 \approx s_p^2 \left(1 - \frac{c_p^2}{c_p^2 - s_p^2} \sum_{n=1} \left[\frac{m_z^2 F_n^2}{m_n^2} - \frac{m_w^2 \left(F_n - F_\psi^{(n)} \right)^2}{m_n^2} \right] + \mathcal{O}(m_n^{-4}) \right)$$

(you can also consider S und T parameter)

Back to the Klebanov-Strassler solution



→ huge constraints!